M&A Advisory and the Merger Review Process

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Abstract
Two firms propose a merger to the antitrust authority. They are uninformed about the efficiencies generated by the merger, but can hire an expert to gather information on their behalf. The authority is also uninformed about the merger's efficiencies, but can run a costly internal investigation to learn them. We analyze the effect of the disclosure of the expert's contract on consumer welfare, and show that consumers are not necessarily better off with disclosure. This negative effect hinges on a free-riding problem between expert and authority in the information acquisition game, and is more relevant in highly competitive industries.

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1 Introduction

M&A advisory fees from completed transactions totalled about 40 billion US dollars in 2017.\(^1\) Goldman Sachs alone recorded total fees of about 3 billion US dollars from M&A deals. CEOs ask for advice to these firms, seeking for specialized help on decisions that are critical for the success of the transaction, like the identification of synergies and the design of the new company’s structure after the deal is closed.\(^2\) Advisors then support clients throughout the process: from the choice of the target to the execution of the due diligence, which may require reporting the transaction to the Antitrust Authority (AA), and disclosing to the Securities and Exchange Commission (SEC) the parties to be compensated and the relative arrangements.

Interestingly, recent legal developments highlight an increasing trend towards more extensive disclosure of advisory fees in M&A deals, including the circumstances in which these fees are payable.\(^3\) Similarly, recent decisions of the Delaware Chancery Court have emphasized the need for greater transparency on advisors’ potential conflicts of interest. The worry is that advisors may have perverse incentives to complete the transaction, possibly in opposition to the interests of shareholders.\(^4\)

Despite it is common for firms to use advisors in M&A transactions, and the greater scrutiny these advisors are subject to, it is unfortunate that little is known about how their presence impacts the success of M&A. What is the relationship between the accuracy of the merger evaluation

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\(^2\)The empirical literature is split on whether advisors provide valuable advice in M&A transactions. On the one hand, Bao and Edmans (2011) show that they have a positive impact on M&A returns, and Golubov et al. (2012) find evidence suggesting that top advisors are able to identify and structure mergers with higher synergies. On the other hand, Servaes and Zenner (1996) find no benefit of hiring an advisor, and the evidence in Rau (2000) is consistent with a negative impact of experts on post-merger returns.


\(^4\)Advisors need not be banks. There are many consulting companies that offer advice in mergers and acquisitions (e.g., McKinsey and International M&A Partnership, among many others). There are also examples in which advisors are industry ‘veterans’. For example, Sumit Banerjee (a cement hotspot for over a decade) has become the reference for funds and strategic players whenever a new target comes into the salemarket in India’s heavily-crowded cement sector (‘ACC confirms exit of Sumit Banerjee’, The Economic Times, 2010).
process, the role of external advisors, and the rules governing the transparency of these deals? Do firms proposing a merger have an incentive to rely on external advice in order to support their proposal? Does the presence of experts alter the process of investigation on the welfare effects of the merger? Should the proponent firms be obliged to disclose advisory fees? This paper proposes a game-theoretic model to answer these questions.

We study a standard static framework à la Farrell and Shapiro (1990) in which two out of \( N \) symmetric firms, competing in a Cournot industry, propose a merger to an AA who adopts a consumer surplus standard. We assume that the basic trade-off is the standard one: the merger may increase market power but also create efficiencies (Williamson, 1968).\(^5\) In the model, the proponent firms are uninformed about the (uncertain) efficiencies of the merger, but can hire an expert to gather information on their behalf. Firms cannot verify the information reported by the expert to the AA — i.e., they lack the technical skills necessary to assess whether such evidence is informative or not (the AA, by contrast, owns these skills). This asymmetry of information creates a moral hazard problem that can be solved by offering the expert an incentive compatible contract that pays a reward only if the merger is approved. The AA is also uninformed about the efficiency improvement of the merger, but can run a costly internal investigation in order to learn its realized value. Hence, the model differs from Besanko and Spulber (1993) and Nocke and Whinston (2013) because firms are not privately informed about the merger characteristics \textit{vis-à-vis} the AA, but both can acquire this information by engaging in a costly information acquisition activity. To make the problem interesting, we follow Besanko and Spulber (1993) in assuming that the merger harms consumer surplus in expected terms — i.e., behind the veil of ignorance the AA always rejects it.

Within this framework, we analyse the welfare effects of the disclosure of the expert’s contract as part of firms’ due diligence. Our main findings follow. First, we show that hiring an advisor does not necessarily improve the chances that a merger is approved. Second, consumers are not necessarily better off when the advisor’s contract is disclosed. These results hinge on the

\(^5\)Efficiency improvements can be measured not only in terms of production costs but also in terms of operational synergies (see, e.g., Gupta and Gerchak, 2002) or dynamic efficiencies and investments (Motta and Tarantino, 2018).
negative effect produced by a free-riding problem that arises between the expert and the AA in the information acquisition game. As we explain below, this problem is particularly relevant in highly competitive industries — i.e., in industries in which there are (relatively) more competitors.

Essentially, since gathering information about the merger’s efficiency improvement is costly, both for the expert and the AA, when the cost of acquiring information is sufficiently large, both have an incentive to free ride on each other. On the one hand, the AA would like the expert to gather information so to base the decision regarding the merger only on the evidence that he collects, while saving on its own information acquisition cost. On the other hand, the expert would like the AA to collect information so to avoid paying the investigation cost, while being rewarded in case the merger is approved. In the region of parameters where this free-riding problem is relevant, disclosure plays a key role for consumer surplus.

Accordingly, the game unfolds differently depending on whether the AA is informed or not about the advisor’s contract:

- Without disclosure the game is *de facto* simultaneous, and has the AA and the expert run into a coordination problem that may lead parties to play mixed strategies in equilibrium — i.e., the merging firms randomize between hiring and not hiring the expert, while the AA randomizes between collecting information and remaining uninformed. In this equilibrium both players collect information and, of course, the accuracy of the AA’s decision increases if both choose to acquire information with a (relatively) high probability.

- With disclosure, the structure of the game becomes sequential. The proponent firms have a first-mover advantage insofar as by committing not to hire an expert,\(^6\) they can force the AA to acquire information (provided that the cost of doing so is not prohibitively high). Hence, in equilibrium, either the AA acquires information or the merger is rejected with certainty even though it may have been beneficial to consumers. Avoiding this risk induces the AA to

\(^6\)Note that this is equivalent to submitting an uninformative report in case the AA asks for additional information during the merger review.
bear alone the cost of being informed.

Comparing the expected consumer surplus in these two regimes, we show that the disclosure of the expert’s contract harms consumers if and only if the cost of acquiring information is sufficiently low. In this case, without disclosure, the probability that the AA takes an informed decision is sufficiently large since both players acquire information with high probability. By contrast, in the regime with disclosure only the AA gathers information and becomes informed with some probability. When the cost of acquiring information is sufficiently large, instead, the probability that the AA takes an informed decision drops in the regime without disclosure and in that case enforcing transparency on the expert’s contract benefits consumers.

Interestingly, the region of parameters in which disclosure harms consumers expands as the market becomes less concentrated. The reason is as follows. When the number of firms in the industry increases, the anticompetitive effect of the merger weakens. Hence, conditional on being informed, the AA rejects the merger with lower probability. Other things being equal, a lower probability of rejection tends to increase consumer surplus more without disclosure than with disclosure, because in the former regime both players acquire information (although they do so with probability less than one) while in the latter only the AA acquires information (with certainty).

Summing up, our model and results not only bring a novel theoretical twist, but they also provide useful insights to better understand how merger review protocols should take into account information that is not directly inherent to the structure of the industry — e.g., demand and market shares — and the particular deal under evaluation — e.g., costs, efficiency improvement, etc.

Inspired by Williamson (1968), the modern approach to horizontal mergers has been developed by Farrell and Shapiro (1990), who formalize the basic welfare trade-off arising from horizontal mergers in a model with Cournot competition and increasing marginal costs (see, e.g., also McAfee and Williams, 1992).\(^7\) These models assume complete information and a passive role for the AA.

