Third parties as incentive to comply

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Abstract

Within an incomplete contract setting, the paper analyses the role of third parties in ameliorating incentive problems arising in the context of financial contracts with costly verification. Contrary to the findings of the bilateral lender-borrower relationship, characterised by no information revelation and a breakdown of the market, it is shown that, in the presence of third parties, an optimal contract exists and has partial information revelation. The importance of third parties is therefore not limited to improving efficiency, as it is when the contract offer comes from the informed party, but to ensure project realisation, and thus to ensure that the surplus that can arise from the project does not get lost.

**Keywords:** Bargaining power; commitment; monitoring

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References
1 Introduction

Several studies in recent years have focused on the design of the optimal principal-agent contract when the agent has private information which is costly to verify and the verification strategy is non-contractible. In these cases the revelation principle does not apply and the contract must be set so as to provide the incentive to monitor (Hart, 1995). The literature has proposed two ways to achieve this: 1. give the incentives to the principal to monitor, by fully reimbursing her of the verification cost incurred even when monitoring detects compliance, so as to induce the agent to truthfully reveal his cash flows (Jost (1996), Persons (1997)); 2. have the agent “misrepresenting” the true state with positive probability, thus using the possibility of punishing the agent for false reporting as an incentive to monitor (Khalil (1997), Persons (1997), Choe (1998), Khalil and Parigi (1998)). When the act of monitoring is not publicly observable, the first of these alternatives is not implementable (Menichini and Simmons, 2002). The incentive to monitor can thus arise only from the possibility of collecting a penalty for detected false reporting, and a contract inducing some misrepresentation in equilibrium will arise, i.e. a contract in which there is diversion of cash flows. The occurrence of this scenario, however, relies either on the assumption that the agent has all the bargaining power, or, when it is the principal who has the bargaining power, that the penalty for non-compliance is set exogenously and can exceed the agent’s total income, as in Khalil (1997).

This paper studies the effects on the properties of the optimal contract of giving all the bargaining power to the uninformed party, while setting the penalty for misreporting endogenously. In particular we study the properties of the contract offered by a risk neutral lender to a risk neutral borrower when the latter has private information about the return of a fixed size investment project which the lender can verify at a cost. We show that, under limited liability, the joint effect of lender’s bargaining power and contractual incompleteness rules out borrower’s compliance and leads to an equilibrium in which

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1To be viable, this route requires repayments to be contingent upon verification: the monitor is fully covered of the verification cost incurred when monitoring detects compliance and the contract induces full information revelation. However, if verification is non-observable or non-verifiable and there is no hard evidence in support of the monitor’s claim, the monitor will always claim to have monitored even if she has not in order to cash the reward for monitoring (the reimbursement of the verification cost). To prevent this, repayments following a truthful monitored low state report cannot be contingent upon monitoring.

2If the agent has the power to set the contract terms, he can hold the principal down to her reservation utility, keeping any residual left in truthful reporting. This increases both the expected cost of deception for the agent and the expected benefit of monitoring for the principal: because the principle of maximum deterrence holds, detected misreporting implies the loss of the entire surplus and its collection by the principal. When the punishment for misreporting is set exogenously and can exceed the agent’s total income, non-compliance involves a net cost for the agent.
the borrower’s reporting strategy is to always claim the worst state to have occurred, while the lender’s monitoring strategy depends only on the size of observation cost. When this is sufficiently high relative to the expected return from monitoring, the lender will no longer have an incentive to monitor and will get a payoff no higher than the worst state cash flows. This has dramatic implications when the lender is called to finance an investment project of fixed size, since the worst state cash flows fall short of the size of the loan: in such circumstances the project has negative NPV for the lender and will not be financed, thus causing a breakdown of the market and a consequential welfare loss. When the observation cost is sufficiently low, knowing that she can catch a lying borrower, the lender has still an incentive to monitor and get the exact repayment. Thus a contract with partial or no information disclosure and deterministic monitoring arises.

We argue that these equilibria are very costly and that it is possible to do better by involving a third party with the role of mitigating the agent’s incentive to lie, even when the monitor has all the bargaining power. In particular, we show that the incentive to cheat is reduced when a second risk averse financier, who has neither bargaining nor monitoring power, is called to cofinance the investment project. The tool for controlling this incentive is the structure of repayments. In the bilateral setup the borrower gets his reservation utility in all states with any rent entirely seized by the monitor. Introducing a third party allows the monitor to structure repayments so as to leave a rent to the borrower in truthfully reported non defaulting states. This increases the borrower’s incentive to comply and, through the higher premium for detected misreporting, the lender’s incentive to monitor, thus inducing partial information revelation and project realisation. Third parties can thus offset the negative effects on the investment decision of a given allocation of bargaining power.

The reason for these results has to do with the number of tools available to control incentives in each of the settings considered. In the bilateral setup, one instrument, the repayments to the monitor, is used to regulate two incentives, the lender’s incentive to monitor and the borrower’s incentive to comply. In particular, to boost the borrower’s incentives to disclose information any low state return is to be transferred to the monitor; however, this worsens her monitoring incentives, as she receives a sure repayment even if she decides not to monitor. Thus, stronger incentives for one side make it harder to provide incentives for the other. When a third party is called to participate to the venture, the repayments to her can be used to better control the incentives. In particular, the third party gets the full right to the low state return, improving both the lender’s incentive to monitor and the borrower’s incentive to comply, and thereby reducing the expected monitoring cost. This in turn liberates resources that can be used to reward the complying borrower, thus further improving reporting and monitoring incentives.

In studying the effects of the allocation of bargaining power on the properties of the optimal lender-
borrower contract when commitment to monitor is assumed away, this paper is close to Choe (1998).
Assuming costly misreporting - e.g. lying involves a falsification cost - he shows that only a misrep-
resentation contract can arise and that the allocation of the bargaining initiative only affects the efficiency
properties of the contract. Within a context in which the principal/lender designs the contract but the
agent/borrower is liable for an exogenous bounded penalty that can exceed his total income in case of
detected misreporting, Khalil (1997) also finds that a misrepresentation contract arises. An exception
in this scenario is Menichini (2001): assuming away falsification cost (as in Choe, 1998) and exogenous
penalty for misreporting (as in Khalil, 1997) and considering endogenous investment size, she finds that a
misrepresentation contract never arises, but only a pooling contract in which the borrower always cheats.
With respect to Menichini (2001), the main contribution of our work is to focus on the role of third
parties and consider not only mixed strategy and pooling equilibria, but all the feasible equilibria which
can be supported at the contracting stage. However, this is not the first paper that analyses multiple
lending relationships as a tool for mitigating incentive problems within this setting. Persons (1997) and
Menichini and Simmons (2002) have already taken up this issue, the former focusing on truth-telling con-
tracts, the latter on misrepresentation contracts. They both have pointed out the role that third parties
play in increasing efficiency, rather than ensuring project realisation, as the present paper. Moreover,
neither of them has looked at the interplay in the design of the optimal contract between the allocation
of bargaining power and multiple parties, as we do.

