Centre for Studies in Economics and Finance

WORKING PAPER NO. 587

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November 2020

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ISSN 2240-9696

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Abstract
This paper studies the interplay between the investor's incentives to delegate her asset allocation choice to a biased financial advisor, and the advisor's decision to acquire information about multiple characteristics of the risky asset. We show that, to prevent unprofitable investments, the investor may delegate to the advisor imposing a cap on the amount of wealth that the advisor can invest. This cap (i) is decreasing in the magnitude of the conflict of interests between the investor and the advisor and (ii) may be lower when the advisor possesses more information. Interestingly, although the investor always prefers a more-informed advisor, the advisor may choose not to acquire full information, and reducing the conflict of interests with the investor may actually induce the advisor to acquire less information.

Keywords: Financial Advice, Asset-Allocation, Delegation, Information Acquisition.

JEL Classification: G11, G21, G51, D40, D82, D83.

Acknowledgements: We would like to thank Nicola Gennaioli, Marco Pagano, Lorenzo Pandolfi, Alessandro Previtero and the seminar participants to the 2020 Quantitative Finance Workshop (QFW).

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1 Introduction

For many individuals, asset allocation decisions represent a challenging problem. The widespread use of investment advisors is therefore not surprising. In the United States, more than 50% of households own mutual funds acquired through an investment professional (Investment Company Institute, 2013). Similarly, in Canada, nearly 50% of households report using financial advisors (The Investment Funds Institute of Canada, 2012), and roughly 80% of the total retail investment assets reside in advisor-directed accounts (Canadian Securities Administrators 2012). For Germany, this percentage is estimated at around 75% (Hackethal *et. al*, 2011).

Financial advisors, in fact, play a crucial role on households' wealth: they facilitate stock market participation and risk-taking (see, among others, Gennaioli, Shleifer, and Vishny, 2015, and Linnainma, Melzer, Previtero and Foester, 2019), help establish and meet retirement savings goals (Lusardi and Mitchell, 2011), and create tax-efficient asset allocations (Bergstresser and Poterba, 2004). Nevertheless, a common criticism to this industry is that the quality of advice is negatively affected by the conflict of interest with investors, raising the cost of financial advice. Since many advisors require no direct payment from their clients but instead obtain commissions on the mutual funds they sell, they may be tempted to recommend products that maximize commissions rather than serving their clients' interests.[1] As documented by Bergstresser, Chalmers, and Tufano (2009) and Foerster, Linnainmaa, Melzer, and Previtero (2017), in fact, advisors provide their clients negative 2% to 3% net returns (relative to passive benchmarks) after fees.

Although financial advice has been extensively studied in models in which the advisor's information regards one aspect or characteristics of the financial product adviced (see, e.g., Piccolo, Puopolo and Vasconcelos 2017), little is known when financial advisors are privately informed about multiple characteristics of the financial products. When the gap of information between informed advisors and investors is large and concerns multiple features of the financial assets, in fact, the advisors' bias towards certain products may increase, thus exacerbating the conflict of interests. In this sense, the existing financial advice models have neglected a salient aspect of the relationship between advisors and their clients.

This paper aims at filling this gap. More precisely, we investigate the households' incentives to delegate their asset allocation to financial advisors when the latters possess different levels of financial information. We consider a model in which an advisor can acquire information about multiple characteristics of a risky asset in which an investor may invest her wealth. Of course, this provides an incentive for the investor to delegate her portfolio decision to the advisor. However, there is a conflict of interests between the investor and the advisor since the latter always prefers to invest an amount of wealth that is larger than

[1] Policymakers in Australia, the United Kingdom, and the United States have responded to this problem by either banning commissions or mandating that advisors act as fiduciaries, placing clients' interests ahead of their own.

the investor's preferred one.[2] In this environment, we analyze: (i) the characteristics of the contract offered by the investor to the advisor and her delegation choice, and (ii) the advisor's choice of the level of information to acquire.

We show that the contract offered by the investor is characterized by a cap that limits the maximum amount of wealth that the advisor is allowed to invest in the risky asset: a higher cap represents more delegation to the advisor and a lower control by the investor over her portfolio. Specifically, the optimal choice of the cap has to balance a trade off for the investor, since a higher cap allows the advisor to choose an investment that better reflects both his superior information — which is positive for the investor — and his biased preferences — which is negative for the investor.

The optimal cap chosen by the investor depends on the information acquired by the advisor and the magnitude of the conflict of interests. We show that, for a given level of information, the optimal cap is always decreasing in the conflict of interests: as the difference between the preferences of the investor and the advisor increases, the investor prefers to reduce the advisor's discretionality, even if this induces the latter to invest the same amount of wealth regardless of his information about the characteristics of the risky asset.

In our setting, delegation is not always optimal for the investor. When the advisor acquires no information or when the conflict of interests is sufficiently high, in fact, the investor prefers to manage her portfolio directly rather than to delegate. By contrast, if the investor delegates, she always obtains a higher utility with an advisor who is more informed, regardless the magnitude of their conflict of interests. Interestingly, dealing with a more-informed advisor does not necessarily induce the investor to increase delegation by allowing more freedom to the advisor: the investor may actually impose a lower cap to an advisor who has more information. Specifically, this happens for intermediate values of the conflict on interests.

Finally, turning to the advisor's choice of the amount of information, we show that the advisor is willing to acquire costly information if and only if this induces the investor to delegate her investment decision, which occurs when the conflict of interests with the investor is not too high. Moreover, when deciding whether to acquire either partial or full information, the advisor prefers to acquire more costly and superior information if both the conflict of interests and the uncertainty about the riskiness of the asset are sufficiently low. Otherwise, the advisor prefers to acquire less information, taking also into account how this will affect the contract offered by the investor.

Our results suggest that a low conflict of interests between investors and financial advisors does not necessarily result in better informed advisors and, hence, does not necessarily guarantee that investment decisions are fully tailored to the actual characteristics of the risky assets. Perhaps surprisingly, reducing the conflict of interest may actually reduce the

[2] This conflict may arise, for example, because of private commissions received by the advisor or because of a lower degree of risk aversion of the advisor.

amount of information acquired by financial advisors and even an extremely low conflict of interests does not necessarily guarantee that advisors acquire the level of information preferred by investors. The reason is that the amount of information acquired by the advisor also affects the characteristics of the contract offered by the investor: dealing with a more informed advisor may actually induce the investor to offer a more restrictive contract that provides tighter limits to the advisor's choices, which in turn discourages the advisor from acquiring more information.

The relationship between investors and financial experts has received an increasing attention over the years. From a theoretical standpoint, earlier adverse-selection models have typically assumed that financial experts are more informed than their clients about the expected return and the riskiness of financial assets, and investigated the optimal transmission of information (e.g., Allen, 1990; Bhattacharya and Pfleiderer, 1985). By contrast, the moral-hazard approach has typically assumed that money managers choose the riskiness and/or the expected return of their client's portfolios, but that this choice is unobservable. Therefore, the investor has to optimally design a (second-best) contract that enforces the manager to choose the right action (e.g., Stoughton, 1993; Adamati and Pfleiderer, 1997; Palomino and Prat, 2003; Palomino and Uhlig, 2007). However, these earlier models of delegated portfolio management did not always provide a clear distinction between the role of portfolio managers and the one of financial advisors.

More recently, Bolton, Freixas and Shapiro (2007) investigate how competition between financial intermediaries affects information disclosure and conflicts of interests. Stoughton, Wu and Zechner (2011), instead, propose a model in which the role of financial advisers is separated from the one of portfolio managers, and investigate the effect of such financial intermediaries (and their compensation schemes) on investors' portfolio decisions, fund returns, management fees and welfare. In addition, Piccolo, Puopolo and Vasconcelos (2017) investigate the implications of non-exclusive financial advice, that is when the financial expert advises multiple clients, on investors' asset-allocation behavior and welfare.

Another important strand of literature has investigated the role of trust and financial advice for stock market participation. In this regard, Guiso, Sapienza, and Zingales (2008) show that lack of trust can lead to low stock market participation by households, whereas Georgarakos and Inderst (2011) highlight that trust in advice matters for investing in the stock market only in the case of households with low financial capability. Furthermore, Gennaioli, Shleifer, and Vishny (2015) provide a model of investors delegating portfolio management to professionals based on trust. In their model, even though financial advisors consistently underperform the market, they reduce households' anxiety and allow them to earn expected returns higher than they would by investing on their own.

From an empirical point of view, several papers have investigated the quality of financial advice. Bergstresser, Chalmers, and Tufano (2009) document significant underperformance of mutual funds sold exclusively by brokers or advisors.[3] Similarly, Hackethal, Haliassos and Jappelli (2012) find that financial advisors end up collecting more in fees and commissions than what they add to their clients' accounts.[4] By investigating the quality of advisor's recommendations using a field experiment, instead, Mullainathan, Noeth, and Schoar (2012) find evidence that advisors direct clients towards more expensive funds and encourage "return chasing". Interestingly, using unique data on Canadian households, Foerster, Linnainmaa, Melzer, and Previtero (2017) show that financial advisors exert substantial influence over their clients' asset allocation, delivering one-size-fits-all portfolios rather than tailoring them to meet their clients' characteristics. Moreover, they also show that, after fees, the average client underperforms passive benchmarks by 2% to 3% per year.

Linnainmaa, Melzer, and Previtero (2019) provide an alternative explanation of costly and low-quality advice based on advisor's misguided beliefs rather than on conflicts of interests. In fact, advisors recommend frequent trading, chasing returns, and expensive-actively managed funds because they believe these portfolios dominate passive benchmarking. More importantly, advisors own portfolios very similarly to those adviced to their clients. In a related paper, Linnainmaa, Melzer, Previtero and Foerster (2019) show that, using the length of the advisor-client relationship as a proxy for trust, financial advisors increase the investors' willingness to assume equity risk. However, conditional on stock market participation, investors' risk taking is not influenced by the presence of advisors. Finally, using a novel database containing the universe of financial advisors in the United States, Egan, Matvos, Seru (2018) document that more than 7% of advisors have been reprimanded for misconduct but only approximately half of them lose their jobs after misconduct.

The rest of the paper is organized as follows. Section 2 describes our model. In Section 3, we characterize the optimal contract offered by the investor depending on the information possessed by the advisor. Section 4 investigates how the investor's incentives to delegate depend on the advisor's information and on the conflict of interests, while Section 5 analyzes the advisor's choice of the amount of information to acquire. Finally, Section 6 concludes. All proofs are in the Appendix.

2 The Model

Players and Environment. Our economy features a (female) investor, I, endowed with initial wealth normalized to 1 without loss of generality. There are two financial assets in which I can invest her wealth. One asset is riskless — e.g., money market account, government bond, etc. — with net return equal to r_f. The other asset is risky — e.g., equity,

[3] French (2008) found that the average investor would have improved his performance by 0.67% per year by switching to a passive market portfolio between 1980 and 2006.

