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The Value of Transparency in Dynamic Contracting with Entry

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Abstract

A manufacturer designs a dynamic contract with a retailer who is privately informed about demand and faces competition by an integrated entrant in a second period. Since the entrant only observes demand after entry and demand is correlated across periods, information about past demand affects the entrant's production. We analyze the incentives of the incumbent players to share information with the entrant and show that the retailer benefits from transparency, but the manufacturer does not. Contrary to what intuition suggests, transparency with an integrated entrant harms consumers. When the entrant is not an integrated firm, whether transparency benefits consumers depends on the degree of demand persistency.

Keywords: Dynamic Adverse Selection, Entry, Information Sharing, Transparency, Vertical Contracting.

JEL Classification: D40, D82, D83, L11.

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1 Introduction

Late or sequential entry is common in many industries (e.g., Geroski, 1995). In concentrated markets, for example, entry often occurs only when regulatory intervention breaks up incumbents' dominant position and creates scope for competition. In emerging markets, high-quality innovators often choose to delay production and allow early entry by low-quality firms, in order to gather information about demand and develop products that better fit consumers' needs (e.g., Dutta *et al.*, 1995). In industries where economies of scale restricts competition, entry is often subsidized by public policy aimed at reducing market concentration (e.g., Bernheim, 1984).

When incumbents anticipate the threat of future entry, they may engage in anticompetitive practices that weaken entrants and protect market power. Limit pricing, excessive patenting, capacity building, exclusive dealings and other forms of vertical restraints are well known examples of barriers to entry that protect monopoly power and damage consumers.¹ Lack of transparency is also considered a source of abuse of dominant position, which benefits incumbents at the expense of potential entrants. In telecoms, gas, electricity and many retail industries, for instance, lack of transparency is cited as an obstacle to competition insofar as it restrains access to the market by new firms (e.g., Kroes, 2007; Coen and H  ritier, 2000).

Surprisingly, even though the existing literature extensively analyzes the welfare effects of information sharing in static models of oligopoly (see, e.g., Vives, 2006, for a survey), little is known on the effects of transparency in dynamic environments, where incumbents may strategically disclose or hide information to new entrants. Hence, even though the common wisdom is that increasing incumbents' transparency should facilitate entry and benefit consumers, there are no models that explicitly analyze this presumption.

Do incumbents have an incentive to share their private information with future competitors? Does communication between incumbents and entrants benefit consumers? How does vertical contracting affect these issues?

We analyze a dynamic vertical contracting environment in which a manufacturer deals with an exclusive retailer for two periods. In the first period the retailer is a monopolist in the downstream market, while in the second period it faces competition by an integrated entrant.² Firms sell a homogeneous product whose demand is uncertain and, in every period, is privately observed by the downstream players — i.e., the retailer and the entrant. The manufacturer designs a long-term contract to elicit the retailer's private information and, since demand is correlated over time, the retailer's second-period production depends on its report in the first period (that the manufacturer uses to update its beliefs about demand) — i.e., the optimal dynamic contract features memory (e.g., Baron and Besanko, 1984; Laffont and Tirole, 1996; Battaglini, 2005).

¹See, e.g., Aghion and Bolton, (1987); Gilbert and Harris, (1984); Hart *et al.*, (1990); Hoppe, (2002); Milgrom and Roberts, (1982); Ordover *et al.*, (1990); Ziss, (1996).

²We also consider entry by a nonintegrated firm — see below.

The long-term contract and the incumbents' profit depend on the information possessed by the entrant. Specifically, because the entrant is uninformed about demand in the first period and the incumbent's production in the second period depends on the retailer's first-period report, sharing information about this report (or directly about first-period production) affects the entrant's production and competition.³ Hence, the long-term interaction between the incumbent manufacturer and retailer creates a contractual link between periods, and the role of information sharing hinges on this link.

We show that the manufacturer and the retailer have diverging incentives to share information: the retailer would like to commit to inform the entrant about its first-period report, while the manufacturer has no incentive to disclose this information. The reason is that the manufacturer's profit and the retailer's information rent are affected by the entrant's production in opposite ways.

When the entrant is informed, it faces lower uncertainty about the incumbent's production and, hence, it produces a relatively less volatile output. This weakens the *competition effect* (highlighted in Martimort, 1996) and increases the retailer's rent in the second period compared to a situation without information sharing. By contrast, the manufacturer has no incentive to share information for two reasons. First, sharing information is detrimental to the manufacturer because it increases the retailer's rent. Second, *ceteris paribus*, the incumbent's profit is lower with information sharing because the entrant is more aggressive on average when it is informed: a business stealing effect.⁴ Moreover, the manufacturer does not disclose information to the entrant even if it can sell it, since the entrant's willingness to pay for information is always lower than the manufacturer's reservation price.

Sharing information reduces the incumbent's production because, other things being equal, it induces the entrant to increase production when the incumbent distorts it for rent extraction reasons. On balance, however, aggregate production is lower with information sharing because, holding constant the incumbent's production, the entrant's decision rule is always efficient regardless of its information (since the entrant equalizes marginal revenue to marginal cost). In other words, although information sharing rebalances production between the incumbent and the entrant, it reduces market efficiency because it increases the retailer's rent via the competition effect. Therefore, consumer surplus and welfare are always lower with information sharing and, contrary to what is commonly believed, with an integrated entrant a welfare maximizing policy should reduce transparency and forbid incumbents to disclose information to entrants.

Our results hold even when the incumbents can disclose noisy information to the entrant,

³Notice that the entrant is only interested in information that signals the quantity produced by the retailer in the second period (like its report or the quantity produced in the first period), rather than information about past demand *per se*.

⁴This echoes the findings of the literature on information sharing in oligopoly — see, e.g., Gal-Or (1985), Li (1985), Shapiro (1986) and Vives (1984) — showing that with Cournot competition firms do not share information about demand because this increases correlation among their decisions, whereby reducing output and profits.

by only letting the entrant observe an imperfect signal (whose precision is chosen by the incumbents) of the retailer’s first-period report or production.

We also consider entry by a manufacturer that sells through its exclusive retailer, rather than by an integrated firm, so that there are two competing hierarchies in the market in the second period. The new entrant does not observe demand in the first period, but demand in the second period is observed by both retailers, so that both manufacturers need to design contracts to elicit truthful information from them.

As in our main model, we show that the incumbent retailer prefers to share information about the retailer’s first-period report, while the incumbent manufacturer does not want to do so. With competing hierarchies, however, transparency may increase consumer surplus and welfare because information sharing allows the entrant manufacturer to elicit information from its retailer at a lower cost, which increases the entrant’s output and tends to increase market efficiency. Since the incumbent reacts by reducing its own production, the effect of information sharing on aggregate production depends on the degree of demand persistency, which affects the entrant’s information about first-period demand without information sharing. Hence, with competing hierarchies transparency has an ambiguous effect on welfare. Moreover, in this case the incumbent may have an incentive to sell information to the entrant.

The rest of the paper is organized as follows. After discussing the existing literature, Section 2 describes the baseline model and Section 3 analyzes the entrant’s problem and discusses benchmarks without asymmetric information and without entry in the second period. Section 4 provides the equilibrium analysis with and without information sharing. In Section 5, we describe the incumbents’ incentives to share information and in Section 5.1 we analyze a market for information. Welfare is discussed in Section 6. We then consider various extensions: Section 7.1 considers competing hierarchies, Section 7.2 analyzes stochastic disclosure rules, Section 7.3 discusses ex post information sharing, and Section 7.4 extends the analysis to large uncertainty. The last section concludes. All proofs are in the Appendix.

Related Literature. We build on and contribute to three strands of literature. First, our paper relates to the literature on dynamic contracting with types correlated over time and full commitment by the principal. Baron and Besanko (1984) first characterize optimal contracts in a two-period environment. Laffont and Tirole (1996) applied dynamic contracting with adverse selection to the regulation of pollution rights and provided an interpretation of the optimal mechanisms in terms of markets with options. More recently, stemming from Battaglini (2005), the literature has evolved to multi-period models (both with discrete and continuous types) to investigate the memory and complexity of optimal dynamic contracts after long histories, convergence to efficiency, the effects of learning by doing, risk aversion and renegotiation, the limits of the ‘first-order’ approach, the impact of dynamics and enforcement risk on the contract incompleteness and stationarity (Arve and Martimort, 2016;

Battaglini and Coate, 2008; Battaglini and Lamba, 2015; Garrett and Pavan, 2012; Gennaioli and Ponzetto, 2017; Eső and Szentes, 2017; Martimort *et al.*, 2017; Pavan *et al.*, 2014).⁵ In our two-period model, most of these technical issues are not present: we chose to analyze a simple contracting environment to focus on the relationship between dynamics, transparency and product market competition.

Second, our analysis is related to the IO literature on information sharing in oligopoly. This literature shows that firms' incentives to share information about their common demand function (Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Gal-Or, 1985) or about their private costs of production (Fried, 1984; Gal-Or, 1986; Shapiro, 1986) depend on the nature of competition. Raith (1996) rationalizes the results of this vast literature in a unified framework. In contrast to our model, this literature typically assumes that firms are *ex ante* symmetric and play simultaneously. By introducing dynamic incentives and sequential entry, we introduce an endogenous asymmetry between firms that depends on the incumbent's contract and information sharing decision, which affect the entrant's behavior.⁶ Hence, the key feature of our environment is vertical contracting, which creates an endogenous relationship between information and competition. Without the contracting dimension, information sharing would play no role since firms' production in every period would only depend on current demand.⁷

Finally, we contribute to the literature on communication between vertical hierarchies, with endogenous information that principals have to obtain from privately informed retailers. Calzolari and Pavan (2006) first study this problem in a sequential common agency model where principals may share the information obtained by contracting with a common agent. They show how the information disclosed by one principal affects the contractual relationships between other players and analyze when a principal wants to offer full privacy to the agent.⁸ When contracts are exclusive, Piccolo and Pagnozzi (2013) show that sharing information about costs affects contracting within competing organizations and induces agents' strategies to be correlated through the distortions imposed by principals to obtain information. In this environment, the incentives to share information depend on the nature of upstream externalities between principals and the correlation of agents' information.⁹ In contrast to our model, both these papers focus on static contracts.

⁵See Bergemann and Pavan (2015) for a survey of the dynamic contracting literature.

⁶While in most of the existing literature on information sharing firms symmetrically exchange information and reciprocally learn about each other's characteristics, in our model the decision to share information is unilaterally taken by the incumbent, who has perfect information about the entrant.

⁷The entrant observes demand in the second period if it is integrated, or it learns it from its retailer.

⁸See also Bennardo *et al.* (2015) and Maier and Ottaviani (2009) for common agency models with moral hazard and communication.

⁹See also Piccolo *et al.* (2015) for a model with moral hazard and communication between competing hierarchies.

2 The Model

Players and Environment. A manufacturer M contracts with an exclusive retailer R for two periods. The manufacturer supplies a fundamental input to the retailer, which is used to produce a final good. There are constant returns to scale and the marginal cost of production is normalized to zero.

In the first period $\tau = 1$, R is a monopolist in the downstream market. In the second period $\tau = 2$, an integrated firm E enters the market.¹⁰ For example, the long term relationship between M and R may arise because of the need to use a retailer with specific skills to customize a new product, requiring a fixed investment that is not worth paying for one period only. By contrast, an entrant may not need a specialized retailers once the product ‘standard’ has been developed by the incumbent. There are no entry costs.¹¹

The inverse demand function in period $\tau = 1, 2$ is

$$P(\theta_\tau, Q_\tau) \triangleq \max\{0, \theta_\tau - Q_\tau\},$$

where Q_1 is R ’s production in the first period and $Q_2 \triangleq q_2 + q_E$ is aggregate production in the second period — i.e., the sum of R ’s second-period production q_2 and E ’s production q_E . The parameters $\theta_\tau \in \Theta \triangleq \{\underline{\theta}, \bar{\theta}\}$ is a measure of the magnitude of demand, with $\Delta\theta \triangleq \bar{\theta} - \underline{\theta} > 0$. The assumption of a linear demand function is standard in the literature on information sharing but it is not necessary for most of our results.¹²

Demand is correlated across periods. We assume that $\Pr[\theta_1 = \bar{\theta}] = \frac{1}{2}$, and let

$$\Pr[\theta_2 = \bar{\theta} | \theta_1 = \bar{\theta}] \triangleq \bar{\nu}, \quad \Pr[\theta_2 = \underline{\theta} | \theta_1 = \underline{\theta}] \triangleq \underline{\nu},$$

where $(\bar{\nu}, \underline{\nu}) \in [0, 1]^2$ and $\Delta\nu \triangleq \bar{\nu} - \underline{\nu}$. The parameters $\bar{\nu}$ and $\underline{\nu}$ can be interpreted as the degree of demand persistency: an increase in $\bar{\nu}$ (resp. $\underline{\nu}$) makes it more likely that demand is high (resp. low) in the second period when it was high (low) in the first period. Notice that demand is positively correlated if

$$\Pr[\theta_2 = \bar{\theta} | \theta_1 = \bar{\theta}] > \Pr[\theta_2 = \bar{\theta} | \theta_1 = \underline{\theta}] \Leftrightarrow \bar{\nu} + \underline{\nu} > 1,$$

and it is negatively correlated otherwise.

In every period, R privately observes θ_τ while the manufacturer does not. The entrant observes θ_2 , but not θ_1 .

Contracts. We assume that M commits to a long-term contract with R . By the (dynamic

¹⁰In Section 7.1, we consider entry both in the upstream and in the downstream market — i.e., a non-integrated entrant.

¹¹In Section 6, we discuss the implications of introducing (fixed) entry costs.

¹²These demand functions arise, for example, if in every period τ there is a representative consumer in the market with utility function $\theta_\tau Q_\tau - \frac{Q_\tau^2}{2} - p_\tau Q_\tau$, where p_τ is the market price.

version of the) Revelation Principle (e.g., Baron and Besanko, 1984, Myerson, 1986) there is no loss of generality in restricting the analysis to incentive-compatible direct revelation mechanisms. Hence, a long term contract is a menu

$$\{q_1(m_1), t_1(m_1), q_2(m_2, m_1), t_2(m_2, m_1)\},$$

where, for every period τ , $m_\tau \in \Theta$ is R 's report about θ_τ ; while $q_\tau(\cdot)$ is the quantity produced by R and $t_\tau(\cdot)$ the transfer paid by R to M , both contingent on R 's current and past reports.¹³

R is protected by limited liability in both periods, which avoids full surplus extraction in the second period (see, e.g., Laffont and Martimort, 2002). The contract is secret, so that E cannot directly observe it.

Communication. The incumbent players (i.e., M or R) can disclose m_1 to E before market competition takes place in period 2. Following the IO literature — e.g., Vives (1984), Raith (1996) and Piccolo and Pagnozzi (2013) among many others — in the baseline model we consider a ‘*all-or-nothing*’ disclosure rule $d \in \{S, N\}$: either m_1 is fully disclosed to E ($d = S$) or it remains private information of R and M ($d = N$). In Section 7.2 we consider more general (stochastic) disclosure rules.

Assuming that firms can share information about m_1 is without loss of generality in our framework, because of the contractual link between R 's first-period report m_1 and its second-period production. Equivalently, this can be interpreted as incumbents disclosing the first-period production to the entrant, which is a natural and realistic form of communication, since quantities are usually verifiable.¹⁴

By contrast, sharing information about θ_1 has no effect in our environment: the entrant is not interested in first-period demand *per se*, but only on R 's report about it, since only this report affects the incumbent's production in the second period. Of course, disclosing m_2 in the second period also has no effect since E directly observes θ_2 .

As standard in the literature, we also assume that once an information sharing decision has been announced, it cannot be renegotiated after uncertainty about θ_1 and θ_2 realizes.¹⁵ Commitment requires, for instance, the presence of a third party (such as a certification intermediary) that verifies communication.

¹³We do not consider more complex franchise contracts, like resale price maintenance (RPM), in order to avoid full extraction of R 's surplus by M — see, e.g., Gal-Or (1991). Even with RPM, however, information rents may still emerge if adverse selection is coupled with a moral hazard problem *à la* Laffont and Tirole (1986). For example, Martimort and Piccolo (2007) show that if the retailer exerts non-verifiable promotional effort, which boosts demand, full surplus extraction is impossible with RPM. For simplicity, we assume that RPM contracts cannot be enforced because prices are too costly to verify.

¹⁴Given that, in practice, long-term contracts consist of menus that specify production in a period as a function of production in previous periods, disclosing information about quantity simply amounts to disclosing the contractual terms agreed between M and R .

¹⁵In Section 7.3 we consider secret renegotiations.

Rather than assuming that a specific incumbent player chooses to share information, we first characterize the equilibrium under each disclosure policy, and then analyze the incumbents' private and joint incentives to share or sell information.

Timing and Profits. The timing is as follows.

1. First period.

- A disclosure policy $d \in \{S, N\}$ is announced.
- R observes θ_1 .
- M offers a contract. If R accepts it, it reports m_1 to M .
- Production occurs, and t_1 is paid.

2. Second period.

- m_1 is disclosed if and only if $d = S$ and E updates its beliefs about θ_1 .
- R and E observe θ_2 .
- R reports m_2 to M .
- Production occurs, and t_2 is paid.

All players are risk neutral and M and R discount future profit at a common discount factor $\delta \in (0, 1)$.¹⁶ Hence, R 's intertemporal payoff is

$$\sum_{\tau=1,2} \delta^{\tau-1} [P(\theta_\tau, Q_\tau) q_\tau - t_\tau],$$

and M 's intertemporal payoff is

$$\sum_{\tau=1,2} \delta^{\tau-1} t_\tau.$$

E 's profit is $P(\theta_2, Q_2) q_E$.

Equilibrium. The solution concept is Perfect Bayesian Equilibrium (PBE). We focus on separating equilibria in which, for any disclosure policy: (i) M offers an incentive compatible contract; (ii) R accepts the contract and truthfully reports demand; (iii) quantity produced by firms in the second period are mutual best responses. We impose passive beliefs off equilibrium path, so that whenever R is offered an unexpected contract, he believes that E still follows its equilibrium strategy. This is a natural assumption since M 's offer should not convey any information about E 's behavior.

We make the following assumptions to simplify the analysis.

¹⁶This can be interpreted as a measure of the length of period 2 relative to period 1.

Assumption 1. Demand persistency is such that $\bar{\nu} \leq 4\underline{\nu} - 1$.

This assumption requires that the degree of demand persistency is neither too low when $\theta_1 = \underline{\theta}$ (i.e., $\underline{\nu} > \frac{1}{4}$) nor too high when $\theta_1 = \bar{\theta}$. These restrictions imply that R 's equilibrium rent in the second period is always positive: a necessary condition for the game to feature a separating equilibrium in the second period.¹⁷

Assumption 2. $\Delta\theta$ is not too large.

This assumption is imposed to obtain closed form solutions when comparing the players' expected profit with and without information sharing (for a similar approach, see Laffont and Tirole, 1988; Martimort, 1999; and Martimort and Piccolo, 2010). We will show that information sharing has relevant welfare effects even in this limit case. In our dynamic framework, focusing on small uncertainty also implies that, in the first period, R only has an incentive to claim that demand is low when it is actually high, and not *vice versa*, yielding the standard 'no distortion at the top' result and that the incumbent never shuts down production. In Section 7.4 we consider the case of large uncertainty.

3 Preliminaries

We first analyze E 's behavior in the second period. Information sharing affects E 's production since R 's second-period production depends on its first-period report — i.e., if m_1 affects $q_2(\cdot)$.

Consider an equilibrium in which R truthfully reports demand in the first-period — i.e., such that $m_1 = \theta_1$ — and E expects R to produce $q_2(\theta_2, \theta_1)$ in the second period. With information sharing, E 's problem is

$$\max_{q_E \geq 0} P(\theta_2, q_E + q_2(\theta_2, \theta_1))q_E,$$

whose solution yields a downward-sloping reaction function

$$q_E(\theta_2, q_2(\theta_2, \theta_1)) \triangleq \frac{\theta_2 - q_2(\theta_2, \theta_1)}{2}, \quad \forall (\theta_2, \theta_1) \in \Theta^2. \quad (1)$$

By contrast, with no information sharing, E must form a belief about θ_1 (which is equal to m_1 in equilibrium), given θ_2 . Bayes' rule implies that E 's posterior beliefs about θ_1 are

$$\Pr[\theta_1 = \bar{\theta} | \theta_2 = \bar{\theta}] = \frac{\bar{\nu}}{1 + \Delta\nu}, \quad \text{and} \quad \Pr[\theta_1 = \underline{\theta} | \theta_2 = \underline{\theta}] = \frac{\underline{\nu}}{1 - \Delta\nu}.$$

¹⁷See, e.g., Gal-Or (1999), Martimort (1996), Kastl *et al.* (2011) and Martimort and Piccolo (2010), for static models with similar assumptions.

Hence, E 's problem is

$$\max_{q_E \geq 0} \sum_{\theta_1} \Pr[\theta_1 | \theta_2] P(\theta_2, q_E + q_2(\theta_2, \theta_1)) q_E,$$

whose solution yields a downward-sloping reaction function

$$q_E(\theta_2, \mathbb{E}[q_2(\cdot) | \theta_2]) \triangleq \frac{\theta_2 - \sum_{\theta_1} \Pr[\theta_1 | \theta_2] q_2(\theta_2, \theta_1)}{2}, \quad \forall \theta_2 \in \Theta. \quad (2)$$

The slope of this function depends on the degree of demand intertemporal correlation: the higher the correlation, the more ‘accurate’ the inference that E can make on θ_1 given θ_2 .