The paper is organised as follows. Section 2 sets up the model assumptions and the time-line of the
game. Section 3 presents the benchmark single investor contract, studying the implications of contractual
incompleteness and the allocation of bargaining power. Section 4 introduces third parties and studies
their role in ameliorating incentive problems. The last section concludes.

2 The Model Assumptions

A risk neutral borrower ($B$) has a fixed size project $I$ for which he needs finance from one or more lenders.
The outcome of the project, $f_s$, $s \in \{H, L\}$, with $s = H$ occurring with probability $p$, is the borrower’s
private information. We assume that $f_H > I > f_L$, and that all parties are protected by limited liability.
The project can be financed by two (groups of) lenders, with (endogenous) investment shares $\alpha$ and $1 - \alpha$
respectively: a monitoring one ($P_M$), that can rely at a cost $\phi$ on a monitoring technology to verify the
outcome of the project, and a non monitoring one ($P_{NM}$), who free-rides on the other lender’s audit.\footnote{Allowing for a single monitor only can be justified on grounds of efficiency. Within our setup, having a second monitor entails only a duplication of monitoring costs and therefore an efficiency loss (Diamond, 1984).}
The monitoring lender is indispensable: under no monitoring, the borrower can only be induced to reveal his private information by setting repayments which are non increasing in cash flows. Because of limited liability, this implies that in any state the highest total transfer lenders can get never exceeds $f_L$, which is less than the total investment outflow $I$. Anticipating that the project has negative NPV, no finance is ever provided by any investor.

The two lenders have different risk attitudes: the monitoring one is risk neutral, while the non monitoring one is risk averse. This can be interpreted in the light of the existing evidence on the use of multiple credit sources both in developed and LDC countries. For example, Petersen and Rajan (1994, 1995) present evidence on multiple credit sources for small businesses in the US: many firms borrow from more than one bank and take trade credit from sellers. In some cases they borrow both from local lenders and well established financial institutions. In LDC’s, credit markets are characterised by formal and informal credit (Hoff and Stiglitz, 1990 and 1997): private money-lenders and family-related informal financial arrangements interacting with banks and formal financial intermediaries. In either case, the various types of lenders have different characteristics in terms of market power, risk attitudes, monitoring ability, ability to raise funds or diversify loans. Although in a very simple and stylised way, our assumptions aim at capturing some of these differences.

**Monitoring technology**

Monitoring is non-verifiable. Thus, neither the act, nor the result of it is observable to third parties. To credibly communicate them, the monitor must be able to produce hard, informative evidence of the borrowers financial position. Because of non-verifiability, such evidence is only available if it differs from the report originally sent by the borrower, that is to say when the borrower reports a bad realisation of the state and the monitor finds that it is in fact good. In such cases the evidence displayed is a proof that monitoring has occurred and repayments are monitoring-contingent. When instead the report of the borrower and the result of monitoring do not differ, it is not possible to say whether monitoring did actually occur and differences in payments cannot be supported in equilibrium. Given non verifiability,

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4 A similar assumption based on lenders’ differential risk attitudes is used also by Bloise and Reichlin (2003).

5 It is not possible to falsify the evidence.

6 If they differed, lender and borrower might collude at the expense of the third party. In particular, if, following a report $\hat{L}$, low state repayments were contingent on claimed monitoring, the monitoring lender would collude with the borrower, claiming to have monitored when in fact she has not, if the repayment received under monitoring is higher than that received under no monitoring, and claiming not to have monitored otherwise.

Collusion issues aside, relaxing the hypothesis of non-verifiability, so as to make also low state repayments monitoring-contingent, would not change the results of the analysis in the bilateral setup: optimally these repayments would be set equal to each other. It would sharpen incentives instead in a multi-investors setup. Thus, these benefits would add up to
the actual frequency of monitoring is also non-observable, which implies that monitoring cannot be contracted upon, i.e. the lender cannot commit to any monitoring strategy at the contract time. This implies that truth-telling cannot be elicited and the lender has to be provided with the incentives to monitor.

**Contracts and repayments structure**

The monitoring lender $P_M$ has a monopoly power which gives her the possibility to set the contract terms. She offers a contract to both the borrower $B$ and to the non-monitoring investor $P_{NM}$, specifying the share of the loan provided by the non-monitoring lender and a set of contingent transfers from the borrower to the monitoring and non-monitoring lenders covering all verifiable states of the world.\(^7\)

The structure of the repayments is as follows. After revenues are realised, the borrower sends a report $\hat{s}$ and a proposed repayment to the lenders. When $s = H$, the borrower can tell the truth or cheat ($\hat{s} \in \{\hat{H}, \hat{L}\}$). If he tells the truth ($\hat{s} = \hat{H}$), which occurs with probability $(1 - l)$, the borrower never monitors\(^8\) and the repayments to $P_M$ and $P_{NM}$ are $R_H, w_H$ respectively. If he cheats ($\hat{s} = \hat{L}$), which occurs with probability $l$, $P_M$ can monitor with probability $m$. In this case the repayments to $P_M$ and $P_{NM}$ are $R_{HL} = R_H + \delta_R, w_{HL} = w_H + \delta_w$, respectively. When $s = L$, the borrower never cheats ($\hat{s} = \hat{L}$)\(^9\), and the transfer to the monitoring and non-monitoring investors are $R_L, w_L$ respectively.\(^10\) Thus, the reporting-monitoring subgame can only take place when the borrower reports low.

**Time-line**

1. $B$ privately observes his type $(H, L)$.

2. $P_M$ proposes a contract to both $B$ (who is fully informed) and $P_{NM}$. They can either accept or reject. If they both accept, they sign the contract and the game continues. If either of them refuses, the next stages of the game do not occur.

3. Upon revenues realisation, $B$ chooses which report to make and, conditional on the report received, $P_M$ decides whether to monitor or not.

4. The relevant transfers are made.

---

\(^7\) Alternative structures might be thought of in this context, which we will describe more thoroughly in the appendix. However, they can all be reconducted in the framework of section 3.

\(^8\) Monitoring a high state report is a dominated strategy for the monitoring investor.

\(^9\) When the low state occurs, it is a dominated strategy to declare high.

\(^10\) Because no hard evidence can be produced showing whether monitoring has taken place, the repayment to the monitoring investor cannot be conditioned on monitoring.
We solve the problem by backward induction. The expected returns to each party are:

\[ E\pi_{B|H} = (1-l)(f_H - R_H - w_H) + l[(1-m)(f_H - R_L - w_L) + m(f_H - R_H - \delta_R - w_H - \delta_w)] \]  
\[ E\pi_{B|L} = f_L - R_L - w_L \]  
\[ E\pi_M = p(1-l)R_H + (1-p+pl)(1-m)R_L + (1-p)m(R_L - \phi) + plm(R_H + \delta_R - \phi) - \alpha I \]  
\[ EU_{NM} = p(1-l)U(w_H - (1-\alpha)I) + plmU(w_H + \delta_w - (1-\alpha)I) + (1-p+pl(1-m))U(w_L - (1-\alpha)I) - \pi \]

where (1) and (2) are the borrower expected profits,\(^{11}\) (3) is the monitoring lender expected profits, and (4) is the non-monitoring lender VNM utility function defined on repayments conditional on each possible state.

