Financial Intermediation, House Prices and the Welfare Effects of the U.S. Great Recession

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Abstract
This paper quantifies the welfare effects of the aggregate house price collapse during the U.S. Great Recession for leveraged and un-leveraged U.S. households. We calibrate a dynamic general equilibrium model to the U.S. economy and simulate the 2007-2009 Great Recession as a contemporaneous shock to interest rate spread and aggregate income that quantitatively account for the observed collapse in house prices. As a consequence of the loss in housing wealth, our estimates show that borrowers lost significantly more than savers in terms of welfare. The worsened conditions in the financial intermediation sector in the Great Recession forced borrowers to deleverage, and generated a pure redistribution from savers to borrowers. This amplified the welfare losses of borrowers while caused a relative welfare gain for savers.

Keywords: Housing Wealth, Heterogeneous Agents, Welfare, Leverage

JEL Classification: D31, D58, D90, E21, E30, E44

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Appendix
1 Introduction

The U.S. Great Recession has been characterized by a large fall in GDP and an unprecedented collapse in the housing market. The drop in aggregate house price between 2007:IV and 2009:II affected a great number of U.S. households,\(^1\) possibly impacting on their consumption and, ultimately, their welfare. The recession has also been coupled with turbulence in the financial markets and, in particular, the financial intermediation sector. This fact has triggered a debate among economists and policy-makers about the consequences of the credit contraction process that followed, and possibly exacerbated, the economic collapse.

Figure 1 summarizes four stylized macroeconomic facts of the US Great Recession.\(^2\) We observe a large drop of around 5.4% in real GDP (panel a) that is mirrored by a collapse in aggregate house prices of about 13% between 2007 and 2009 and of about 19% between 2007 and 2010 (panel c).

The years that followed the recession have been also characterized by changes in the financial intermediation sector. We observe that, from mid-2008, the spread between the mortgage interest rate and the federal funds rate jumped to a level of about 4.5%; this increase followed a period of steady decline (panel b) when the spread reached a value close to 0%. Interpreting this last evidence, we share the view of Bordo (2008) that fluctuations in spreads largely reflected disturbances in the financial markets’ assessments of credit risk which possibly resulted from banks’ balance sheet effects of monetary policy (see also Adrian and Shin, 2010). The increase in the spread from 2008 coincided with the decrease in households’ aggregate leverage (panel d). In particular, while households’ leverage

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\(^1\)Iacoeliello (2011) shows that housing wealth represents about half of total household net worth in 2008 and almost two third of median household total wealth.

\(^2\)Throughout the paper we will refer to the US Great Recession as the period defined by NBER recession dates: 2007:IV - 2009:II.
increased between 2001-2007, it skyrocketed from mid-2007 (possibly as a consequence of the housing market collapse) and started declining in 2009 when the turbulence in the financial markets resulted in decreased supply of credit to the household sector.

This paper builds a model that is consistent with the stylized facts documented in figure 1 and uses it to quantitatively examine the effects of an exogenous income and an exogenous interest rate spread shock on endogenous housing wealth. While we are agnostic about the primitive nature of the shocks from which the recession originated, we want to ultimately estimate the effects of the joint dynamics of house prices, leverage, and the wealth distribution on households’ welfare.
To address this question we use a stochastic dynamic general equilibrium model with heterogeneous households and endogenous collateral constraints. In the model, households differ in their level of patience. This heterogeneity results into two types of agents: borrowers - potentially financial constrained - and financial unconstrained savers. The descriptive statistics presented in table 1 for U.S. households motivate our choice of this cross-sectional heterogeneity across households. Using the 2007-2009 panel data from the Survey of Consumer Finances (SCF), the table shows that households with a positive net saving position in 2007 - labeled savers - show an average drop in housing wealth of 12.5% between 2007 and 2009. This is significantly lower than the -19.0% experienced by the group of households with a negative net savings position in 2007 - labeled borrowers.

Table 1: Summary Statistics from SCF panel 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>Savers</th>
<th>Borrowers</th>
<th>All households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ2007−2009housing wealth</td>
<td>-12.5%</td>
<td>-19.0%</td>
<td>-15.9%</td>
</tr>
<tr>
<td>Housing wealth in 2007</td>
<td>405,429</td>
<td>342,200</td>
<td>372,533</td>
</tr>
<tr>
<td>Wage income in 2007</td>
<td>69,449</td>
<td>64,424</td>
<td>66,834</td>
</tr>
<tr>
<td>Age in 2007</td>
<td>60</td>
<td>47</td>
<td>53</td>
</tr>
</tbody>
</table>

The average values are weighted by using sample weights provided by the Survey of Consumer Finances panel 2007-2009. Housing wealth and wage income are measured in US dollars in 2007. Age refers to the head of the household. A more detailed description of the data analysis is in Appendix A.

We interpret this as evidence of redistribution of housing between the two types of agents for a given drop in aggregate house prices. We are aware of the possibility that housing markets can be segmented among households that belong to different percentiles.

4 The structure of the economy is similar to Iacoviello (2014) and Justiniano, Primiceri, and Tambalotti (2013).

5 In table 1, the saver/borrower-status refers to households in 2007. Savers and borrowers are defined here - and throughout the paper- as households that show respectively a positive or a negative net asset position. The net asset position has been calculated using households’ level data from the SCF panel 2007-2009. For a detailed description of the definition please refer to Appendix A.
of the wealth and age distribution (Landvoigt, Piazzesi, and Schneider, 2012); however, table 1 documents that average borrowers and average savers belong to close wage income, housing wealth and age classes in 2007; this suggests that, at least to some extent, the two groups were active in the same housing market.

Given that individual households' housing consumption is not observed in the data but we observe aggregate house prices and housing wealth before and after the recession, we employ a structural model that matches observed moments from microeconomic and macroeconomic data; we use this model to study the evolution of the agents' housing consumption as well as consumption of non-durable goods and, ultimately, compute the welfare effects. The structural nature of our exercise allows us to conduct counter-factual experiments in order to disentangle the quantitative effects of income and intermediation shocks on aggregate house prices and agents' welfare.

In the model economy, agents are fully rational and derive utility from both the consumption of perishable goods and of housing services coming from housing stock. Housing is the only physical asset in the economy and it is fixed in supply. This is motivated by the fact that previous and during the Great Recession, house prices were most volatile in geographical areas where the supply of houses was relatively fixed.\(^6\)

Financial frictions arise for two reasons. First, agents have to collateralize debt by a fraction of their available housing stock. Second, we introduce a competitive financial intermediation sector. All saving and borrowing is conducted through this sector whose efficiency is subject to exogenous shocks.\(^7\) This shock gives rise to a time-varying spread between the borrowing and lending rate; therefore, in some states of the world, borrowers might find it too expensive to borrow so that the collateral constraint does not necessarily bind. In other words, it generates endogenous changes in households' leverage. The second disturbance is a standard aggregate income shock that impacts the households'...
endowment of the perishable good.\textsuperscript{8}

We calibrate the model to the US economy and simulate the Great Recession as a contemporaneous negative income and financial intermediation shock that follows a period of moderate economic growth accompanied by a credit expansion and increasing leverage. This characterization is motivated by the facts highlighted in figure 1; in panels a) and b) both income and financial intermediation were high before 2007, and low in the recession. In order to calibrate our key parameters we consider moments from both micro and macro data. In particular, we match the borrowers’ mortgage debt to housing ratio and the wealth share of borrowers relative to savers from the Survey of Consumer Finances. This calibration strategy - yet different from the approach of most existing papers in the literature that target macro moments only - results in calibrated parameters that are compatible with recent contributions (e.g. Iacoviello and Guerrieri, 2012).

