Entry by Takeover: Auctions vs. Negotiations

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**Abstract**
We compare two mechanisms through which a potential entrant can take over an incumbent in a market with asymmetric firms: auctions (where other incumbents can bid for the target) and bilateral negotiations between the entrant and the target. The entrant’s choice of target depends on the mechanism, and it may not maximize ex-post profit or consumer welfare. In an auction, the entrant pays a higher price to take over a target with higher synergies, because they impose stronger negative externalities on incumbents and increase their willingness to pay for preventing entry. This provides a new rationale for takeover premia. Auctions increase the price obtained by the target, but reduce welfare compared to negotiation because they may discourage the entrant from acquiring a target with higher synergies.

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1. Introduction

Firms often use mergers and acquisitions to enter new industries. Indeed, in some industries barriers for de novo entry are so high that acquiring an incumbent is the only profitable way to enter.¹ For example, in 1988 Phillip Morris entered the packaged-foods industry via the acquisition of Kraft for about $13 billion. More recently, Microsoft acquired Skype Technologies, creator of the VoIP service Skype, for $8.5 billion in 2011 and, in September 2013, announced its intent to acquire the mobile hardware division of Nokia in a deal worth $7.2 billion.²

Similarly, firms can find it more convenient to enter a foreign market by taking over one of the existing local firms. For example, Hennart and Park (1993) find that between 1981 and 1989, 36% of all market entries in the U.S. by Japanese companies took place by merger. More recently, in January 2014 the leading automobile manufacturer in Italy, FIAT, acquired Chrysler, one of the three major American automobile manufacturers.

In all these situations, the presence of desirable acquisition targets and the entrant’s choice of target affect the industry structure and consumer welfare. Therefore, the analysis of entry by takeover may have important policy implications.

Conditional on acquisition being the mode of entry, what factors affect a potential entrant’s choice of the incumbent to acquire? How does this choice depend on the mechanism through which the entrant is allowed to acquire an incumbent? In order to address this issue, we consider the choice of a takeover target in a market where asymmetric firms compete à la Cournot and have different levels of synergies with the potential entrant. We compare two alternative takeover mechanisms: an auction for the target between the entrant and the other incumbents, and bilateral bargaining between the entrant and the target.³ Hence, in a takeover by auction the other incumbents can react to the attempted entry and bid against the entrant to acquire the target, while with bargaining they cannot.⁴ Bilateral bargaining can also be interpreted as a private negotiation whose terms cannot be observed by outsiders.

We assume that the takeover mechanism is exogenously fixed. For example, auctions may not be used in the presence of high bidding costs for incumbents, or when incumbents are not allowed to merge. Alternatively, if relationship-specific investments are necessary for the takeover to be profitable, the target may have to enter an exclusive dealing arrangement with the entrant (for example, through the use of break-up fees) in order to induce it to submit a serious takeover offer (see, e.g., Boone and Mulherin, 2007). By contrast, an auction may be the only possible mechanism when a takeover target is legally required to solicit offers from all potentially interested acquirers.

In our model, three factors affect the entrant’s choice of the incumbent to acquire: (i) the

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¹Using data from the U.S. commercial bank industry, Uetake and Watanabe (2012) find evidence of high entry barriers, causing a significant fraction of entries to happen via mergers and acquisitions.
²The acquisition of Nokia, pending regulatory approval, is scheduled to close in early 2014.
³For an analysis of the choice between direct entry and entry via acquisition, see Gilbert and Newbery (1992) and McCordle and Viswanathan (1994). Both these papers, however, only analyze takeovers by bargaining.
⁴Although it is arguably uncommon to observe incumbents that prevent entry by merging with a takeover target in the real world, we will show that it is the mere possibility of them acquiring the target that affects the entrant’s choice (by forcing it to pay a higher price for the target in an auction).
incumbents’ market shares before entry; (ii) the level of synergies that the entrant can realize with the incumbents, and (iii) the price that the entrant has to pay to acquire an incumbent. While the first two factors depend solely on the primitives of the model (the number of incumbents, their marginal costs, and the level of synergies), the third one depends on the specific takeover mechanism. Hence, the choice of which incumbent to acquire depends on the takeover mechanism.

With bargaining, the choice of a takeover target is determined by a trade-off between efficiency gains (that depend on the synergies) and the incumbents’ market shares (that depend on their costs), which determine their reserve prices — i.e., the minimum prices that the entrant has to pay to acquire the incumbents. On the one hand, if all incumbents had the same market share, the entrant would always take over the one with the highest synergies. On the other hand, if it experienced the same synergies with all incumbents, the entrant would always take over the one with the larger market share. With asymmetric incumbents and target-specific synergies, the choice of the incumbent to acquire depends on which of these two effects dominate.

In an auction, instead, the entrant may have to pay a price higher than the target’s reserve price in order to outbid other incumbents. As a takeover results in a new firm producing at a lower marginal cost, entry imposes a negative externality on other incumbents and reduces their profit. Hence, other incumbents are willing to bid more than the target’s intrinsic value in order to prevent entry.\footnote{This is an example of the auctions with downstream interaction among bidders analyzed by Funk (1996) and Jehiel and Moldovanu (2000).} This provides a justification for “takeover premia” that raiders pay for targets: a takeover target may be paid a price higher than its intrinsic value in an auction.\footnote{An empirical implication of our model is that we should observe smaller premia (or no premia at all) for uncontested acquisitions.}

We show that an auction may induce the entrant to choose a less efficient target, resulting in a takeover that generates a lower consumer surplus. The reason is that takeovers that generate higher consumer surplus (by creating a more efficient firm ex-post) also generate stronger negative externalities on other incumbents. Hence, other incumbents are willing to bid more aggressively to prevent entry, so that these takeovers are especially costly for the entrant with auctions. By contrast, when the takeover takes place through a bilateral negotiation between the entrant and the targeted incumbent, the negative externalities that the takeover imposes on other incumbents do not affect the price that the entrant has to pay. Indeed, we show that the entrant may select the target that maximizes consumer surplus with bargaining and a different target in an auction, but not vice versa. Hence, takeovers by bargaining always result in a (weakly) higher consumer surplus than takeovers by auctions.

In addition, since entry increases welfare because of synergies, takeovers by auctions also generate inefficiencies if they allow incumbents to outbid the entrant and acquire the target. In fact, this reduces consumer surplus by preventing entry and increases market concentration by reducing the number of firms. By contrast, entry always takes place with takeovers by bargaining and it does not affect market concentration.

Therefore, the takeover mechanism affects the entrant’s choice of target and consumer surplus.
Which takeover mechanism is likely to prevail in the real world? Of course, a target always prefers auctions, since they result in a higher takeover price. Similarly, other incumbents also prefer auctions, since they may allow them to prevent entry. But our analysis suggests that a regulator should not necessarily prefer auctions.

Under Delaware law (the predominant corporate law in the US), when a potential buyer makes a serious bid for a target, the target’s board of directors is required to act as would “auctioneers charged with getting the best price for the stockholders at a sale of the company” (Cramton, 1998). Indeed, auctions are not only advised (see, for example, Cramton and Schwartz, 1991 and Bulow and Klemperer, 2009), but also widely used in takeovers (for empirical evidence, see Boone and Mulherin, 2007 and Malmendier, Moretti and Peters, 2012). However, we uncover a tension between maximizing the target shareholders’ surplus from the takeover, which is achieved through auctions, and maximizing consumers’ welfare in the market, which is achieved through bargaining. To the best of our knowledge, our paper is the first to bring to light this trade-off.

Since antitrust authorities in the U.S., the European Union, and many other jurisdictions apply a consumer-welfare standard when evaluating potential mergers and acquisitions, our analysis suggests that they may want to favour takeovers by bargaining and prevent incumbent firms from competing with an entrant in a takeover contest. For example, this may be achieved by only allowing bilateral negotiations between an entrant and the selected target, or by forbidding mergers between incumbents.

Although in our main model we consider a bargaining mechanism in which the entrant makes a take-it-or-leave-it offer to the selected target, we show that all our results also hold with a more general bargaining mechanism in which both parties obtain some positive shares of the gains from trade in case of successful entry. We also analyze how the possibility of collusion among the incumbents that bid against the entrant in an auction affects the entrant’s choice of target and the auction outcome.

Our paper is related to the work by Funk (1996), Jehiel and Moldovanu (2000), and Das Varma (2002) who analyze auctions with allocative and informational externalities created by ex-post interaction among bidders. Subsequent papers discuss how externalities may arise in takeover auctions. Specifically, Inderst and Wey (2004) show that a takeover is more likely to succeed under Bertrand (resp. Cournot) competition if goods are substitutes (resp. complements); Ding et al. (2013) compare cash and profit-share auctions with bidder-specific synergies. In contrast to ours, these papers only consider mergers among incumbents with an exogenous target and do not analyze negotiations.

Our paper is also related to the literature on endogenous mergers. Fridolfsson and Stennek (2005) show that, with negative externalities on outsiders, an unprofitable merger may occur to prevent the target from merging with a rival. Similarly, Qiu and Zhou (2007) find that merger waves may arise because firms which merge early free-ride on subsequent increases in the market.

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7 More generally, Jehiel et al. (1999) study mechanism design in the presence of externalities. For an experimental analysis of auctions with negative externalities, see Hu et al. (2012).
price caused by further mergers.\footnote{For analysis of endogenous merger waves, see also Gowrisankaran (1999), Fauli-Oller (2000), Gowrisankaran and Holmes (2004) and Nocke and Whinston (2010). These papers endogenize merger decisions in dynamic model, but they do not address the question of “with whom” to merge.} In an environment where a “pivotal” firm chooses to propose one among several mutually exclusive mergers, Nocke and Whinston (2013) show that, in order to maximize consumer surplus, an antitrust authority commits to imposing tougher standards on mergers involving firms with a larger market shares.