3.1 Benchmarks

Consider two useful benchmarks. First, suppose that, in every period, θ_τ is common knowledge. Then M fully extracts R 's surplus and the optimal contract implements the monopoly outcome in the first period — i.e., $q^*(\theta_1) \triangleq \frac{\theta_1}{2}$ — and the symmetric Cournot outcome in the second period — i.e., both firms produce $q^C(\theta_2) \triangleq \frac{\theta_2}{3}$.

Second, suppose that there is no entry in the second period. Assume that in both periods only the incentive compatibility constraint of the high-demand type matters,¹⁸ and let $U_1(\cdot)$ be R 's equilibrium rent in the first period. Using a standard change of variables (e.g., Laffont and Martimort, 2002), M offers the contract that solves the following intertemporal problem:

$$\max_{q_1(\cdot), q_2(\cdot), U_1(\cdot)} \mathbb{E} \left[\sum_{\tau=1,2} \delta^{\tau-1} P(\theta_\tau, q_\tau(\cdot)) q_\tau(\cdot) \right] - \sum_{\theta_1} \Pr[\theta_1] [U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] \Delta \theta q_2(\underline{\theta}, \theta_1)],$$

subject to $U_1(\underline{\theta}) \geq 0$ and

$$U_1(\bar{\theta}) \geq \underbrace{U_1(\underline{\theta}) + \Delta \theta q_1(\underline{\theta})}_{\text{Static rent}} + \underbrace{\delta \bar{\nu} \Delta \theta [q_2(\underline{\theta}, \underline{\theta}) - q_2(\underline{\theta}, \bar{\theta})]}_{\text{Intertemporal rent}}. \quad (3)$$

It can be verified that, in the optimal dynamic contract, first-period quantities are

$$q_1^M(\bar{\theta}) = q^*(\bar{\theta}), \quad \text{and} \quad q_1^M(\underline{\theta}) = q^*(\underline{\theta}) - \frac{\Delta \theta}{2},$$

while second-period quantities are

$$q_2^M(\underline{\theta}, \underline{\theta}) = q_1^M(\underline{\theta}) - \underbrace{\frac{1 + \Delta \nu}{2 \nu} \Delta \theta}_{\text{Intertemporal distortion}},$$

¹⁸It can be checked that this is always the case under Assumption 2.

and $q_2(\theta_2, \theta_1) = q^*(\theta_2)$ in all other states.

Hence, R always produces the monopoly quantity in a period in which demand is high — i.e., there is ‘no distortion at the top’. By contrast, there is a standard (static) downward distortion of production in the first period when demand is low, while production in the second period is distorted only when demand is low in both periods. This intertemporal distortion arises because a higher quantity in state $(\underline{\theta}, \underline{\theta})$ increases R ’s rent both in the second period and in the first period (since it makes it more attractive for R to report low demand in the first period, *ceteris paribus*).

The intertemporal distortion increases with $\bar{\nu}$ and decreases with $\underline{\nu}$. First, a high $\underline{\nu}$ implies a high probability of low demand in the second period following low demand in the first period, which reduces M ’s willingness to distort production. Second, other things being equal, a higher $\bar{\nu}$ increases R ’s intertemporal rent and induces M to increase quantity distortion to trade off efficiency and rent minimization.

4 Equilibrium Analysis

We now characterize the optimal contract offered by M without information sharing (Section 4.1) and with information sharing (Section 4.2).

4.1 No Information Sharing

With no information sharing, E ’s production depends on its expectation of the quantity produced by R , which depends on m_1 through the contract chosen by M (that E correctly expects in equilibrium). Let $q_E^N(\theta_2)$ be E ’s equilibrium production and denote by

$$\Delta q^N \triangleq q_E^N(\bar{\theta}) - q_E^N(\underline{\theta})$$

the difference between E ’s production with high and low demand in the second period.

4.1.1 Retailer’s Rent

Let $U_2(\cdot)$ be R ’s equilibrium rent in the second period. Following Martimort (1996) we first assume that R only has an incentive to under-report demand and then verify this conjecture *ex post*. Given a report m_1 , R ’s relevant incentive and participation constraints in the second period are

$$U_2(\bar{\theta}, m_1) \geq U_2(\underline{\theta}, m_1) + (\Delta\theta - \Delta q^N) q_2(\underline{\theta}, m_1), \quad \forall m_1 \in \Theta,$$

$$U_2(\underline{\theta}, m_1) \geq 0, \quad \forall m_1 \in \Theta.$$

Since limited liability implies that $U_2(\underline{\theta}, m_1) = 0$ for every m_1 , R 's second period rent is

$$U_2(\bar{\theta}, m_1) \triangleq \underbrace{\Delta\theta q_2(\underline{\theta}, m_1)}_{\text{Information rent}} - \underbrace{\Delta q^N q_2(\underline{\theta}, m_1)}_{\text{Competition effect}}, \quad \forall m_1 \in \Theta. \quad (4)$$

This expression embeds two contrasting effects. First, R has an incentive to report low demand in the second period in order to pay a lower transfer. Other things being equal, this secures R a (standard) information rent which is increasing in the quantity produced when demand is low — see, e.g., Mussa and Rosen (1978) and Maskin and Riley (1985). Second, there is a *competition effect* (see, e.g., Gal-Or, 1999; Martimort, 1996; Martimort and Piccolo, 2010): when R under-reports demand in the second period, E produces more than M expects and the transfer offered to R does not take this effect into account. Hence, R 's incentive to under-report demand is weaker than without entry. As a result, competition in the downstream market reduces R 's information rent, and makes it less costly for M to elicit truthful information from R .

Consider now the first period. Let R 's rent in the first period be

$$U_1(\theta_1, m_1) \triangleq P(\theta_1, q_1(m_1)) q_1(m_1) - t_1(m_1),$$

and $U_1(\theta_1) \triangleq U_1(\theta_1, m_1 = \theta_1)$, $\forall m_1 \in \Theta$. Taking into account its rent in the second period (4), R 's intertemporal incentive constraint (ensuring that R truthfully reports demand in the first period) is

$$U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q^N) q_2(\underline{\theta}, \theta_1) \geq \max_{m_1 \in \Theta, m_1 \neq \theta_1} \{U_1(\theta_1, m_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q^N) q_2(\underline{\theta}, m_1)\}, \quad \forall \theta_1 \in \Theta.$$

Assuming that the constraint only binds when demand is high,¹⁹ the relevant first-period incentive compatibility constraint is

$$U_1(\bar{\theta}) \geq \underbrace{U_1(\underline{\theta}) + \Delta\theta q_1(\underline{\theta})}_{\text{Static Rent}} + \underbrace{\delta \bar{\nu} (\Delta\theta - \Delta q^N) [q_2(\underline{\theta}, \underline{\theta}) - q_2(\underline{\theta}, \bar{\theta})]}_{\text{Intertemporal Rent}}, \quad (5)$$

while the relevant first-period participation constraint is $U_1(\underline{\theta}) \geq 0$.

R 's incentive to under-report demand in the first period depends on two terms: the static rent of a single period relationship and the intertemporal rent that R obtains when demand is high in the second period, which happens with probability $\bar{\nu}$. The sign of this second term depends on how the first-period report affects production in the second period when demand is low. If $q_2(\underline{\theta}, \underline{\theta}) > q_2(\underline{\theta}, \bar{\theta})$, R 's second-period rent is higher when it reports low rather than high demand in the first period and eliciting truthful information is more costly

¹⁹In the Appendix we check that under Assumption 2 this conjecture is verified in equilibrium.

than in a static environment. By contrast, when $q_2(\underline{\theta}, \underline{\theta}) < q_2(\underline{\theta}, \bar{\theta})$, it is less costly for M to elicit truthful information. Of course, as the competition effect becomes stronger — i.e., as Δq^N increases — R 's second-period rent decreases and $q_2(\underline{\theta}, \underline{\theta}) - q_2(\underline{\theta}, \bar{\theta})$ has a weaker effect on R 's first-period rent.

4.1.2 Optimal Long Term Contract

After a standard change of variables, M 's intertemporal (relaxed) maximization problem is

$$\begin{aligned} \max_{q_1(\cdot), q_2(\cdot), U_1(\cdot)} \mathbb{E} \left[\sum_{\tau=1,2} \delta^{\tau-1} P(\theta_\tau, Q_\tau^N(\cdot)) q_\tau(\cdot) \right] + \\ - \sum_{\theta_1} \Pr[\theta_1] [U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q^N) q_2(\underline{\theta}, \theta_1)], \end{aligned} \quad (6)$$

subject to (5) and the relevant participation constraint, where

$$Q_2^N(\theta_2, \theta_1) \triangleq q_2(\theta_2, \theta_1) + q_E^N(\theta_2).$$

Since both constraints bind at the optimum, it can be shown that first-period production is as in a market without entry, with no distortion at the top and downward distortion at the bottom — i.e., using the superscript N to denote the optimal quantities chosen by the manufacturer with no information sharing, $q_1^N(\bar{\theta}) = q^*(\bar{\theta})$ and that $q_1^N(\underline{\theta}) = q_1^M(\underline{\theta})$.

Differentiating the objective function with respect to $q_2(\bar{\theta}, \theta_1)$,

$$P'(\bar{\theta}, Q_2^N(\bar{\theta}, \theta_1)) q_2(\bar{\theta}, \theta_1) + P(\bar{\theta}, Q_2^N(\bar{\theta}, \theta_1)) = 0, \quad \forall \theta_1 \in \Theta. \quad (7)$$

Hence, when demand is high in the second period, R 's production is not distorted (compared to the benchmark without incomplete information) regardless of the level of demand in the first period — i.e., $q_2^N(\bar{\theta}, \theta_1) = q^C(\bar{\theta})$ for every θ_1 — so that E 's best response is $q_E^N(\bar{\theta}) = q^C(\bar{\theta})$.

Differentiating the objective function with respect to $q_2(\underline{\theta}, \bar{\theta})$,

$$P'(\underline{\theta}, Q_2^N(\underline{\theta}, \bar{\theta})) q_2(\underline{\theta}, \bar{\theta}) + P(\underline{\theta}, Q_2^N(\underline{\theta}, \bar{\theta})) = 0. \quad (8)$$

Hence, if demand is high in the first period, the optimal dynamic contract rewards R in the second period even if demand is low in the second period — i.e., production is determined by the equalization of marginal revenues to marginal cost (which is normalized to zero).

Finally, differentiating the objective function with respect to $q_2(\underline{\theta}, \underline{\theta})$,

$$P'(\underline{\theta}, Q_2^N(\underline{\theta}, \underline{\theta})) q_2(\underline{\theta}, \underline{\theta}) + P(\underline{\theta}, Q_2^N(\underline{\theta}, \underline{\theta})) = (\Delta\theta - \Delta q^N) \frac{1 + \Delta\nu}{\underline{\nu}}. \quad (9)$$

As without entry, increasing the second-period output in state $(\underline{\theta}, \underline{\theta})$ has two effects: a higher $q_2(\underline{\theta}, \underline{\theta})$ increases both R 's second-period rent when demand is high in the second period and low in the first period, and R 's intertemporal rent when demand is high in the first period. Both effects make it more profitable for R to under-report demand in the first period in order to enjoy higher rents in the future. Hence, a higher $\underline{\nu}$ reduces both the static and the intertemporal distortion, while a higher $\bar{\nu}$ increases the intertemporal distortion, which induces a higher distortion when demand is low in both periods.

Substituting $q_2^N(\bar{\theta}, \theta_1) = q_E^N(\bar{\theta}) = q^C(\bar{\theta})$ into (2), (8), (9) yields the following result.

Proposition 1 *Without information sharing, $q_2^N(\underline{\theta}, \underline{\theta}) < q_2^N(\underline{\theta}, \bar{\theta}) < q^C(\underline{\theta}) < q_E^N(\underline{\theta})$.*

Hence, M distorts production downward (compared to a benchmark without incomplete information) when demand is low in both periods in order to optimally trade off efficiency and rent extraction. This distortion induces E to increase production when demand is low because E expects R to under-produce with positive probability. As a consequence, R faces a more aggressive competitor when demand is low in the second period, regardless of first-period demand, which induces it to reduce production.

4.2 Information Sharing

With information sharing, E 's equilibrium production $q_E^S(\theta_2, m_1)$ depends both on demand in the second period and on M 's report in the first period. This impacts R 's second-period rent, and therefore it affects R 's equilibrium production through the distortions chosen by M in order to trade off efficiency and (intertemporal) rent extraction. For any m_1 , let

$$\Delta q^S(m_1) \triangleq q_E^S(\bar{\theta}, m_1) - q_E^S(\underline{\theta}, m_1)$$

be the difference between E 's production with high and low demand in the second period.

4.2.1 Retailer's Rent

R 's binding incentive compatibility constraint in the second period is²⁰

$$U_2(\bar{\theta}, m_1) = \underbrace{\Delta \theta q_2(\underline{\theta}, m_1)}_{\text{Information rent}} - \underbrace{\Delta q^S(m_1) q_2(\underline{\theta}, m_1)}_{\text{Competition effect}}, \quad \forall m_1 \in \Theta.$$

In contrast to the case of no information sharing, the competition effect and, hence, R 's intertemporal rent now depends on the effect of R 's first-period report on E 's production.

²⁰For simplicity, we use the same notation for R 's rent as in Section 4.1.1.

R 's intertemporal incentive constraint is

$$U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] [\Delta\theta - \Delta q^S(\theta_1)] q_2(\underline{\theta}, \theta_1) \geq \max_{m_1 \in \Theta, m_1 \neq \theta_1} \{U_1(\theta_1, m_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q^S(m_1)) q_2(\underline{\theta}, m_1)\}, \quad \forall \theta_1 \in \Theta.$$

As before, we assume (and verify ex post) that R only has an incentive to misreport demand when demand is high. Hence, the relevant first-period incentive compatibility constraint is

$$U_1(\bar{\theta}) \geq \underbrace{U_1(\underline{\theta}) + \Delta\theta q_1(\underline{\theta})}_{\text{Static rent}} + \underbrace{\delta \bar{v} [(\Delta\theta - \Delta q^S(\underline{\theta})) q_2(\underline{\theta}, \underline{\theta}) - (\Delta\theta - \Delta q^S(\bar{\theta})) q_2(\underline{\theta}, \bar{\theta})]}_{\text{Intertemporal rent}}, \quad (10)$$

while the relevant first-period participation constraint is $U_1(\underline{\theta}) \geq 0$.

Other things being equal, R 's intertemporal rent is increasing in $\Delta q^S(\bar{\theta})$ and decreasing in $\Delta q^S(\underline{\theta})$. The stronger is the competition effect when a high demand is reported in the first period, the higher is R 's intertemporal rent because second-period rents are higher (in equilibrium). By contrast, the stronger is the competition effect when a low demand is reported in the first period, the lower is R 's intertemporal rent, which *ceteris paribus* reduces R 's incentive to mimic in the first period.

4.2.2 Optimal Long Term Contract

After a standard change of variables, M 's intertemporal (relaxed) maximization problem is

$$\max_{q_1(\cdot), q_2(\cdot), U_1(\cdot)} \mathbb{E} \left[\sum_{\tau=1,2} \delta^{\tau-1} P(\theta_\tau, Q_\tau^S(\cdot)) q_\tau(\cdot) \right] + \sum_{\theta_1} \Pr[\theta_1] [U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q^S(\theta_1)) q_2(\underline{\theta}, \theta_1)], \quad (11)$$

subject to (10) and the relevant participation constraint, where

$$Q_2^S(\theta_2, \theta_1) \triangleq q_2(\theta_2, \theta_1) + q_E^S(\theta_2, \theta_1).$$

Since both constraints bind at the optimum, it is easy to show that first-period production is the same as without information sharing, and that in the second period there is no distortion at the top regardless of level of demand in the first period — i.e., using the superscript S to denote the optimal quantities chosen by the manufacturer with information sharing, $q_2^S(\bar{\theta}, \theta_1) = q_E^S(\bar{\theta}, \theta_1) = q^C(\bar{\theta})$ for every θ_1 .

Differentiating with respect to $q_2(\underline{\theta}, \bar{\theta})$,

$$P'(\underline{\theta}, Q_2^S(\underline{\theta}, \bar{\theta})) q_2(\underline{\theta}, \bar{\theta}) + P(\underline{\theta}, Q_2^S(\underline{\theta}, \bar{\theta})) = 0. \quad (12)$$

With information sharing, since E 's output depends on first-period demand, neither firm distorts production when demand is low in the first period and high in the second — i.e., $q_2^S(\underline{\theta}, \bar{\theta}) = q_E^S(\underline{\theta}, \bar{\theta}) = q^C(\underline{\theta})$. Differentiating with respect to $q_2(\underline{\theta}, \underline{\theta})$,

$$P'(\underline{\theta}, Q_2^S(\underline{\theta}, \underline{\theta})) q_2(\underline{\theta}, \underline{\theta}) + P(\underline{\theta}, Q_2^S(\underline{\theta}, \underline{\theta})) = (\Delta\theta - \Delta q^S(\underline{\theta})) \frac{1 + \Delta\nu}{\underline{\nu}}. \quad (13)$$

Hence, the effects of the intertemporal distortions on production when demand is low in both periods are as in the case of no information sharing.

Substituting and solving jointly with E 's first-order condition (1) yields the following result.

Proposition 2 *With information sharing, $q_2^S(\underline{\theta}, \underline{\theta}) < q^C(\underline{\theta}) < q_E^S(\underline{\theta}, \underline{\theta})$.*

As intuition suggests, E produces more than R when demand is repeatedly low since M distorts production downward to reduce R 's intertemporal rent. By contrast, when demand is high in the first period and low in the second period, firms produce the same quantities because M does not distort production.

5 Incentives to Share Information

To analyze the effects of information sharing on the manufacturer's and the retailer's expected profits,²¹ we start by comparing E 's production with and without information sharing. Since production is never distorted when demand is high, we can focus on the quantity produced by E when demand is low.

Proposition 3 *When demand is low in the second period, E 's production is higher (lower) with information sharing than without if demand is low (high) in the first period — i.e., $q_E^S(\underline{\theta}, \bar{\theta}) < q_E^N(\underline{\theta}) < q_E^S(\underline{\theta}, \underline{\theta})$. Moreover, E 's average production is higher with information sharing than without — i.e., $\mathbb{E}[q_E^S(\underline{\theta}, \theta_1) | \underline{\theta}] > q_E^N(\underline{\theta})$.*

With information sharing, E knows the quantity that R produces in the second period. When demand is low in both periods, R 's second-period production is distorted for rent extraction reasons and, since reaction functions are downward sloping, information sharing allows E to produce more. By contrast, without information sharing E is uncertain about R 's production and has a lower incentive to expand its own production. On the other hand, when demand is low in the second period and high in the first period, E 's production is lower with information sharing because R 's second-period production is not distorted, and this induces E to produce less when he is informed.

²¹Notice that while we consider information about demand, similar effects arises with information about costs. In fact, information about θ_1 allows the entrant to learn whether R 's production will be distorted, which is analogous to knowing whether a competitor has high or low cost of production.

Therefore, with information sharing R faces tougher (weaker) competition from E when demand is low (high) in the first period. Without information sharing, E responds to uncertainty about R 's production by producing an intermediate quantity, between $q_E^S(\underline{\theta}, \bar{\theta})$ and $q_E^S(\underline{\theta}, \underline{\theta})$. In expectation, information sharing increases E 's production since the first effect discussed above dominates, so that the entrant obtains a larger market share when it is informed.

To analyze players' incentives to share information, we now compare R 's expected rents and M 's expected profits with and without information sharing. In order to obtain closed-form solutions, we restrict to the case of small uncertainty (by Assumption 2).²²

Proposition 4 *R wants to share information with E , while M does not.*

R would like to disclose m_1 because letting E know the quantity that R produces in the second period reduces the variability of E 's production — i.e., the difference between $q^C(\bar{\theta})$ and E 's production when demand is low. In fact, by Proposition 3, with information sharing E expands production when demand is low, compared to the case without information sharing, whereby reducing its (equilibrium) output variability — i.e.,

$$q_E^S(\underline{\theta}, \underline{\theta}) > q_E^N(\underline{\theta}) \quad \Rightarrow \quad \Delta q^S(\underline{\theta}) < \Delta q^N.$$

This weakens the competition effect (relative to the case without information sharing) and increases rents in the second period. Hence, R prefers to face an informed rather than an uninformed competitor in the second period.

For the manufacturer, by contrast, disclosing m_1 to E has two negative effects. First, since $q_E^N(\underline{\theta}) < \mathbb{E}[q_E^S(\underline{\theta}, \theta_1) | \underline{\theta}]$, the entrant is (on average) more aggressive with information sharing. Hence, M can extract a lower surplus from R when E is informed about first-period demand: a *business stealing* effect. Second, holding revenues constant, sharing information is detrimental to M because it increases R 's expected rent.

