Inefficient Taxation of Sin Goods

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Abstract
Within an O. Donoghue and Rabin (2006) style model, we study the optimal sin taxes that a government wants to implement when consumers are time-inconsistent, and taxation is inefficient in terms of administrative, collection and compliance costs. We find that, if the inefficiency of taxation is not too large, the optimal tax is positive and it may be higher or lower than the first best depending on the elasticity of demand with respect to taxation. Finally, the extent of the distortion depends on the degree of inefficiency of taxation.

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1 Introduction

Recent economic literature has investigated the effect of sin taxes, i.e., taxes on goods which are enjoyable to consume but create negative health consequences in the future (O’Donoghue and Rabin, 2003, 2006; Gruber and Koszegi, 2004). This literature has provided strong arguments for taxation to correct not just the externalities associated with the consumption of the sin good, but rather the “internalities” generated by consumers’ time-inconsistency. However, there are inefficiencies associated with taxation in terms of administrative and compliance costs. The former are those incurred by the tax authority to collect taxes and enforce regulations, while the latter are those incurred by taxpayers to comply with tax regulations. As regards administration cost, a recent OECD study reports an estimate of roughly 0.5% of net revenue collection for US, with a median of about 1% for OECD countries (2010). As for compliance costs, a study by Pricewaterhouse Coopers (2015) for 189 countries across the world reports that the number of hours spent to comply for consumption tax (sales and VAT) amounts on average to 99 hours, with 55 and 60 hours for EU-EFTA and North-America area respectively.\(^1\) We incorporate the inefficiencies associated to taxation in an O’Donoghue and Rabin style model with identical agents to study whether and how the optimal tax is affected by them.

Our first result is that the optimal tax is positive, provided the inefficiency of taxation is not too large. Moreover, it may be higher or lower than the first best depending on the elasticity of demand with respect to taxation. Finally, the degree of inefficiency of taxation affects the extent to which taxation is driven away from its first-best level. In particular, the higher the inefficiency, the higher the distortion.

The paper is related to the literature on time-inconsistency and hyperbolic discounting (Ainslee, 1992; Laibson, 1996), and in particular to the literature studying the welfare effects of sin taxes (Gruber and Koszegi, 2001, 2004; Gruber and Mullainathan, 2005).

\(^1\)Strictly speaking, consumption taxes compliance costs are incurred by firms. They nevertheless represent a burden for the system that ultimately impacts on prices and undermines efficiency. For an extensive survey of the literature on the relevance of tax operating costs, see Evans (2003).
2 Model

Players and Environment. We consider an O’Donoghue and Rabin style model where consumers have quasi-hyperbolic discounting. Their intertemporal utility is given by

\[ U^t(u_t, ..., u_T) = u_t + \beta \delta \sum_{s=t+1}^{T} \delta^{s-t} u_s, \]

where \( u \) is the instantaneous utility function, \( \delta \) is the discount factor, which we assume to be one for simplicity, and \( \beta \in [\overline{\beta}, 1] \), with \( \beta > 0 \), is the preference for immediate gratification.

The instantaneous utility function is quasi-linear with respect to the addictive good, \( x \), and a composite good which acts as a numeraire, \( z \). The addictive good increases the consumer’s current utility, but reduces future utility, because it creates health damages. Specifically,

\[ u_t = v(x_t) - c(x_{t-1}) + z_t. \]

The function \( v \) represents the immediate benefit from current sin good consumption and satisfies Inada conditions. The function \( c \) represents the negative health consequences from past sin good consumption and is such that \( c_x > 0, c_x(0) = 0, \) and \( v_{xx} - c_{xx} < 0. \)

Notice that the cost of addiction occurs only in the period following consumption, since the stock of past consumption does not affect time \( t \) consumption. This implies that the individual faces a series of independent decisions. In particular, at any period the consumer maximizes:

\[ u^a \equiv v(x) - \beta c(x) + z, \quad (1) \]

subject to the budget constraint \( I = px + z \), where \( I \) is the per-period income earned by the consumer and \( p \) is the price of the addictive good. There is no borrowing or lending, markets are competitive and the marginal cost of producing the sin good is normalized to one.