[4] Carlin and Manso (2011) show that when investors are uninformed, they may either pay excessive fees or invest in "bad assets". Hence, providers of financial products earn higher rents from investors who are unsophisticated and more uninformed.

common fund, etc. — and provides a stochastic net return $\widetilde{r}$ with mean $\mu \in \Omega \triangleq \{\underline{\mu}, \overline{\mu}\}$ and variance $\sigma^2 \in \Sigma \triangleq \{\underline{\sigma}^2, \overline{\sigma}^2\}$, where $\overline{\mu} > \underline{\mu}$ and $\overline{\sigma}^2 > \underline{\sigma}^2$. Accordingly, there are four possible distributions of $\widetilde{r}$ according to the realization of the pair $(\mu, \sigma^2) \in \Omega \times \Sigma$, which we refer to as the *states of nature*. We assume that μ and σ^2 are independent and that $\Pr[\mu = \overline{\mu}] = \Pr[\sigma^2 = \overline{\sigma}^2] = \frac{1}{2}$. Hence, for example, high risk may not necessarily imply high expected return. Finally, we assume that $\underline{\mu} > r_f$, so that the risk premium is always positive.[5]

Due for example to a lack of financial literacy or limited access to detailed information on the asset's characteristics, I cannot assess with certainty the mean μ and the variance σ^2. However, there is a (male) financial advisor, A, who can acquire information about the realizations of μ and/or σ^2. The investor can either delegate her asset allocation problem to A, to benefits from his superior knowledge of the financial market, or invest on her own — i.e., without financial advice. As we shall explain below, with delegation, the investor leaves A in full control over the actual portfolio composition under certain limits: an extremely simple form of delegated portfolio management. We assume that, when indifferent, the investor chooses to invest on her own.

Information Acquisition. The financial advisor can correctly assess the mean and the variance of the risky asset by acquiring information. We consider three different information acquisition policies (or level of information) denoted by d:

- Full information, $d = \mathcal{F}$: A acquires private information about both μ and σ^2, so that he is able to disentangle any realized pair (μ, σ^2).

- Partial information, $d = \mathcal{P}$: A acquires private information only about the mean of the risky asset, so that he is able to disentangle only the realizations of μ.[6]

- No information, $d = \varnothing$: A does not acquire any information and remains uninformed.

Gathering information is costly for the advisor. The fixed cost of the information acquisition policy d is denoted by $\kappa_d \geqslant 0$. We normalize $\kappa_\varnothing = 0$ and assume that $\kappa_\mathcal{F} > \kappa_\mathcal{P} > 0$, with $\Delta\kappa \triangleq \kappa_\mathcal{F} - \kappa_\mathcal{P}$.

Investor's Preferences. I is risk-averse with respect to wealth and exhibits mean-variance preferences. Specifically, given an amount α invested in the risky asset and the realized state of nature (μ, σ^2), the investor's utility is

$$\mathcal{U}\left(\alpha, \mu, \sigma^2\right) \triangleq \alpha\left(\mu - r_f\right) + r_f - \frac{\gamma}{2}\alpha^2\sigma^2,$$

[5] As explained later, this assumption ensures that, in our mean-variance framework, the investor never short-sells the risky asset.

[6] Typically, it is easier to evaluate the mean of the risky asset rather than its variance. In fact, estimates of the variance require estimates of the mean. For example, it may be very difficult to estimate the return volatility of private equity funds, since their net asset values are not known with regularity.

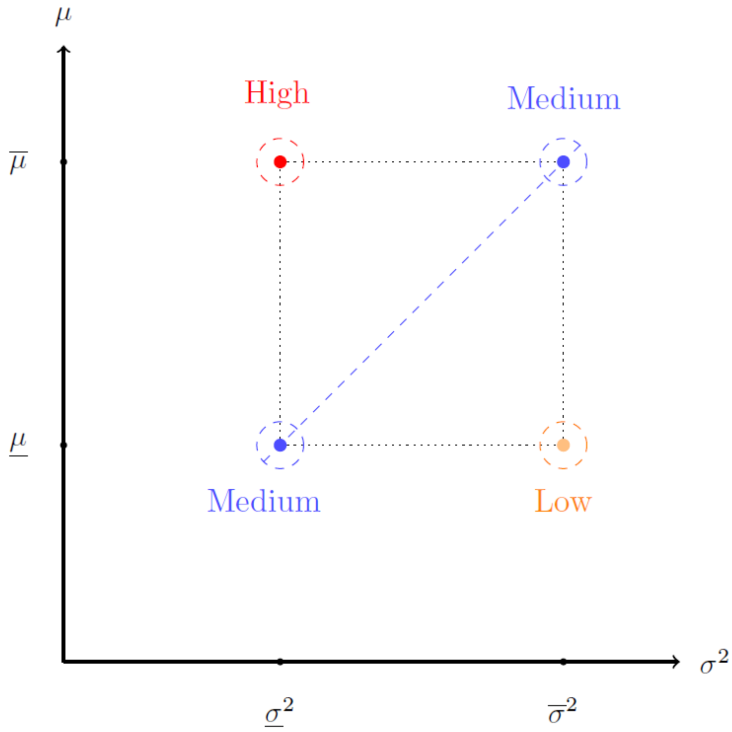


Figure 1: Investor's preferred allocation with full information.

where $\gamma > 0$ is the coefficient of risk aversion.

The investor's preferred investment maximizes her utility based on the level of information. Specifically, if I had access to the level of information d, her preferred investment is the standard mean-variance allocation that maximizes $\mathcal{U}$:

$$\alpha_d = \begin{cases} \alpha_{\mathcal{F}}\left(\mu, \sigma^2\right) \triangleq \frac{\mu - r_f}{\gamma\sigma^2} & \text{if } d = \mathcal{F}, \\ \alpha_{\mathcal{P}}\left(\mu\right) \triangleq \frac{\mu - r_f}{\gamma E[\sigma^2]} & \text{if } d = \mathcal{P}, \\ \alpha_{\varnothing} \triangleq \frac{E[\mu] - r_f}{\gamma E[\sigma^2]} & \text{if } d = \varnothing. \end{cases} \tag{1}$$

Hence, if I is fully informed about the state of nature, her preferred asset allocation is tailored to the realizations of both μ and σ^2, as in the canonical mean-variance portfolio problem. On the contrary, in the absence of full information, I's optimal investment is obtained by replacing the unknown parameter(s) in the mean-variance allocation with the corresponding expected value(s). This result stems from the linearity of the function $\mathcal{U}(\cdot)$ in the mean and the variance of the risky asset.

Notice that the investor's optimal allocations can be ordered as follows:

$$\alpha_{\mathcal{F}}\left(\overline{\mu}, \underline{\sigma}^2\right) > \alpha_{\mathcal{P}}\left(\overline{\mu}\right) > \alpha_{\varnothing} > \alpha_{\mathcal{P}}\left(\underline{\mu}\right) > \alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right).$$

Moreover, in order to simplify the analysis, we assume that $\alpha_{\mathcal{F}}\left(\underline{\mu}, \underline{\sigma}^2\right) = \alpha_{\mathcal{F}}\left(\overline{\mu}, \overline{\sigma}^2\right)$.[7] This assumption implies that, with full information, there are only three possible amounts that I wants to invest in the risky asset, depending on the state of nature: a High, a Medium, and a Low amount (see Figure 1).

Advisor's Preferences and Conflict of Interests. If the investor does not delegate her asset allocation, A obtains a negative utility normalized to $-K$, where K is sufficiently large for the advisor to always prefer delegation. By contrast, if I delegates and the advisor chooses the asset allocation α, an advisor with information level d obtains utility equal to

$$\mathcal{V}_d\left(\alpha\right) = -\left|\alpha - \left(1+\lambda\right)\alpha_d\right|. \tag{2}$$

In this linear loss specification, the advisor's preferred investment is $\left(1+\lambda\right)\alpha_d$.[8] This reflects an environment where, given his information acquisition policy d, A wants to invest an amount close to I's preferred one — for example, because of long-term reputation concerns — but where the preferences of the advisor and the investor are not perfectly aligned.

The parameter λ captures the conflict of interests between the advisor and the investor. Following the earlier literature (e.g., Inderst and Ottaviani, 2012, and Piccolo, Puopolo and Vasconcelos, 2017, among many others), we assume that $\lambda > 0$ which implies that A would like his client to overinvest in the risky asset (compared to I's actual preferences). Higher values of λ indicate a more self-serving advisor. This conflict of interest may arise, for example, because of commissions that the financial advisor receives for selling certain types of financial products. Alternatively, the preferred investment $\left(1+\lambda\right)\alpha_d$ may reflect an advisor characterized by mean-variance preferences and a degree of risk aversion which is lower than the investor's one.

Hence, with delegation, in order to maximize his utility (based on his information) A chooses the portfolio composition α that minimizes the weighted (linear) distance between the delegated portfolio composition and the investor's preferred allocation defined in (1). Finally, the advisor chooses an information acquisition policy in order to maximize his expected utility.

Contracts. We focus on a simple class of indirect mechanisms where the investor caps the amount of wealth that the advisor can invest in the risky asset to $\overline{\alpha}$, and then delegates her portfolio choice to A by letting him pick any allocation up to the cap $\overline{\alpha}$. We refer to this mechanism as *partial delegation*.[9] The investor chooses a mechanims to maximize her expected utility.

Restricting attention to this type of indirect mechanisms is without loss of generality in

[7] This also implies that $\alpha_{\mathcal{F}}\left(\underline{\mu}, \underline{\sigma}^2\right) = \alpha_{\mathcal{F}}\left(\overline{\mu}, \overline{\sigma}^2\right) = \alpha_{\varnothing}$.

[8] This type of loss utility function is common in the cheap talk literature (e.g., Crawford and Sobel, 1982; Morgan and Stocken, 2003; among many others) and allows to obtain (tractable) closed-form solutions.

[9] In this environment, by picking an allocation within the cap imposed by the investor, the advisor indirectly reveals ex post part or all of the information he possesses.

our framework because, for any equilibrium of a direct mechanism with truthtelling, there exists an equivalent equilibrium in our class of indirect mechanisms with partial delegation.

Timing. The timing of the game is as follows:

1. A publicly chooses the information acquisition policy d and privately learns the corresponding information.
2. I chooses whether to directly manage her portfolio or rely on the advisor by announcing and committing to a cap $\overline{\alpha}$.
3. With delegation, A chooses an asset allocation up to the cap $\overline{\alpha}$. Without delegation, I chooses her asset allocation.
4. Asset returns and payoffs materialize.

The solution concept is Perfect Bayesian Equilibrium (PBE).

3 Optimal Contract

We begin by analyzing the properties of the optimal contract proposed by the investor to the advisor — that is, the cap imposed by I on the amount of wealth invested into the risky asset. This optimal contract is shaped by the information possessed by the advisor and the degree of conflict between the two parties.

Notice that a higher cap $\overline{\alpha}$ implies more delegation to the advisor and, hence, a lower control by the investor over the portfolio composition. In fact, a higher cap provides A with more flexibility and this allows the advisor to better tailor the investment in the risky asset to his information on the state of nature, which is desirable for the investor, but also to choose an allocation closer to his preferred one rather than the investor's one, which is undesirable for the investor.

The following lemma characterizes the advisor's investment behavior.

Lemma 1 *For any information acquisition policy d and any cap $\overline{\alpha}$ imposed by the investor, the advisor's optimal strategy is to invest an amount equal to* $\min\{\overline{\alpha}, (1+\lambda)\,\alpha_d\}$.

The advisor can invest his preferred amount of wealth into the risky asset only if he is allowed to do so by I. Assume first that the cap is lower than A's preferred asset allocation — i.e., $\overline{\alpha} < (1+\lambda)\alpha_d$.[10] In this case, the investment strategy that maximizes A's utility coincides with the cap, since this is the highest amount of wealth that A can invest in the

[10] Recall that the investor's preferred allocation α_d may vary according to the realized state of nature. Therefore, for a given cap $\overline{\alpha}$, there may be states of nature where $\overline{\alpha} < (1+\lambda)\alpha_d$ and states of nature where, instead, $\overline{\alpha} > (1+\lambda)\alpha_d$.

risky asset and it minimizes the distance between the actual investment and his preferred allocation. In this case, we say that the cap binds.