A delicate issue for the calibration exercise is what time frame to use, and in particular, whether to incorporate recession data for targeted moments. We take the following stance. Our main goal is to maintain a close link between the model and the research question. We study the Great Recession as a state-contingent exogenous event that hit the US economy in late 2007, following a period characterized by banking innovation and increasing household leverage. Therefore, we consider the Great Recession as a low probabilistic event. For this reason, we calibrate the model to data including the years 2007-2009.

The benchmark model is calibrated such that it generates a drop in house prices during the Great Recession that is compatible with the data. This results from our choice of the risk aversion parameter. This parameter value is within the parameter interval usually considered in the macroeconomic and macro-finance literature (see e.g. Glover, Heathcote, Krueger, and Ríos-Rull, 2014; Piazzesi, Schneider, and Tuzel, 2007). We find that about 80\% of the drop in house prices is attributed to income shocks while 20\% is attributed to the financial intermediation shock. This goes in the same direction as the findings in

\textsuperscript{8}This may be interpreted as a reduced form way to capture the cyclical behavior of productivity shocks.
Justiniano, Primiceri, and Tambalotti (2013) where shocks to the financial sector have a limited effect on house prices in the US economy before and during the Great Recession.

As for the welfare effects, we highlight two main findings. First, borrowers significantly lose more than savers in the Great Recession. Second, the negative financial intermediation shock is the main driver behind the de-leveraging process that started in 2009. We show that the de-leveraging process has amplified the negative welfare effect for borrowers; the opposite is true for savers, who relatively benefited from aggregate de-leveraging. The mechanism behind these findings is the following. A negative realization of the income shock leads to a reduction in the aggregate demand for normal goods and a deflation pressure on house prices. The drop in the collateralized assets generate a credit contraction for borrowers who have access only to collateralized debt as a source of external financing. If the reduction in debt is sufficiently large, borrowers need to reduce their housing stock. For a given supply of housing, house prices must further decrease. This causes borrowers to suffer in terms of both wealth and expected lifetime utility. Similarly savers suffer from the aggregate drop in house prices (negative wealth effect). However, given that they are unconstrained and expect house prices to rise again in the future, they smooth their consumption by buying houses when prices are low; this results in a smaller housing wealth drop. If the income contraction is coupled with high spreads - which is what we observe during the Great Recession -, interest rates on mortgages are high. The consequence is that debt becomes more costly and borrowers optimally reduce their demand for debt, for their collateral asset (housing), and possibly move away from the collateral constraint. This exacerbates the redistribution of the housing asset from borrowers to savers as described above; as a consequence, savers relatively gain more in terms of housing consumption and suffer less than borrowers in terms of expected lifetime utility compared to a recession where mortgage spreads would have stayed low. Notice that the effect of the financial intermediation shock results from endogenous movements in household leverage. In our model, in fact, borrowers can choose the pace at which to reduce their debt, unlike the case in models with an always-binding constraint where the
leverage ratio is by construction constant.

Relative to the SCF data summarized in table 1, we find that the simulated drop in housing wealth for borrowers (savers) in the benchmark model is slightly too large (small); the reason is that borrowers find it optimal to reduce their housing stock and - in absence of transaction costs - savers are happy to buy those houses, so the model suggests that there is too much housing redistribution relative to the data. In order to bring the model closer to the data, we introduce adjustment costs for housing for both agents and we set the parameters so that the drop in housing wealth in the Great Recession for both agents is broadly in line with the SCF data. In this modified version, we find that the equilibrium house price drop is quantitatively slightly lower and financial intermediation shocks imply quantitatively slightly smaller redistributive effects between borrowers and savers due to the smaller reaction of housing and leverage.

The present study is related to two important strands of literature. First, we relate to the recent literature that explore the effects of financial frictions on US macroeconomy in the Great Recession (Hall (2011) and Quadrini and Urban (2012)). Huo and Ríos-Rull (2014) study the quantitative power of financial frictions in explaining the drop in house and stock prices; Justiniano, Primiceri, and Tambalotti (2014) highlight that credit supply shocks played a pre-eminent role in qualitatively explaining the dynamics of house prices before and following the Great Recession; although similar in spirit, our contribution is the focus on welfare and the distributive implications of financial shocks. Guerrieri and Lorenzoni (2011) find that a shock to the spread between the interest rate on borrowings and the interest rate on savings, in the presence of a collateral constraint that links debt to the level of durables, generates a decrease in the borrowers’ consumption; however their analysis abstracts from the wealth and collateral effects coming from movements in aggregate house prices.

Second, our analysis relates to studies on the welfare effects of financial innovation and of the Great Recession. Our focus is on the effects of the recession on housing and welfare. In this respect, we complement the work by Campbell and Hercowitz (2009)
who study the welfare implications of increasing leverage for US households in the years previous to the Great Recession. Compared to Glover, Heathcote, Krueger, and Rios-Rull (2014) - a study on intergenerational redistribution during the Great Recession - we focus on a different dimension of agent heterogeneity and welfare, namely, redistribution between constrained agents (borrowers) and unconstrained agents (savers). Similar to Hur (2012), we find that the constrained agents always lose more than unconstrained agents. Both of the aforementioned studies are silent about the inherent redistributive nature of financial shocks, the focus of this paper.

The rest of the paper is structured as follows: In the following section we present the model. Section 3 presents the quantitative analysis. Section 4 concludes.

2 Model

The modeling framework is a heterogeneous agents environment with borrowing constraints similar to the one studied by Kiyotaki and Moore (1997) and extended to a stochastic setting, for example, by Iacoviello (2005). Time is discrete. There are two types of atomistic agents, impatient and patient households; impatient households discount the future more than patient agents, so that there are potential gains from borrowing and lending between the two groups. Both types of households demand non-durable consumption and housing services that stem from real estate which also serves as collateral for negative asset positions. We assume that the aggregate supply of houses is fixed which guarantees a variable price of housing. In addition, there are financial intermediaries that collect deposits and can give out loans that we call mortgages; the intermediaries ability to transform savings into loans is subject to shocks and drives a wedge between the real interest rate on savings and the mortgage interest rate.

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9 Hur (2012) considers an overlapping generations model with collateral constraints; he finds that the constrained agents are mostly from the young cohort, and that these agents suffer the most during a recession.

