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Competition along a river :
Decentralizing hydropower production

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Stefan Ambec* and Joseph A. Doucet**

Abstract

We analyze the production of electricity from *n* power stations situated along a river in a dynamic model. Each power station's production of electricity is constrained by the quantity of water available to it (capacity constraint) as well as limitations of reservoir capacity (storage constraint). Due to the water flow, production from one power station affects the production capacity of the next downstream power station. We show that when no constraint (capacity or storage) is binding, competition dominates monopoly. We then provide some examples in which, because one power station is constrained, monopoly dominates competition. Finally, we illustrate the model with an empirical example.

Keywords: Hydropower, Electricity, Competition, Regulation, Water.

JEL Classification. L10, L50, L94, Q48

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1 Introduction

This paper is motivated by two relatively recent phenomena that are of great importance for many regulators both in North America and around the world: electricity restructuring initiatives and water resource management related to hydroelectricity generation. While restructuring initiatives generally emphasize the introduction of competition in the generation segment of the industry and hence call upon decentralized market mechanisms, water resource management in a river system with hydroelectric production often leads to some form of centralized decision making because of the externalities involved in water flows and water management decisions. It would thus appear that river systems with multiple hydroelectric plants are not amenable to competition between plants along the river.

To illustrate the use of this line of reasoning, we refer to two prominent North American hydroelectric systems. In Québec 94% of the province's electricity production capacity is hydro. What's more, this production capacity is fairly concentrated: 74% is situated in 3 major river systems and 89% in 6 river systems.[1] It has long been argued that since ownership of sites along a river can't be separated, and since capacity is highly concentrated in a small number of river systems, introduction of an efficient market mechanism is infeasible for Québec.[2]

Under California's electricity restructuring plan, Pacific Gas and Electric (PG&E), one of the State's three major utilities, plans to sell the 3,900 megawatts (MW) of hydroelectric capacity that it owns. PG&E's proposed sale offers the capacity in 20 different groups, which it refers to as "bundles". The bundles are made up of plants, equipment and water rights along particular rivers or river systems. Environmental analysis of the proposed sale suggests

[1] Source: Hydro-Québec web site: http://www.hydroquebec.com/

[2] As voiced by Raphals and Dunsky(1997): "The vast majority of Hydro-Québec's production comes from hydroelectric "complexes", each of which is composed of many plants situated along the same hydrographic system. These complexes must therefore be managed in an integrated manner, since otherwise the management of an upstream plant could induce negative impacts on the management of downstream plants. In a competitive market, day to day management issues and the possibility of market power for an upstream plant would become even more important" (page 33, our translation).

that a change in ownership might lead to changes in the timing of release of water, which could change levels of reservoirs. In addition, analysis by an administrative law judge has raised concerns that the change in ownership might lead to market power.[3]

In general, the argument supporting decentralization or restructuring of electricity markets rests on the welfare benefit due to competition in the electricity markets. Consistent with this, in our model competition tends to move the allocation of electricity closer to consumers' tastes, thereby increasing welfare. The cost involved in decentralization is the loss of coordination in reservoir management along a river system. As a result of less coordination, production plans for individual plants along the river might face supplementary constraints than they would under the coordinated scenario. This can then affect strategic competition between firms, inducing potential inefficiencies. As a result, coordination failures tend to reduce the welfare.

Building on the links between water management along a river and electricity markets, this paper develops a model that highlights the tradeoffs involved. Electricity is produced during two periods from power generation plants located along a river. Each plant's production is constrained by the quantity of water available. Water comes from natural inflows, upstream release and reservoir storage. Due to the flow of water, production from one plant affects the production capacity of the next downstream plant.

More specifically, this paper quantifies the two arguments. We show that when coordination costs are zero, decentralizing hydro-power production increases welfare. We provide sufficient conditions on technical parameters for which decentralization benefits consumers. Then, in a duopoly example, we identify other conditions for which decentralization decreases welfare because of coordination failure.

Many papers address the issue of strategic competition in electricity markets. Scott and Read (1996) and Bushnell (1998) analyze competition among hydro-power producers located in different hydro basins. Crampes and Moreaux (1999) study competition between a thermal power producer and a hydro power producer. Our paper analyzes competition among firms located in the same hydro basin, thereby linking competition on the electricity market with

[3] Information on the proposed sale can be found at the California Public Utility Commission site: http://cpuc-pgehydro.support.net/

water management.

The rest of the paper proceeds as follows. We present the model in Section 2. We derive successively the first-best (Section 3), monopoly (Section 4) and competitive (Section 5) allocations of electricity. Section 6 compares these three situations. Section 7 presents and explains some situations where competition actually decreases welfare with respect to monopoly. Section 8 discusses the policy implications of our analysis. Section 9 presents a numerical example of the modeling approach using data from a large river system in Québec. Finally, the concluding section suggests an open research question. All proofs are relegated to the Appendix.

2 The model

Consider a dynamic model with two periods of production.[4] Let $u_t(Q_t)$ be the total utility from consuming Q_t units of electricity during period t. The utility function is assumed increasing and concave. The discount factor is normalized to 1. The total utility from consuming $Q = (Q_1, Q_2)$ units of electricity is denoted $U(Q) = u_1(Q_1) + u_2(Q_2)$. To account for seasonal differences in taste, we allow $u_1(Q_1) \neq u_2(Q_2)$, when $Q_1 = Q_2$.

When facing prices P_1 and P_2, the consumer chooses demand levels of electricity such that utility is maximized subject to the budget constraint. The solution to this program defines a (inverse) demand function for electricity $P_t(Q_t) = u'(Q_t)$, henceforth denoted P_t. We make the following assumption:

$$P_t'' Q_t + 2P_t' < 0.$$

This assumption guarantees that the profit of the monopoly is strictly concave and therefore avoids unrealistic corner solutions.

[4]Two periods are sufficient in order to highlight the dynamic nature of the problem and the implicit link between decision periods that storage capacity creates. In addition, the practice of management of hydro systems usually focuses on some element of a peak versus off-peak problem, and in this sense a two-period model is appropriate. As such, the proposed model is sufficiently general to capture either seasonal or shorter term storage problems (daily, weekly), but not both simultaneously.

Electricity is produced at n hydro plants located on a river. Plants are identified by their location along the river and numbered from upstream to downstream: $i < j$ means that i is upstream from j. Plant i produces q_{it} units of energy during period t by using water flowing to it. For simplicity, it is assumed that one unit of water used at plant i yields α_i units of energy ($\alpha_i > 0$).[5] Operating costs are normalized to zero. It is assumed that the unit cost of producing more electricity than there is available water is infinite. The total production of electricity during period t is $Q_t = \sum_{i \in N} q_{it}$.