Therefore, in our environment transparency arises if information sharing is chosen by the downstream incumbent, but not if it is chosen by the upstream incumbent that prefers to face an uninformed entrant. This highlights a conflict of interest between upstream and downstream firms in vertical relations facing entry: whether information is shared with entrants depends on which player owns privacy rights, and is accordingly entitled to disclose information within a vertical hierarchy.²³

We now consider the effect of information sharing on the (expected) joint profit of M and R , to analyze whether their conflict of interest can be solved by ex ante contracting.

²²Hence, expected rents and profits are approximated by a first-order Taylor expansion for $\Delta\theta$ close to 0.

²³The fact that the retailer prefers to disclose information is in line with the literature finding that Cournot competition firms want to exchange information about their stochastic costs (see, e.g., Shapiro, 1986).

Can M and R jointly agree to an information sharing decision with compensation for the damaged party, before demand realizes?²⁴

Proposition 5 *The ex ante joint profit of M and R is lower with information sharing than without.*

Hence, with small uncertainty, the manufacturer and the retailer jointly gain by not sharing information with the entrant: M obtains privacy rights by offering an ex ante payment to R which compensates its loss when no information is disclosed. When ex ante contracting is not possible, however, either because M is capital constrained or because privacy rights cannot be easily transferred, it is unlikely that M can prevent R from disclosing information to E .

Since the entrant can always (commit to) disregard the information received by the incumbent and implement the same outcome as without information sharing, we have the following result.

Proposition 6 *E obtains higher profit with information sharing than without.*

5.1 Market for Information

Since information about the first-period report by the retailer is valuable, the entrant is willing to pay for it. Do incumbent players have any incentive to sell information to the entrant, rather than simply share it at no cost? Coherently with our full commitment assumption, we assume that the incumbent can commit at the outset of the game to a price that the entrant has to pay in order to acquire information.

Of course, by Propositions 4 and 6, E and R have a joint interest to trade information, since they are both better off with information sharing. By contrast, since M 's profits are lower with information sharing by Propositions 4, M has an incentive to sell information to E only if the highest price that E is willing to pay for information is higher than M 's loss for facing an informed competitor — i.e.,

$$\underbrace{\Pi_E^S - \Pi_E^N}_{E's \text{ willingness to pay}} \geq \underbrace{\Pi_2^N - \Pi_2^S}_{M's \text{ reservation price}},$$

where, given a disclosure policy $d \in \{S; N\}$, Π_E^d denotes E 's expected profit and Π_2^d denotes M 's second-period expected profit.

Under Assumption 2, we have the following result.

Proposition 7 *It is never profitable for M to sell information to E .*

²⁴This is equivalent to analyzing whether M and R can agree, *behind the veil of ignorance*, to a system of ex ante transfers that harmonizes their interests, with R paying M to disclose m_1 to E , or vice versa. Of course, in order for this agreement to be feasible, players must not be capital constrained.

As discussed above, M 's loss for information sharing is shaped by two effects: by sharing information M induces the entrant to be more aggressive and increases R 's rent. Since the highest price that E is willing to pay for information can internalize the first effect, but not the second one, M has no incentive to sell information.

Finally, if M and R maximize joint profits, since R 's rent is just a transfer between M and R , then the entrant can pay a price for information that fully compensates M 's loss.

Proposition 8 *M and R have an incentive to jointly sell information to E .*

Trading information is jointly profitable for the incumbents and the entrant because it maximizes total profit in the industry: it allows firms to extract more surplus from consumers (as we are going to show), and it rebalances production from a less efficient firm (the incumbent who faces agency costs) to a more efficient one (the entrant who faces no agency costs and, on average, produces more when it is informed).

6 Welfare

In order to study the welfare effects of information sharing, since first-period production is the same with and without information sharing, we analyze how the incumbent's decision to disclose information impacts aggregate production in the second period.

Proposition 9 *Expected aggregate production is lower with information sharing than without.*

Information sharing reduces the incumbent's production and allows E to increase production when the incumbent distorts it. On balance, however, aggregate production is lower than without information sharing because, holding constant the incumbent's production, E 's decision is always efficient regardless of its information (since it equalizes marginal revenue to marginal costs). In other words, information reduces market efficiency because it increases R 's information rent via the competition effect, whereby reducing the incumbent's overall efficiency.

In the limit of small uncertainty, information sharing has an analogous effect on consumer surplus and total welfare.

Proposition 10 *Consumer surplus and total welfare are lower with information sharing than without.*

This suggests that information sharing between incumbents and new entrants should be forbidden, and that incumbents should not be allowed to sell information to future competi-

tors. Hence, our analysis highlights a potential drawback of imposing transparency about past performance to incumbents.²⁵

Notice that the result in Proposition 10 hinges on the absence of a fixed cost of entry. With a sufficiently high entry cost, not sharing information may foreclose entry, which always harms consumers. With a stochastic entry cost, however, the net effect of information sharing on consumer surplus depends on the relative likelihood of entry being blocked without information sharing.

7 Extensions

7.1 Competing Hierarchies

Suppose that the entrant is a vertical hierarchy rather than an integrated firm (see, e.g., Caillaud *et al.*, 1995; and Martimort, 1996): in period 2 a new manufacturer M_E enters the market and sells through its exclusive retailer R_E , who is privately informed about θ_2 but does not know θ_1 . For example, this may happen when the entrant is a foreign firm that needs a local retailer in order to enter the market and distribute its product.

M_E offers to R_E a direct revelation mechanism

$$\{q_E(m_E, s), t_E(m_E, s)\},$$

which specifies a production level $q_E(\cdot)$ and a transfer $t_E(\cdot)$ contingent on R_E 's report m_E about θ_2 and on the information $s \in \Theta \cup \{\emptyset\}$ revealed by the incumbent (with $s = \emptyset$ denoting no information).²⁶ To focus on separating equilibria, we impose a condition equivalent to Assumption 1.

Assumption 3. The degree of demand persistency is such that $\bar{\nu} \leq \bar{\nu}^* \triangleq \min\{4\underline{\nu}^2 - 1, \frac{1}{2}\}$. Moreover, δ is not too large.

The assumption requires that the degree of demand persistency is neither too low when $\theta_1 = \underline{\theta}$ (i.e., $\underline{\nu} > \frac{1}{2}$) nor too high when $\theta_1 = \bar{\theta}$. This guarantees that, in the second period, retailers have an incentive to mis-report demand only if demand is high, and that retailers' information rent is positive (see, e.g., Kastl *et al.*, 2011). Moreover, the assumption on δ ensures that intertemporal rents are positive — i.e., that in the first period retailers have an

²⁵Of course, transparency may be welfare beneficial in other contexts. For example, improving price and quality transparency unambiguously benefit consumers — e.g., Varian (1980), Schultz (2009) and Gu and Wenzel (2011). But while these models focus on firms' ability to inform consumers about product characteristics, in our environment communication is about past demand or performance.

²⁶Consistent with our main model, we assume passive beliefs off equilibrium path so that, whenever a retailer receives an unexpected offer, it believes that the other players follow equilibrium strategies.

incentive to mimic only when demand is high (see Laffont and Martimort, 2002).²⁷

M 's maximization problem is the same as in our main model, regardless of whether information is shared or not. By contrast, the entrant solves a different maximization problem, since M_E is uninformed about θ_2 and has to induce R_E to truthfully report it. Hence, the entrant's production is also distorted for rent extraction reasons, and this distortion crucially depends on whether the incumbent shares information or not.

7.1.1 No Information Sharing

Without information sharing, when R truthfully reveals its private information, R_E 's expected utility from truthfully reporting θ_2 is

$$U_E(\theta_2) \triangleq \sum_{\theta_1} \Pr[\theta_1|\theta_2] P(\theta_2, Q_2^N(\theta_2, \theta_1)) q_E(\theta_2) - t_E(\theta_2),$$

where aggregate quantity is

$$Q_2^N(\theta_2, \theta_1) \triangleq q_E(\theta_2) + q_2^N(\theta_2, \theta_1).$$

Let

$$\Delta q_2^N \triangleq \mathbb{E}[q_2^N(\bar{\theta}, \theta_1) | \theta_2 = \bar{\theta}] - \mathbb{E}[q_2^N(\underline{\theta}, \theta_1) | \theta_2 = \underline{\theta}].$$

Conjecturing that only the incentive compatibility constraint in the high demand state binds, R_E 's information rent is determined by

$$U_E(\bar{\theta}) \geq U_E(\underline{\theta}) + (\Delta\theta - \Delta q_2^N) q_E(\underline{\theta}), \quad (14)$$

which reflects a competing contracts effect (averaged over θ_1).

By standard techniques, M_E 's (relaxed) maximization problem is

$$\max_{q_E(\cdot)} \sum_{\theta_2} \Pr[\theta_2] \sum_{\theta_1} \Pr[\theta_1|\theta_2] P(\theta_2, Q_2^N(\theta_2, \theta_1)) q_E(\theta_2) - \Pr[\theta_2 = \bar{\theta}] (\Delta\theta - \Delta q_2^N) q_E(\underline{\theta}).$$

Differentiating with respect to $q_E(\bar{\theta})$ and using the first-order conditions (7) it follows that, in equilibrium, R_E 's production is not distorted when demand is high in the second period. Since R 's production is also efficient, in equilibrium $q_E^N(\bar{\theta}) = q_2^N(\bar{\theta}, \theta_1) = q^C(\bar{\theta})$ for every θ_1 .

²⁷This is a sufficient condition that does not affect the main results of the analysis since only second-period outputs matter to determine the incentives to share/sell information, and the welfare effects of this choice.

By contrast, differentiating with respect to $q_E(\underline{\theta})$ yields

$$\begin{aligned} \sum_{\theta_1} \Pr[\theta_1 | \theta_2 = \underline{\theta}] [P'(\underline{\theta}, Q_2^N(\underline{\theta}, \theta_1)) q_E(\underline{\theta}) + P(\underline{\theta}, Q_2^N(\underline{\theta}, \theta_1))] = \\ = \underbrace{\Delta\theta - [q^C(\bar{\theta}) - \mathbb{E}[q_2^N(\underline{\theta}, \theta_1) | \theta_2 = \underline{\theta}]]}_{\text{Distortion without information}}. \end{aligned}$$

Hence, other things being equal, R_E 's production is downward distorted when demand is low in the second period. Together with (8) and (9), this condition determines equilibrium production in the second period.

Proposition 11 *Without information sharing, $q_E^N(\underline{\theta}) < q^C(\underline{\theta}) < q_2^N(\underline{\theta}, \underline{\theta}) = q_2^N(\underline{\theta}, \bar{\theta})$.*

Therefore, the incumbent overproduces (compared to the benchmark without incomplete information). Since M can exploit demand correlation to reduce R 's rent while M_E cannot, without information sharing the entrant always produces less than the incumbent. This asymmetry provides the incumbent with a competitive edge that completely offsets the potential distortion stemming from asymmetric information and enables M to increase production when demand is low.

7.1.2 Information Sharing

With information sharing, when R truthfully reports its information, R_E 's equilibrium utility is

$$U_E(\theta_2, \theta_1) \triangleq P(\theta_2, Q^S(\theta_2, \theta_1)) q_E(\theta_2, \theta_1) - t_E(\theta_2, \theta_1),$$

where, slightly abusing notation, aggregate quantity is

$$Q^S(\theta_2, \theta_1) \triangleq q_E(\theta_2, \theta_1) + q_2^S(\theta_2, \theta_1).$$

As before, we assume (and verify ex post) that only the incentive compatibility constraint in the high demand state matters. Let

$$\Delta q_2^S(\theta_1) \triangleq q_2^S(\bar{\theta}, \theta_1) - q_2^S(\underline{\theta}, \theta_1), \quad \forall \theta_1 \in \Theta.$$

R_E 's information rent is then determined by the following inequality

$$U_E(\bar{\theta}, \theta_1) \geq U_E(\underline{\theta}, \theta_1) + (\Delta\theta - \Delta q_2^S(\theta_1)) q_E(\underline{\theta}, \theta_1), \quad \forall \theta_1 \in \Theta. \quad (15)$$

With information sharing, the strength of the ‘competing contracts effect’ depends on demand in the first period. Hence, for every θ_1 , M_E 's (relaxed) maximization problem is

$$\max_{q_E(\cdot, \theta_1)} \sum_{\theta_2} \Pr[\theta_2 | \theta_1] P(\theta_2, Q(\theta_2, \theta_1)) q_E(\theta_2, \theta_1) - \Pr[\theta_2 = \bar{\theta} | \theta_1] (\Delta\theta - \Delta q_2^S(\theta_1)) q_E(\underline{\theta}, \theta_1).$$

Differentiating with respect to $q_E(\bar{\theta}, \theta_1)$ it follows that, in equilibrium, R_E 's production is not distorted when demand is high in the second period. Since R 's second-period production is also efficient, $q_E^S(\bar{\theta}) = q_2^S(\bar{\theta}, \theta_1) = q^C(\bar{\theta})$. By contrast, in the low demand state R_E 's production is distorted for rent extraction reasons. Differentiating with respect to $q_E(\underline{\theta}, \theta_1)$ yields

$$P'(\underline{\theta}, Q_2^S(\underline{\theta}, \theta_1)) q_E(\underline{\theta}, \theta_1) + P(\underline{\theta}, Q_2^S(\underline{\theta}, \theta_1)) = \underbrace{\frac{\Pr[\theta_2 = \bar{\theta}|\theta_1]}{\Pr[\theta_2 = \underline{\theta}|\theta_1]} (\Delta\theta - \Delta q_2^S(\theta_1))}_{\text{Distortion with information}}, \quad \forall \theta_1 \in \Theta.$$

Together with (12) and (13) this condition determines the equilibrium production in states $(\underline{\theta}, \underline{\theta})$ and $(\underline{\theta}, \bar{\theta})$. While M only distorts R 's production when first-period demand is low, R_E 's production is always distorted downward because M_E has an incentive to reduce R_E 's static rent by reducing $q_E(\underline{\theta})$. The magnitude of this distortion depends on the likelihood ratio

$$\mathcal{L}(\theta_1) \triangleq \frac{\Pr[\theta_2 = \bar{\theta}|\theta_1]}{\Pr[\theta_2 = \underline{\theta}|\theta_1]} = \begin{cases} \frac{\bar{\nu}}{1-\bar{\nu}} & \text{if } \theta_1 = \bar{\theta} \\ \frac{1-\nu}{\nu} & \text{if } \theta_1 = \underline{\theta} \end{cases} \quad (16)$$

Indeed, M_E distorts production more when the information about $m_1 = \theta_1$ received by the incumbent indicates that demand in the second period is relatively more likely to be high than low.

Proposition 12 *With information sharing, $q_2^S(\underline{\theta}, \bar{\theta}) > q_E^S(\underline{\theta}, \bar{\theta})$ and $q_2^S(\underline{\theta}, \underline{\theta}) < q_E^S(\underline{\theta}, \underline{\theta})$. Moreover, $q_2^S(\underline{\theta}, \bar{\theta}) > q^C(\underline{\theta})$, while $q_2^S(\underline{\theta}, \underline{\theta}) < q^C(\underline{\theta})$ if and only if $\bar{\nu} \geq \frac{(1-\nu)(1-2\nu)}{3\nu-1}$.*

Hence, the incumbent produces more than the entrant in the second period if and only if demand in the first period is high. Moreover, when demand is persistent enough — i.e., when $\bar{\nu}$ is sufficiently large — the information that M_E obtains from observing m_1 is more precise, so that it pays lower rents and can expand production. Hence, the incumbent underproduces if it faces an informed rival. By contrast, when the information conveyed by m_1 is less precise, M_E is more uncertain about demand, pays higher rents, and distorts production more in order to trade off efficiency and rent extraction. This, in turn, induces M to expand production (relatively to the complete information benchmark).

7.1.3 Value of Information

Consider the effects of information sharing on players' profits.

Proposition 13 *With information sharing, the entrant always produces more than without information sharing — i.e., $q_E^N(\underline{\theta}) < \min\{q_E^S(\underline{\theta}, \bar{\theta}), q_E^S(\underline{\theta}, \underline{\theta})\}$.*

Since knowing θ_1 allows M_E to elicit R_E 's private information at a lower cost, information sharing induces the entrant to increase production, so that $\mathbb{E}[q_E^S(\underline{\theta}, \theta_1) | \underline{\theta}] > q_E^N(\underline{\theta})$. Hence, we have the following result (as in our baseline model).

Proposition 14 *R wants to share information, while M does not. Moreover, M_E obtains higher profit when it is informed.*

7.1.4 Welfare

To analyze the effects of information sharing on consumer surplus and total welfare, consider aggregate (expected) production.

Proposition 15 *There exist two thresholds $\underline{\nu}_0$ and $\bar{\nu}_0(\underline{\nu}) \leq \bar{\nu}^*$ such that aggregate production, consumer surplus and welfare are higher with information sharing if: (i) $\underline{\nu} \geq \underline{\nu}_0$ or (ii) $\underline{\nu} < \underline{\nu}_0$ and $\bar{\nu} \leq \bar{\nu}_0(\underline{\nu})$. Otherwise, aggregate production, consumer surplus and welfare are higher without information sharing.*

Figure 1 illustrates the region of parameters, identified in Proposition 15, where consumers prefer information sharing. In contrast to the case in which the entrant is an integrated firm, with competing hierarchies information sharing may improve market efficiency because it reduces the entrant's cost. Specifically, information sharing reduces R_E 's information rent, whereby allowing M_E to distort production less, which tends to increase aggregate production. Since reaction functions are downward sloping, however, the increase of the entrant's production triggers a reduction of the incumbent's production. The net effect on aggregate production depends on the degree of demand persistency, which measures the precision of the information that the entrant obtains on θ_2 when it learns θ_1 . When demand is sufficiently persistent in state $\theta_1 = \underline{\theta}$ (i.e., $\underline{\nu}$ is large) and it is not too persistent in state $\theta_1 = \bar{\theta}$ (i.e., $\bar{\nu}$ is small), the likelihood ratio $\mathcal{L}(\cdot)$ is small. In this case, information sharing has a stronger impact on the entrant's production than on the incumbent's production, thus increasing aggregate production. By contrast, when the information obtained by M_E on θ_1 does not result in a sufficiently large increase in R_E 's production, information sharing reduces aggregate production.

The effect of information sharing on aggregate production (in the limit of small uncertainty) also determines its impact on consumer surplus and total welfare. Hence, in contrast to the case of an integrated entrant, with competing hierarchies transparency standards improve efficiency only if demand is sufficiently persistent in bad times and/or not too persistent in good times.

Finally, allowing M and R to contract ex ante reduces welfare since it induces them not to share information.

Proposition 16 *The ex ante joint profit of M and R is lower with information sharing than without.*

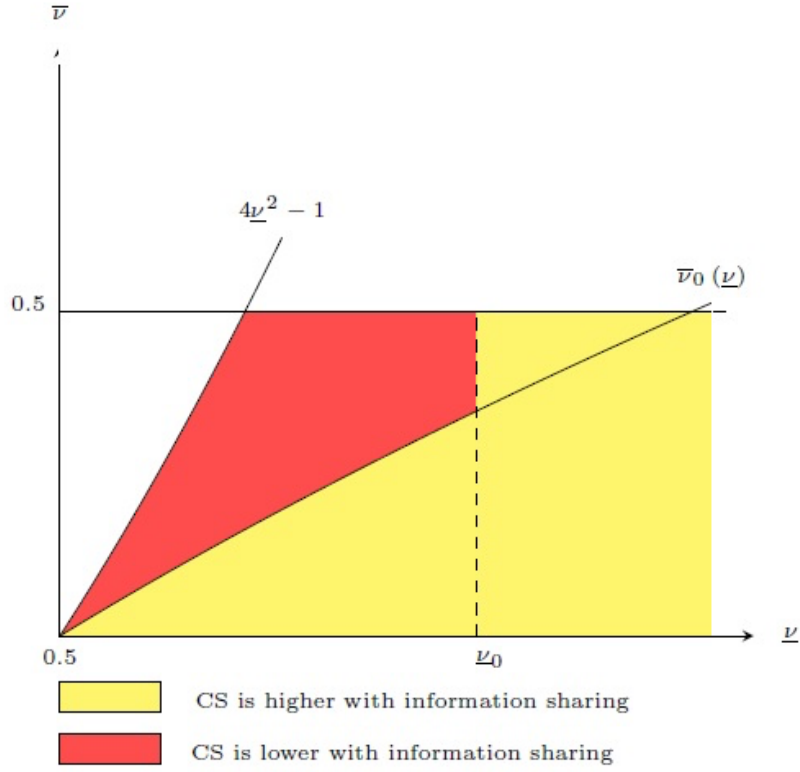


Figure 1: Effect of information sharing on consumer surplus.

7.1.5 Market for Information

Assume now that incumbents can commit to a price at which they sell information to the entrant. Of course, R and M_E have a joint incentive to trade information, since they obtain a higher profit with information sharing. Moreover, as in our baseline model, the highest price that M_E is willing to pay for information cannot internalize M 's loss due to the higher information rent for R .

Proposition 17 *It is never profitable for M to sell information to M_E .*

Suppose now that M and R maximize joint profits when selling information to M_E .

