The flotation of companies on the stock market
A coordination failure model*

Marco Pagano

Università Bocconi, Milan, Italy and CEPR, London, UK

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In some countries the size of the stock market and the number of listed companies have persistently lagged behind the growth of the economy. Trading externalities can help explain this fact. An entrepreneur who goes public increases risk sharing opportunities for others. This externality can lead to an inefficiently low number of listed companies, and also generate multiple equilibria when flotation decisions are positively correlated across entrepreneurs. This happens if these face borrowing constraints and lack liquidity, and thus cannot diversify their portfolios unless they go public. It also happens if trading shares is costly, so that a sufficient number of issues must be listed to lure investors into the market.

1. Introduction

Why are the domestic equities listed on the U.K. stock market worth more than three times as much as those of all the German exchanges and almost six times those of the Milan exchange? And why were there fewer publicly traded companies in Italy in 1979 than in 1929? One rarely sees this sort of question addressed — or even raised — by economists. Yet these conspicuous differences point to more general, large-scale phenomena that should be of interest to economic analysis. First, the size of the stock market differs greatly between countries that are at the same stage of economic development, even after adjusting for differences in country scale. Second, in some countries the stock market has failed to keep pace with real economic growth for long periods of time, suffering secular stagnation.

These facts, which are set out in section 2 below, raise two important

Correspondence to: Marco Pagano, Via Catullo 64, 80122 Napoli, Italy.

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0014–2921/93/$06.00 © 1993—Elsevier Science Publishers B.V. All rights reserved
questions. If the size of the stock market is not necessarily related to the size of the economy, what are the factors that determine it? And what are the welfare implications of differences in stock market size?

The most obvious determinants of stock market size are the relevant institutional and regulatory arrangements. Many point to the involvement of the government and of banks in the market for corporate control, observing that in countries where the stock market is less developed, the government or the banks often have controlling shareholdings in many large companies. Others argue that the stock markets in Continental Europe are less attractive to ordinary investors than in the Anglo-Saxon countries because regulators are less resolute in repressing insider trading and protection to minority shareholders is feebler. While these conjectures are likely to contain some elements of truth, they are not completely persuasive. For instance, it is not clear if the relatively large role of the government and banks in the capital markets of Continental Europe is the cause or rather the effect of underdeveloped stock markets. Similarly, the repression of insider trading does not necessarily increase stock market liquidity, and thus participation in the stock market: Dennert (1992) shows that regulatory intervention that restrains but does not altogether eradicate insider trading may actually decrease market liquidity, and increase price volatility and risk premiums.

Another explanation for the large international differences in stock market size turns on the idea that they may reflect a multiplicity of equilibria, arising from 'thick market externalities' among actual or potential stock market participants: one investor's participation in a stock market makes it more attractive for others to participate, so that in equilibrium the decision to enter the market is correlated across individuals. A stock market may thus be trapped in persistent stagnation, where everyone expects low participation and this expectation is validated in equilibrium, although high participation equilibria also exist. Note that this approach potentially complements, rather than contradicting, explanations based on institutional and regulatory differences. In a world with multiple equilibria, institutional or regulatory innovations can have powerful effects, if they succeed in coordinating the expectations of market participants on a different equilibrium.

This insight has been elaborated in several models of asset markets where participation externalities give rise to coordination failures. In these models, market participation moderates the riskiness of the security [Chatterjee (1988), Allen and Gale (1991)] or the sensitivity of prices to the order flow [Pagano (1989a)] or both [Pagano (1989b)]. The number of market participants in turn depends on their expectations about the riskiness of the security or about price sensitivity to orders: the safer the asset and the more liquid the market, the greater the number of traders. This feedback can produce multiple Pareto-ranked equilibria – participation being positively correlated with price stability or market liquidity across equilibria.
The present paper focuses on another 'thick market externality' that may generate multiple equilibria in stock market size, and particularly in the number of listed companies. The externality stems from the fact that when an entrepreneur floats his company's shares on the stock market, he opens up new risk-sharing opportunities for other investors, letting them better diversify their equity portfolios. If going public entails private costs (such as the danger of a takeover and loss of the private benefits of control), the equilibrium number of traded companies may be inefficiently low (section 3.1). Moreover, in the presence of capital market imperfections, an entrepreneur's decision to go public can induce others to do the same, creating the potential for multiple equilibria in the number of traded companies.