There are natural inflows of water to the river along its entire length. e_{it} denotes the exogenous (perfectly forecasted) volume of water supplied in the hydrographic basin controlled by firm i during period t. $e_{i1} + e_{i2}$ denotes the total volume of natural water inflows at site i during the two periods. It is assumed that water is scarce enough not to be wasted. In other words, over the two periods each plant i uses all of the water inflows coming from upstream $\sum_{j \leq i}(e_{j1} + e_{j2})$ to produce electricity up to its capacity $\bar{q}_i$:[6]

$$q_{i1} + q_{i2} = \bar{q}_i = \alpha_i \sum_{j \leq i}(e_{j1} + e_{j2}). \tag{1}$$

This implies that total production during the two periods equals total production capacity of the system $\bar{Q}$:

$$Q_1 + Q_2 = \bar{Q} = \sum_{i \in N} \bar{q}_i = \sum_{i \in N} \alpha_i \sum_{j \leq i}(e_{j1} + e_{j2}). \tag{2}$$

[5] α_i is a productivity parameter which is exogenous and constant in the current setting. In practice, its value can vary as a function of system parameters, most notably the water head (height of the water fall from the intake to the turbines). However, this quite simple linear production function mapping water releases to power production is consistent with Wood and Wollenberg (1996) 's statement that:"flow of 1 ft^3 falling 100 ft has the power equivalent of approximately 8.5 kW.[...] The power equivalent for a flow of 1 ft^3 of water per second with a net drop of $100 - 5$, or 95 ft, would have the power equivalent of slightly more than 8 kW (8.5x95%)."

[6] Assuming that producers exhaust their inflows of water over the two periods obviously simplifies the problem. However, on average in a long term equilibrium hydro plants cannot have net positive or negative accumulation of water. The assumption reflects this physical limitation. Alternatively, we can suppose that market demand is such that the marginal revenue of electricity is always positive, which implies no incentive to produce less than the maximum capacity. Modifying this assumption to allow for "wastage" of water does not qualitatively change the results.

Water available to i during period 1 can be used to produce electricity during the first period, or can be stored for use in the second period. In the first period, Plant i relies on water in its hydrographic basin and on water released by its predecessor in the river during this first period.[7] The latter depends on the predecessor's production, $q_{(i-1)1}$. For notational convenience q_{0t} is normalized to 0, for $t = 1, 2$. Hence, the amount of water available for any arbitrary firm $i \in N$ in period 1 is $e_{i1} + \frac{q_{(i-1)1}}{\alpha_{i-1}}$. It follows that any arbitrary plant $i \in N$ is able to produce at no cost during the first period any quantity q_{i1} such that

$$q_{i1} \leq \alpha_i(e_{i1} + \frac{q_{(i-1)1}}{\alpha_{i-1}}). \tag{3}$$

Equation 3 represents Plant i's capacity constraint for the first period.[8] Plant i's capacity is explicitly constrained by the production plan of its predecessor. This yields an upper bound on the first-period production. For Plant 1, the constraint is simply $q_{11} \leq \alpha_1 e_{11}$.

The volume of water stored in the reservoir during the first period $e_{i2} - (\frac{q_{i1}}{\alpha_i} - \frac{q_{(i-1)1}}{\alpha_{i-1}})$ is used to produce electricity in the second period (Recall that no water is wasted). This volume is bounded by the reservoir capacity denoted $\bar{s}_i$. In terms of first-period production, this storage constraint writes:

$$q_{i1} \geq \alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}}{\alpha_{i-1}}). \tag{4}$$

Constraint 4 tells us that what is not produced in the first period must be stored. This yields a lower bound on the first-period production.

A *production plan* is any vector $q = (q_1, q_2) \in R^{2n+}$, where $q_t = (q_{1t}, ..., q_{it}, ..., q_{nt})$ for each period $t = 1, 2$, which satisfies equations 1, 3, 4 for every $i \in N$.

3 First Best

In order to better measure the impact on welfare, a benchmark case is developed. A first best allocation of electricity production is a production plan q^* which maximizes the total utility

[7]As such, decisions made by upstream plants define the strategy space of downstream plants. This characteristic of the problem is discussed further in the text.

[8]Note that it is implicitly assumed that Plant i possesses the technology (i.e. the turbines) to use this quantity of water. As such, there is no upper limit on the production capacity other than 3.

$u_1(Q_1) + u_2(Q_2)$. Since constraint 1 determines the second-period production levels q^*_{i2}, only the first-period production levels q^*_{i1} need to be characterized. A first-best production plan can be found by solving,

$$\begin{array}{lll} \max_{q_1} & u_1(\sum_{i\in N} q_{i1}) + u_2(Q - \sum_{i\in N} q_{i1}) & \text{subject to} \\ & q_{i1} \leq \alpha_i(e_{i1} + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \\ & q_{i1} \geq \alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \text{for every } i \in N. \end{array}$$

Denoting $\bar{\lambda}_i$ and $\underline{\lambda}_i$ the Langrangian multipliers associated with, respectively, the upper-bound constraint (the capacity constraint) and the lower-bound constraint (the storage constraint) on q_{i1}, the first order conditions yield,

$$\begin{aligned} u_1'(Q_1^*) - u_2'(Q_2^*) &= \bar{\lambda}_i - \underline{\lambda}_i, \\ \lambda_i(q^*_{i1} - \alpha_i(e_{i1} + \frac{q^*_{(i-1)1}}{\alpha_{i-1}})) &= 0, \\ \underline{\lambda}_i(\alpha_i(e_{i1} - \bar{s}_i + \frac{q^*_{(i-1)1}}{\alpha_{i-1}}) - q^*_{i1}) &= 0, \text{ for every } i \in N. \end{aligned}$$

Three separate cases need to be examined. First, if the capacity constraints are binding then $u_1'(Q_1^*) > u_2'(Q_2^*)$ and all plants produce up to their capacity (available water) in each period.[9] Consumption levels are $Q_1^* = \sum_{i\in N} \alpha_i \sum_{j\leq i} e_{j1}$ and $Q_2^* = \sum_{i\in N} \alpha_i \sum_{j\leq i} e_{j2}$. Second, if the storage constraints are binding, then $u_1'(Q_1^*) < u_2'(Q_2^*)$, and all plants store water up to their reservoir capacity. Consumption levels are $Q_1^* = \sum_{i\in N} \alpha_i \sum_{j\leq i}(e_{j1} - \bar{s}_j)$ and $Q_2^* = \sum_{i\in N} \alpha_i \sum_{j\leq i}(e_{j2} + \bar{s}_j)$. Third, suppose that neither constraint is binding. Then, we have

$$u_1'(Q_1^*) = u_2'(Q_2^*). \tag{5}$$

Equation 5 equalizes the marginal utilities of consumption for the two periods. By equalizing the marginal utilities (when feasible), the optimal allocation of electricity smoothes consumers' consumption. Writing 5 in terms of the demand functions yields:

$$P_1^* = P_2^*. \tag{6}$$

[9] In the first best solution, either all capacity constraints will be binding or none will be. The same is true for the storage constraints. Notice also that the strict concavity of the utility function excludes corner solutions.

Equal prices decentralize the first-best allocation. This efficient condition is illustrated in figure 1.

[Figure 1]

Figure 1 represents the demand functions for the two periods on the same graph. The first and second period prices are, respectively, on the left and the right axis. The length of the horizontal axis is equal to the total capacity of production $\bar{Q}$. Take any point on this axis. The distance from the left (right) represents the first (second) period consumption level. The first-best allocation divides the total production capacity at O, where the two demand functions cross. Prices are equal and more electricity is consumed during the peak period. This choice maximizes the consumers' total utility represented by the area under the curve AOB.[10] Any other division of Q induces a dead-weight loss of utility.