On the contrary, a cap imposed by I that does not bind and is higher than A's preferred allocation — i.e., $\overline{\alpha} \geqslant (1+\lambda)\,\alpha_d$ — provides A with enough flexibility to invest his preferred amount of wealth into the risky asset. In this case, the advisor's optimal investment strategy coincides with his preferred asset allocation and maximizes his utility, yielding a zero loss.[11]

In what follows, we investigate the optimal cap imposed by the investor under different information acquisition policies. As we will show, the magnitude of the conflict of interests plays a key role in the rest of the analysis: the decision to acquire information affects the investor's choice of cap and the advisor's investment behavior; hence (*ceteris paribus*) it also influences the decision on whether to rely on the financial advisor.

3.1 Contract with No Information

When A decides not to acquire any information and remains uninformed, that is $d = \varnothing$, he does not possess any information advantage with respect to I. Hence, in this case, the optimal contract set by I features a cap equal to:

$$\alpha_{\varnothing} \triangleq \max_{\overline{\alpha}\in\mathbb{R}} \mathcal{U}\left(\overline{\alpha}, E\left[\mu\right], E\left[\sigma^2\right]\right) = \frac{E\left[\mu\right] - r_f}{\gamma E\left[\sigma^2\right]}. \tag{3}$$

The intuition is the following. Under no information acquisition, A cannot tailor the asset allocation to the state of nature and thus, by Lemma 1, he invests the lowest between the cap and his preferred allocation $(1+\lambda)\,\alpha_{\varnothing}$. Therefore, it is optimal for the investor to set a cap equal to her preferred allocation $\alpha_{\varnothing}$, which is binding and induces A to invest $\alpha_{\varnothing}$. Such cap is chosen behind the veil of ignorance — i.e. without knowing the actual mean and variance of the risky asset, but only their expectations.

Since (3) also represents I's optimal investment when she does not consult the financial advisor and directly manages her portfolio, I has no incentive to delegate to an advisor who does not acquire any information about the characteristics of the risky asset. Hence, the decision to rely on the financial advisor crucially depends on his information acquisition policy.

3.2 Contract with Partial Information

When A acquires information only about the mean of the risky asset but not about its variance, that is $d = \mathcal{P}$, the optimal cap may result in different investments depending on

[11] Notice that, when the cap does not bind, I perfectly learns the information possessed by the advisor ex-post, by observing the chosen allocation. Hence, A's investment strategy implicitly depends on the investor's ability to commit to the cap and not to change ex-post the portfolio allocation that A has chosen on her behalf.

the magnitude of the conflict of interests between the advisor and the investor. The investor takes into account two aspects in choosing the optimal cap. First, since A is informed about the mean of the risky asset, his investment strategy can solely be tailored to the realization of μ. Second, the investor's linear utility function $\mathcal{U}$ implies that the variance can be taken in expected value, because both the investor and the advisor are uninformed about it. Therefore, to maximize her expected utility, I chooses the cap that solves the following problem:

$$\max_{\overline{\alpha}\in\mathbb{R}} \sum_{\mu\in\Omega} \Pr[\mu]\, \mathcal{U}\left(\min\left\{\overline{\alpha}, (1+\lambda)\,\alpha_{\mathcal{P}}(\mu)\right\}, \mu, E\left[\sigma^2\right]\right). \tag{4}$$

The following lemma characterizes the properties of the optimal contract under partial information acquisition: even if A only possesses partial information, the investor may still have an incentive to delegate the advisor.

Lemma 2 *When the advisor acquires partial information, there exists a threshold $\lambda_{\mathcal{P}} > 0$ such that:*

- *For $\lambda \leqslant \lambda_{\mathcal{P}}$, the optimal cap imposed by the investor is $\alpha_{\mathcal{P}}(\overline{\mu})$ and it only binds when $\mu = \overline{\mu}$.*
- *For $\lambda > \lambda_{\mathcal{P}}$, the optimal cap imposed by the investor is $\alpha_{\varnothing} < \alpha_{\mathcal{P}}(\overline{\mu})$ and it binds both when $\mu = \underline{\mu}$ and $\mu = \overline{\mu}$.*

Figure 2 depicts the result of Lemma 2.There are two forces that shape the optimal cap: I may choose a low cap that induces A to invest the same amount regardless of his information about μ, or a high cap that induces A to choose different amounts depending on his information. Both alternatives depart from the asset allocation that I would choose if she had information about μ. On the one hand, a low cap equal to $\alpha_{\varnothing}$ binds both when $\mu = \underline{\mu}$ and $\mu = \overline{\mu}$, whereby inducing A to always invest this cap: this is costly to I because the investment is not tailored to the expected return of the risky asset. On the other hand, a high cap equal to $\alpha_{\mathcal{P}}(\overline{\mu})$ grants to A the flexibility to invest his ideal amount when $\mu = \underline{\mu}$ but is costly to I because this amount is larger than her preferred allocation. The relative cost of these two alternatives depends on the magnitude of the conflict of interests λ, which therefore determines whether partial delegation occurs.

When λ is small, the advisor's preferences are closer to the investor's ones so that the investor is more keen to delegate her investment decision. In this case, the optimal cap corresponds to I's preferred asset allocation when the expected return on the risky asset is high, $\alpha_{\mathcal{P}}(\overline{\mu})$. As a result, I achieves her preferred investment when μ is high but, at the same time, she provides A with enough flexibility to invest according to his preferences when μ is low. With this partial delegation, I correctly learns the expected return of the risky asset ex-post, upon observing the portfolio chosen by the advisor.

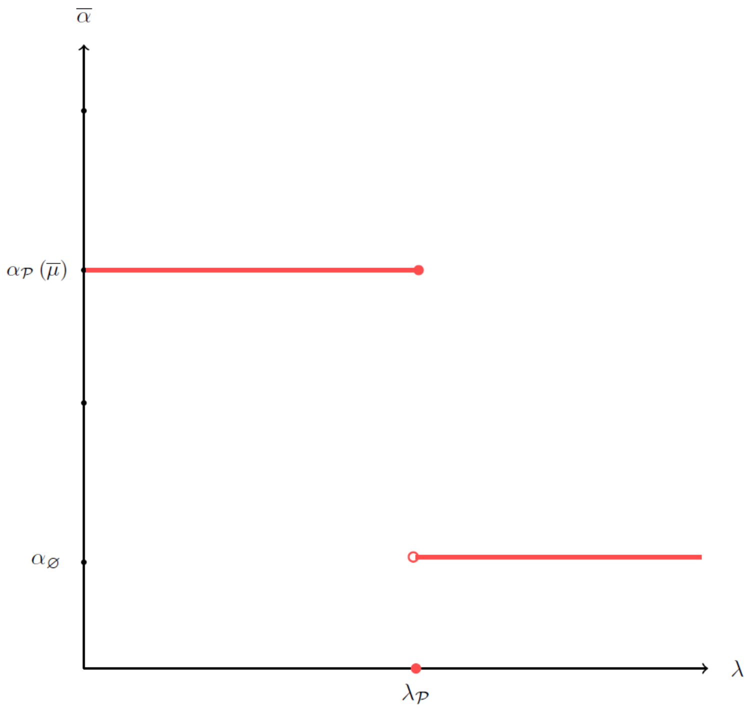


Figure 2: Optimal Contract under Partial Information Acquisition.

By contrast, when λ is large, the conflict of interests is stronger, which makes partial delegation more costly for I, even when the mean of the risky asset is low. Hence, I designs a contract in which the cap always binds, thus inducing A to invest the same amount regardless the realization of μ. In this case, since A does not tailor the investment to her information, it is optimal for the investor to set a cap $\alpha_{\varnothing}$, which is I's preferred one when she has no information. In this case, I learns nothing about the expected return of the risky asset ex-post, upon observing the portfolio chosen by A.

3.3 Contract with Full Information

Suppose now that the advisor acquires information about both the mean and the variance of the risky asset. This allows him to disentangle all realized pairs (μ, σ^2) and, thus, to better tailor the investment to the state of nature, which is desirable for the investor. As explained above, however, delegating the asset allocation decision to A also comes with a cost for I, since the advisor's preferred allocation is larger than the investor's one. In order to design an optimal contract, the investor trades off these two opposite forces.

Hence, to maximize her expected utility, I chooses the cap that solves the following problem:

$$\max_{\overline{\alpha}\in\mathbb{R}} \sum_{(\mu,\sigma^2)\in\Omega\times\Sigma} \Pr\left[\mu,\sigma^2\right] \mathcal{U}\left(\min\left\{\overline{\alpha},(1+\lambda)\,\alpha_{\mathcal{F}}\left(\mu,\sigma^2\right)\right\},\mu,\sigma^2\right). \tag{5}$$

In contrast to the partial information case investigated in the previous section, under full information the investor's problem explicitly depends on μ and σ^2, since now the advisor knows both characteristics of the risky assets.

With a slight abuse of notation, we denote all states of nature except $\left(\underline{\mu},\overline{\sigma}^2\right)$ by

$$\left(\underline{\mu},\overline{\sigma}^2\right)^c \triangleq \left\{\left(\underline{\mu},\underline{\sigma}^2\right),\left(\overline{\mu},\overline{\sigma}^2\right),\left(\overline{\mu},\underline{\sigma}^2\right)\right\}.$$

The following lemma characterizes the properties of the optimal contract under full information acquisition.

Lemma 3 *Let*

$$\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c \triangleq \frac{E\left[\mu\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right]-r_f}{\gamma E\left[\sigma^2\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right]}.$$

When the advisor acquires full information, there exists two thresholds $\underline{\lambda}_{\mathcal{F}}<\overline{\lambda}_{\mathcal{F}}$ such that:

- *For $\lambda\leqslant\underline{\lambda}_{\mathcal{F}}$, the optimal cap imposed by the investor is $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$ and it only binds in state $\left(\overline{\mu},\underline{\sigma}^2\right)$.*
- *For $\underline{\lambda}_{\mathcal{F}}<\lambda\leqslant\overline{\lambda}_{\mathcal{F}}$, the optimal cap imposed by the investor is $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c<\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$ and it binds in all states except $\left(\underline{\mu},\overline{\sigma}^2\right)$.*
- *For $\overline{\lambda}_{\mathcal{F}}<\lambda$, the optimal cap imposed by the investor is $\alpha_{\varnothing}<\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c$ and it binds in all states.*

Figure 3 summarizes the result of Lemma 3 and compares the optimal caps with partial and full information. Under full information, the investor has three viable options. First, as in the case of partial information, I may exercise more control over the portfolio composition and impose a low cap $\alpha_{\varnothing}$. In this case, A always invests $\alpha_{\varnothing}$ and the investment is not tailored to the state of nature. Second, I may impose an intermediate cap $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c>\alpha_{\varnothing}$ and grant some flexibility to the advisor. By doing so, she partially delegates the investment decision to A to exploit his knowledge about the states of nature. Third, I may exercise a lower control over the portfolio composition and grant even more flexibility to the advisor by imposing a higher cap $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$.[12] As with partial information, the choice between these options depends on the magnitude of the conflict of interests λ.

When the conflict of interests between A and I is sufficiently low, that is $\lambda\leqslant\underline{\lambda}_{\mathcal{F}}$, the benefits of a better-tailored asset allocation dominate the costs of misalignment of preferences. Therefore, I trusts the advisor more and chooses the high cap $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$, which only binds in state $\left(\overline{\mu},\underline{\sigma}^2\right)$. As a result, A invests his preferred amount in all states except $\left(\overline{\mu},\underline{\sigma}^2\right)$ and I obtains her preferred allocation in this state. Notice that, in this case, I always learns ex post the realizations of the mean and variance of the risky asset.

Instead, when the conflict of interests between A and I takes an intermediate value, that is $\underline{\lambda}_{\mathcal{F}}<\lambda\leqslant\overline{\lambda}_{\mathcal{F}}$, the costs of misalignment of preferences increase and reduce the benefits of

[12] Of course, it is never optimal for I to choose a cap higher than $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$.