10 Another distinguishing element of our analysis to Hur (2012) and Guerrieri and Lorenzoni (2011), is that they consider the recession as an unanticipated event while, in our economy, agents take into account the probability of negative aggregate shocks when making decisions about the future.
Households. There are two types of households which differ with respect to the rate at which they discount the future. The different discounting implies that impatient agents - in equilibrium - are borrowing from the patient households. We therefore refer to the impatient agents as ‘borrowers’ and to the patient agents as ‘savers’. Savers, denoted with a subscript \( s \), represent a share \( n_s \) of the total population; borrowers are denoted by a subscript \( b \) and represent a share \( n_b = 1 - n_s \) of the total population. The respective discount factors satisfy \( \beta_s > \beta_b \), where \( \beta_i \in (0, 1) \). At time 0, each type’s representative household \( i = b, s \) maximizes her expected lifetime utility

\[
\max_{\{c_{it}, h_{it}\}} \sum_{t=0}^{\infty} \beta_t^t \left( \frac{c_{it}^{\phi \cdot t \cdot h_{it}^{1-\phi}}} {1 - \sigma} \right) - 1
\]

where \( c_{it} \) is per capita consumption of the perishable consumption good and \( h_{it} \) is type \( i \)’s stock of houses chosen in period \( t \); \( h_{it} \) units of houses entitles the owner to a service stream of \( 1 \cdot h_{it} \). Other than the service stream, houses do not yield any dividend payments. Houses are the only physical asset in the economy and are in fixed net supply. This assumption ensures a variable relative price of houses and is motivated by the fact that (i) the share of new houses is small relative to the total housing stock and (ii) that - in the years 2005 to 2009 - house prices in the U.S. were most volatile in counties where the supply of houses remained relatively fixed (Mian and Sufi, 2010).

Each period, agents receive per-capita income \( y_t \), in units of the perishable consumption good; we assume that income is exogenous and subject to an aggregate shock (i.e. the same for both types of households), so \( y_t \) is not indexed by the household type. Denote with \( q_t \) the relative housing price. Assume that households save \( s_{it} \geq 0 \) and receive back \( R_{t-1} s_{t-1} \) where \( R_{t-1} \) is the real interest rate on savings between \( t - 1 \) and \( t \). Similarly households can take up a loan \( l_{it} \leq 0 \) and pay back \( R_{Lt-1} l_{it-1} \) where \( R_{Lt-1} \) is the real interest rate on loans between \( t - 1 \) and \( t \). We make this distinction of savings and loans because - as will be clear below - the interest rates differ in equilibrium. The flow of
funds of households is

\[ c_t + q_t h_{it} + s_{it} + l_{it} = y_t + q_t h_{it-1} + R_{t-1}s_{it-1} + R_{L,t-1}l_{it-1} + \Upsilon_t, \]  

(1)

where \( \Upsilon_t \) are per capita lump-sum profits received from the financial intermediaries (described below).

As in Kiyotaki and Moore (1997) we assume limits on debt obligations. Houses are distinguished from other assets by the fact that they are widely used as collateral for debt obligations (mortgages). We assume that agents can borrow at most a fraction \( m \) of the value of the housing stock they own, so the constraint takes the form:\(^{11}\)

\[ l_{it} + mq_{it}h_{it} \geq 0. \]  

(2)

**Financial Intermediaries.** Intermediaries finances the supply of debt \( L_t \) by deposits (i.e. savings), denoted by \( S_t \).\(^{12}\) For each unit lent the intermediary earns a gross interest of \( R_{Lt} \) and pays \( R_t \) for one unit deposited at the intermediary. The collateral constraints make sure that agents do not default in equilibrium and debt is risk free. The key distortion in the intermediation sector is modeled as in Cooper and Ejarque (2000).\(^{13}\) We assume that each period only a fraction of savings can be transformed into loans, denoted by \( \theta_t \in [0, 1] \); \( \theta_t \) is governed by a stochastic process that is the same for all intermediaries.

This exogenous financial shock represents a reduced form way to model the risk-bearing capacity of the financial sector. In particular, changes in the intermediation efficiency \( \theta_t \) potentially reflect changes in the value of equity associated with a risky asset portfolio or changes in monitoring by the bank managers as a consequence of changes in

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\(^{11}\)This constraint is often stated as a requirement that the promised debt value (interest plus principal) does not exceed the expected housing value. In order to avoid problems with multiplicity of equilibria when expectations in the collateral constraints are involved, we choose the contemporaneous formulation. We experimented with different formulations of this constraint and numerically the differences are negligible, so we decided to stick to the simplest case.

\(^{12}\)In the remainder of the analysis, we use the labels ‘savings’ and ‘deposits’ as substitutes.

\(^{13}\)Another example for the inclusion of a supply-sided friction in the banking sector into an international macro model is Kalemli-Ozcan, Papaioannou, and Perri (2012).
risk aversion. Consequently, while we remain agnostic about the exact foundation of \( \theta_t \), we point out that the observed variations in the spread series in the period 2005-2009 mainly reflect changes in the banks’ price for risk rather than changes in the default risk.

Financial intermediaries are otherwise risk neutral and maximize profits on their portfolio, that is,

\[
\max_{L_t, S_t \geq 0} R_{Lt}L_t - R_t S_t
\]

subject to the constraint

\[
L_t \leq \theta_t S_t
\]

Because intermediaries operate in a competitive market with free entry, equilibrium interest rates are such that intermediaries make zero profits:

\[
R_{Lt}\theta_t = R_t.
\]

This last relation implies that there is a spread between loan and deposit rates in this economy. In other words, the interest rate on debt is always at least as big as the interest rate on savings, or \( R_{Lt} \geq R_t \).

**Transfers from the financial intermediaries to the households.** Completing the description model, we specify the lump-sum per capita transfers \( \Upsilon_t \). The intermediation process as outlined above implies an aggregate intermediation loss in terms of real resources that, in equilibrium, is given by \( (1 - \theta_t)S_t \). This can be easily verified by combining the households budget constraints, using market clearing conditions in the debt and savings markets, and the zero profit condition of financial intermediaries:

\[
\sum_{i=b,s} n_i c_{it} + (1 - \theta_t)S_t = y_t + \Upsilon_t
\]

On the left hand side, we have the borrowers’ and savers’ consumption plus the resources ‘eaten up’ by the financial sector. On the right hand side we have aggregate income.
plus transfers. In order to keep the intermediation process as a purely re-distributive distortion, we choose $\Upsilon_t$ such that all resources ‘lost’ in the intermediation sector are redistributed back to the agents, so that aggregate consumption is a function of aggregate income only. Therefore, we assume

$$\Upsilon_t \equiv (1 - \theta_t)S_t$$  \hspace{1cm} (6)$$

We interpret this transfer as income generated by the intermediation sector that is redistributed back to the households because they are either the managers of the bank or the residual claimants on the portfolio revenues of the bank. The inclusion of the transfer function has two advantages. The first is that any effect of a intermediation shock on house prices and welfare comes through the effect on interest rates, and is not generated by an aggregate loss of resources. The second advantage is computational, as the re-distribution of resources makes sure that aggregate consumption is a function of aggregate endowment only, an essential requirement for the application of the concept of wealth recursive equilibria proposed by Kubler and Schmedders (2003) to our framework.