4 Monopoly

We now solve the monopolist's problem. A monopoly production plan q^m maximizes the monopoly's total profit $\Pi(Q_1, Q_2) = \pi_1(Q_1) + \pi_2(Q_2) = P_1(Q_1)Q_1 + P_2(Q_2)Q_2$. Once again, constraint 1 determines the second-period production levels q_{i2}^m. The monopoly production plan q^m can be found by solving,

$$\begin{array}{lll} \max_{q_1} & \pi_1(\sum_{i \in N} q_{i1}) + \pi_2(\bar{Q} - \sum_{i \in N} q_{i1}) & \text{subject to} \\ & q_{i1} \leq \alpha_i(e_{i1} + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \\ & q_{i1} \geq \alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \text{for every } i \in N. \end{array}$$

Denoting $\bar{\lambda}_i$ and $\underline{\lambda}_i$ the Langrangian multipliers associated with, respectively, the upper-bound constraint (the capacity constraint) and the lower-bound constraint (the storage constraint)

[10] Recall that the demand function of any level of electricity Q equals the consumer's marginal utility. Hence, the utility of consuming Q units of electricity equals the integral of the demand function up to Q. That is the surface of the area below the demand function from the corresponding origin (on the left if Q produced in the first period, on the right otherwise) to Q.

on q_{i1}, the first order conditions yield,

$$\begin{aligned} \pi_1'(Q_1^m) - \pi_2'(Q_2^m) &= \bar{\lambda}_i - \underline{\lambda}_i, \\ \bar{\lambda}_i(q_{i1}^m - \alpha_i(e_{i1} + \frac{q_{(i-1)1}^m}{\alpha_{i-1}})) &= 0, \\ \underline{\lambda}_i(\alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}^m}{\alpha_{i-1}}) - q_{i1}^m) &= 0, \text{ for every } i \in N. \end{aligned}$$

Three separate cases need to be examined. First, if the capacity constraints are binding then $\pi_1'(Q_1^m) > \pi_2'(Q_2^m)$, and all plants produce up to their capacity (available water) in each period. Consumption levels are $Q_1^m = \sum_{i \in N} \alpha_i \sum_{j \leq i} e_{j1}$ and $Q_2^m = \sum_{i \in N} \alpha_i \sum_{j \leq i} e_{j2}$. Second, if the storage constraints are binding, then $\pi_1'(Q_1^m) < \pi_2'(Q_2^m)$, and all plants store water up to their reservoir capacity. Consumption levels are $Q_1^m = \sum_{i \in N} \alpha_i \sum_{j \leq i}(e_{j1} - \bar{s}_j)$ and $Q_2^m = \sum_{i \in N} \alpha_i \sum_{j \leq i}(e_{j2} + \bar{s}_j)$. Third, suppose that neither constraint is binding. Then, we have,

$$\pi_1'(Q_1^m) = \pi_2'(Q_2^m). \tag{7}$$

Equation 7 equalizes marginal profits for the two periods. In terms of revenues, 7 yields:

$$P_1^{m\prime} Q_1^m + P_1^m = P_2^{m\prime} Q_2^m + P_2^m. \tag{8}$$

Equation 8 equalizes marginal revenues for the two periods. (Recall that costs are normalized to zero.) The result is illustrate in figure 2.

[Figure 2]

The dashed lines represent the marginal profits (or revenues, since costs are assumed zero). They cross at the point M which divides the total production capacity Q. To extract more surplus from consumers, the monopoly fixes the peak period price above the first-best price. Consumers incur a dead-weight loss represented by the shaded area.

5 Competition

Suppose now that each plant is owned by an individual firm, Firm i being the owner of plant i. In each period the n firms compete in quantity in a single electricity market. Consider the following game. At the beginning of the first period, the firms observe the future water supply e_{it} at every $i \in N$ and $t = 1, 2$. Then, at each stage $t = 1, 2$ of the game, firms simultaneously chose their production level $q_{it} \in R^+$. We derive the perfect subgame equilibria of this game.

In this game, the strategy of player $i = 1, 2$ is defined by a vector of production levels (q_{i1}, q_{i2}). As usual, (q_{-i1}, q_{-i2}) denotes the strategies of the other players and the sum of these strategies $Q_{-it} = \sum_{j \neq i} q_{jt}$ for every $t \in \{1, 2\}$. Player i's payoff is the sum of the two period profits $\pi_{it}(q_{it}, q_{-it})$ for $t = 1, 2$ where $\pi_{it}(q_{it}, q_{-it}) = P_t(q_{it} + Q_{-it})q_{it}$ for $i = 1, 2$, $t = 1, 2$.

However, because it is assumed that all water is used in the two periods[11], first period production entirely determines second period production. Hence, each firm has only one strategic decision variable, second period production being a residual. Therefore, using 1, the second-period equilibrium strategy for each firm $i \in N$ is defined by:

$$\hat{q}_{i2} = \bar{q}_i - q_{i1}, \tag{9}$$

During the first period, Firm i maximizes its two-period profit subject to its first period capacity and storage constraints and given the strategy Q_{-i1} of the other players' and the anticipated second-period best replies. Firm i's first-period program is thus:

$$\begin{array}{ll} \max_{q_{i1}} \; P_1(q_{i1} + Q_{-i1})q_{i1} + P_2(\hat{q}_{i2} + \hat{Q}_{-i2})\hat{q}_{i2} & \text{subject to} \\ q_{i1} \leq \alpha_i(e_{i1} + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \\ q_{i1} \geq \alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}}{\alpha_{i-1}}) & \text{for every } i \in N. \end{array}$$

Denoting $\bar{\lambda}_i$ and $\underline{\lambda}_i$ the Langrangian multipliers associated with, respectively, the upper-bound constraint (capacity) and the lower-bound constraint (storage) on q_{i1}, the first order conditions yield,

$$\begin{aligned} P_1^{c\prime} q_{i1}^c + P_1^c + \left[P_2^{c\prime} q_{i2}^c (1 + \frac{d\hat{Q}_{-i2}}{dq_{i2}}) + P_2^c \right] \frac{d\hat{q}_{i2}}{dq_{i1}} &= \bar{\lambda}_i - \underline{\lambda}_i, \\ \bar{\lambda}_i (q_{i1}^c - \alpha_i(e_{i1} + \frac{q_{(i-1)1}^c}{\alpha_{i-1}})) &= 0, \\ \underline{\lambda}_i (\alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}^c}{\alpha_{i-1}}) - q_{i1}^c) &= 0 \;\; \text{for every } i \in N. \end{aligned}$$

[11] Clearly, this is the case in a subgame Nash equilibrium: as long as marginal profits are positive in the second period, each firm will produce up to its capacity.