Proposition 18 *There exist two thresholds \underline{v}_1 and $\bar{v}_1(\underline{v}) \leq \bar{v}^*$ such that M and R have a joint incentive to sell information to M_E if and only if $\underline{v} \leq \underline{v}_1$ and $\bar{v} \geq \bar{v}_1(\underline{v})$.*

Figure 2 illustrates the region of parameters where M and R have a joint incentive to sell information. When \bar{v} is sufficiently large, firms trade information in order to gain market power vis-à-vis consumers, who are harmed by information sharing. By contrast, M and R do not sell information when (ceteris paribus) $\underline{v} \leq \underline{v}_1$ because an informed M_E distorts

production more when $\underline{\nu}$ is small (see the expression of $\mathcal{L}(\theta)$ in (16)). In this case, it is less costly for the incumbents to face an informed competitor, which lowers the reservation price at which they are willing to sell information.²⁸

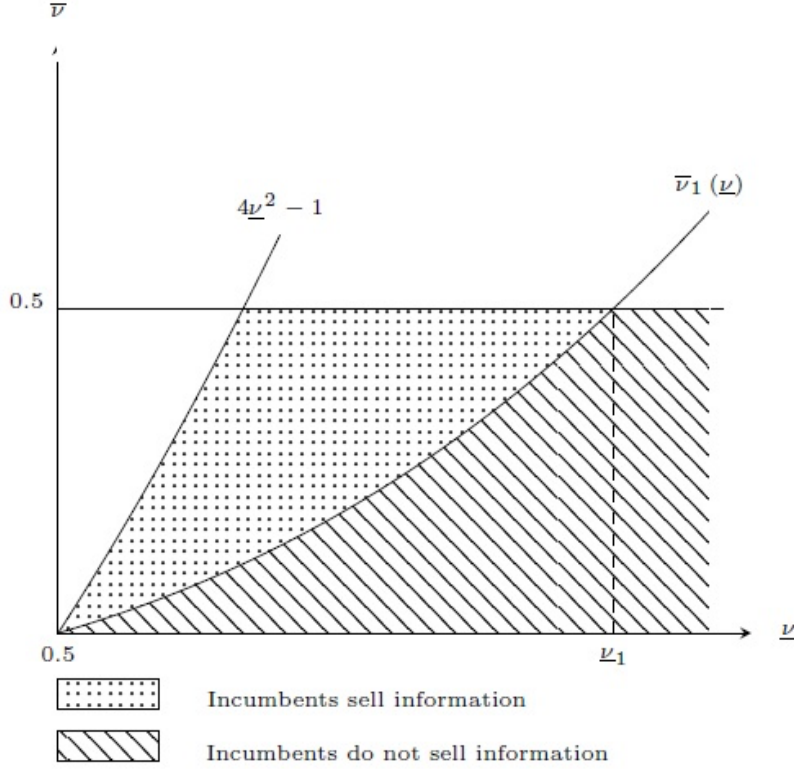


Figure 2: Incumbents' incentive to sell information.

Finally, the next result shows when the presence of a market where firms can trade information harms consumers.

Proposition 19 *When M and R have a joint incentive to sell information to M_E , information sharing harms consumers if $\underline{\nu} < \underline{\nu}_0$ and $\bar{\nu} > \bar{\nu}_0(\underline{\nu})$. When, M and R have no incentive to sell information to M_E , information sharing always benefits consumers.*

Figure 3 graphically summarizes the result of Proposition 19. When incumbents sell information to the entrant, the welfare effect depends on the degree of demand persistency. By contrast, consumers are always worse off when incumbents do not sell information. Hence, a social planner should force incumbents to sell information when they are not willing to do so, despite the presence of a market for information.

²⁸Of course, M_E 's willingness to pay for information also depends on the fact that information sharing reduces R_E 's rent.

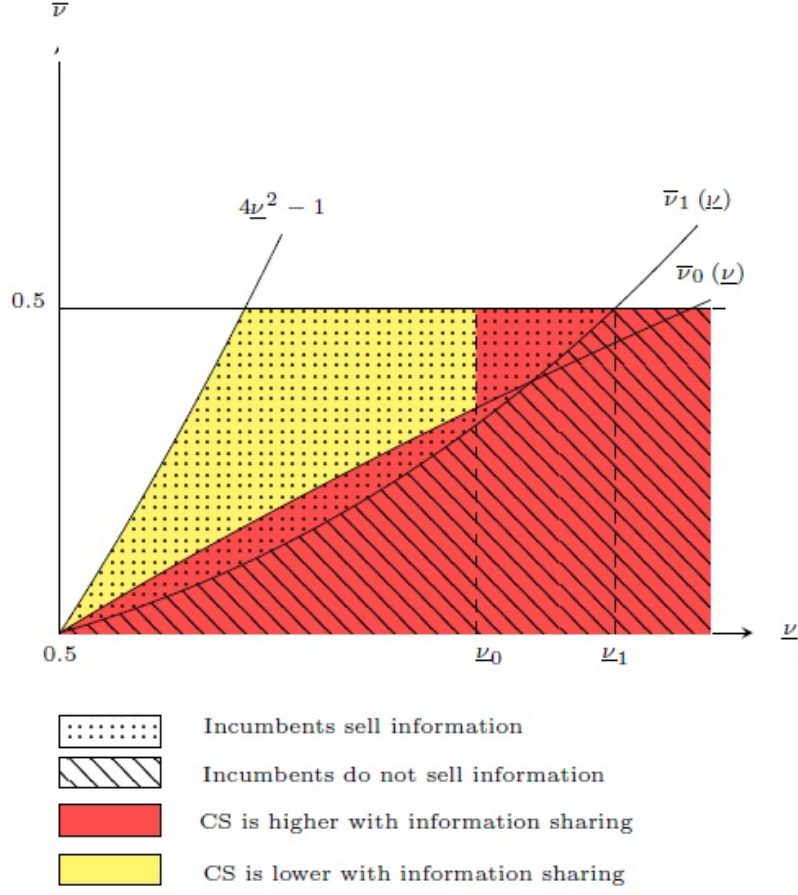


Figure 3: Does trading information harm consumers?

7.2 Stochastic Disclosure Rule

In our main model we have assumed an all-or-nothing disclosure rule. In this section, we consider a more sophisticated communication protocol that relies on a stochastic structure.²⁹ To simplify the analysis, we assume that $\bar{\nu} = \underline{\nu} = \nu$.

If it shares information, the incumbent commits to a disclosure rule that consists of a binary experiment with two signals, $\sigma \in \{\bar{\sigma}, \underline{\sigma}\}$, such that $\Pr[\sigma = \bar{\sigma} | m_1 = \bar{\theta}] \triangleq \alpha$ and $\Pr[\sigma = \underline{\sigma} | m_1 = \underline{\theta}] \triangleq \beta$ (see, e.g., Bergemann *et al.*, 2017 and Kastl *et al.*, 2017). The parameters α and β measure the informativeness, or accuracy, of the experiment. As a convention (and without loss of generality), we assume that $\alpha + \beta \geq 1$.³⁰ Consistent with the assumption of verifiable information in our main model, the outcome of the experiment is public — i.e., there are no further information frictions between the incumbents and the entrant. An experiment with $\alpha = \beta = 1$ is fully informative, which is equivalent to $d = S$ in

²⁹As in our main model, we assume the entrant is an integrated firm.

³⁰This is just a labelling of signals that ensures that upon observing signal $\bar{\sigma}$ (resp. $\underline{\sigma}$), the entrant assigns higher probability to $m_1 = \bar{\theta}$ (resp. $\underline{\theta}$).

our main model; while an experiment with $\alpha + \beta = 1$ is uninformative, which is equivalent to $d = N$ in our main model.

The long term contract is a menu

$$\{q_1(m_1), t_1(m_1), q_2(m_2, m_1, \sigma), t_2(m_2, m_1, \sigma)\},$$

that specifies a quantity and a transfer in the second period that are also contingent on the realized signal σ .

Since σ and θ_2 are independent conditionally on m_1 , with information sharing E observes two independent signals on m_1 , that it uses to infer the quantity produced by the incumbent in the second period. Hence, the entrant's posterior is

$$\begin{aligned} \Pr[m_1 = \bar{\theta} | \sigma, \theta_2] &= \frac{\Pr[\sigma, \theta_2 | m_1 = \bar{\theta}] \Pr[m_1 = \bar{\theta}]}{\Pr[\sigma, \theta_2 | m_1 = \bar{\theta}] \Pr[m_1 = \bar{\theta}] + \Pr[\sigma, \theta_2 | m_1 = \underline{\theta}] \Pr[m_1 = \underline{\theta}]} \\ &= \frac{\Pr[\theta_2 | \theta_1 = \bar{\theta}] \Pr[\sigma | \theta_1 = \bar{\theta}]}{\Pr[\theta_2 | \theta_1 = \bar{\theta}] \Pr[\sigma | \theta_1 = \bar{\theta}] + \Pr[\theta_2 | \theta_1 = \underline{\theta}] \Pr[\sigma | \theta_1 = \underline{\theta}]}, \end{aligned}$$

where we have used the fact that, in a truthful equilibrium where $m_1 = \theta_1$, $\Pr[m_1 = \bar{\theta}] = \Pr[\theta_1 = \bar{\theta}] = \frac{1}{2}$. And, for any realization (σ, θ_2) , the entrant's problem is

$$\max_{q_E \geq 0} \sum_{\theta_1} \Pr[\theta_1 | \sigma, \theta_2] P(\theta_2, q_E + q_2(\theta_2, \theta_1, \sigma)) q_E,$$

whose first order condition yields

$$q_E(\theta_2, \sigma) \triangleq \frac{\theta_2 - \sum_{\theta_1} \Pr[\theta_1 | \theta_2, \sigma] q_2(\theta_2, \theta_1, \sigma)}{2}, \quad \forall (\sigma, \theta_2).$$

Consider now the incumbent's problem. For any σ , let

$$\Delta q^S(\sigma) \triangleq q_E^S(\bar{\theta}, \sigma) - q_E^S(\underline{\theta}, \sigma)$$

be the difference between E 's output with high and low demand in the second period. R 's binding incentive compatibility constraint in the second period (see the Appendix) is

$$U_2(\bar{\theta}, m_1, \sigma) = \Delta \theta q_2(\bar{\theta}, m_1, \sigma) - \underbrace{[q_E(\bar{\theta}, \sigma) - q_E(\underline{\theta}, \sigma)]}_{\Delta q(m_1, \sigma)} q_2(\underline{\theta}, m_1, \sigma),$$

where the competition effect now depends on the signal σ . Hence, the relevant first-period

incentive compatibility constraint is

$$U_1(\bar{\theta}) = \underbrace{\Delta\theta q_1(\underline{\theta})}_{\text{Static rent}} + \underbrace{\delta\nu \left[\sum_{\sigma} \Pr[\sigma|\underline{\theta}] (\Delta\theta - \Delta q(\underline{\theta}, \sigma)) q_2(\underline{\theta}, \underline{\theta}, \sigma) - \sum_{\sigma} \Pr[\sigma|\bar{\theta}] (\Delta\theta - \Delta q(\bar{\theta}, \sigma)) q_2(\underline{\theta}, \bar{\theta}, \sigma) \right]}_{\text{Intertemporal rent}},$$

which is equivalent to (5) when $\alpha + \beta = 1$ and to (10) when $\alpha = \beta = 1$.

The incumbent's maximization problem is

$$\max_{q_1(\cdot), q_2(\cdot), U_1(\cdot)} \mathbb{E} \left[\sum_{\tau=1,2} \delta^{\tau-1} P(\theta_{\tau}, Q_{\tau}(\cdot)) q_{\tau}(\cdot) \right] + \sum_{\theta_1} \Pr[\theta_1] \left[U_1(\theta_1) + \delta \Pr[\theta_2 = \bar{\theta}|\theta_1] \sum_{\sigma} \Pr[\sigma|\theta_1] [\Delta\theta - \Delta q(\theta_1, \sigma)] q_2(\underline{\theta}, \theta_1, \sigma) \right],$$

where

$$Q_2(\theta_2, \theta_1, \sigma) \triangleq q_2(\theta_2, \theta_1, \sigma) + q_E(\theta_2, \sigma).$$

In the Appendix, we show that the first-order conditions of this problem are analogous to those in our main model. The difference is that the entrant's production now depends on the signal produced by the experiment.

Proposition 20 *The optimal experiment offered by M is uninformative — i.e., it features $\alpha + \beta = 1$. The optimal experiment offered by R is fully informative — i.e., it features $\alpha = \beta = 1$. The uninformative experiment maximizes consumer surplus and welfare.*

Hence, even with a more complex information structure, our main qualitative results obtained with the all-or-nothing disclosure rule, and their policy implications, hold.

7.3 Secret Renegotiation and Ex-post Disclosure

In our main model, we assumed that the incumbent players can commit ex-ante to an information disclosure rule. Even if commitment is a standard hypothesis in the existing literature on information sharing (see, e.g., Vives, 2006), one may wonder whether our results are robust to the possibility that the incumbent players (secretly) renege on their ex ante commitment to share or not information. In this section we show that, when at the beginning of period 2 — i.e., before learning θ_2 — the incumbent players can renege on the information sharing decision, but not on the terms of the optimal long term contract, only the equilibrium with information sharing survives.

Proposition 21 *The equilibrium with information sharing characterized in Section 4.2 is robust to ex post renegotiation, while the equilibrium without information sharing is not.*

The reason why the equilibrium with information sharing is robust to ex-post renegotiation of the information sharing decision is straightforward. Consider an equilibrium in which the incumbent players commit to share information and M offers the long term contract characterized in Section 4.2. First, M has no incentive to renege on this commitment since players cannot modify the contractual terms and, hence, by refusing to share information M cannot increase the second-period transfer. But then R has no profitable deviation either, since the optimal long term contract is incentive compatible.

By contrast, the equilibrium without information sharing is not robust to ex-post renegotiation because R has a unilateral incentive to disclose information when demand in the first period is high. In fact, other things being equal, this reduces E 's production relative to the no information sharing outcome characterized in Section 4.1, whereby increasing R 's revenue.

This result suggests that when incumbent players can secretly renege on their ex-ante commitment not to share information, there is an even stronger incentive from a welfare point of view to ban communication with entrants.

7.4 Large Uncertainty

Our results hinge of the assumption of small uncertainty — i.e., $\Delta\theta$ small (Assumption 2) — that allowed us to analytically solve for players' expected profit and rents in Section 5. In this section we use numerical simulations to analyze the effects of large uncertainty.

To simplify the analysis, we assume that $\underline{\theta} = 1$ and $\bar{\nu} = \underline{\nu} = \nu$, so that Assumption 1 implies $\nu \geq \frac{1}{3}$. Moreover, we impose a no-shut down condition ensuring that $\Delta\theta$ is not too large to induce the incumbent to shut down production when demand is repeatedly low. In the Appendix we show that the incumbent never shuts down production if $\Delta\theta \leq \Delta\theta_0(\nu) \triangleq \frac{3\nu-1}{4}$.³¹

Hence, the two parameters of interest are ν and $\Delta\theta$ and we compare profits and rents with and without informations sharing when $\nu \geq \frac{1}{3}$ and $\Delta\theta \leq \Delta\theta_0(\nu)$ (see the Appendix for details).

Figure 4 shows that the retailer wants to share information if and only if uncertainty is sufficiently small — i.e.,

$$\Delta\theta \leq \Delta\theta_u(\nu) \triangleq \frac{2\nu(3\nu-1)}{14\nu+9\nu^2-3}.$$

Since $q_2^S(\underline{\theta}, \underline{\theta}) < q_2^N(\underline{\theta}, \underline{\theta})$, holding E 's production constant R prefers not to share information because rents are increasing with quantity. However, E 's production and, hence, the compe-

³¹In fact, for $\nu \geq \frac{1}{3}$,

$$q_2^S(\underline{\theta}, \underline{\theta}) = \frac{1}{3} - \frac{4}{3(3\nu-1)}\Delta\theta < q_2^N(\underline{\theta}, \underline{\theta}) = \frac{1}{3} - \frac{3+\nu}{6\nu}\Delta\theta,$$

and $q_2^S(\underline{\theta}, \underline{\theta}) \geq 0$ if and only if $\Delta\theta \leq \Delta\theta_0(\nu)$.

tion effect also depend on the entrant's information. When $\Delta\theta$ is small, R 's rent is mainly shaped by the competition effect because the difference in the incumbent's quantities with an without information sharing only has a second order effect — i.e., $q_2^S(\underline{\theta}, \underline{\theta}) - q_2^N(\underline{\theta}, \underline{\theta}) \rightarrow 0$ as $\Delta\theta \rightarrow 0$. By contrast, when $\Delta\theta$ grows large, the difference in the incumbent's quantities have a larger effect on R 's rent and overcome the competition effect, so that R prefers not to share information. And the effect of the incumbent's quantities magnifies when demand is more persistent.

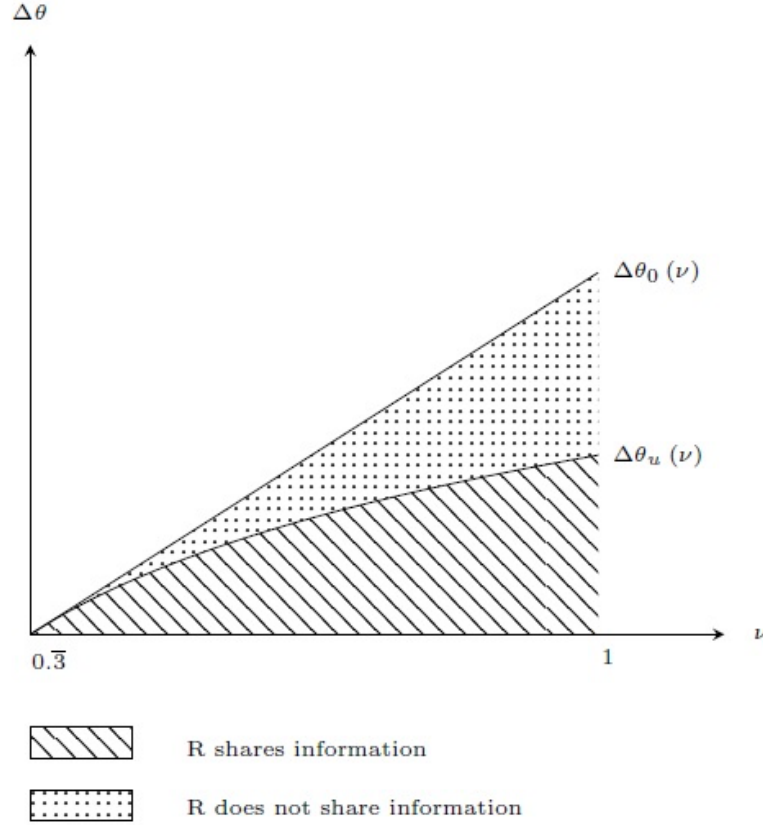


Figure 4: R 's incentive to share information with large uncertainty

The manufacturer's incentive to share information is illustrated in Figure 5. M prefers not to share information if and only if $\Delta\theta$ is sufficiently small — i.e.,

$$\Delta\theta \leq \Delta\theta_\pi(\nu) \triangleq \frac{16\nu(3\nu - 1)}{38\nu + 63\nu^2 - 9}.$$

Of course, even with large uncertainty, sharing information allows E to be more aggressive, which harms M . As $\Delta\theta$ grows large, however, R 's rent is higher without information sharing, as discussed above. Hence, for $\Delta\theta$ sufficiently large M has an incentive to share information.

Finally, since $\Delta\theta_\pi(\nu) \geq \Delta\theta_u(\nu)$, M and R have a joint incentive not to share information for $\Delta\theta \in [\Delta\theta_u(\nu), \Delta\theta_\pi(\nu)]$ even if they do not contract ex ante, as shown in Figure 6.

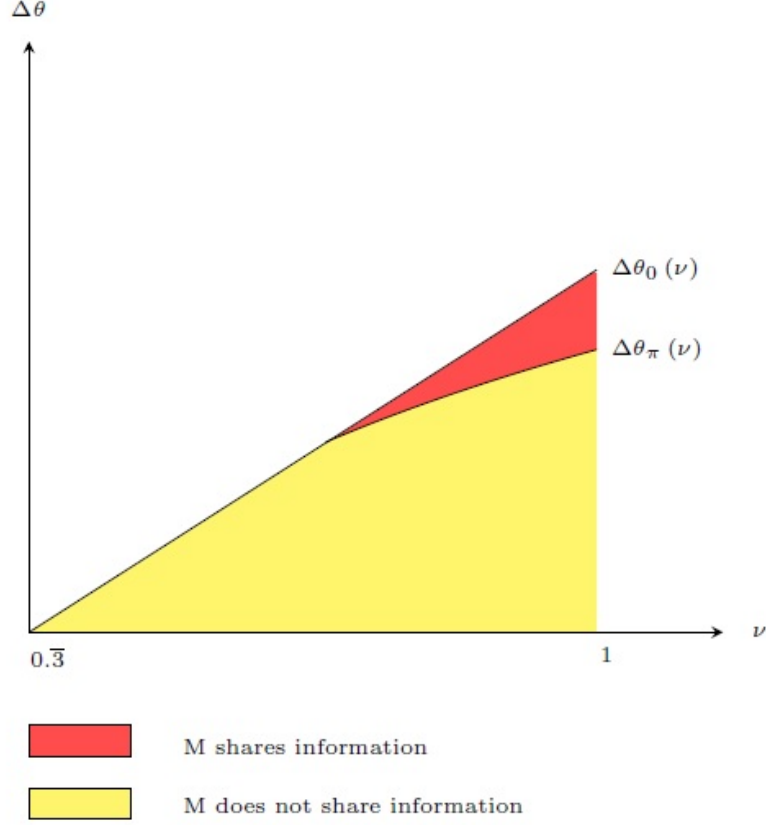


Figure 5: M 's incentive to share information with large uncertainty

8 Conclusions

It is commonly believed that forcing incumbents to be more transparent with entrants intensifies competition and increases consumer surplus, efficiency and total welfare. This presumption may be incorrect, however, when competition takes place between vertical hierarchies. Specifically, when incumbents contract over time with privately informed retailers or downstream units, forcing them to share information about past demand with an entrant may actually lower consumer surplus and total welfare. Interestingly, while downstream firms are willing to disclose their private information to entrants, upstream firms do not want to do so.