**Equilibrium definition.** A competitive equilibrium is a collection of allocations and a collection of prices such that - taken as given these prices - households and intermediaries solve their respective problems, financial intermediaries make zero profits, and markets clear:

- **House market**

  $$1 = \sum_{i=b,s} n_i h_{it}$$

- **Loan market**

  $$L_t = - \sum_{i=b,s} n_i l_{it}$$
Savings market

\[ S_t = \sum_{i=b,s} n_i s_{it} \]

**Wealth Recursive Equilibrium.** For the quantitative exercise, we define a wealth recursive formulation in the spirit of Kubler and Schmedders (2003). Since we have only two agents, the relative wealth of one agent can be summarized by a single value on the unit interval; we therefore use as a state variable the borrowers' beginning-of-period wealth-share, defined as the aggregate value of borrowers' beginning of period portfolio divided by aggregate wealth (which is equal to the value of housing):

\[ \omega_{bt} = \frac{q_t h_{bt} + R_t h_{bt} + R_t s_{bt}}{q_t} \]  

(7)

Note that the collateral constraints, the constraints on asset holdings, and the utility functions satisfying Inada-conditions, imply that the relative wealth share lies in the unit interval, \( \omega_b \in [0, 1] \). By definition, \( \omega_{st} = 1 - \omega_{bt} \). The equilibrium policy function is then a function of the exogenous state variables \((y_t, \theta_t)\) and the financial wealth distribution \( \Omega = (\omega_b, 1 - \omega_b) \). To solve for an equilibrium numerically, we follow Kubler and Schmedders (2003). For the approximation of the equilibrium policy functions we adopt the time-iteration algorithm with linear interpolation proposed by Grill and Brumm (2010). That is, we approximate the equilibrium policy on a fine grid for the borrowers’ wealth share. For points outside the grid we use linear piecewise interpolation. See appendix B for a detailed description of the numerical procedure.

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14To see this, use the market clearing conditions for the housing, debt, and savings markets and the fact that financial intermediaries make zero-profits in equilibrium.
3 Quantitative Analysis

This section studies the quantitative effects of the Great Recession on house prices and households’ welfare. The Great Recession is modeled as contemporaneous negative shocks to both aggregate income and financial intermediation (interest rate spread). In this respect, our simulation is driven by the macroeconomic facts that motivated our research question. The next subsection outlines the calibration strategy. We then present the quantitative results on welfare.

3.1 Calibration

In the benchmark calibration we set the risk aversion parameter equal to $\gamma = 5$. While the business cycle literature usually features a log-separable utility function with risk aversion equal to unity, recent contributions on asset pricing and on the distributive effects of the Great Recession focus on a broader set of parameter values for risk aversion. Piazzesi, Schneider, and Tuzel (2007), in a capital asset pricing model with housing, find that a model featuring a higher level of risk aversion better performs in matching the moments of housing returns. Relatedly, Glover, Heathcote, Krueger, and Rios-Rull (2014) set the risk aversion equal to 2.5 in their benchmark case and then conduct a sensitivity analysis. They find that the magnitude of equilibrium price responses increase monotonically as risk aversion increases.\footnote{Along the same lines, we find that the elasticity of house prices with respect to income drop increases less than proportionally with the risk aversion parameter. The results are available upon request.} In line with these contributions, in our setting we pick a risk aversion equal to 5 because, conditional on the calibrated exogenous income and financial intermediation shocks, the model generates a house price drop in the simulated Great Recession that is close to the drop observed in the data.

The parameter $\phi$ is the expenditure share of non-durable consumption. We pick the value to match the average housing wealth over GDP in the data during the period 2002-2009; in the data, aggregate housing wealth is the sum of the value of owner occupied real estate of private households plus the residential housing wealth of non-financial non-
corporate private business. The savers’ discount factor $\beta_s$ is set so that the average interest rate on savings in the model matches the average yearly return on savings, equal to 2% in the period 1980-2009. The borrowers’ discount factor $\beta_b$ and $m$ are calibrated to match the average wealth share and the leverage ratio of borrowers, respectively, from the Survey of Consumer Finances in 2007. Since there is not necessarily a one-to-one mapping between the parameters and their targets, we follow an iterative procedure to jointly find values for $\beta_s$, $\beta_b$, $m$ and $\phi$. That is, we first guess values for the parameters and then compare the computed moments to their counterparts in the data. If they do not match, we change the values and repeat until they do. The procedure leads to a quite satisfactorily match between model and data moments.\(^{16}\)

The relative population size of borrowers is set to 52%, corresponding to the fraction

\(^{16}\)The variable definitions used to calculate the data moments are as close as possible to the definition of the model counterparts. For a detailed description of how we compute the relative wealth share and the leverage ratio in the data, see appendix A.
of borrowers in the SCF 2007 when using the weighted average share of households with a negative net asset position as defined in appendix A.

The stochastic processes for the exogenous state variables $y_t$ and $\theta_t$ are assumed to be independent. This is in line with the correlation in the data.\footnote{We conducted a VAR analysis for GDP growth and spreads for different lag-lengths and orderings and found only weak evidence for significant spillover terms and no evidence for a contemporaneous correlations between GDP innovations and innovations to mortgage spreads. Only in one specification, the null of Granger-causality of output growth on spreads is rejected, for spreads the Null is never rejected. The VAR results are available upon request.} We assume that both aggregate income and the intermediation spread shock follow a two-state Markov process whose realization take two values each, that is $y_t = \{y_L, y_H\}$ and $\theta_t = \{\theta_L, \theta_H\}$, respectively. For the intermediation shock, we assume that the transition probabilities are given by:

$$\pi_{ij} = (1 - \rho)\pi_j + \delta_{ij}\rho \quad \text{for } i, j = H, L$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise; $\pi_j > 0$ is the unconditional probability of being in state $j$, and by definition we have $\sum_j \pi_j = 1$. The parameter $\rho$ governs the persistence of the shock.\footnote{See Backus, Gregory, and Zin (1989) and Mendoza (1991).} The unconditional probability of a high intermediation efficiency, $P(\theta = \theta_H)$, is set to 0.565, the fraction of quarters in which the U.S. experienced low spreads between 1998:I and 2009:II. We set $\theta_L = 0.9578$, $\theta_H = 0.995$, and $\rho_{\theta} = 0.868$ so that we match the autocorrelation and the highest and lowest realization of the spreads in the data.

For the income shock, we choose $y_H$ and $y_L$ to match the mean, normalized to $E(y) = 1$, and an average peak-to-trough drop in GDP of 5% during a recession. The transition probabilities are chosen as follows. The conditional probability of the low realization of $y$ being in a recession today $\pi_{yLL}^y$ is chosen to match an average duration of a recession equal to 1.5 years: this is in line with the NBER recession dates between 1980:I and 2009:II. The transition probability of the high income realization conditional on high income today, $\pi_{yHH}^y = 1 - (1 - \pi_{yLL}^y)^{1-\pi_{yH}^y/\pi_{yL}^y}$, is obtained by setting the unconditional probability of a recession equal to 15% ($\pi_{yH}^y = 0.85$). This is also in line with NBER recession dates between 1980:I and 2009:II.
To summarize, the exogenous state space is given by \( \Sigma = \{(y_H, \theta_H), (y_L, \theta_H), (y_H, \theta_L), (y_L, \theta_L)\} \) and - given the assumption that income and intermediation processes are uncorrelated - the transition matrix for the exogenous process is just the Kronecker product of the individual transition probability matrices for the income shock and the intermediation shock, respectively. Table 2 summarizes the calibrated parameter values, the implied simulated moments, and the data targets.