Finally, there is a large empirical literature in corporate finance documenting that stockholders of target firms receive large takeover premia. Theoretical explanations of this empirical anomaly include Roll’s (1986) hubris hypothesis, Jensen’s (1986) theory of free cash flows, Shleifer and Vishny’s (1990) managerial entrenchment hypothesis, Shleifer and Vishny’s (2003) and Rhodes-Kropf and Viswanathan’s (2004) models of market misvaluation, Jovanovic and Braguinsky’s (2004) theory of learning about investment opportunities, and Malmendier and Tate’s (2008) theory of overconfident CEOs.\footnote{For analysis of bidding behavior in takeovers with different types of offers (cash vs. securities) see Fishman (1988, 1989), Eckbo et al. (1990), Rhodes-Kropf and Viswanathan (2000), Che and Lewis (2007), Brusco et al. (2007), Ekmekci and Kos (2012), and Ekmekci et al. (2013).} In contrast to these papers, by considering how the industry structure affects takeover bidding, we provide an explanation of takeover premia based on the negative externality that the takeover itself imposes on other firms in the market.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we consider the difference between a profitable takeover target for an entrant and an efficient one. Sections 4 and 5 analyze takeovers by bargaining and auction, respectively. In Section 6 we show how the choice of a takeover target by the entrant depends on the takeover mechanism. Sections 7 considers various extensions of the main model and Section 8 concludes. All proofs are in the appendix.

2. The Model

**Players and environment.** Consider a market with \(n\) incumbent firms producing a homogeneous good and competing \(à la\) Cournot. The marginal cost of firm \(i\) is \(c_i, i = 1, ..., n\). Fixed costs of production are equal to zero. We assume that firms \(2, ..., n\) are symmetric and have the same marginal cost \(c_2 = ... = c_n\), while firm 1 is a dominant firm that produces at a lower marginal cost \(c_1 < c_2\).\footnote{The model could be easily extended to accommodate for more than two levels of marginal cost; however, this would increase the number of cases to consider, complicating the analysis without providing any additional insights.} This can be thought of as a market in which there is a dominant firm with a technological advantage and \(n - 1\) smaller competitors with (approximately) equal market shares.\footnote{For example, in 2006 Google entered the video sharing industry by acquiring YouTube for $1.65 billion. At the time of the takeover, YouTube had already established itself as the leader among video sharing websites; however, there were also other smaller and older firms like Metacafe or Vimeo who were operating (and still are) in the same industry.}

The inverse linear demand function is \(P(Q) = A - Q\), where \(Q\) is the total quantity produced
in the market.\footnote{We consider a linear demand function for simplicity, in order to obtain closed-form solutions, but we conjecture that none of our qualitative results hinge on this assumption and that they hold as long as the Cournot game is “stable” and has “well-behaved” comparative statics — i.e., $P' (Q) < 0$, $P'' (Q) + Q P''' (Q) < 0$ and $\lim_{Q \to -\infty} P (Q) = 0$.} Therefore, firm $i$’s initial equilibrium profit is

$$\pi_n \left( c_i; \sum_{k \neq i} c_k \right) = \left( \frac{A - nc_i + \sum_{k \neq i} c_k}{n + 1} \right)^2, \quad i = 1, ..., n,$$

where $n$ indicates the number of firms active in the market.

There is a potential entrant $E$ that can enter the market only by taking over an incumbent, for example because of legal or technological reasons (e.g., $E$ lacks a necessary input for production), or because of high fixed costs to enter the market as a new competitor. Without loss of generality, we assume that $E$ can take over either firm 1 or firm 2 (since all other incumbents are identical to firm 2). There are firm-specific synergies: if $E$ takes over firm $i$, the resulting firm has marginal cost $c_i - s_i, \ i = 1, 2$, where $s_i \in [0, c_i]$ represents the strength of the synergy between the entrant and firm $i$. Marginal costs and synergies are common knowledge among players.\footnote{Our qualitative results also hold in an environment where the entrant is privately informed about synergies at the takeover stage, and synergies with different incumbents are drawn from different distributions (see footnote 31).}

Let

$$\Phi_i \equiv A - nc_i + \sum_{k \neq i} c_k.$$

To ensure that firms always produce non-negative quantities, we assume that $\Phi_i > s_j, \ \forall i, j$ and $j \neq i$.

**Takeover.** We consider two different (exogenously fixed) takeover mechanisms:

- **Bargaining.** $E$ makes a take-it-or-leave-it offer to its chosen target.

- **Auction.** $E$ competes with the other incumbents in an ascending auction for the takeover target.\footnote{In an ascending auction the price raises continuously and bidders who wish to be active at the current price keep a button pressed. When a bidder releases the button, he is withdrawn from the auction. The the auction ends when only one active bidder is left. For the interpretation of takeover contests as ascending auctions see Fishman (1988), Bulow et al. (1999), Inderst and Wey (2004) and Bulow and Klemperer (2009).}

When the takeover takes place through an auction, once $E$ selects a takeover target, the other incumbents can react and compete to acquire it. Hence, an incumbent can prevent $E$’s takeover by merging with the target itself. By contrast, when the takeover takes place through bargaining, other incumbents cannot prevent the takeover — e.g., because they are not allowed to acquire the target for antitrust reasons (since a merger between incumbents reduces the number of competitors in the market).

In our main model, we assume that $E$ has full bargaining power in a takeover by bargaining but, in Section 7.1, we show that all our result also hold with a more general bargaining mechanism in which the takeover target obtains some positive share of the gains from trade generated by entry.
To characterize the price paid by $E$ in an auction for firm 1, we restrict attention to equilibria in pure and weakly undominated strategies (played by the other incumbents).\textsuperscript{15} We also assume that, when indifferent, players always participate in an auction and bid up to their willingness to pay (because bidding is costless). This is relevant because, in our model with complete information, a bidder knows whether he will win or lose an auction before participating.

**Timing.** The timing of the game is as follows:

- **Period 1.** $E$ selects the takeover target.
- **Period 2.** Takeover by auction or bargaining for the target.
- **Period 3.** Market competition among the remaining firms.

For simplicity, we assume that if $E$ fails to take over its chosen target, then it cannot take over another incumbent (for example, because there are high sunk costs associated with each takeover attempt that make it unprofitable for the entrant to make more than one attempt).

**Period 3’s Profits.** If there is no takeover, in period 3 firm $i$ continues to earn its current profit. Therefore, if firm $i$ is selected by $E$ as the takeover target in period 1, its reserve price (or reservation value) is

$$r^i = \pi_n \left( c_i; \sum_{k \neq i} c_k \right),$$

both with bargaining and in an auction. If $E$ takes over firm $i$, then in period 3 its profit is $\pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right)$ and the profit of an incumbent who is not taken over is $\pi_n \left( c_j; \sum_{k \neq j} c_k - s_i \right)$, $j \neq i$.

If two incumbents merge, we assume that the resulting firm’s marginal cost is the minimum of the two incumbents’ initial marginal costs (see Fauli-Oller, 2000, Stennek, 2003, and Qiu and Zhou, 2007).\textsuperscript{16} Hence, if firm $i$ and firm $j$ merge, their joint profit in period 3 is $\pi_{n-1} \left( \min \{c_i, c_j\}; \sum_{k \neq i,j} c_k \right)$, while the profit of a firm $l$ that does not merge is $\pi_{n-1} \left( c_l; \sum_{k \neq i,j,l} c_k + \min \{c_i, c_j\} \right)$.

Notice that, before $E$ attempts a takeover, a merger between two of the symmetric firms is never profitable.\textsuperscript{17} Furthermore, we also assume that the dominant firm 1 has no incentive to merge with another firm ex-ante — i.e., that\textsuperscript{18}

$$\pi_n (c_1; (n-1)c_2) + \pi_n (c_2; c_1 + (n-2)c_2) > \pi_{n-1} (c_1; (n-2)c_2) \tag{2.1}$$

$$\iff A > \frac{2c_2(2n^2-n-1)-c_1(3n^2-1)}{n^4-2n-1}. \tag{2.2}$$

\textsuperscript{15}See Jehiel and Moldovanu (2000) for a discussion of the problems arising when constructing equilibria in auctions with negative externalities.

\textsuperscript{16}Even without synergies, horizontal mergers may increase social welfare through production rationalization — e.g., if the resulting firm relocates production from less efficient plants to more efficient ones.

\textsuperscript{17}In fact, $2\pi_n (c_2; c_1 + (n-2)c_2) > \pi_{n-1} (c_2; c_1 + (n-3)c_2)$.

\textsuperscript{18}Inequality (2.1) represents a quadratic equation in $A$, whose relevant solution is (2.2).
This ensures that the original market structure is “stable” and that two incumbents may only want to merge to block \( E \)'s entry into the market. In Section 7.3 we analyze the effects of relaxing this assumption.

Condition (2.1) is more likely to hold when the size of the market, as captured by \( A \), is large, when the difference between \( c_2 \) and \( c_1 \) is small, and when \( n \) is large. To see the intuition, notice that when all firms are symmetric — i.e., when \( c_i = c, \forall i \) — they do not have any incentive to merge.\(^{19}\) On the other hand, if two firms are sufficiently asymmetric they may have an incentive to merge in order to produce a higher quantity at the lowest of their pre-merger marginal costs. However, as \( A \) and/or \( n \) increase, asymmetries in marginal costs become relatively less important for firms’ profits, which tend to be more similar to each other, thus reducing the incentive to merge.

