Taxing and Regulating Vices

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March 2016
Abstract
We study the sin taxes and regulatory measures that it is optimal to implement when consumers are time-inconsistent and there are inefficiencies associated with the use of either instrument. For high inefficiency of regulation, only taxation is used and it may be higher or lower than the first-best depending on the price elasticity of demand. For high inefficiency of taxation, only regulation is used to an extent which depends on its effectiveness in terms of quantity reduction relative to the disutility it generates. For moderate inefficiency of either instrument, taxation and regulation are both optimally used.

JEL Classification: D03, H21, L51.

Keywords: Hyperbolic preferences, Taxation, Regulation, Consumption restrictions, Sin goods.

Acknowledgement: We thank Alberto Bennardo, Emilio Calvano, Marco Ottaviani and Antonio Rosato for useful discussions. We also thank the seminar participants in Naples Federico II. The usual disclaimer applies. The paper extends and replaces a previous working paper entitled “Inefficient taxation of sin goods”.

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References
1 Introduction

Recent economic literature has investigated the effect of sin taxes, i.e., taxes on goods which are enjoyable to consume but create negative health consequences in the future (O’Donoghue and Rabin, 2003, 2006; Gruber and Koszegi, 2004). This literature has provided strong arguments for taxation to correct not just the externalities associated with the consumption of the sin good, but rather the “internalities” generated by consumers’ time-inconsistency. It has nevertheless failed to see that there are inefficiencies associated with taxation in terms of administrative and compliance costs. The former are those incurred by the tax authority to collect taxes and enforce fiscal laws, while the latter are those incurred by taxpayers to comply with tax obligations. As regards administration cost, a recent OECD study reports an estimate of roughly 0.5% of net revenue collection for US, with a median of about 1% for OECD countries (2011). As for compliance costs, a study by Pricewaterhouse Coopers (2015) for 189 countries across the world reports that the number of hours spent to comply for consumption tax (sales and VAT) amounts on average to 99 hours, with 55 and 60 hours for EU-EFTA and North-America area respectively.\(^1\)

Besides taxation, the government can try and affect directly and/or indirectly the demand of sin goods through regulatory measures. Bans on junk food, smoking bans, bans on alcohol purchase or on gambling are only but few examples of regulation of consumption widely used in many countries that have proven to be an effective means of affecting the consumption of sin goods.\(^2\)

Despite the widespread policy makers’ reliance on both such measures, the economic literature has not investigated so far the joint impact of tax and regulatory measures on the consumption of sin goods.

We incorporate the inefficiencies associated to taxation in a model in which the consumption choice of identical agents with self-control problems may be affected also by regulatory measures. We focus on a simple quasi-linear economy in which, in addition to a composite good, there is

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\(^1\)Strictly speaking, consumption taxes compliance costs are incurred by firms. They nevertheless represent a burden for the system that ultimately impacts on prices and undermines efficiency. For an extensive survey of the literature on the relevance of tax operating costs, see Evans (2003).

\(^2\)An ample description of some regulatory measures and of the literature testing their effectiveness is provided in the next section.
a sin good. Regulatory measures affect the agents’ utility through a reduction in the actual immediate benefit from consuming the sin good. In some cases this reduction is high. For example, clean air regulations, by restricting the choice freedom and constraining addicted consumers to smoke in uncomfortable situations (e.g., out in the cold), may generate a large disutility. Similarly, bans on fatty food such as crisps in schools, by obliging to postpone consumption, impose a significant reduction in the utility from fatty food consumption. In other cases, this utility-reduction effect is low. An example is given by warning labels on tobacco, alcohol or fatty food.

Within this setting, the paper analyzes three different cases. Those in which each instrument, taxation and regulation, is used in isolation, dealt with in Sections 4 and 5, respectively, and the case in which the two instruments are jointly used, dealt with in Section 6.

When only taxation is used, our first result is that the optimal tax is positive, provided the inefficiency associated with it is not too large. Moreover, it may be higher or lower than the first-best—where the first-best taxation is the level that induces the agent to consume the first-best level of the sin good—depending on the price elasticity of demand. Finally, the degree of inefficiency of taxation affects the extent to which taxation is driven away from its first-best level. In particular, the higher the inefficiency, the higher the distortion.

The intuition for taxation exceeding the first-best level despite the inefficiency associated with it is the following. Taxing the sin good reduces its demand and, through the lump-sum transfer from the tax proceeds, increases the demand of the numeraire. However, such increase is mitigated or even offset by the leakage of resources implied by the inefficiency of the tax system. This leakage of resources, and the subsequent reduction in the consumption of the numeraire, represents the cost of the tax measure. This cost may increase or decrease with taxation depending on the elasticity of demand. When the elasticity is high, an increase in taxation implies a reduction in the consumption of the sin good (quantity effect) that is sufficiently large to mitigate the deadweight loss implied by the extra-expenditure driven by the tax increase. Thus, to mitigate its inefficiency, taxation is set above the first-best. Conversely, when the elasticity is low, an increase in taxation implies a reduction in the consumption of the sin good that is small relative to the extra-expenditure implied by the tax increase. To
mitigate such negative effect, it is best not to increase the tax and set it below the first-best. In other words, decreasing taxation generates an increase in the consumption of the sin good that is nevertheless small relative to the saving (in the deadweight loss) implied by the tax decrease.

While the result just described shows that the direction of the distortion due to taxation (with respect to the first-best) depends on the elasticity of demand, the extent to which taxation is driven away from its first-best level depends on the degree of inefficiency of taxation. More specifically, the higher the inefficiency, the higher the leakage of resources following a tax increase, the larger the upward (or downward) distortion in taxation that is needed to compensate it.

To determine whether it is optimal to regulate the consumption of the sin good, we compare the relative inefficiency of the regulatory instrument with the internality that it aims to correct. Similar to the case with taxation, we find that regulation is optimal, when it is the only instrument (Section 5), as long as its inefficiency, i.e., the disutility it generates, is not too high. In addition, the extent to which it is relied upon relative to the first-best—where the first-best regulation is the level that induces the agent to consume the first-best level of the sin good—depends on whether the positive effect of regulation in terms of quantity reduction is large (small) with respect to the negative one induced by the loss in utility.

When both instruments are available, the problem becomes one of choosing a control scheme so as to minimize the total social costs, these being the sum of the inefficiency of taxation and the disutility on consumption implied by regulation. In particular, when the inefficiency of taxation is low (high) relative to the inefficiency of regulation, taxation (regulation) is preferred. When the inefficiency associated with both instruments is not too high, both instruments are used. This is because the social cost of using taxation and regulation together are convex. All the previous results and some further ones are illustrated either through parametrical or numerical examples.

A last remark is in order. In real world, there may be other than purely efficiency-based reasons for favouring either tax or regulation as control instruments. As argued by Weitzman (1974, p. 479), “these reasons might involve ideological, political, legal, social, historical, administrative, motivational, informational, monitoring, enforcing, or other considerations. But there is little of what might be called a system-free character.” Thus, our model aims at providing
a normative, rather than a positive theory of taxing and regulating vices. We will come back to this point in the concluding discussion.

The paper is related to the literature on time-inconsistency and hyperbolic discounting (Ainslee, 1992). Time-inconsistency has been recently applied in the context of savings decisions (Laibson, 1996; Laibson, Repetto and Tobacman, 1998; O’Donoghue and Rabin, 1999), retirement decisions (Diamond and Koszegi, 2003), and economic growth (Barro, 1999). Other papers have studied the welfare effects of sin taxes (Gruber and Koszegi, 2001, 2004; Gruber and Mullainathan, 2005).

The paper is also related to the literature on externality. Starting from the seminal contribution of Weitzman (1974), this literature has investigated whether price or quantity instruments should be used to correct the externalities. In a setting of imperfect information, the use of both instruments, by affecting the schedule of revenues of the firms, is shown to provide higher expected welfare gains than an approach relying on either policy instrument alone. The intuition is that, because of uncertainty, one instrument “can protect against the failings of the other” (Roberts and Spence, 1976). In our setting, the instruments used by the regulator affect the social cost of regulation and taxation and they are jointly used so as to elicit an optimal response by consumers in terms of reduced consumption of the sin good. This is due to the convexity of the social cost in the use of the two instruments.

In the real world, both taxation and regulation have been extensively used to correct the externalities and the internalities associated to the consumption of sin goods. As regards taxation, there is ample evidence that the actual level of taxation exceeds what would be justified by the correction of the externality they create (Chaloupka and Warner, 1998; Evans et al., 1999a,b; Gruber, 2001). Similarly, there are examples of regulatory measures showing that these measures are mainly devoted to correct internalities rather than externalities, like education initiatives. This will be dealt with in the next section.