### 2.1 The reporting-monitoring game

With non-contractible monitoring and the above repayment structure, repayments that induce truth-telling fail to give commitment to monitor. Instead, at the interim stage, \(l\) and \(m\) will be determined as a BNE of the game between the borrower and the monitoring lender.

From (1) and (3), the best responses of the borrower and the monitor are set by:

\[
\begin{align*}
    l &= 0 & \in [0,1] & \text{if } R_H + w_H - (1-m)(R_L + w_L) - m(R_H + \delta_R + w_H + \delta_w) \leq 0 \\
    l &= 1 & \\
    m &= 0 & \in [0,1] & \text{if } pl(R_H + \delta_R - \phi) - (1-p+pl)R_L + (1-p)(R_L - \phi) \leq 0 \\
    m &= 1
\end{align*}
\]

Among the possible equilibria, we focus here on a mixed strategy one, with random monitoring and partial information revelation. In the appendix we consider all the other possible equilibria relative to the single investor case. Of these, some are infeasible, as they involve the borrower always repaying \(R \leq f_L\), which is insufficient to repay the lenders' investment \(I\).\(^{12}\)

\(^{11}\)Notice that, because the borrower has private information at the contracting stage, his expected profits are set in ex-post terms.

\(^{12}\)A complete list of all equilibria for the case in which there is a single monitoring investor is provided by Simmons and Garino (2003) in a context in which it is the borrower and not the lender who has the power to set the contract terms and monitoring is observable. Because of these differences, the set of feasible equilibria is not the same across the two cases.
A Nash equilibrium in interior mixed strategies requires:

\[
\frac{\partial E\pi_B}{\partial l} = R_H + w_H - (1 - m) (R_L + w_L) - m (R_H + \delta_R + w_H + \delta_w) = 0 \quad (5)
\]

\[
\frac{\partial E\pi_M}{\partial m} = pl (R_H + \delta_R - R_L - \phi) - (1 - p)\phi = 0. \quad (6)
\]

Notice that in order to maximise the lender’s incentive to monitor \( \delta_w = 0 \), i.e. \( w_{HL} = w_H \). This is because the marginal benefit of monitoring (6) is increasing in \( \delta_R \), while the marginal benefit of lying (5) is decreasing in the total penalty for detected misreporting. Thus, while setting \( \delta_R \) as high as possible (and \( \delta_w \) as low as possible) has a neutral effect on (5), it makes the lender strictly prefer to monitor requiring \( l \) to fall to restore indifference.

Setting \( \delta_w = 0 \), the interior mixed strategy is defined by:

\[
l = \frac{(1 - p)\phi}{p(R_H + \delta_R - R_L - \phi)}; \quad (7)
\]

\[
m = \frac{R_H + w_H - R_L - w_L}{R_H + \delta_R + w_H - R_L - w_L}. \quad (8)
\]

Then, if it is optimal to have a mixed strategy, the following conditions must hold: \( m \in [0,1] \) requires \( \delta_R > 0 \) and \( R_H + w_H > R_L + w_L \); \( l \in [0,1] \) requires \( R_H + \delta_R - R_L > \frac{\phi}{p} \).

We can use the equilibrium strategies to solve for the optimal contract: anticipating time three probabilities of lying and monitoring (7) and (8), the monitoring lender \( P_M \) chooses repayments to maximise her expected profits, subject to the borrower and to the non-monitoring investor participation constraints ((1), (2) and (4)), and to the limited liability condition \( f_H \geq R_H + w_H + \delta_R \).

### 2.2 The contract problem

Using the indifference conditions (5) and (6) (and \( \delta_w = 0 \)), the optimal contract conditional on a mixed strategy equilibrium is the solution to the following programme \( P_{M*} \): 

\[
\max_{\alpha,R_H,R_L,\delta_R,w_H,w_L} \quad p(1 - l)R_H + (1 - p + pl)R_L - \alpha I \quad (9)
\]

\[
\text{s.t.} \quad f_H - w_H - R_H \geq 0 \quad (10)
\]

\[
f_L - w_L - R_L \geq 0 \quad (11)
\]

\[
f_H - w_H - R_H - \delta_R \geq 0 \quad (12)
\]

\[
\mu U (w_H - (1 - \alpha) I) + (1 - \mu) U (w_L - (1 - \alpha) I) \geq \bar{u} \quad (13)
\]

where \( \mu = p - pl + plm \), (10) and (11) are the ex post participation constraints for the borrower, (12) is a limited liability condition for the borrower,\(^{13}\) (13) is the participation constraint for the non-monitoring

\(^{13}\text{This assumption (Sappington, 1983) plays a similar role as the borrower’s risk aversion. It implies that, in spite of risk neutrality, a first best contract is not implementable.}\)
lender and \( l \) and \( m \) are the equilibrium probabilities of lying and monitoring as defined by (7) and (8).

The solution to this programme is postponed until section 4. In the next section we consider the benchmark case in which there is a single monitoring investor.\(^{14}\)

3 The benchmark: the single investor contract

In this case the monitor entirely finances the project. We thus set \( w_H = w_L = 0, \alpha = 1,\)\(^{15}\) and get the following programme \( P_{SI} \):

\[
\begin{align*}
\max_{R_H, R_L, \delta R} & \quad p(1-l)R_H + (1-p + pl)R_L - I \\
\text{s.t.} & \quad f_H - R_H \geq 0 \\
& \quad f_L - R_L \geq 0 \\
& \quad f_H - R_H - \delta R \geq 0
\end{align*}
\]

where the constraints have the same meaning as in programme \( P_{SI} \) and the probabilities of lying and monitoring are defined as \( l = \frac{(1-p)\phi}{p(R_H + \delta R - R_L - \phi)}, m = \frac{R_H - R_L}{R_H + \delta R - R_L}. \)

We first see that the low state participation constraint binds: if not, the monitor could raise \( R_L \) to her benefit.\(^{16}\) Hence \( R_L = f_L \). Moreover, the objective function is increasing both in \( \delta R \) and \( R_H \), but it increases faster in \( R_H \).\(^{17}\) The participation constraint (15) sets however a limit on the size of \( R_H \): it cannot exceed \( f_H \), which implies, using (17), that the problem involves a corner solution with \( \delta R = 0 \), as shown in figure 1:

\[\text{Figure 1: } \delta_R = 0\]

\(^{14}\)This case has been studied by Menichini (2001) within a setup with endogenous project size.

\(^{15}\)This case corresponds to the decentralised contractual structure represented in figure 2 in appendix.