I consider two examples where this occurs. In the first, the credit market is imperfect: one cannot borrow to buy shares. Then, if entrepreneurs have no liquid wealth, they must sell off part of their own shares to diversify their portfolio. But the attractiveness of floating their own shares depends on the behaviour of other entrepreneurs. If few of these are expected to float their shares, correspondingly few will demand shares, and the revenue from flotation will be low; in addition, the gains from diversification will be modest because few issues can be bought on the market. Thus, everyone will be reluctant to float his shares. Conversely, if many companies are expected to go public, the incentive to do so will be comparatively high. Thus equilibria with different numbers of traded companies can result, depending on expectations (section 3.2.1).

In the second example, instead, the friction is located in the stock market: trading shares is costly. Then, the number of investors buying shares depends on the magnitude and variety of the supply of shares. If few issues are expected to be listed on the stock exchange, the number of buyers will be expected to be low, making flotation unattractive -- and the converse will happen if many issues are expected to be listed on the exchange (section 3.2.2).

In the absence of some coordinating mechanism, a stock market may thus be trapped at a low-level equilibrium with very few companies listed, regardless of potential entrants. As always when externalities and strategic complementarities result in multiple equilibria, high-level equilibria Pareto-dominate low-level equilibria [Cooper and John (1988)]: everyone gains from superior risk-sharing in the equilibria where more entrepreneurs decide to float.

While these results parallel those of the participation externality models mentioned above, the analysis presented here turns on a different externality. In those models, additional investors reduce the riskiness of a security that is already traded or render its market more liquid. Here, instead, the decision to list new securities enhances risk sharing opportunities, and possibly induces more companies to list their own securities. A similar 'contagion
mechanism' among security issuers is actually found in a recent paper by Gale (1992), which centers on the external benefits from issuing 'standard securities'. Being issued by many companies, standard securities are well-known, and investors can diversify away their idiosyncratic risk by drawing on a rich menu of issues. This creates a potential coordination failure: no single company will want to issue a non-standard security, but a large number of companies may find it worthwhile, effectively transforming it into a new standard security. Investors may be willing to bear the cost of becoming informed about the new security if they anticipate offsetting gains from buying a large and diversified portfolio of the new issues. Thus the new security may sell at an attractive price, provided the issuers reach a critical mass.

On another front, this paper is related to the growing literature on why companies decide to go public or to stay private. Here the choice turns on the trade-off between the advantages of diversification for the initial owner and the costs of floating. The latter are not modelled explicitly, and can be conceived of as deriving from registration and underwriting fees, underpricing of the initial public offering, or heightened danger of an undesired loss of control.

Others have proposed different explanations for the decision to go public. Yosha (1992) notes that publicly traded companies are required to disclose private information, which may benefit their competitors. In the presence of flotation costs, he shows that this may deter companies with sensitive information from going public. In Holmström and Tirole (1992), instead, publicly traded companies benefit from investors' monitoring activity: exploiting the information reflected in stock prices, their shareholders can design more efficient managerial compensation schemes. But market monitoring also entails a cost: the initial share price must be low enough to compensate uninformed traders for their losses to the informed speculators. Finally, Zingales (1992) models the decision to go public as an optimal strategy to surrender corporate control. He shows that selling off a stake in the company to a dispersed ownership may increase the surplus that the initial owner can extract from the private sale of the company, if there are private benefits to control. The decision to go public and the amount to be floated depend on the relative value that the initial owner and the subsequent potential buyer place on the verifiable profits of the company and on the private benefits from control.