Once again, three separate cases need to be examined. First, if the capacity constraint is binding then $q^c_{i1} = \alpha_i(e_{i1} + \frac{q^c_{(i-1)1}}{\alpha_{i-1}})$ and $q^c_{i2} = \alpha_i(e_{i2} + \frac{q^c_{(i-1)2}}{\alpha_{i-1}})$. Second, if the storage constraint is binding, then $q^c_{i1} = \alpha_i(e_{i1} - \bar{s}_i + \frac{q^c_{(i-1)1}}{\alpha_{i-1}})$ and $q^c_{i2} = \alpha_i(e_{i2} + \bar{s}_i + \frac{q^c_{(i-1)2}}{\alpha_{i-1}})$. Third, suppose that neither constraint is binding. Then, we have $\bar{\lambda}_i = \underline{\lambda}_i = 0$. Furthermore, 9 implies $\frac{d\hat{q}_{i2}}{dq_{i1}} = -1$ and $\frac{d\hat{Q}_{-i2}}{dq_{i2}} = 0$. Hence, the first order conditions imply,

$$P_1^{c\prime} q^c_{i1} + P^c_1 = P_2^{c\prime} q^c_{i2} + P_2. \tag{10}$$

Equation 10 equalizes Firm i's marginal revenues (or marginal profits) for the two periods. Sum up 10 for all firms $i \in N$ to obtain:

$$P_1^{c\prime} Q^c_1 + nP^c_1 = P_2^{c\prime} Q^c_2 + nP^c_2. \tag{11}$$

Equation 11 is an aggregated equilibrium condition. It allows us to compare the competitive outcome with the monopoly and first best outcomes. It is interesting to note that the monopoly condition of equality of marginal receipts as well as the optimal condition of equality of prices appear as special cases of 11. Indeed fixing $n = 1$ yields 8. Dividing 11 by n and increasing n to infinity (i.e. perfect competition) yields 6.

6 Comparison of the first-best, the monopoly and the competitive outcomes

In this economy consumers' utility measures welfare. The comparison between outcomes, in terms of welfare, is simplified through the use of the following lemma.

Lemma *For any two production plans q^a and q^b, let l be such that $Q^a_l > Q^b_l \geq Q^*_l$. Then $U(Q^b) > U(Q^a)$.*

The lemma tells us that to compare any two outcomes, we need only rank their corresponding consumption levels with the efficient levels. Indeed, if l is such that $Q^a_l > Q^b_l \geq Q^*_l$, then, because all the production capacity is used, $h \neq l$ must be such that $Q_h^a < Q_h^b \leq Q_h^*$. Hence, moving from one outcome to another increases welfare if the consumption levels get closer to the optimal one.

The lemma is equivalent to saying that for any production plans q^a and q^b such that $P_h^a > P_h^b \geq P_h^* = P_l^* \geq P_l^b > P_l^a$, then $U(Q^b) > U(Q^a)$. It implies that, for any production plans q^a and q^b such that $|P_1^a - P_2^a| > |P_1^b - P_2^b|$, then $U(Q^b) > U(Q^a)$. In other words, a production plan decreases the welfare if the distance between the equilibrium prices (or the marginal utilities) increases. As a consequence, the distance between prices is a measure of the welfare loss.

We now state sufficient conditions for which a competitive outcome dominates the monopoly outcome.

Proposition *If neither constraint is binding, then competition dominates monopoly.*

When firms' capacity and storage are large enough, competition increases total welfare compared to a private monopoly situation. The result is intuitive. The optimal allocation of electricity equalizes prices or marginal utilities in the two periods. When the demand is heterogenous[12], the monopoly increases the distance between prices. Competition decreases this distance. At the limit, perfect competition leads to equality between prices and therefore to optimality.

Figure 3 illustrates the result for the case of linear demand functions $P_t(Q_t) = a_t - b_t Q_t$ for $t = 1, 2$. In this case, 11 becomes:

$$a_1 - (1 + \frac{1}{n}) b_1 Q_1^c = a_2 - (1 + \frac{1}{n}) b_2 Q_2^c. \tag{12}$$

Each side of 12 is the equation of a dotted line represented in Figure 3.

[Figure 3]

[12]Note that, if the demand is the same in the two periods, the monopoly outcome divides the total production capacity equally between the two periods and is therefore optimal.

The left-hand (right-hand) side of 12 is drawn from the left hand (right-hand) axis in figure 3. Each line lies between the demand function ($P_t(Q_t) = a_t - b_tQ_t$) (solid line) and the monopoly marginal profit function ($P_t'(Q_t) = a_t - 2b_tQ_t$) (dashed line). The competitive outcome splits total capacity Q where the two dotted lines cross, namely point C. Since C is located between O and M with respect to the horizontal axis, the competitive outcome increases total welfare compared to the monopoly outcome. Moreover, as n increases the slope of each line decreases and therefore C gets closer to O.

In this example of linear demand, it is easy to compute the competitive unconstrained production plan q^c:

$$q_{i1}^c = \frac{1}{1+n}\frac{a_1 - a_2}{b_1 + b_2} + \frac{b_2}{b_1 + b_2}\bar{q}_i,$$

and,

$$q_{i2}^c = \frac{1}{1+n}\frac{a_2 - a_1}{b_1 + b_2} + \frac{b_1}{b_1 + b_2}\bar{q}_i,$$

for every plant $i \in N$. This outcome arises when parameters are such that $q_{i1}^c \leq \alpha_i(e_{i1} + \frac{q_{(i-1)1}^c}{\alpha_{i-1}})$ and $q_{i1}^c \geq \alpha_i(e_{i1} - \bar{s}_i + \frac{q_{(i-1)1}^c}{\alpha_{i-1}})$ for every $i \in N$.

In the next section, we show that the proposition is not necessarily true if one firm is constrained, i.e. competition can decrease the welfare.

7 Analysis of particular configurations in a duopolistic competition

As shown in the previous section, in the unconstrained case the introduction of competition unambiguously increases welfare with respect to the monopoly allocation. This section focuses attention on particular configurations where, because of coordination failure, competition may in fact reduce welfare (always with respect to the monopoly allocation). We restrict attention to the duopoly case.

In order to highlight the results of interest, the following parameters values are used:

- $a_2 > a_1$ (period 1 is off-peak and period 2 is peak);
- $b_1 = b_2 = b$ (both demands have the same slope);
- $e_{12} = e_{22} = 0$ (all water inflows occur in period 1, the off-peak).

The above parameters conceptually fit the situation in many hydroelectric systems (such as Québec, Manitoba, British Columbia and the Pacific Northwest) where water inflows arrive (principally) in the off-peak season (spring-summer).

For these parameters, where most water arrives in period 1 while demand is higher in period 2, a first best production plan requires storage of water in the reservoirs from period 1 to period 2. In this sense, the first best is obtained through "perfect" coordination and use of storage. Note that the first best is only actually attained if the physical configuration of the river system permits sufficient storage.

The monopolist also benefits from "perfect" coordination and use of storage. However, since the monopolist's objective differs from that of the consumers', the former's allocation of production over the two periods differs from the first best. Specifically, for the above parameters, as illustrated in Figure 3, the monopolist produces more in the first period than the first best, i.e. $Q_1^m > Q_1^*$.

The allocations attained in the first best and the monopoly outcomes use system storage to shift water from period 1 to period 2. In both of these cases coordination implies combined or coordinated use of the reservoir capacity of both plants. Should one of these reservoirs be too small, coordination allows the planner to use capacity at the other reservoir in order to achieve the global objective (first best or monopoly). Competition, on the other hand, decouples the decisions on utilization of the two reservoirs. If one reservoir is too small, limiting one firm's actions, it is not necessarily in the interest of the other firm to modify its reservoir operation as a result. Because of this coordination failure, first period total production may increase vis-à-vis the monopoly case, resulting in decreased welfare. The following examples illustrate conditions under which the above occurs. (This is obviously not an exhaustive list of cases.)