### 3.2 Welfare effects of the Great Recession

In this section we show the main quantitative exercise, the estimation of welfare effects of the Great Recession. We construct an event window around the Great Recession. We define the Great Recession as a state of the world with low income and high spreads that is preceded by a state of the world where income is high and spreads are low (i.e. intermediation is high). We then go along the equilibrium path of the simulated economy and select all sequences that match these criteria. In figure 2, we plot the average of these sequences including five years preceding the crisis and five years after the crisis. In the figure, we compare the Great Recession to a counter-factual scenario: we ask what would happen if spreads were low before and stayed low during the recession; this corresponds to the dashed line in 2 which we label as \textit{low-spreads scenario}. This experiment helps us to evaluate the effects of a negative financial intermediation shock in the Great Recession.

The solid lines in panels (a) to (d) are the simulated model counterparts to the empirical series shown in figure 1; panel (a) of figure 2 shows the evolution of income. In all scenarios, income first increases previous to the recession and then drops by 5 percent in period 0 when the recession hits. In the Great Recession, as shown by the solid line in panel (b), the mortgage spread first decreases towards its lowest value in period \(-1\) and then jumps to 4.5 percent in period 0. In the \textit{low-spreads} counter-factual scenario spreads decline and stay in their lowest realization in periods \(-1\) and 0 and then return towards their long-run mean. Panel (c) show the dynamics of house-prices;
Figure 2: Great Recession (solid) versus counterfactual recession with high financial intermediation (dashed)
the drop in the model is about 16%;\textsuperscript{19} comparing this value to the house price drop in the counter-factual low-spread scenario (about 13 percent), we find that the relative contribution of the income shock and spread shock to aggregate house prices is about 80% and 20%, respectively. Panel (d) plots the behavior of leverage defined as the ratio between debt and housing wealth of borrowers. When spreads are low, borrowers leverage up by increasing their debt holdings faster than their housing wealth. This means they move towards the collateral constraint that binds; from panel (e) you may notice that the multiplier on the borrowing constraint is positive. When spreads increase in period 0, it becomes too costly for borrowers to roll-over their mortgages and they de-leverage; the multiplier goes towards zero. In the low-spreads scenario, borrowers stay leveraged also in period 0 and then de-leverage slowly following the path of spreads; the value of the multiplier spikes because borrowers try to smooth the recession by borrowing up to the limit (which is tighter because the house price drops in the recession).

Panels (f) and (g) show the paths for housing wealth for borrowers and savers, respectively. This figures illustrate the following. If mortgage spreads would have stayed low during the recession (low-spreads case), borrowers would have lost less in terms of housing wealth than in the benchmark scenario, whereas savers would have lost more housing wealth. Together with borrowers’ leverage, this affects the evolution of wealth distribution across agents. This is clearly reflected by the evolution of borrowers’ wealth share, shown in panel (h). In this panel, the solid line displays a much larger drop than the dashed line. Importantly, the wealth share recovers much slower after the Great Recession compared to the case when mortgage spreads would have stayed low during the crisis. This means that the negative wealth shock for borrowers is quite persistent in the Great Recession.

Panels (i) and (j) show the dynamics of non-durable (per-capita) consumption for borrowers and savers, respectively. Notice that aggregate non-durable consumption is

\textsuperscript{19}This value is consistent with the average housing wealth drop showed in table 1; furthermore the value is in between the observed drop in the period 2007-2009 (13\%) and in the period 2007-2010 (19\%) from figure 1.
equal to aggregate endowment $y_t$, so that in this model the dynamics of aggregate non-durable consumption are identical to the income dynamics as shown in panel (a). The panels reveal that borrowers’ per-capita consumption of non-durable goods decreases slightly more than the per-capita non-durable consumption for savers. In the low spreads scenario (dashed line), borrowers would cut down more on non-durable consumption than in the Great Recession but would recover faster afterwards; for the savers, the opposite is the case. Finally, panels (k) and (l) show the time paths for the interest rates on mortgages and deposits, respectively. Both interest rates are higher in the simulated Great Recession than in the low-spread scenario. The fact that mortgage and deposit rates move together comes - by construction - from the zero-profit condition of the financial intermediaries. Interest rates on deposits go up more in the Great Recession because savers’ consumption decreases more.

The welfare effects are summarized in table 3. The table compares the model predictions with the data: the statistics in table 1 (and reported for convenience in the first row of table 3) show that we observe the aggregate change in housing wealth, as well as the change in housing wealth for borrowers and savers, between 2007 and 2009; however, given that we do not observe the level of housing consumption in the data, we recover its dynamics from the model simulations. The main objects of interest are the change in borrowers’ wealth share, denoted by $\Delta \omega_b$, which is a direct measure of re-distribution in our model and the welfare gains/losses in the recession for the two types of households, denoted by $\lambda_b$ and $\lambda_s$, respectively. We define welfare gains of the recession as the compensation - in terms of total consumption (i.e. the aggregate of housing and non-durable consumption) - that is needed each period for all future periods to make agents indifferent between the expected life-time utility in period $-1$ (i.e. the year that precedes the recession) and expected life-time utility in period $0$ (i.e. the year when the recession hits). Negative numbers therefore reflect welfare losses of the recession. We refer to these estimates as ‘welfare gains’.

Based on figure 2 and estimates from table 3 in the two scenarios, we can summarize
Table 3: Welfare effects of the US Great Recession with adjustment costs

<table>
<thead>
<tr>
<th></th>
<th>$\Delta q$</th>
<th>$\Delta (q_{h_b})$</th>
<th>$\Delta (q_{h_s})$</th>
<th>$\Delta \omega_b$</th>
<th>$\lambda_b$</th>
<th>$\lambda_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF Data</td>
<td>-15.9</td>
<td>-19.0</td>
<td>-12.5</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Welfare gains relative to pre-recession</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Great Recession</td>
<td>-16.3</td>
<td>-28.5</td>
<td>-11.2</td>
<td>-2.7</td>
<td>-3.7</td>
<td>-1.0</td>
</tr>
<tr>
<td>Low spreads</td>
<td>-13.2</td>
<td>-22.8</td>
<td>-9.1</td>
<td>-2.1</td>
<td>-3.2</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

Notes: Column one shows the change in aggregate housing wealth between date $-1$ and date $0$. Columns two and three tabulate the percentage change in housing wealth between date $-1$ and $0$ for borrowers and savers, respectively. Column four tabulates the absolute change of the borrowers’ wealth share between date $-1$ and date $0$. Columns five and six show the welfare gains of the recession in total consumption equivalents (relative to expected utility in period $-1$) for borrowers and savers, respectively. The Great recession is defined as a contemporaneous drop in income and financial intermediation in period $0$. The counterfactual in row three assumes that financial intermediation is low in both periods $-1$ and $0$.

the following two key findings:

1. Borrowers lost significantly more than savers in the Great Recession.
2. A negative financial intermediation shock, when coupled with a negative income shock, results in higher (smaller) welfare losses for borrowers (savers).