3. Efficient and Profitable Targets

The profitability of an incumbent as a potential target depends both on its marginal cost and on its synergies with the entrant. \( E \) obtains a higher gross surplus (neglecting the takeover price) in period 3 by taking over firm \( i \) rather than firm \( j \) if and only if

\[
\pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right) > \pi_n \left( c_j - s_j; \sum_{k \neq j} c_k \right) \quad \iff \quad n (s_i - s_j) > (n + 1) (c_i - c_j).
\]

\( (3.1) \)

This condition is satisfied either if firm \( i \)'s synergy is sufficiently larger than firm \( j \)'s synergy, or if firm \( i \)'s marginal cost is sufficiently smaller than firm \( j \)'s marginal cost, or both. In this case, we say that firm \( i \) is the profitable target. Notice that firm 1 may be the profitable target even if \( E \) would be able to produce at a lower marginal cost by taking over firm 2 (while the converse is impossible) — i.e., \( E \) may prefer to produce at a higher marginal cost but face less efficient competitors.\(^{20}\)

If \( E \) takes over firm \( i \), the total market output is \( \frac{1}{n+1} (nA - \sum_k c_k + s_i) \). Therefore, in order to maximize consumers’ surplus, \( E \) should take over the firm with the strongest synergies — i.e., with the highest \( s_i \). We define this firm as the efficient target. Although the efficient target is not necessarily the one that maximizes total welfare (Farrell and Shapiro, 1990), we consider it an appropriate benchmark for evaluating the desirability of a takeover. This is consistent with the “consumer-welfare standard” adopted by competition authorities in many countries. According to this standard, when evaluating potential mergers and acquisitions, efficiency gains are only taken into account to the extent that they are passed on to consumers as lower prices.\(^{21}\)

There are three possible cases, which are displayed in Figure 3.1 (where the top line represents condition (3.1)):
Figure 3.1: Efficient and profitable target

(i) If and \( s_2 < s_1 \), firm 1 is the profitable and the efficient target.

(ii) If \( s_2 > \frac{n+1}{n} (c_2 - c_1) \) \( s_1 \leq s_2 \), firm 1 is the profitable target but firm 2 is the efficient target.

(iii) If \( s_1 \leq s_2 > \frac{n+1}{n} (c_2 - c_1) \), firm 2 is the profitable and the efficient target.

In case (i), firm 1 has both the highest synergies and the lowest marginal cost. In case (ii), the profitable target differs from the efficient one because, even though firm 2 has higher synergies, \( E \) still obtains a higher profit by taking over firm 1 because of its lower marginal cost. In case (iii), firm 2’s synergies are so high that \( E \) obtains a higher profit by taking over firm 2, even if firm 1 has a lower marginal cost.

Notice that in Figure 3.1, if \( c_2 \rightarrow c_1 \) or if the number of incumbents (of any type) increases, the top line moves closer to the 45-degree line, and the two lines coincide if \( c_2 = c_1 \). Indeed, when all incumbents have the same marginal cost, a target is profitable if and only if it is efficient (i.e., it has the largest synergies with \( E \)). Moreover, as \( n \) increases in condition (3.1), the difference in the incumbents’ marginal costs becomes relatively less important than the difference in the synergies, so that it is less likely that the efficient target is not the profitable one.

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22 In fact, if firm \( i \) has a lower marginal cost and is the efficient target, then it is also the profitable one: \( c_j > c_i \) and \( s_i > s_j \) imply (3.1).

23 When \( s_j > s_i \) and (3.1) holds, firm \( j \) is the efficient target while firm \( i \) the profitable one. Notice that it is not possible that firm 1 is the efficient target but firm 2 is the profitable one.

24 In fact, if firm \( j \) has a higher marginal cost but is the profitable target, then it is also the efficient one: \( c_j > c_i \) and \( n (s_j - s_i) > (n + 1) (c_j - c_i) \) imply that \( s_j > s_i \).
To summarize, in markets where incumbents are asymmetric, the profitable target may differ from the efficient one. The larger is the asymmetry among incumbents or the smaller is the number of firms (i.e., the less competitive is the market), the more likely it is that the profitable target differs from the efficient one. Notice that, in order for profitable and efficient targets to differ, firms must have both different synergies and different marginal costs.

4. Takeover by Bargaining

In this section, we analyze $E$’s choice of target when the takeover takes place through bargaining. In this case, the price that $E$ has to pay in order to acquire incumbent $i$ is equal to the reserve price $r_i$ and, by taking over firm $i$, $E$ obtains surplus

$$\pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right) - \pi_n \left( c_i; \sum_{k \neq i} c_k \right).$$

Since this is strictly positive, $E$ has an incentive to take over any of the incumbents with bargaining and, hence, it always enters the market.

Of course, $E$’s preferred target is the one that allows it to obtain the highest surplus. This choice is determined by a trade-off between efficiency gains (that depend on the synergies) and the incumbents’ pre-merger market shares, which determine their reserve prices (and depend on the difference between the incumbents’ initial costs).

**Proposition 1.** When the takeover takes place through bargaining, $E$ takes over firm 1 rather than firm 2 if and only if

$$s_1^2 - s_2^2 > \frac{2}{n} (s_2 \Phi_2 - s_1 \Phi_1).$$

Condition (4.1) requires $s_2$ to be sufficiently low.$^{25}$ If all incumbents had the same market share ($\Phi_1 = \Phi_2$), the entrant would always take over the one with the highest synergies. On the other hand, if it experiences the same synergies with all incumbents ($s_1 = s_2$), the entrant takes over the one with the larger pre-merger market share. With asymmetric incumbents and target-specific synergies, the choice of which incumbent to take over depends on which of these two effects dominate.

If $s_1 > s_2$, then condition (4.1) holds, so that $E$ takes over firm 1 when it is both the profitable and the efficient target; if $s_1 \leq s_2 - \frac{n+1}{n} (c_2 - c_1)$, then condition (4.1) does not hold (see the proof of Lemma 1), so that $E$ takes over firm 2 when it is both the profitable and the efficient target. Hence, we have the following result.

**Lemma 1.** When the takeover takes place through bargaining, $E$ always takes over a firm that is both the profitable and the efficient target.

$^{25}$Specifically, condition (4.1) requires that $s_2 < \frac{\sqrt{n^2 \Phi_1^2 + 2\Phi_1 s_1 \Phi_2} - \Phi_2}{n}$. 

The intuition for the above result is straightforward. If firm 1 is both the profitable and the efficient target, it provides $E$ with the larger synergies as well as the larger market share. Both these effects induce $E$ to acquire firm 1. If firm 2 is both the profitable and the efficient target, it provides $E$ with extremely large synergies, even if it has a lower pre-merger market share than firm 1. Hence, the efficiency-gain effect dominates and induces $E$ to acquire firm 2. By contrast, when $s_2 > s_1 > s_2 - \frac{n+1}{n} (c_2 - c_1)$ so that firm 1 is the profitable target while firm 2 is the efficient one, $E$ prefers to take over the profitable (resp. efficient) firm 1 (resp. 2) if condition (4.1) holds (resp. fails); that is, $E$ takes over the more efficient firm if and only if the efficiency-gain effect dominates the market-share effect.

Figure 4.1 displays condition (4.1), represented by the green curve: $E$ takes over firm 1 (resp. 2) if $s_2$ is below (resp. above) the green curve.\textsuperscript{26} Therefore, for values of $s_1$ and $s_2$ between the top line and the green curve, $E$ takes over firm 2 that is the efficient but not the profitable target; for values of $s_1$ and $s_2$ between the green curve and the 45-degree line, $E$ takes over firm 1 that is the profitable but not the efficient target.

Notice that, as $c_2 \rightarrow c_1$, the green curve moves towards the 45-degree line, and if $c_2 = c_1$ the green curve coincides with the 45-degree line so that $E$ takes over firm $i$ if and only if $s_i > s_j$. Therefore, the smaller the asymmetry between incumbents, the less likely it is that the entrant does not take over the efficient firm. On the other hand, as the number of symmetric firms increases, it

\textsuperscript{26}As in Figure 3.1, the top line represents condition (3.1). It is straightforward to show that the green curve lies between this line and the 45-degree line.
can be shown that the green curve becomes steeper while the top black line shifts downward, so that the area between the two shrinks. Therefore, in less concentrated markets, it is less likely that the profitable and efficient targets differ; however, if they do differ, then it is more likely that the entrant does not take over the efficient target.

5. Takeover by Auction

In this section, we analyze $E$’s choice of target when the takeover takes place through action. Hence, other incumbents can bid for the target against $E$ and, if they are successful, prevent entry into the market.

In an auction for firm $i, i = 1, 2$, firm $j$’s willingness to pay for blocking $E$’s takeover and merging with firm $i$ is

$$v_j^i \equiv \pi_{n-1} \left( \min \{c_i, c_j\}; \sum_{k \neq i,j} c_k \right) - \pi_n \left( c_j; \sum_{k \neq j} c_k - s_i \right).$$

This is increasing in firm $j$’s profit if it merges with firm $i$, and decreasing in firm $j$’s profit if firm $i$ is taken over by $E$. Hence, firm $j$’s willingness to pay depends on two effects:

1. The increase in firm $j$’s profit (with respect to its current profit) if it merges with firm $i$ — i.e., $\pi_{n-1} \left( \min \{c_i, c_j\}; \sum_{k \neq i,j} c_k \right) - \pi_n \left( c_j; \sum_{k \neq j} c_k \right)$.

2. The reduction in firm $j$’s profit (with respect to its current profit) if $E$ takes over firm $i$ — i.e., $\pi_n \left( c_j; \sum_{k \neq j} c_k \right) - \pi_n \left( c_j; \sum_{k \neq j} c_k - s_i \right)$.

The second effect is a negative externality created by $E$’s takeover of firm $i$: following the takeover, firm $j$ faces a more efficient competitor in period 3 and earns a lower profit. This externality is increasing in $s_i$. The larger is the externality, the higher is the price that firm $j$ is willing to pay to prevent the takeover.