Although we developed our arguments in a manufacturer-retailer framework, the scope of our analysis is broader. Our insights apply to any environment involving entry by a competing organization with horizontal externalities, where principals deal with exclusive and privately informed agents, like procurement contracting, manufacturer-retailer relations, executive compensations, patent licensing, and insurance or credit relationships.

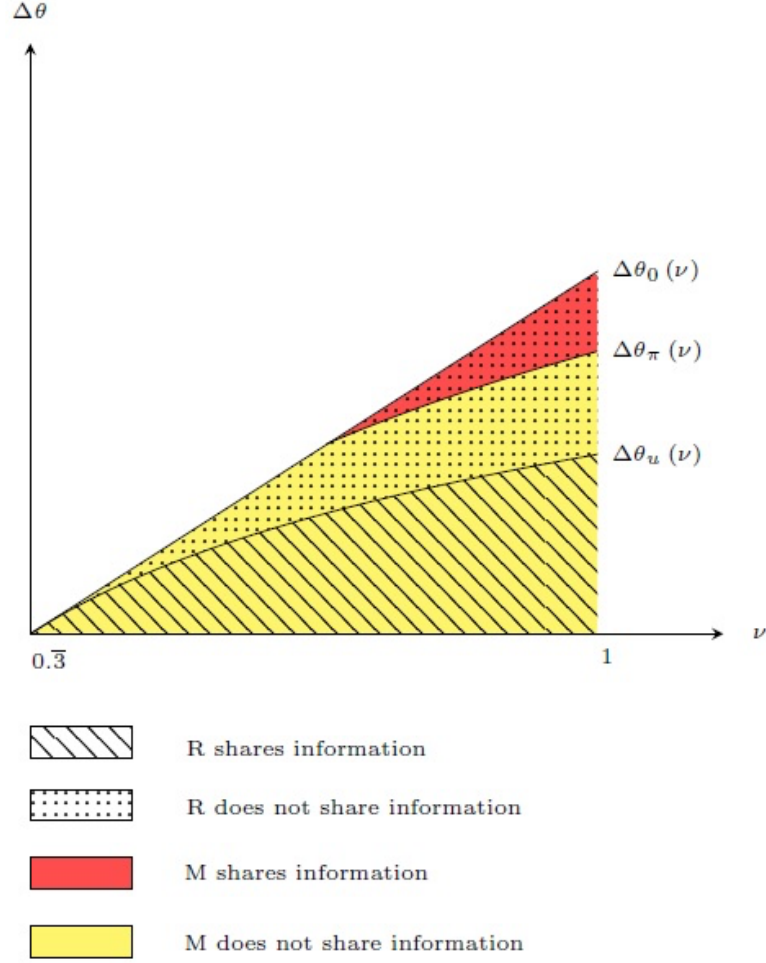


Figure 6: Joint incentives to share information with large uncertainty

A Appendix

Posterior Probabilities. Since in a separating equilibrium M 's report is truthful, — i.e., $m_1 = \theta_1$ — E 's beliefs are computed through the Bayes rule:

$$\Pr[\theta_1 = \bar{\theta} | \theta_2 = \bar{\theta}] \triangleq \frac{\Pr[\theta_2 = \bar{\theta} | \theta_1 = \bar{\theta}] \Pr[\theta_1 = \bar{\theta}]}{\sum_{\theta_1} \Pr[\theta_2 = \bar{\theta} | \theta_1] \Pr[\theta_1]} = \frac{\bar{\nu}}{1 + \Delta\nu},$$

$$\Pr[\theta_1 = \bar{\theta} | \theta_2 = \underline{\theta}] \triangleq \frac{\Pr[\theta_2 = \underline{\theta} | \theta_1 = \bar{\theta}] \Pr[\theta_1 = \bar{\theta}]}{\sum_{\theta_1} \Pr[\theta_2 = \underline{\theta} | \theta_1] \Pr[\theta_1]} = \frac{1 - \bar{\nu}}{1 - \Delta\nu},$$

with $\Pr[\theta_1 = \underline{\theta} | \theta_2] = 1 - \Pr[\theta_1 = \bar{\theta} | \theta_2]$ for every θ_2 .

Proof of Proposition 1. Both constraints (5) and $U_1(\underline{\theta}) \geq 0$ bind at the optimum.

Maximizing (6) with respect to $q_1(\bar{\theta})$ and $q_1(\underline{\theta})$ yields

$$q_1^N(\bar{\theta}) = \frac{\bar{\theta}}{2} > q_1^N(\underline{\theta}) = \frac{\underline{\theta} - \Delta\theta}{2}.$$

Maximizing (6) with respect to $q_2(\bar{\theta}, \theta_1)$, $q_2(\underline{\theta}, \bar{\theta})$ and $q_2(\underline{\theta}, \underline{\theta})$ yields

$$\bar{\theta} - 2q_2(\bar{\theta}, \theta_1) - q_E^N(\bar{\theta}) = 0, \quad \forall \theta_1 \in \Theta, \quad (\text{A1})$$

$$\underline{\theta} - 2q_2(\underline{\theta}, \bar{\theta}) - q_E^N(\underline{\theta}) = 0, \quad (\text{A2})$$

$$\underline{\theta} - 2q_2(\underline{\theta}, \underline{\theta}) - q_E^N(\underline{\theta}) - \frac{1 + \Delta\nu}{\underline{\nu}} (\Delta\theta - \Delta q^N) = 0. \quad (\text{A3})$$

Using E 's reaction function (2), we obtain $q_2^N(\bar{\theta}, \theta_1) = q_E^N(\bar{\theta})$ and

$$q_E^N(\underline{\theta}) = q^C(\underline{\theta}) + \frac{1 + \Delta\nu}{3(1 - 2\Delta\nu)} \Delta\theta,$$

$$q_2^N(\underline{\theta}, \bar{\theta}) = q^C(\underline{\theta}) - \frac{1 + \Delta\nu}{6(1 - 2\Delta\nu)} \Delta\theta,$$

$$q_2^N(\underline{\theta}, \underline{\theta}) = q^C(\underline{\theta}) - \frac{(1 + \Delta\nu)(3(1 - \Delta\nu) + \underline{\nu})}{6\underline{\nu}(1 - 2\Delta\nu)} \Delta\theta.$$

It is then immediate to verify that

$$q_2^N(\underline{\theta}, \bar{\theta}) < q^C(\underline{\theta}) < q_E^N(\underline{\theta}),$$

and, by Assumption 1,

$$q_2^N(\underline{\theta}, \underline{\theta}) - q_2^N(\underline{\theta}, \bar{\theta}) = -\frac{(1 + \Delta\nu)(1 - \Delta\nu)}{2\underline{\nu}(1 - 2\Delta\nu)} \Delta\theta < 0.$$

R 's second-period rent is strictly positive since, by Assumption 1,

$$\Delta\theta - \Delta q^N = \frac{1 - \Delta\nu}{1 - 2\Delta\nu} \Delta\theta > 0.$$

R 's first-period rent is

$$U_1^N(\bar{\theta}) = \Delta\theta q_1^N(\underline{\theta}) + \delta\bar{\nu}(\Delta\theta - \Delta q^N) [q_2^N(\underline{\theta}, \underline{\theta}) - q_2^N(\underline{\theta}, \bar{\theta})].$$

By first-order Taylor approximation around $\Delta\theta = 0$,

$$U_1^N(\bar{\theta}) \approx \lim_{\Delta\theta \rightarrow 0} U_1^N(\bar{\theta}) + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial U_1^N(\bar{\theta})}{\partial \Delta\theta},$$

where $\lim_{\Delta\theta \rightarrow 0} U_1^N(\bar{\theta}) = 0$. Letting $\underline{\theta} = \bar{\theta} - \Delta\theta$,

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial U_1^N(\bar{\theta})}{\partial \Delta\theta} = q^M(\bar{\theta}).$$

Therefore, for $\Delta\theta$ small (Assumption 2) R 's first-period rent is also strictly positive. ■

Proof of Proposition 2. Both constraints (10) and $U_1(\underline{\theta}) \geq 0$ are binding at the optimum. Maximizing (11) with respect to $q_1(\cdot)$, it is straightforward to show that first-period production is the same as without information sharing. Maximizing (11) with respect to $q_2(\bar{\theta}, \theta_1)$, $q_2(\underline{\theta}, \bar{\theta})$ and $q_2(\underline{\theta}, \underline{\theta})$ yields

$$\bar{\theta} - 2q_2(\bar{\theta}, \theta_1) - q_E^S(\bar{\theta}, \theta_1) = 0, \quad \forall \theta_1 \in \Theta, \quad (\text{A4})$$

$$\underline{\theta} - 2q_2(\underline{\theta}, \bar{\theta}) - q_E^S(\underline{\theta}, \bar{\theta}) = 0, \quad (\text{A5})$$

$$\underline{\theta} - 2q_2(\underline{\theta}, \underline{\theta}) - q_E^S(\underline{\theta}, \underline{\theta}) - \frac{1 + \Delta\nu}{\underline{\nu}} \times [\Delta\theta - \Delta q^S(\underline{\theta})] = 0. \quad (\text{A6})$$

Using E 's reaction function (1), we obtain

$$q_2^S(\underline{\theta}, \underline{\theta}) = q^C(\underline{\theta}) - \frac{4(1 + \Delta\nu)}{3(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta,$$

$$q_E^S(\underline{\theta}, \underline{\theta}) = q^C(\underline{\theta}) + \frac{2(1 + \Delta\nu)}{3(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta.$$

Moreover,

$$q_2^S(\bar{\theta}, \theta_1) = q_E^S(\bar{\theta}, \theta_1) = q^C(\bar{\theta}), \quad \forall \theta_1 \in \Theta,$$

and $q_2^S(\underline{\theta}, \bar{\theta}) = q_E^S(\underline{\theta}, \bar{\theta}) = q^C(\underline{\theta})$. By Assumption 1,

$$q_2^S(\underline{\theta}, \underline{\theta}) - q_2^S(\underline{\theta}, \bar{\theta}) = -\frac{4(1 + \Delta\nu)}{3(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta < 0,$$

$$q_E^S(\underline{\theta}, \underline{\theta}) - q_E^S(\underline{\theta}, \bar{\theta}) = \frac{2(1 + \Delta\nu)}{3(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta > 0.$$

Hence, E produces more than R when demand is repeatedly low.

R 's second-period rent is strictly positive since, by Assumption 1,

$$\Delta\theta - \Delta q^S(\bar{\theta}) = \frac{2}{3} \Delta\theta > 0, \quad \Delta\theta - \Delta q^S(\underline{\theta}) = \frac{2\underline{\nu}}{4\underline{\nu} - \bar{\nu} - 1} \Delta\theta > 0.$$

R 's first-period rent is

$$U_1^S(\bar{\theta}) = \Delta\theta q_1^S(\underline{\theta}) + \delta\bar{\nu} \{ [\Delta\theta - \Delta q^S(\underline{\theta})] q_2^S(\underline{\theta}, \underline{\theta}) - [\Delta\theta - \Delta q^S(\bar{\theta})] q_2^S(\underline{\theta}, \bar{\theta}) \}.$$

By a first-order Taylor approximation around $\Delta\theta = 0$,

$$U_1^S(\bar{\theta}) \approx \lim_{\Delta\theta \rightarrow 0} U_1^S(\bar{\theta}) + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial U_1^S(\bar{\theta})}{\partial \Delta\theta},$$

where $\lim_{\Delta\theta \rightarrow 0} U_1^S(\bar{\theta}) = 0$. Letting $\underline{\theta} = \bar{\theta} - \Delta\theta$,

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial U_1^S(\bar{\theta})}{\partial \Delta\theta} = q^M(\bar{\theta}) + \delta\bar{\nu} [\dot{q}_E^S(\underline{\theta}, \underline{\theta}) - \dot{q}_E^S(\underline{\theta}, \bar{\theta})] q^C(\bar{\theta}).$$

Hence, for $\Delta\theta$ small,

$$U_1^S(\bar{\theta}) \approx q^M(\bar{\theta})\Delta\theta + \delta\bar{\nu} [\dot{q}_E^S(\underline{\theta}, \underline{\theta}) - \dot{q}_E^S(\underline{\theta}, \bar{\theta})] q^C(\bar{\theta})\Delta\theta.$$

Since $\dot{q}_E^S(\underline{\theta}, \underline{\theta}) > 0$ and $\dot{q}_E^S(\underline{\theta}, \bar{\theta}) < 0$ by Assumption 1, for $\Delta\theta$ small R 's first-period rent is strictly positive. ■

Proof of Proposition 3. We compare E 's equilibrium quantities with and without information sharing. When demand is low in both periods, by Assumption 1,

$$q_E^S(\underline{\theta}, \underline{\theta}) - q_E^N(\underline{\theta}) = \frac{(1 + \Delta\nu)(1 - \bar{\nu})}{(4\underline{\nu} - \bar{\nu} - 1)(1 - 2\Delta\nu)} \Delta\theta > 0.$$

When demand is low only in the second period, by Assumption 1,

$$q_E^S(\underline{\theta}, \bar{\theta}) - q_E^N(\underline{\theta}) = -\frac{1 + \Delta\nu}{3(1 - 2\Delta\nu)} \Delta\theta < 0.$$

For the last part of the proposition,

$$\begin{aligned} \mathbb{E}[q_E^S(\underline{\theta}, \theta_1) | \underline{\theta}] - q_E^N(\underline{\theta}) &= \Pr[\theta_1 = \bar{\theta} | \theta_2 = \underline{\theta}] q_E^S(\underline{\theta}, \bar{\theta}) + \Pr[\theta_1 = \underline{\theta} | \theta_2 = \underline{\theta}] q_E^S(\underline{\theta}, \underline{\theta}) - q_E^N(\underline{\theta}) \\ &= \frac{(1 - \bar{\nu})(1 + \Delta\nu)^2}{3(4\underline{\nu} - \bar{\nu} - 1)(1 - \Delta\nu)(1 - 2\Delta\nu)} \Delta\theta, \end{aligned}$$

which is strictly positive by Assumption 1. ■

Proof of Proposition 4. First, we compare R 's ex ante rent with and without information sharing. R 's rent without information sharing is

$$\begin{aligned} \mathcal{V}^N &\triangleq \frac{\delta}{2} (1 - \underline{\nu}) (\Delta\theta - \Delta q^N) q_2^N(\underline{\theta}, \underline{\theta}) \\ &+ \frac{1}{2} \{ \underbrace{\Delta\theta q^M(\underline{\theta}) + \delta\bar{\nu} (\Delta\theta - \Delta q^N) [q_2^N(\underline{\theta}, \underline{\theta}) - q_2^N(\underline{\theta}, \bar{\theta})]}_{\triangleq U_1^N(\bar{\theta})} + \delta\bar{\nu} (\Delta\theta - \Delta q^N) q_2^N(\underline{\theta}, \bar{\theta}) \}. \end{aligned}$$

By a first-order Taylor approximation around $\Delta\theta = 0$,

$$\mathcal{V}^N \approx \lim_{\Delta\theta \rightarrow 0} \mathcal{V}^N + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{V}^N}{\partial \Delta\theta},$$

where $\lim_{\Delta\theta \rightarrow 0} \mathcal{V}^N = 0$ and, letting $\underline{\theta} = \bar{\theta} - \Delta\theta$,

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{V}^N}{\partial \Delta\theta} = \frac{1}{2} q^M(\bar{\theta}) + \frac{\delta}{2} q^C(\bar{\theta}) [\bar{\nu} (1 + \dot{q}_E^N(\underline{\theta})) + (1 - \underline{\nu}) (1 + \dot{q}_E^N(\underline{\theta}))].$$

Hence,

$$\mathcal{V}^N \approx \frac{1}{2} q^M(\bar{\theta}) \Delta\theta + \frac{\delta}{2} q^C(\bar{\theta}) [\bar{\nu} (1 + \dot{q}_E^N(\underline{\theta})) + (1 - \underline{\nu}) (1 + \dot{q}_E^N(\underline{\theta}))] \Delta\theta. \quad (\text{A7})$$

Similarly, R 's rent with information sharing is

$$\begin{aligned} \mathcal{V}^S \triangleq & \frac{\delta}{2} (1 - \underline{\nu}) [\Delta\theta - \Delta q^S(\underline{\theta})] q_2^S(\underline{\theta}, \underline{\theta}) \\ & + \frac{1}{2} \{ \underbrace{\Delta\theta q_1^M(\underline{\theta}) + \delta \bar{\nu} [(\Delta\theta - \Delta q^S(\underline{\theta})) q_2^S(\underline{\theta}, \underline{\theta}) - (\Delta\theta - \Delta q^S(\bar{\theta})) q_2^S(\underline{\theta}, \bar{\theta})]}_{\triangleq U_1^S(\bar{\theta})} \\ & + \delta \bar{\nu} (\Delta\theta - \Delta q^S(\bar{\theta})) q_2^S(\underline{\theta}, \bar{\theta}) \} . \end{aligned}$$

As before, $\lim_{\Delta\theta \rightarrow 0} \mathcal{V}^S = 0$ and

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{V}^S}{\partial \Delta\theta} = \frac{1}{2} q^M(\bar{\theta}) + \frac{\delta}{2} q^C(\bar{\theta}) [\bar{\nu} (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta})) + (1 - \underline{\nu}) (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta}))] .$$

Hence,

$$\mathcal{V}^S \approx \frac{1}{2} q^M(\bar{\theta}) \Delta\theta + \frac{\delta}{2} q^C(\bar{\theta}) [\bar{\nu} (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta})) + (1 - \underline{\nu}) (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta}))] \Delta\theta. \quad (\text{A8})$$

Comparing (A7) and (A8),

$$\begin{aligned} \mathcal{V}^N - \mathcal{V}^S & \approx \frac{\delta}{2} q^C(\bar{\theta}) [\bar{\nu} (\dot{q}_E^N(\underline{\theta}) - \dot{q}_E^S(\underline{\theta}, \underline{\theta})) + (1 - \underline{\nu}) (\dot{q}_E^N(\underline{\theta}) - \dot{q}_E^S(\underline{\theta}, \underline{\theta}))] \Delta\theta \\ & = -\frac{\delta q^C(\bar{\theta}) (1 - \bar{\nu}) (1 + \Delta\nu)^2}{2 (1 - 2\Delta\nu) (4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta < 0, \end{aligned} \quad (\text{A9})$$

where we have used equilibrium quantities from Propositions 1 and 2 and Assumption 1.