Result 1 says that, when there is a drop in aggregate house prices, financially constrained households (borrowers) are less able to smooth the wealth loss and de-leverage. By de-leveraging they allow unconstrained agents to buy the deflated housing; as a result savers can buffer the negative welfare effects of the housing wealth drop.

Result 2 comes from the comparison of the simulated Great Recession with the counter-factual scenario that we labeled low-spreads case. We observe that, in the counter-factual economy, aggregate house prices drop about 20% less but also the redistribution is smaller; the absolute change in the borrowers’ wealth share is -2.7 percentage points in the Great Recession while it is -2.1 percentage points in the counter-factual case. With respect to welfare, the borrowers’ welfare gain is -3.7 percent of total consumption in the Great Recession and -3.2 percent in the low-spread scenario; in relative terms, borrowers would have lost about 13% less in the low-spread scenario compared to the Great Recession.
Recession. Conversely, savers lost about 20% more (-1.2 versus -1 percent), even though aggregate house prices go down less. The reason is related to the question whether a financial intermediation shock, and the subsequent de-leveraging process, induces a larger redistribution of wealth. When house prices fall and there is a contemporaneous negative intermediation shock, borrowers face a higher interest rate on debt, which makes it more costly to roll over the debt and they choose to move away from the collateral constraint. At the same time, borrowers decrease their stock of housing that is bought by savers, who are financially unconstrained.

This set of results leads to the conclusion that, while both types of agents experienced a welfare loss in the Great Recession, savers could cushion themselves from the negative impact of the negative aggregate shocks by substituting their savings with depreciated houses. This conclusion, while qualitatively comparable with the recent findings of Hur (2012), highlights a different mechanism that comes from the financial intermediation shock. Finally, regarding the size of the welfare losses, our estimates are in the same order of magnitude as in Glover, Heathcote, Krueger, and Rios-Rull (2014) who found welfare losses for different age cohorts ranging from -1.67% to -8.3%.

### 3.3 Adjustment costs

As discussed in the previous subsection, the benchmark model slightly overestimates the drop in housing wealth for borrowers compared to the SCF data and slightly underestimates it for savers.\(^{20}\) This results from the fact that we assumed a perfectly liquid housing market in the model. The goal of this section is to show that the observed changes in housing wealth from the SCF can be reconciled with the view that the Great Recession was a joint shock to income and financial intermediation if housing markets show some degree of rigidity. Some authors interpret housing market rigidities as constraints on

\(^{20}\)For borrowers the change in housing wealth is -28.5 percent in the model versus -19 percent in the data while for savers it is -11.2 percent in the model versus 12.5 percent in the data, see table 3.
the housing segment in which agents are active\textsuperscript{21} rather than housing adjustment costs. However, since our model does not distinguish different qualities of housing services, we interpret housing market rigidities as restricting the amount of housing agents can buy or sell.\textsuperscript{22} More specifically, we assume that households incur convex adjustment costs:

\begin{equation}
q_t \frac{\eta}{2} (h_{it} - \bar{h}_i)^2.
\end{equation}

We specify the cost as deviation from the exogenous fixed targets $\bar{h}_i$ which are set equal to the long-run average of the per-capita housing stock held by agent $i$, so this parameter is specific to borrowers and savers, respectively. We choose this specification in order to keep the computational complexity of the model as simple as possible.\textsuperscript{23} We assume that agents pay the cost in terms of the value of the house rather than in terms of the size of the house but this is of minor importance for the quantitative implications. The key parameter is $\eta$. With a positive value of $\eta$, households incur adjustment costs whenever their housing stock differs from the long-run average. This means that agents adjust slower upwards as well as downwards. The cost increases with higher $\eta$. Note that - even though we assumed that $\eta$ is the same for both groups of agents, effective adjustment costs might still differ for borrowers and savers because borrowers in addition face the collateral constraint. We set $\eta$ such that the drop in housing wealth for borrowers and savers in the model are close to the sample moments from the SCF reported in table 1, while leaving all other parameters at the benchmark calibration. This procedure implies a value of $\eta = 1.5$. The targets $\bar{h}_i$, $i = b, s$ are set so that adjustment costs for both household types are zero on average.\textsuperscript{24}

In figure 3 we compare the simulated Great Recession for the model with adjustment

\textsuperscript{21}See, for example, Landvoigt, Piazzesi, and Schneider (2012) and Justiniano, Primiceri, and Tambalotti (2014).

\textsuperscript{22}Housing adjustment costs are a standard assumption in structural models of housing demand, see Bajari, Chan, Krueger, and Miller (forthcoming).

\textsuperscript{23}Ideally, one would like to use a more standard adjustment cost function as $q_t \frac{\eta}{2} (h_{it} - h_{it-1})^2$. This formulation, however, would introduce an additional state variable so that the borrowers’ wealth share is no longer a sufficient statistic for the wealth distribution across agents. As a consequence, we could no longer exploit the wealth recursive structure of the model to compute the equilibrium of the economy.

\textsuperscript{24}This yields values of $\bar{h}_b = 0.5208$ and $\bar{h}_s = 1.5191$, respectively.
Figure 3: Great Recession: benchmark (solid) versus adjustment cost (dashed)
cost (dashed lines) to the benchmark case (solid lines). From panel (c) we notice that in the model with adjustment costs, house prices are slightly higher in the boom previous to the recession and they decline less on impact of the recession, as compared to the benchmark. The substantial difference between the two models, however, is that, in the adjustment cost case, the equilibrium interest rates go down in the Great Recession and even become negative on impact (see panels (k) and (l)). These dynamics can be rationalized by the following considerations. In the model with adjustment costs, borrowers cut down less in housing compared to the benchmark because it is costly to do so. They rather prefer to cut back non-durable consumption because they can avoid the adjustment cost. In fact, as shown in panel (i) borrowers’ non-durable consumption decreases more in the adjustment cost case compared to the benchmark case. In equilibrium this means that savers have to reduce non-durable consumption less in the adjustment cost case compared to the benchmark. In other words, they will consume relatively more today compared to the benchmark case. Ceteris paribus, to support this in equilibrium, real interest rates on deposits have to fall, so that savers find it optimal to reduce non-durable consumption today relatively less as they would do in absence of adjustment costs.