The incumbents’ willingness to pay for blocking a takeover determines their bids in an auction. In order to acquire an incumbent, $E$ has to pay the highest between the other incumbents’ bids and the reserve price. Firm $j$’s willingness to pay for firm $i$, $v_j^i$, is higher than the reserve price, $r^i$, if and only if

$$\pi_{n-1} \left( \min \{c_i, c_j\}; \sum_{k \neq i,j} c_k \right) > \pi_n \left( c_j; \sum_{k \neq j} c_k - s_i \right) + \pi_n \left( c_i; \sum_{k \neq i} c_k \right).$$

(5.1)

When this condition is not satisfied for any firm $j$, we say that the reserve price of firm $i$ is binding.

The next lemma characterizes the highest bid by an incumbent depending on the takeover target, and compares it with the target’s reserve price.

**Lemma 2.** In order to acquire firm 2 in an auction, $E$ has to pay a price equal to:
• \( v_1^2 \) if \( s_2 \geq \hat{s}_2 \);  
• \( r^2 \) if \( s_2 < \hat{s}_2 \), where \( \hat{s}_2 = \Phi_1 - \frac{\sqrt{n^2(\Phi_1^2 - \Phi_2^2) + 2n(\Phi_1 + \Phi_2)}}{n} \).

In order to acquire firm 1 in an auction, \( E \) has to pay a price equal to:

• \( v_1^1 \) if \( s_1 \geq \hat{s}_1 \);  
• \( r^1 \) if \( s_1 < \hat{s}_1 \), where \( \hat{s}_1 = \Phi_2 - \frac{\sqrt{\Phi_2(\Phi_2 + 2n\Phi_1)}}{n} \).

Furthermore, \( \hat{s}_1 > \hat{s}_2 \).

In the proof of Lemma 2, we show that the highest bid by an incumbent in an auction for firm 2 is \( v_2^2 \), which is the bid by the dominant firm 1. Therefore, the price that \( E \) pays to take over firm 2 is the maximum between \( v_1^2 \) and \( r^2 \).

In an auction for firm 1, all other incumbents have the same willingness to pay to block \( E \)'s takeover (because they have the same marginal cost) — i.e., \( v_1^1 \). In the proof of Lemma 2, we show that there is no equilibrium (in pure and weakly undominated strategies) in which all symmetric incumbents drop out at a price lower than \( v_1^1 \).\(^{27}\) Hence, this is the highest bid by an incumbent in an auction for firm 1,\(^{28}\) and the price that \( E \) pays to take over firm 1 is the maximum between \( v_1^1 \) and \( r^1 \).

Finally, the reserve price is binding in an auction for firm \( i \) if \( s_i \) is sufficiently low, because the incumbents’ willingness to pay is increasing in the negative externality of the takeover and, hence, in the level of \( E \)'s synergies with target. Moreover, since firm 1’s pre-merger profit is higher than firm 2’s, the threshold on synergies for firm 1’s reserve price to bind is higher than for firm 2’s (see condition (5.1)).

Of course, \( E \)'s preferred target is the one that allows it to obtain the highest net surplus. The next proposition characterizes \( E \)'s choice of the target.

**Proposition 2.** If the takeover takes place through auction:

(i) When \( s_1 \leq \hat{s}_1 \) and \( s_2 \leq \hat{s}_2 \), \( E \) takes over firm 1 (resp. 2) if condition (4.1) holds (resp. fails).

(ii) When \( s_1 \leq \hat{s}_1 \) and \( s_2 > \hat{s}_2 \), \( E \) takes over firm 1 (resp. 2) if the following condition holds (resp. fails):

\[
s_1^2 - s_2^2 > \frac{2}{n} (s_2\Phi_2 - s_1\Phi_1) + \frac{s_2}{n^2} (s_2 - 2\Phi_1) - \Phi_2 [\Phi_2 + n (2\Phi_1 - n\Phi_2)].
\]

\(^{27}\)Although there are many possible equilibria in mixed or weakly dominated strategies, equilibria in which there is at least one incumbent who bids up to his willingness to pay are the natural ones. This is consistent with the results obtained by Jehiel and Moldovanu (2000) in a set-up with incomplete information. In fact, they prove that in the unique symmetric equilibrium of a second-price auction with negative externalities and two bidders, each bidder submits a bid equal to his willingness to pay, inclusive of the externality term. Furthermore, this strategy profile is still an equilibrium with more than two bidders in “symmetric settings” (i.e., when the function capturing a bidder’s payoff if an opponent wins the auction is symmetric with respect to the type of all the other losing bidders), as it is the case in our model.

\(^{28}\)It is straightforward to show that no symmetric incumbent bids more than \( v_2^2 \).
(iii) When \( s_1 > \hat{s}_1 \), \( E \) takes over firm 1 (resp. 2) if the following condition holds (resp. fails):

\[
s_1^2 - s_2^2 > 2 \left( \frac{n s_2 + s_1}{n^2} \right) \Phi_2 - 2 \left( \frac{n s_1 + s_2}{n^2} \right) \Phi_1.
\]  

(5.3)

If both reserve prices bind, the choice of the target is the same as with bargaining. When at least one reserve price does not bind (cases (ii) and (iii) in Proposition 2), the entrant takes over firm 2 if \( s_2 \) is sufficiently high (see the proof of Proposition 2). Therefore, although firm 1 has a lower marginal cost than firm 2, the entrant takes over firm 2 when its synergies are sufficiently higher than firm 1’s.

When an incumbent is both the profitable and the efficient target, we have the following result.

**Lemma 3.** When the takeover takes place through auction, \( E \) always takes over a firm that is both the profitable and the efficient target.

By Lemmas 1 and 3, when a firm is both the profitable and the efficient target, the takeover mechanism is irrelevant for \( E \)’s choice of target. The next section, however, shows that this is not the case when the profitable target differs from the the efficient one.

Figure 5.1 displays conditions (4.1), (5.2) and (5.3), represented by the red curve: \( E \) takes over firm 1 (resp. 2) if \( s_2 \) is below (resp. above) the red curve.\(^{29}\) Therefore, for values of \( s_1 \) and

\(^{29}\)As in Figure 3.1, the top line represents condition (3.1). It is straightforward to show that the red curve lies between this line and the 45-degree line.
s_2$ between the top line and the red curve, $E$ takes over firm 2 that is the efficient but not the profitable target; for values of of $s_1$ and $s_2$ between the red curve and the 45-degree line, $E$ takes over firm 1 that is the profitable but not the efficient target.

As $c_2 \to c_1$, the red curve moves towards the 45-degree line, and if $c_2 = c_1$ the red curve coincides with the 45-degree line so that $E$ takes over firm $i$ if and only if $s_i > s_j$. Therefore, the smaller the asymmetry between incumbents, the less likely it is that the entrant does not take over the efficient firm.

Notice that, in an auction for a takeover target, the entrant always has a higher willingness to pay for the target than other incumbents, so that it always outbids them. Hence, as with bargaining, the entrant always acquires its preferred target in a takeover by auction. This happens in our model because of the presence of complete information and because of the assumption that a merger between incumbents is not profitable ex-ante (see Section 7.3). Of course, in a richer model with incomplete information, an incumbent may outbid the entrant in an auction for the target.\footnote{For example, an incumbent may win a takeover auction if there is uncertainty about the level of synergies, and only the entrant learns its synergy with an incumbent once it has committed to bid for it.}

6. Auction vs. Bargaining

In this section, we compare $E$’s choice of target in a takeover by auction, when incumbents can react and acquire the target, with its choice in a takeover by bargaining. We show that these choices may differ (Proposition 3) and that takeovers by auction yield a (weakly) lower consumers’ surplus than takeovers by bargaining (Corollary 1 of Proposition 4).

The next result describes when the entrant chooses a different target depending on the takeover mechanism.

**Proposition 3.** $E$ takes over firm 1 in an auction and firm 2 with bargaining if and only if:

- either $s_1 > \tilde{s}_1$ and
  \[
  \frac{2}{1+n} (\Phi_1 s_2 - \Phi_2 s_1) - \frac{2}{n(1+n^2)} (s_1 \Phi_1 - s_2 \Phi_2) > s_2^2 - s_1^2 - \frac{2}{n} (s_1 \Phi_1 - s_2 \Phi_2) > 0; \tag{6.1}
  \]

- or $s_1 \leq \tilde{s}_1$ and
  \[
  \frac{s_2}{n^2} (2\Phi_1 - s_2) + \frac{\Phi_2}{n} [\Phi_2 + n (2\Phi_1 - n\Phi_2)] > s_2^2 - s_1^2 - \frac{2}{n} (s_1 \Phi_1 - s_2 \Phi_2) > 0. \tag{6.2}
  \]

$E$ never takes over firm 2 in an auction and firm 1 with bargaining.

Hence, $E$ may prefer to take over the incumbent with the highest marginal cost if it can make a take-it-or-leave-it offer, without the other incumbents reacting, and the incumbent with the lowest marginal cost if it has to compete against other incumbents, but not vice versa. In other words,
when the choice of a takeover target differs in the two mechanisms, the entrant always chooses firm 1, the incumbent with the lowest marginal cost, in an auction.

Conditions (6.1) and (6.2) require that $s_2$ is higher than $s_1$, but not too much so, in order for the choice of takeover target to differ in the two mechanisms. Indeed, $s_2$ has to be relatively high for the entrant to take over firm 2 with bargaining, but not too high for the entrant to take over firm 1, rather than firm 2, in an auction.

Recall that, when a firm is the efficient and the profitable target, $E$ takes it over both in an auction and with bargaining (Lemmas 1 and 3). The following proposition shows that $E$ may choose a different target depending on the takeover mechanism when the profitable and efficient targets differ.