Second, we compare M 's expected profit with and without information sharing. By a first-order Taylor approximation around $\Delta\theta = 0$, M 's expected profit without information sharing is

$$\Pi^N \approx \lim_{\Delta\theta \rightarrow 0} \Pi^N + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial \Pi^N}{\partial \Delta\theta},$$

where

$$\lim_{\Delta\theta \rightarrow 0} \Pi^N = q^M(\bar{\theta})^2 + \delta q^C(\bar{\theta})^2,$$

and, using $\underline{\theta} = \bar{\theta} - \Delta\theta$ and the Envelope Theorem,

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \Pi^N}{\partial \Delta\theta} = -q^M(\bar{\theta}) - \delta [1 + \dot{q}_E^N(\underline{\theta})] q^C(\bar{\theta}).$$

Hence,

$$\Pi^N \approx q^M(\bar{\theta})^2 + \delta q^C(\bar{\theta})^2 - [q^M(\bar{\theta}) + q^C(\bar{\theta}) \delta (1 + \dot{q}_E^N(\underline{\theta}))] \Delta\theta. \quad (\text{A10})$$

With information sharing, since

$$\lim_{\Delta\theta \rightarrow 0} \Pi^S = q^M(\bar{\theta})^2 + \delta q^C(\bar{\theta})^2,$$

and

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \Pi^S}{\partial \Delta\theta} = -q^M(\bar{\theta}) - q^C(\bar{\theta})\delta [\bar{\nu}(1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta})) + (1 - \bar{\nu})(1 + \dot{q}_E^S(\underline{\theta}, \bar{\theta})) + (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta}))],$$

M 's expected profit is

$$\begin{aligned} \Pi^S &\approx q^M(\bar{\theta})^2 + \delta q^C(\bar{\theta})^2 - q^M(\bar{\theta})\Delta\theta + \\ &\quad - q^C(\bar{\theta})\delta [\bar{\nu}(1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta})) + (1 - \bar{\nu})(1 + \dot{q}_E^S(\underline{\theta}, \bar{\theta})) + (1 + \dot{q}_E^S(\underline{\theta}, \underline{\theta}))] \Delta\theta. \end{aligned}$$

Comparing this with (A10),

$$\begin{aligned} \Pi^N - \Pi^S &\approx q^C(\bar{\theta})\delta \left[\dot{q}_E^S(\underline{\theta}, \underline{\theta}) - \dot{q}_E^N(\underline{\theta}) - \frac{1 - \bar{\nu}}{2} (\dot{q}_E^S(\underline{\theta}, \underline{\theta}) - \dot{q}_E^S(\underline{\theta}, \bar{\theta})) \right] \Delta\theta \\ &= \frac{2\delta q^C(\bar{\theta})(1 - \bar{\nu})(1 + \Delta\nu)^2}{3(4\underline{\nu} - \bar{\nu} - 1)(1 - 2\Delta\nu)} \Delta\theta > 0, \end{aligned} \quad (\text{A11})$$

where we have used equilibrium quantities from Propositions 1 and 2 and Assumption 1. ■

Proof of Proposition 5. For any $d \in \{S, N\}$, the ex ante joint profit of M and the R is $\Pi^d + \mathcal{V}^d$. For $\Delta\theta$ small, using Taylor approximations and the results of Proposition 4,

$$(\Pi^N + \mathcal{V}^N) - (\Pi^S + \mathcal{V}^S) \approx \frac{\delta q^C(\bar{\theta})(1 - \bar{\nu})(1 + \Delta\nu)^2}{6(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta.$$

This is strictly positive by Assumption 1. ■

Proof of Proposition 6. Let Π_E^d , $d \in \{S, N\}$, be the entrant's ex ante profit. Let $\underline{\theta} = \bar{\theta} - \Delta\theta$. By Taylor approximations around $\Delta\theta = 0$ we have

$$\begin{aligned} \Pi_E^N &\approx \lim_{\Delta\theta \rightarrow 0} \Pi_E^N + \lim_{\Delta\theta \rightarrow 0} \frac{\partial \Pi_E^N}{\partial \Delta\theta} \Delta\theta \\ &= q^C(\bar{\theta})^2 - \frac{1}{2} [(1 - \bar{\nu})(1 + \dot{q}_2^N(\underline{\theta}, \bar{\theta})) + \underline{\nu}(1 + \dot{q}_2^N(\underline{\theta}, \underline{\theta}))] q^C(\bar{\theta}) \Delta\theta, \end{aligned}$$

and

$$\begin{aligned} \Pi_E^S &\approx \lim_{\Delta\theta \rightarrow 0} \Pi_E^S + \lim_{\Delta\theta \rightarrow 0} \frac{\partial \Pi_E^S}{\partial \Delta\theta} \Delta\theta \\ &= q^C(\bar{\theta})^2 - \frac{1}{2} [(1 - \bar{\nu})(1 + \dot{q}_2^S(\underline{\theta}, \bar{\theta})) + \underline{\nu}(1 + \dot{q}_2^S(\underline{\theta}, \underline{\theta}))] q^C(\bar{\theta}) \Delta\theta. \end{aligned}$$

Hence,

$$\begin{aligned}\Pi_E^N - \Pi_E^S &\approx \frac{1}{2} [(1 - \bar{\nu}) (\dot{q}_2^S(\underline{\theta}, \bar{\theta}) - \dot{q}_2^N(\underline{\theta}, \bar{\theta})) + \underline{\nu} (\dot{q}_2^S(\underline{\theta}, \underline{\theta}) - \dot{q}_2^N(\underline{\theta}, \underline{\theta}))] q^C(\bar{\theta}) \Delta\theta \\ &= -\frac{q^C(\bar{\theta}) (1 + \Delta\nu)^2 (1 - \bar{\nu})}{3(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta,\end{aligned}$$

which is negative under Assumption 1. ■

Proof of Proposition 7. Let ρ be the price for information about θ_1 , that E pays to M . E is willing to buy information if and only if

$$\rho \leq \Pi_E^S - \Pi_E^N.$$

M is willing to sell information if and only if

$$\rho \geq \Pi_2^N - \Pi_2^S.$$

Hence, M and E are willing to trade at price $\rho > 0$ if and only if

$$\Pi_2^N - \Pi_2^S \leq \Pi_E^S - \Pi_E^N.$$

Under Assumption 2 this condition simplifies to

$$\frac{2q^C(\bar{\theta}) (1 - \bar{\nu}) (1 + \Delta\nu)^2}{3(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta \leq \frac{q^C(\bar{\theta}) (1 - \bar{\nu}) (1 + \Delta\nu)^2}{3(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta,$$

which is never satisfied. ■

Proof of Proposition 8. M and R are willing to trade information if and only if

$$\mathcal{J}_2^N - \mathcal{J}_2^S \leq \Pi_E^S - \Pi_E^N,$$

where J_2^d is the joint profit of M and R in the second period. Using the results of Propositions 5 and 6, under Assumption 2 this condition simplifies to

$$\frac{q^C(\bar{\theta}) (1 - \bar{\nu}) (1 + \Delta\nu)^2}{6(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta \leq \frac{q^C(\bar{\theta}) (1 + \Delta\nu)^2 (1 - \bar{\nu})}{3(1 - 2\Delta\nu)(4\underline{\nu} - \bar{\nu} - 1)} \Delta\theta,$$

which is always satisfied. ■

Proof of Proposition 9. For any disclosure policy $d \in \{S, N\}$, expected aggregate production in the second period is

$$\mathcal{Q}^d \triangleq \sum_{\theta_1 \in \Theta} \Pr[\theta_1] \sum_{\theta_2 \in \Theta} \Pr[\theta_2 | \theta_1] Q_2^d(\theta_2, \theta_1).$$

For $\Delta\theta$ small

$$\mathcal{Q}^d \approx \lim_{\Delta\theta \rightarrow 0} \mathcal{Q}^d + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{Q}^d}{\partial \Delta\theta},$$

with $\lim_{\Delta\theta \rightarrow 0} \mathcal{Q}^S = \lim_{\Delta\theta \rightarrow 0} \mathcal{Q}^N = 2q^C(\bar{\theta})$, and

$$\begin{aligned}\lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{Q}^N}{\partial \Delta\theta} &= \frac{1}{2}[(1 - \bar{\nu}) \underbrace{(\dot{q}_2^N(\underline{\theta}, \bar{\theta}) + \dot{q}_E^N(\underline{\theta}))}_{\dot{Q}^N(\underline{\theta}, \bar{\theta})} + \underline{\nu} \underbrace{(\dot{q}_2^N(\underline{\theta}, \underline{\theta}) + \dot{q}_E^N(\underline{\theta}))}_{\dot{Q}^N(\underline{\theta}, \underline{\theta})}], \\ \lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{Q}^S}{\partial \Delta\theta} &= \frac{1}{2}[(1 - \bar{\nu}) \underbrace{(\dot{q}_2^S(\underline{\theta}, \bar{\theta}) + \dot{q}_E^S(\underline{\theta}, \bar{\theta}))}_{\dot{Q}^S(\underline{\theta}, \bar{\theta})} + \underline{\nu} \underbrace{(\dot{q}_2^S(\underline{\theta}, \underline{\theta}) + \dot{q}_E^S(\underline{\theta}, \underline{\theta}))}_{\dot{Q}^S(\underline{\theta}, \underline{\theta})}].\end{aligned}$$

Hence,

$$\begin{aligned}\mathcal{Q}^S - \mathcal{Q}^N &\approx \left[(1 - \bar{\nu}) \left(\dot{Q}^S(\underline{\theta}, \bar{\theta}) - \dot{Q}^N(\underline{\theta}, \bar{\theta}) \right) + \underline{\nu} \left(\dot{Q}^S(\underline{\theta}, \underline{\theta}) - \dot{Q}^N(\underline{\theta}, \underline{\theta}) \right) \right] \frac{\Delta\theta}{2} \\ &= -\frac{(1 - \bar{\nu})(1 + \Delta\nu)^2}{6(4\underline{\nu} - \bar{\nu} - 1)(1 - 2\Delta\nu)} \Delta\theta < 0,\end{aligned}\tag{A12}$$

where we have used equilibrium quantities from Propositions 1 and 2 and Assumption 1. ■

Proof of Proposition 10. Without loss of generality, we focus on the second period, since production in the first period is the same with and without information sharing. For any $d \in \{S, N\}$, since the inverse demand is linear, expected consumer surplus is

$$\mathcal{CS}^d = \sum_{\theta_1 \in \Theta} \Pr[\theta_1] \sum_{\theta_2 \in \Theta} \Pr[\theta_2|\theta_1] \frac{Q_2^d(\theta_2, \theta_1)^2}{2}.$$

For $\Delta\theta$ small

$$\mathcal{CS}^d \approx \lim_{\Delta\theta \rightarrow 0} \mathcal{CS}^d + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{CS}^d}{\partial \Delta\theta},$$

with $\lim_{\Delta\theta \rightarrow 0} \mathcal{CS}^d = 2q^C(\bar{\theta})^2$ and

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{CS}^d}{\partial \Delta\theta} = q^C(\bar{\theta}) \left[(1 - \bar{\nu}) \dot{Q}^d(\underline{\theta}, \bar{\theta}) + \underline{\nu} \dot{Q}^d(\underline{\theta}, \underline{\theta}) \right].$$

Hence,

$$\mathcal{CS}^N - \mathcal{CS}^S \approx \frac{q^C(\bar{\theta})(1 - \bar{\nu})(1 + \Delta\nu)^2}{3(4\underline{\nu} - \bar{\nu} - 1)(1 - 2\Delta\nu)} \Delta\theta,$$

which is strictly positive by Assumption 1.

Total (expected) welfare in the second period — i.e., the sum of M 's expected profit, R 's expected rent, E 's expected profit and the expected consumer surplus — is

$$\mathcal{TW}^d \triangleq \sum_{\theta_1 \in \Theta} \Pr[\theta_1] \sum_{\theta_2 \in \Theta} \Pr[\theta_2|\theta_1] \left[\theta_2 Q_2^d(\theta_2, \theta_1) - \frac{1}{2} Q_2^d(\theta_2, \theta_1)^2 \right].$$

For $\Delta\theta$ small, using a first-order Taylor approximation around $\Delta\theta = 0$,

$$\mathcal{TW}^d \approx \lim_{\Delta\theta \rightarrow 0} \mathcal{TW}^d + \Delta\theta \lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{TW}^d}{\partial \Delta\theta},$$

with $\lim_{\Delta\theta \rightarrow 0} \mathcal{TW}^d = 4q^C(\bar{\theta})^2$ and

$$\lim_{\Delta\theta \rightarrow 0} \frac{\partial \mathcal{TW}^d}{\partial \Delta\theta} = \sum_{\theta_1} \Pr[\theta_1] \sum_{\theta_2} \Pr[\theta_2 = \underline{\theta} | \theta_1] \left[q^C(\bar{\theta}) \dot{Q}_2^d(\underline{\theta}, \theta_1) - 2q^C(\bar{\theta}) \right].$$

Hence,

$$\begin{aligned} \mathcal{TW}^N - \mathcal{TW}^S &\approx q^C(\bar{\theta}) \left[(1 - \bar{\nu}) \left(\dot{Q}_E^N(\underline{\theta}, \bar{\theta}) - \dot{Q}_E^S(\underline{\theta}, \bar{\theta}) \right) + \nu \left(\dot{Q}_E^N(\underline{\theta}, \underline{\theta}) - \dot{Q}_E^S(\underline{\theta}, \underline{\theta}) \right) \right] \frac{\Delta\theta}{2}, \\ &= \frac{1}{2} [\mathcal{CS}^N - \mathcal{CS}^S] > 0. \end{aligned}$$

■

Proof of Proposition 11. M 's maximization problem does not depend on the information sharing decision. Without information sharing, M_E 's (relaxed) maximization program is

$$\begin{aligned} &\max_{q_E(\cdot), U_E(\cdot)} \sum_{\theta_2 \in \Theta} \Pr[\theta_2] \sum_{\theta_1 \in \Theta} \Pr[\theta_1 | \theta_2] P(\theta_2, q_E(\theta_2) + q_2^N(\theta_2, \theta_1)) q_E(\theta_2) + \\ &- \Pr[\theta_2 = \bar{\theta}] \left[\Delta\theta - \sum_{\theta_1 \in \Theta} \Pr[\theta_1 | \theta_2 = \bar{\theta}] q_2^N(\bar{\theta}, \theta_1) + \sum_{\theta_1 \in \Theta} \Pr[\theta_1 | \theta_2 = \underline{\theta}] q_2^N(\underline{\theta}, \theta_1) \right] q_E(\underline{\theta}). \end{aligned}$$

Maximizing with respect to $q_E(\bar{\theta})$ and $q_E(\underline{\theta})$ yields

$$\begin{aligned} &\underbrace{\frac{\bar{\nu}}{1 + \Delta\nu}}_{\Pr[\theta_1 = \bar{\theta} | \theta_2 = \bar{\theta}]} \times [\bar{\theta} - 2q_E(\bar{\theta}) - q_2^N(\bar{\theta}, \bar{\theta})] + \underbrace{\frac{1 - \nu}{1 + \Delta\nu}}_{\Pr[\theta_1 = \underline{\theta} | \theta_2 = \bar{\theta}]} \times [\bar{\theta} - 2q_E(\bar{\theta}) - q_2^N(\bar{\theta}, \underline{\theta})] = 0, \\ &[\underline{\theta} - 2q_E(\underline{\theta})] - [\Delta\theta - q^C(\bar{\theta})] - 2 \left[\underbrace{\frac{1 - \bar{\nu}}{1 - \Delta\nu}}_{\Pr[\theta_1 = \bar{\theta} | \theta_2 = \underline{\theta}]} \times q_2^N(\underline{\theta}, \bar{\theta}) + \underbrace{\frac{\nu}{1 - \Delta\nu}}_{\Pr[\theta_1 = \underline{\theta} | \theta_2 = \underline{\theta}]} \times q_2^N(\underline{\theta}, \underline{\theta}) \right] = 0. \end{aligned}$$

Using (A1)-(A3), it follows that $q_E^N(\bar{\theta}) = q_2^N(\bar{\theta}, \theta_1)$, $q_E^N(\underline{\theta}) = q^C(\underline{\theta}) - \frac{2}{3}\Delta\theta$ and

$$q_2^N(\underline{\theta}, \bar{\theta}) = q_2^N(\underline{\theta}, \underline{\theta}) = q^C(\underline{\theta}) + \frac{1}{3}\Delta\theta.$$

By direct comparison of these quantities,

$$q_E^N(\underline{\theta}) < q^C(\bar{\theta}) < q_2^N(\underline{\theta}, \bar{\theta}) = q_2^N(\underline{\theta}, \underline{\theta}).$$

R obtains a non-negative rent in the second period since $\Delta\theta - \Delta q^N = 0$. Similarly, R_E rent is

$$\left[\Delta\theta - \sum_{\theta_1} \Pr[\theta_1 | \theta_2 = \bar{\theta}] q_2^N(\bar{\theta}, \theta_1) + \sum_{\theta_1} \Pr[\theta_1 | \theta_2 = \underline{\theta}] q_2^N(\underline{\theta}, \theta_1) \right] q_E^N(\underline{\theta}) = \frac{2}{3}\Delta\theta q_E^N(\underline{\theta}),$$

which is strictly positive. Finally, R 's rent in the first period is

$$U_1^N(\bar{\theta}) = \Delta\theta q_1^N(\underline{\theta}) + \delta\bar{\nu} [\Delta\theta - \Delta q^N] [q_2^N(\underline{\theta}, \underline{\theta}) - q_2^N(\underline{\theta}, \bar{\theta})],$$

which is positive for δ not too large. ■

Proof of Proposition 12. M 's maximization problem is the same as in the baseline model. With information sharing, M_E 's (relaxed) maximization problem is

$$\begin{aligned} \max_{q_E(\cdot), U_E(\cdot)} \sum_{\theta_2 \in \Theta} \Pr[\theta_2 | \theta_1] & P(\theta_2, q_E(\theta_2, \theta_1) + q_2^S(\theta_2, \theta_1)) q_E(\theta_2, \theta_1) + \\ & - \Pr[\theta_2 = \bar{\theta} | \theta_1] [\Delta\theta - (q_2^S(\bar{\theta}, \theta_1) - q_2^S(\underline{\theta}, \theta_1))] q_E(\underline{\theta}, \theta_1). \end{aligned}$$

Maximizing with respect to $q_E(\bar{\theta}, \bar{\theta})$ and $q_E(\underline{\theta}, \bar{\theta})$ yields

$$\begin{aligned} \bar{\theta} - 2q_E(\bar{\theta}, \bar{\theta}) - q_2^S(\bar{\theta}, \bar{\theta}) &= 0, \\ (1 - \bar{\nu})(\underline{\theta} - 2q_E(\underline{\theta}, \bar{\theta}) - q_2^S(\underline{\theta}, \bar{\theta})) - \bar{\nu}(\Delta\theta - \Delta q_2^S(\bar{\theta})) &= 0. \end{aligned}$$

Using (A4) and (A5), it follows that $q_E^S(\bar{\theta}, \bar{\theta}) = q_2^S(\bar{\theta}, \bar{\theta}) = q^C(\bar{\theta})$,

$$\begin{aligned} q_E^S(\underline{\theta}, \bar{\theta}) &= q^C(\underline{\theta}) - \frac{4\bar{\nu}}{3(3 - 4\bar{\nu})} \Delta\theta, \\ q_2^S(\underline{\theta}, \bar{\theta}) &= q^C(\underline{\theta}) + \frac{2\bar{\nu}}{3(3 - 4\bar{\nu})} \Delta\theta. \end{aligned}$$

Maximizing with respect to $q_E(\bar{\theta}, \underline{\theta})$ and $q_E(\underline{\theta}, \underline{\theta})$ yields

$$\begin{aligned} \bar{\theta} - 2q_E(\bar{\theta}, \underline{\theta}) - q_2^S(\bar{\theta}, \underline{\theta}) &= 0, \\ \underline{\nu}(\underline{\theta} - 2q_E(\underline{\theta}, \underline{\theta}) - q_2^S(\underline{\theta}, \underline{\theta})) - (1 - \underline{\nu})(\Delta\theta - \Delta q_2^S(\underline{\theta})) &= 0. \end{aligned}$$

Using (A4) and (A6), it follows that $q_E^S(\bar{\theta}, \underline{\theta}) = q_2^S(\bar{\theta}, \underline{\theta}) = q^C(\bar{\theta})$,

$$\begin{aligned} q_E^S(\underline{\theta}, \underline{\theta}) &= q^C(\underline{\theta}) + \frac{2(2\underline{\nu}^2 - 3\underline{\nu} + \bar{\nu} + 1)}{3(4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta, \\ q_2^S(\underline{\theta}, \underline{\theta}) &= q^C(\underline{\theta}) - \frac{2(3\underline{\nu} - 2\underline{\nu}^2 - \bar{\nu} - 1 + 3\bar{\nu}\underline{\nu})}{3(4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta. \end{aligned}$$

Direct comparison of these outputs together with Assumption 3 yields

$$\begin{aligned} q_2^S(\underline{\theta}, \underline{\theta}) - q_E^S(\underline{\theta}, \underline{\theta}) &= -\frac{2\bar{\nu}\underline{\nu}}{4\underline{\nu}^2 - \bar{\nu} - 1} \Delta\theta < 0, \\ q_2^S(\underline{\theta}, \bar{\theta}) - q_E^S(\underline{\theta}, \bar{\theta}) &= \frac{2\bar{\nu}}{3 - 4\bar{\nu}} \Delta\theta > 0, \\ q^C(\underline{\theta}) - q_2^S(\underline{\theta}, \bar{\theta}) &= -\frac{2\bar{\nu}}{3(3 - 4\bar{\nu})} \Delta\theta < 0, \end{aligned}$$

and

$$q^C(\underline{\theta}) - q_2^S(\underline{\theta}, \underline{\theta}) = \frac{2(3\underline{\nu} - 2\underline{\nu}^2 - \bar{\nu} - 1 + 3\bar{\nu}\underline{\nu})}{3(4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta > 0 \quad \Leftrightarrow \quad \bar{\nu} \geq \frac{(1 - \underline{\nu})(1 - 2\underline{\nu})}{3\underline{\nu} - 1}.$$

In order to show that retailers obtain strictly positive rents in the second period, notice that

$$\begin{aligned} U_2^S(\bar{\theta}, \bar{\theta}) &\triangleq [\Delta\theta - \Delta q_E^S(\bar{\theta})] q_2^S(\underline{\theta}, \bar{\theta}) = \frac{2 - 4\bar{\nu}}{3 - 4\bar{\nu}} q_2^S(\underline{\theta}, \bar{\theta}) \Delta\theta, \\ U_2^S(\bar{\theta}, \underline{\theta}) &\triangleq [\Delta\theta - \Delta q_E^S(\underline{\theta})] q_2^S(\underline{\theta}, \underline{\theta}) = \frac{2\underline{\nu}(2\underline{\nu} - 1)}{4\underline{\nu}^2 - \bar{\nu} - 1} q_2^S(\underline{\theta}, \underline{\theta}) \Delta\theta, \\ U_E^S(\bar{\theta}, \bar{\theta}) &\triangleq [\Delta\theta - \Delta q_2^S(\bar{\theta})] q_E^S(\underline{\theta}, \bar{\theta}) = \frac{2(1 - \bar{\nu})}{3 - 4\bar{\nu}} q_E^S(\underline{\theta}, \bar{\theta}) \Delta\theta, \\ U_E^S(\bar{\theta}, \underline{\theta}) &\triangleq [\Delta\theta - \Delta q_2^S(\underline{\theta})] q_E^S(\underline{\theta}, \underline{\theta}) = \frac{2\underline{\nu}(2\underline{\nu} - \bar{\nu} - 1)}{4\underline{\nu}^2 - \bar{\nu} - 1} q_E^S(\underline{\theta}, \underline{\theta}) \Delta\theta, \end{aligned}$$

which are all strictly positive under Assumption 3.