The paper is organized as follows. In the next section, we provide some motivating evidence on the regulation of sin goods. In Section 3, we set up the model. In Section 4, we consider the case in which only taxation can be used, while in Section 5 we focus on the case in which only regulation can be used. In Section 6, we consider the case in which both instruments can
be used. In the last section we provide some concluding remarks and discuss the normative implications of the model. All the proofs are in the Appendix.

2 Evidence on regulation of sin goods

Everyday life provides us with several examples of initiatives aimed at forbidding, limiting or deterring the consumption of sin goods. These initiatives can be categorized into two groups: educational initiatives, aimed at increasing the awareness of the health effects of the consumption of sin goods, and policy initiatives, aimed at directly limiting the availability and provision of sin goods.

Education initiatives, like information/awareness campaigns (mass media campaigns, social marketing), are widespreadly used. In many countries, for example, health warning labels, often mandatory, appear on fatty food, tobacco or alcohol products pointing to the health risks associated with their consumption. In some cases they can take the form of recommendations, like in responsible drinking campaigns to prevent alcoholism or drunk driving. Recommendations to moderate/responsible consumption also close advertisements campaigns of alcohol products and gambling (e.g., gamble responsibly). On the packaging of cigarettes and other tobacco products appear a variety of textual and pictorial warnings covering, within a black frame, a large part of the surface of the pack and concerning the health effects of tobacco products consumption. For fatty food, some countries (e.g., UK) have developed a system of front of pack nutritional labels that associates colors with information on fat, salt, sugar, and calories contained in food products, to help people making healthier choices. Those warnings, especially pictorial ones, may have an emotional impact, thus altering the pleasure that one may get from the act of consumption and subtly influencing choice.

Education initiatives seem to be effective in increasing the consumers’ knowledge and attitude about the health consequences of the consumption of sin goods. For tobacco and alcohol products, for example, comprehensive review studies have provided evidence showing the effectiveness of strategies and interventions aimed at preventing smoking uptakes (Thomas et al., 2013) or alcohol related problems (Babor et al., 2003). Although much more limited, some evidence is also available for food products. For example, a study by Cioffi et al. (2015)
has shown that the introduction of food labels on a sample of pre-packaged food items results in a reduction of the average calories purchased from the labelled foods.

Besides education initiatives, governments rely on several policy initiatives aimed at altering the availability and provision of the good. Examples are bans on fatty food in schools cafeteria, clean-air regulations for tobacco products, alcohol consumption regulations placing controls on the type of alcohol available, on the quantity, or on the days and hours of sale, vending machines and point-of-sale restrictions for tobacco or alcohol products, gambling regulations restrictions on the number of gambling venues, location, and hours of operation.

The idea behind these measures is that the difficulty of accessing a good may generate such a high disutility on consumption to induce a reduction of the same. There is ample evidence that this is indeed the case. For example, several studies indicate that laws and ordinances restricting smoking in public places can be an effective tobacco control strategy (Gruber and Zinman, 2001; Wasserman et al., 1991; Chaloupka, 1988; Keeler et al., 1991). Similarly, workplace smoking bans are associated with both reduced smoking prevalence and reduced daily consumption among smokers (Evans et al., 1999; Wakefield et al., 1992; Chapman et al., 1999; among others).

Regarding alcohol consumption restrictions, several studies show that alcohol availability is positively associated with higher levels of consumption, which is in turn correlated with higher levels of alcohol-related problems (Cook, 2007; Anderson et al., 2009). This finding is confirmed whether restrictions on retail access to alcohol are considered, like limits on the number of alcohol outlets (Campbell et al., 1999; Livingston et al., 2007), or changes to permitted hours of sale (Stockwell and Chikritzhs, 2009; Hahn et al., 2010). It indicates that restricting the availability of alcohol can prevent excessive consumption.³ In a comprehensive review of the educational and policy initiatives aimed at preventing problem gambling, Williams et al. (2008) report significant increases in the population prevalence of problem/pathological gambling following the expansion of legalized gambling in the 1980s and 1990s in several countries. This suggests that restricting gambling availability, by lowering its incidence, can reduce problem gambling.⁴

³When consumer welfare is considered, Hinnosaar (2015) finds that alcohol sales restrictions are preferable to increases in taxes.

⁴Another important regulation tool is to impose restrictions on who can consume the sin good. A typical example is given by restrictions on youth access to tobacco or alcohol products, such as minimum age of purchase; photo identification; vending-machine availability; free distribution; random inspections. In assessing the effectiveness of various policy measures on youth smoking in the US, Gruber and Zinman (2001) find that
As regards food products, while there are some recent examples of health related food taxes (i.e., taxes on food items that are considered unhealthy) being levied in some countries, the evidence on regulatory measures being put in place and the effectiveness of such measures is rather limited (Datar and Nicosia, 2012).

3 The Model

We consider a model in which consumers have quasi-hyperbolic discounting. Their intertemporal utility is given by

$$U^t(u_t, \ldots, u_T) = u_t + \beta \delta \sum_{s=t+1}^{T} \delta^{s-t} u_s,$$

where $u$ is the instantaneous utility function, $\delta$ is the discount factor, which we assume to be one for simplicity, and $\beta \in (0, 1]$, with $\beta > 0$, is the preference for immediate gratification.

The instantaneous utility function is quasi-linear with respect to the sin good ($x$) and a composite good which acts as a numeraire ($z$). The sin good increases the consumer’s current utility, but reduces future utility, because it creates health damages. Specifically,

$$u_t = v(x_t) - c(x_{t-1}) + z_t.$$

The function $v$ represents the immediate benefit from current sin good consumption and satisfies Inada conditions. The function $c$ represents the negative health consequences from past sin good consumption and is such that $c_x > 0$, $c_{xx} > 0$, and $c_x(x) = 0$ when $x = 0$.

Additive separability of the benefits and costs of consumption implies that the individual faces a series of independent decisions. In particular, at any period the consumer maximizes

$$u^a = v(x) - \beta c(x) + z,$$

subject to the budget constraint $I = p_x x + p_z z$, where $I$ is the exogenous income earned by the consumer, $p_x$ and $p_z$ are the price of the sin good and of the numeraire, respectively. We assume access restrictions measures reduce the intensity of youth smoking, although not smoking participation. In reviewing scientific studies on policies aimed at reducing tobacco and alcohol use in young people, Toumbourou et al. (2007) document the effectiveness of restricting settings of use and raising legal purchase age. Similarly, a series of studies have documented the sensitivity of youthful drinking to minimum-purchase-age (MPA) laws (Coate and Grossman, 1987; Laixuthai and Chaloupka, 1993; Cook and Moore, 2001; Wagenaar and Toomey, 2002).

\textsuperscript{5}Taxes of this type have been recently introduced in a number of countries, like Finland, France, Hungary, and Mexico.

\textsuperscript{6}Those assumptions guarantee that the consumer’s problem is well-behaved. In particular, the assumptions that $\lim_{x \to -\infty} v(x) = \infty$, $\lim_{x \to -\infty} v_x(x) = 0$, and $c_x(x) = 0$ when $x = 0$ ensure that the sin good demand is strictly positive for any price $p_x < \infty$ and are made to simplify the exposition.
that there is no borrowing or lending, that markets are competitive and that the marginal cost of producing both goods is equal to one, so that the price of each good is also one.\footnote{We assume that $I$ is large relative to the sin good consumption, so as to avoid corner solutions for $x$.}

Because of the self-control problem ($\beta < 1$), the consumers’ behavior may not maximize their own welfare, measured by the long-run utility function

$$u^* = v(x) - c(x) + z.$$ (1)

Following O’Donoghue and Rabin (2006), we call decision utility the utility function that explains the choice of the agent ($u^*$), and experienced utility the utility function that reflects the welfare of the agent ($u^*$).

The first-best consumption, which we denote by ($x^*, z^*$), maximizes the experienced utility (1) subject to the budget constraint $I = x + z$. It is such that $x^*$ satisfies the first order condition $v_x(x^*) - c_x(x^*) = 1$, and $z^* = I - x^*$.

In the absence of policy measures aimed at affecting the consumers’ behavior, the actual consumption of the sin good, $x^a$, satisfies the first order condition $v_x(x^a) - \beta c_x(x^a) = 1$. Since $v_x$ is decreasing in $x$ and $c_x$ increasing in $x$ and $v_x(x) - c_x(x)$ is lower than $v_x(x) - \beta c_x(x)$ for any $x$, $x^a$ is larger than $x^*$. Moreover, $z^a = I - x^a < z^*$. Thus, the agent consumes too much of the sin good and too little of the numeraire.

There are two instruments available to the social planner to correct the consumers’ irrational behavior: a linear tax $\tau$ on the consumption of the sin good, and the regulation of the same consumption activity, captured by the parameter $\rho \in [0, \overline{\rho}]$, where $\rho = 0$ means absence of regulatory measures and $\rho = \overline{\rho} < \infty$ implies zero sin good consumption. The effect of the tax is to increase the consumer price of the sin good, that becomes $p_x = 1 + \tau$. The effect of regulating the consumption of the sin good is to reduce the immediate benefit from consuming the sin good.