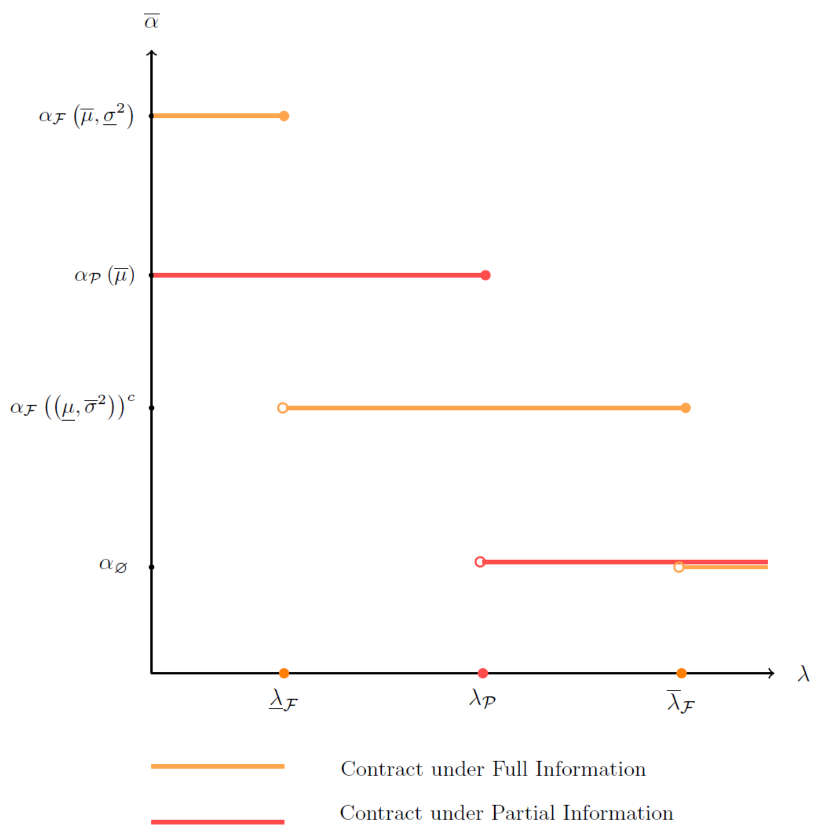


Figure 3: Optimal Contract under Full Information Acquisition.

a better tailored investment. Therefore, I reduces delegation to the advisor and sets a cap that binds in all states except $\left(\underline{\mu},\overline{\sigma}^2\right)$. As a result, in state $\left(\underline{\mu},\overline{\sigma}^2\right)$, A invests his preferred amount and I learns information ex post; in all other states, A is capped by the amount $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c$, which is the investor's preferred allocation when she knows that the state of nature is not $\left(\underline{\mu},\overline{\sigma}^2\right)$, and I does not learn any information ex-post.

Finally, when λ is sufficiently large, the investor cannot benefit from A's information because the misalignment of preferences is too large. Therefore, I sets a cap that binds in all states to constraint A's behavior, whereby inducing A to invest the same amount regardless the information possessed. As a result, I never learns the characteristics of the risky asset ex-post, upon observing the portfolio chosen by the advisor.

Comparing the characteristics of the contracts offered by the investor with full and partial information, we have the following result:

Lemma 4 *The cap set by the investor under partial information is higher than the cap under full information if and only if* $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \lambda_{\mathcal{P}}$.

When the advisor only knows the mean of the risky asset, the investor delegates her investment decision to the advisor up to the cap $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ if $\lambda < \lambda_{\mathcal{P}}$, and manages the portfolio on her own otherwise. By contrast, when the advisor knows both the mean and the variance of the risky asset, delegation takes place with two different degrees of flexibility: either up to $\alpha_{\mathcal{F}}(\overline{\mu}, \underline{\sigma}^2)$ or up to $\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c$ (see Figure 3). Interestingly, when the misalignment of preferences is low, $\lambda \leqslant \underline{\lambda}_{\mathcal{F}}$, it is optimal for the investor to grant more flexibility to a fully informed advisor while as the misalignment of preferences increases it is optimal to grant more flexibility to a partially informed advisor.

4 Incentives to Delegate

Our analysis highlights two forces that shape the magnitude of the optimal cap, thus affecting the investor's incentives to delegate her investment decision to the advisor: A's preferences towards a larger investment in the risky asset λ, and the information acquisition policy d.

When $d = \varnothing$, the advisor does not acquire any information about the characteristics of the risky asset. Hence, I prefers to directly manage her portfolio rather than to delegate, because the financial advice would be uninformative. Moreover, even when the advisor is more informed than the investor — i.e., $d = F, P$ — I does not delegate when the conflict of interests is sufficiently high. In this case, the optimal cap with delegation would be $\alpha_{\varnothing}$ and would induce A to always invest the amount preferred by I without information; hence the investor obtains no benefit from the advisor. This implies the following result:

Proposition 1 *The investor delegates her portfolio decision to the financial advisor when $d = P$ and $\lambda < \lambda_{\mathcal{P}}$ or $d = F$ and $\lambda < \overline{\lambda}_{\mathcal{F}}$.*

We now compare the investor's ex-ante utility of delegation — i.e., before the state of nature realizes — to analyze her preferences between different levels of information possessed by the advisor.

Proposition 2 *With delegation, the investor always prefers a fully informed advisor to a partially informed one.*

Hence, regardless the degree of misalignment of preferences, an investor always prefers to delegate her asset allocation to an advisor who is informed about both characteristics of the risky asset. The reason is that, although costly, delegation is desirable because it allows the investor to achieve an asset allocation that is more tailored to the advisor's superior information about the state of nature.

In this regard, notice that portfolio customization is positively related to: (i) the magnitude of the cap — since a higher cap provides more freedom to the advisor — and (ii) the amount of information possessed by A — since a fully informed advisor can tailor the investment to both the mean and the variance of the risky asset. Hence, the investor's preferences for different information policies depend on both these elements. When the conflict of interests is low, $\lambda \leqslant \underline{\lambda}_{\mathcal{F}}$, the investor prefers a fully informed advisor both because he has an information advantage and because he is granted more freedom, as shown in Figure 3. By contrast, when $\underline{\lambda}_{\mathcal{F}} < \lambda \leqslant \overline{\lambda}_{\mathcal{F}}$, the information advantage of a fully informed advisor is counterbalanced by the higher cap of a partially informed advisor. Nevertheless, the first effect dominates and the investor still prefers a fully informed advisor.

5 Information Acquisition

Since information is valuable for the investor, one may wonder whether information is also valuable for the advisor. This section addresses this issue and investigates A's costs and benefits of acquiring information.

Recall that A has to pay a fixed price $\kappa_{\mathcal{F}}$ (resp. $\kappa_{\mathcal{P}}$) in order to acquire full (resp. partial) information and that the cost of information is increasing in its level — i.e., $\kappa_{\mathcal{F}} > \kappa_{\mathcal{P}}$. Since A obtains a large negative utility without delegation, A has an incentive to pay the cost of acquiring information if and only if this induces I to delegate her investment decision — which occurs when the conflict of interests is low. Therefore, the advisor remains uninformed when the misalignment of preferences is high, because in this case I prefers to directly manage her portfolio (so that costly information would provide no benefit to A).

We now compare A's utility with full and partial information acquisition. On the one hand, full information benefits the advisor because it may allow him to choose a portfolio that is closer to his preferred allocation (which depends on both μ and σ^2), thus increasing his utility. On the other hand, acquiring more information is more costly for the advisor and also affects the cap imposed by the investor. Therefore, A's incentives to acquire different types of information depend on the trade-off between a more tailored asset allocation and the cost of information acquisition.

Proposition 3 *When $\lambda < \underline{\lambda}_{\mathcal{F}}$, there exists a threshold $\sigma^*(\Delta\kappa)$ such that A prefers to acquire full information if $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} < \sigma^*(\Delta\kappa)$ and partial information otherwise. This threshold is increasing in $\Delta\kappa$ and such that $0 < \sigma^*(\Delta\kappa) < 3$.*

When $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \lambda_{\mathcal{P}}$, A always prefers to acquire partial rather than full information. When $\lambda_{\mathcal{P}} \leqslant \lambda < \overline{\lambda}_{\mathcal{F}}$, A always prefers to acquire full rather than partial information.

The advisor prefers to acquire full, rather than partial, information if and only if the expected incremental value of additional information is larger than its incremental cost — i.e.,

$$E\left[\mathcal{V}_{\mathcal{F}} - \mathcal{V}_{\mathcal{P}}\right] \geqslant \Delta\kappa. \tag{6}$$

Hence, a higher cost of full information acquisition relative to the partial information one makes it more likely that A acquires less information.

First, when the conflict of interests is large — i.e., When $\lambda_{\mathcal{P}} \leqslant \lambda < \overline{\lambda}_{\mathcal{F}}$ — the advisor prefers to acquire full information. The reason is that, in this case, partial information

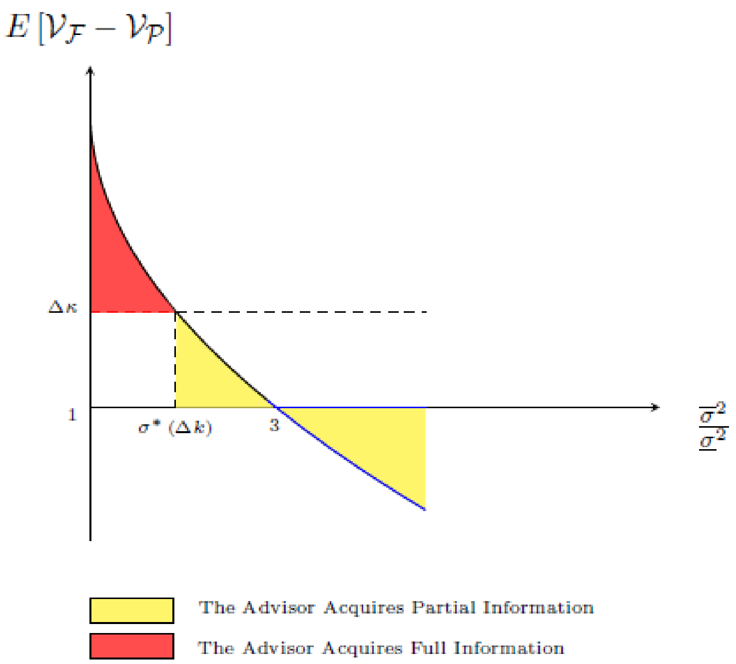


Figure 4: Advisor's costs and benefits of acquiring more information

acquisition does not induce I to delegate while full informatio acquisition does. Second, for intermediate values of the the conflict of interests — i.e., when $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \lambda_{\mathcal{P}}$ — the advisor prefers to acquire partial information. The reason is that, in this case, partial information acquisition induces I to choose a higher cap and is less costly than full information.

Third, notice also that, if $\lambda < \underline{\lambda}_{\mathcal{F}}$, condition (6) can be written as

$$-\Pr\left(\overline{\mu},\underline{\sigma}^2\right)\left|\underbrace{\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)-(1+\lambda)\,\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)}_{<0}\right|+\Pr\left(\overline{\mu}\right)\left|\underbrace{\alpha_{\mathcal{P}}\left(\overline{\mu}\right)-(1+\lambda)\,\alpha_{\mathcal{P}}\left(\overline{\mu}\right)}_{<0}\right|\geqslant\Delta\kappa$$
$$\Leftrightarrow \quad \lambda\left[\Pr\left(\overline{\mu}\right)\alpha_{\mathcal{P}}\left(\overline{\mu}\right)-\Pr\left(\overline{\mu},\underline{\sigma}^2\right)\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)\right]\geqslant\Delta\kappa. \tag{7}$$

When the cap binds, the advisor suffers a loss because he does not achieve his preferred asset allocation. Condition (7) highlights that, if the conflict of interests is low, this loss is proportional to the cap imposed by the investor multiplied the corresponding probability that the cap binds. In this case, the cap imposed by the investor with full information is always larger than with partial information — i.e., $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right) > \alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ — but this cap is more likely to bind with partial information — i.e., $\Pr\left(\overline{\mu}\right) > \Pr\left(\overline{\mu},\underline{\sigma}^2\right)$. Therefore, the choice of the information acquisition policy depends on the relative magnitude of these two factors.