These considerations also help us to understand why borrowers - in the model with adjustment cost - stay leveraged during the Great Recession and only de-leverage one year later, as shown by the dashed line in panel (d) of figure 3. The reason is that the spread between the borrowing and lending rate is determined solely through the zero profit conditions of financial intermediaries (see equation (5)); so if the interest rates on deposits falls, the interest rate on loans increases less in the model with adjustment cost compared to the benchmark case (in fact, here it even becomes negative). This, in turn, makes it easier for borrowers to roll over their outstanding debt and stay leveraged. They do so up to the collateral constraint. This constraint is less tight than in the benchmark case because house prices decrease less and borrowers reduce their housing stock by a smaller amount, so aggregate debt and savings stays high on impact. As a result, the magnitude of the redistribution of housing across agents is smaller both before
and following the recession.

| Table 4: Welfare effects of the US Great Recession |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                  | $\Delta q$      | $\Delta (qh_b)$ | $\Delta (qh_s)$ | $\Delta \omega_b$ | $\lambda_b$ | $\lambda_s$ |
| SCF Data         | -15.9           | -19.0            | -12.5            | ?                | ?            | ?            |
| Welfare gains relative to pre-recession: Great Recession |
| Benchmark        | -16.3           | -28.5            | -11.2            | -2.7             | -3.7         | -1.0         |
| Adjustment cost  | -13.9           | -19.8            | -11.6            | -2.1             | -3.3         | -1.1         |
| Welfare gains relative to pre-recession: Low spreads |
| Benchmark        | -13.2           | -22.8            | -9.1             | -2.1             | -3.2         | -1.2         |
| Adjustment cost  | -10.3           | -15.6            | -8.3             | -1.5             | -3.0         | -1.2         |

Notes: Column one shows the change in aggregate housing wealth between date $-1$ and date 0. Columns two and three tabulate the percentage change in housing wealth between date $-1$ and 0 for borrowers and savers, respectively. Column four tabulates the absolute change of the borrowers' wealth share between date $-1$ and date 0. Columns five and six show the welfare gains of the recession in total consumption equivalents (relative to expected utility in period $-1$) for borrowers and savers, respectively. The Great recession is defined as a contemporaneous drop in income and financial intermediation in period 0. The low-spreads case assumes that financial intermediation is low in both periods $-1$ and 0.

Finally, what implications do the previous considerations have for the welfare effects? A comparison between the second and third row of table 4 shows that the welfare gains for borrowers are less negative than compared to the benchmark case, while savers experience higher welfare losses. This implies that in the model with adjustment costs there is less re-distribution of the Great Recession.

The comparison of the Great Recession with the low-spread scenario in the model with adjustment costs (third and fifth row of table 4) confirms the finding of the benchmark simulations that a negative financial intermediation shock redistributes housing among agents and results in higher welfare losses for borrowers and smaller welfare losses for savers. The magnitude of this re-distribution is however quantitatively lower in the model with adjustment costs (though still quantitatively significant); this relates to the above mentioned ability of borrowers to roll over their debt on impact and de-leverage slower afterwards in the model with adjustment costs, which is not possible in the benchmark
In this respect, the model with adjustment cost gets closer to a model with always binding collateral constraint; however, a model with always binding collateral constraint would imply a constant aggregate leverage over time, that is not what we observe in the US Great Recession. In particular, in the model with adjustment cost, although changes in housing consumption become costly, we still find a quantitative role for changes in leverage in shaping the welfare of agents. Consequently, a model with always binding collateral constraint (e.g. as in Iacoviello, 2005) would miss the quantitative relevance of the redistribution implied by financial intermediation shocks.

4 Conclusion

Using a dynamic general equilibrium model calibrated to the US economy, we evaluate the welfare effects of the Great Recession for leveraged (borrowers) and un-leveraged agents (savers). This analysis complements recent contributions (Glover, Heathcote, Krueger, and Ríos-Rull, 2014; Hur, 2012) and adds a new mechanism stemming from shocks to financial intermediation. Our set-up is well suited for the evaluation of the welfare consequences of credit supply shocks in a recession, and explores the effects of financial intermediation shocks in a model with endogenous collateral constraints.

Our simulation results show that in the Great Recession, modeled as a simultaneous shock to aggregate income and financial intermediation, all agents in the economy experience a welfare loss, and borrowers lose more than savers. This finding comes from the fact that savers, being unconstrained, change their portfolio allocations and smooth the negative shock by buying the deflated asset (housing). We find that, as a consequence of the negative intermediation shock that resulted in high credit spreads in the Great Recession, borrowers’ de-leveraging amplified the re-distribution from savers to borrowers and this translated into higher welfare losses for borrowers; we estimate that borrowers would have lost 10 to 20 percent less in terms of welfare if mortgage spreads would have
stayed low during the 2007-2009 recession, while savers would have lost the same amount more.

Finally, we show that the observed moments from micro data can be reconciled with our view of the Great Recession as a joint income and financial intermediation shock if we introduce housing adjustment costs to the model. The main findings regarding the households’ welfare remain unchanged while we find slightly smaller redistribution between borrowers and savers in this case.

Our paper focuses on the welfare effects of the Great Recession on borrowers and savers while abstracting from the possibility that borrowers can default on their debt obligations. While this could potentially benefit borrowers at the expense of their creditors (savers), empirical evidence suggests that this feature of the U.S. Great Recession was restricted to a subset of borrowers, the sub-primers, who are not explicitly modeled here. Adding this third form of heterogeneity to the analysis is, in our opinion, an interesting avenue for future research.

References


Appendix

A Data

The following series have been used in Figure 1: the federal funds rate, the one year mortgage interest rate (released by the Primary Mortgage Market Survey by Freddie Mac), mortgage (defined as home mortgages from the balance sheet of U.S. households and nonprofit organizations) and real estate (defined as the market value of real estate from the balance sheet of U.S. households and nonprofit organizations). All series are at yearly frequency. The source is Federal Reserve System, flow of funds data. The series for house prices is the National Composite Home Price Index for the United States (the release is by S&P/Case-Shiller Home Price Indices). In the calibration section, we calculate housing wealth as percentage of US nominal GDP (yearly) by using historical data of the flows of funds tables from the Board of Governors. US nominal GDP is from the Bureau of Economic Analysis. Our definition of housing wealth includes the market value of real estate belonging to households, non-profit and non-financial non-corporate business. The micro-data used for the calibration of the relative wealth distribution of borrowers and the leverage ratio are provided by the 2007-2009 panel of Survey of Consumer Finances (SCF). Unfortunately, the SCF does not provide information on the precise date at which
households were interviewed. Consequently, we assume that the observed portfolios in 2009 reflect the distribution of household net worth at the end of the recession. Surveyed households have been partitioned into borrowers and savers depending on their net asset position. The net asset position is defined as the sum of savings bonds, directly held bonds, the cash value of life insurances, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts minus the debt secured by primary residence, the debt secured by other residential property, credit card debt balance, debt lines on primary residences and other forms of debt. If the net asset position is positive, we consider the household to be a saver in our model economy, otherwise we consider the household to be a borrower. The reason to use a broad definition of aggregate deposits and debt in the data counterpart is that it is difficult to target borrowers and savers by strictly restricting attention to particular classes of debt. We moreover define net wealth per capita as the sum of the net asset position and the value of the primary residence and other residential properties, for both net borrowers and net savers. Finally, we aggregate the net wealth of both groups (borrowers and savers) and we calculate the relative net wealth of borrowers as the ratio between their net wealth over the total net wealth in the economy. The leverage ratio of the borrowers is instead obtained as the weighted average mean (using SCF sample weights) of the net asset position over the value of primary and secondary residences. The reference values that are matched by the model are obtained by cutting the 5% tails of the distribution of net worth (as defined in SCF) and by excluding households that show zero housing wealth (which are most likely the renters). This is done to cut the extreme observations that may bias the average values of net worth in the US economy. We want, in fact, to avoid the possibility of including in the range of borrower households that maintain large positions in the stock or housing markets and hold little savings.
B Numerical Details

This appendix describes the numerical solution of the equilibrium policy functions for the competitive equilibrium. The algorithm employed is an adoption of the time-iteration procedure with linear interpolation used in Grill and Brumm (2010). As we have only two agents, a fine grid for wealth is enough to deliver satisfactorily small Euler errors. For this reason, we do not adapt the grid around the points where the collateral constraint is binding, as proposed by Grill and Brumm (2010).