**Proposition 4.** When firm 2 is the efficient target but firm 1 the profitable target, $E$ may take over the profitable target in an auction and the efficient target with bargaining, but $E$ never takes over the efficient target in an auction and the profitable target with bargaining.

Hence, whenever the efficient and profitable targets differ, $E$ may select the efficient target in a takeover by bargaining and the profitable one in a takeover by auction, but not vice versa.

The intuition for this result is that, if incumbents are allowed to react to a takeover attempt by an entrant, their willingness to pay to block the takeover is increasing in the production efficiency of the firm resulting from the takeover. And the higher is the incumbents’ willingness to pay, the more likely the entrant is to prefer a different target. Hence, in a takeover by auction the entrant is less likely to select a target with whom it has higher synergies than in a takeover by bargaining.\(^{31}\)

By Proposition 4, when the entrant does not take over the efficient target with bargaining, it does not take over the efficient target in an auction either. However, when the entrant does not take over the efficient target in an auction, it may do so with bargaining. Hence, we have the following result.

**Corollary 1.** Takeovers by auctions (weakly) reduce consumer surplus compared to takeovers by bargaining.

Figure 6.1 displays how $E$’s choice of target depends on the takeovers mechanism. If a firm is both the profitable and the efficient target — that is, for values of $s_1$ and $s_2$ below the 45-degree line or above the black line — the actual takeover mechanism is irrelevant, because $E$ always takes over this target with both mechanisms. However, for values of $s_1$ and $s_2$ between the black line and the 45-degree line (so that firm 2 is the efficient target but firm 1 is the profitable one), $E$ takes over the efficient target in an auction above the red curve, whereas $E$ takes over the efficient target with bargaining above the green curve. Therefore, $E$ takes over a different firm depending on the

\(^{31}\)If the entrant is privately informed about synergies at the takeover stage, and synergies with different incumbents are drawn from different distributions, an incumbent’s willingness to pay to prevent entry is based on expected synergies. In this environment, taking over firm 2 in an auction is even more costly, since this choice signals relatively higher synergies with the target and, hence, increases other incumbents’ willingness to pay by a larger amount. Therefore, we expect the entrant to be even more likely to take over firm 1 in an auction and firm 2 with bargaining.
takeover mechanism when \( s_1 \) and \( s_2 \) lie between the green and the red curves, and in this case it takes over the efficient target with bargaining and the profitable target in an auction.

Our analysis suggests that an antitrust authority that can control takeover mechanisms and aims to maximize consumer surplus should favour bargaining mechanisms, in which incumbents cannot bid for the target against the entrant, when the takeover generates efficiency gains. Of course, \( E \) always prefers takeovers by bargaining as they allow it to acquire a target at its reserve price, while in an auction the target price is weakly higher than the reserve price. For the same reason, potential targets, conditional on being acquired, always prefer takeovers by auctions. In the next section we show that non-targeted incumbents prefer takeovers by auctions since they may allow them to prevent entry.

7. Extensions

In our model, the negative externality that entry imposes on incumbents induces them to bid aggressively in an auction for a takeover target, and this may force the entrant to choose a different target than the one that it would choose with bargaining, when the entrant makes a take-it-or-leave-it offer for the target. One may wonder whether this result depends on the specific bargaining mechanism that we have considered. In Section 7.1, we show that our results also hold with a more general bargaining mechanism.

Moreover, we also highlight other sources of inefficiency of takeovers by auctions. Since an
incumbent may win a takeover auction and acquire a competitor, takeovers by auction may reduce welfare by preventing entry in the market and increasing market concentration. We discuss two cases in which this may happen. First, even though a single incumbent may be unable to outbid the entrant in an auction, incumbents may profitably collude to block entry by jointly bidding more than the entrant’s willingness to pay for the target. Hence, the threat of entry may induce incumbents to merge, even when incumbents have no incentive to merge in the absence of a potential entrant. Second, an incumbent may outbid the entrant in “small markets,” where incumbents have incentive to merge even without the threat of entry. These sources of inefficiency never arise with bargaining, when entry always occurs because the entrant obtains a positive surplus from entering the market and other incumbents cannot block the takeover.

7.1. Bargaining Weights

Assume that, in a takeover by bargaining, the entrant has bargaining power \((1 - \beta)\) and the takeover target has bargaining power \(\beta\), where \(\beta \in [0, 1]\). (If the entrant makes a take-it-or-leave-it-offer as in our main model, \(\beta = 0\).) The outcome of bargaining between the entrant and the target is given by the Nash bargaining solution, where the disagreement point is represented by players’ current profits.\(^{32}\) Therefore, the entrant obtains a share \((1 - \beta)\) of the gains from trade if it takes over an incumbent, while the incumbent obtains a share \(\beta\).

In a takeover by bargaining of firm \(i\): firm \(i\)’s disagreement point is \(\pi_n \left( c_i; \sum_{k \neq i} c_k \right)\); \(E\)’s disagreement point is 0; the gains from trade are \(\pi_o \left( c_i - s_i; \sum_{k \neq i} c_k \right) - \pi_n \left( c_i; \sum_{k \neq i} c_k \right)\). Hence, if it takes over firm \(i\), \(E\) pays a price equal to

\[
\pi_n \left( c_i; \sum_{k \neq i} c_k \right) + \beta \left[ \pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right) - \pi_n \left( c_i; \sum_{k \neq i} c_k \right) \right],
\]

and obtains surplus

\[
(1 - \beta) \left[ \pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right) - \pi_n \left( c_i; \sum_{k \neq i} c_k \right) \right].
\]

By inspection, as in our main model, \(E\) takes over firm 1 rather than firm 2 with bargaining if and only if condition (4.1) is satisfied. Therefore, all our results from Sections 4 and 6 are unchanged.

Because an entrant with a lower bargaining power pays a higher price in takeover by bargaining, one may conjecture that this price may be higher than in a takeover by auction. However, in this case, it is arguably natural to assume that the target has the ability to commit to a reserve price higher than its current profit in an auction. Hence, as in our main model, it remains true that the takeover price is higher in an auction than with bargaining.

\(^{32}\) In a strategic model of alternating offers, players’ current profits can be interpreted as their “impasse points” or “inside options” in case bargaining continues forever without agreement being reached or the negotiation being abandoned — see, e.g., Binmore et al., (1986, 1989).
7.2. Collusion among Incumbents

In this section we analyze the possibility that, in a takeover by auction, non-targeted incumbents form a bidding ring and jointly bid against the entrant. We show that the ring willingness to pay to block the takeover may be higher than the entrant willingness to pay for the target. In this case, incumbents jointly manage to block entry, even though no single incumbent would be able to do so.

If the takeover of firm 1 is successful, the total profits of non-targeted incumbents are

\[(n - 1) \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_1 \right) .\]

If instead one of the incumbents wins the auction for firm 1, so that E’s entry is blocked, the total industry profits are

\[\pi_{n-1} \left( c_1; \sum_{k \neq 1} c_k \right) + (n - 2) \pi_{n-1} \left( c_2; \sum_{k \neq 2} c_k \right) .\]

Hence, the total willingness to pay of non-targeted incumbents to block the takeover of firm 1 is equal to the difference between these two profits, and non-targeted incumbents can prevent entry if and only if this is higher than E’s willingness to pay for firm 1, which requires that \(s_1\) is sufficiently low. Similarly, the total willingness to pay of non-targeted incumbents to block the takeover of firm 2 is higher than E’s willingness to pay for firm 2 if and only if \(s_2\) is sufficiently low.\(^{33}\)

If the entrant knows that non-targeted incumbents will form a ring, the choice of the takeover target also depends on which of the incumbents can be acquired by the entrant, if any, because colluding incumbents may outbid the entrant. When the entrant can acquire both incumbents, it chooses the one that yields a higher surplus, taking into account the willingness to pay of the ring.

**Proposition 5.** If non-targeted incumbents can collude in a takeover by auction, there exist two thresholds \(s_1^*\) and \(s_2^* > s_1^*\) such that:

(i) When either \(s_1 > s_1^*\) or \(s_2 > s_2^*\), E takes over firm 1 (resp. 2) if the following condition holds (resp. fails):

\[s_1^2 - s_2^2 > \frac{2\Phi_2 [2s_2 + (n - 1) s_1]}{n^2 + n - 1} - \frac{2\Phi_1 (s_2 + ns_1)}{n^2 + n - 1} .\]

(ii) When \(s_1 \leq s_1^*\) and \(s_2 \leq s_2^*\), E does not enter the market.

Figure 7.1 displays how E’s choice of target depends on the takeovers mechanism: with bargaining E takes over firm 1 (resp. 2) if \(s_2\) is below (resp. above) the green curve (as in our main model); with auctions E prefers to take over firm 1 (resp. 2) if \(s_2\) is below (resp. above) the purple curve. Therefore, similarly to our main model: for values of \(s_1\) and \(s_2\) between the purple curve and the 45-degree line, in an auction with collusion E takes over firm 1 that is the profitable but

\(^{33}\)See the Appendix for details. In the Appendix, we also show how firms can design side payments that support collusion to prevent entry by making it individually rational for all non-targeted incumbents to join a ring.
not the efficient target; for values of $s_1$ and $s_2$ between the purple curve and the green curve, $E$ takes over the efficient target with bargaining, but not with auctions.