\(^7\)Relatedly, Deneckere and Davidson (1985) study mergers in a model with price competition and constant
Besanko and Spulber (1993) were the first to examine the role of the AA in a model with asymmetric information. They assume that merging firms are more informed than the AA about the merger’s efficiency gains and show that, by committing to a conservative consumer surplus standard, the AA may increase aggregate surplus.\(^8\) A different form of asymmetric information is considered in Armstrong and Vickers (2010), who determine the optimal permission set in a model in which the AA can verify the characteristics of the received merger proposal, but does not know which other merger projects were available to the firms (see, e.g., also Lyons, 2003). Building on the same idea, Nocke and Whinston (2013) analyse the optimal merger approval policy in a model with a single acquirer who can make a merger proposal to one of several heterogeneous firms. In particular, they assume that the AA observes the characteristics of the proposed merger only, which is the result of a bargaining process among the firms, and find that the AA optimally commits to a policy that imposes tougher standards on mergers involving firms with a larger pre-merger market share.

Finally, closest to us is Sørgard (2009), who studies a framework where the AA sets an activity level (how many mergers to investigate). He shows that an active merger control can lead to an adverse selection of proposed mergers, which in turn has implications for the possible decision errors made on investigated mergers. The optimal activity level trades off the possible mistakes from enforcement with the gains from deterring a potentially harmful merger.\(^9\)

None of these papers studies the role of experts and explicitly models the information acquisition process.\(^{10}\) By introducing both these ingredients, our model delivers policy implications on the relationship between the accuracy of the merger evaluation process, the role of external advisors

\(^8\) A parallel strand has emphasized the role of uncertainty in efficiency gains (see, e.g., Choné and Linnemer, 2008; Amir et al., 2009; Cunha et al., 2014).

\(^9\) Katsoulacos and Ulph (2009) also combine the decision-error framework with deterrence and procedural effects.

\(^{10}\) In a recent work, Vasu (2017) studies the optimal contract that a seller and a buyer offer to their respective advisors who can identify the value of the synergies arising from the merger. However, the paper focuses on the bargaining process between the merging firms and is thus not related to the problem of merger review, to which our work contributes.
and the rules governing how much transparent their deals should be.

The article proceeds as follows. We set up the model in Section 2 and first develop the analysis for the regime with disclosure (Section 3.2) and then the regime without disclosure (Section 3.3). In Section 3.4, we study the effect of disclosure on consumer surplus. Section 4 discusses some extensions of the model. Section 5 concludes. Proofs are in the Appendix.

2 The model

Environment. Suppose that two firms (1 and 2 without loss of generality) contemplate a merger in a \(N\)-firm Cournot industry, but in order to do so they need approval by an AA. Before the merger, all firms produce at a (constant) marginal cost \(c_0 > 0\). Conditional on the merger being rejected, let \(\pi_0\) and \(w_0\) denote each firm’s profit and the consumer surplus (hereafter, CS), respectively. Similarly, once the merger takes place, depending on the (uncertain) post-merger marginal cost \(c \leq c_0\) (more below), the merged firm obtains a profit \(\pi_M(c)\) and the consumer surplus is \(w_M(c)\). Define the change in the merged entity profits and in the consumer surplus as \(\pi(c) \equiv \pi_M(c) - 2\pi_0\) and \(w(c) \equiv w_M(c) - w_0\), respectively. The merger is said to be profit-increasing if \(\pi(c) \geq 0\) and CS-increasing if \(w(c) \geq 0\). To make the problem interesting for our purposes, in line with Besanko and Spulber (1993), assume that \(\mathbb{E}[w(c)] < 0\) — i.e., behind the veil of ignorance the AA always rejects the merger.\(^{11}\)

Information acquisition. The (constant) marginal cost \(c\) of the merged entity is uncertain, and is distributed uniformly on the support \([0, c_0]\). The difference \(c_0 - c \geq 0\) captures the efficiency improvement (cost synergies) produced by the merger. The AA is active (Sørgard, 2009), meaning that the AA can carry out an internal investigation in order to learn the realization of \(c\).\(^{12}\) The investigation technology is all-or-nothing: it is successful with probability \(p \in [0, 1]\), while nothing

\(^{11}\)Hence, on average, the post merger price increase is not compensated by the positive effect of cost synergies.

\(^{12}\)Evidence of active merger policies is discussed in Sørgard (2009).
is learned with probability $1 - p$.\textsuperscript{13} Before merging, the two firms have no skills to learn the state of nature $c$, but can delegate an expert (advisor) in order to gather information on their behalf. Conditional on paying the information acquisition cost, the expert learns $c$ with probability $p$, and remains uninformed otherwise. For simplicity, we assume that the expert cannot falsify information \textit{vis-à-vis} the AA. Hence, he can either disclose the true state of nature if he knows it, or produce an uninformative report. The decision of whether gathering information or not is private information of the expert: a moral hazard problem.

Gathering information is costly both for the AA and the expert, whose (sunk) cost of acquiring information is (for simplicity) identical and equal to $\psi \geq 0$ (see, e.g., Gromb and Martimort, 2007). We assume on purpose that the expert and the AA own the same information acquisition technology in order to isolate results from the direction of such asymmetry (we discuss the role of asymmetries in the information acquisition technologies in Section 4).

This information acquisition process is meant to capture the idea that the merger review process, in most jurisdictions, is articulated in two phases. In the first phase the AA carries out a quick review to examine whether the deal entails any potentially serious issues. If this is not the case, the merger is cleared; otherwise, the process enters into the second phase in which the AA requires the proponent firms to produce evidence on the merger efficiency improvements. During this phase, the AA’s lawyers and economists conduct internal analysis, request and review documents from parties and non-parties, and interviews take place with interested entities (such as customers, competitors, suppliers, etc.), with the merging parties, and sometimes with business people, the counsel and outside economists.\textsuperscript{14}

\textit{Contracts.} The expert’s contract consists of a two-part tariff: a fixed payment $F \geq 0$ plus a linear component $\alpha \in [0,1]$ contingent on the deal completion (i.e., the merger being approved). In

\textsuperscript{13}In Sørgard (2009) the parameter $p$ is interpreted as the probability with which a merger is investigated.

\textsuperscript{14}See, for example, the merger investigation process conducted at the Federal Trade Commission (FTC) (www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/mergers/premerger-notification-merger-review).
addition, we assume that this contingent fee is given by a proportion of the change in the merged entity aggregate profits (more below). This remuneration structure is commonly observed in the real world: many of the contracts seen in practice are linearly increasing in transaction value — see, e.g., Rau (2000).

Disclosure regimes and timing. We consider two alternative contract disclosure regimes: one in which the expert’s contract must be disclosed to the AA before the decision is taken, the other in which the contract is kept secret. The timing of the game is as follows:

\[ t=1 \] The state of nature \( c \) realizes.

\[ t=2 \] Firms 1 and 2 decide whether to propose the merger and (if so) they also decide whether to hire an expert and which contract to offer him. The contract is observed by the AA if its disclosure is mandatory.

\[ t=3 \] If hired, the expert decides whether to gather information or not, and so does the AA.

\[ t=4 \] The expert submits a report to the AA, who learns the outcome of the internal investigation, updates beliefs and approves or rejects the merger.

\[ t=5 \] If the merger is approved, the post-merger outcome realizes (more below), and the fees are paid to the expert; otherwise, the status quo remains.

Hence, the AA exerts an activity level (information acquisition) with the aim of discovering the merger’s welfare effect (see, e.g, Sørgard, 2009). As mentioned above, AAs typically involve market insiders — i.e., competitors and third parties — during the merger review process, to gather information on the market or get comments on the draft of the Statement of Objections (Giebe and Lee, 2015).\(^\text{15}\)

\(^{15}\)Giebe and Lee (2015) show that in the EU the number of cases where competitors were given the opportunity to voice their opinions has sharply increased between 1990 and 2013. In the U.S., competitors’ claims were traditionally treated restrictively, but both the Department of Justice and the Federal Trade Commission have recently started to widen the extent of competitor participation in merger proceedings by conducting an ‘open door’ policy.
More generally, the timing of the model captures the simple idea that merger decisions may take quite a long time, and that discovering and documenting efficiency gains might be costly both for the proponent firms and the AA. In contrast to Nocke and Whinston (2010, 2013) and Sørgard (2009), in our model the AA does not commit to a merger policy, but makes a decision on the basis of the information elicited throughout the review process. This seems to reflect real world practices where a decision is made either right away (i.e., in Phase 1 of the review process) or in Phase 2 after evaluation of the merger (see Section 4 for a discussion of this issue).