We now introduce and motivate the two main assumptions of our analysis.

When taxation is imposed, the tax proceeds $\tau x$ are redistributed in a lump sum way to consumers. However, we assume that one euro tax revenues translates in less than one euro transfer for consumers due to the inefficiency of the fiscal system. Formally:
Assumption 1 (Inefficiency of the tax system) The per-capita transfer from tax proceeds is given by:

\[ l = (1 - \lambda)\tau x, \]  

(2)

where \( \lambda \in [0,1] \) is the direct inefficiency of the tax system, reflecting the loss in the economy from collecting one euro tax revenues.

In the case of linear tax \( \tau \) and lump sum transfer \( l \), the actual consumption maximizes the decision utility \( v(x) - \beta c(x) + z \) subject to the budget constraint \( I + l = (1 + \tau) x + z \). The consumption of the sin good satisfies the first order condition

\[ v_x(x) - \beta c_x(x) = 1 + \tau. \]  

(3)

Let \( x(\tau) \) be the agent’s consumption rule of the sin good defined from condition (3). From the concavity of the utility function, \( v_x - \beta c_x \) is decreasing in \( x \). This implies that \( x(\tau) \) is lower than \( x^a \) and decreasing in \( \tau \). Indeed, the first derivative of \( x(\tau) \) with respect \( \tau \) is

\[ x_x(\tau) \equiv \frac{1}{v_x(x(\tau)) - \beta c_x(x(\tau))}, \]

which is negative for all \( \tau \). Moreover, from the budget constraint,

\[ z(\tau) = I - x(\tau) + l - \tau x(\tau) = I - (1 + \lambda \tau) x(\tau), \]

whence we immediately see that the inefficiency of taxation has an adverse effect on the consumption of the numeraire. This can even fall short of \( z^a \) if \( x^a - x(\tau) < \tau x(\tau) - l = \lambda \tau x(\tau), \) i.e., if the inefficiency of taxation \( \lambda \) is sufficiently high. Thus, the reduction in the consumption of the numeraire can be seen as a measure of the inefficiency of taxation.

To see why, let us consider the effect of the tax on the sin good. It increases the price of the same and determines a reduction in the demand of both the sin good and the numeraire. However, there is an income effect due to fact that the proceeds from taxation are transferred to consumers \([ l = (1 - \lambda)\tau x]\). Because of the quasi-linearity of the agent’s utility function, this income effect is zero for the sin good (its demand depends only on the relative price) and positive for the numeraire, whose demand increases. It turns out that the extent of such increase depends on the inefficiency of the tax system. The higher the inefficiency, the more the demand of the numeraire is distorted away from the level it would have under an efficient tax system.

This inefficiency may thus prompt for alternative instruments to correct the excess consumption driven by the time-inconsistency. One such instrument is to impose regulatory
measures aimed either at creating the conditions for the consumers to voluntarily limit consumption, like information/awareness campaigns, or at directly forbidding the consumption of the sin good through restrictions on the general availability of the good, or on who can consume it. The effect of imposing such measures is to reduce the immediate benefit from consuming the sin good, which becomes \( v(x) - k(x, \rho) \). This is introduced by the following assumption:\(^8\)

**Assumption 2 (The effect of regulatory measures)** The cost of regulation is given by a continuous function \( k(x, \rho) \) defined on \( \mathbb{R}_+ \times (0, \bar{\rho}] \) such that: \( \lim_{\rho \to 0} k(x, \rho) = 0, k_{\rho} > 0, k_{x\rho} > 0 \).

When regulation is imposed, the agent’s experienced utility becomes

\[
    u^*(x, \rho) = v(x) - k(x, \rho) - c(x) + z,
\]

and the agent’s decision utility becomes

\[
    u^a(x, \rho) = v(x) - k(x, \rho) - \beta c(x) + z,
\]

in which we see that \( \rho \) reduces the immediate benefit of consumption.

An increase in \( \rho \) has a negative effect both on the level of the utilities \((u^*_\rho = u^a_\rho = -k_\rho < 0)\) and on the marginal utilities \((u^*_{x\rho} = u^a_{x\rho} = -k_{x\rho} < 0)\). Moreover, to get a strictly positive level of consumption for all \( \rho \in (0, \bar{\rho}) \) and zero consumption for \( \rho = \bar{\rho} \), we introduce the following technical assumptions: (A1) \( k_x(0, \rho) < \infty \) for all \( \rho \in (0, \bar{\rho}) \), (A2) \( \lim_{x \to 0}(v_x(x) - k_x(x, \bar{\rho})) \leq 1 \) and (A3) \( v_{xx}(x, \rho) - k_{xx}(x, \rho) < 0 \) for all \( x \) and \( \rho \).

In the case of regulation \( \rho \), the actual consumption, which we denote by \((x(\rho), z(\rho))\), maximizes (5) subject to \( z = I - x \). Then, the agent’s consumption rule of the sin good, \( x(\rho) \), satisfies the first order condition

\[
    v_x(x) - k_x(x, \rho) - \beta c_x(x) = 1.
\]

Clearly, \( z(\rho) = I - x(\rho) \). Since regulation reduces the marginal utility of consumption of the sin good and since the utility function is concave, it follows that \( x(\rho) \) is lower than \( x^a \) and \( z(\rho) \) is higher than \( z^a \) for any \( \rho \). In this case, unlike what obtains with taxation, the inefficiency does not affect the budget constraint and thus the consumption of the numeraire.

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\(^8\)For a psychological argument justifying Assumption 2, see Loewenstein and O’Donoghue (2006).
Finally, it is immediate to notice that the optimal sin good consumption is decreasing in $\rho$. Indeed, the first derivative of $x(\rho)$ with respect $\rho$ is 
\[
\frac{d}{d\rho} x(\rho) = \frac{k_{xx}(x(\rho), \rho)}{v_{xx}(x(\rho)) - k_{xx}(x(\rho), \rho) - \beta c_x(x(\rho))},
\]
that is negative.

In the case of linear tax $\tau$, lump sum transfer $l$ and regulation $\rho$, the actual consumption maximizes the agent’s decision utility $v(x) - k(x, \rho) - \beta c(x) + z$ subject to the budget constraint $z = I + l - (1 + \tau) x$. Then, the optimal sin good consumption, $x$, satisfies the first order condition
\[
v_x(x) - k_x(x, \rho) - \beta c_x(x) = (1 + \tau).
\]
Clearly, $z = I + l - (1 + \tau) x$. Since regulation reduces the marginal utility of consumption of the sin good and the marginal utility is decreasing in $x$, the agent’s consumption rule defined by (7) is lower than both $x(\tau)$ and $x(\rho)$.

In the next sections we study the choice problem of the social planner. We first focus on the case in which only taxation can be used, then on the case in which there is only regulation, and last we consider the case in which both instruments are available and determine the conditions under which it is optimal to use them both.

4 Taxing vices

The programme $P_{\rho}$ that the social planner solves is to choose the level of taxation $\tau$ that maximizes the experienced utility function (1) subject to the budget constraint $z = I + l - (1 + \tau) x$, the lump-sum transfer constraint (2) and the consumption rule $x(\tau)$ defined by condition (3).

By substituting the budget constraint, the lump-sum transfer constraint and the consumption rule in the experienced utility function, the objective function reads as:
\[
\Omega(\tau) = \frac{[v(x(x(\tau))) - c(x(x(\tau))) + I - x(x(\tau))] - \lambda x(x(\tau))}{BT(\tau)} - \frac{CT(\tau)}{CT(\tau)}.
\]

The term $BT(\tau)$ represents the benefit of taxation, and is given by the utility that would be obtained by inducing a level of consumption $x(\tau) \leq x^a$ and there was no inefficiency associated with taxation ($\lambda = 0$). The second term, $CT(\tau)$, represents the resources lost due to the inefficiency of taxation, that reduce the consumption of the numeraire. The social planner’s problem is to choose $\hat{\tau}$ that maximizes the distance between the benefits and costs of taxation.
Proposition 1 states that if $\lambda$ is not too high, then the optimal tax, $\hat{\tau}$, is strictly positive when $\beta < 1$.

**Proposition 1** Suppose that taxation is inefficient ($\lambda > 0$). Then, if $\beta = 1$, the optimal tax is $\hat{\tau} = 0$; if $\beta < 1$ and $\frac{\lambda x^a}{x_r(0)} < (1-\beta)c_x(x^a)$, the optimal tax is $\hat{\tau} > 0$.

The above proposition extends O’Donoghue and Rabin’s Proposition 1 to the case in which taxation features inefficiencies in terms of administrative and compliance costs.