Compared to our main model, when non-targeted incumbents collude, takeovers by auctions create two additional inefficiencies. First, when $s_1 \leq s_1^*$ and $s_2 \leq s_2^*$, $E$ does not enter the market with auctions (but it does enter with bargaining). This is inefficient because $E$’s entry always increases consumer surplus. Second, when $s_1 > s_1^*$ and $s_2 > s_2^*$, takeovers by auctions are more likely to reduce consumer surplus than in our main model. The reason is that, when non-targeted incumbents collude, $E$ is more likely to take over the efficient firm 2 with bargaining and the profitable firm 1 in an auction — that is, the purple curve in Figure 7.1 is strictly higher than the red curve in Figure 6.1. The intuition is that the presence of colluding incumbents increases the price that $E$ has to pay in an auction for firm 2. Hence, it is more likely that $E$ is discouraged from acquiring firm 2.

7.3. Small Markets

In this section we consider a situation in which incumbents have an incentive to merge even if $E$ does not attempt to enter the market. Therefore, we assume that condition (2.2) is not satisfied, so that a merger between firm 1 and one of the symmetric incumbents is profitable in period 1.\footnote{A possible interpretation is that of an industry where firms face technology shocks that result in the possibility of a profitable merger among incumbents and, at the same time, of profitable entry by an outsider.}
This is more likely to happen in small markets — i.e., if $n$ is small — and with more asymmetric firms — i.e., if the difference between $c_2$ and $c_1$ is large.

Because condition (2.2) does not affect a target’s reserve price, the analysis of takeovers by bargaining in small markets is the same as in our main model. In a takeover by auction, instead, when condition (2.2) is not satisfied an incumbent’s willingness to pay to block $E'$’s entry may be higher than $E'$’s willingness to pay for the target, so that the incumbent may outbid $E$ and prevent entry. Specifically, in an auction for firm $i$, $E$’s entry is blocked if and only if

$$v^i_j \geq \pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right), \quad i, j = 1, 2, \quad i \neq j.$$ 

This condition requires that $s_i$ is sufficiently low.\footnote{See the proof of Proposition 6 for details.}

Moreover, a target reserve price is never binding in an auction.\footnote{When condition (2.1) is not satisfied, $v^i_j = \pi_n \left( c_i - s_i; \sum_{k \neq i} c_k \right) = \pi_n \left( c_j; \sum_{k \neq i} c_k - s_j \right) > \pi_n \left( c_j; \sum_{k \neq i} c_k \right) = r^j$.}

Therefore, when incumbents cannot outbid the entrant regardless of the target, the choice of the takeover target in an auction is the same as in our main model. When incumbents can outbid the entrant, the choice of the takeover target depends on which of the incumbents can be acquired by the entrant, if any.

**Proposition 6.** If condition (2.2) is not satisfied, in a takeover by auction there exist two thresholds $\tilde{s}_1$ and $\tilde{s}_2 > \tilde{s}_1$ such that:

(i) When either $s_1 > \tilde{s}_1$ or $s_2 > \tilde{s}_2$, $E$ takes over firm 1 (resp. 2) if condition (5.3) holds (resp. fails),

(ii) When $s_1 \leq \tilde{s}_1$ and $s_2 \leq \tilde{s}_2$, $E$ does not enter the market.

Figure 7.2 displays how $E$’s choice of target depends on the takeovers mechanism: with bargaining $E$ takes over firm 1 (resp. 2) if $s_2$ is below (resp. above) the green curve (as in our main model); with auctions $E$ prefers to take over firm 1 (resp. 2) if $s_2$ is below (resp. above) the blue curve. Therefore, similarly to our main model, for values of $s_1$ and $s_2$ between the blue curve and the 45-degree line, $E$ takes over firm 1 that is the profitable but not the efficient target in an auction; for values of $s_1$ and $s_2$ between the blue curve and the green curve, $E$ takes over the efficient target with bargaining, but not with auctions.

As for collusion, in small markets takeovers by auctions create two additional inefficiencies: (i) when $s_1 \leq \tilde{s}_1$ and $s_2 \leq \tilde{s}_2$, $E$ does not enter the market with auctions; (ii) when $s_1 > \tilde{s}_1$ and $s_2 > \tilde{s}_2$, takeovers by auctions are more likely to reduce consumer surplus than in our main model — that is, the blue curve in Figure 7.2 is weakly higher than the red curve in Figure 6.1.

8. Conclusions

We have analyzed a model of entry by takeover with endogenous target choice and compared two alternative takeover mechanisms: (i) bilateral bargaining between the entrant and the selected
target, and (ii) an auction for the selected target in which the entrant competes against other incumbents. With bargaining, the entrant pays the target’s reserve price. By contrast, because entry imposes negative externalities on them, in an auction non-targeted incumbents bid aggressively to prevent entry and the entrant may pay more than the target’s reserve price. This offers a new justification for the takeover premia observed in the real world.

The choice of which incumbent to acquire depends on the takeover mechanism. Specifically, an auction may induce the entrant to choose a less efficient target than the one chosen with bargaining, resulting in a takeover that generates a lower consumer surplus. The reason is that takeovers that generate higher consumer surplus also generate stronger negative externalities on other incumbents and, hence, they are especially costly for the entrant with auctions. Moreover, takeovers by auction also reduce consumer welfare when incumbents outbid the entrant by merging with the target, since this increases industry concentration and prevents the entry of a more efficient firm. Therefore, antitrust authorities that aim to maximize consumer surplus should favour bargaining mechanisms for takeovers by new entrants in a market, when entry generates efficiency gains.

We conclude by discussing two possible avenues for future research. A generalization of our model would allow the entrant to make new takeover attempts if the first one is unsuccessful. In this case, a target’s reserve would depend on its profits if a different incumbent is taken over in a later attempt.\textsuperscript{37} Moreover, as in our model the target obtains a larger surplus than other incumbents, it might be interesting to analyze a model where firms compete for being targeted.

\textsuperscript{37} However, because preparing a takeover offer requires significant effort and expenses for the entrant (for example for carrying out industry studies, or for obtaining legal and consulting advice) we feel that our assumption that the entrant can only attempt one takeover is realistic.
A. Proofs

Proof of Proposition 1. When the takeover takes place through bargaining, \( E \) prefers to take over firm 1 rather than firm 2 if and only if

\[
\pi_n \left( c_1 - s_1; \sum_{k \neq 1} c_k \right) - \pi_n \left( c_1; \sum_{k \neq 1} c_k \right) > \pi_n \left( c_2 - s_2; \sum_{k \neq 2} c_k \right) - \pi_n \left( c_2; \sum_{k \neq 2} c_k \right)
\]

\[
\iff \left( \frac{\Phi_1 + ns_1}{n + 1} \right)^2 - \left( \frac{\Phi_1}{n + 1} \right)^2 > \left( \frac{\Phi_2 + ns_2}{n + 1} \right)^2 - \left( \frac{\Phi_2}{n + 1} \right)^2.
\]

Rearranging yields the statement. \( \blacksquare \)

Proof of Lemma 1. First, if \( s_1 > s_2 \), condition (4.1) is satisfied since the left-hand side is positive while the right-hand side is negative.

Second, if \( s_1 = s_2 - \frac{n+1}{n} (c_2 - c_1) \) (using the fact that \( \Phi_1 - \Phi_2 = (n + 1) (c_2 - c_1) \)) condition (4.1) simplifies to

\[
\left[ \frac{s_2 - \frac{n+1}{n} (c_2 - c_1)}{n+1} \right] - 2 \frac{s_2}{n} \Phi_2 + 2 \frac{s_2}{n} \Phi_1 \left[ \frac{s_2 - \frac{n+1}{n} (c_2 - c_1)}{n+1} \right] > 0
\]

\[
\iff 2 \Phi_1 < (n + 1) (c_2 - c_1),
\]

which is never satisfied since

\[
\underbrace{A - nc_1 + (n-1) c_2}_{\Phi_1} > (n + 1) (c_2 - c_1) \iff \underbrace{A - 2c_2 + c_1}_{\Phi_2} > 0.
\]

And if condition (4.1) is not satisfied when \( s_1 = s_2 - \frac{n+1}{n} (c_2 - c_1) \), it is not satisfied for smaller values of \( s_1 \) either. \( \blacksquare \)

Proof of Lemma 2. We first derive the highest bid by an incumbent when \( E \) attempts to take over firm \( i = 1, 2 \) and then we compare it with firm \( i \)'s reserve price.

First, consider an auction for firm 2. Firm 1's willingness to pay for blocking the takeover of firm 2 is

\[
v_1^2 = \frac{\left[ A - (n-1) c_1 + (n-2) c_2 \right]^2 - \left[ A - nc_1 + (n-1) c_2 - s_2 \right]^2}{(n + 1)^2}.
\]

Firm \( j \)'s willingness to pay, \( j > 2 \), for blocking the takeover of firm 2 is

\[
v_j^2 = \frac{\left[ A - (n-1) c_2 + (n-3) c_2 + c_1 \right]^2 - \left[ A - nc_2 + (n-2) c_2 + c_1 - s_2 \right]^2}{(n + 1)^2}.
\]

Therefore,

\[
v_1^2 - v_j^2 = \frac{2}{n} \frac{2 c_2 - c_1}{n + 1} (A - 2c_2 + c_1 + ns_2),
\]

which is strictly positive since \( \Phi_2 = A - 2c_2 + c_1 > 0 \) by assumption.
Next, we show that firm $j$’s willingness to pay, $j > 2$, for blocking the takeover is lower than $E$’s willingness to pay to take over firm 2 — i.e.,

$$\pi_n \left( c_2 - s_2; \sum_{k \neq 2} c_k \right) > v_j^2 \iff \frac{(\Phi_2 + ns_2)^2}{(n + 1)^2} > \frac{(\Phi_2 - s_2)^2}{n^2}$$

$$\iff \left( n^2 - 2n - 1 \right) \Phi_2^2 + n^2 s_2^2 (n^2 + 1) + 2n^2 (n - 1) \Phi_2 s_2 > 0,$$

which is always verified when $n \geq 3$. Therefore, only firm 1 can block the takeover of firm 2 and it is a weakly dominant strategy for firm 1 to bid his willingness to pay in an ascending auction for firm 2.