Figure 4 displays $E\left[\mathcal{V}_{\mathcal{F}}-\mathcal{V}_{\mathcal{P}}\right]$ as a function of the variance ratio $\frac{\overline{\sigma}^2}{\underline{\sigma}^2}$, when $\lambda < \underline{\lambda}_{\mathcal{F}}$. This function is decreasing because the higher is the ratio $\frac{\overline{\sigma}^2}{\underline{\sigma}^2}$, the wider is the distance between the caps $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$ and $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$, and hence the lower is the incremental benefit of acquiring full, rather than partial, information.[13] When $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} < \sigma^*(\Delta\kappa)$, the two caps $\alpha_{\mathcal{F}}(\overline{\mu}, \underline{\sigma}^2)$ and $\alpha_{\mathcal{P}}(\overline{\mu})$ are relatively similar and, since full information grants the advisor more freedom by reducing the probability that the cap binds, the benefits of additional information overcome its cost.

By contrast, when $\sigma^*(\Delta\kappa) \leqslant \frac{\overline{\sigma}^2}{\underline{\sigma}^2} < 3$, the increase in A's ex-ante utility due to full information acquisition is lower since the distance between the caps is wider. In this case, although the additional benefit of knowing both characteristics of the risky asset is positive, it is not sufficient to cover the additional cost of full information. Moreover, when $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} \geqslant 3$, the distance between the caps is so large that A's ex ante utility is always higher with partial rather than full information; hence A prefers partial information, regardless of the information costs.

Therefore, although the investor always prefers the advisor to acquire more information, the advisor does so only if: (i) the conflict of interest with the investor is relatively large, or (ii) both the conflict of interests with the investor and the uncertainty about the asset's volatility are sufficiently low. Otherwise the investor offers a less attractive contract to a better informed advisor, which discourages the advisor from acquiring more information. This suggests that a lower conflict of interests between the investor and the financial advisor does not necessarily result in a more informed advisor, and hence in investment choices that are better tailored to the actual characteristics of the risky assets. Precisely: (i) reducing the conflict of interest below $\lambda_{\mathcal{P}}$ (but not below $\underline{\lambda}_{\mathcal{F}}$) reduces the amount of information acquired by the advisor and (ii) even when the conflict of interest is relatively low (below $\underline{\lambda}_{\mathcal{F}}$) the advisor does not acquire the level of information preferred by the investor if $\frac{\overline{\sigma}^2}{\underline{\sigma}^2}$ is high.

Remark. Notice that the difference $E[V_{\mathcal{F}} - V_{\mathcal{P}}]$ can also be interpreted as the maximum price that an advisor is willing to pay in order to switch from partial to full information, i.e. $\overline{\Delta\kappa}$. Obviously, when such a price is larger than the actual switching cost $\Delta\kappa$, i.e. when $\overline{\Delta\kappa} > \Delta\kappa$, the advisor prefers to acquire full information. This case is represented by the red area in Figure 4. Otherwise, the advisor prefers to acquire partial information.

6 Conclusions

In this paper we study the interplay between the investor's incentives to delegate her asset allocation choice to a biased financial advisor, and the advisor's decision to acquire information about multiple characteristics of the risky asset. We assume there is a conflict of interests between the investor and the advisor due, for example, to private commissions re-

[13] In fact, notice that $\alpha_{\mathcal{F}}(\overline{\mu}, \underline{\sigma}^2) - \alpha_{\mathcal{P}}(\overline{\mu})$ can be written as

$$\frac{\overline{\mu} - r_f}{\gamma\underline{\sigma}^2} - \frac{2(\overline{\mu} - r_f)}{\gamma\left[\underline{\sigma}^2 + \overline{\sigma}^2\right]} = \frac{\overline{\mu} - r_f}{\gamma}\left[\frac{1}{\underline{\sigma}^2} - \frac{2}{\left(\underline{\sigma}^2 + \overline{\sigma}^2\right)}\right] = \frac{\overline{\mu} - r_f}{\gamma}\left[\frac{\overline{\sigma}^2 - \underline{\sigma}^2}{\underline{\sigma}^2\left(\underline{\sigma}^2 + \overline{\sigma}^2\right)}\right].$$

ceived by the advisor or different preferences: the advisor always prefers to invest in the risky product an amount of wealth that is larger than the investor's preferred one.

We show that, when the investor delegates her asset allocation decision to the (informed) advisor, she imposes a cap on the amount of wealth that the latter is allowed to invest in the risky asset. The cap, in fact, allows the investor to balance the benefits stemming from more informed advice with the costs arising from the advisor's biased preferences. In this environment, we find that the optimal cap is always decreasing in the conflict of interests — that is, the wider the difference between the advisor's and the investor's preferences, the lower is the advisor's discretionality in choosing the asset allocation — and depends on the information acquired by the advisor.

Delegation, however, is not always optimal. In fact, when the advisor is not informed about the characteristics of the risky asset or when the conflict of interests is sufficiently high, the investor prefers to manage her portfolio directly rather than to delegate. Turning to the advisor's choice regarding the level of information to acquire, we show that the advisor is willing to acquire costly information if and only if this induces the investor to delegate her investment decision. Moreover, the advisor prefers to acquire more costly and superior information if the conflict of interest is sufficiently high or if both the conflict of interests and the variance ratio of the asset are sufficiently low. Otherwise, the advisor prefers to acquire less information, taking also into account how this will affect the contract offered by the investor.

Our results suggest that a low conflict of interests between investors and financial advisors does not necessarily result in better informed advisors that produce investment decision that are tailored to the actual characteristics of the risky assets. Perhaps surprisingly, reducing the conflict of interest may actually reduce the amount of information acquired by financial advisors and even an extremely low conflict of interests does not necessarily guarantee that advisors acquire the level of information preferred by investors. This is because the amount of information acquired by an advisor also affects the characteristics of the contract offered by the investor: dealing with a more informed advisor may actually induce the investor to offer a more restrictive contract that provides tighter limits to the advisor's choices, thus discouraging the advisor from acquiring more information.

7 Appendix

Proof of Lemma 1. Suppose that I announces a cap $\overline{\alpha}$ and commits to it. Then, for any information level $d \in \{\mathcal{F}, \mathcal{P}, \varnothing\}$, the advisor's optimization problem is

$$\min_{\alpha \in [0,\overline{\alpha}]} |\alpha - (1+\lambda)\,\alpha_d| \,.$$

The result follows immediately because A optimally chooses to invest his first-best allocation $(1+\lambda)\,\alpha_d$ if and only if $(1+\lambda)\,\alpha_d \leqslant \overline{\alpha}$. By contrast, loss minimization requires A to invest the cap $\overline{\alpha}$ if and only if the cap is binding, that is $\overline{\alpha} < (1+\lambda)\,\alpha_d$. ■

Proof of Lemma 2. The proof of the lemma is structured as follows. We first characterize the properties that a (candidate) equilibrium with cap $\overline{\alpha}$ needs to satisfy. Second, we show that I cannot profitably deviate by offering any other cap $\alpha \neq \overline{\alpha}$ in the region of parameters under consideration.

Step 1. We start by finding the cap imposed by I within various regions of parameters.

Region (i). Suppose that I commits to a cap $\overline{\alpha}$ such that $\overline{\alpha} \leqslant (1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$. Using the advisor's investment strategy from Lemma 1, A invests $\overline{\alpha}$ in all states of nature since the cap binds in both states $\overline{\mu}$ and $\underline{\mu}$. Hence, in this region, I's optimization problem is

$$\max_{\overline{\alpha} \in \left[0,(1+\lambda)\alpha_{\mathcal{P}}(\underline{\mu})\right]} \sum_{\mu \in \Omega} \Pr(\mu) \left\{ \overline{\alpha}\,(\mu - r_f) - \frac{\gamma}{2}\overline{\alpha}^2 E\left[\sigma^2\right] \right\}. \tag{A1}$$

Maximizing (A1) with respect to $\overline{\alpha}$, we obtain

$$\alpha_{\varnothing} = \frac{\mathrm{E}[\mu] - r_f}{\gamma \mathrm{E}[\sigma^2]} > 0.$$

Since,

$$\alpha_{\varnothing} < (1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right) \quad \Leftrightarrow \quad \lambda > \lambda_0 \triangleq \frac{\overline{\sigma}^2 - \underline{\sigma}^2}{2\underline{\sigma}^2},$$

I imposes a cap equal to $\alpha_{\varnothing}$, which is unresponsive to any state of nature only if $\lambda > \lambda_0$. Instead, if $\lambda \leqslant \lambda_0$, $\alpha_{\varnothing}$ is outside the interval of interest, and hence, I's preferred asset allocation becomes $(1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$ (that coincides with A's preferred investment in state $\underline{\mu}$).

As a result, in this region of parameters, the optimal cap is

$$\overline{\alpha} = \min\left\{\alpha_{\varnothing}, (1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)\right\}.$$

Region (ii). Suppose now that I commits to a cap $\overline{\alpha}$ such that $(1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right) \leqslant \overline{\alpha} < (1+\lambda)\,\alpha_{\mathcal{P}}(\overline{\mu})$. Using the advisor's investment strategy from Lemma 1, A invests $\overline{\alpha}$ in state $\overline{\mu}$ and his preferred asset allocation $(1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$ in state $\underline{\mu}$. Hence, in this region, I's optimization problem is

$$\max_{\overline{\alpha} \in \left[(1+\lambda)\alpha_{\mathcal{P}}\left(\underline{\mu}\right),(1+\lambda)\alpha_{\mathcal{P}}(\overline{\mu})\right)} \frac{1}{2}\left\{\overline{\alpha}\,(\overline{\mu} - r_f) - \frac{\gamma}{2}\overline{\alpha}^2 \mathrm{E}\left[\sigma^2\right]\right\}$$
$$+ \frac{1}{2}\left\{(1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)\left(\underline{\mu} - r_f\right) - \frac{\gamma}{2}(1+\lambda)\,\alpha_{\mathcal{P}}^2\left(\underline{\mu}\right)\mathrm{E}\left[\sigma^2\right]\right\},$$

which, maximizing with respect to $\overline{\alpha}$, yields

$$\alpha_{\mathcal{P}}\left(\overline{\mu}\right)=\frac{\overline{\mu}-r_f}{\gamma \mathrm{E}\left[\sigma^2\right]}.$$

Since

$$\alpha_{\mathcal{P}}\left(\overline{\mu}\right)<\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\overline{\mu}\right), \quad \forall \lambda>0,$$

and

$$\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\underline{\mu}\right)<\alpha_{\mathcal{P}}\left(\overline{\mu}\right) \quad \Leftrightarrow \quad \lambda<\lambda_1 \triangleq \frac{\overline{\sigma}^2-\underline{\sigma}^2}{\underline{\sigma}^2},$$

in this region of parameters I sets a cap equal to $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ — her preferred asset allocation in state $\overline{\mu}$ — if $\lambda<\lambda_1$. Otherwise, when $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ lies outside the region of interest, I sets a cap equal to $\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ — A's preferred asset allocation in state $\overline{\mu}$. Hence, in this region of parameter, the optimal cap is

$$\overline{\alpha}=\max\left\{\alpha_{\mathcal{P}}\left(\overline{\mu}\right),\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\underline{\mu}\right)\right\}.$$

Region (iii). Finally, suppose that I commits to a cap $\overline{\alpha}$ such that $\overline{\alpha}\geqslant\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$. Using the advisor's investment strategy from Lemma 1, since the cap is not binding in any state, A always invests his preferred asset allocation and bears no loss — i.e., full delegation by the investor. By contrast, in this region the investor never achieves her preferred investment. Hence, as we show below, a cap in this region is never optimal for the investor.