B.1 Equilibrium conditions

We want to describe the equilibrium in our economy in terms of policy functions that map the current state into current policies. Furthermore, we want to focus on recursive mappings - that is, time-invariant functions that satisfy the period-by-period first-order equilibrium conditions. In what follows, we characterize these equilibrium conditions in every detail. For each agent \( i = b, s \), denote by \( \nu_i(w, z) \) the Lagrange multiplier with respect to her budget constraint and by \( \phi_i(w, z) \) the Kuhn-Tucker multiplier attached to her collateral constraint. In addition, we treat saving and debt as two separate assets: saving is an asset in which the agent can only take long positions, \( s_i \geq 0 \); debt (loans) is an asset with return \( R_L \) in which agents can only take short positions, \( l_i \leq 0 \). Denote the Kuhn-Tucker multipliers attached to these inequalities as \( \chi_i \) and \( \mu_i \), respectively. Then, for each tuple consisting of wealth and exogenous state today \( \sigma = (w, z) \), the (time-invariant) policy and pricing functions have to satisfy the following system of equations (we will show below how to solve for these time-invariant functions):
• Agent’s first order conditions

\[ u_1(c_i(\sigma), h_i(\sigma)) - \nu_i(\sigma) = 0 \]

\[ u_2(c_i(\sigma), h_i(\sigma)) - q(\sigma)\nu_i(\sigma) = 0 \]

\[ -\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R(\sigma) + \chi_i(\sigma) = 0, \quad i = s, b \]

\[ -\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R_L(\sigma) + \phi_i(\sigma) - \mu_i(\sigma) = 0 \]

\[ -\nu_i(\sigma)q(\sigma) + u_2(c_i(\sigma), h_i(\sigma)) + \beta^i E[\nu_i(\sigma^+)]q(\sigma^+)|\sigma] + \phi_i(\sigma)mq(\sigma) = 0 \]

• Agent’s budget constraints

\[ y(s) + \Upsilon(\sigma) + \frac{\omega \cdot q(\sigma)}{n_b} - l_b(\sigma) - s_b(\sigma) - q(\sigma)h_b(\sigma) - c_b(\sigma) = 0 \]

\[ y(s) + \Upsilon(\sigma) + \frac{(1 - \omega) \cdot q(\sigma)}{n_s} - l_s(\sigma) - s_s(\sigma) - q(\sigma)h_s(\sigma) - c_s(\sigma) = 0 \]

NB: Here we have already used the definition for the borrower’s wealth share and rewritten the budget constraints in these terms (see the law of motion for wealth below as a reminder of how we defined the wealth share).

• Zero profits in the financial sector

\[ \theta(s) \cdot R_L(\sigma) - R(\sigma) = 0 \]

• Market clearing in housing and financial sector

\[ h_s(\sigma) + h_b(\sigma) - 1 = 0 \]

\[ l_b(\sigma) + l_s(\sigma) + \theta(s) \cdot (s_b(\sigma) + s_s(\sigma)) = 0 \]
• Transfers

\[ \Upsilon(\sigma) - (1 - \theta(s))(s_b(\sigma) + s_s(\sigma)) = 0 \]

• Complementary slackness conditions

\[
\begin{align*}
\mu_i(\sigma) &\geq 0, \quad d_i(\sigma) \geq 0, \quad \mu_i(\sigma) \perp d_i(\sigma) \\
\chi_i(\sigma) &\geq 0, \quad s_i(\sigma) \geq 0, \quad \chi(\sigma) \perp s_i(\sigma), \quad i = s, b \\
\phi_i(\sigma) &\geq 0, \quad CC_i(\sigma) \geq 0, \quad \phi_i(\sigma) \perp CC_i(\sigma)
\end{align*}
\]

where \( CC_i(\cdot) \) is the collateral constraint of agent \( i \), that is,

\[ CC_i(\sigma) \equiv l_i(\sigma) + mq(\sigma) h_i(\sigma) \geq 0 \]

• Implicit “Law of motion” for borrower’s wealth share

\[
w^+(\sigma, z^+) \equiv n_b \frac{R_L(\sigma)l_b(\sigma) + R(\sigma)s_b(\sigma) + q(w^+(\sigma, z^+), z^+)h_b(\sigma)}{q(w^+(\sigma, z^+), z^+)}.\]

B.2 Algorithm

The structure of the above period-by-period equilibrium conditions can be summarized as follows: Given a guess for the policy and pricing functions in the next period - denoted by \( f^{\text{prime}} \) - we can compute the expectations in the agents’ first order conditions. The functions that map current states to current policies - denoted by \( f \) - are then obtained by solving the static system of non-linear given in the previous subsection. More formally, the structure of the problem can be summarized as follows. For all tuples \( \sigma = (w, z) \), we have

\[
\psi(f^{\text{prime}}(\sigma, f(\sigma), \mu(\sigma)) = 0, \quad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.
\]

The system of equations \( \psi[f^{\text{prime}}](\cdot) \) contains first order conditions of agents and the
financial sector and market clearing conditions. The function $\zeta(\cdot)$ contains the sign restrictions and collateral constraints. $\mu(\cdot)$ denotes the respective Kuhn-Tucker multipliers. A recursive policy function $f$ then solves $\psi(f)(\sigma, f(\sigma)\mu(\sigma)) = 0$ such that the complementary slackness conditions are satisfied. The time iteration algorithm defined below finds the approximate recursive policy function iteratively.

In each iteration, taking as given a guess for $f^{\prime}$, we obtain $f$ by solving the above system of equations and then updating our guess by interpolating the obtained policy function on the implicitly defined next period wealth. The following box summarizes our algorithm in a form of Pseudo-code:

1. Select a grid $W$, an initial guess $f^{init}$ and an error tolerance $\epsilon$. Set $f^{\prime} = f^{init}$.

2. Make one time-iteration step:

   (a) For all $\sigma = (w, z)$, where $w \in W$, find the function $f(\sigma)$ that solves

   \[ \psi(f^{\prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \quad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0. \]

   (b) Use the solution $f$ and the guess $f^{\prime}$ to update wealth tomorrow and interpolate $f$ on the obtained values for wealth tomorrow.

3. If $||f - f^{\prime}|| < \epsilon$, go to step 4. Else set $f^{\prime} = f$ and repeat step 2.

4. Set numerical solution $\hat{f}$ equal to the solution of the infinite horizon problem, $\hat{f} = f$. 

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