2. Some puzzling facts

The size of national stock markets, measured by the total worth of domestic companies traded, varies widely (table 1, column 2). This is to be expected, since the size of a nation's stock market should be related to the
Table 1
International differences in stock market value and number of publicly traded companies (1987, end of year values).*

<table>
<thead>
<tr>
<th>Stock exchange</th>
<th>Total market value of domestic shares (millions of U.K. pounds)</th>
<th>Total market value of domestic shares as percent of GDP</th>
<th>Number of domestic companies traded</th>
<th>Average value of domestic companies traded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam</td>
<td>46,106</td>
<td>35.4</td>
<td>248</td>
<td>186</td>
</tr>
<tr>
<td>Athens</td>
<td>2,393</td>
<td>8.5</td>
<td>117</td>
<td>20</td>
</tr>
<tr>
<td>Australia b</td>
<td>54,532</td>
<td>45.7</td>
<td>1,488</td>
<td>37</td>
</tr>
<tr>
<td>Brussels</td>
<td>23,013</td>
<td>26.4</td>
<td>192</td>
<td>120</td>
</tr>
<tr>
<td>Copenhagen</td>
<td>10,840</td>
<td>17.5</td>
<td>269</td>
<td>40</td>
</tr>
<tr>
<td>Germany b</td>
<td>116,710</td>
<td>17.0</td>
<td>507</td>
<td>230</td>
</tr>
<tr>
<td>Helsinki</td>
<td>10,642</td>
<td>19.6</td>
<td>49</td>
<td>217</td>
</tr>
<tr>
<td>Madrid</td>
<td>38,399</td>
<td>21.7</td>
<td>327</td>
<td>117</td>
</tr>
<tr>
<td>Milan</td>
<td>64,425</td>
<td>14.0</td>
<td>204</td>
<td>315</td>
</tr>
<tr>
<td>New Zealand</td>
<td>9,292</td>
<td>49.7</td>
<td>291</td>
<td>32</td>
</tr>
<tr>
<td>Oslo</td>
<td>6,560</td>
<td>14.3</td>
<td>138</td>
<td>48</td>
</tr>
<tr>
<td>Paris</td>
<td>92,117</td>
<td>17.1</td>
<td>481</td>
<td>191</td>
</tr>
<tr>
<td>Stockholm</td>
<td>41,156</td>
<td>41.8</td>
<td>150</td>
<td>274</td>
</tr>
<tr>
<td>Tokyo</td>
<td>1,451,322</td>
<td>99.4</td>
<td>1,532</td>
<td>947</td>
</tr>
<tr>
<td>Toronto</td>
<td>108,773</td>
<td>47.9</td>
<td>1,147</td>
<td>95</td>
</tr>
<tr>
<td>U.K.</td>
<td>363,170</td>
<td>87.0</td>
<td>2,135</td>
<td>170</td>
</tr>
<tr>
<td>Vienna</td>
<td>3,991</td>
<td>5.6</td>
<td>70</td>
<td>57</td>
</tr>
<tr>
<td>USA c</td>
<td>1,360,676</td>
<td>49.2</td>
<td>6,829</td>
<td>166</td>
</tr>
<tr>
<td>Zurich</td>
<td>68,301</td>
<td>65.3</td>
<td>166</td>
<td>411</td>
</tr>
</tbody>
</table>

*Sources: Stock market data are drawn from the Quality of Markets Quarterly, The International Stock Exchange, Spring 1988, Appendix 5, p. 68, except for Toronto, New Zealand and Oslo (where they are drawn from Morgan Stanley Capital International Perspectives). The GDP data are drawn from the OECD Economic Outlook, 1990.

bAssociation of Exchanges.

cNew York Stock Exchange, American Stock Exchange and NASDAQ.

size of the host economy. But the enormous international variation cannot be wholly explained by the differing sizes of the corresponding economies. Even after scaling the value of national stock markets by GDP, the international variation remains striking (column 3). In the OECD countries outside Europe, the value of the stock market ranges between 45 and 100 percent of GDP. The corresponding figure in most European countries is in the order of 20 percent or less; exceptions being the U.K., Switzerland, Sweden and the Netherlands. In economies whose level of economic development is comparable, then, financial intermediation appears to rely to vastly differing extent on the stock market: in the U.K. the ratio of stock market value to GDP is five times larger than in Germany, France, Denmark and Finland, and six times larger than in Italy and Norway.