Following the behavioral economics literature, we refer to \( \beta < 1 \) as the “self-control problem” because it reflects a short-term desire that the person disapproves of at every other moment in her life. We assume that the social planner treats this as an error and, in order to correct the

\[ ^2 \text{Those assumptions guarantee that the problem is well-behaved.} \]

\[ ^3 \text{We assume that } I \text{ is large relative to the sin good consumption.} \]
consumers’ irrational behavior, maximizes their long-run utility function, i.e.,

\[ u^* = v(x) - c(x) + z, \]  

(2)

The first-best consumption, which we denote by \((x^*, z^*)\), maximizes (2) subject to the budget constraint \(I = x + z\). Hence, \(x^*\) satisfies the first order condition \(v_x(x^*) - c_x(x^*) = 1\).

The effect of the tax is to increase the price of the addictive good, that becomes \(p = 1 + \tau\). The proceeds from taxation \(\tau x\) are redistributed in a lump sum way to consumers. However, one euro of tax translates in a transfer of less than one euro to consumers due to the inefficiency of the fiscal system. Formally, the per-capita transfer \(l\) from tax proceeds is given by:

\[ l = (1 - \lambda)\tau x \]  

(3)

where \(\lambda \in [0, 1)\) is the direct inefficiency of the tax system, reflecting how many cents are lost in the economy to collect one extra euro tax revenues.

In the absence of taxes, the actual consumption of the sin good, \(x^a\), satisfies the first order condition \(v_x(x^a) - \beta c_x(x^a) = 1\). Since \(v_x\) is decreasing, \(c_x\) is increasing in \(x\) and \(v_x(x) - c_x(x)\) is lower than \(v_x(x) - \beta c_x(x)\) for any \(x, x^a > x^*\). Moreover, \(z^a = I - x^a < z^*\). Thus, the agent consumes too much of the sin good and too little of the numeraire.

In the case of linear tax \(\tau\) and lump sum transfer \(l\), the actual consumption \((x(\tau), z(\tau))\) maximizes (1) subject to

\[ I + l = (1 + \tau)x + z. \]  

(4)

The consumption of the sin good \(x(\tau)\) satisfies the first order condition

\[ v_x(x(\tau)) - \beta c_x(x(\tau)) = 1 + \tau. \]  

(5)

From the concavity of the utility function, \(v_x - \beta c_x\) is decreasing in \(x\). This implies that \(x(\tau)\) is lower than \(x^a\). Moreover, from the budget constraint, \(z(\tau) = I - x(\tau) + l - \tau x(\tau)\). This is greater than \(z^a\) if \(x^a - x(\tau) > \tau x(\tau) - l = \lambda \tau x(\tau)\), given that \(l = (1 - \lambda)\tau x\), i.e., if the inefficiency of taxation \(\lambda\) is not too high. Thus, the inefficiency of taxation has adverse effects on the effectiveness of taxation as a means to control the consumption of the sin good.
In the next section we study the optimal \( \tau \) that the social planner will choose to maximize the consumers’ long run utility.

### 3 Taxing vices

The programme \( P_{\tau} \) that the social planner solves is to choose the level of taxation \( \tau \) that maximizes (2) subject to the budget constraint (4), the lump-sum transfer constraint (3) and the consumption rule \( x(\tau) \) defined by condition (5).

By substituting (3) and (4) in (2), the objective function reads as:

\[
\Omega(\tau) = \frac{v(x(\tau)) - c(x(\tau)) + I - x(\tau)) - (\lambda \tau x(\tau))}{BT(\tau)}
\]

The term \( BT(\tau) \) represents the benefit of taxation, and is given by the utility that would be obtained by inducing a level of consumption \( x(\tau) < x^a \) and there was no inefficiency associated with taxation (\( \lambda = 0 \)). The second term, \( CT(\tau) \), represents the reduction in the consumption of the numeraire due to the inefficiency of taxation. The social planner’s problem is to choose \( \hat{\tau} \) that maximizes the distance between the benefits and costs of taxation.

Proposition 1 states that if \( \lambda \) is not too high, then the optimal \( \tau \), \( \hat{\tau}(\lambda,\beta) \equiv \hat{\tau} \), is strictly positive when \( \beta < 1 \).

**Proposition 1** Suppose that taxation is inefficient (\( \lambda > 0 \)). Then,

1. if \( \beta = 1 \), the optimal tax is \( \hat{\tau} = 0 \).

2. if \( \beta < 1 \) and \( \lambda < \lambda^M \), with \( \lambda^M \equiv \frac{(1 - \beta)c(x^a)}{v_{xx}(x^a) - \beta c_{xx}(x^a)x^a} \), the optimal tax is \( \hat{\tau} > 0 \).

The above proposition extends O’Donoghue and Rabin’s Proposition 1 to the case in which taxation features inefficiencies in terms of administrative and compliance costs.