Step 2. We now compare the investor's utility across the regions defined above.

For any $\overline{\alpha}$, the investor's ex-ante utility $\mathcal{U}_{\mathcal{P}}\left(\overline{\alpha}\right)$ can be written as

$$\mathcal{U}_{\mathcal{P}}\left(\overline{\alpha}\right)=\sum_{\widehat{\mu}\in\Omega}\Pr\left[\widehat{\mu}\right]\left\{\overline{\alpha}\left(\widehat{\mu}-r_f\right)-\frac{\gamma}{2}\overline{\alpha}^2E\left[\sigma^2\right]\right\}+$$
$$+\sum_{\mu\neq\widehat{\mu}}\Pr\left[\mu\right]\left\{\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\mu\right)\left(\mu-r_f\right)-\frac{\gamma}{2}\left[\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\mu\right)\right]^2E\left[\sigma^2\right]\right\},$$

where the first term denotes I's ex-ante utility in the states $\widehat{\mu}\in\Omega$ where the cap binds, whereas the second term captures I's ex-ante utility in the remaining states $\mu\neq\widehat{\mu}$ (i.e., when the cap does not bind).

Notice that setting a cap $\overline{\alpha}\geqslant\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ is never optimal for the investor since I can increase her ex-ante utility by imposing $\overline{\alpha}=\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$. In fact, $\overline{\alpha}=\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ ensures her first-best in state $\overline{\mu}$. Hence, in what follows we rule out *Region (iii)* and focus on $\overline{\alpha}\leqslant\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$.

Using the results obtained in Step 1 (see Figure 5), there are three (sub)cases to consider to determine the optimal cap under partial information: (1) $\lambda<\lambda_0$; (2) $\lambda>\lambda_1$; and (3) $\lambda_0<\lambda<\lambda_1$.

First consider case (1). As explained above, there are two candidate caps to consider, that is $\overline{\alpha}=\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ and $\overline{\alpha}=\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$. It is straightforward to see that

$$\mathcal{U}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right)-\mathcal{U}_{\mathcal{P}}\left(\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\underline{\mu}\right)\right)=\left(\underline{\mu}-r_f\right)^2\frac{\left(\overline{\sigma}^2-\underline{\sigma}^2-\underline{\sigma}^2\lambda\right)^2}{2\gamma\left(\overline{\sigma}^2+\underline{\sigma}^2\right)}>0,$$

as claimed. This implies that, when $\lambda<\lambda_0$, the optimal cap is $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$.

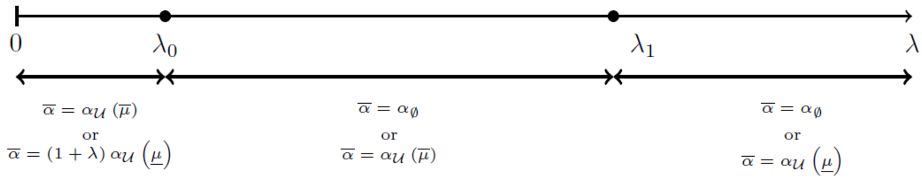


Figure 5: Step 1: Properties of a Candidate Equilibrium.

Next, consider case (2). The candidate caps to consider are $\overline{\alpha} = \alpha_{\varnothing}$ and $\overline{\alpha} = (1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$. Setting a cap equal to $\overline{\alpha} = (1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)$, however, is suboptimal for I since

$$\mathcal{U}_{\mathcal{P}}\left(\alpha_{\varnothing}\right) - \mathcal{U}_{\mathcal{P}}\left((1+\lambda)\,\alpha_{\mathcal{P}}\left(\underline{\mu}\right)\right) = \left(\underline{\mu} - r_f\right)^2 \frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2 - 2\underline{\sigma}^2\lambda\right)^2}{4\gamma\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)} > 0.$$

Hence, when $\lambda_1 < \lambda$, the optimal cap is $\alpha_{\varnothing}$.

Finally, consider case (3). Here the candidate caps are $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$ and $\alpha_{\varnothing}$. It is straightforward to show that

$$\mathcal{U}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right) - \mathcal{U}\left(\alpha_{\varnothing}\right) = \left(\underline{\mu} - r_f\right)^2 \frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2 - 2\left(\underline{\sigma}^2\right)^2\lambda^2}{4\gamma\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)}.$$

The sign of this expression depends on the sign of the numerator

$$\beta\left(\lambda, \sigma^2\right) \triangleq \left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2 - 2\left(\underline{\sigma}^2\right)^2\lambda^2, \tag{A2}$$

where

$$\frac{\partial\beta\left(\lambda,\sigma^2\right)}{\partial\lambda} = -4\lambda\left(\underline{\sigma}^2\right)^2.$$

Setting (A2) equal to zero and solving for λ, we obtain

$$\lambda_{\mathcal{P}} \triangleq \frac{\overline{\sigma}^2 - \underline{\sigma}^2}{\underline{\sigma}^2\sqrt{2}},$$

which is strictly positive. Moreover,

$$\lambda_{\mathcal{P}} - \lambda_0 = \left(\sqrt{2} - 1\right)\frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)}{2\underline{\sigma}^2} > 0,$$

and

$$\lambda_1 - \lambda_{\mathcal{P}} = \left(\sqrt{2} - 1\right)\frac{\overline{\sigma}^2 - \underline{\sigma}^2}{\sqrt{2}\underline{\sigma}^2} > 0.$$

Hence, if $\lambda \leqslant \lambda_{\mathcal{P}}$, the optimal cap is $\alpha_{\mathcal{P}}\left(\overline{\mu}\right)$; by contrast, if $\lambda > \lambda_{\mathcal{P}}$, the optimal cap is $\alpha_{\varnothing}$, as desired. ■

Proof of Lemma 3. As in the proof of Lemma 2, we first characterize the properties that a (candidate) equilibrium with cap $\overline{\alpha}$ needs to satisfy, and then show that I cannot profitably deviate by offering any other cap $\alpha \neq \overline{\alpha}$.

Step 1. To begin with, we find I's delegation strategy within the various regions of parameters.

Region (i). Suppose that I commits to a cap $\overline{\alpha} \leqslant (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)$. By Lemma 1, A invests $\overline{\alpha}$ in all states of nature since the cap always binds. Hence, I's optimization problem is

$$\max_{\overline{\alpha}\in\left[0,(1+\lambda)\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\right]} \sum_{(\mu,\sigma^2)\in\Omega\times\Sigma} \left\{\overline{\alpha}\,(\mu - r_f) - \frac{\gamma}{2}\overline{\alpha}^2\sigma^2\right\}. \tag{A3}$$

The first-order condition yields

$$\alpha_{\varnothing} = \frac{E\,[\mu] - r_f}{\gamma E\,[\sigma^2]}.$$

Since,

$$\alpha_{\varnothing} < (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right) \quad \Leftrightarrow \quad \lambda > \lambda_1 \triangleq \frac{\overline{\sigma}^2 - \underline{\sigma}^2}{\underline{\sigma}^2},$$

I imposes a cap $\alpha_{\varnothing}$, which is unresponsive to any states of nature, if $\lambda > \lambda_1$. Instead, if $\lambda \leqslant \lambda_1$, then $\alpha_{\varnothing}$ is outside the interval of interest, and hence I's desired cap becomes $(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)$ (that coincides with A's preferred investment in state $\left(\underline{\mu},\overline{\sigma}^2\right)$). As a result, in this region of parameters, the optimal cap is

$$\overline{\alpha} = \min\left\{\alpha_{\varnothing}, (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\right\}.$$

Region (ii). Suppose that I commits to a cap such that

$$(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right) \leqslant \overline{\alpha} < (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right).$$

In this region of parameters, the cap is binding in all states except $\left(\underline{\mu},\overline{\sigma}^2\right)$. Using the advisor's investment strategy from Lemma 1, the investor's maximization problem is

$$\begin{aligned}\max_{\overline{\alpha}\in\left[(1+\lambda)\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right),(1+\lambda)\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)\right)} &\sum_{(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)} \tfrac{3}{4}\left\{\overline{\alpha}\,(\mu - r_f) - \frac{\gamma}{2}\overline{\alpha}^2\sigma^2\right\} \\ &+ \tfrac{1}{4}\left\{(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\left(\underline{\mu} - r_f\right) - \frac{\gamma}{2}\left((1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\right)^2\overline{\sigma}^2\right\}.\end{aligned}$$

The solution to this problem is

$$\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c \triangleq \frac{E\left[\mu\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right] - r_f}{\gamma E\left[\sigma^2\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right]},$$

where $E\left[\mu\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right] = \frac{2\overline{\mu}+\underline{\mu}}{3}$ and $E\left[\sigma^2\left|(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)\right.\right] = \frac{2\underline{\sigma}^2+\overline{\sigma}^2}{3}$ are the expected mean and variance of the risky asset conditional on states $(\mu,\sigma^2)\neq\left(\underline{\mu},\overline{\sigma}^2\right)$.

Notice that

$$(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right) < \alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c \quad \Leftrightarrow \quad \lambda < \lambda_2 \triangleq \frac{2\left(\overline{\sigma}^4 - \underline{\sigma}^4\right)}{\underline{\sigma}^2\left(2\underline{\sigma}^2 + \overline{\sigma}^2\right)},$$

and

$$\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c < (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right), \quad \forall\lambda > 0.$$

Hence, if $\lambda < \lambda_2$, then I imposes a cap equal to $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c$, which binds in states $(\mu,\sigma^2) \neq \left(\underline{\mu},\overline{\sigma}^2\right)$. Instead, if $\lambda \geqslant \lambda_2$, I's preferred asset allocation lies outside the interval of interest, and hence the cap becomes $(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)$ — A's preferred asset allocation in state $\left(\underline{\mu},\overline{\sigma}^2\right)$. Hence, in this region of parameter, the optimal cap is

$$\overline{\alpha} = \max\left\{\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c, (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\right\}.$$

Finally, it is straightforward to show that $\lambda_1 < \lambda_2$.