One may wonder if these differences are mainly due to the number of
companies in each market or to their average size. There is no clearcut answer to this question (columns 4 and 5). In Greece, Denmark, Norway and Austria, the size of public companies is quite small by international standards. But in France, Germany, Italy and Finland, the relatively modest size of the stock market appears to be related mainly to an abnormally small number of traded companies. In these countries, the average listed company is larger than its counterpart in the U.K., U.S., Canada, Australia or New Zealand.

These international statistics convey the impression that the size of the stock market and the number of public companies do not bear a clear relationship to the size of the corresponding economy. One gathers the same impression from secular data for Italy, where the development of the stock market appears largely decoupled from that of the real economy. Fig. 1 shows the number of listed companies from 1860 to 1992. After a steep rise around the turn of the century, the number has stagnated around the same value for over 70 years. It is striking that while the rise at the start of the century coincided with the first spurt of Italian industrialization, no comparable increase in the number of listed companies accompanied the tremendous growth of manufacturing and GDP during Italy's 'economic miracle' of the
The number of companies traded started growing again only in the late 1980s, concomitantly with the introduction of mutual funds. The funds brought diversified stock portfolios within the reach of small investors and thus triggered an increase in the demand for equities and a rise in stock prices, which in turn lured new companies into the stock market.

Not only has the number of publicly listed companies stagnated for the better part of a century, but the total worth of companies traded has not kept pace with the economy as a whole. In fact, the ratio of total stock market value to GDP has been on a downward secular trend. This is apparent from fig. 2, displaying data for the last 54 years, and from the fact that at the turn of the century the value of the Italian stock exchange as a proportion of GDP was much higher than it is now: in 1906, it stood at 26.3 percent of GDP (not shown in fig. 2), compared with 12.6 percent in 1991.

It should be noted that Italy is not the only country where the number of listed companies has stagnated for so long. From 1939 to 1988 the number of domestic share issues officially traded in Frankfurt has fluctuated between a minimum of 193 and a maximum of 295, without any appreciable trend. And on average it has been lower than in the early decades of the century: the domestic share issues traded in Frankfurt were 291 in 1912, 318 in 1920, 281 in 1930 and 370 in 1935, to be compared with an average of 262 issues for the 1954–1988 period. This evidence for Italy and Germany is in sharp contrast with the record of the New York stock exchange, where listed companies have increased steadily from 855 in 1930 to 1774 in 1990. This adds a time dimension to the cross-country data reported in table 1. The long-run evidence for Italy and Germany suggests that the comparative smallness of most European stock markets reflects their inability to keep pace with the growth of the corresponding economies. Why can this happen?

The illustrative model proposed below offers a partial explanation in the

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1 In the decade 1897–1907 real GNP grew at a yearly rate of 3.8 percent, the highest growth rate achieved until after World War II; over the same period, the number of publicly listed companies increased by 21.1 percent a year (from 24 to 163). By contrast, during the early postwar period real GDP grew by a staggering 17.4 percent per year in 1945–1950 and 5.8 percent in 1950–1960; but the annual increase in the number of listed companies remained modest, between 1.1 and 1.8 percent (from 116 to 127 in 1945–1950, and from 127 to 142 in 1950–1960). Real GDP data are drawn from Banca d'Italia (1990, table 46).

2 The 1906 figure for the value of the Italian stock market is drawn from Bonelli (1971), and the corresponding GDP estimate is from table XII.4.1, in Fua (1969).

3 These data are drawn from the Frankfurt Stock Exchange Statistics, 1988, pp. 8 and 51. Unfortunately data for companies – as opposed to share issues – listed in Frankfurt are available only from 1970 onwards.