**Payoffs.** The expert is risk-neutral, but is protected by limited liability — i.e., the fixed fee $F$ cannot be negative — and (without loss of generality) his outside option is normalized to zero. Hence, the expert’s payoff is

$$u_e(\cdot) \triangleq \max\{0, F + \alpha \pi(c) d(c) - \psi e\},$$

where, for any state $c$, $d(c) \in \{0, 1\}$ is an index that takes value 1 if the merger is approved and 0 otherwise. Of course, $d(c) = 0$ when the AA’s decision is made with no information available about the state of nature since the merger is not CS-increasing in expected terms. Similarly, let $e \in \{0, 1\}$ be an index that takes value 1 if the expert gathers information and 0 otherwise.

The joint payoff of the proponent firms is

$$u_f(\cdot) \triangleq \max\{0, (1 - \alpha) \pi(c) d(c) - F\},$$

where, of course, $\alpha = F = 0$ if the expert is not hired.

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16 A study conducted in 2017 by Allen & Overy, an international law company, shows that the cases that are cleared unconditionally in the first phase obtain clearance within 30 working days. By contrast, periods to receive unconditional clearances following an in-depth investigation range from an average of 58 working days (Turkey) to 144 working days (the Netherlands), while prohibitions generally took over 100 working days, with the in-depth investigation period for deals blocked in the U.S. averaging 467 working days. In Europe, the European Commission makes regular use of its stop the clock powers, suspending the review by asking the parties for additional information. In the U.S., the average duration of in-depth investigations is consistently increasing over recent years, thus leading the DOJ (Department of Justice) to announce that it wants to increase the speed and reduce the burden of merger reviews.
The AA’s preferences are determined by a loss function which takes into account, together with the cost of acquiring information, the probability (computed from ex-ante point of view) of making a wrong decision, weighted by the respective damage to consumers. For any state \( c \in [0, c_0] \), let \( s(c) \in \{0, 1\} \) be an index that takes value 1 if, in that state, the merger is CS-increasing and 0 otherwise. With the all-or-nothing information structure considered above, for any disclosure regime \( j = D, N \), the AA maximizes the following objective function:

\[
U^j(\cdot) \triangleq - \int_{c \in \{c: s(c) = 1\}} \Pr [d(c) = 0|s(c) = 1, j] w(c) \, dc - \psi a.
\]

where \( a \in \{0, 1\} \) is an index that takes value 1 if the AA acquires information, and 0 otherwise.

In words, the AA is only concerned with a type-I error — i.e., the event in which a CS-increasing merger is rejected — since this is the only error that can happen with the all-or-nothing information acquisition technology introduced before (we discuss type-II errors in Section 4).

Market structure and technical assumptions. In order to obtain closed form solutions, we consider a (linear) inverse demand function

\[
P(\cdot) \triangleq \max \left\{ 0, A - \sum_{i=1}^{N} q_i \right\},
\]

with \( c_0 < A \). We argue in Section 4 that our results hold qualitatively with a more general demand function.

Firm \( i \)'s production is denoted by \( q_i \). Hence, in the pre-merger equilibrium, each firm produces \( q_0 \triangleq \frac{A - c_0}{N+1} \). Accordingly, each firm’s profit is

\[
\pi_0 \triangleq \left( \frac{A - c_0}{N+1} \right)^2,
\]

and the consumer surplus is

\[
w_0 \triangleq \frac{N^2 \pi_0}{2}.
\]
Next, consider the scenario in which the merger is approved. The merged entity will compete in the industry with the remaining \( N - 2 \) firms. To simplify the analysis, we assume that once the merger has occurred the cost \( c \) becomes common knowledge to its competitors — i.e., a standard Cournot equilibrium is played in this event (in other words, we abstract from considering additional effects driven by the fact that the merged entity owns private information on its cost structure). Accordingly, following the merger, the equilibrium of the market game is such that the merged entity produces

\[
q_M(c) \triangleq \frac{A - (N - 1)c + (N - 2)c_0}{N},
\]

while each other competitor produces

\[
q_i(c) \triangleq \frac{A - 2c_0 + c}{N}, \quad \forall i \geq 3.
\]

The profit of the merged firm is

\[
\pi_M(c) \triangleq \frac{(A + (N - 2)c_0 - (N - 1)c)^2}{N^2},
\]

and the consumer surplus is

\[
w_M(c) \triangleq \frac{[(N - 1)A - (N - 2)c_0 - c]^2}{2N^2}.
\]

For simplicity and without loss of insights, in the following analysis we normalize \( c_0 \) to 1.

Conditional on the merger being CS-increasing, let

\[
w \triangleq \text{E}[w(c) \mid s(c) = 1],
\]

11
denote the expected change in consumer surplus. Similarly, let

\[ \pi \triangleq \mathbb{E}[\pi(c) | s(c) = 1], \]

the expected change in the merged firm’s profit. As we will see, the disclosure regime plays a role in our model if information acquisition technology is sufficiently accurate — i.e.,

\[ p \geq \frac{\pi - 2w}{\pi - w}. \quad (A1) \]

In the Appendix we show that this assumption implies that the number of firms in the industry is not too small.

As a standard tie breaking condition, in all pure strategy equilibria of the game we assume that, whenever the expert or the AA are indifferent between acquiring information and remaining uninformed, they prefer the first option.

Finally, since the game is sequential and there is incomplete information, our equilibrium concept will be PBE.

3 Equilibrium analysis

In this Section, we characterize the equilibrium of the game with and without disclosure of the expert’s contract. Then, in Section 3.4, we compare the expected consumer surplus in the two regimes.

3.1 Preliminaries

We first state a few preliminary results that will be useful in the subsequent analysis. First, comparing the outcomes with and without the merger, it is easy to show that the merger is profit-
increasing — i.e., $\pi(c) \geq 0$ — if and only if $c \leq \hat{c}$, where

$$\hat{c} \triangleq \frac{N^2 + N(\sqrt{2} - 1) - 2 - A[N(\sqrt{2} - 1) - 1]}{N^2 - 1}.$$ 

Analogously, the merger is CS-increasing — i.e., $w(c) \geq 0$ — if and only if $c \leq \hat{c}$, where

$$\hat{c} \triangleq \frac{N + 2 - A}{N + 1}.$$ 

Notice that $\hat{c} < \tilde{c}$, so that if the merger is CS-increasing, it is also profit-increasing, but not the other way around.\footnote{Such a result holds true in much more general Cournot models (see, e.g., Nocke and Whinston, 2010).} Hence, without loss of generality, in the following we assume that the merger is always proposed. The reason is that whenever the AA approves the merger because it is CS-increasing, then it must also be profitable for the proponent firms.

Of course, in order to make the problem interesting, we focus on the region of parameters where $\hat{c} > 0$. In the Appendix, we show that together with $E[w(c)] < 0$, these inequalities are satisfied for $N \in [A - 2, 2(A - 2)]$, which we will assume throughout.

Next, since $s(c) = 1$ if and only if $c \leq \hat{c}$, the following results hold true.

**Lemma 1** $\pi$ is decreasing in $N$, while $\hat{c}$, $w$ and $\hat{c}\pi$ are increasing in $N$. Moreover, $\pi > 2w$.