Finally, R 's rent in the first period is

$$U_1^S(\bar{\theta}) = \Delta\theta q_1^S(\underline{\theta}) + \delta\bar{\nu} [(\Delta\theta - \Delta q^S(\underline{\theta})) q_2^S(\underline{\theta}, \underline{\theta}) - (\Delta\theta - \Delta q^S(\bar{\theta})) q_2^S(\underline{\theta}, \bar{\theta})],$$

which is positive for δ not too large. ■

Proof of Proposition 13. When demand is low only in the second period, under Assumption 3,

$$\begin{aligned} q_E^N(\underline{\theta}) - q_E^S(\underline{\theta}, \bar{\theta}) &= -\frac{2 - 4\bar{\nu}}{3 - 4\bar{\nu}} \Delta\theta < 0 \\ q_E^N(\underline{\theta}) - q_E^S(\underline{\theta}, \underline{\theta}) &= -\frac{2\underline{\nu}(2\underline{\nu} - 1)}{4\underline{\nu}^2 - \bar{\nu} - 1} \Delta\theta < 0. \end{aligned}$$

■

Proof of Proposition 14. Using (A11) and the equilibrium quantities in Propositions 11 and 12,

$$\Pi^N - \Pi^S \approx q^C(\bar{\theta}) \delta \left[\frac{-2\bar{\nu}^3 + \bar{\nu}^2(4\underline{\nu} + 1) + \bar{\nu}(-14\underline{\nu}^2 + \underline{\nu} + 2) + 10\underline{\nu}^2 - 3\underline{\nu} - 1}{(3 - 4\bar{\nu})(4\underline{\nu}^2 - \bar{\nu} - 1)} \right] \Delta\theta. \quad (\text{A13})$$

Since the denominator is positive by Assumption 3, the sign of (A13) depends on the sign of

$$\xi(\bar{\nu}, \underline{\nu}) \triangleq -2\bar{\nu}^3 + \bar{\nu}^2(4\underline{\nu} + 1) + \bar{\nu}(-14\underline{\nu}^2 + \underline{\nu} + 2) + 10\underline{\nu}^2 - 3\underline{\nu} - 1,$$

with

$$\frac{\partial \xi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} = -6\bar{\nu}^2 + \bar{\nu}(8\underline{\nu} + 2) - 14\underline{\nu}^2 + \underline{\nu} + 2.$$

It can be shown that $\frac{\partial \xi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} < 0$ in our relevant region of parameters. Since $\xi(4\underline{\nu}^2 - 1, \underline{\nu}) =$

$4\underline{\nu}^3 (2\underline{\nu} - 1) (7 - 16\underline{\nu}^2) > 0$ and $\xi(0.5, \underline{\nu}) = \frac{3}{2}\underline{\nu} (2\underline{\nu} - 1) > 0$, (A13) is positive and, hence, M does not want to share information.

Using (A9) and the equilibrium quantities in Propositions 11 and 12,

$$\mathcal{V}^N - \mathcal{V}^S \approx -\frac{q^C(\bar{\theta}) \delta \underline{\nu} (2\underline{\nu} - 1) (1 + \Delta \underline{\nu})}{4\underline{\nu}^2 - \bar{\nu} - 1} \Delta \theta, \quad (\text{A14})$$

which is negative under Assumption 3. Hence, R 's ex ante rent is higher with information sharing.

We now compare M_E 's expected profit with and without information sharing. Notice that

$$\lim_{\Delta \theta \rightarrow 0} \Pi_E^N = \lim_{\Delta \theta \rightarrow 0} \Pi_E^S = q^C(\bar{\theta})^2,$$

and

$$\lim_{\Delta \theta \rightarrow 0} \frac{\partial \Pi_E^N}{\partial \Delta \theta} = -q^C(\bar{\theta}) [\Pr[\theta_1 = \bar{\theta} | \theta_2 = \underline{\theta}] (1 + \dot{q}_2^N(\underline{\theta}, \bar{\theta})) + \Pr[\theta_1 = \underline{\theta} | \theta_2 = \underline{\theta}] (1 + \dot{q}_2^N(\underline{\theta}, \underline{\theta}))],$$

$$\lim_{\Delta \theta \rightarrow 0} \frac{\partial \Pi_E^S}{\partial \Delta \theta} = -q^C(\bar{\theta}) \left[1 + \frac{1}{2} (\dot{q}_2^S(\underline{\theta}, \bar{\theta}) + \dot{q}_2^S(\underline{\theta}, \underline{\theta})) \right].$$

Hence, using a Taylor approximation around $\Delta \theta = 0$ and the equilibrium quantities from Proposition 11 and 12,

$$\begin{aligned} \Pi_E^S - \Pi_E^N &\approx q^C(\bar{\theta}) \left[\frac{1 - \bar{\nu}}{1 - \Delta \underline{\nu}} \dot{q}_2^N(\underline{\theta}, \bar{\theta}) + \frac{\underline{\nu}}{1 - \Delta \underline{\nu}} \dot{q}_2^N(\underline{\theta}, \underline{\theta}) - \frac{1}{2} (\dot{q}_2^S(\underline{\theta}, \bar{\theta}) + \dot{q}_2^S(\underline{\theta}, \underline{\theta})) \right] \Delta \theta \\ &= q^C(\bar{\theta}) \frac{\bar{\nu}^2 (3 - 4\underline{\nu}) - \bar{\nu} (4\underline{\nu}^2 + \underline{\nu} - 1) + 2\underline{\nu}^2 + 3\underline{\nu} - 2}{(3 - 4\bar{\nu}) (4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta \theta. \end{aligned} \quad (\text{A15})$$

Since the denominator is positive by Assumption 3, the sign of (A15) depends on the sign of

$$\mu(\bar{\nu}, \underline{\nu}) \triangleq \bar{\nu}^2 (3 - 4\underline{\nu}) - \bar{\nu} (4\underline{\nu}^2 + \underline{\nu} - 1) + 2\underline{\nu}^2 + 3\underline{\nu} - 2,$$

where it can be shown that, in the relevant region of parameters,

$$\frac{\partial \mu(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} = \bar{\nu} (6 - 8\underline{\nu}) - 4\underline{\nu}^2 - \underline{\nu} + 1 < 0.$$

Hence, since

$$\mu(4\underline{\nu}^2 - 1, \underline{\nu}) = 2\underline{\nu}^2 (2\underline{\nu} - 1) (7 - 16\underline{\nu}^2) > 0$$

and

$$\mu(0.5, \underline{\nu}) = \frac{3}{4} (2\underline{\nu} - 1) > 0,$$

M_E 's expected profit is higher with information sharing. ■

Proof of Proposition 15. Using (A12) and the equilibrium quantities in Propositions 11

and 12,

$$Q^N - Q^S \approx \frac{2\bar{\nu}^3 - \bar{\nu}^2 (16\underline{\nu}^2 - 4\underline{\nu} + 1) + \bar{\nu} (16\underline{\nu}^3 + 2\underline{\nu}^2 + \underline{\nu} - 2) - 12\underline{\nu}^3 + 8\underline{\nu}^2 - 3\underline{\nu} + 1}{2(3 - 4\bar{\nu})(4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta. \quad (\text{A16})$$

The sign of (A16) depends on the sign of the numerator

$$\chi(\bar{\nu}, \underline{\nu}) \triangleq 2\bar{\nu}^3 - \bar{\nu}^2 (16\underline{\nu}^2 - 4\underline{\nu} + 1) + \bar{\nu} (16\underline{\nu}^3 + 2\underline{\nu}^2 + \underline{\nu} - 2) - 12\underline{\nu}^3 + 8\underline{\nu}^2 - 3\underline{\nu} + 1.$$

Following the proof of Proposition 12, first let $\underline{\nu} \leq 0.6$ so that $\bar{\nu}^* = 4\underline{\nu}^2 - 1$. In this case,

$$\begin{aligned} \chi(0, \underline{\nu}) &= -(2\underline{\nu} - 1)(6\underline{\nu}^2 - \underline{\nu} + 1) < 0, \\ \chi(4\underline{\nu}^2 - 1, \underline{\nu}) &= 2\underline{\nu}^2 (7 - 16\underline{\nu}^2) (2\underline{\nu} - 1)^2 > 0. \end{aligned}$$

Moreover,

$$\frac{\partial \chi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} = 6\bar{\nu}^2 - \bar{\nu} (32\underline{\nu}^2 - 8\underline{\nu} + 2) + 16\underline{\nu}^3 + 2\underline{\nu}^2 + \underline{\nu} - 2.$$

Setting this equation equal to 0 and solving for $\bar{\nu}$ yields the critical points

$$\begin{aligned} \bar{\nu}_{\min} &\triangleq \frac{8}{3}\underline{\nu}^2 + \frac{1}{6}\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} + \frac{1}{6} > 0, \\ \bar{\nu}_{\max} &\triangleq \frac{8}{3}\underline{\nu}^2 - \frac{1}{6}\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} + \frac{1}{6} > 0. \end{aligned}$$

Since

$$\begin{aligned} \lim_{\bar{\nu} \rightarrow \bar{\nu}_{\min}} \frac{\partial^2 \chi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}^2} &= 2\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} > 0, \\ \lim_{\bar{\nu} \rightarrow \bar{\nu}_{\max}} \frac{\partial^2 \chi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}^2} &= -2\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} < 0, \end{aligned}$$

$\chi(\bar{\nu}, \underline{\nu})$ has a relative minimum at $\bar{\nu} = \bar{\nu}_{\min}$ and relative maximum at $\bar{\nu} = \bar{\nu}_{\max}$. Finally, for $\underline{\nu} \leq 0.6$ the critical points are outside the interval of interest — i.e.,

$$\begin{aligned} \bar{\nu}_{\min} - (4\underline{\nu}^2 - 1) &= \frac{1}{6}\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} - \frac{4}{3}\underline{\nu}^2 + \frac{7}{6} > 0, \\ \bar{\nu}_{\max} - (4\underline{\nu}^2 - 1) &= -\frac{1}{6}\sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} - \frac{4}{3}\underline{\nu}^2 + \frac{7}{6} > 0. \end{aligned}$$

Hence, by the mean-value theorem there exists a unique $\bar{\nu}_0$ such that $\chi(\bar{\nu}, \underline{\nu}) < 0$ (so that aggregate production is higher with information sharing) if and only if $\bar{\nu} \leq \bar{\nu}_0(\underline{\nu})$.

Second, consider the case where $\underline{\nu} > 0.6$ so that $\bar{\nu} \leq \frac{1}{2}$. Notice that

$$\chi(0.5, \underline{\nu}) = \frac{1}{2}\underline{\nu} (2\underline{\nu} - 1) (3 - 4\underline{\nu}) < 0 \quad \Leftrightarrow \quad \underline{\nu} > \underline{\nu}_0 \triangleq 0.75.$$

Let $\underline{\nu} \geq \underline{\nu}_0$. The function $\chi(\bar{\nu}, \underline{\nu})$ has two critical points $\bar{\nu}_{\min}$ and $\bar{\nu}_{\max}$ and is always negative

because

$$\begin{aligned}\bar{\nu}_{\min} - \frac{1}{2} &= \frac{1}{6} \sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} + \frac{8}{3}\underline{\nu}^2 - \frac{1}{3} > 0, \\ \bar{\nu}_{\max} - \frac{1}{2} &= -\frac{1}{6} \sqrt{256\underline{\nu}^4 - 224\underline{\nu}^3 + 36\underline{\nu}^2 - 14\underline{\nu} + 13} - \frac{2}{3}\underline{\nu} - \frac{4}{3}\underline{\nu}^2 + \frac{2}{3} > 0.\end{aligned}$$

Hence, in this region of parameters, consumers are better off with information sharing. Next, let $\underline{\nu} < \underline{\nu}_0$. Since $\chi(0, \underline{\nu}) < 0$ and $\chi(0.5, \underline{\nu}) > 0$, when $\bar{\nu} < \frac{1}{2}$ the function $\chi(\bar{\nu}, \underline{\nu})$ crosses the $\bar{\nu}$ -axis at least once. This point is unique because the relative maximum and minimum are outside the interval of interest — i.e., $\bar{\nu}_{\min} > \frac{1}{2}$ and $\bar{\nu}_{\max} > \frac{1}{2}$. Hence, if $\underline{\nu} < \underline{\nu}_0$ there exist a unique threshold $\bar{\nu}_0$ such that aggregate production is higher with information sharing if $\bar{\nu} \leq \bar{\nu}_0(\underline{\nu})$. Using numerical approximations, Figure 1 illustrates the region of parameters where consumers benefit from information sharing.

Finally, using Taylor approximations (see the proof of Proposition 10), there are linear relationships between total welfare and aggregate production,

$$\mathcal{TW}^N - \mathcal{TW}^S = q^C(\bar{\theta}) [\mathcal{Q}^N - \mathcal{Q}^S],$$

and between consumer surplus and aggregate production,

$$\mathcal{CS}^N - \mathcal{CS}^S = 2q^C(\bar{\theta}) [\mathcal{Q}^N - \mathcal{Q}^S].$$

Hence, whenever information sharing increases aggregate production, it also increases consumer surplus and total welfare. ■

Proof of Proposition 16. For any $d \in \{S, N\}$, the ex ante joint profit of M and R is $\mathcal{J}^d = \diamond^d + \mathcal{V}^d$. For $\Delta\theta$ small, using Taylor approximations and the results of Proposition 14, we have

$$[\diamond^N + \mathcal{V}^N] - [\diamond^S + \mathcal{V}^S] \approx \frac{\delta q^C(\bar{\theta}) (-2\bar{\nu}^3 + \bar{\nu}^2(8\underline{\nu}^2 + 1) - \bar{\nu}(8\underline{\nu}^3 + 8\underline{\nu}^2 - 2) + 6\underline{\nu}^3 + \underline{\nu}^2 - 1)}{(3 - 4\bar{\nu})(4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta.$$

The sign of this expression depends on the numerator

$$\Psi(\bar{\nu}, \underline{\nu}) \triangleq -2\bar{\nu}^3 + \bar{\nu}^2(8\underline{\nu}^2 + 1) - \bar{\nu}(8\underline{\nu}^3 + 8\underline{\nu}^2 - 2) + 6\underline{\nu}^3 + \underline{\nu}^2 - 1,$$

with

$$\frac{\partial \Psi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} = -6\bar{\nu}^2 + \bar{\nu}(16\underline{\nu}^2 + 2) - 8\underline{\nu}^3 - 8\underline{\nu}^2 + 2.$$

It can be shown that $\frac{\partial \Psi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} < 0$ in the relevant region of parameters. Hence, since

$$\Psi(4\underline{\nu}^2 - 1, \underline{\nu}) = \underline{\nu}^2(2\underline{\nu} - 1)(7 - 16\underline{\nu}^2) > 0$$

and

$$\Psi(0.5, \underline{\nu}) = \underline{\nu}^2(2\underline{\nu} - 1) > 0,$$

the ex ante joint profit of M and R is higher without information sharing. ■

Proof of Proposition 17. Let ρ be the price for information about θ_1 , that M_E pays to

M . M_E is willing to buy information if and only if

$$\rho \leq \Pi_E^S - \Pi_E^N.$$

M is willing to sell information if and only if

$$\rho \geq \Pi_2^N - \Pi_2^S.$$

Hence, M and M_E are willing to trade at price $\rho > 0$ if and only if

$$\Pi_2^N - \Pi_2^S \leq \Pi_E^S - \Pi_E^N.$$

Under Assumption 3, this inequality simplifies to

$$\frac{q^C(\bar{\theta}) (2\bar{\nu}^3 - \bar{\nu}^2 (8\underline{\nu} - 2) - \bar{\nu} (-10\underline{\nu}^2 + 2\underline{\nu} + 1) - 8\underline{\nu}^2 + 6\underline{\nu} - 1)}{(3 - 4\bar{\nu}) (4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta \geq 0. \quad (\text{A17})$$

The sign of this expression depends on the numerator

$$\kappa(\bar{\nu}, \underline{\nu}) \triangleq 2\bar{\nu}^3 - \bar{\nu}^2 (8\underline{\nu} - 2) - \bar{\nu} (-10\underline{\nu}^2 + 2\underline{\nu} + 1) - 8\underline{\nu}^2 + 6\underline{\nu} - 1,$$

with

$$\frac{\partial \kappa(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} = 6\bar{\nu}^2 - \bar{\nu} (16\underline{\nu} - 4) + 10\underline{\nu}^2 - 2\underline{\nu} - 1.$$

It can be shown that in the relevant region of parameters $\frac{\partial \kappa(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} > 0$. Hence, since

$$\kappa(4\underline{\nu}^2 - 1, \underline{\nu}) = -2\underline{\nu}^2 (7 - 16\underline{\nu}^2) (2\underline{\nu} - 1)^2 < 0$$

and

$$\kappa(0.5, \underline{\nu}) = -\frac{3}{4} (2\underline{\nu} - 1)^2 < 0,$$

inequality (A17) is never satisfied. ■

Proof of Proposition 18. M_E , M and R have a joint incentive to trade information if and only if

$$\mathcal{J}_2^N - \mathcal{J}_2^S \leq \Pi_E^S - \Pi_E^N.$$

Under Assumption 3, using the results of Proposition 14, this inequality simplifies to

$$\frac{q^C(\bar{\theta}) [2\bar{\nu}^3 - \bar{\nu}^2 (8\underline{\nu}^2 + 4\underline{\nu} - 2) - \bar{\nu} (-8\underline{\nu}^3 - 4\underline{\nu}^2 + \underline{\nu} + 1) - 6\underline{\nu}^3 + \underline{\nu}^2 + 3\underline{\nu} - 1]}{(3 - 4\bar{\nu}) (4\underline{\nu}^2 - \bar{\nu} - 1)} \Delta\theta \geq 0. \quad (\text{A18})$$

The sign of this expression depends on the sign of the numerator

$$\varpi(\bar{\nu}, \underline{\nu}) \triangleq 2\bar{\nu}^3 - \bar{\nu}^2 (8\underline{\nu}^2 + 4\underline{\nu} - 2) - \bar{\nu} (-8\underline{\nu}^3 - 4\underline{\nu}^2 + \underline{\nu} + 1) - 6\underline{\nu}^3 + \underline{\nu}^2 + 3\underline{\nu} - 1.$$

First, let $\underline{\nu} \leq 0.6$, so that $\bar{\nu}^* = 4\underline{\nu}^2 - 1$. In this case,

$$\begin{aligned}\varpi(0, \underline{\nu}) &= -(2\underline{\nu} - 1)(\underline{\nu} + 3\underline{\nu}^2 - 1) < 0, \\ \varpi(4\underline{\nu}^2 - 1, \underline{\nu}) &= \underline{\nu}^2(2\underline{\nu} - 1)(7 - 16\underline{\nu}^2) > 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \varpi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}} &= 6\bar{\nu}^2 - \bar{\nu}(16\underline{\nu}^2 + 8\underline{\nu} - 4) + 8\underline{\nu}^3 + 4\underline{\nu}^2 - \underline{\nu} - 1 = 0 \\ \Leftrightarrow \begin{cases} \bar{\nu}_{\min} \triangleq \frac{2}{3}\underline{\nu} + \frac{1}{6}\sqrt{2}\sqrt{32\underline{\nu}^4 + 8\underline{\nu}^3 - 20\underline{\nu}^2 - 5\underline{\nu} + 5} + \frac{4}{3}\underline{\nu}^2 - \frac{1}{3} > 0, \\ \bar{\nu}_{\max} \triangleq \frac{2}{3}\underline{\nu} - \frac{1}{6}\sqrt{2}\sqrt{32\underline{\nu}^4 + 8\underline{\nu}^3 - 20\underline{\nu}^2 - 5\underline{\nu} + 5} + \frac{4}{3}\underline{\nu}^2 - \frac{1}{3} > 0. \end{cases}\end{aligned}$$

Since

$$\begin{aligned}\lim_{\bar{\nu} \rightarrow \bar{\nu}_{\min}} \frac{\partial^2 \varpi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}^2} &= 2\sqrt{2}\sqrt{-5\underline{\nu} - 20\underline{\nu}^2 + 8\underline{\nu}^3 + 32\underline{\nu}^4 + 5} > 0, \\ \lim_{\bar{\nu} \rightarrow \bar{\nu}_{\max}} \frac{\partial^2 \varpi(\bar{\nu}, \underline{\nu})}{\partial \bar{\nu}^2} &= -2\sqrt{2}\sqrt{-5\underline{\nu} - 20\underline{\nu}^2 + 8\underline{\nu}^3 + 32\underline{\nu}^4 + 5} < 0,\end{aligned}$$

$\varpi(\bar{\nu}, \underline{\nu})$ has a relative minimum at $\bar{\nu} = \bar{\nu}_{\min}$ and relative maximum at $\bar{\nu} = \bar{\nu}_{\max}$. The relative minimum is outside the interval of interest — i.e.,

$$\bar{\nu}_{\min} - (4\underline{\nu}^2 - 1) = -\frac{2}{3} \left(-\underline{\nu} - \frac{1}{4}\sqrt{2}\sqrt{-5\underline{\nu} - 20\underline{\nu}^2 + 8\underline{\nu}^3 + 32\underline{\nu}^4 + 5} + 4\underline{\nu}^2 - 1 \right) > 0.$$

Hence, by the mean-value theorem, there exists a unique $\bar{\nu}_1$ such that $\varpi(\bar{\nu}, \underline{\nu}) > 0$ if and only if $\bar{\nu}_1 \leq \bar{\nu}$.