\(^{16}\)The objective function (14) is increasing in \( R_L \) \( \frac{\partial \text{obj}}{\partial R_L} = 1-p+pl-p(R_H - R_L) \frac{\partial l}{\partial R_L} = 1-p+pl+\frac{pl(R_H - R_L)}{(R_H + \delta R - R_L - \phi)} > 0. \)

\(^{17}\)A rise in \( \delta R \) has two effects on the objective function: a direct one - the objective varies by \( p(1-l) \), the probability of receiving a high state report - and an indirect one, through the effect that such a rise has in reducing the probability of lying. A rise in \( \delta R \), instead, affects only indirectly the objective function by reducing in the probability of lying. Thus:

\[
\frac{\partial \text{obj}}{\partial R_H} = p(1-l) + \frac{pl(R_H - R_L)}{R_H + \delta R - R_L - \phi} > \frac{pl(R_H - R_L)}{R_H + \delta R - R_L - \phi} = \frac{\partial \text{obj}}{\partial R_L}.
\]
If $\delta_R = 0$, lying is costless, which, under the assumption that the lender adopts a random monitoring strategy, makes the borrower strictly prefer to lie: the marginal benefit of lying is strictly positive - (5) becomes $(1 - m) (R_H - R_L) > 0$ - which implies that $l = 1$. Thus, no interior mixed strategy is possible in this case.

The only other feasible equilibria which can be supported at the contracting stage, so long as the observation cost is not too high, are the following:\(^{18}\)

- $m = l = 1$, with observation cost $\phi$;
- $m = 1$, $0 < l < 1$, with expected observation cost $(1 - p + pl) \phi$.

We thus derive the following result:

**Proposition 1** Under the described forms of contractual incompleteness and lender’s bargaining power, for sufficiently low observation cost, only contracts with deterministic monitoring are implementable with partial or no information revelation. No other contract is implementable for higher observation cost.

Partial information revelation can thus only be elicited under deterministic monitoring. The intuition behind this result is the following: by designing the contract, the lender can hold the borrower down to his reservation utility in each state. This reduces the opportunity cost of misreporting and increases the incentive to cheat: the only way for the borrower to get a rent is by lying, independently of the lender’s decision to monitor. When the observation cost is not too high, anticipating that she will always catch a lying borrower, the lender has still an incentive to monitor and get the exact repayment. When the observation cost is higher than the expected return from monitoring, the lender has no longer an incentive to monitor. Anticipating that she will never recoup the investment outlay,\(^{19}\) there is no contract that she is willing to offer to the borrower and hence a breakdown of the market.

This result crucially depends on the assumption of fixed project size and on endogenous punishment for misreporting. In particular, if the size of the loan was determined endogenously, then $I$ could be chosen so as to ensure the feasibility of other equilibria,\(^{20}\) although not of a mixed strategy one. If instead the punishment for misreporting was imposed exogenously, for example by asking the entrepreneur to provide a collateral as a guarantee for the loan received, it could be even possible to restore a mixed strategy equilibrium. Because the collateral could only be seized if monitoring occurs, the lender could not substitute the punishment for misreporting ($\delta_R$) with a higher payoff for truthful high state reports.

\(^{18}\) The full derivation of the results is in the appendix.

\(^{19}\) If the borrower reports low and the lender never monitors, the most she gets is $f_L < I$.

\(^{20}\) Menichini (2001) has discussed the case of endogenous project size in which $I$ is reduced sufficiently so as to ensure the feasibility of a single repayment (pooling) contract paying $f_L(I)$ in each state.
(\(R_H\)), as when the punishment is endogenous. This would restore the lender’s incentive to monitor and mitigate the entrepreneur’s incentive to lie.

We propose here an alternative way-out to restore the monitoring incentives and elicit information revelation, thereby increasing efficiency: allow for multiple investors.

4 Multiple investors

This section shows that, even with endogenous penalty for misreporting, the introduction of a second investor allows the monitor to implement a contract with random monitoring and partial information revelation dominating any two-party contract with deterministic monitoring.

Solving programme \(P_{MI}\), we derive the following proposition:

**Proposition 2** In the presence of third uninformed parties, the optimal contract conditional on a mixed strategy Nash equilibrium is characterised by:

- positive share of finance provided by each lender: \(\alpha > 0\);
- maximum punishment to the borrower in non-truthful audited states (\(R_{HL} = R_H + \delta_R = f_H - w_H\)); positive rent in high reported states (\(f_H - R_H - w_H > 0\)) and zero rent in low reported states, if audited or not (\(f_L = R_L + w_L\));
- negative correlation between repayments to each lender, i.e. \(R_H > R_L = 0\) and \(w_H < w_L = f_L\).

**Proof.** In Appendix. ■

With a second non monitoring lender who can become informed following the other lender’s audit, the strategies chosen by the parties involve random monitoring and partial information revelation. It is then still possible to prevent the borrower from always cheating by rewarding him for compliance.

These results are striking in the light of those obtained in Proposition 1 where it is shown that, in a bilateral setup, information can be disclosed only by monitoring deterministically, and thus at very high observation cost. The reason for these results is that there are not enough tools to control all incentives. One instrument, the repayments to the monitoring investor \(R_s\), is used to provide the incentive to monitor, the incentive to comply and to meet the investor’s participation (ensure non-negative profits): in particular, \(R_{HL} = R_H + \delta_R\) is used to provide the incentive to monitor and the incentive to comply, while \(R_H, R_L\) are used to meet participation. However, while an increase in \(R_H\) does not affect the incentive to monitor,\(^{21}\) an increase in \(R_L\) has an adverse effect on it. Thus \(R_L\) should be reduced to

\(^{21}\)In detected low states the monitor is interested in \(R_{HL} = R_H + \delta_R\).
boost the monitoring incentive, but this is not a viable solution as it not only reduces the investor’s profits, but also increases the return from cheating (marginal benefit of lying).\(^{22}\) Thus, stronger incentives for one side make it harder to provide incentives for the other.

When a third party is called to participate to the venture, the repayments to her can be used to better control the incentives. The monitor can reduce her financial participation and thus lower her stake into the venture. This implies that \(R_H\) and \(R_L\) can be lowered to meet the non monitoring investor participation, while still ensuring non-negative profits to the monitor. In particular, because the incentive to monitor (6) decreases in \(R_L\),\(^{23}\) while the incentive to lie (5) increases in it, \(R_L\) can be reduced relative to \(f_L\) and the saving can be used to repay the non monitoring investor in low states \((w_L)\). Limited liability and the monitor’s risk neutrality imply then that \(R_L\) can be set to zero and thus \(w_L = f_L\). Analogously, \(R_H\) can be lowered and \(w_H\) can be increased. Setting \(w_H < w_L\) increases the incentive to monitor and requires a lower \(l\) to keep the monitor’s reaction function unchanged.\(^{24}\) For a given reservation utility equal to the rent obtained in the bilateral setup, the reduction in \(l\) decreases the expected monitoring costs \(m (1 - p + pl) \phi\), liberating resources that can be used to reward the complying borrower \(\delta_R = f_H - R_H - w_H > 0\). This implies that the initial decrease in \(R_H\) is only partially compensated by the rise in \(w_H\). Through the rise in \(\delta_R\), compliance becomes more attractive, thus making the equilibrium value of \(m\) fall, further increasing the saving in expected monitoring costs and making even more resources available to raise the rent for compliance/penalty for misreporting. Thus, the presence of the third party acts as a commitment for the monitor not to extract all the rent from the borrower.