If $\beta = 1$, there is no conflict of interest between the social planner and the agent about the consumption level of $x$ and the optimal tax is zero. If $\beta < 1$, the agent’s consumption is too large and the optimal tax will be positive as long as the relative inefficiency associated with it is not too large relative to the time inconsistency problem that it wants to correct. To measure the relative inefficiency of taxation we take the ratio between the marginal cost of an increase in tax rate in terms of deadweight loss $\lambda$ on each unit of the sin good, $\lambda x^a$, and the marginal benefit in terms reduced consumption, evaluated in $\tau = 0$, $x_r(0)$. The higher the ratio, the higher the inefficiency of the instrument. To measure the time inconsistency problem, we take the difference between the social marginal cost associated to sin good consumption, $c_x$, and the private marginal cost, $\beta c_x$, again evaluated in $\tau = 0$. This difference represents a measure of the extent to which the individual underestimates the long term consequences of sin good consumption. As the time inconsistency increases ($\beta$ falls), the internality associated to sin good consumption increases. When the inefficiency of the instrument is sufficiently smaller than the inefficiency induced by the internality problem, it is optimal to impose a tax on the sin good. To simplify the exposition, we will assume throughout that the condition in Proposition 1 is satisfied. This is a sufficient condition for a maximum.

Define with $\tau^*$ the level of taxation that induces the agent to consume the first-best level of the sin good $x^*$. This is such that the agent’s first order condition (3) is satisfied with equality when $x(\tau)$ is equal to $x^*$, i.e., $\tau^* = (1-\beta)c_x(x^*)$. When there is no efficiency loss associated with taxation ($\lambda = 0$), the social planner’s problem (8) simplifies to maximizing $BT(\tau)$ and the optimal tax chosen by the social planner $\hat{\tau}$ coincides with the level $\tau^*$. When $\lambda > 0$, the...

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9This term can be interpreted as a measure of the effectiveness of the instrument. The higher the reduction in consumption following an increase in $\tau$, the more effective the instrument.

10This can be better understood by noticing that in the social planner’s problem (8) the benefit from...
cost component $CT(\tau)$ of the social planner’s problem (8) is positive and the optimal tax is displaced from its first-best level. However, despite what one may think, it will not necessarily be set below the first-best. Proposition 2 shows that, when $\lambda > 0$, the optimal tax rate can exceed or fall short of the first-best, depending on the elasticity of $x(\tau)$ with respect to $\tau$, i.e.,

$$\eta_{x,\tau} = \frac{x_\tau(\tau)}{x(\tau)}.$$ 

**Proposition 2** Assume that $\beta$ is strictly lower than 1 and that $\Omega(\tau)$ is concave. Then $\hat{\tau} \leq \tau^*$ if and only if $\eta_{x,\tau} \geq -1$.

The intuition is the following. When the inefficiency of taxation is strictly positive, the tax has a cost given by the lower consumption of the numeraire, equal to $CT(\tau) = \lambda \hat{\tau} x(\hat{\tau})$. The marginal cost of taxation is given by $CT_{\tau} = \lambda [x(x) + \tau x_{\tau}]$. This is the result of two opposite effects. A negative direct effect due to the higher price paid on each unit of sin good purchased, which subtracts resources that could have been spent on the numeraire (extra-expenditure effect), and a positive indirect effect due to the reduction in the consumption of the sin good, that frees resources to be used in the numeraire (quantity effect). When the demand is highly elastic ($\eta_{x,\tau} < -1$), the quantity effect prevails on the extra-expenditure effect and the marginal cost of taxation is decreasing. Thus, in order to exploit the positive impact that taxation has on the consumption of the numeraire, it is best to set it above the first-best level ($\hat{\tau} > \tau^*$). When the demand is inelastic ($\eta_{x,\tau} > -1$), the quantity effect is offset by the extra-expenditure effect and the marginal cost of taxation is increasing. To mitigate the very negative impact that taxation has on the consumption of the numeraire it is best to set it below the first-best ($\hat{\tau} < \tau^*$). Finally, if $\eta_{x,\tau} = -1$ the optimal tax $\hat{\tau}$ is $\tau^*$, regardless of $\lambda$.

In Proposition 2 we point to the existence of an upward or downward distortion in taxation, relative to the first-best, necessary to compensate the negative effect of the inefficiency of taxation. In Proposition 3 we quantify the magnitude of such distortion.

**Proposition 3** For all $\lambda$ such that $\hat{\tau} \equiv \hat{\tau}(\lambda)$ is differentiable, the distance between $\hat{\tau}$ and $\tau^*$, $|\hat{\tau} - \tau^*|$, is zero when $\lambda = 0$ and increases as $\lambda$ increases.

The taxation $BT(\tau)$ is maximum when the agent consumes the first-best level of the sin good, $x^*$. Indeed, $BT_{\tau} = [v_\tau(x(\tau)) - c_\tau(x(\tau)) - 1] x_\tau(\tau)$ is zero when $x(\tau) = x^*$. 

13
The above result can be better understood by considering that, when the elasticity is high, a higher inefficiency \( \lambda \) implies a higher leakage of resources following a tax increase, thus calling for a larger upward distortion in taxation relative to its first-best level to compensate it. When the demand is inelastic, instead, taxation is to be set below the first-best. In these circumstances, a higher \( \lambda \) calls for a larger downward distortion in \( \tau \), so as to have a larger saving in resources (the reduced deadweight loss following the tax decrease) and offset the less than proportional increase in the demand of the sin good.

### 4.1 A closed form example

In this section, we extend the example in O’Donoghue and Rabin (2006) to the case in which the tax system is inefficient (\( \lambda > 0 \)). We assume that the future cost from consumption is linear in the amount consumed, that is \( c(x) = cx \), where \( c > 0 \) represents the magnitude of the future health cost relative to the cost of production. We also assume that the utility from consumption, \( v(x) \), takes the following functional form:

\[
v(x) = \frac{x^{1-\gamma}}{1-\gamma},
\]

with \( 0 < \gamma < 1 \). By solving the agent’s optimization problem, the demand for the sin good becomes

\[
x(\tau) = \left( \frac{1}{\beta c + 1 + \tau} \right)^{\frac{1}{\gamma}}.
\]

The first-best consumption of the sin good, obtained when \( \beta = 1 \) and \( \tau = 0 \), is

\[
x^* = \left( \frac{1}{c + 1} \right)^{\frac{1}{\gamma}}.
\]

The first-best level of taxation can be obtained by solving \( x(\tau) = x^* \), and it is equal to

\[
\tau^* = (1 - \beta) c.
\]

Substituting the consumption rule (10) in (8), the social welfare function becomes:

\[
\Omega(\tau) = I + \frac{\gamma - c(1 - \beta - \gamma) + \tau (1 - (1 - \gamma) \lambda)}{(1 - \gamma) (1 + \tau + c\beta)^{\frac{1}{\gamma}}}.
\]

with optimal taxation

\[
\hat{\tau} = \frac{c(1 - \beta (1 + \gamma \lambda)) - \gamma \lambda}{(1 - (1 - \gamma) \lambda)}.
\]
that is positive whenever \( \lambda < \lambda^M \), with \( \lambda^M = \frac{(1-\beta)c}{\gamma(1+\kappa\beta)} \), increasing in the marginal health cost of consumption, \( c \), and increasing in the time inconsistency, \( 1-\beta \).

The example confirms the results obtained in Proposition 1. First, from (12), we see that there is no taxation if \( \beta = 1 \). Moreover, \( \hat{\tau} \) is strictly positive whenever there is time-inconsistency (\( \beta < 1 \)), and, at \( \tau = 0 \), the inefficiency of taxation is not too high relative to the time inconsistency problem that it wants to correct (\( \lambda < \lambda^M \)), as stated in the inequality in Proposition 1. Finally, if taxation is efficient (\( \lambda = 0 \)), the optimal tax is strictly positive for any \( \beta < 1 \).

Furthermore, depending on the elasticity of \( x(\tau) \) with respect to \( \tau \), \( \hat{\tau} \) can be either larger or smaller than \( \tau^* \), thus confirming Proposition 2. In particular, \( \eta_{x,\tau} = \frac{-\tau}{\gamma(1+\tau+\beta c)} \) is increasing in \( \gamma \) so that \( \hat{\tau} \) is lower than \( \tau^* \) if \( \gamma \) is low (\( \eta_{x,\tau} < -1 \)), and higher in the opposite case. Consequently, also the optimal level of consumption \( x(\hat{\tau}) \) can be higher or lower than \( x^* \), depending on the magnitude of \( \gamma \).