4 Their number has declined slightly only during the Great Depression and the decade of the oil crisis – between 1975 and 1985. This impression of steady progression is reinforced by the longer time series available for the number of share issues listed in New York, which have risen almost uninterruptedly from 136 issues in 1864 to 2,284 in 1990. The relevant data are drawn from the NYSE Fact Book, various issues.
form of positive externalities among stock market participants. Given such externalities, the rational owner of a private company will make the decision to float his company's stock in correlation with decisions by other entrepreneurs. Depending on the expectations of each about the behaviour of others, the market may settle at an equilibrium with few public companies (or even none), or at one with many public companies. Although the model proposed here is static, it can be stretched into the historical description of an economy by iterating it over time. The model then indicates that as long as expectations do not change the stock market may remain trapped in a low-level equilibrium; when they do change, possibly because of an institutional innovation, a substantial number of companies will move into the stock market together. This is consistent with the historical pattern in Italy, where the number of listed companies has held roughly constant for decades at a time, and then suddenly soared, as in the first decade of this century and again in the 1980s.

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**Fig. 2.** Value of the Italian stock market as a percent of GDP. Sources: The data for the value of the stock market are drawn from *Annuario Italiano di Statistica* (until 1954), Pivato (1965, 1972), Pivato and Seognamiglio (1972) and the *Listino Ufficiale della Borsa Valori di Milano*, various issues. The GDP data are drawn from Table XII.4.1.A in Fuà (1969) until 1953, from the *International Financial Statistics* of the IMF for 1954–1959, and from the *Economic Outlook* of the OECD for 1960–1992.
3. The model

The model posits that a company's choice to go public turns on the trade-off between the advantages of diversification for the initial owner and the fixed cost of floating the stock. In fact, floating shares on the stock market is generally a lengthy and expensive business: there are legal and brokerage fees to be paid,\(^5\) costs stemming from the underpricing of initial public offerings,\(^6\) administrative expenses due to the obligation of certifying balance sheets and disclosing other information, and so on. Most of these are fixed costs, so that the total cost of flotation does not increase in proportion to the size of the firm applying for listing.

Beside these, there is another once-and-for-all cost of going public: the greater danger of an undesired loss of control via a hostile take-over bid. One could object that rational agents should not value corporate control per se: the stock market should allocate the control of each company to the investors that maximize its value, and if the initial owners are forced to surrender control to a more efficient group, the changing of the guard is in everybody's interest, as it will raise the value of all shareholders' wealth. In practice, however, the control group often appropriates profits at the expense of minority interests in the form of managerial perks. These private benefits of control can outweigh the gains from selling to a more efficient control group; or the owner of a firm may enjoy the power and prestige associated with its management for their own sake, and thus choose to forgo the pecuniary gains.

The calculation of this individual trade-off neglects that the flotation of private companies may also yield a collective benefit in the form of improved risk-sharing. The owner of an unlisted company has an incentive to free-ride on the others, buying their shares on the market without floating his own company. The decentralized outcome may be inefficient compared with the first-best solution, which weighs the resource costs of listing against the collective gains from improved risk-sharing (section 3.1).

In and of itself, this inefficiency does not imply the existence of multiple equilibria. Multiple equilibria result when the choices of the entrepreneurs feature also strategic complementarity: each entrepreneur not only gains when the other companies float, but has also a greater incentive to float his own company's shares as a result. In the model, such a strategic complemen-

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\(^5\) Ritter (1987) estimates that in the United States these costs average 14% of the value of the equities floated.

\(^6\) The underpricing of initial public offerings varies considerably from country to country, but is never negligible. In percentage terms, estimates concerning the underpricing of the first trading day after the public offering range from 8.6% in the U.K. [Levis (1990)] and 9.3% in Canada [Jog and Riding (1987)] to 29.2% in Australia [Finn and Hingham (1988)] and 27.3% in Italy [Cherubini and Ratti (1991)]. Intermediate values are found in the U.S., where Ritter (1987) estimates underpricing to be 14.8% for 'firm commitment'; and in Germany, where Wittleder and Wasserfallen (1990) place it at 17.6%.
tarity arises in the presence of frictions in the credit market (borrowing constraints) or in the stock market (transaction costs) (section 3.2). The resulting equilibria can be Pareto-ranked, those with more public companies corresponding to superior allocations.