Second, let $\underline{\nu} > 0.6$, so that $\bar{\nu} \leq \frac{1}{2}$. Notice that

$$\varpi(0.5, \underline{\nu}) = \frac{1}{4}(2\underline{\nu} - 1)(3 - 4\underline{\nu}^2) < 0 \quad \Leftrightarrow \quad \underline{\nu} > \underline{\nu}_1 \triangleq 0.87.$$

When $\underline{\nu} > \underline{\nu}_1$, $\varpi(\bar{\nu}, \underline{\nu})$ has two critical points $\bar{\nu}_{\min}$ and $\bar{\nu}_{\max}$ and is always negative because

$$\begin{aligned}\bar{\nu}_{\min} - \frac{1}{2} &= \frac{2}{3} \left(\underline{\nu} + \frac{1}{4}\sqrt{2}\sqrt{-5\underline{\nu} - 20\underline{\nu}^2 + 8\underline{\nu}^3 + 32\underline{\nu}^4 + 5} + 2\underline{\nu}^2 - \frac{5}{4} \right) > 0, \\ \bar{\nu}_{\max} - \frac{1}{2} &= \frac{2}{3} \left(\underline{\nu} - \frac{1}{4}\sqrt{2}\sqrt{-5\underline{\nu} - 20\underline{\nu}^2 + 8\underline{\nu}^3 + 32\underline{\nu}^4 + 5} + 2\underline{\nu}^2 - \frac{5}{4} \right) > 0.\end{aligned}$$

Therefore, condition (A18) is not satisfied and players do not have incentive to sell information.

When $\underline{\nu} \leq \underline{\nu}_1$, $\varpi(0, \underline{\nu}) < 0$ and $\varpi(0.5, \underline{\nu}) > 0$. Hence, the function $\varpi(\bar{\nu}, \underline{\nu})$ crosses the $\bar{\nu}$ axis at least once. This point is unique because the relative minimum is outside the interval of interest — i.e., $\bar{\nu}_{\min} > \frac{1}{2}$.

Summing up, there exist a unique $\bar{\nu}_1(\underline{\nu})$ such that M_E , M and R have a joint incentive to trade information if $\underline{\nu} \leq \underline{\nu}_1$ and $\bar{\nu} \geq \bar{\nu}_1(\underline{\nu})$. The region of parameters where M and R sell information to M_E is illustrated in Figure 2 by numerical approximations. ■

Proof of Proposition 19. Using numerical approximations of the implicit functions defined by $\varpi(\bar{\nu}, \underline{\nu}) = 0$ and $\chi(\bar{\nu}, \underline{\nu}) = 0$, Figure 3 shows that $\bar{\nu}_1(\underline{\nu}) \leq \bar{\nu}_0(\underline{\nu})$ for $\underline{\nu} \leq \underline{\nu}_1$, which proves

the result. (The coding for the numerical approximations is available upon request.) ■

Proof of Proposition 20. Differentiating M 's objective function, it is easy to show that $q_2(\bar{\theta}, \theta_1, \sigma) = q_E(\bar{\theta}, \sigma) = q^C(\bar{\theta})$ for every θ_1 and σ . While, for every $\sigma \in \{\bar{\sigma}, \underline{\sigma}\}$ we have

$$P'(\underline{\theta}, Q_2^S(\underline{\theta}, \bar{\theta}, \sigma)) q_2(\underline{\theta}, \bar{\theta}, \sigma) + P(\underline{\theta}, Q_2^S(\underline{\theta}, \bar{\theta}, \sigma)) = 0, \quad (\text{A19})$$

and

$$P'(\underline{\theta}, Q_2^S(\underline{\theta}, \underline{\theta}, \sigma)) q_2(\underline{\theta}, \underline{\theta}, \sigma) + P(\underline{\theta}, Q_2^S(\underline{\theta}, \underline{\theta}, \sigma)) = \frac{\Delta\theta - \Delta q^S(\underline{\theta}, \sigma)}{\nu}. \quad (\text{A20})$$

Solving (A19) and (A20) together with E 's first-order conditions,

$$\begin{aligned} q_2(\underline{\theta}, \bar{\theta}, \bar{\sigma}) &= q^C(\underline{\theta}) + \frac{1 - \beta}{3(1 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta \\ q_2(\underline{\theta}, \bar{\theta}, \underline{\sigma}) &= q^C(\underline{\theta}) - \frac{\beta}{3(3 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta \\ q_2(\underline{\theta}, \underline{\theta}, \underline{\sigma}) &= q^C(\underline{\theta}) - \frac{3(1 - \nu)(1 - \alpha) + 4\beta\nu}{3\nu(3 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta \\ q(\underline{\theta}, \underline{\theta}, \bar{\sigma}) &= q^C(\underline{\theta}) + \frac{3\alpha(1 - \nu) + 4\nu(1 - \beta)}{3\nu(1 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta \end{aligned}$$

and

$$\begin{aligned} q_E(\underline{\theta}, \underline{\sigma}) &= q^C(\underline{\theta}) + \frac{2\beta}{3(3 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta \\ q(\underline{\theta}, \bar{\sigma}) &= q^C(\underline{\theta}) - \frac{2(1 - \beta)}{3(1 - \beta - 3(\alpha(1 - \nu) + \nu(1 - \beta)))} \Delta\theta. \end{aligned}$$

Substituting these quantities into M 's expected profit, maximizing with respect to α and β , respectively, and assuming that $\Delta\theta \rightarrow 0$, in an interior solution we have

$$(\alpha + \beta - 1)(3(1 - \nu)(2\beta - 1)\alpha + (1 - \beta)(3(1 - \nu) - 2\beta(1 - 3\nu))) = 0, \quad (\text{A21})$$

and

$$(\alpha + \beta - 1)((6\beta\nu - 9\nu - 2\beta + 7)\alpha - (1 - 3\nu)(1 - \beta) - 6(1 - \nu)\alpha^2) = 0. \quad (\text{A22})$$

Solving with respect to α and β , the system of equations (A21)-(A22) features two critical points $(\alpha = 0, \beta = 1)$ and $(\alpha = 1, \beta = 0)$. Let M 's expected profit in the second period be

$$\begin{aligned} \Pi(\alpha, \beta) &\triangleq \frac{1 - \nu}{2} \left[\alpha q_2(\underline{\theta}, \bar{\theta}, \bar{\sigma})^2 + (1 - \alpha) q_2(\underline{\theta}, \bar{\theta}, \underline{\sigma})^2 \right] + \\ &\quad + \frac{\nu}{2} \left[\beta q(\underline{\theta}, \underline{\theta}, \underline{\sigma})^2 + (1 - \beta) q(\underline{\theta}, \underline{\theta}, \bar{\sigma})^2 \right] \end{aligned}$$

Notice that these two solutions are payoff-equivalent since they both imply an uninformative experiment — i.e.,

$$\Pi(\alpha = 0, \beta = 1) = \Pi(\alpha = 1, \beta = 0).$$

Moreover, it can be shown that

$$\Pi(\alpha = 1, \beta = 1) - \Pi(\alpha = 1, \beta = 0) \approx -\frac{2(1-\nu)}{9(3\nu-1)}\bar{\theta}\Delta\theta < 0.$$

Hence, M prefers not to disclose information since $\nu > \frac{1}{3}$ by Assumption 1.

Consider now R 's incentive to share information. R 's second-period expected rent is

$$\begin{aligned} \mathcal{U}(\alpha, \beta) \triangleq & \frac{1-\beta}{2} [\Delta\theta - (q^C(\underline{\theta}) - q(\underline{\theta}, \bar{\sigma}))] q(\underline{\theta}, \underline{\theta}, \bar{\sigma}) + \\ & + \frac{\beta}{2} [\Delta\theta - (q^C(\underline{\theta}) - q(\underline{\theta}, \underline{\sigma}))] q_2(\underline{\theta}, \underline{\theta}, \underline{\sigma}). \end{aligned}$$

Maximizing with respect to α and β , respectively, and assuming $\Delta\theta \rightarrow 0$, in an interior solution, we have

$$(\alpha + \beta - 1)(3(1-\nu)(1-2\beta) - (1-\beta)(3(1-\nu) - 2\beta(1-3\nu))) = 0, \quad (\text{A23})$$

and

$$(\alpha + \beta - 1)((3\nu - 1)(1 - \beta) - 6(1 - \nu)\alpha^2 + (7 - 9\nu - 2\beta(1 - 3\nu))\alpha) = 0. \quad (\text{A24})$$

The system of equations (A23)-(A24) features two payoff-equivalent solutions ($\alpha = 0, \beta = 1$) and ($\alpha = 1, \beta = 0$). Notice, however, that

$$\mathcal{U}(\alpha = 1, \beta = 1) - \mathcal{U}(\alpha = 0, \beta = 1) = \frac{1-\nu}{3(3\nu-1)}\bar{\theta}\Delta\theta > 0.$$

Hence, $\mathcal{U}(\alpha, \beta)$ has a global maximum at $\alpha = \beta = 1$, so that R would like to share information perfectly.

Finally, consider the effect of information sharing in consumer surplus. As before, it can be easily shown that for $\Delta\theta$ small, the effect on consumer and total welfare is equivalent to the effect on aggregate quantity — i.e.,

$$\begin{aligned} \mathcal{Q}(\alpha, \beta) \triangleq & \frac{1-\nu}{2} (\alpha (q_2(\underline{\theta}, \bar{\theta}, \bar{\sigma}) + q(\underline{\theta}, \bar{\sigma})) + (1-\alpha) (q_2(\underline{\theta}, \bar{\theta}, \underline{\sigma}) + q(\underline{\theta}, \underline{\sigma}))) + \\ & + \frac{\nu}{2} (\beta (q_2(\underline{\theta}, \underline{\theta}, \underline{\sigma}) + q(\underline{\theta}, \underline{\sigma})) + (1-\beta) (q_2(\underline{\theta}, \underline{\theta}, \bar{\sigma}) + q(\underline{\theta}, \bar{\sigma}))). \end{aligned}$$

Maximizing with respect to α and β , respectively, in an interior solution for $\Delta\theta \rightarrow 0$ we have

$$(\alpha + \beta - 1)(3(1-\nu)(1-2\beta)\alpha - (1-\beta)(3(1-\nu) - 2\beta(1-3\nu))) = 0, \quad (\text{A25})$$

and

$$(\alpha + \beta - 1)((3\nu - 1)(1 - \beta) + (6\beta\nu - 9\nu - 2\beta + 7)\alpha - 6(1 - \nu)\alpha^2) = 0. \quad (\text{A26})$$

The system of equations (A25)-(A26) features two payoff-equivalent solutions ($\alpha = 0, \beta = 1$)

and $(\alpha = 1, \beta = 0)$. Moreover,

$$\mathcal{Q}(\alpha = 1, \beta = 1) - \mathcal{Q}(\alpha = 1, \beta = 0) \approx -\frac{1}{6} \frac{1 - \nu}{3\nu - 1} \Delta\theta < 0,$$

so that consumer surplus is maximized by an uninformative experiment. ■

Proof of Proposition 21. Consider first the outcome without information sharing characterized in Section 4.1. In order to show that it is not robust to ex-post renegotiation, consider R and suppose that: (i) $\theta_1 = \bar{\theta}$, and (ii) $m_1 = \bar{\theta}$. Then, R has an incentive to disclose m_1 to E if and only if

$$\underbrace{\bar{\nu} [\Delta\theta - (q^C(\bar{\theta}) - q_E^N(\underline{\theta}))] q_2^N(\underline{\theta}, \bar{\theta})}_{\text{Second-period equilibrium rent}} < \underbrace{\sum_{\theta_2} \Pr[\theta_2 | \bar{\theta}] [P(\theta_2, q_2^N(\theta_2, \bar{\theta}) + q_E^R(\theta_2, \bar{\theta})) q_2^N(\theta_2, \bar{\theta}) - t_2^N(\theta_2, \bar{\theta})]}_{\text{Deviation profit}},$$

where

$$q_E^R(\theta_2, \bar{\theta}) = \arg \max_{q_E} \{P(\theta_2, q_E + q_2^N(\theta_2, \bar{\theta})) q_E\}.$$

Notice that, by definition

$$t_2^N(\bar{\theta}, \bar{\theta}) \equiv P(\bar{\theta}, q_2^N(\bar{\theta}, \bar{\theta}) + q_E^R(\bar{\theta}, \bar{\theta})) q_2^N(\bar{\theta}, \bar{\theta}) - [\Delta\theta - (q^C(\bar{\theta}) - q_E^N(\underline{\theta}))] q_2^N(\underline{\theta}, \bar{\theta}),$$

and

$$t_2^N(\underline{\theta}, \bar{\theta}) \equiv P(\underline{\theta}, q_2^N(\underline{\theta}, \bar{\theta}) + q_E^R(\underline{\theta}, \bar{\theta})) q_2^N(\underline{\theta}, \bar{\theta}).$$

R 's incentive to disclose m_1 then rewrites as

$$0 < (1 - \bar{\nu}) q_2^N(\underline{\theta}, \bar{\theta}) (q_E^N(\underline{\theta}) - q_E^R(\underline{\theta}, \bar{\theta})). \quad (\text{A27})$$

It can be shown that

$$q_E^R(\underline{\theta}, \bar{\theta}) \equiv \frac{\underline{\theta} - q_2^N(\underline{\theta}, \bar{\theta})}{2} = q^C(\underline{\theta}) + \frac{(1 + \Delta\nu)(3(1 - \Delta\nu) + \underline{\nu})}{12\underline{\nu}(1 - 2\Delta\nu)} \Delta\theta,$$

so that

$$q_E^N(\underline{\theta}) - q_E^R(\underline{\theta}, \bar{\theta}) = \frac{1 + \Delta\nu}{4(1 - 2\Delta\nu)},$$

which is positive under Assumption 1. Hence, (A27) holds.

The equilibrium with information sharing is robust to ex-post renegotiation because M cannot improve its profit from concealing information to E since, by assumption, the second period transfer cannot be renegotiated. Hence, M does not deviate. This implies that, by the intertemporal incentive compatibility constraint, R cannot deviate either. ■

Large uncertainty. We derive the functions plotted in Figures 4 and 5. Under the parametric restrictions imposed in Section 7.4, the first-order conditions without information

sharing imply

$$q_E^N(\bar{\theta}) = \frac{1}{3} + \frac{1}{3}\Delta\theta, \quad q_2^N(\bar{\theta}, \theta_1) = \frac{1}{3} + \frac{1}{3}\Delta\theta, \quad \forall \theta_1 \in \Theta,$$

$$q_E^N(\underline{\theta}) = \frac{1}{3} + \frac{1}{3}\Delta\theta, \quad q_2^N(\underline{\theta}, \bar{\theta}) = \frac{1}{3} - \frac{1}{6}\Delta\theta, \quad q_2^N(\underline{\theta}, \underline{\theta}) = \frac{1}{3} - \frac{3+\nu}{6\nu}\Delta\theta.$$

Similarly, the first-order conditions with information sharing imply

$$q_E^S(\bar{\theta}, \theta_1) = \frac{1}{3} + \frac{1}{3}\Delta\theta, \quad q_2^S(\bar{\theta}, \theta_1) = \frac{1}{3} + \frac{1}{3}\Delta\theta, \quad \forall \theta_1 \in \Theta,$$

$$q_E^S(\underline{\theta}, \bar{\theta}) = q_2^S(\underline{\theta}, \bar{\theta}) = \frac{1}{3}, \quad q_E^S(\underline{\theta}, \underline{\theta}) = \frac{1}{3} + \frac{2}{3(3\nu-1)}\Delta\theta, \quad q_2^S(\underline{\theta}, \underline{\theta}) = \frac{1}{3} - \frac{4}{3(3\nu-1)}\Delta\theta.$$

Notice that $q_2^S(\underline{\theta}, \underline{\theta}) < q_2^N(\underline{\theta}, \underline{\theta})$ for $\nu \geq \frac{1}{3}$. Hence, we need to impose $\Delta\theta \leq \Delta\theta_0(\nu) \triangleq \frac{3\nu-1}{4}$ to guarantee that the incumbent does not shut down production when demand is repeatedly low.

Consider now R 's expected rent. With no information sharing, R 's second-period rent is strictly positive since

$$U_2^N(\bar{\theta}, \theta_1) = \Delta\theta - \Delta q^N = \Delta\theta > 0.$$

R 's first-period rent with no information sharing is

$$U_1^N(\bar{\theta}) = \frac{\Delta\theta}{2} - \Delta\theta^2,$$

which is strictly positive for $\Delta\theta \leq \Delta\theta_0(\nu)$, where $\Delta\theta_0(\nu) > 0$.

With information sharing, R 's second-period rent is strictly positive since, by Assumption 4,

$$U_2^S(\bar{\theta}, \bar{\theta}) = \Delta\theta - \Delta q^S(\bar{\theta}) = \frac{2}{3}\Delta\theta > 0,$$

$$U_2^S(\bar{\theta}, \underline{\theta}) = \Delta\theta - \Delta q^S(\underline{\theta}) = \frac{2\nu}{3\nu-1}\Delta\theta > 0,$$

R 's first-period rent is

$$U_1^S(\bar{\theta}) = \frac{(93\nu^2 - 58\nu + 9)\Delta\theta - (129\nu^2 - 54\nu + 9)\Delta\theta^2}{18(3\nu-1)^2}.$$

The sign of this expression depends on the numerator

$$\sigma(\nu, \Delta\theta) \triangleq -\Delta\theta^2(129\nu^2 - 54\nu + 9) + \Delta\theta(93\nu^2 - 58\nu + 9),$$

with

$$\frac{\partial\sigma(\nu, \Delta\theta)}{\partial\Delta\theta} = -\Delta\theta(258\nu^2 - 108\nu + 18) + 93\nu^2 - 58\nu + 9 > 0$$

in the relevant region of parameters. Hence, since $\frac{\sigma(\nu,0)}{\Delta\theta} = 0$, and

$$\frac{\sigma\left(\nu, \frac{3\nu-1}{4}\right)}{\Delta\theta} = \frac{1}{16} (3\nu-1)^3 (45-43\nu) > 0,$$

R 's first-period information rent with information sharing is positive.

We now compare R 's ex ante rent with and without information sharing. Under Assumption 5, for any $d \in \{S, N\}$ R 's ex ante rent is

$$\mathcal{V}^d = \sum_{\theta_1} \Pr[\theta_1] \left[U_1^d(\theta_1) + \sum_{\theta_2} \Pr[\theta_2 = \bar{\theta}|\theta_1] U_2^d(\bar{\theta}, \theta_1) \right]. \quad (\text{A28})$$

Using the first and second period information rents just derived, it can be shown that

$$\mathcal{V}^N - \mathcal{V}^S = \frac{(1-\nu)(9\nu^2 + 14\nu - 3)\Delta\theta + (1-\nu)(2\nu - 6\nu^2)}{12\nu(3\nu-1)^2} \Delta\theta.$$

Setting the numerator equal to 0 and solving for $\Delta\theta$ yields

$$\Delta\theta_u(\nu) \triangleq \frac{2\nu(3\nu-1)}{14\nu + 9\nu^2 - 3},$$

which is positive in the relevant region of parameters. Figure 4 plots the threshold $\Delta\theta_u(\nu)$ such that R 's ex ante rent is the same with and without information sharing — i.e., $\mathcal{V}^N = \mathcal{V}^S$.

Second, we compare M 's expected profit with and without information sharing. Without loss of generality, we focus on the second period, since production in the first period is the same with and without information sharing. For any disclosure policy $d \in \{S, N\}$, M 's expected profit is

$$\Pi^d = \sum_{\theta_1} \Pr[\theta_1] \sum_{\theta_2} \Pr[\theta_2|\theta_1] q_2^d(\theta_2, \theta_1)^2.$$

Hence,

$$\Pi^N - \Pi^S = \frac{((1-\nu)(9-63\nu^2-38\nu)\Delta\theta + (48\nu^2-16\nu)(1-\nu))}{72\nu(3\nu-1)^2} \Delta\theta.$$

Setting the numerator equal to 0 and solving for $\Delta\theta$ yields

$$\Delta\theta_\pi(\nu) \triangleq \frac{16\nu(3\nu-1)}{38\nu + 63\nu^2 - 9},$$

which is positive in the relevant region of parameters. Figure 5 plots the threshold $\Delta\theta_\pi(\nu)$ such that M 's expected profit is the same with and without information sharing — i.e., $\Pi^N = \Pi^S$.

Finally, showing that $\Delta\theta_\pi(\nu) \geq \Delta\theta_u(\nu)$ is immediate. ■

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