To see this, suppose for example that \( \beta = 0.7 \), \( c = 2.1 \) and \( \lambda = 0.2 \). The first-best level of taxation is \( \tau^* = 0.63 \), which corresponds to the first-best consumption \( x^*_{\gamma_L} = 0.0019 \) if \( \gamma = \gamma_L \), where \( \gamma_L = 0.18 \) (\( \eta_{x,\tau} = -1.15 \)), and to \( x^*_{\gamma_H} = 0.24 \) if \( \gamma = \gamma_H \), where \( \gamma_H = 0.80 \) (\( \eta_{x,\tau} = -0.11 \)). In the first case, \( \hat{\tau}_{\gamma_L} = 0.65 > \tau^* \), and the optimal consumption is \( x(\hat{\tau}_{\gamma_L}) = 0.0029 < x^*_{\gamma_L} \). In the second case, \( \hat{\tau}_{\gamma_H} = 0.2446 > \tau^* \), and the optimal consumption is \( x(\hat{\tau}_{\gamma_H}) = 0.287 > x^*_{\gamma_H} \). Last, notice that the distance between \( \hat{\tau} \) and \( \tau^* \) is increasing in \( \lambda \), as predicted by Proposition 3. Indeed, if \( \lambda' = 0.3 > \lambda \), \( \hat{\tau}'_{\gamma_L} = 0.66 > \hat{\tau}_{\gamma_L} \), and \( \hat{\tau}'_{\gamma_H} = 0.04 < \hat{\tau}_{\gamma_H} \).

5 Regulating vices

The program \( P_{\rho} \) that the social planner solves is to choose the level of regulation \( \rho \) that maximizes the experienced utility function (4) subject to the budget constraint \( I - x = z \) and the consumption rule \( x(\rho) \) defined by condition (6).

By substituting the budget constraint and the consumption rule in the experienced utility function, the latter can be written as the difference between the benefit and the cost of regulation:

\[
\Omega(\rho) = \Omega = \left[ v(x(\rho)) - c(x(\rho)) + I - x(\rho) \right] - k(x(\rho), \rho).
\]
The term in square brackets, $BR(\rho)$, represents the benefit of regulation and is given by the utility that would be obtained by inducing a level of consumption $x(\rho) < x^0$ if there was no cost associated with regulation. The second term, $CR(\rho)$, represents the cost induced by the inefficiency of regulation, namely, the reduction in the immediate benefit of current consumption due to the disutility it generates. The social planner’s problem is to choose $\hat{\rho}$ that maximizes the distance between the benefits and costs of regulation.

Similar to the case with taxation, to determine whether it is optimal to regulate the consumption of the sin good, we compare the relative inefficiency of the regulatory instrument with the internality that it aims to correct. To measure the relative inefficiency of the regulatory instrument we consider the ratio between the marginal cost, $k_\rho$, and the marginal benefit of regulation, $x_\rho$: the higher the ratio, the more inefficient the instrument. The internality associated to sin good consumption is again measured by the difference between the social marginal cost associated to sin good consumption, $c_x$, and the private marginal cost, $\beta c_x$.

Proposition 4 states that if at $\rho = 0$ the relative inefficiency of the regulatory instrument is lower than the internality due to the time inconsistency problem, then the optimal regulation, $\hat{\rho}$, is strictly positive.

**Proposition 4** Suppose that regulation is costly ($k_\rho > 0$). Then, if $\beta = 1$, the optimal regulation is $\hat{\rho} = 0$; if $\beta < 1$ and $k_\rho(x^0,0) < (1 - \beta)c_x(x^0)$, the optimal regulation is $\hat{\rho} \in (0,\bar{\rho})$.

If $\beta = 1$, there is no conflict of interest between the social planner and the agent about the consumption level of $x$ and the optimal regulation is zero. If $\beta < 1$, instead, the agent’s consumption is too high and the optimal regulation will be positive as long as the inefficiency of regulation is not too high relative to the time inconsistency problem that it wants to correct.

As for the case with taxation, the benefit from regulation is maximum when the agent consumes the first-best level of the sin good, $x^*$. Define the first-best regulation as the level that induces the agent to consume $x^*$, and denote it by $\rho^*$. Since regulation is costly, one may be induced to think that the social planner will set it to a level below the first-best. This is not always the case. To see why, consider that an increase in $\rho$ has two effects on the disutility

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11 Indeed, $BR_\rho = [v_x(x(\rho)) - c_x(x(\rho)) - 1] x_\rho(\rho)$, and the term in square brackets is zero when $x(\rho) = x^*$.

12 Such $\rho^*$ exists since $x(\rho)$ is a continuous function of $\rho$, $x(0) = x^0$, $x(\rho) = 0$, and $x^* \in [0,x^0]$. 

16
of regulation: a positive indirect effect due to the reduction of the actual immediate benefit from consuming the sin good, $k_x x < 0$; and a negative direct effect, $k_\rho > 0$. If the positive indirect effect prevails on the negative direct one, the disutility of regulation is decreasing and the regulation level that maximizes the distance between the benefits and costs of regulation is larger than $\rho^*$.

**Proposition 5** Assume that $\beta$ is strictly lower than 1 and that $\Omega(\rho)$ is concave. Then $\hat{\rho} \geq \rho^*$ if and only if

$$\frac{k_\rho(x^*, \rho^*)}{|x_\rho(\rho^*)|} \leq (1 - \beta) c_x(x^*).$$

The left hand side of (15) represents the relative inefficiency of the regulatory instrument in the first-best. The right hand side of (15) measures the internality due to the time inconsistency problem, again in the first-best.

Proposition 5 has implications on the levels of consumption. Indeed, since $x(\rho)$ is decreasing in $\rho$, $x(\hat{\rho})$ can be higher or lower than $x^*$ depending on whether $\hat{\rho}$ is higher or lower than the first-best level. Thus, regulation may exceed or fall short of the first-best level according to how rapidly the direct cost of regulation increases relative to its benefit.

5.1 A closed form example

In this section, we modify the above example assuming that the only instrument available to affect the consumption of the sin good is regulation. In particular, the utility from consumption, $v(x) - k(x, \rho)$, takes the following functional form:

$$(1 - \rho) \frac{x^{1-\gamma}}{1-\gamma} - k\rho,$$

with $0 < \gamma < 1$, $k > 0$, and $0 \leq \rho \leq 1$. By solving the agent’s optimization problem, the demand for the sin good becomes

$$x(\rho) = \left( \frac{1 - \rho}{\beta c + 1} \right) \frac{1}{\gamma}.$$  

(16)

The first-best level of regulation can be obtained by solving $x(\rho) = x^*$, where $x^*$ is defined in Eq. (11), and is equal to

$$\rho^* = \frac{(1 - \beta) c}{1 + c}.$$  

(17)
Substituting the consumption rule (16) in (14), the social welfare function becomes:

$$\Omega(\rho) = I + (1 - \rho)^{\frac{1}{\gamma}} \Psi - k\rho,$$

with $$\Psi = \frac{\gamma - c(1 - \beta - \gamma)}{(1 - \gamma)(\beta c + 1)^{\frac{1}{\gamma}}}.$$ If strictly positive, the optimal regulation $$\hat{\rho}$$ can be obtained by solving the first order condition, and it is equal to

$$\hat{\rho} = 1 - \left(\frac{k(1 - \gamma)\gamma(\beta c + 1)^{\frac{1}{\gamma}}}{c(1 - \gamma - \beta - \gamma)}\right)^{\frac{1}{1 - \gamma}}.$$

increasing in the time inconsistency, $$1 - \beta$$.

The example confirms the results obtained in Proposition 4. First, from (17), we see that there is no regulation if $$\beta = 1$$. Moreover, from simple algebra, $$\hat{\rho}$$ is strictly positive whenever there is time-inconsistency ($$\beta < 1$$), and at $$\rho = 0$$ the inefficiency of regulation is not too high relative to the time inconsistency problem that it wants to correct (i.e., $$k < k^M$$, with $$k^M \equiv -\Psi$$), as stated in Proposition 4.

Furthermore, depending on the value of parameters, $$\hat{\rho}$$ can be either larger or smaller than $$\rho^*$$, thus confirming Proposition 5. In particular, it is lower than $$\rho^*$$ if $$k$$ is high, and higher in the opposite case. Consequently, also the optimal level of consumption $$x(\hat{\rho})$$ can be higher or lower than $$x^*$$, depending on the magnitude of the regulation cost. To see this, suppose that $$\gamma = 0.2$$, $$\beta = 0.7$$, $$c = 2.1$$. The first-best level of regulation is $$\rho^* = 0.2032$$, which corresponds to the first-best consumption $$x^* = 0.0035$$. If the regulation cost is small, say $$k = k_L$$, where $$k_L = 0.00007$$, then $$\hat{\rho}_{k_L} = 0.4377 > \rho^*$$, and the optimal consumption is $$x(\hat{\rho}_{k_L}) = 0.0006 < x^*$$. But, if the regulation cost is high, say $$k = k_H$$, where $$k_H = 0.0006$$, then $$\hat{\rho}_{k_H} = 0.026 < \rho^*$$, and the optimal consumption is $$x(\hat{\rho}_{k_H}) = 0.0095 > x^*$$.