7.1 Upstream storage constraint

In this case, at equilibrium Firm 1 is assumed constrained by its storage capacity whereas Firm 2 is not. In other words, given system parameters and Firm 2's decision, Firm 1 is unable to store as much water (for use in the second period) as it would want to. This occurs when $\bar{s}_1 < \frac{e_{11}}{2} + \frac{a_2 - a_1}{3\alpha_1 b}$. Clearly Firm 1 will be storage constrained when its storage capacity is "small" relative to the difference of inflows of water and of peak and off-peak demands. However, hitting the constraint does not necessarily lead to a decrease in welfare. As long as $\bar{s}_1$ is not "too small", i.e. $\bar{s}_1 \geq \frac{e_{11}}{2}$, the benefits of competition on the electricity market compensate for the coordination costs. When $\bar{s}_1 < \frac{e_{11}}{2}$, competition is unambiguously worse than monopoly for consumers because of the high coordination cost.

This situation is illustrated in figure 4 in the limit case of no storage capacity, i.e. $\bar{s}_1 = 0$.

[Figure 4]

Firm 1 produces electricity only in the first period. Its production level is $q_{11}^c = \alpha_1 e_{11}$ units. Firm 2 is thus able to act as a monopoly on the first period residual demand and on the entire second period demand. Equalizing marginal revenues on the demands it faces, it splits its total capacity $\bar{q}_2$ at C'. As compared to monopoly, more production is assigned to the first period, where competition is stronger, and less to the second period. This move goes against consumers' taste and therefore diminishes welfare.

7.2 Downstream storage constraint and Cournot competition

Now Firm 2 is assumed to be constrained by its storage capacity. In other words, given system parameters and Firm 1's decision, Firm 2 is unable to store as much water (for use in the second period) as it would want to. The key difference with the previous case (upstream storage constraint) is that Firm 2 turbines water inflows released by Firm 1. This renders the analysis more complex. To keep it tractable, we assume that one unit of water yields one unit of electricity at each power plant, that is $\alpha_1 = \alpha_2 = 1$. Firm 2 is constrained by its storage capacity if $\bar{s}_2 < \frac{e_{11} + e_{21}}{2}$. However, as long as $\bar{s}_2 \geq \frac{e_{11} + e_{21}}{2} - \frac{a_2 - a_1}{4b}$, the benefit of competition compensates for the lack of coordination. If $\bar{s}_2 < \frac{e_{11}+e_{21}}{2} - \frac{a_2 - a_1}{4b}$, the coordination costs are so high that the competitive outcome is worse than the monopoly outcome.

7.3 Downstream storage constraint and Stackelberg competition

In the downstream storage constraint configuration, the quantity of water released by the upstream firm determines the production of the downstream firm. If Firm 1 increases its production by one unit, then Firm 2's production increases by $\frac{\alpha_2}{\alpha_1}$ units during the same period. In this sense, Firm 1 can (locally) manipulate Firm 2's production. In the previous section, we assumed that Firm 1 acts myopically, taking Firm 2's production as given. In this section, we suppose that Firm 1 takes into account Firm 2's automatic response to its strategy. In other words, Firm 1 acts as a Stackelberg leader. This sophisticated behavior makes sense in particular when Firm 2 has no storage capacity at all and relies entirely on Firm 1's water releases.

Firm 1's first order condition becomes:

$$P_1^{c\prime}(1+\frac{\alpha_2}{\alpha_1})q_{11}^c + P_1^c = P_2^{c\prime}(1+\frac{\alpha_2}{\alpha_1})q_{12}^c + P_2. \tag{13}$$

Unlike in the previous section, Firm 1 now realizes that it suffers from its own competition on the electricity market. For each additional unit produced by Firm 1, total production increases by $1+\frac{\alpha_2}{\alpha_1}$ units. Acting as a monopoly on its residual demand, Firm 1 equalizes marginal revenues in 13. In this case, competition decreases the welfare if $s_2 < \frac{e_{21}}{4}$. This condition on storage capacity is less stringent that in the Cournot case, which is not surprising. Precisely because Firm 1 internalizes the effect that its own water releases have on the demand, through Firm 2's production, Firm 1 increases coordination.

8 Policy implications

What are the policy implications of the above analysis? First of all, the analysis demonstrates that competition need not necessarily increase total welfare. More importantly, it suggests caution in the manner in which competition might be implemented. For instance, in many existing hydro systems there is some potential for increasing water flows to some plants along a river by diverting neighboring rivers.[13] A priori, increasing water flows to the constrained plant might be looked on favorably by a regulator seeking to "level" the playing field. The two cases presented above suggest caution. Increasing water flows to the plant which is already constrained would only increase the production in the first period, thus exacerbating the divergence from the first best.[14] A regulator interested in increasing total welfare, in the presence of competition, would want to first increase storage capacity at the constrained plant before increasing water flows to the plant in question.

A similar recommendation applies to any regulation of water management on a river. Fixing minimum and maximum levels of water release along the river, which is often an important aspect of environmental regulation (e.g. Edwards and Al. 1999), would add constraints to the corresponding firm's program. This may have a non-trivial effect on competition, the resulting outcome and, finally, on welfare. This strategic effect should be taken into account when evaluating the social benefit of such regulation. Our model provides a simple and realistic framework for this purpose.

Finally, one can argue that regulating competition could restore the efficiency of the competitive outcome. Indeed, traditional responses to externalities are Pigouvian taxes or the creation of markets. In our model taxing electricity during one period can actually shift production to the other period and therefore move the production closer to consumers' tastes. However, this argument applies in the monopoly outcome as well: a tax on electricity during one period can induce the private monopoly to produce the first best outcome. Therefore, when a suitable regulation can be designed and implemented, decentralization of hydroelectric production is not an issue. It arises on the policy agenda precisely because of regulatory

[13] This is certainly the case in Québec. Expansion plans for the Churchill Falls hydroelectric complex in Labrador, which have been the subject of negotiations between Québec and Newfoundland for several years, have included diversion projects to increase water flow to existing plants.

[14] Of course a more complete analysis of the welfare implications would be necessary since the total amount of water available over the two periods would increase.

failure. With respect to the creation of a market, it is far from clear that a market for water will restore efficiency. Hydroelectric production is a non-consumptive use of water which has public good characteristics (what is turbined by firm i is freely available to firm $i+1$). It is well known that, in this case, competitive markets fail to provide an efficient allocation.[15]

9 Empirical example

As mentioned in the introduction, the electricity system in the province of Québec presents an ideal test case for the proposed model. Hydro-Québec, the government owned electric utility, owns and controls all of the segments of the vertically integrated provincial electric utility. While 94% of the province's electricity production capacity is hydro, 74% of this capacity is situated in 3 major river systems. Clearly, any move to a more competitive electricity system would necessarily need to address the issue of market power in the generation segment, leading to the central issue of this paper, decentralized control of hydro generation along a river.[16]

The following example provides an illustration of the application of the model, rather than an attempt to actually analyze decentralization in Québec. The latter would require a full model of Québec's electricity system and clearly goes beyond the scope and ambition of the current paper.