If \( \beta = 1 \), there is no conflict of interest between the social planner and the agent about the consumption level of \( x \) and the optimal tax is zero. If \( \beta < 1 \), the agent’s consumption is too large and the optimal tax will be positive as long as the inefficiency of taxation is not too large. To
simplify the exposition, we will assume throughout that \( \lambda < \lambda^M \). This is a sufficient condition for a maximum.

Define with \( \tau^* \) the level of taxation that induces the agent to consume the first-best level of the sin good. This is such that the agent’s first order condition (5) is satisfied with equality when \( x(\tau) \) is equal to \( x^* \), i.e., \( \tau^* = (1 - \beta)c_x(x^*) \). When there is no efficiency loss associated with taxation (\( \lambda = 0 \)), the social planner’s problem (6) simplifies to maximizing \( BT(\tau) \) and the optimal tax chosen by the social planner \( \hat{\tau} \) coincides with the level \( \tau^* \) that induces the agent to consume the first best level of the sin good. This can be better understood by noticing that in the social planner’s problem (6) the benefit from taxation \( BT(\tau) \) is maximum when the agent consumes the first-best level of the sin good, \( x^* \).

However, when \( \lambda > 0 \), the cost component \( CT(\tau) \) of the social planner’s problem (6) is positive. One may thus think that she will (always) choose a level of taxation lower than the first best. Corollary 1 shows that this is not so and that, when \( \lambda > 0 \), the optimal tax rate can exceed or fall short of the first best, depending on the elasticity of \( x(\tau) \) with respect to \( \tau \), i.e.,

\[
\eta_{x,\tau} = \frac{x^*}{x(\tau)}.
\]

**Corollary 1** \( \hat{\tau} \leq \tau^* \) if and only if \( \eta_{x,\tau} \geq -1 \).

The intuition is the following. When the inefficiency of taxation is strictly positive, the tax has a cost given by the lower consumption of the numeraire, equal to \( \lambda \hat{\tau} x(\hat{\tau}) \). An increase in the sin good tax has two opposite effects on the consumption of the numeraire good: a negative direct effect due to the higher price paid on each unit of sin good purchased, and a positive indirect effect due to the distortionary impact of taxation on quantities. When the demand is highly elastic (\( \eta_{x,\tau} < -1 \)), the positive effect prevails on the negative one and the optimal taxation exceeds the first best (\( \hat{\tau} > \tau^* \)). Conversely, when the elasticity is low (\( \eta_{x,\tau} > -1 \)), the negative effect prevails on the positive one and taxation has a very negative impact on the consumption of the numeraire. To mitigate such impact, taxation has to be set lower than its first-best level (\( \hat{\tau} < \tau^* \)). Finally, if \( \eta_{x,\tau} = -1 \) the optimal tax \( \hat{\tau} \) is \( \tau^* \), regardless of \( \lambda \).

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\(^4\)Indeed, \( BT^*(\tau) = [v_x(x(\tau)) - c_x(x(\tau)) - 1]x^*(\tau) \) is zero when \( x(\tau) = x^* \).
In Corollary 1 we point to the existence of an upward or downward distortion of taxation relative to the first best. In Proposition 2 we quantify the magnitude of such distortion.

**Proposition 2** For all $\lambda$ such that $\hat{\tau} \equiv \hat{\tau}(\lambda)$ is differentiable, the distance between $\hat{\tau}$ and $\tau^*$, $|\hat{\tau} - \tau^*|$, is zero when $\lambda = 0$ and increases as $\lambda$ increases.

Thus, in Proposition 2 we show that the extent to which $\hat{\tau}$ is displaced from its first best level $\tau^*$ depends on the degree of inefficiency of taxation, and it is larger, the larger the inefficiency of taxation $\lambda$. Thus, $\lambda$ only affects the size of the distortion and not its direction, which depends on the elasticity of demand with respect to $\tau$.