6 Taxing and regulating vices

The problem $$\mathcal{P}_{P(\rho, \tau)}$$ that the social planner solves is to choose the consumption bundle $$(x, z)$$, the level of tax $$\tau$$ and the level of regulation $$\rho$$ that maximizes the experienced utility function (4) subject to the budget constraint $$z = I + l - (1 + \tau)x$$, the lump-sum transfer constraint (2) and the consumption rule defined by condition (7).

\textsuperscript{13}$$\Omega(\rho)$$ is concave (and $$\Psi$$ is negative) for all $$\rho \in [0, 1]$$ whenever $$\gamma + \beta < 1$$ and $$c > \frac{\gamma}{1 - \gamma - \beta}.$$
By substituting the budget constraint and the lump-sum transfer constraint in the objective function, the latter reads as:

$$\Omega (x, \rho, \tau) = \left[ v(x) - c(x) + I - x \right] - B(x) - \frac{\lambda \tau x}{CT(x, \tau)} - k(x, \rho).$$ (18)

For any sin good consumption level $x$, and policy pair $(\rho, \tau)$ implementing $x$, the term in square brackets represents the benefit of consumption, $B(x)$. The second term, $C_{r}(x, \tau)$, represents the cost of taxation, while the third term, $C_{\rho}(x, \rho)$, represents the cost of regulation.

Solving the social planner maximization problem is a complex task that requires several technical and unintuitive assumptions, given that the problem might not be quasi-concave. We will therefore simplify the analysis by focusing on the problem of choice of the optimal taxation and regulation policy $(\hat{\rho}(x), \hat{\tau}(x))$ that minimizes the social cost of inducing a level of consumption of the sin good $x \leq x^a$, defined by Eq. (7), i.e.,

$$\left(\hat{\rho}(x), \hat{\tau}(x)\right) = \text{arg min}_{\tau \geq 0, \rho \in [0, \hat{\rho}]} \{CT(x, \tau) + CR(x, \rho) : v_x(x) - k_x(x, \rho) - \beta c_x(x) - (1 + \tau) = 0\}. \quad (19)$$

Notice that the objective function in (19) can be interpreted as an isocost function, i.e., the set of all policy combinations generating equal cost, while the constraint can be interpreted as an isoquant, that is the set of all possible combinations of policies that induce the agent to consume the level $x$ of sin good.

For the problem to be well-behaved, we assume that the effect of regulation on the agent’s level of experienced utility is decreasing in $\rho$ ($u_{\rho}^s = -k_{\rho} < 0$), and that the effect of regulation on the agent’s marginal decision utility is increasing in $\rho$ ($u_{\rho}^a = -k_{\rho} > 0$). These assumptions imply that the isocost curves are decreasing and concave, while the isoquant is decreasing and convex.

In the $\rho, \tau$—plane, the solution $(\hat{\tau}(x), \hat{\rho}(x))$ of the social planner minimization problem lies on the isocost curve that intersects the isoquant closest to the origin.

Denote by $\hat{\tau}(x)$ and $\hat{\rho}(x)$ respectively the level of taxation and regulation needed to induce the consumption $x$ when only one instrument is used, that is $\hat{\tau}(x)$ solving the isoquant constraint when $\rho = 0$, and $\hat{\rho}(x)$ solving the same constraint when $\tau = 0$.

---

14 We leave the full characterization of the social planner maximization problem to a numerical example.

15 Notice that while the constraint is convex (the isoquant), the objective function (the iso-cost) is non-linear, because of the concavity of the cost of regulation, and it is a function of $x$. 

19
An interior solution obtains when the isocost closest to the origin is tangent to the isoquant, i.e., when

\[
\frac{k(x, \rho(x))}{\lambda x} = k_{xp}(x, \rho(x)).
\]

A corner solution with \( \rho = 0 \) obtains when the slope of the isocost closest to the origin is higher than the slope of the isoquant, i.e., when

\[
\frac{k(x, 0)}{\lambda x} > k_{xp}(x, 0).
\]

This allows us to define a lower bound for \( \lambda \) below which only taxation is used, \( \lambda_L(x) \equiv k(x, 0) / (xk_{xp}(x, 0)) \).

Last, a corner solution with \( \tau = 0 \) obtains when the slope of the isocost closest to the origin is lower than the slope of the isoquant, i.e., when

\[
\frac{k(x, \hat{\rho}(x))}{\lambda x} < k_{xp}(x, \hat{\rho}(x)).
\]

This allows us to define an upper bound for \( \lambda \) above which only regulation is used, \( \lambda_H(x) \equiv k(x, \hat{\rho}(x)) / (xk_{xp}(x, \hat{\rho}(x))) \).

From the above, Proposition 6 can be derived.

**Proposition 6** Assume that \( \beta \) is strictly lower than 1 and that the social planner cost minimization problem is well-behaved. Then, for any \( x \in (0, x^\alpha] \), if

1. \( \lambda \geq \lambda_H(x) \) only regulation is used by the social planner and \( \rho = \hat{\rho}(x) \);
2. \( \lambda_L(x) < \lambda < \lambda_H(x) \), taxation and regulation are both used by the social planner are both smaller than the levels that would be used when each instrument is used in isolation;
3. \( \lambda \leq \lambda_L(x) \) only taxation is used by the social planner and \( \tau = \hat{\tau}(x) \).

From Proposition 6 it follows that, depending on the cost of taxation, different scenarios can arise. For \( \lambda \) sufficiently low (\( \lambda < \lambda_L(x) \)), the regulator uses only taxation, as the inefficiency of taxation is not too high to justify the use of regulation. As \( \lambda \) increases (\( \lambda_L(x) \leq \lambda \leq \lambda_H(x) \)), there is an interval of values of \( \lambda \) in which the regulators prefers to introduce regulation. This is because the inefficiency of taxation is such that the regulator may save on these costs by using regulation. As \( \lambda \) increases beyond \( \lambda_H(x) \), the inefficiency of taxation is so high that the regulator prefers to give up taxation altogether and use only regulation.
6.1 A closed form example

We now modify the previous examples assuming that both taxation and regulation may be used to affect the consumption of the sin good. We develop a numerical example to illustrate the results of the cost minimization problem (19) and to derive the full characterization of the social planner maximization problem. The utility from consumption, \( v(x) - k(x, \rho) \), takes the following functional form:

\[
A \frac{x^{1-\gamma}}{1-\gamma} + (1 - \rho) \frac{x^\alpha}{\alpha} - k\rho^2,
\]

where \( A > 0 \), \( \alpha > 1 \), and the future cost from consumption is quadratic in the amount consumed, that is \( c(x) = cx^2 \), where \( c > 0 \). Using these functional forms, the cost of taxation is \( CT(x, \tau) = \lambda \tau x \), while the cost of regulation is \( CR(x, \rho) = \rho \left( \frac{x^\alpha}{\alpha} + k\rho \right) \). The isoquant constraint in programme (19), i.e., the set of all possible combinations of policies that induce the agent to consume the level \( x < x^a \), is given by:

\[
Ax^{-\gamma} + (1 - \rho)x^{\alpha-1} - 2\beta cx - (1 + \tau) = 0.
\]

By solving the isoquant constraint when \( \rho = 0 \), gives \( \hat{\tau}(x) = x^{\alpha-1} - (1 + 2\beta cx - A x^{-\gamma}) \), and by solving it when \( \tau = 0 \) gives \( \hat{\rho}(x) = 1 - x^{1-\alpha}(1 + 2\beta cx - A x^{-\gamma}) \).

By solving the cost minimization problem, we find two threshold values, \( \lambda_L(x) = 1/\alpha \) and \( \lambda_H(x) = 1/\alpha + \rho(x)2kx^{-\alpha} \) such that:

\[
(\hat{\tau}(x), \hat{\rho}(x)) = \begin{cases} 
(\hat{\tau}(x), 0) & \text{for all } \lambda \leq \lambda_L(x); \\
(\tau(x), \rho(x)) & \text{for all } \lambda_L(x) < \lambda \leq \lambda_H(x); \\
(0, \rho(x)) & \text{for all } \lambda > \lambda_H(x).
\end{cases}
\]

In order to find a numerical solution, we assume that \( A = 10 \), \( \gamma = 0.2 \), \( \alpha = 3 \), \( \beta = 0.7 \), \( c = 13 \), \( k = 0.0007 \). Under these parameter values, \( x^a = 0.5767 \), \( \lambda_L(x) = \frac{1}{3} \), and \( \lambda_H(x) = \frac{1}{3} + \frac{7}{50} \left( x^{-\frac{26}{5}} - x^{-\frac{3}{10}} - \frac{91x^{-4}}{50} + x^{-3} \right) \). From the above we see that if, for example, the quantity that the social planner wants to implement is \( x = 0.565 \), then \( \lambda_H(0.565) = 0.3931 \), and the taxation and regulation policy \((\hat{\tau}(x), \hat{\rho}(x))\) that minimizes the social costs of inducing \( x = 0.565 \) will be \((0.2458, 0)\) if \( \lambda = 0.30 \); \((0.1773, 0.2147)\) if \( \lambda = 0.35 \); and \((0, 0.8589)\) if \( \lambda = 0.40 \).