The La Grande River in northern Québec currently has 8 hydro-electric generating stations with total production capacity of 15,200 MW. The key data input for the empirical example is the hydrological data. Three different hydrological data sets were available. The data was collected during three periods, 1950-1979, 1942-1987 and 1950-1995, and at different locations along the river.[17] We limit our analysis to three generation stations with locations that were common to all three data sets: LG-4, LG-3 and LG-2 (from upstream to downstream). Table

[15]Notice that even in the presence of consumptive use of water, it is not so easy to create a market for water as a solution to externalities. In a model of water extraction from a river, Ambec and Sprumont have stressed how assigning property rights on water can be complex (Ambec and Sprumont, 2000).

[16]Note that at the present time there is no push within Québec for major restructuring of the electricity industry.

[17]The data is explained in more detail in Lemieux (2001).

1 describes the production capacity (represented by $\bar{q}_i$), the head (proportional to α_i) and the reservoir capacity ($\bar{s}_i$ in the model) of each power plant. [18]

[Table 1]

The water flow data for the river is "collapsed" upstream and downstream of the three stations of interest. In spite of the differences in the data sets, they provide relatively similar results as is seen in the analysis that follows.

In order to apply the paper's methodology to the data, the water flows are separated into peak and off-peak periods. The peak period is defined as November-April, when 58% of the annual energy load is produced. We assume that for the individual generation stations this intertemporal allocation between the peak and off-peak periods remains unchanged under (unconstrained) competition. This allows us to avoid estimation of the demand, which would be of limited interest for the simple example presented here.[19] For each period and for each of the 3 data sets 3 different levels of water flows are calculated: low-flow (L), average (A) and high-flow (H). The high-flow (resp. low-flow) represents the flow where the average is taken over monthly maximum (resp. minimum) flows for each data set. Using 3 levels of water flows is obviously important in order to test the constraints of the system.

The results are presented in two tables where (-) indicates that the model constraint is question is satisfied (non-binding) while (B) indicates that the model constraint is question is binding.

[Tables 2 and 3]

We now briefly discuss the results of this empirical example. First, it is important to remind the reader of the proper interpretation to be given to the constraints. If the constraints

[18] Information on Hydro- Québec's production units is available at http://www.hydroquebec.com/generation/.

[19] Or, put differently, we assume that the demand belongs to a class of demand functions which includes, for instance, $P_t(Q_t) = a - b_t(Q_t)$ for $b_1 \neq b_2$.

are non-binding (-) then the competitive outcome dominates the monopolistic outcome. Conversely, a binding constraint (B) indicates that the monopolistic outcome may (though need not necessarily) dominate the competitive outcome.

The capacity constraints are always binding for the low-flow data and never binding for the high-flow data, which is certainly consistent with the expected results of the model. With low flows of water, which in practice can be due to lower than average rainfall, environmental constraints on water release, etc. it is not surprising that competition could lead to reduced total welfare. The model confirm this.

The storage constraints on the other hand are binding only for the LG-4 station (which has the lowest reservoir capacity) and here only in the high-flow case. This suggests that, for the case examined here, storage capacity does not appear to be an impediment to the introduction of competition (or at the very least is much less problematic than is production capacity). Given the nature of the La Grande River system, a very large inter-annual hydro complex with huge reservoirs, it should come as no surprise that the capacity constraints would bind more than the storage constraints. The reverse would likely occur for run of the river type systems.

The above empirical example clearly illustrates the relevance and applicability of our framework. The model clearly can be made operational and be used to understand an important aspect of restructuring of hydro systems. Under our assumptions, we found little evidence of binding constraints with average flow levels, suggesting that competition would not lead to reduced total welfare. But this is not necessarily true with extreme flows of water. However, given the simplifying assumptions employed here, the analysis, though interesting, is certainly not sufficiently detailed to base an opinion on restructuring of Québec's electricity industry.

10 Conclusion

The model developed in this paper suggests how certain key characteristics of hydroelectric systems influence the welfare impacts of different market organizations. The central observation is that the tradeoff involved in introducing competition, namely the benefit from the reduction of monopoly power versus the cost of reduced coordination, can be significant in hydroelectric systems which are generally designed and built to take advantage of coordination of storage capacity. In the case where plants are not constrained, competition unambiguously increases welfare with respect to monopoly, as expected. However, when one of the plants becomes constrained in the competitive scenario, the welfare loss due to the lack of coordination in the use of storage capacity may result in a welfare loss with respect to monopoly.

We now conclude with a last comment. Beyond the case of the hydroelectric industry, our paper sheds new light on deregulation of network industries. Railroad, telecommunications and electricity share a common feature: local congestion of the network during the peak periods may limit some operators' production capacity (through their physical link to the market) while other operators remain unconstrained. This can affect the level or type of competition in the market. Likewise, our model clearly shows that because capacity constraints affect strategic competition between firms, the allocation of production between peak and off-peak periods can be even worse under competition than with a private monopoly. Policy makers should be aware of this effect when decentralizing production in network industries.

A Proof of the lemma

Since $Q_l^a = \bar{Q} - Q_h^a$ and $Q_l^b = \bar{Q} - Q_h^b$, then $Q_l^a > Q_l^b \geq Q_l^*$ implies $Q_h^* \geq Q_h^b > Q_h^a$. Strict concavity of u_t for $t = l, h$ implies:

$$u_l(Q_l^a) - u_l(Q_l^b) < u_l'(Q_l^b)(Q_l^a - Q_l^b)$$

and,

$$u_h(Q_h^b) - u_h(Q_h^a) > u_h'(Q_h^b)(Q_h^b - Q_h^a)$$

Which in turn imply:

$$u_l(Q_l^b) + u_h(Q_h^b) > u_l(Q_l^a) + u_h(Q_h^a) + u_l'(Q_l^b)(Q_l^b - Q_l^a) + u_h'(Q_h^b)(Q_h^b - Q_h^a) \tag{14}$$

By assumption, $Q_l^b \geq Q_l^*$ and $Q_h^* \geq Q_h^b$, therefore $u_l'(Q_l^b) \leq u_l'(Q_l^*) = u_h'(Q_h^*) \leq u_h'(Q_h^b)$. Combining with 14 yields:

$$U(Q^b) > U(Q^a) + u_l'(Q_l^b)[Q_l^b + Q_h^b - (Q_l^a + Q_h^a)]$$

Or, since $Q_l^b + Q_h^b = Q_l^a + Q_h^a = \bar{Q}$,

$$U(Q^b) > U(Q^a)$$

B Proof of the proposition

Before beginning the proof, it is convenient to rewrite 11 as

$$\pi_1'(Q_1^c) - \pi_2'(Q_2^c) = (n-1)(P_2^c - P_1^c). \tag{15}$$

In order to prove the proposition, all we need to show is that under the specified conditions $\exists l \in \{1,2\}$ such that $Q_l^m \geq Q_l^c \geq Q_l^*$. Fix, without loss of generality, $l = 1$ and $h = 2$. Now suppose that the above is not true. Suppose first that $Q_1^c > Q_1^m \geq Q_1^*$. This implies $Q_2^* \geq Q_2^m > Q_2^c$ by 2. We claim that the monopoly can increase its profit by producing q^c rather than q^m, contradicting the assertion that q^m is a monopoly production plan. Denote $\Delta\Pi = \Pi(Q_1^c, Q_2^c) - \Pi(Q_1^m, Q_2^m)$. We show that $\Delta\Pi > 0$.