A last remark for why the share of finance provided by each party is strictly positive. We cannot have the monitor financing the whole project \((\alpha = 1)\) since non-trivial three-party contracts require the pure investor getting positive transfers in each state of nature, which is costly to the monitor. This is not the case if the pure investor cofinances the project with a share equal to the expected payoff she is paid to create incentives. However, the burden of financing the project cannot entirely fall on the third party either \((\alpha = 0)\), given that optimally the contract has negative correlation between repayments, which

\(^{22}\)A reduction in \(R_L\) affects negatively the investor’s profits via two channels: 1) directly, as it increases the return from cheating \(f_H - R_L\) and tightens the incentive constraint; 2) indirectly, as a lower \(R_L\) requires a higher \(R_H\) to remunerate the monitoring investor for the capital provided, thus lowering the reward for compliance and further tightening the incentive constraint. The tighter incentive constraint calls for a higher \(m\) to restore indifference, thus causing an increase in expected observation cost.

\(^{23}\)\(R_L\) can be seen as the opportunity cost of monitoring: whenever the supervisor receives a low state report and decides to monitor she gives up a sure payoff of \(R_L\) and incurs a direct cost \(\phi\).

\(^{24}\)What is important in our setup to boost incentives is that \(w_H\) is strictly less than \(w_L\). If \(w_H \geq w_L\), the positive effect on incentives of not repaying the monitor in unmonitored low states \((R_L = 0)\) is entirely or more than entirely offset by the lower repayment in monitored states \((R_{HL} = R_H + \delta_R = f_H - w_H)\).

11
implies that $w_H < w_L = f_L < I$, thus violating the participation constraint. Hence $\alpha$ has to be strictly positive.

Having derived the properties of the three-party contract, we are left with comparing the welfare properties of this contract form with either of the bilateral ones listed in Proposition 1. This has lead to the following proposition:

**Proposition 3** The two investor contract dominates either of the bilateral contracts with $m = 1$ and $0 < l \leq 1$, as it involves unambiguously lower observation cost than either of them.

**Proof.** In appendix.

This proposition shows that the only feasible bilateral contracts, when they are implementable, i.e. for sufficiently low observation cost, are dominated by a three-party contract involving higher information revelation and lower monitoring probability. Thus, the involvement of third parties into the venture gives a result which is at least Pareto superior to either of the single monopolistic lender contracts listed in Proposition 1. When these are not implementable, third parties not only increase efficiency, but ensure project realisation.

5 Concluding Remarks

We have studied the properties of the contract among an informed borrower and one or two uninformed lenders, one of whom *is* endowed with the right to audit, under limited liability, non-verifiable and non-contractible monitoring. Within a setting in which the bargaining initiative is held by the monitoring lender, the novelty of the paper has been to allow for multiple financing sources.

Taking as a benchmark the model with a single monopolistic lender, in which information revelation can never be elicited because of the inability of the lender to commit not to extract all the rent from the borrower, we have shown that third uninformed lenders with no monitoring and no bargaining power can ameliorate this incentive problem: in particular, the monitoring lender can “use” the non-monitoring one and set the transfers so as to reduce their spread across states, thus leaving the borrower with some rent in truthful high state reports. This mitigates the borrower’s incentive to cheat and the lender’s incentive to audit, thus restoring partial information revelation. As a consequence of the smoothing out

---

25 For example because the observation costs are too high for $m = 1$ to be optimal.

26 One may wonder whether reducing project size so as to ensure the feasibility of a single repayment contract with $l = 1$ and $m = 0$ dominates employing a third party. This really depends on observation cost and on the degree of risk aversion of the third party. Although interesting, the issue is beyond the point of this study, which has analysed the impact of a given allocation of bargaining power when the investment size is fixed.
of repayments, we also find that the two lender scenario is Pareto superior to the single monopolistic lender contract, when this exists, i.e. when the observation cost is not too high to rule out also a contract with deterministic monitoring. When the observation cost is so high relative to the expected return from monitoring that the lender has no longer an incentive to monitor, there is a breakdown of the market. In such circumstances, the importance of the third party is not limited to improving efficiency, as it is when the contract offer comes from the informed party, but to ensure project realisation, and thus to ensure that the surplus that can arise from the project does not get lost.

The results show the crucial importance of the allocation of the bargaining initiative in shaping the lender-borrower relationship and gives to third parties the role of improving the incentives for compliance and monitoring. Of course, as the problem arises from the interplay of the lender’s bargaining power and her inability to commit to monitor, various other devices could be used in alternative to (or jointly with) multiple financing sources to improve incentives. Increasing the transparency of the audit procedures could be a useful step ahead; asking the entrepreneur to provide a collateral as a guarantee for the loan received, or increase the competition among monitoring lenders, would be other viable, and maybe more direct routes.

Finally, we have not analysed collusion problems arising in this setting. Generally the costly state verification literature has focused on those arising when a third party, usually a supervisor, is used to perform the monitoring (Tirole (1986), Kofman and Lawarrée (1993), Strausz (1997)). In our context, collusion might arise between the monitor and the borrower: the first might give up her audit right in exchange for a higher repayment at expense of the other lender. However, the particular structure of repayments, namely the negative correlation between repayments, implies that such an agreement is never going to hurt the non monitoring lender, thus strongly limiting the scope for collusion. Of course other agreements might be thought of, but we leave the analysis of this and other interesting cases for future research.

A Appendix

A.1 Alternative contractual structures

The paper has analysed the case in which there is a monopolistic lender who offers a contract to a borrower and to a non-monitoring lender (centralised contractual structure in figure 2). Alternative structures might be thought of in this context: in the first structure (a in fig. 2), the monitoring lender acts as an intermediary between the borrower and the depositors, by collecting funds from depositors and
lending them to the borrower. Because at the contracting stage only the borrower has private information, the intermediary and the depositor contract under symmetric information. Since the intermediary is risk neutral while depositors are risk averse, there is perfect risk sharing and depositors get a flat return across states. Asymmetric information has therefore no impact on depositors but only on intermediaries who, having raised capital from depositors, contract with ex ante informed borrowers. This case has been analysed in section 3.

The second setup (structure b in fig. 2) is one in which the monitoring lender contracts with the borrower who in turns contracts with the non-monitoring lender. If monitoring is unobservable, the only way for the borrower to persuade the non-monitoring lender to grant him credit is by proposing her a contract with repayments non-increasing in cash flows.²⁷ The transfer that maximises the share of finance provided by the non-monitoring lender is a flat transfer equal to low state cash flows \( f_L \). The residual is therefore provided by the monitor and the problem becomes again one of studying the financial contract between the monitoring lender and the borrower as the one that has been studied in section 3, with the peculiarity that \( f_L \) is zero, as it is seized by the non-monitoring lender, and the investment provided by the monitor is less than \( I \).