---

\[\text{All our main assumptions are satisfied for } \rho \in [0, 1] \text{ and } x \text{ low enough.}\]

\[\text{The interior value of the policies is given by } \tau(x) = x^{\alpha-1} - (1 + 2\beta cx - A x^{-\gamma}) - \frac{x^{2\alpha-1}(a\lambda - 1)}{2ak} \text{ and } \rho(x) = \frac{x^{\alpha}(a\lambda - 1)}{2ak}.\]
To conclude, assume that $\lambda = 0.35$ and analyze the social planner maximization problem. $\lambda_H(x) \geq \lambda$ for all $x \leq 0.5732$. Therefore, for all $x \leq 0.5732$, the minimum cost function, $\Gamma(x)$, can be obtained by substituting $\tau(x)$ and $\rho(x)$ in $CT(x, \tau) + CR(x, \rho)$, and is given by

$$\Gamma(x) \equiv 3.5x^\frac{4}{5} - 0.35x - 6.37x^2 + 0.35x^3 - 0.0099x^6.$$ 

By substituting $v(x)$, $c(x)$, and $\Gamma(x)$ in the social planner objective function (18), the latter reads as

$$\Omega(x) = 9x^\frac{4}{5} - 0.65x - 6.63x^2 - 0.0167x^3 + 0.0099x^6,$$

that is maximum when $x = 0.5598$. The optimal policy to induce $x = 0.5598$ is $\tau(0.5598) = 0.2899$ and $\rho(0.5598) = 0.2088$.

7 Discussion and conclusion

We have studied the optimal sin taxes and regulatory measures that a social planner wants to implement when consumers are time-inconsistent. There is an inefficiency associated with regulation, in terms of the disutility it generates on consumption, and an inefficiency associated with taxation, in terms of administrative, collection and compliance costs. We find that both instruments should be used, provided the inefficiency associated with either is not too high.

We thus provide a theory that rationalizes the widespread evidence of the joint use of taxation and regulation, as witnessed, for instance, by plenty of examples on tobacco, alcohol consumption or gambling.

The analysis so far has been carried out ignoring other factors that might influence the effectiveness of each instrument. First, a sin tax is effective only if it is fully transmitted to the consumer (only if it succeeds in limiting consumption). Whether or not this happens depends not only on demand price-elasticity, but also on various other factors, like the supply price elasticity, the availability of substitutes, the supplier response to the tax. For example, if the supply is relatively inelastic, an increase in tax rate will be borne largely by firms and will have small effects in terms of quantity reduction. Similarly, if the demand is inelastic, a tax will have a small quantity effect and a large price effect. In such circumstances, regulatory measures can be more effective in reducing harmful consumption.
Second, as argued above, government action is not driven solely by efficiency considerations, but also by general public pressures. As shown by Johnson and Meier for US states (1990), regulatory measures and tax rates on sin goods are affected by cultural and religious factors and by industry lobbying. For instance, the tobacco industry has had a strong negative effect on cigarette tax rates. As regards cultural and religious factors, they show that the prevalence of Catholic groups are associated with legalized gambling and low alcohol taxes.

A noteworthy episode of the impact of public pressures on government action is the introduction in Denmark of the world’s first fat tax, in line with the evidence suggesting a larger responsiveness of eating behaviour to price increases than to nutritional education (Horgen and Brownell, 2002). The policy consisted in imposing a tax on saturated fats in foods with saturated fat content larger than 2.3g/100 g (The Economist, 2012). Despite the encouraging evidence on the positive effect of the tax in reducing consumption (10-15 % reduction in consumption of fats, as documented by Jensen and Smed, 2013), in 2012, following the campaign organized by a coalition of Danish food businesses, the Danish government dropped the fat tax and abandoned an impending tax sugar. According to Nestle (2012), the reason for the tax drop was to appease business interests.19

Finally, since our model can be seen as a normative rather than a positive theory of taxing and regulating vices, it is an empirical issue to verify how much efficiency considerations are the driving force of the legislators action. To verify the extent to which these considerations are central to the government action, one could test whether countries with more inefficient tax systems (λ high) rely more heavily on regulatory instruments relative to those with more efficient tax systems (λ low). Alternatively, for given tax systems (λ given), one could compare different interventions aimed at affecting the consumption of different sin goods. We leave the empirical verification of these predictions to future research.

19 Similarly, more recently the Finnish government rescinded a planned tax increase for 2015 in the sweets tax due to pressures coming from the Finnish Food and Drinks Industries Federation.
Appendix

Proof of Proposition 1. We will first prove that if $\beta = 1$ the optimal tax is 0, and then we will show that if $\beta < 1$ and $\lambda$ is small, it is strictly positive. For all $\tau \geq 0$, as long as $v$ and $c$ are thrice differentiable, $\Omega(\tau)$ is continuous and twice differentiable. If strictly positive, $\hat{\tau}$ satisfies the first order condition

$$\partial \Omega/\partial \tau = \Omega_x(\tau)x_x(\tau) - \lambda x(\tau) = 0,$$

(20)

where $\Omega_x(\tau) = [v_x(x(\tau)) - c_x(x(\tau)) - 1 - \lambda \tau]$. From (3) we derive $\Omega_x(\tau) = \tau(1 - \lambda) - (1 - \beta)c_x(x(\tau))$. If $\beta = 1$, $\partial \Omega/\partial \tau = \tau(1 - \lambda)x_x(\tau) - \lambda x(\tau) < 0$ for all $\tau \geq 0$, and so the optimal tax is the corner solution $\hat{\tau} = 0$. Suppose instead $\beta < 1$. In this case, $\Omega(\tau)$ may not be quasi-concave. However, when $\tau = 0$, $\partial \Omega/\partial \tau = -(1 - \beta)c_x(x^a)/(v_{xx}(x^a) - \beta c_{xx}(x^a)) - \lambda x^a$, which is positive if $\lambda x^a/|x_x(0)| < (1 - \beta)c_x(x^a)$. Hence, $\tau = 0$ cannot be a corner solution of the social planner maximization problem. Moreover, Inada conditions for $v(x)$ together with $c_x(0) = 0$ imply $\lim_{\tau \to -\infty} x(\tau) = 0$ and $\lim_{\tau \to -\infty}(\partial \Omega/\partial \tau) = -\infty$. Hence, by continuity of $\Omega(\tau)$, there exists at least one $\hat{\tau} > 0$ satisfying condition (20).

Proof of Proposition 2. The first order condition (20) can be written as

$$[v_x(x(\hat{\tau})) - c_x(x(\hat{\tau}) - 1] = \lambda \hat{\tau} \left( \frac{1}{\eta_{x,\tau}} + 1 \right),$$

(21)

where $\eta_{x,\tau} = \frac{x_x(\tau)}{x_x(\hat{\tau})}$. Since $x_x(\tau) < 0$, the right-hand side of (21) equals 0 iff $\eta_{x,\tau} = -1$, is positive iff $\eta_{x,\tau} < -1$ and negative in the opposite case. The left-hand side of (21) equals 0 iff $\tau = \tau^*$, is positive iff $\tau > \tau^*$ and negative in the opposite case.

Proof of Proposition 3. Assume $\lambda = 0$. Substituting in (20) gives $d\Omega/d\tau = \Omega_x(\tau)x_x(\tau) = 0$, with $\Omega_x(\tau) = \tau - (1 - \beta)c_x(x(\tau))$. Since $\tau^* = (1 - \beta)c_x(x(\tau^*))$, then $d\Omega(\tau^*)/d\tau = 0$ and $\hat{\tau} = \tau^*$.