$$\Delta\Pi = \pi_1(Q_1^c) - \pi_1(Q_1^m) + \pi_2(Q_2^c) - \pi_2(Q_2^m). \tag{16}$$

Now, since π_t is strictly concave and increasing in $[0, \bar{Q}]$, we have for every a and b such that $\bar{Q} \geq b > a > 0$,

$$\pi_t(b) - \pi_t(a) < \pi_t'(a)(b-a) \quad \text{and} \quad \pi_t(b) - \pi_t(a) > \pi_t'(b)(b-a)$$

Hence, $Q_1^c > Q_1^m$ and $Q_2^m > Q_2^c$ imply,

$$\begin{aligned}
\pi_1(Q_1^c) - \pi_1(Q_1^m) &> \pi_1'(Q_1^c)(Q_1^c - Q_1^m), \\
\pi_2(Q_2^m) - \pi_2(Q_2^c) &< \pi_2'(Q_2^c)(Q_2^m - Q_2^c).
\end{aligned}$$

The last two inequality and 16 imply:

$$\Delta\Pi > \pi_1'(Q_1^c)(Q_1^c - Q_1^m) - \pi_2'(Q_2^c)(Q_2^m - Q_2^c). \tag{17}$$

Let us define $\epsilon = Q_1^c - Q_1^m = Q_2^m - Q_2^c > 0$ (Remember that $Q_1^c + Q_2^c = Q_1^m + Q_2^m = \bar{Q}$). Substitute in 15 and use 17 to obtain:

$$\Delta\Pi \geq (n-1)(P_2^c - P_1^c)\epsilon. \tag{18}$$

By assumption, the consumption levels are such that $P_2^c > P_2^* = P_1^* > P_1^c > 0$. Moreover, since $n > 1$, then the left-hand side of 18 is strictly positive.

Suppose now that $Q_1^m > Q_1^* > Q_1^c$. This implies $Q_2^c > Q_2^* > Q_2^m$. Then prices and profits are respectively ranked as follow: $P_1^c > P_1^* = P_2^* > P_2^c > 0$ and $\pi_1'(Q_1^c) > \pi_1'(Q_1^m) = \pi_2'(Q_2^m) > \pi_2'(Q_2^c) > 0$. Therefore, $P_2^c - P_1^c < 0$ while $\pi_1'(Q_1^c) - \pi_2'(Q_2^c) > 0$ which contradict 15. A similar proof contradicts $Q_1^c > Q_1^* > Q_1^m$.

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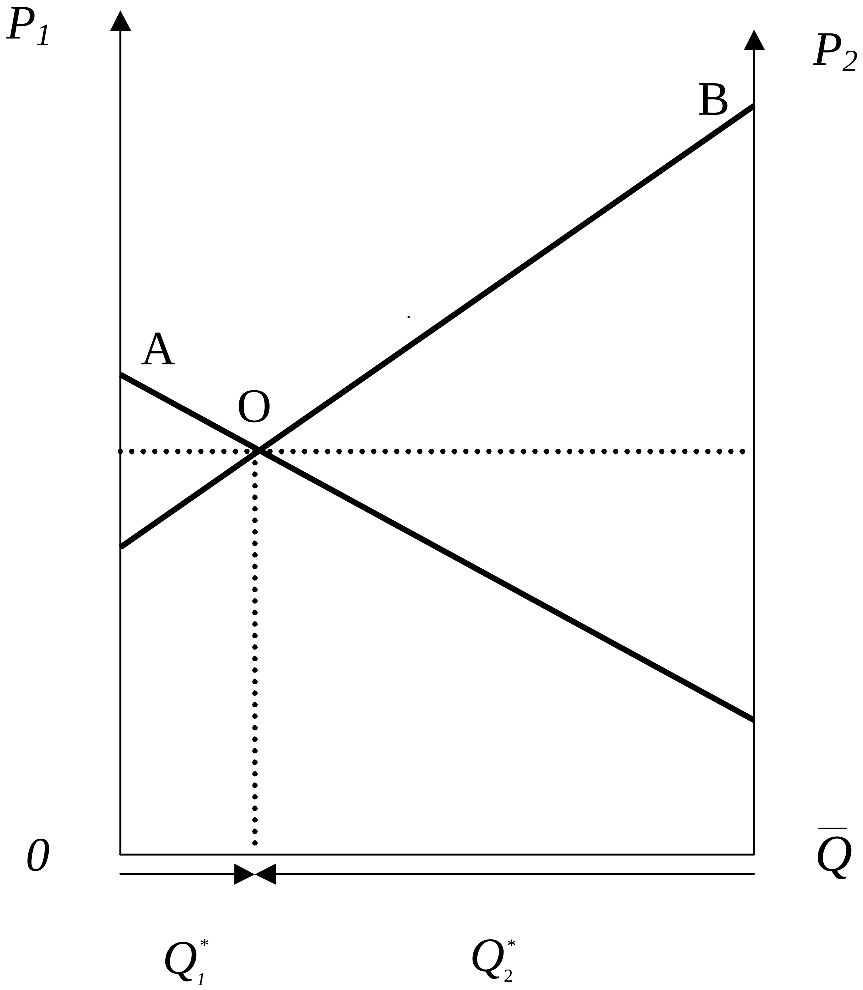