Region (iii). Suppose that I commits to a cap such that

$$(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right) \leqslant \overline{\alpha} < (1+\lambda)\,\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right).$$

In this region of parameters, the cap is binding only in state $(\overline{\mu},\underline{\sigma}^2)$. Using the A's investment strategy from Lemma 1, the investor's maximization problem is

$$\max_{\overline{\alpha}\in\left[(1+\lambda)\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right),(1+\lambda)\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)\right)} \frac{1}{4}\left(\overline{\alpha}\left(\overline{\mu}-r_f\right)-\frac{\gamma}{2}\overline{\alpha}\underline{\sigma}^2\right)+$$
$$+\sum_{(\mu,\sigma^2)\neq(\overline{\mu},\underline{\sigma}^2)}\frac{3}{4}(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)\left[(\mu-r_f)-\frac{\gamma}{2}(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)\sigma^2\right].$$

The solution to this problem is

$$\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right) = \frac{\overline{\mu}-r_f}{\gamma\underline{\sigma}^2}.$$

Notice that

$$\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right) < (1+\lambda)\,\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right), \quad \forall\lambda > 0,$$

and

$$(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right) < \alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right) \quad \Leftrightarrow \quad \lambda < \lambda_1 = \frac{\overline{\sigma}^2-\underline{\sigma}^2}{\underline{\sigma}^2}.$$

Hence, if $\lambda < \lambda_1$, I imposes a cap equal to $\alpha_{\mathcal{F}}(\overline{\mu},\underline{\sigma}^2)$, which binds only in state $(\overline{\mu},\underline{\sigma}^2)$. Instead, if $\lambda \leqslant \lambda_1$, the allocation $\alpha_{\mathcal{F}}(\overline{\mu},\underline{\sigma}^2)$ lies outside the interval of interest, and hence I's preferred asset allocation becomes $(1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)$ (which coincides with A's preferred investment in state $\left(\underline{\mu},\overline{\sigma}^2\right)$). As a result, in this region of parameter, the optimal cap is

$$\overline{\alpha} = \max\left\{\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right), (1+\lambda)\,\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)\right\}.$$

Region (iv). Suppose that the investor chooses a cap $\overline{\alpha} \geqslant (1+\lambda)\,\alpha_{\mathcal{F}}(\overline{\mu},\underline{\sigma}^2)$. In this region, the advisor invests his ideal amount of wealth in all states without any loss. Hence, the investor's delegation strategy leaves full control to the advisor and the investor obtains the same expected utility for any $\overline{\alpha} \geqslant (1+\lambda)\,\alpha_{\mathcal{F}}(\overline{\mu},\underline{\sigma}^2)$. As we will show below, this region is not optimal for the investor.

Step 2. We now compare the investor's utility across the regions defined above.

As in the proof of Lemma 2, for any $\overline{\alpha}$ the investor's ex ante utility is

$$\mathcal{U}_{\mathcal{F}}\left(\overline{\alpha}\right) = \sum_{\left(\widehat{\mu},\widehat{\sigma}^2\right)\in\Omega\times\Sigma} \Pr\left(\widehat{\mu},\widehat{\sigma}^2\right)\left(\overline{\alpha}\left(\widehat{\mu}-r_f\right)-\frac{\gamma}{2}\overline{\alpha}^2\widehat{\sigma}^2\right)+$$
$$+\sum_{\left(\mu,\sigma^2\right)\neq\left(\widehat{\mu},\widehat{\sigma}^2\right)} \Pr\left(\mu,\sigma^2\right)\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\mu,\sigma^2\right)\left(\mu-r_f\right)-\frac{\gamma}{2}\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\mu,\sigma^2\right)\right)^2\sigma^2\right),$$

where the first term denotes I's ex ante utility in the states $\left(\widehat{\mu},\widehat{\sigma}^2\right)$ where the cap binds, whereas the second term denotes I's ex ante utility in the remaining states $\left(\mu,\sigma^2\right)\neq\left(\widehat{\mu},\widehat{\sigma}^2\right)$ (i.e., when the cap does not bind).

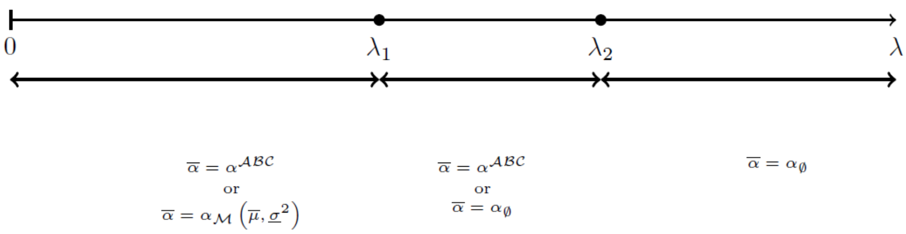
Notice that I never sets a cap equal to $\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)$ since she always obtains a higher ex-ante utility by setting the cap to $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c$ — i.e.,

$$U_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c\right)-U_{\mathcal{F}}\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)\right)$$
$$=\left(\underline{\mu}-r_f\right)^2\frac{\left(2\left(\overline{\sigma}^2+\underline{\sigma}^2\right)\left(\overline{\sigma}^2-\underline{\sigma}^2\right)-\lambda\underline{\sigma}^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)\right)^2}{8\gamma\left(\overline{\sigma}^2\right)^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)}>0,\quad\forall\lambda.$$

Similarly, I never sets a cap equal to $\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)$ since she always obtains a higher ex-ante utility by setting a cap euqal to $\alpha_{\varnothing}$ — i.e.,

$$\mathcal{U}_{\mathcal{F}}\left(\alpha_{\varnothing}\right)-\mathcal{U}_{\mathcal{F}}\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\underline{\mu},\underline{\sigma}^2\right)\right)=\left(\underline{\mu}-r_f\right)^2\frac{\left(\overline{\sigma}^2+\underline{\sigma}^2\right)\left(\left(\overline{\sigma}^2-\underline{\sigma}^2\right)-\lambda\underline{\sigma}^2\right)^2}{4\gamma\left(\overline{\sigma}^2\right)^2\left(\underline{\sigma}^2\right)^2}>0.$$

Given this, there are three cases to consider: (1) $\lambda<\lambda_1$; (2) $\lambda>\lambda_2$; and (3) $\lambda_1<\lambda<\lambda_2$.



Consider first case (1). In this region of parameters, optimality requires that I sets a cap equal to either $\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)$ or $\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c$ and we have that

$$\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)\right)-\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c\right)=\left(\underline{\mu}-r_f\right)^2\left(\overline{\sigma}^2+\underline{\sigma}^2\right)\frac{-\underline{\sigma}^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)\lambda^2+\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2}{8\gamma\left(\underline{\sigma}^2\right)^3\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)}. \tag{A4}$$

Since the denominator is strictly positive, the sign of (A4) depends on the sign of the numerator

$$\vartheta\left(\lambda,\sigma^2\right)\triangleq-\underline{\sigma}^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)\lambda^2+\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2,$$

with

$$\frac{\partial\vartheta\left(\lambda,\sigma^2\right)}{\partial\lambda}=-2\lambda\underline{\sigma}^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)<0.$$

Setting the numerator equal to zero and solving for λ, yields

$$\underline{\lambda}_{\mathcal{F}} \triangleq \frac{\overline{\sigma}^2 - \underline{\sigma}^2}{\sqrt{\left(2\underline{\sigma}^2 + \overline{\sigma}^2\right)\underline{\sigma}^2}},$$

which is strictly positive. Since

$$\lambda_1 - \underline{\lambda}_{\mathcal{F}} = \frac{\Delta\sigma^2\sqrt{\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)}}{\underline{\sigma}^2\sqrt{\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)}} > 0,$$

it is optimal set a cap equal to $\alpha_{\mathcal{F}}\left(\overline{\mu}, \underline{\sigma}^2\right)$ if and only if the advisor's bias is sufficiently small — i.e., $\lambda < \underline{\lambda}_{\mathcal{F}}$, as claimed.

Next, consider case (2). In this region of parameters, optimality requires that I sets a cap equal to either $\alpha_{\varnothing}$ or $\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c$. We have that

$$\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c\right) - \mathcal{U}_{\mathcal{F}}\left(\alpha_{\varnothing}\right) = \left(\underline{\mu} - r_f\right)^2 \frac{2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2 - \left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\left(\underline{\sigma}^2\right)^2\lambda^2}{8\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\gamma\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)}. \tag{A5}$$

The sign of (A5) depends on the sign of the numerator

$$\zeta\left(\lambda, \sigma^2\right) \triangleq 2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2 - \left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\left(\underline{\sigma}^2\right)^2\lambda^2,$$

with

$$\frac{\partial\zeta\left(\lambda, \sigma^2\right)}{\partial\lambda} = -2\lambda\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\left(\underline{\sigma}^2\right)^2 < 0.$$

Setting the numerator equal to zero and solving for λ yields

$$\overline{\lambda}_{\mathcal{F}} \triangleq \frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)\sqrt{2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)}}{\underline{\sigma}^2\sqrt{\overline{\sigma}^2 + 2\underline{\sigma}^2}} > 0.$$

Since

$$\overline{\lambda}_{\mathcal{F}} - \lambda_1 = \frac{\Delta\sigma^2\sqrt{\overline{\sigma}^2}}{\underline{\sigma}^2\sqrt{\overline{\sigma}^2 + 2\underline{\sigma}^2}} > 0,$$

and

$$\overline{\lambda}_{\mathcal{F}} - \lambda_2 = -\frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)\sqrt{2\overline{\sigma}^2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)}}{\underline{\sigma}^2\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)} < 0,$$

it is optimal to set a cap equal to $\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c$ if and only if $\lambda_1 \leqslant \lambda < \overline{\lambda}_{\mathcal{F}}$.

Finally, consider case (3). In this region of parameters, optimality requires that I sets a cap $\alpha_{\varnothing}$, since she cannot profitably deviate by offering a different cap. Therefore, if $\lambda > \lambda_2$, the optimal cap is $\alpha_{\varnothing}$.

In sum: when A's bias is sufficiently small — i.e., $\lambda < \underline{\lambda}_{\mathcal{F}}$ — the optimal cap is $\alpha_{\mathcal{F}}\left(\overline{\mu}, \underline{\sigma}^2\right)$; when A's bias is neither too small nor too large — i.e., $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \overline{\lambda}_{\mathcal{F}}$ — the optimal cap for is $\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c$; when A's bias is very large, the optimal cap is $\alpha_{\varnothing}$. ■

Proof of Lemma 4. We compare the optimal caps imposed by the investor under different information acquisition policies.

Suppose that $\lambda \leqslant \underline{\lambda}_{\mathcal{F}}$. Using the results of Lemmas 2 and 3, in this region of parameters, we have

$$\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)-\alpha_{\mathcal{P}}\left(\overline{\mu}\right)=\left(\underline{\mu}-r_f\right)\frac{\overline{\sigma}^2\left(\overline{\sigma}^2-\underline{\sigma}^2\right)}{\gamma\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2+\underline{\sigma}^2\right)}>0.$$

Suppose now that $\underline{\lambda}_{\mathcal{F}}<\lambda\leqslant\lambda_{\mathcal{P}}$. In this region of parameters,

$$\alpha_{\mathcal{F}}\left(\underline{\mu},\overline{\sigma}^2\right)^c-\alpha_{\mathcal{P}}\left(\overline{\mu}\right)=-\left(\underline{\mu}-r_f\right)\frac{\overline{\sigma}^2-\underline{\sigma}^2}{\gamma\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)\left(\overline{\sigma}^2+\underline{\sigma}^2\right)}<0.$$

Hence, when $\lambda\leqslant\underline{\lambda}_{\mathcal{F}}$, the investor delegates a higher amount of wealth to an advisor who acquired full information, while when $\underline{\lambda}_{\mathcal{F}}<\lambda\leqslant\lambda_{\mathcal{P}}$, the investor delegates a higher amount of wealth to an advisor who acquired partial information. ■

Proof of Proposition 2. As in the proofs of Lemmas 2 and 3, given the optimal cap $\overline{\alpha}$, the investor's ex-ante utility with full information acquisition is

$$\begin{aligned}\mathcal{U}_{\mathcal{F}}\left(\overline{\alpha}\right)=&\sum_{\left(\widehat{\mu},\widehat{\sigma}^2\right)\in\Omega\times\Sigma}\Pr\left(\widehat{\mu},\widehat{\sigma}^2\right)\left(\overline{\alpha}\left(\widehat{\mu}-r_f\right)-\frac{\gamma}{2}\overline{\alpha}^2\widehat{\sigma}^2\right)+\\&+\sum_{\left(\mu,\sigma^2\right)\neq\left(\widehat{\mu},\widehat{\sigma}^2\right)}\Pr\left(\mu,\sigma^2\right)\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\mu,\sigma^2\right)\left(\mu-r_f\right)-\frac{\gamma}{2}\left(\left(1+\lambda\right)\alpha_{\mathcal{F}}\left(\mu,\sigma^2\right)\right)^2\sigma^2\right),\end{aligned}$$

while the investor's ex-ante utility with with partial information acquisition is

$$\begin{aligned}\mathcal{U}_{\mathcal{P}}\left(\overline{\alpha}\right)=&\sum_{\widehat{\mu}\in\Omega}\Pr\left(\widehat{\mu}\right)\left(\overline{\alpha}\left(\widehat{\mu}-r_f\right)-\frac{\gamma}{2}\overline{\alpha}^2\sigma^2\right)+\\&+\sum_{\mu\neq\widehat{\mu}}\Pr\left(\mu\right)\left(\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\mu\right)\left(\mu-r_f\right)-\frac{\gamma}{2}\left(\left(1+\lambda\right)\alpha_{\mathcal{P}}\left(\mu\right)\right)^2\sigma^2\right).\end{aligned}$$