The intuition is straightforward. Clearly, as the number of firms in the market grows larger, the merger becomes less profitable because, once merged, the proponent firms will be exposed to more competition by the non-merging firms. By contrast, as the market becomes more competitive, it is more likely that the merger is CS-increasing — i.e., $\hat{c}$ being increasing in $N$ — because the market power of the merged firm becomes weaker in regards to the efficiency gain it brings. The same intuition explains why the expected change in consumer surplus $w$ is increasing in $N$.

We are then ready to characterize the expert’s reporting behavior and the AA’s decision rule.
Proposition 1 The informed expert sends a truthful report to the AA if and only if \( c \leq \hat{c} \), otherwise he produces an uninformative report. The AA clears the merger if and only if the evidence gathered internally or by the expert is such that \( c \leq \hat{c} \).

The intuition is straightforward. Conditional on the expert having learned the state of nature, he has an incentive to produce an informative report if and only if this proves that the merger is CS-increasing because, in this case, the merger is also profit-increasing. Otherwise, he produces an uninformative report. In this case, however, the AA anticipates the expert’s strategic behavior and rejects the proposal unless the evidence collected internally shows that the merger is CS-increasing.

3.2 The disclosure regime

When the expert’s contract is observable, the game is sequential. Hence, the proponent firms act as a Stackelberg leader to influence the AA’s behavior. To save on notation, hereafter we normalize the expert’s fixed payment \( F \) to zero since he is protected by limited liability.

Information acquisition. We start by analysing the expert’s incentive to gather information. The key point to notice is that because the proponent firms are unable to assess the evidence collected by the expert, they may end up paying him even when he has not gathered information, but the merger was approved only thanks to the evidence collected by the AA. Hence, the expert’s incentive to gather information depends on the AA’s behavior: other things being equal, the expert may want to save on the cost of acquiring information and let the AA learn the state of nature on his behalf. This creates a standard free-riding problem which will be key to our analysis.

If the AA is expected not to acquire information, the expert has an incentive to gather information if and only if

\[ \alpha p \hat{c} \pi - \psi \geq 0 \iff \alpha \geq \alpha_L \triangleq \frac{\psi}{p \hat{c} \pi} . \]

This inequality has a simple interpretation. When only the expert gathers information, the merger is approved if the expert learns the state of nature (with probability \( p \)) and if the merger brings
strong enough cost synergies — i.e., if $c \leq \hat{c}$. When, instead, neither the expert nor the AA gather information, the merger is always rejected (by assumption) and the expert never gets rewarded.

If the AA is expected to acquire information, the expert gathers information if and only if

$$\alpha (1 - (1 - p)^2) \hat{c} \pi - \psi \geq \alpha p \hat{c} \pi \quad \Leftrightarrow \quad \alpha \geq \alpha_H \equiv \frac{\psi}{p(1 - p) \hat{c} \pi}.$$ 

The right-hand side of the inequality is the expert’s expected utility if he does not gather information. As mentioned before, conditional on the merger being approved, he can be rewarded even if he has not gathered information. The left-hand side of the inequality is, instead, the expert’s expected utility when he gathers information. In this case, what matters is the probability that (conditional on both the expert and the AA gathering information) at least one learns the state of nature — i.e., $1 - (1 - p)^2$.

It can be easily verified that $\alpha_H > \alpha_L$ and that both these values are decreasing in $N$. The reason is simple. Clearly, $\alpha_H$ exceeds $\alpha_L$ because the expert has less incentive to acquire information when the AA acquires information, owing to the free-riding problem mentioned before. Moreover, holding the AA’s behavior constant, as the market becomes less concentrated, the anticompetitive effect of the merger weakens compared to the efficiency gains it may bring. Hence, the probability of the merger being approved increases so that it is less risky for the expert to incur the information acquisition cost, which in turn weakens his incentive compatibility constraint.

We can then show the following:

**Lemma 2** The expert’s behavior is as follows:

(i) If $\alpha \geq \alpha_H$ he gathers information regardless of the AA’s behavior.

(ii) If $\alpha < \alpha_L$ he does not gather information regardless of the AA’s behavior.

(iii) If $\alpha \in [\alpha_L, \alpha_H)$ the expert gathers information if and only if he expects the AA not to do so.
Since acquiring information is costly, the expert and the AA face a standard free-riding problem. Other things being equal, the expert would like the AA to gather information since this would allow him to save on the cost of getting informed and nevertheless obtain a reward with probability $p\hat{c}$. Yet, if the AA does not collect information, the expert has an incentive to do so on his own as long as the bonus that he receives in case the merger is approved is sufficiently large.

Using the same logic, we now analyse the incentive of the AA to acquire information. Since acquiring information is costly, the AA’s objective depends on whether the expert (in the equilibrium) gathers or not information.

First, consider the case in which the expert gathers information. The AA has an incentive to gather information if and only if

$$-(1 - p)^2\hat{c} - \psi \geq -(1 - p)\hat{c} \Leftrightarrow \psi \leq \psi^a_L \triangleq p(1 - p)\hat{c}w.$$  

Notice that $(1 - p)^2$ is the probability that neither the expert nor the AA are able to learn the state of nature, while $\hat{c}$ is the probability that the merger is indeed CS-increasing. Hence, $(1 - p)^2\hat{c}$ is the probability of rejecting a CS-increasing merger. Clearly, the AA acquires information if the cost of doing so is not too large. Notice, however, that this incentive is stronger when the expected gain in consumer surplus from the merger is higher — i.e., when *ceteris paribus* $\hat{c}w$ increases — and when the probability that at least one between the expert and the AA learns the state of nature increases — i.e., when $p(1 - p)$ grows larger.

Second, consider the case in which the expert does not gather information. The AA has an incentive to gather information if and only if

$$-(1 - p)\hat{c} - \psi \geq -\hat{c} \Leftrightarrow \psi \leq \psi^a_H \triangleq p\hat{c}w.$$  

Once again, the AA would like to be informed if the cost of doing so is (relatively) small and if the expected gain in consumer surplus is large enough. However, when the expert is uninformed,
only the probability $p$ that the AA learns the state of nature matters.

It is immediate to see that $\psi^a_L < \psi^a_H$, which again reflects the free-riding problem between the AA and the expert — i.e., the AA has less incentive to get informed when it expects the expert to gather information. We can thus show the following result.

**Lemma 3** The AA’s information acquisition behavior is as follows:

(i) If $\psi \leq \psi^a_L$ the AA gathers information regardless of the expert’s behavior.

(ii) If $\psi > \psi^a_H$ the AA does not gather information regardless of the expert’s behavior.

(iii) If $\psi \in [\psi^a_L, \psi^a_H)$ the AA gathers information if and only if the expert does not.

Clearly, if the cost of acquiring information is sufficiently small, the AA always tries to get informed regardless of the expert’s behavior; by contrast, if this cost is too high, the AA prefers to remain uninformed no matter what the expert does. The most interesting case is the region of parameters in which $\psi$ takes intermediate values — i.e. where the free riding problem bites. In this case, the AA would like to invest in information acquisition, but only if the expert is uninformed.

**Advisory contract, hiring decision and equilibrium.** We can now turn to characterize the contract offered by the firms to the expert, and then the equilibrium of the game. Since we normalized the expert’s outside option to zero and $\tilde{c} < \tilde{c}$, it follows that any bonus $\alpha \geq 0$ is accepted by the expert. Hence, the optimal $\alpha$ satisfies the expert’s incentive compatibility constraint as equality — i.e., it makes him indifferent between gathering information and remaining uninformed (for given AA’s information acquisition choice). Accordingly, $\alpha = \alpha_H$, if (in the equilibrium) the AA is expected to acquire information, and $\alpha = \alpha_L$ otherwise.