Comparing firm 1’s bid with the reserve price of firm 2,

$$v_1^2 \leq r^2 \iff \frac{[A - (n - 1) c_1 + (n - 2) c_2]^2}{n^2} \leq \frac{(A - 2c_2 + c_1)^2}{(n + 1)^2}.$$

The relevant solution of this inequality is

$$s_2 \leq \Phi_1 - \frac{\sqrt{n^2 (\Phi_1^2 - \Phi_2^2) + 2n\Phi_1 \Phi_2 + \Phi_2^2}}{n} \equiv \tilde{s}_2. \quad (A.1)$$

Consider now an auction for firm 1. Since all other incumbents have the same willingness to pay to block the takeover of firm 1, there is no equilibrium in pure and undominated strategies in which $E$ wins the auction at a price lower than $v_2^1$ — i.e., in which no incumbent bids up to his willingness to pay.

Comparing firm $j$’s bid, $j > 2$, with the reserve price of firm 1,

$$v_j^1 \leq r^1 \iff \frac{[A - (n - 1) c_1 + (n - 2) c_2]^2}{n^2} \leq \frac{[A - nc_2 + (n - 2) c_2 + c_1 - s_1]^2}{(n + 1)^2}.$$

The relevant solution of this inequality is

$$s_1 \leq \Phi_2 - \frac{\sqrt{\Phi_2 (\Phi_2 + 2n\Phi_1)}}{n} \equiv \tilde{s}_1.$$

Finally,

$$\tilde{s}_1 > \tilde{s}_2 \iff n^2 \Phi_2 (\Phi_1 - \Phi_2)^2 (2\Phi_1 n + \Phi_2 - \Phi_2 n^2) < 0$$

$$\iff A > \frac{2c_2 (2n^2 - n - 1) - c_1 (3n^2 - 1)}{n^2 - 2n - 1},$$

which is satisfied by condition (2.2).

**Proof of Proposition 2.** (i) Assume that $s_1 \leq \tilde{s}_1$ and $s_2 \leq \tilde{s}_2$. Since reserve prices are binding for both targets, $E$’s choice of target is the same as with bargaining.

(ii) Assume that $s_1 \leq \tilde{s}_1$ and $s_2 > \tilde{s}_2$: firm 1’s reserve price is binding, whereas firm 2’s is not.
Therefore, $E$ takes over firm 1 if and only if
\[
\pi_n(c_1 - s_1; (n - 1)c_2) - r^1 > \pi_n(c_2 - s_2; (n - 2)c_2 + c_1) - v_1^2
\]
\[
\iff \pi_n(c_1 - s_1; (n - 1)c_2) + \pi_{n-1}(c_1; (n - 2)c_2) > 
\pi_n(c_2 - s_2; (n - 2)c_2 + c_1) + \pi_n(c_1; (n - 1)c_2) + \pi_n(c_1; (n - 1)c_2 - s_2).
\]
Substituting and rearranging yield the statement. Notice that condition (5.2) requires that
\[
s_2 < \frac{\Phi_1 - n\Phi_2 + \sqrt{\Phi_1^2 + \frac{s_2^2}{n^2} + \Phi_1s_2 + n\Phi_1s_1 + n^2\Phi_1^2 + 2n\Phi_2s_1 + 2n^2\Phi_1s_1 + 2n^3\Phi_1s_1}}{n^2 + 1}.
\]
(iii) Assume that $s_1 > \hat{s}_1$: reserve prices are not binding in an auction. In order to take over firm 2, $E$ has to pay $v_2^3$ by Lemma 2. In order to take over firm 1, $E$ has to pay firm $j$’s willingness to pay, $j \geq 2$, for blocking the takeover, $v_j^3$. Therefore, $E$ takes over firm 1 if and only if
\[
\pi_n(c_1 - s_1; (n - 1)c_2) - v_j^3 > \pi_n(c_2 - s_2; (n - 2)c_2 + c_1) - v_j^2
\]
\[
\iff \pi_n(c_1 - s_1; (n - 1)c_2) + \pi_n(c_2; (n - 2)c_2 + c_1 - s_1) > 
\pi_n(c_2 - s_2; (n - 2)c_2 + c_1) + \pi_n(c_1; (n - 1)c_2 - s_2).
\]
Substituting and rearranging yield the statement. Notice that condition (5.3) requires that
\[
s_2 < \frac{\Phi_1 - n\Phi_2 + \sqrt{\Phi_1^2 + \frac{s_2^2}{n^2} + \Phi_1s_2 + n\Phi_1s_1 + n^2\Phi_1^2 - 2n\Phi_2s_1 + 2n^2\Phi_1s_1 + \Phi_1^2 - 2\Phi_2s_1 + s_2^2}}{n^2 + 1}.
\]
Finally, it can be shown that conditions (5.2) and (5.3) coincide if $s_1 = \hat{s}_1$ and conditions (4.1) and (5.2) coincide if $s_2 = \hat{s}_2$. (This implies that the red curve in Figure 5.1 is continuous.) \[\blacksquare\]

**Proof of Lemma 3.** Assume that $s_1 > s_2$, so that firm 1 is both the profitable and the efficient target. Conditions (4.1) and (5.3) hold by inspection. Condition (5.2) holds as well because, when (4.1) holds, $E$ takes over firm 1 with bargaining and firm 1’s price in an auction is the same as in bargaining, whereas firm 2’s price is higher. Hence, $E$ takes over firm 1.

Assume that $s_2 > s_1 + \frac{n+1}{n}(c_2 - c_1)$, so that firm 2 is both the profitable and the efficient target. In this case, conditions (4.1), (5.2) and (5.3) are not satisfied, so that $E$ takes over firm 2. First, condition (4.1) is not satisfied by Lemma 1. Second, when $s_2 > \hat{s}_2$ condition (5.2) is not satisfied since
\[
\frac{s_2^2 - s_2^2}{H} < \frac{2}{n}(s_2\Phi_2 - s_1\Phi_1) + \frac{s_2}{n^2}(s_2 - 2\Phi_1) - \frac{\Phi_2}{n^4}[\Phi_2 + n(2\Phi_1 - n\Phi_2)],
\]
where $H < K$ (because condition (4.1) is not satisfied) and $J > 0$ (using the definition of $\hat{s}_2$ in (A.1) and the assumption that $s_2 > \hat{s}_2$). Third, condition (5.3) requires that
\[
s_2 < \frac{\Phi_1 - n\Phi_2 + \sqrt{s_2^2(n^2 + 1)^2 + 2s_1(n^2 + 1)(n\Phi_1 - \Phi_2) + (\Phi_1 - n\Phi_2)^2}}{n^2 + 1}
\]
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(which is the only positive root of condition (5.3)). However,

\[ \bar{s}_2 < s_1 + \frac{n+1}{n} (c_2 - c_1) \iff s_1 < \Phi_2 + \frac{n^2-1}{2n} (c_2 - c_1), \]

which is always satisfied because of the assumption that \( s_1 < \Phi_2 \). Hence, \( s_2 > \bar{s}_2 \) when \( s_2 > s_1 + \frac{n+1}{n} (c_2 - c_1) \), as we have assumed. ■

**Proof of Proposition 3.** If \( s_1 > \bar{s}_2 \), condition (6.1) follows from conditions (5.3) and (4.1). If \( s_1 \leq \bar{s}_1 \), condition (6.2) follows from conditions (5.2) and (4.1).

Finally, we prove that \( E \) never takes over firm 2 in an auction and firm 1 with bargaining. In order to do this, we show that when \( s_2 - \frac{n+1}{n} (c_2 - c_1) < s_1 < s_2 \) (so that the choices of target in auction and bargaining may differ): (i) if \( s_1 > \bar{s}_1 \), the opposite of condition (6.1) cannot hold; (ii) if \( s_1 \leq \bar{s}_1 \), the opposite of condition (6.2) cannot hold.

First, the opposite of condition (6.1) requires that

\[
\frac{2}{n} (s_1 \Phi_1 - s_2 \Phi_2) > s_2^2 - s_1^2 > \frac{2}{1+n^2} [(n s_1 + s_2) \Phi_1 - (n s_2 + s_1) \Phi_2]. \tag{A.2}
\]

Since \( s_2 > s_1 \),

\[
(ns_2 - s_1) \Phi_1 > (ns_1 - s_2) \Phi_2 \iff \frac{(ns_2 - s_1)}{1+n^2} \Phi_1 + s_1 \Phi_1 - s_2 \Phi_2 > \frac{(ns_1 - s_2)}{1+n^2} \Phi_2 + s_1 \Phi_1 - s_2 \Phi_2
\]

\[
\iff \frac{2}{1+n^2} [(n s_1 + s_2) \Phi_1 - (n s_2 + s_1) \Phi_2] > \frac{2}{n} (s_1 \Phi_1 - s_2 \Phi_2),
\]

which contradicts (A.2).