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The usual disclaimer applies.
Appendix

Proof of Proposition 1 We will first prove that if $\beta = 1$ the optimal tax is 0, and then we will show that if $\beta < 1$ and $\lambda$ is small, it is strictly positive. For all $\tau \geq 0$, as long as $v$ and $c$ are thrice differentiable, $\Omega(\tau)$ is continuous and twice differentiable. If strictly positive, $\hat{\tau}$ satisfies the first order condition

$$\frac{\partial \Omega}{\partial \tau} = \Omega_x(\tau)x_\tau(\tau) - \lambda x(\tau) = 0,$$  

(7)

where $\Omega_x(\tau) = [v_x(x(\tau)) - c_x(x(\tau)) - 1 - \lambda r]$ and $x_\tau(\tau) = 1/(v_{xx}(x(\tau)) - \beta c_{xx}(x(\tau))) < 0$. From (5) we derive $\Omega_x(\tau) = \tau(1 - \lambda) - (1 - \beta)c_x(x(\tau))$. If $\beta = 1$, $\partial \Omega/\partial \tau = \tau(1 - \lambda)x_\tau(\tau) - \lambda x(\tau) < 0$ for all $\tau \geq 0$, and so the optimal tax is the corner solution $\hat{\tau} = 0$. Suppose instead $\beta < 1$. In this case, $\Omega(\tau)$ may not be quasi-concave. However, if $\lambda x^a < a^a$, $\tau = 0$ cannot be a corner solution of the social planner maximization problem and there exists at least one $\hat{\tau} > 0$ that solves equation (7). Indeed, when $\tau = 0$, $\partial \Omega/\partial \tau = -(1 - \beta)c_x(x^a)/(v_{xx}(x^a) - \beta c_{xx}(x^a)) - \lambda x^a$, which is positive for all $\lambda x^a < a^a$. Moreover, Inada conditions for $v(x)$ together with $c_x(0) = 0$ imply $\lim_{\tau \to -\infty} x(\tau) = 0$ and $\lim_{\tau \to -\infty}(\partial \Omega/\partial \tau) = -\infty$. Hence, by continuity of $\Omega(\tau)$, there exists at least one $\hat{\tau} > 0$ satisfying condition (7).

Proof of Corollary 1. The first order condition (7), can be written as

$$[v_x(x(\hat{\tau})) - c_x(x(\hat{\tau}) - 1] = \lambda \hat{\tau} \left( \frac{1}{\eta_{x,\tau}} + 1 \right),$$  

(8)

where $\eta_{x,\tau} = \frac{x_x(\hat{\tau}) x(\hat{\tau})}{x(\hat{\tau})}$. Since $x_\tau(\tau) < 0$, the right-hand side of (8) equals 0 iff $\eta_{x,\tau} = -1$, is positive iff $\eta_{x,\tau} > -1$ and negative in the opposite case. The left-hand side of (8) equals 0 iff $\tau = \tau^*$, is positive iff $\tau > \tau^*$ and negative in the opposite case.

Proof of Proposition 2. Assume $\lambda = 0$. Substituting in (7) gives $d\Omega/d\tau = \Omega_x(\tau)x_\tau(\tau) = 0$, with $\Omega_x(\tau) = \tau - (1 - \beta)c_x(x(\tau))$. Since $\tau^* = (1 - \beta)c_x(x(\tau^*))$, then $d\Omega(\tau^*)/d\tau = 0$ and $\hat{\tau} = \tau^*$.

Assume $\lambda > 0$. The derivative of $|\hat{\tau} - \tau^*|$ with respect to $\lambda$ is

$$\frac{\partial |\hat{\tau} - \tau^*|}{\partial \lambda} = \frac{\partial \hat{\tau}}{\partial \lambda} \text{sgn}(\hat{\tau} - \tau^*).$$

By the envelope theorem,

$$\frac{\partial \hat{\tau}}{\partial \lambda} = \frac{x_\tau(\hat{\tau})}{x_\tau} \frac{v_{xx}(x(\hat{\tau}) - \beta c_{xx}(x(\hat{\tau}))}{v_{xx}(x(\hat{\tau}) - \beta c_{xx}(x(\hat{\tau}))} (v_x - c_x - 1 - \lambda \hat{\tau} - 2\lambda).$$
The denominator is negative by the local concavity of the objective function into a neighborhood of \( \hat{\tau} \), and then \( \partial \hat{\tau} / \partial \lambda \geq 0 \) iff \( x(\hat{\tau}) + \hat{\tau} x_r(\hat{\tau}) \leq 0 \). Moreover, since \( x(\hat{\tau}) + \hat{\tau} x_r(\hat{\tau}) \leq 0 \) iff \( \eta_{x,\tau} < -1 \), by Corollary 1 \( \partial \hat{\tau} / \partial \lambda \geq 0 \) iff \( \hat{\tau} \) is larger than \( \tau^* \). Hence, \( \text{sgn}(\partial \hat{\tau} / \partial \lambda) = \text{sgn}(\hat{\tau} - \tau^*) \) and the derivative of the absolute value of the distance between \( \hat{\tau} \) and \( \tau^* \) is always positive. ■
References