But, do firms have an incentive to rely on the expert? The answer depends on the AA’s information acquisition behavior. If the AA remains uninformed, the expected profit of the proponent
firms is $p(1 - \alpha_L) \hat{c}\pi$ when the expert gathers information.\textsuperscript{18} Hence, he will be hired if and only if
\[ p(1 - \alpha_L) \hat{c}\pi \geq 0 \iff \psi \leq \psi_H^e \triangleq p\hat{c}\pi. \]

By contrast, if the AA gathers information, the expected profit of the proponent firms is $(1 - (1 - p)^2)(1 - \alpha_H) \hat{c}\pi$ if they hire the expert, and $p\hat{c}\pi$ otherwise. Hence, the expert is hired if and only if
\[ (1 - (1 - p)^2)(1 - \alpha_H) \hat{c}\pi \geq p\hat{c}\pi \iff \psi \leq \psi_L^e \triangleq \frac{p(1 - p)^2}{2 - p} \hat{c}\pi < \psi_H^e. \]

Notice that $\psi_L^e < \psi_H^e$ because when the AA does not gather information the merger is rejected with probability 1 if the expert is not hired. In the Appendix we show that, under Assumption A1,
\[ \psi_L^a < \psi_L^e < \psi_H^a < \psi_H^e. \]

The following result characterizes the equilibrium of the game with disclosure.

**Proposition 2** With disclosure of the expert’s contract, the equilibrium of the game has the following features.

(i) The proponent firms hire the expert if either $\psi \leq \psi_L^a$ or $\psi \in (\psi_H^a, \psi_H^e]$. Moreover:

- If $\psi \leq \psi_L^a$ the contingent fee is equal to $\alpha_H$ and both the expert and the AA gather information;

- If $\psi \in (\psi_H^a, \psi_H^e]$ the contingent fee is equal to $\alpha_L$ and only the expert gathers information.

(ii) The proponent firms do not hire the expert if either $\psi \in (\psi_L^a, \psi_H^a]$ or $\psi > \psi_H^e$. Moreover:

- If $\psi \in (\psi_L^a, \psi_H^a]$ the AA gathers information;

\[ \text{\textsuperscript{18}There is no reason, of course, for hiring the expert when he will not be induced to gather information.} \]
– If $\psi > \psi^e_H$ the AA remains uninformed.

With disclosure of the expert’s contract, the game is *de facto* sequential with the proponent firms acting as a Stackelberg leader *vis-à-vis* the AA. In particular, in the region of parameters where the cost of gathering information is large enough to create a free-riding problem but not too large to make information acquisition unprofitable, the commitment not to hire the expert forces the AA to gather information, which resolves the free-riding problem in favor of the proponent firms.

### 3.3 The no-disclosure regime

When the expert’s contract is not disclosed to the AA, the game is simultaneous: the proponent firms choose whether or not hiring the expert and the AA decides whether or not starting an internal investigation, simultaneously. Hence, when the free-riding problem arises, the AA and the proponent firms may fail to coordinate on an equilibrium in which at least one of them gathers information. This potential coordination failure generates scope for a mixed strategy equilibrium, which will be at the heart of the welfare analysis developed in the next Section.

**Proposition 3** Without disclosure of the expert’s contract, the equilibrium of the game has the following features.

- If $\psi \in (\psi^a_L, \psi^e_L]$ only the expert gathers information in equilibrium and is offered a bonus $\alpha_L$.
- If $\psi \in (\psi^e_L, \psi^a_H)$ there are two pure strategy equilibria: one in which only the expert gathers information (and is offered $\alpha_L$), the other in which the AA gathers information and the expert is not hired ($\alpha = 0$). Moreover, there is also a mixed strategy equilibrium in which the expert is hired with some probability while the AA randomizes between gathering information and remaining uninformed. Specifically:
- When the expert is hired he is offered a bonus

\[ \alpha^* \triangleq \frac{\psi(1-p) + \sqrt{4p^2 \psi \hat{c}\pi + \psi^2 (1-p)^2}}{2p\hat{c}\pi} \in [0,1], \]

- The AA collects information with probability

\[ \sigma_A^* \triangleq \frac{1 - \alpha^*}{1 - (1-p)(1-\alpha^*)} \in (0,1). \]

- The expert is hired with probability

\[ \sigma_F^* \triangleq \frac{1}{p} - \frac{\psi}{p^2 \hat{c}w} \in (0,1). \]

- Otherwise — i.e., for any \( \psi \leq \psi^a_L \) and \( \psi \geq \psi^a_H \) — the equilibrium is as in the game with disclosure.

Hence, without disclosure the game features a larger equilibrium set. The reason is that when the expert’s contract is not disclosed, the AA has to form a conjecture on whether the expert has been hired or not, while the proponent firms must conjecture the AA’s information gathering decision. This uncertainty leads to the emergence of multiple equilibria in the region of parameters where the AA and the expert have an incentive to free-ride on one another — i.e., for \( \psi \in (\psi^a_L, \psi^a_H) \). In this region of parameters the game without disclosure also features an equilibrium in mixed strategies in which the AA and the proponent firms randomize. The existence of multiple equilibria brings out a selection issue. While there are good reasons to focus on the mixed strategy equilibrium, it also worth remarking that this multiplicity does not reduce the generality of our results. The reason is that, if one of the pure strategy equilibria were played instead, then the games with and without disclosure would yield the same results, meaning that our welfare conclusions, and thus the policy implications, are valid independently of which equilibrium is played.
We now explain our interest in the mixed strategy equilibrium. First, from a competition policy perspective, it is the most interesting equilibrium because the probability of rejecting a CS-increasing merger depends on the market structure (i.e., the number of firms in the industry) through the players’ indifference conditions (which pin down the probabilities $\sigma^*_A$ and $\sigma^*_F$). Second, from a game theoretic point of view, the reasons to select the mixed strategy equilibrium is twofold. A standard argument builds on the so called ‘purification’ argument (Harsanyi, 1973). According to this logic, mixed strategy equilibria are explained as being the limit of pure strategy equilibria for a perturbed game of incomplete information in which the payoff of every player is his own private information.\footnote{\textsuperscript{19}The mixed nature of the strategy can be seen as the result of each player playing a pure strategy with threshold values that depend on the ex-ante distribution over the continuum of payoffs that a player can have. For example, one could think of our mixed strategy equilibrium as being the equilibrium of a game where the cost of acquiring information ($\psi$) is uncertain and private information of each player — i.e., the AA and the expert privately observe their (random) costs of acquiring information.} Alternatively, according to evolutionary game theory, mixed strategy equilibria can be selected based on the idea of a large population of players (see, e.g., Fudenberg and Tirole, 1991, Ch. 1).\footnote{\textsuperscript{20}This logic postulates that in a large population of players (firms in our case) some always play one strategy (i.e., in our case, do not hire the expert) while the others play the alternative strategy (hire the expert).}

The mixed strategy equilibrium has an interesting comparative statics, which we summarize in the following result.

**Proposition 4** The impact of $N$ on the mixed strategy equilibrium characterized above is as follows:

- The bonus $\alpha^*$ is decreasing in $N$;
- The probability $\sigma^*_A$ is decreasing in $N$, whereas $\sigma^*_F$ is increasing in $N$.

As discussed before, when the market becomes less concentrated, the anticompetitive effect of the merger weakens. This increases the probability of the merger being approved, whereby making the expert not only more willing to acquire information, but also to do so at a (relatively) lower
bonus. To understand why $\sigma^*_A$ is decreasing in $N$, recall that this probability makes the proponent firms indifferent between hiring and not hiring the expert: the higher $\sigma^*_A$ the lower incentive the proponent firms have to hire the expert since they can free ride on the AA. When $N$ increases, the proponent firms have a higher expected benefit because the merger is approved with higher probability (recall that $\hat{c}_\pi$ is increasing in $N$ by Lemma 1). Hence, in order to keep the proponent firms indifferent between hiring and not hiring the expert, the probability $\sigma^*_A$ has to drop as $N$ increases so to enhance the incentive of the proponent firms to hire the expert. The same type of argument explains (mutatis mutandis) why $\sigma^*_F$ is increasing in $N$: other things being equal, as $N$ grows larger the merger is more likely to be CS-increasing, which increases the AA’s incentive to acquire information. A higher $\sigma^*_F$ counterbalances this higher incentive to acquire information because it exacerbates the AA’s incentive to free-ride on the proponent firms.