The only other possibility is then to have the monitor offering a contract to both the borrower and the non-monitoring lender (centralised structure in fig. 2). As shown in section 4, the results of this contractual structure are in sharp contrast with those of the previous two, as summarised in section 3.

²⁷ Otherwise the borrower would always claim the state to be low.
A.2 The equilibria of the reporting-monitoring game

To show which are the feasible equilibria of the reporting-monitoring game between the lender and the borrower, we use (1) and (3) setting \( w_H = w_L = \delta_w = 0 \).

(i) \( l = 0, 0 < m \leq 1 \) requires \( \frac{\partial E\pi}{\partial m} \big|_{l=0} = - (1 - p) \phi \geq 0 \) to give \( m \) positive, which is impossible.

(ii) \( m = l = 0 \) requires \( \frac{\partial E\pi}{\partial l} \big|_{m=0} = p(R_H - R_L) < 0 \) to give \( l = 0 \). This leads to \( R_H < R_L \leq f_L \), which falls short of the size of the loan as \( f_L < I \). Anticipating this, no contractual agreement can be reached.

(iii) \( m = 0, 0 < l < 1 \) requires \( \frac{\partial E\pi}{\partial l} \big|_{m=0} = R_H = R_L \leq f_L \), which implies that the borrower pays out at most \( f_L \) in any state. Again, this does not suffice to repay the loan as \( f_L < I \) and thus no contract will be signed.

(iv) \( m = 0, l = 1 \) implies that, by lying all the time and never being monitored, the borrower pays out at most \( f_L \) in any state, and thus no contractual agreement can be reached.

(v) \( l = 1, 0 < m < 1 \) requires

\[
\frac{\partial E\pi}{\partial l} \big|_{0 < m < 1} = R_H - (1 - m) R_L - m (R_H + \delta_R) > 0 \\
\frac{\partial E\pi}{\partial m} \big|_{l=1} = p (R_H + \delta_R - R_L) - \phi = 0
\]

This equilibrium cannot be supported at the contracting stage either. Using \( l = 1 \), the lender’s problem reduces to:

\[
\max_{R_H, R_L, m} (1 - m) R_L + (1 - p) m (R_L - \phi) + pm (R_H + \delta_R - \phi) - I \\
f_H - (1 - m) R_L - m (R_H + \delta_R) \geq 0 \\
f_L \geq R_L, \quad R_H > R_L
\]

and \( p (R_H + \delta_R - R_L) = \phi \). Using this, we get:

\[
\max_{R_H, R_L, m} R_L - I \\
f_H - (1 - m) R_L - m (R_H + \delta_R) \geq 0 \\
f_L \geq R_L, \quad R_H > R_L
\]

As for the case in which \( m = 0 \) and \( l = 1 \), this equilibrium is infeasible. From the objective, because the low state repayment is no higher than \( f_L \) and \( f_L < I \), the payoff to the lender is negative, which implies that no contractual agreement can be reached under these circumstances.
(vi) \( l = m = 1 \) requires
\[
\frac{\partial E_{\pi_B}}{\partial m} \big|_{m=1} = \delta_R > 0
\]
\[
\frac{\partial E_{\pi_M}}{\partial m} \big|_{l=1} = p (R_H + \delta_R - R_L) - \phi > 0.
\]
Using \( m = l = 1 \), the problem becomes
\[
\max_{R_H, R_L, m} p (R_H + \delta_R) + (1 - p) R_L - \phi - I
\]
\[
f_H - R_H - \delta_R \geq 0
\]
\[
f_L \geq R_L
\]
Because the terms of the contract are such that the borrower always cheats and the lender always monitors, the lender either gets the low state payoff \( (R_L) \), or the penalty for misreporting \( (R_H + \delta_R) \). The borrower receives no rent in either state, otherwise the monitor could raise \( R_H + \delta_R \) and \( R_L \) to her benefit. The expected payoff to the monitor is therefore given by \( p f_H + (1 - p) f_L - I - \phi \), with observation cost \( \phi \).

(vii) \( m = 1; 0 < l < 1 \) requires
\[
\frac{\partial E_{\pi_B}}{\partial m} \big|_{m=1} = -\delta_R = 0
\]
\[
\frac{\partial E_{\pi_M}}{\partial m} \big|_{0 < l < 1} = pl (R_H + \delta_R - R_L) - (1 - p + pl) \phi > 0.
\]
Using \( m = 1 \) and \( \delta_R = 0 \), the problem reduces to
\[
\max \quad p R_H + (1 - p) R_L - I - (1 - p + pl) \phi
\]
\[
f_H - R_H \geq 0
\]
\[
f_L - R_L \geq 0
\]
Because of deterministic monitoring, in each state the monitor receives the correct state transfer. The borrower receives no rent in either state, otherwise the monitor could raise \( R_H + \delta_R \) and \( R_L \) to her benefit. The expected payoff to the monitor is therefore given by \( p f_H + (1 - p) f_L - I - (1 - p + pl) \phi \), with expected observation cost \( (1 - p + pl) \phi \).

**Proof of Proposition 2.** To prove the existence of a mixed strategy equilibrium in programme \( P_{M^2} \), it suffices to show that there is a positive penalty associated with misreporting. The proof proceeds as follows: first we prove that \( \alpha > 0 \), then that \( w_H < w_L \); next that the borrower gets a rent only in high truthfully reported states, i.e. that \( \delta_R > 0 \), and no rent in any other state. Setting up the Lagrangian:
\[
\Lambda = p(1 - l) R_H + (1 - p + pl) R_L - \alpha I + \lambda_1 (f_H - w_H - R_H) + \lambda_2 (f_L - w_L - R_L)
\]
\[
+ \lambda_3 (f_H - w_H - R_H - \delta_R) + \lambda_4 [\mu U_H + (1 - \mu) U_L - \pi] + \tau_1 \alpha + \tau_2 (1 - \alpha)
\]
where \( \mu = p(1 - l) + plm, U_H = U(w_H - (1 - \alpha) I) \) and \( U_L = U(w_L - (1 - \alpha) I) \), we get the following FOC's:

\[
\frac{d\Lambda}{dR_H} : p - pl + \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi} + \lambda_4 \frac{pl(U_H - U_L)}{R_H + w_H + \delta_R - R_L - w_L} = \lambda_1 + \lambda_3 \tag{19}
\]

\[
\frac{d\Lambda}{dR_L} : 1 - p + pl - \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi} - \lambda_4 \frac{pl(U_H - U_L)}{R_H + w_H + \delta_R - R_L - w_L} = \lambda_2 \tag{20}
\]

\[
\frac{d\phi}{d\Lambda} : \lambda_4 \left( \mu U_H' + \frac{pl(1-m)(U_H - U_L)}{R_H + w_H + \delta_R - R_L - w_L} \right) = \lambda_1 + \lambda_3 \tag{22}
\]

\[
\frac{d\lambda}{d\phi} : -I \left( 1 - \lambda_4 \left( \mu U_H' + 1 - \mu \right) U_L' \right) = \tau_1 - \tau_2 \tag{24}
\]

- \( \lambda_4 > 0 \)

From \( \frac{d\Lambda}{dR_H} + \frac{d\Lambda}{dR_L} \) and \( \frac{d\Lambda}{d\phi} + \frac{d\Lambda}{d\phi} \), we have:

\[
\lambda_1 + \lambda_2 + \lambda_3 = 1 = \lambda_4 [\mu U_H' + (1 - \mu) U_L'] \tag{25}
\]

The non monitoring lender is thus held down to her reservation utility level. Moreover, from \( \frac{\partial\Lambda}{\partial R_H} - \frac{\partial\Lambda}{\partial \phi} : \lambda_1 = p(1 - l) + \lambda_4 \frac{pl(U_H - U_L)}{R_H + w_H + \delta_R - R_L - w_L} \),

whence we deduce that a necessary condition for \( \lambda_1 = 0 \) is that \( w_H < w_L \).