Similarly, the opposite of condition (6.2) — i.e.,

\[
\frac{s_2}{n^2} (2\Phi_1 - s_2) + \frac{\Phi_2}{n^2} [\Phi_2 + n (2\Phi_1 - n \Phi_2)] < s_2^2 - s_1^2 - \frac{2}{n} (s_1 \Phi_1 - s_2 \Phi_2) < 0,
\]

does not hold since

\[
\frac{s_2}{n^2} (2\Phi_1 - s_2) + \frac{\Phi_2}{n^2} [\Phi_2 + n (2\Phi_1 - n \Phi_2)]
\]

is strictly positive when \( s_2 > \bar{s}_2 \) (using the definition of \( \bar{s}_2 \) in (A.1)). ■

**Proof of Proposition 4.** Suppose that \( s_1 + \frac{n+1}{n} (c_2 - c_1) > s_2 > s_1 \), so that firm 1 is the profitable target but firm 2 is the efficient one. By Proposition 3, \( E \) cannot take over firm 2 in an auction and firm 1 with bargaining, but may take over firm 1 in an auction and firm 2 with bargaining. ■

**Collusion among incumbents.** The total willingness to pay of non-targeted incumbents to block the takeover of firm 1 is

\[
\Delta(s_1) \equiv \pi_{n-1} \left( c_1; \sum_{k \neq 1} c_k \right) + (n-2) \pi_{n-1} \left( c_2; \sum_{k \neq 2} c_k \right) - (n-1) \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_1 \right)
\]

\[
= \frac{\Phi_2 + n (c_2 - c_1)^2}{n^2} + (n-2) \frac{\Phi_2^2}{n^2} - (n-1) \frac{(\Phi_2 - s_1)^2}{(n+1)^2}.
\]

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Non-targeted incumbents can prevent entry if and only if their total willingness to pay is higher than \( E \)'s willingness to pay for firm 1 — i.e.,

\[
\Delta (s_1) \geq \pi_n \left( c_1 - s_1; \sum_{k \neq 1} c_k \right) \iff s_1 \leq s_1^*, \tag{A.3}
\]

where

\[
s_1^* = \frac{n(n-1)\Phi_2 - n^2\Phi_1 + \sqrt{n^4\Phi_1^2 + 2n\Phi_1\Phi_2(-n^3 + 2n^2 + n - 1) + \Phi_2^2(2n^4 - 4n^3 - 4n^2 + 2n + 1)}}{n^3 + n^2 - n}.
\]

The total willingness to pay of non-targeted incumbents to block the takeover of firm 2 is

\[
\Delta (s_2) \equiv \frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} + \frac{(\Phi_2 - s_2)^2}{(n + 1)^2} - \frac{\Phi_1 - s_2}{(n + 1)^2} - \frac{(\Phi_2 - s_2)^2}{(n + 1)^2} - (n - 2) \frac{(\Phi_2 - s_2)^2}{(n + 1)^2}.
\]

Non-targeted incumbents can prevent entry if and only if

\[
\Delta (s_2) \geq \pi_n \left( c_2 - s_2; \sum_{k \neq 2} c_k \right) \iff s_2 \leq s_2^*, \tag{A.4}
\]

where

\[
s_2^* = \frac{2n\Phi_1 + \Phi_2(-3n + n^2 - 1)}{n^3 + n^2 - n}.
\]

We consider a collusive mechanism such that a ring is formed if and only if all non-targeted incumbents join it, and a ring center collects and distributes payments among ring members before the auction. We show how incumbents can design side payments to support collusion when conditions (A.3) and (A.4) are satisfied. Let firm \( i \) be the non-targeted incumbent selected to bid against the entrant and let \( t_i^j \) be the transfer between incumbent \( j \) and the selected firm \( i \). We consider symmetric transfers, so that \( t_i^j = t_i, \forall j \). Let \( T_i = \sum_{j \neq i} t_i^j = (n - 2) t_i \).

First, consider an auction for firm 1. In order to outbid \( E \) and block entry, firm \( i \) has to bid at least \( \frac{(\Phi_1 + ns_1)^2}{(n + 1)^2} \). In this case, firm \( i \)'s surplus is

\[
\frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} - \frac{(\Phi_1 + ns_1)^2}{(n + 1)^2} + T_i,
\]

and firm \( j \)'s surplus, \( j \neq i \), is

\[
\frac{\Phi_2^2}{n^2} - t_i.
\]

Consider side payments such that all non-targeted incumbents obtain the same surplus — i.e.,

\[
\frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} - \frac{(\Phi_1 + ns_1)^2}{(n + 1)^2} + T_i = \frac{\Phi_2^2}{n^2} - t_i \iff t_i^* = \frac{\Phi_2^2}{n^2} - \frac{(\Phi_1 + ns_1)^2}{(n + 1)^2} + \frac{\Phi_2^2}{n^2} - \frac{\Phi_2^2}{n^2} - \frac{2n\Phi_1 + \Phi_2(-3n + n^2 - 1)}{n^3 + n^2 - n}.
\]

If a non-targeted incumbents does not join the ring, collusion fails and \( E \) acquires firm 1. The ring

\[\text{\footnotesize 38} \text{Notice that the total industry profits if the takeover is blocked do not depend on the identity of the non-targeted incumbent that wins the auction because, in this case, the industry always includes } n - 2 \text{ firms with cost } c_2 \text{ and 1 firm with cost } c_1.\]
is stable if and only if non-targeted incumbents prefer to join the ring — i.e.,

$$\frac{\Phi_2^2}{n^2} - t_j^* > \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_1 \right).$$

Substituting and re-arranging yield condition (A.3).

Second, consider an auction for firm 2 and suppose that firm 1 is selected to bid against the entrant. (A similar analysis applies to the case in which a different incumbent is the selected bidder.) In order to outbid $E$ and block entry, firm 1 has to bid at least $\frac{(\Phi_2 + ns_2)^2}{(n+1)^2}$. The ring is stable if and only if non-targeted incumbents prefer to join the ring (rather than let $E$ acquire firm 2). This individual rationality constraint for firm 1 is

$$\frac{[\Phi_2 + n (c_2 - c_1)]^2}{n^2} - \frac{(\Phi_2 + ns_2)^2}{(n+1)^2} + T_1 \geq \pi_n \left( c_1; \sum_{k \neq 1} c_k - s_2 \right),$$

(A.5)

and for firm $j$ is

$$\frac{\Phi_j^2}{n^2} - t_j \geq \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_2 \right), \quad j = 3, \ldots, n. \quad \text{(A.6)}$$

Adding up these constraints yields condition (A.4). Hence, there exist symmetric transfers between other non-targeted incumbents and firm 1 that support collusion (although in this case firm 1 may have to obtain a larger share of the collusive profits than other incumbents because of its higher outside option).

**Proof of Proposition 5.** If $s_1 > s_1^*$ and $s_2 > s_2^*$, non-targeted incumbents cannot block the takeover of any target. Hence, $E$ takes over firm 1 if and only if

$$\pi_n (c_1 - s_1; (n-1)c_2) - \Delta(s_1) > \pi_n (c_2 - s_2; (n-2)c_2 + c_1) - \Delta(s_2)$$

$$\iff \pi_n (c_1 - s_1; (n-1)c_2) + (n-1)\pi_n (c_2; c_1 + (n-2)c_2 - s_1) >$$

$$> \pi_n (c_2 - s_2; (n-2)c_2 + c_1) + (n-2)\pi_n (c_2; c_1 + (n-2)c_2 - s_2) + \pi_n (c_1; (n-1)c_2 - s_2).$$

Substituting and rearranging yield the statement. The other statement follows directly from the definition of $s_1^*$ and $s_2^*$.

Finally,

$$s_2^* > s_1^* \iff 2n (\Phi_1 - \Phi_2) (n+1) (2n\Phi_1 - \Phi_2 - 3n\Phi_2 + n^2 \Phi_2) > 0,$$

which holds since $\Phi_1 > \Phi_2$ and $n > 2$.

**Proof of Proposition 6.** In an auction for firm 1, $E$’s entry is blocked if and only if

$$u_2^1 \geq \pi_n \left( c_1 - s_1; \sum_{k \neq 1} c_k \right) \iff s_1 \leq \tilde{s}_1,$$

where $\tilde{s}_1 = \frac{n\Phi_2 - n^2\Phi_1 + \sqrt{\Phi_2^2(2n^3 + 2n^2 + 2n + 1) - n^2\Phi_1(\Phi_1 + 2n\Phi_2) - n(n^2 + 1)\Phi_2(\Phi_1 - \Phi_2)(2\Phi_2 + n\Phi_1 + n\Phi_2)}}{n + n^3}$. In an auction
for firm 2, \(E\)'s entry is blocked if and only if

\[
v_1^2 \geq \pi_n \left( c_2 - s_2; \sum_{k \neq 2} c_k \right) \iff s_2 \leq \tilde{s}_2,
\]

where \(\tilde{s}_2 \equiv \frac{n\Phi_1 - n^2\Phi_2 + \sqrt{\Phi_1^2(n^4 + 2n^3 + n^2 + 2n + 1) - n^3\Phi_1(2\Phi_2 + n\Phi_1) - n(n^2 + 1)(\Phi_1 - \Phi_2)(2\Phi_2 + n\Phi_1 + n\Phi_2)}}{n + n^2}.\)

Part (i) of the statement follows directly from the definition of \(\tilde{s}_1\) and \(\tilde{s}_2\) and part (iii) of Proposition 2 (since reserve prices are never binding, \(\tilde{s}_1 < 0\)). We now show that \(\tilde{s}_2 > \tilde{s}_1\). Using the definitions of \(\tilde{s}_1\) and \(\tilde{s}_2\),

\[
C + 2 \left( n\Phi_1 - \Phi_2 \right) \tilde{s}_1 + \tilde{s}_1^2 (1 + n^2) = 0
\]

and

\[
C + 2 \left( n\Phi_2 - \Phi_1 \right) \tilde{s}_2 + \tilde{s}_2^2 (1 + n^2) = 0,
\]

where \(C \equiv \Phi_1^2 + \Phi_2^2 - (n + 1)^2 \pi_{n-1} (c_1; (n - 2) c_2) < 0\) by assumption. The result follows since \(n\Phi_1 - \Phi_2 > n\Phi_2 - \Phi_1\). \(\blacksquare\)

\footnote{Of course, as in our main model, in an auction for firm 2 \(v_1^2 > v_j^2, \forall j > 2\).}
References