### 3.4 Optimal disclosure rule

In this section, we characterize the optimal disclosure rule. In doing so, following most of the literature (see, e.g., Whinston, 2007, Nocke and Whinston, 2013), we adopt a consumer surplus standard. Accordingly, we compare across the relevant parameter regions the expected loss in consumer surplus in the equilibria of the two games with and without disclosure.

The first important point to notice is that disclosure plays no role in the region of parameters where the free riding problem does not arise. For the same reasons, it has no role when one of the pure equilibrium strategies is played absent disclosure.\footnote{Of course, if one considers only pure strategy equilibria the disclosure of the expert’s contract has no effect on consumers. This is because the game with disclosure is just the sequential version of the game without disclosure — i.e., every SPNE of the game with disclosure is also a Nash equilibrium of the game without disclosure. Moreover, notice that an equilibrium in which only the AA acquires information is equivalent, from the consumers’ point of view, to an equilibrium in which only the expert acquires information since both players have the same information acquisition technology. Hence, regardless of which player acquires information in equilibrium, the expected loss for consumers is the same with and without disclosure of the expert’s contract.} Obviously, disclosure is neutral to consumer surplus when neither the AA nor the proponent firms acquire information — i.e., for $\psi > \psi^*_H$. In this case, no information is acquired in the equilibrium of both games, and the merger is
always rejected. Similarly, disclosure has no effect on consumer surplus in the region of parameters where both the expert and the AA acquire information regardless of what the other does — i.e., for $\psi \leq \psi_L^\alpha$. In this case, the merger is approved and rejected with the same probabilities in both games.

The most interesting region of parameters is that in which the free-riding problem arises — i.e., when each player remains uninformed when it expects the other to acquire information, and vice versa. Indeed, if in this region of parameters players coordinate on the mixed strategy equilibrium when disclosure is not mandated, the probability with which the AA learns the state of nature is endogenous and depends on the market structure. This dependency yields interesting results as we are going to show.

In the no disclosure regime, the expected loss in consumer surplus is

$$\text{Pr}[d(c)=0,s(c)=1,N]w = -\alpha^*\psi$$

which, from Proposition 4, is increasing in $N$, since it is decreasing in $\alpha^*$.

By contrast, the loss in consumer surplus under disclosure is

$$\text{Pr}[d(c)=0,s(c)=1,D]w = -\frac{(1-p)\hat{c}}{p(p+(1-p)\alpha^*)}$$

which, from Lemma 1, is clearly decreasing in $N$. Comparing these expressions we have:

**Proposition 5** Let $\psi \in (\psi_L^\epsilon, \psi_H^\epsilon)$. There exists a threshold $\psi^* \in (\psi_L^\epsilon, \psi_H^\epsilon)$ such that the regime in which the expert’s contract is disclosed to the AA increases consumer surplus relative to the regime without disclosure if and only if $\psi \geq \psi^*$. The threshold $\psi^*$ is increasing in $N$.

The intuition of this result is as follows. When the cost of acquiring information is small ($\psi$ low), the expert and the AA have (other things being equal) a relatively strong incentive to invest in information acquisition. In particular, in the regime without disclosure the two probabilities
\(\sigma^*_A\) and \(\sigma^*_F\) are (relatively) high. Hence, consumers prefer the expert’s contract not to be disclosed because this regime minimizes the probability that none of them gets informed (which is equivalent to the probability of rejecting a CS-increasing merger) compared to the case of disclosure where only the AA invests in information acquisition. By contrast, when \(\psi\) is large enough, the AA and the expert have a relatively low incentive to be informed when they play mixed strategies in the regime without disclosure — i.e., \(\sigma^*_A\) and \(\sigma^*_F\) are small — while with disclosure the AA acquires information with certainty. As a result, consumers prefer the regime with disclosure because this minimizes the risk of rejecting a CS-increasing merger.

Interestingly, the region of parameters in which disclosure harms consumers expands as the market becomes less concentrated. The reason is as follows. When the number of firms in the industry increases, the anticompetitive effect of the merger weakens. Hence, conditional on being informed, the AA rejects the merger with lower probability. Other things being equal, a lower probability of rejection tends to increase consumer surplus more without disclosure than with disclosure because in the former regime both players acquire information (although they do so with probability less than one) while in the latter only the AA acquires information (with certainty).

In order to provide more insights on the effect of \(N\) on the optimal disclosure rule, below we provide a few intuitive illustrations of the comparative statics just discussed.

[Figure 1 about here.]

For fixed values of \(A, p\) and \(\psi\), in Figure 1 we plot the expected CS loss, in absolute value, in the disclosure regime (red line) and in the no-disclosure regime (black line), showing that there exists a threshold \(N^*\) above which consumers prefer the regime in which the expert’s contract is not disclosed. First notice that, as argued before, in the case of disclosure, the loss in consumer surplus is (in absolute value) increasing with \(N\), while the opposite holds in the regime without disclosure. The reason is simple. When competition increases, the loss in consumer surplus caused by the rejection of a CS-increasing merger increases since the positive effect of the merger on the
(equilibrium) price is diluted by a more intense competitive pressure — i.e., $\hat{cw}$ is increasing in $N$. This effect is present both with and without disclosure. In the latter case, however, there is also the effect of the market structure on the probabilities with which players randomize in the mixed strategy equilibrium. A stronger competitive pressure in this case reduces the expected loss in consumer surplus since it increases the probability with which the proponent firms hire the expert (since $\sigma_F^*$ is increasing in $N$). This effect, in fact, outweighs the negative effects of $N$ on $\sigma_A^*$ and the positive effect of $N$ on $\hat{cw}$ discussed above.

[Figure 2 about here.]

In Figure 2, the value assigned to $\psi$ drops and the region in which disclosure harms consumers expands. This is because, as information acquisition becomes less costly, the AA and the expert get informed with higher probability in the mixed strategy equilibrium of the regime without disclosure, while the probability of getting informed in the regime with disclosure is constant.

[Figure 3 about here.]

Finally, in Figure 3, the value assigned to $p$ drops — i.e., the individual probability of learning the state of nature decreases. Again, compared to Figure 1, the area in which consumers prefer the regime without disclosure expands. The reason is simple: as $p$ drops, the probability of remaining uninformed in the regime with disclosure increases at a faster rate than in the regime without disclosure in which both players acquire information (although they do so randomly).

4 Extensions and further directions

Our model and results suggest some interesting questions for future research. For example, one may wonder whether our conclusions hold with a more general downward-sloping demand function. It is well known (Nocke and Whinston, 2010) that in a general Cournot model, under standard assumptions, any merger among $M \geq 2$ firms that the antitrust authority would be willing to
approve is strictly profitable for the merging parties. Hence, \( \hat{c} < \check{c} \) even with a more general demand function, so that our main results would hold true qualitatively.

Moreover, since we assumed an all-or-nothing technology, the AA is only concerned with a type-I error (i.e., the probability of rejecting a CS-increasing merger). Would results change when a type-II error — i.e., the probability of accepting a CS-decreasing merger — also matters? To address this issue, we should introduce a more sophisticated information acquisition technology which admits ‘errors’ — i.e., the possibility of underestimating the post-merger cost. In this case it may well happen that the merger is approved even if it is detrimental to consumers (like, for example, in Besanko and Spulber, 1993, and Sørgard, 2009). However, as long as acquiring information is costly both for the AA and the expert, the free-riding problem emphasized above would still be present, and the regime without disclosure would still feature a coordination issue and equilibria in mixed strategies. Hence, the qualitative insights of our model should hold true even with type-II errors. Yet, as for the effect of the market structure on the optimal disclosure policy, it should be noted that the cost of committing a type-II error is decreasing (in absolute value) in the number of firms: accepting a merger that is not CS-increasing is clearly less problematic as the industry becomes more competitive. Therefore, the effect of the market structure on the optimal disclosure rule may be ambiguous when also type-II errors matter, even if our results still hold when the probability of such error is not too large.