- \( 0 < \alpha < 1 \)

Using (25) we deduce from (24) that \( \tau_1 = \tau_2 \). Thus, either the multipliers are both positive, or they are both zero. \( \tau_1 > 0 \) implies \( \alpha = 0 \), and \( \tau_2 > 0 \) implies \( \alpha = 1 \), which is impossible. Thus both multipliers must be zero, which implies that the share of funding provided by each lender is strictly positive.

- \( w_H < w_L \)

From the non monitoring lender’s participation constraint, in first best \( (EU_{FB}) \) and in second best \( (EU_{SB}) \), we have respectively:

\[
pU(w_H - (1 - \alpha) I) + (1 - p)U(w_L - (1 - \alpha) I) = \pi
\]

\[
\mu U(w_H - (1 - \alpha) I) + (1 - \mu)U(w_L - (1 - \alpha) I) = \pi.
\]

whence, in the \( w_L - w_H \) space

\[
\frac{d\phi}{d\phi} \bigg|_{EU_{FB}} = -\frac{p^l U_H' + (1-p)U_L'}{(1-p)U_L'} \tag{27}
\]

\[
\frac{d\phi}{d\phi} \bigg|_{EU_{SB}} = -\frac{\mu U_H' + (U_H - U_L) \frac{\partial u}{\partial w}}{(1-\mu)U_L' + (U_H - U_L) \frac{\partial u}{\partial w}} \tag{28}
\]
On the 45° line, the slope of the non monitoring lender utility contour respectively in first best and second best is \( \frac{\partial H}{\partial w_H} = -\frac{p}{1-p} \) and \( \frac{\partial L}{\partial w_L} = -\frac{p}{1-p} \). Using \( \mu < p \), we deduce that

\[ -\frac{p}{1-p} < -\frac{\mu}{1-\mu} \]  

(29)

i.e. \( EU_{FH} \) crosses \( EU_{SB} \) from above.

Taking the ratio of \( \frac{\partial A}{\partial w_H} \) and \( \frac{\partial A}{\partial w_L} \):

\[ -\frac{\mu U_H' + (U_H - U_L) \frac{\partial H}{\partial w_H}}{(1-\mu)U_L' + (U_H - U_L) \frac{\partial H}{\partial w_L}} = -\frac{\lambda_1 + \lambda_3}{\lambda_2} \]  

(30)

From \( \frac{\partial A}{\partial R_H} \) and \( \frac{\partial A}{\partial R_L} \), solving for \( \lambda_1 + \lambda_3 \) and \( \lambda_2 \) respectively (assuming they are positive), substituting out in (30) and assuming that \( w_H = w_L \), we have:

\[ -\frac{\mu}{1-\mu} = -\frac{p - pl + \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}}{1 - p + pl - \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}} \]  

(31)

where \( -\frac{\mu}{1-\mu} \) is the marginal rate of substitution for the non monitoring lender between \( w_H \) and \( w_L \) at \( w_H = w_L \). The right hand side can be interpreted as the marginal rate of substitution for the monitor between \( R_H \) and \( R_L \). The borrower does not seem to take part in this repayments determination problem: actually he is not interested in the distribution of repayments between the two lenders, but only in (minimising) their total amount \( (R_H + w_H, R_L + w_L) \). Conversely for the lenders there is a trade off between the remuneration each of them can get with respect to the other. For example, minimising the repayments to the non monitoring lender allows the monitoring one to maximise her own remuneration.

We can represent in an Edgeworth box the distribution of revenues between the lenders and the trade off between transfers (Fig. 3). In the \( w_H, w_L \) space we represent the first best and second best utility contours for the non monitoring lender as derived from (27) and (28). Opposite the \( w_H, w_L \) space, we represent the \( R_H, R_L \) space and the isoprofit contour of the monitor:

\[ \frac{dR_L}{dR_H} \mid E \pi_{FM} = -\frac{p - pl + \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}}{1 - p + pl - \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}} \]  

(32)

The monitor is better off the higher the transfers she receives and therefore the lower the transfers received by the non monitoring lender. If \( w_H = w_L \), then from (31) we have \(-\phi = 0\) which is a contradiction. Hence:

\[ 0 > -\phi \Rightarrow -\frac{\mu}{1-\mu} < -\frac{p - pl + \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}}{1 - p + pl - \frac{p(R_H - R_L)}{R_H + s_R - R_L - \phi}} \]  

(33)

i.e.\(^{28}\)

\[ \frac{dR_L}{dR_H} \mid E \pi_{FM} > \frac{dR_L}{dR_H} \mid E \pi_{FM} \]

\(^{28}\)From (33) descends that the RHS is non-positive, whence it follows that, because the numerator is positive, also the denominator is positive.
which means that along the 45° line, in the $w_H, w_L$ space, the monitor’s isoprofit contour is steeper than the non monitoring lender’s indifference curve; for well-behaved functions, the equilibrium must lie to the left of the 45° line, whence $w_H < w_L$ and $R_H > R_L$ (fig. 3).

![Fig. 3: The repayments space](image)

- $\lambda_2 > 0$

Using $w_H < w_L$ in $\frac{\partial \Lambda}{\partial R_L}$ and the result in footnote 28, we get:

$$\lambda_2 = 1 - p + pl - \frac{pl(R_H - R_L)}{R_H + \delta_R - R_L - \phi} - \lambda_4 \frac{p(U_H - U_L)}{R_H + w_H + \delta_R - R_L - w_L} > 0$$

whence $\lambda_2 > 0$.

- $\lambda_1 = 0$, $\lambda_3 > 0 \Rightarrow \delta_R > 0$

We can first use a direct argument to show that at least one of the borrower’s high state participation constraints (audited or non-audited) is binding, i.e. either $f_H - w_H - R_H - \delta_R > 0$ and $f_H - w_H - R_H = 0$, or $f_H - w_H - R_H - \delta_R = 0$ and $f_H - w_H - R_H > 0$. If they were both slack, then the monitor could raise either $R_H$ or $\delta_R$ to her benefit and still satisfy the borrower’s utility.