Assume $\lambda > 0$. The derivative of $|\hat{\tau} - \tau^*|$ with respect to $\lambda$ is

$$\frac{\partial |\hat{\tau} - \tau^*|}{\partial \lambda} = \frac{\partial \hat{\tau}}{\partial \lambda} \text{sgn}(\hat{\tau} - \tau^*).$$

By the envelope theorem,

$$\frac{\partial \hat{\tau}}{\partial \lambda} = \frac{x(\hat{\tau}) + \hat{\tau}x_x(\hat{\tau})}{x_x[\eta_{xx}-c_{xx} + \eta_{xx}^2/c_{xx}(v_x - c_x - 1 - \lambda \hat{\tau}) - 2\lambda]}.$$

The denominator is negative by the local concavity of the objective function into a neighborhood of $\hat{\tau}$. Then $\partial \hat{\tau}/\partial \lambda \geq 0$ iff $x(\hat{\tau}) + \hat{\tau}x_x(\hat{\tau}) \leq 0$. Moreover, since $x(\hat{\tau}) + \hat{\tau}x_x(\hat{\tau}) \leq 0$ iff $\eta_{x,\tau} < -1$,
by Proposition 2 $\partial \hat{\tau} / \partial \lambda \geq 0$ iff $\hat{\tau}$ is larger than $\tau^*$. Hence, $\text{sgn}(\partial \hat{\tau} / \partial \lambda) = \text{sgn}(\hat{\tau} - \tau^*)$ and the derivative of the absolute value of the distance between $\hat{\tau}$ and $\tau^*$ is always positive. ■

**Proof of Proposition 4.** We will first prove that if $\beta = 1$ the optimal regulation is 0, and then we will show that if $\beta < 1$ and $k_p(x^a,0)/|x_\rho(0)| < (1 - \beta)c_x(x^a)$, it is strictly positive. For all $\rho \in [0,\overline{\rho}]$, as long as $v$, $k$, and $c$ are thrice differentiable, $\Omega(\rho)$ is continuous and twice differentiable. If strictly positive, $\hat{\rho}$ satisfies the first order condition

$$
\frac{\partial \Omega}{\partial \rho} = \Omega_x(\rho)x_\rho(\rho) - k_p(x(\rho),\rho) = 0,
$$

where $\Omega_x(\rho) = [v_x(x(\rho)) - k_x(x(\rho),\rho) - c_x(x(\rho)) - 1]$. From (6), we can derive $\Omega_x(\rho) = (1 - \beta)c_x(x(\rho))$. If $\beta = 1$, $\Omega_x(\rho) = 0$ and $\partial \Omega / \partial \rho = -k_p(x(\rho),\rho) < 0$ by assumption. Thus, $\hat{\rho} = 0$. Suppose instead $\beta < 1$. In this case, $\Omega(\rho)$ may not be quasi-concave. However, when $\rho = 0$, $\partial \Omega / \partial \rho = -(1 - \beta)c_x(x^a)x_\rho(0) - k_p(x^a,0)$, which is positive whenever $k_p(x^a,0)/|x_\rho(0)| < (1 - \beta)c_x(x^a)$. Hence, $\rho = 0$ cannot be a corner solution of the social planner maximization problem. Moreover, Assumptions (A2) and (A3) together with $c_x(0) = 0$ imply that when $\rho = \overline{\rho}$, $\partial \Omega / \partial \rho < 0$ for all $x$. Hence, by continuity of $\Omega(\rho)$, there exists at least one $\hat{\rho} \in (0,\overline{\rho})$ satisfying condition (22). ■

**Proof of Proposition 5.** Let be $CR_\rho(\rho) = k_x(x(\rho),\rho)x_\rho + k_p(x(\rho),\rho)$ and assume that $\beta < 1$ and $\partial^2 \Omega / \partial \rho^2 \leq 0$. We will prove the proposition in two steps. In the first step we will show that $k_p(x^*,\rho^*)/|x_\rho(\rho^*)| < (1 - \beta)c_x(x^*)$ if and only if $CR_\rho(\rho^*)$ is positive. In the second step we will prove that $\hat{\rho} < \rho^*$ if and only if $CR_\rho(\rho^*)$ is positive.

**Step 1.** $k_p(x^*,\rho^*)/|x_\rho(\rho^*)| < (1 - \beta)c_x(x^*) \iff CR_\rho(\rho^*) > 0$ : From condition (6), $k_x(x^*,\rho^*) = v_x(x^*) - 1 - \beta c_x(x^*) = (1 - \beta)c_x(x^*)$ since $v_x(x^*) - 1 = c_x(x^*)$ by definition of $x^*$. Hence, $CR_\rho(\rho^*)$ can be written as $(1 - \beta)c_x(x^*)x_\rho(\rho^*) + k_p(x^*,\rho^*)$, that is positive if and only if $k_p(x^*,\rho^*)/|x_\rho(\rho^*)| < (1 - \beta)c_x(x^*)$.

**Step 2.** $CR_\rho(\rho^*) > 0 \iff \hat{\rho} < \rho^*$: To prove the step we will first show that $CR_\rho(\rho^*) > 0$ implies $\hat{\rho} < \rho^*$, and then we will prove that if $\hat{\rho} < \rho^*$ then $C_\rho(\rho^*) > 0$. Assume $C_\rho(\rho^*) > 0$. Since the problem is concave by assumption, $\hat{\rho} \in [0,\overline{\rho})$ solving program $P_{BR}(\rho)$ exists. If $k_p(x^a,0)/|x_\rho(0)| \geq (1 - \beta)c_x(x^a)$, then $(\partial \Omega / \partial \rho)(0) = BR_\rho(0) - CR_\rho(0) < 0$ and $(\partial \Omega / \partial \rho)(\rho) = BR_\rho(\rho) - CR_\rho(\rho) < 0$ for all $\rho \geq 0$ since $\partial^2 \Omega / \partial \rho^2 < 0$ by assumption. Hence, $\hat{\rho} = 0 < \rho^*$. From Proposition 4 we know that if $k_p(x^a,0)/|x_\rho(0)| < (1 - \beta)c_x(x^a)$, then $\hat{\rho}$ is
strictly positive and satisfies the first order condition (22), that can be written as

$$BR_{\rho} (\hat{\rho}) = CR_{\rho} (\hat{\rho}),$$

(23)

where $BR_{\rho} (\rho) = [v_x (x (\rho)) - c_x (x (\rho)) - 1] x_{\rho}$. Since $x_{\rho}(\rho) < 0$, the left side of (23) is equal to 0 in $\rho = \rho^*$, it is positive if $\rho < \rho^*$ and negative in the opposite case. The function $(\partial \Omega / \partial \rho) (\rho) = BR_{\rho} (\rho) - CR_{\rho} (\rho)$ is continuous and decreasing. Since $(\partial \Omega / \partial \rho) (\rho^*) = -CR_{\rho} (\rho^*) < 0$ by assumption, and $(\partial \Omega / \partial \rho) (0) > 0$ when $k_{\rho} (x^a, 0) / |x_{\rho} (0)| < (1 - \beta) c_x (x^a)$, there exists $\rho' \in (0, \rho^*)$ such that $(\partial \Omega / \partial \rho) (\rho') = 0$ and then $\hat{\rho} = \rho' < \rho^*$. Now assume $\hat{\rho} < \rho^*$. By the first order condition $(\partial \Omega / \partial \rho) (\hat{\rho}) \leq 0$. Since $\partial^2 \Omega / \partial \rho^2 < 0$ by assumption, $(\partial \Omega / \partial \rho) (\rho) < 0$ for all $\rho > \hat{\rho}$. This implies $(\partial \Omega / \partial \rho) (\rho^*) = -CR_{\rho} (\rho^*) < 0$, and then $CR_{\rho} (\rho^*) > 0$. From Step 1 it follows that $\hat{\rho} < \rho^*$ if and only if $k_{\rho} (x^a, \rho^*) / |x_{\rho} (\rho^*)| < (1 - \beta) c_x (x^a)$.

**Proof of Proposition 6.** Assume that $k_{pp} > 0$, and $k_{xpp} < 0$. By substituting the consumption rule in the objective function, the social planner problem becomes to choose $(\hat{\rho} (x), \hat{\tau} (x))$ such that:

$$\hat{\rho} (x) \in \arg \min \{ \nu (\rho) \equiv \lambda x (\hat{\tau} (x) - k_x (x, \rho)) + k (x, \rho) : \rho \in [0, \hat{\rho} (x)] \},$$

(24)

and

$$\hat{\tau} (x) = \tau (x) - k_x (x, \hat{\rho} (x)),$$

(25)

A solution for problem (24) always exists since the objective function is convex ($-\lambda x k_{xpp} + k_{pp} > 0$ by assumption) and the constraint is a compact set. A necessary and sufficient condition for an interior solution is

$$-\lambda x k_{xp} (x, \rho) + k_{\rho} (x, \rho) = 0.$$  

(26)

Then, if $\lambda \leq \lambda_L (x)$, $\partial \nu (\rho) / \partial \rho \geq 0$ for all $\rho$, and $\hat{\rho} (x) = 0$ and $\hat{\tau} (x) = \hat{\tau} (x)$; if $\lambda < \lambda_L (x)$, $\partial \nu (\rho) / \partial \rho < 0$ at $\rho = 0$ and $\partial \nu (\rho) / \partial \rho > 0$ at $\rho = \hat{\rho} (x)$, and the optimal policy is $\hat{\rho} (x) \in (0, \hat{\rho} (x))$ satisfying condition (26) and $\hat{\tau} \in (0, \hat{\tau} (x))$ given by condition (25); if $\lambda \geq \lambda_H (x)$, $\partial \nu (\rho) / \partial \rho \leq 0$ for all $\rho$, and $\hat{\rho} (x) = \hat{\rho} (x)$ and $\hat{\tau} (x) = 0$. ■
References