Relatedly, one may wonder what happens when the expert and the AA differ in their ability to acquire information. Once again, the free-riding problem driving our results should not vanish as long as the information acquisition technologies of the two players are not too different. Of course, if acquiring information is costless for one player, in equilibrium this player should acquire information. As intuition suggests, in this case disclosure would have no impact on consumer surplus. Yet, in light of the constraints that antitrust authorities face in real world, it seems implausible to assume that information acquisition is costless for them, and since time is valuable it should be costly for the experts too. In addition, it is hard to imagine why the ability of the
AA and the expert should be different in the real world. Indeed, in addition to the fact that merger evaluations are guided by same theory and data that are available to both the AA and the experts, antitrust authorities and consulting companies around the world often recruit people (e.g., graduate students) with the same background and from the same market.

Finally, motivated by the real world practice, we have ruled out the possibility of a merger policy — i.e., an ex ante commitment of the AA to approve only certain mergers. Besanko and Spulber (1993), Sørgard (2009) and Nocke and Whinston (2010, 2013) argue that an AA acting in the consumers’ interest may benefit from committing ex ante to a clearance rule or to an activity level that is different from the ex-post optimal rule. This result is likely to hold also in our framework since any commitment ex ante of the AA would modify not only its own incentive to gather information ex post but also that of the proponent firms. In this sense the AA could exploit the commitment option as a way to exert a first mover advantage that in our model with disclosure is exerted by the proponent firms. Commitment ex ante to a merger policy that is ex post inefficient may however be subject to renegotiations issues that should be considered carefully in the analysis. The same ‘time inconsistency’ issue would emerge if the AA could commit (before the review process) not to gather information in order to solve the free riding problem with the proponent firms. This is clearly outside the scope of the present paper, we hope to address these questions in future research.

5 Concluding Remarks

We have developed a simple model showing that when two merging firms, who are uninformed on the uncertain post-merger cost efficiencies, have an incentive to hire an external expert who collects information on their behalf, consumers surplus may drop when the antitrust authority in charge of approving or rejecting the merger is informed about the expert’s contract. The negative effect of disclosure on consumer surplus hinges on a novel free-riding problem between the expert and the
AA in the information acquisition game, and it is more relevant in highly competitive industries. This suggests that mandatory rules forcing firms that wish to merge to disclose if they are advised by external experts (and how much they pay for such advice) should be used in (relatively) more concentrated markets. By contrast, in competitive industries the AA should actually commit not to learn this information. Given the relevance of advice in M&A practices, this result not only brings a novel theoretical twist but it also provides useful guidelines to know how an optimal review process should take into account information not directly linked to the structure of the industry.

Finally, although the model is very stylized and framed in a typical IO framework, its novel implications are likely to hold in more general environments where firms compete in a Cournot fashion. In addition, its structure can be used to model not only mergers but also other types of regulatory issues in which there are not monetary transfers between the authority and the firms.
References


Appendix

**Proof of Lemma 1.** Since \(c\) uniformly distributes over \([0, 1]\), it is immediate to show that \(\hat{c} > 0\) if and only if \(N > A - 2\), and that \(E[W(c)] < 0\) if and only if

\[
A^2(2N^2 - 1) - 3A(N^3 + 5N^2 - N - 3) + 3N^3 + 8N^2 - 5N - 7 > 0,
\]

which is always satisfied for \(N \leq 2(A - 2)\), provided that \(A \geq 3\). Moreover, it can be immediately seen that

\[
\frac{\partial \hat{c}}{\partial N} = \frac{A - 1}{(1 + N)^2} > 0
\]

We then compute:

\[
\pi = \frac{A^2(N^2 + 4N + 1) + 4A(N^3 - 3N - 1) + N^4 - 4N^3 - 3N^2 + 8N + 4}{3N^2(N + 1)^2}
\]

\[
w = \frac{-A^2(3N^2 - 1) + A(3N^3 + 9N^2 - 2N - 4) - (N + 2)(3N^2 - N - 2)}{6N^2(N + 1)^2}
\]

To obtain the comparative static results, first notice that \(\hat{c} \geq 0\) and \(E[W(c)] \leq 0\) for \(A \in [\hat{A}, N+2]\), where:

\[
\hat{A} \triangleq \frac{(N + 1)\sqrt{3(3N^4 + 2N^2 - 1)} + 3(N^3 + 5N^2 - N - 3)}{6(2N^2 - 1)}
\]

We compute:

\[
\frac{\partial w}{\partial N} = \frac{A^2(6N^3 - 4N - 2) - A(3N^4 + 15N^3 - 6N^2 - 18N - 8) + 3N^4 + 7N^3 - 12N^2 - 20N - 8}{6N^3(1 + N)^3}
\]

which is positive if

\[
A > \frac{3N^4 + 15N^3 - 6N^2 - 18N - 8 + N(N + 1)\sqrt{9N^4 + 12N^2 + 24N + 4}}{4(3N^3 - 2N - 1)}
\]
This inequality is indeed implied by $A \geq \hat{A}$. Hence, in the relevant range of the parameters, 
\[
\frac{\partial w}{\partial N} > 0.
\]
We then have:
\[
\frac{\partial \pi}{\partial N} = -\frac{2(A^2(N^3 + 6N^2 + 4N + 1) + 2A(N^4 - N^3 - 9N^2 - 7N - 2) - 3N^4 - N^3 + 4(3N^2 + 3N + 1))}{3N^3(N + 1)^3}
\]
which is negative if
\[
A > \frac{N(N + 1)\sqrt{N^4 - N^3 + 3N^2 + 5N + 1} - N^4 + N^3 + 9N^2 + 7N + 2}{N^3 + 6N^2 + 4N + 1}.
\]
This inequality is indeed implied by $A \geq \hat{A}$. Hence, in the relevant range of the parameters, 
\[
\frac{\partial \pi}{\partial N} < 0.
\]
Next, we compute:
\[
\frac{\partial \hat{c}\pi}{\partial N} = \frac{x_3A^3 + x_2A^2 + x_1A + x_0}{3N^3(1 + N)^4}
\]
where
\[
x_3 \triangleq 3N^3 + 16N^2 + 9N + 2
\]
\[
x_2 \triangleq 3(2N^4 - 7N^3 - 28N^2 - 17N - 4)
\]
\[
x_1 \triangleq -3(N^5 + 6N^4 - 13N^3 - 48N^2 - 32N - 8)
\]
\[
x_0 \triangleq 5N^5 + 18N^4 - 17N^3 - 80N^2 - 60N - 16
\]
We have:
\[
\frac{\partial(x_3A^3 + x_2A^2 + x_1A + x_0)}{\partial A} = 3A^2(3N^3 + 16N^2 + 9N + 2) + 6A(2N^4 - 7N^3 - 28N^2 - 17N - 4) - 3(N^5 + 6N^4 - 13N^3 - 48N^2 - 32N - 8),
\]
which is positive for
\[
A > \frac{-2N^4 + 7N^3 + 28N^2 + 17N + 4 + N(N + 1)\sqrt{7N^4 - 8N^3 + 12N^2 + 12N + 1}}{3N^3 + 16N^2 + 9N + 2},
\]
which is again implied by \(A \geq \hat{A}\). Thus, \(\frac{\partial \pi}{\partial A}\) is increasing in \(A\). Moreover, it can be easily checked that \(\frac{\partial \pi}{\partial N}\bigg|_{A=\hat{A}} > 0\). Hence, we conclude that, under our parametric restrictions, \(\frac{\partial \pi}{\partial N} > 0\).

Finally, we compute:
\[
\pi - 2w = 4A^2 + A(N - 10) + N^2 - 2N + 4
\]
\[
\frac{3N(N + 1)}{3N(N + 1)}
\]

Notice that, since
\[
\frac{\partial (\pi - 2w)}{\partial A} = \frac{8A + N - 10}{3N(N + 1)} > 0
\]
and
\[
(\pi - 2w)|_{A=2} = \frac{N}{3(N + 1)} > 0,
\]
it follows that \(\pi > 2w\).