Suppose then that $\lambda_3 = 0$ and $\lambda_1 > 0$, that is to say that $\delta_R < 0$ ($f_H - w_H - R_H - \delta_R > 0$ and $f_H - w_H - R_H = 0$). From $\frac{\partial \Lambda}{\partial R_H}/\frac{\partial \Lambda}{\partial \phi}$, solving for $\lambda_1$:

$$\lambda_1 = \frac{1}{R_H + \delta_R - R_L - \phi - m} \frac{p(1-\delta_R - p(R_H - R_L))}{R_H + \delta_R - R_L - \phi} - pm(1-\ell)$$

From (26), solving for $\lambda_4$ we get:

$$\lambda_4 = \frac{(R_H - R_L)(R_H + w_H + \delta_R - R_L - w_L)}{(U_H - U_L)(\delta_R - m(R_H + \delta_R - R_L - \phi))} < 0$$
which is negative, so long as $R_H + \delta_R + w_H - R_L - w_L > 0$ and is a contradiction. Hence $\delta_R < 0$.

Another possibility is then to have $\lambda_3 > 0$ and $\lambda_1 > 0$, i.e. $\delta_R = 0$ ($f_H - w_H - R_H - \delta_R = 0$ and $f_H - w_H - R_H = 0$). We will prove that this is also impossible. The proof proceeds as follows: we consider the slope of the utility functions of the borrower ($B$) and the two lenders ($P_M, P_{NM}$) when $\delta_R = 0$ and show that $\lambda_1$ is never positive:

$$
\frac{d\delta_R}{dw_H} \bigg|_{P_M} = 0
$$

$$
\frac{d\delta_R}{dw_H} \bigg|_{P_{NM}} = -\frac{\mu U'_H + \frac{p(U_H - U_L)}{U_H - U_L} \left[ 1 - m + R_H + \delta_R - R_L - \phi - m \right]}{R_H + w_H + \delta_R - R_L - w_L} \frac{\delta_R}{R_H + \delta_R - R_L - w_L - \phi - m} < 0
$$

$$
\frac{d\delta_R}{dw_H} \bigg|_{B} = -1.
$$

Notice that the sign of the non monitoring lender’s indifference curve is ambiguous. However the slope of this function evaluated at $\delta_R = 0$ is unambiguously negative:

$$
\frac{d\delta_R}{dw_H} \bigg|_{P_M} \bigg|_{\delta_R = 0} > \frac{d\delta_R}{dw_H} \bigg|_{P_{NM}} \bigg|_{\delta_R = 0} > \frac{d\delta_R}{dw_H} \bigg|_{B}.
$$

Hence for $\delta_R = 0$ one of the following must hold (see fig. 4):

1. the non monitoring lender utility function is flatter than the borrower’s, i.e. $\frac{d\delta_R}{dw_H} \bigg|_{P_{NM}} \bigg|_{\delta_R = 0} > \frac{d\delta_R}{dw_H} \bigg|_{B}$;

2. the non monitoring lender utility function is steeper than the borrower’s, i.e. $\frac{d\delta_R}{dw_H} \bigg|_{P_M} \bigg|_{\delta_R = 0} < \frac{d\delta_R}{dw_H} \bigg|_{B}$.

We will find contradictions for each of these cases.

**Case 1.** In this case the slope of the non monitoring lender utility function is flatter than the borrower’s,$^{29}$ i.e.:

$$
\frac{pU'_H}{R_H + w_H - R_L - w_L} > -1.
$$

$^{29}$Because of this, the borrower’s utility cannot be maximal in $\delta_R = 0$, but it is possible to increase it by setting $\delta_R > 0$.  

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From $\frac{\partial \lambda}{\partial \delta} / \frac{\partial \lambda}{\partial w}$, substituting out $\lambda_1 + \lambda_3$ from $\frac{\partial \lambda}{\partial R_H}$ and solving for $\lambda_1$ (recalling that $\delta_R = 0$):

$$\lambda_1|_{\delta_R = 0} = \frac{1}{pU_H} \left\{ \frac{p(U_H - U_L)}{R_H + w_H - R_L - w_L} \left[ p - pl + \frac{pl(R_H - R_L)}{R_H - R_L - \phi} \right] \right\} + p(1 - l) \quad (39)$$

Rearranging, $\lambda_1$ becomes:

$$\lambda_1|_{\delta_R = 0} = p(1 - l) \left( 1 + \frac{p(U_H - U_L)}{R_H + w_H - R_L - w_L} \right) + \frac{p(U_H - U_L)}{pU_H} \left( \frac{p(R_H - R_L)}{R_H - R_L - \phi} \right), \quad (40)$$

which, using (38), is negative and hence impossible.

**Case 2.** In this case the slope of the non monitoring lender utility function is steeper than the borrower’s, i.e.:

$$pU_H' < -\frac{p(U_H - U_L)}{R_H + w_H - R_L - w_L}. \quad (41)$$

The sign of $\lambda_1$ as from (39) is now ambiguous. However, after some manipulations, $\lambda_1$ can be rewritten as:

$$\lambda_1|_{\delta_R = 0} = -\frac{p(R_H - R_L)}{R_H - R_L - \phi} + \frac{1}{pU_H} \left( pU_H' + \frac{p(U_H - U_L)}{R_H + w_H - R_L - w_L} \right) \left( p - pl + \frac{p(R_H - R_L)}{R_H - R_L - \phi} \right) \quad (42)$$

which is negative and rules also this latter case out as a possible solution. Hence $\delta_R \not\leq 0$.

Last, notice that a tangency might also occur at $\delta_R = 0$ between the non monitoring lender and the borrower’s utility functions, i.e. $\frac{\partial \delta_R}{\partial w_L}|_{\delta_R = 0} = \frac{\partial \delta_R}{\partial w_L}|_{B}$. We can rule out this case as the possible solution since, if a tangency occurred at $\delta_R = 0$, that is to say: $-pU_H' = \frac{p(U_H - U_L)}{R_H + w_H - R_L - w_L}$, then by (39) $\lambda_1$ could be written as:

$$\lambda_1|_{\delta_R = 0} = -\frac{p(R_H - R_L)}{R_H - R_L - \phi} < 0 \quad (43)$$

which is a contradiction. ■

**Proof of Proposition 3.**

To prove this we compare the expected observation cost with multiple investors when a mixed strategy equilibrium arises, and those with a single investor. We denote the former as $EMC_2 = m_2 (1 - p + pl_2) \phi$. As for the latter, in section 1 (appendix) it has been shown that with a single investor and fixed loan size the only feasible equilibria involve deterministic monitoring ($m_1 = 1$) and random (or deterministic) misrepresentation ($0 < l_1 \leq 1$), with expected observation cost $EMC_1 = (1 - p + pl_1) \phi$.

From (18), using $R_H = f_H$ and $R_L = f_L$, we see that $l_1 \geq \frac{(1-p)\phi}{pl(f_H - f_L - \phi)}$, which is strictly higher than $l_2 = \frac{(1-p)\phi}{pl(f_H - w_H - \phi)}$, given that $w_H < f_L$. Because $m_2 < 1 = m_1$, it follows that $EMC_2 < EMC_1$. ■
References