Figure 1

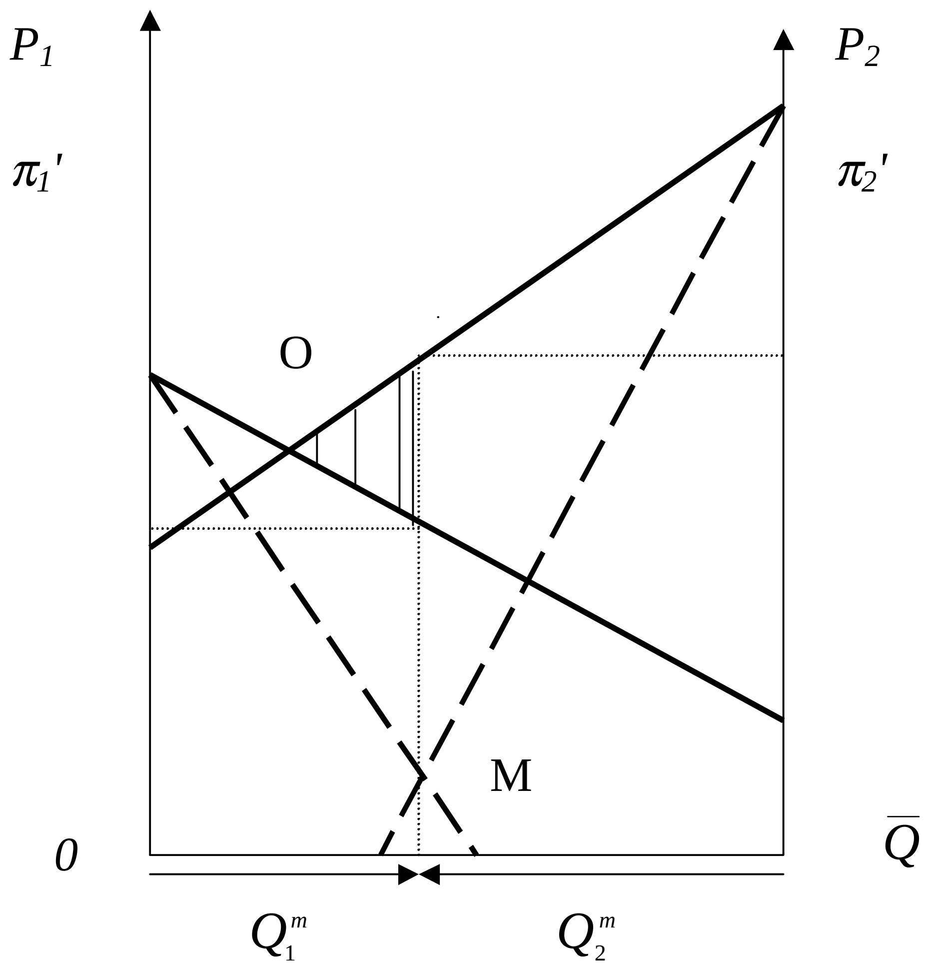

Figure 2

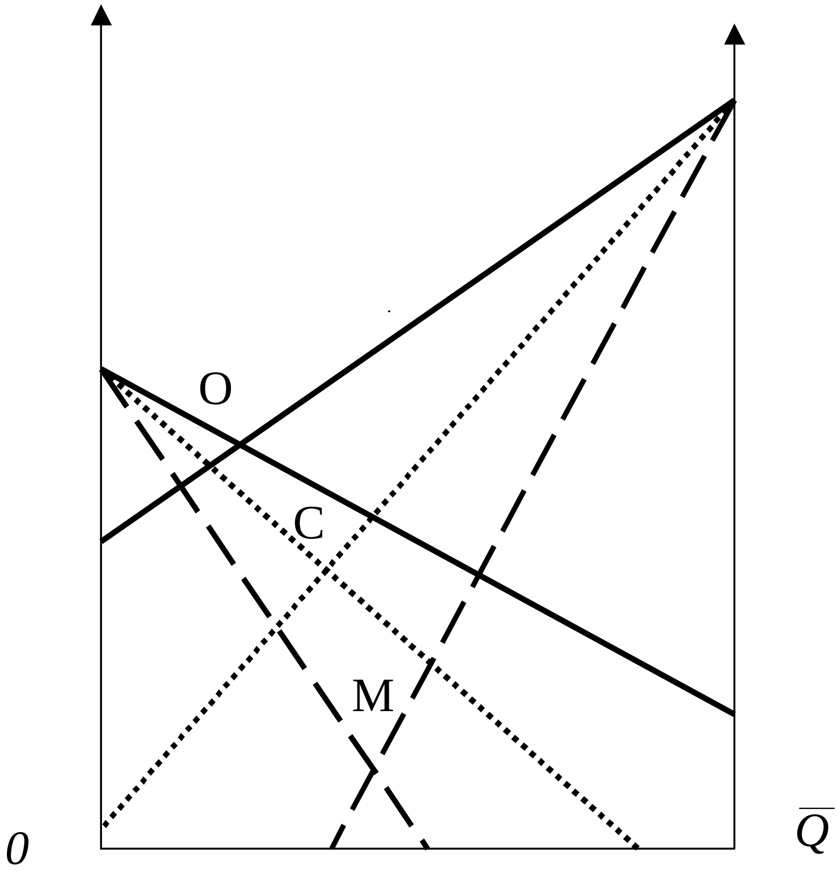

Figure 3

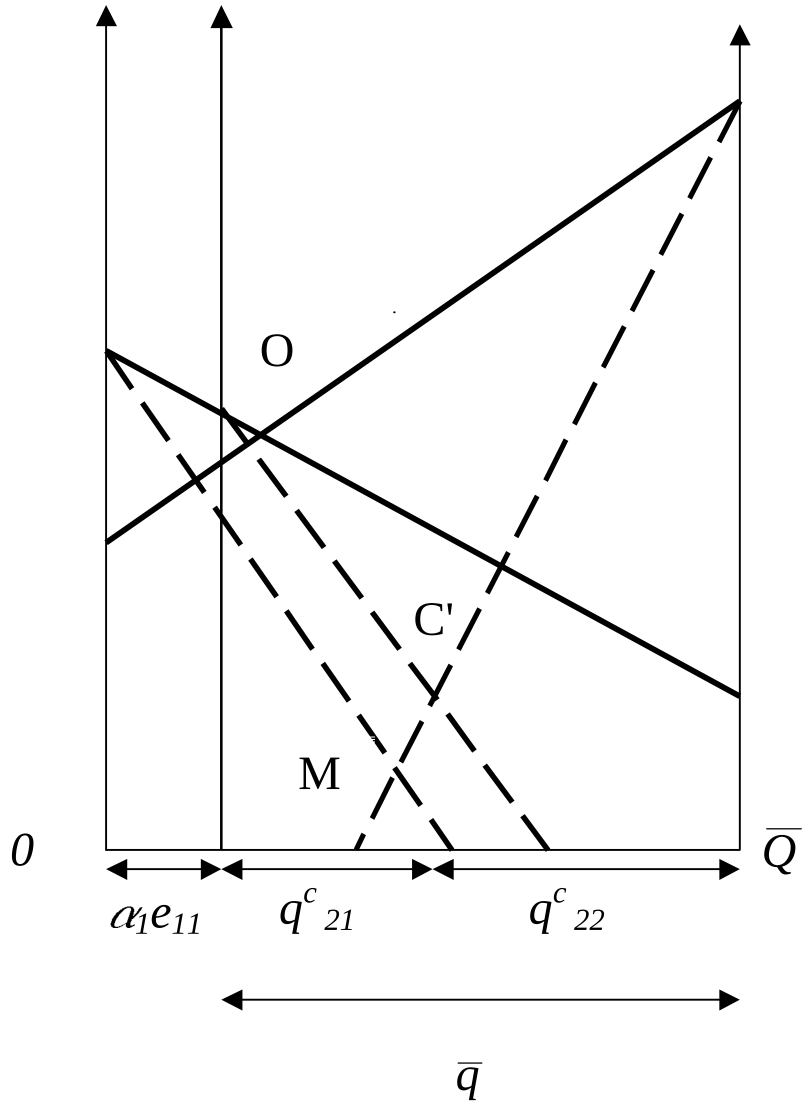

Figure 4

Power Plant	LG4	LG3	LG2
Installed capacity (MW)	2,650	2,304	7,326
Head (m)	117	79	137
Reservoir (10^9 m^3)	7.16	25.2	19.4

Table 1: Technological parameters

Power Plant	LG4			LG3			LG2		
Water Flow Level	L	A	H	L	A	H	L	A	H
Data set 1	B	-	-	B	-	-	B	-	-
Data set 2	B	-	-	B	-	-	B	B	-
Data set 3	B	-	-	B	-	-	B	B	-

Table 2: Capacity constraints

Power Plant	LG4			LG3			LG2		
Water Flow Level	L	A	H	L	A	H	L	A	H
Data set 1	-	-	B	-	-	-	-	-	-
Data set 2	-	-	B	-	-	-	-	-	-
Data set 3	-	-	B	-	-	-	-	-	-

Table 3: Storage constraints