To see why Assumption \text{A1} entails that the lower bound on \(N\) must be higher than \(A - 2\), notice that the considered threshold \(\frac{\pi - 2w}{\pi - w}\) is decreasing in \(N\) (since it is increasing in \(\pi\) and decreasing in \(w\)) and it approaches 1 as \(N \to A - 2\): hence, for a given value of \(p\), Assumption \text{A1} is satisfied for \(N\) sufficiently high. ■

**Proof of Proposition 1.** If the AA learns the state of nature the merger is approved if and only if \(c \leq \hat{c}\), regardless of the expert’s choice. Next, suppose that the AA has learned nothing. Of course, the merger is approved if the expert reports \(c \leq \hat{c}\). Thus, the expert finds it optimal to disclose his information in this case. Clearly, if instead he learns that \(c > \hat{c}\), then he does not disclose this information. However, when the expert submits an uninformative report, the merger is rejected. This is because the AA draws a negative inference — i.e., it knows that the expert has
either discovered that $c > \hat{c}$ or has not found any hard evidence — and we assumed that behind the veil of ignorance the merger is CS-decreasing in expectation. ■

**Proof of Proposition 2.** From Lemma 2 and Lemma 3, it follows that the pure strategy Nash equilibria of the static game between the expert and the AA are as follows.

1. If $\alpha < \alpha_L$, then:
   - for $\psi \leq \psi^a_H$, only the AA gathers information
   - for $\psi > \psi^a_H$, neither the expert nor the AA gather information

2. If $\alpha_L \leq \alpha < \alpha_H$, then:
   - for $\psi \leq \psi^a_L$, only the AA gathers information
   - for $\psi^a_L < \psi \leq \psi^a_H$ there are two pure strategy equilibria: one in which only the AA gathers information, the other in which only the expert gathers information
   - for $\psi > \psi^a_H$, only the expert AA gathers information

3. If $\alpha \geq \alpha_H$, then:
   - for $\psi \leq \psi^a_L$, both the expert and the AA gather information
   - for $\psi > \psi^a_L$, only the expert gathers information

Therefore,

- For $\psi \leq \psi^a_L$, the proponent firms find it optimal to hire the expert if $\psi \leq \psi^e_L$.
- For $\psi^a_L < \psi \leq \psi^a_H$, the proponent firms never find it optimal to hire the expert.
- For $\psi > \psi^a_H$, the proponent firms find it optimal to hire the expert if $\psi \leq \psi^e_H$.  

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To get the SPNE of the game, we must be able to compare the thresholds on the effort cost parameter. From Lemma 1, it follows $\psi^a_H < \psi^e_H$. Moreover, under Assumption A1, we have $\psi^a_L < \psi^e_L$ and $\psi^c_L < \psi^a_H$. 

**Proof of Proposition 3.** Assume that the AA gathers information with probability $\sigma_A \in [0, 1]$ and the firms hire the expert with probability $\sigma_F \in [0, 1]$.

It is easy to see that, given the firms’ strategy, the AA best reply correspondence is:

$$
\sigma_A(\sigma_F) = \begin{cases} 
1 & \text{if } \sigma_F < \sigma^*_F \\
[0, 1] & \text{if } \sigma_F = \sigma^*_F \\
0 & \text{if } \sigma_F > \sigma^*_F 
\end{cases}
$$

where $\sigma^*_F$ is given by:

$$
\sigma^*_F \triangleq \frac{1}{p} - \frac{\psi}{p^2 \hat{c}w}
$$

and it holds: $\sigma^*_F \in (0, 1)$ for $\psi^a_L < \psi < \psi^a_H$, $\sigma^*_F > 1$ for $\psi < \psi^a_L$, $\sigma^*_F < 0$ for $\psi > \psi^a_H$.

Analogously, the firms’ best reply correspondence is:

$$
\sigma_F(\sigma_A) = \begin{cases} 
1 & \text{if } \sigma_A < \sigma^*_A \\
[0, 1] & \text{if } \sigma_A = \sigma^*_A \\
0 & \text{if } \sigma_A > \sigma^*_A 
\end{cases}
$$

where $\sigma^*_A$ is given by:

$$
\sigma^*_A \triangleq \frac{1 - \alpha}{1 - (1 - p)(1 - \alpha)}
$$

and it holds: $\sigma^*_A > 0$ for all $\alpha < 1$, $\sigma^*_A < 1$ for $\alpha > \frac{1-p}{2-p} \in [0, \frac{1}{2}]$.

Then, $\alpha^*$ is computed by imposing the expert’s incentive compatibility constraint to be binding

The latter inequality is indeed satisfied for $p > \frac{2\pi - w - \sqrt{w(w + 4\pi)}}{2\pi}$, which is implied by Assumption A1.
when $\sigma_A = \sigma_A^* \in (0, 1)$:

$$\sigma_A^*(1 - (1 - p)^2)\hat{c}\alpha\pi + (1 - \sigma_A^*)p\hat{c}\alpha\pi - \psi = \sigma_A^*p\hat{c}\alpha\pi + (1 - \sigma_A^*)0$$

We have:

$$\alpha^* < 1 \iff \psi < \min\left(\frac{2p\hat{c}\pi}{1-p}, \psi_H^e\right) = \psi_H^e$$

$$\alpha^* > \frac{1-p}{2-p} \iff \psi > \min\left(\frac{2p\hat{c}\pi}{2-p}, \psi_L^e\right) = \psi_L^e$$

Thus, for $\psi < \psi_L^e$: $\sigma_A^* > 1$, and the value of $\alpha$ which makes the expert’s incentive compatibility constraint binding is given by $\alpha_H$; for $\psi > \psi_H^e$: $\sigma_A^* < 0$, and the value of $\alpha$ which makes the expert’s incentive compatibility constraint binding is given by $\alpha_L$.

By combining the two best reply correspondences, we get the Nash Equilibria of the game in the no-disclosure regime. ■

**Proof of Proposition 4.** We compute:

$$\frac{\partial \alpha^*}{\partial \hat{c}\pi} = -\frac{(2p^2[\hat{c}\pi] + \psi(1-p)^2)\sqrt{\psi(4p^2[\hat{c}\pi] + \psi(1-p)^2 + \psi(1-p)(4p^2[\hat{c}\pi] + \psi(1-p)^2)}}{2p[\hat{c}\pi]^2(4p^2[\hat{c}\pi] + \psi(1-p)^2)} < 0$$

Thus, from Lemma 1, $\alpha^*$ is decreasing in $N$. Consequently, since $\sigma_A^*$ is decreasing in $\alpha^*$, it must be increasing in $N$. Finally, from Lemma 1, it immediately follows that $\sigma_F^*$ is increasing in $N$. ■

**Proof of Proposition 5.** It is easy to get that the consumers’ expected loss with disclosed contract is higher than in the no-disclosure regime if:

$$\psi^3 - p(1-p)^2\hat{c}w\psi^2 - p^4(1-p)^2\hat{c}^3w^2\psi > 0$$

To establish our result, first notice that the derivative with respect to $\psi$ of the left hand side of the above inequality is positive for $\psi > \frac{2}{3}p(1-p)^2\hat{c}w$, which is satisfied for all $\psi > \psi_L^e$. Moreover, after
simple algebra, it follows that, if \((1 - p)^2 \pi - (2 - p)w < 0\), which is satisfied under Assumption A1, the left hand side of the above inequality takes a negative value when evaluated at \(\psi = \psi_L^c\) and a positive value when evaluated at \(\psi = \psi_H^a\). This proves the existence a threshold \(\psi^* \in (\psi_L^c, \psi_H^a)\) such that consumers are better off in the no-disclosure regime if and only if \(\psi < \psi^*\).

Finally, to see that \(\psi^*\) is increasing in \(N\), it is sufficient to notice that the left hand side of (5) is decreasing in \(N\). ■
Figure 1: Baseline example. Parameters’ values: $A = 10, p = 0.8, \psi = 0.025$. Consumer expected loss (in absolute value) as function of $N$ in the game with (resp. without) disclosure in red (black).
Figure 2: Parameters’ values: $A = 10, p = 0.8, \psi = 0.0125$. 
Figure 3: Parameters’ values: $A = 10, p = 0.7, \psi = 0.025$. 