First, let $\lambda\leqslant\underline{\lambda}_{\mathcal{F}}$. In this case,

$$\begin{aligned}&\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\overline{\mu},\underline{\sigma}^2\right)\right)-\mathcal{U}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right)\\&=\left(\underline{\mu}-r_f\right)^2\frac{-\underline{\sigma}^2\left(\left(\overline{\sigma}^2\right)^3+\left(\underline{\sigma}^2\right)^3+2\left(\overline{\sigma}^2\right)^2\underline{\sigma}^2-2\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\right)\lambda^2+\left(\left(\overline{\sigma}^2\right)^2+\left(\underline{\sigma}^2\right)^2\right)\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2}{8\gamma\overline{\sigma}^2\left(\underline{\sigma}^2\right)^3\left(\overline{\sigma}^2+\underline{\sigma}^2\right)},\end{aligned}$$

whose sign depends on the sign of the numerator,

$$\nu\left(\lambda,\sigma^2\right)\triangleq-\underline{\sigma}^2\left(\left(\overline{\sigma}^2\right)^3+\left(\underline{\sigma}^2\right)^3+2\left(\overline{\sigma}^2\right)^2\underline{\sigma}^2-2\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\right)\lambda^2+\left(\left(\overline{\sigma}^2\right)^2+\left(\underline{\sigma}^2\right)^2\right)\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2.$$

Notice that

$$\frac{\partial\nu\left(\lambda,\sigma^2\right)}{\partial\lambda}=-2\underline{\sigma}^2\left(\left(\overline{\sigma}^2\right)^3+\left(\underline{\sigma}^2\right)^3+2\left(\overline{\sigma}^2\right)^2\underline{\sigma}^2-2\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\right)\lambda<0,$$

$$\lim_{\lambda\to0}\nu\left(\lambda,\sigma^2\right)=\left(\left(\overline{\sigma}^2\right)^2+\left(\underline{\sigma}^2\right)^2\right)\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2>0,$$

and

$$\lim_{\lambda \to \underline{\lambda}_{\mathcal{F}}.} \nu\left(\lambda, \sigma^2\right) = \left(\underline{\sigma}^2\right)^2 \left(3\overline{\sigma}^2 + \underline{\sigma}^2\right) \frac{\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2}{\overline{\sigma}^2 + 2\underline{\sigma}^2} > 0,$$

Hence, when $\lambda \leqslant \underline{\lambda}_{\mathcal{F}}$, I's ex-ante utility is higher with full information than with partial information.

Second, if $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \lambda_{\mathcal{P}}$,

$$\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c\right) - \mathcal{U}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right) = \left(\underline{\mu} - r_f\right)^2 \frac{\left(3\overline{\sigma}^2 - \underline{\sigma}^2\right)\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\lambda^2 + 2\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2}{8\gamma\overline{\sigma}^2\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)},$$

which is always positive. Hence, I's ex ante utility is higher with full information than with partial information.

Third, if $\lambda_{\mathcal{P}} \leqslant \lambda < \overline{\lambda}_{\mathcal{F}}$,

$$\mathcal{U}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c\right) - \mathcal{U}_{\mathcal{P}}\left(\alpha_{\varnothing}\right) = \left(\underline{\mu} - r_f\right)^2 \frac{-\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + 2s\right)\lambda^2 + 2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2}{8\gamma\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)}, \quad \text{(A6)}$$

whose sign depends on the sign of the numerator,

$$\kappa\left(\lambda, \sigma^2\right) \triangleq -\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + 2s\right)\lambda^2 + 2\left(\overline{\sigma}^2 + \underline{\sigma}^2\right)\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2.$$

Notice that

$$\frac{\partial \kappa\left(\lambda, \sigma^2\right)}{\partial \lambda} = -2\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + 2s\right)\lambda < 0,$$

$$\kappa\left(\lambda_{\mathcal{P}}, \sigma^2\right) = \left(\underline{\mu} - r_f\right)^2 \frac{\left(3\overline{\sigma}^2 + 2\underline{\sigma}^2\right)\left(\overline{\sigma}^2 - \underline{\sigma}^2\right)^2}{16\gamma\overline{\sigma}^2\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2 + 2\underline{\sigma}^2\right)} > 0,$$

and

$$\kappa\left(\overline{\lambda}_{\mathcal{F}}, \sigma^2\right) = 0.$$

Hence, I's ex ante utility is higher with full information than without information.

Summing up, for $0 < \lambda < \overline{\lambda}_{\mathcal{F}}$, the investor's ex-ante utility is always higher with full information than with partial information and without information.■

Proof of Proposition 3 Without loss of generality, we focus on the region where $\lambda \leqslant \lambda_{\mathcal{P}}$ since when $\lambda > \lambda_{\mathcal{P}}$ the advisor never acquires partial information.

For a given cap $\overline{\alpha}$, the advisor's ex ante utility from paying $\kappa_{\mathcal{F}}$ and acquiring full information is

$$\mathcal{V}_{\mathcal{F}} = -\sum_{\left(\widehat{\mu}, \widehat{\sigma}^2\right) \in \Omega \times \Sigma} \Pr\left(\widehat{\mu}, \widehat{\sigma}^2\right)\left|\overline{\alpha} - (1+\lambda)\,\alpha_{\mathcal{F}}\left(\widehat{\mu}, \widehat{\sigma}^2\right)\right| - \kappa_{\mathcal{F}},$$

where $\left(\widehat{\mu}, \widehat{\sigma}^2\right)$ indicates the state(s) where the cap binds so that A obtains non-zero utility. Similarly, the advisor's ex ante utility from paying $\kappa_{\mathcal{P}}$ and acquiring partial information is

$$\mathcal{V}_{\mathcal{P}} = -\sum_{\widehat{\mu} \in \Omega} \Pr\left(\widehat{\mu}\right)\left|\overline{\alpha} - (1+\lambda)\,\alpha_{\mathcal{P}}\left(\widehat{\mu}\right)\right| - \kappa_{\mathcal{P}},$$

where $\widehat{\mu}$ denotes the state(s) where the cap binds. Therefore, the advisor has an incentive to acquire full information if and only if

$$\mathcal{V}_{\mathcal{F}} - \mathcal{V}_{\mathcal{P}} \geqslant \Delta\kappa \triangleq \kappa_{\mathcal{F}} - \kappa_{\mathcal{P}}. \tag{A7}$$

Suppose that $\lambda \leqslant \underline{\lambda}_{\mathcal{F}}$. Using the optimal caps from Lemmas 2 and 3, condition (A7) simplifies to

$$\mathcal{V}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\overline{\mu}, \underline{\sigma}^2\right)\right) - \mathcal{V}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right) = \left(\underline{\mu} - r_f\right)\lambda \frac{\left(3 - \frac{\overline{\sigma}^2}{\underline{\sigma}^2}\right)\overline{\sigma}^2}{4\gamma\left(\underline{\sigma}^2\right)^2\left(\frac{\overline{\sigma}^2}{\underline{\sigma}^2} + 1\right)} \geqslant \Delta\kappa. \tag{A8}$$

The sign of the left-hand-side of (A8) depends on the sign of the numerator

$$\beta\left(\overline{\sigma}^2, \underline{\sigma}^2\right) \triangleq \left(3 - \frac{\overline{\sigma}^2}{\underline{\sigma}^2}\right)\overline{\sigma}^2,$$

which is positive if and only if $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} \leqslant 3$. Moreover,

$$\frac{\partial\beta\left(\overline{\sigma}^2, \underline{\sigma}^2\right)}{\partial\left(\frac{\overline{\sigma}^2}{\underline{\sigma}^2}\right)} = -\overline{\sigma}^2 < 0.$$

First, let $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} \leqslant 3$. Hence, condition (A8) is satisfied if and only if

$$\left(\underline{\mu} - r_f\right)\lambda \frac{\left(3 - \frac{\overline{\sigma}^2}{\underline{\sigma}^2}\right)\overline{\sigma}^2}{4\gamma\left(\underline{\sigma}^2\right)^2\left(\frac{\overline{\sigma}^2}{\underline{\sigma}^2} + 1\right)} - \Delta\kappa \geqslant 0.$$

Setting this equal to zero and solving for the variance ratio we have

$$\sigma^*\left(\Delta\kappa\right) \triangleq \frac{3\overline{\sigma}^2\left(\underline{\mu} - r_f\right)\lambda - 4\gamma\left(\underline{\sigma}^2\right)^2\Delta\kappa}{\left(\underline{\mu} - r_f\right)\overline{\sigma}^2\lambda + 4\gamma\left(\underline{\sigma}^2\right)^2\Delta\kappa} < 3,$$

which is positive and is decreasing in $\Delta\kappa$, as expected. This implies that condition (A8) is satisfied if and only if $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} < \sigma^*\left(\Delta\kappa\right)$, and hence A has an incentive to acquire full information. Showing that A has no incentive to acquire full information when $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} \geqslant \sigma^*\left(\Delta\kappa\right)$ is straightforward.

Second, let $\frac{\overline{\sigma}^2}{\underline{\sigma}^2} > 3$ so that the left hand side of (A8) is negative. In this case, condition (A8) is never satisfied and A has no incentive to acquire full information.

Finally, suppose that $\underline{\lambda}_{\mathcal{F}} \leqslant \lambda < \lambda_{\mathcal{P}}$. In that case, A has incentive to acquire full information if and only if

$$\mathcal{V}_{\mathcal{F}}\left(\alpha_{\mathcal{F}}\left(\underline{\mu}, \overline{\sigma}^2\right)^c\right) - \mathcal{V}_{\mathcal{P}}\left(\alpha_{\mathcal{P}}\left(\overline{\mu}\right)\right) \geqslant \Delta\kappa.$$

As above, using the optimal caps from Lemmas 2 and 3, this condition simplifies to

$$-\left(\underline{\mu}-r_f\right)\frac{\left(\left(\overline{\sigma}^2\right)^2\left(\overline{\sigma}^2+\underline{\sigma}^2\right)+4\left(\underline{\sigma}^2\right)^3\right)\lambda+\left(\overline{\sigma}^2+\underline{\sigma}^2\right)\left(\overline{\sigma}^2-\underline{\sigma}^2\right)^2}{16\gamma\left(\underline{\sigma}^2\right)^2\left(\overline{\sigma}^2+2\underline{\sigma}^2\right)\left(\overline{\sigma}^2+\underline{\sigma}^2\right)}\geqslant\Delta\kappa,$$

which is never satisfied. Therefore, when the conflict of interest is large — i.e., $\underline{\lambda}_{\mathcal{F}}\leqslant\lambda<\lambda_{\mathcal{P}}$ — the advisor has no incentive to acquire full information.

In sum, A has an incentive to acquire full information if and only if $\lambda<\underline{\lambda}_{\mathcal{F}}$ and $\frac{\overline{\sigma}^2}{\underline{\sigma}^2}<\sigma^*$. Otherwise, he prefers to acquire partial information. ■

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