3.1. The baseline model: Inefficiency of the decentralized outcome

There are \( M + N \) individuals in the economy, indexed by \( j = 1, \ldots, M + N \). The first \( M \) of these individuals are 'entrepreneurs'; each of them is endowed with equity in a risky project, of which he is the sole owner – a 'family business', which for simplicity will carry the same index \( j \) as its owner. The remaining \( N \) individuals are 'ordinary investors', who are born with no risk capital, but with \( w_0 \) units of 'money' that can be invested in whichever stocks are traded on the market and in a safe asset yielding a gross return \( R \).

All individuals live for two periods. In period 0, entrepreneurs choose if they want to float their shares, determining the menu of traded securities; then, conditional on this menu, both entrepreneurs and ordinary investors choose their optimal portfolios and trade accordingly on the stock exchange. In period 1, returns are paid to security holders: debt is paid off, and shares pay a liquidating dividend.

Each company’s shares are equal to its units of real capital, an exogenous number \( K_i \) for company \( i \). A share in company \( i \) trades at price \( p_i \) and yields a gross return \( R_i \), which is normally distributed with mean \( \mu_i \) and variance \( \sigma_i^2 \). Returns are uncorrelated across companies.

Initially, it is assumed that there are no restrictions on short sales or borrowing. Anyone can borrow any amount at the interest rate \( r = R - 1 \). Later we shall see that removing this assumption opens the door to the existence of multiple equilibria (see section 3.2, below).

To go public, each entrepreneur must pay a fixed cost \( c \). If he does, he can sell any amount of his shares at the price it will command, and reinvest the proceeds of the sale on the market. If he chooses not to float, he will be unable to trade his shares at any price: of course, this is an extreme assumption, intended to stress the very limited marketability of companies whose equity is not publicly traded.

All the individuals in the economy have identical preferences, which are represented by a negative exponential expected utility function in terminal wealth \( w_{t_{ij}} \) for individual \( j \). Let \( k_{ij} \) denote the number of shares in firm \( i \) demanded by person \( j \). The set of publicly traded firms, \( \mathcal{F}^* \), is endogenous, since it is determined by the equilibrium choices of the \( M \) entrepreneurs.

The choice problem of entrepreneurs consists of two sequential stages: the flotation choice and the portfolio choice. When he decides about flotation, entrepreneur \( j \) anticipates that a certain set of securities will be traded, that he will choose his portfolio optimally (given this set of traded securities), and
that markets will clear. Moreover, he assumes that his decision about flotation of company \( j \) has no effect on the corresponding choices of other entrepreneurs. Thus, the problem must be solved by backward induction: first, one finds the optimal portfolio choice rule for an arbitrary set of traded securities; second, one determines the optimal criterion for the flotation choice, conditioning on optimal portfolio choice, thus pinning down the set of traded securities. If expectations are rational, the latter coincides with that assumed at the portfolio choice stage.

Now consider entrepreneur \( j \) at the moment he chooses whether floating his company’s shares. He expects another \( T \) companies, forming a subset \( \mathcal{I} \) of the \( M \) companies, to go public, whether his own firm does or not. So he expects the set of traded companies to be formed by \( \mathcal{I} \) if company \( j \) stays private, or by the union of \( \mathcal{I} \) and company \( j \) if it goes public. Thus he anticipates his portfolio choice problem to be

\[
\begin{align*}
\max_{\{\tilde{w}_{ij}\}} E[V(\tilde{w}_{1j})] &= E[-Ae^{-b\tilde{w}_{ij}}], & A > 0, \ b > 0, \ (1)
\end{align*}
\]

subject to

\[
\begin{align*}
\tilde{w}_{ij} &= \sum_{i \in \mathcal{I}, j} (\tilde{R}_i - R_{ij})k_{ij} + [(\tilde{R}_j - R_{ij})k_{jj} + R_{pj}K_j - c]1_j \\
&\quad + \tilde{R}_jK_j(1-I_j) \quad \text{for } j = 1, \ldots, M, \quad (2a)
\end{align*}
\]

where the indicator function \( I_j \) equals 1 if entrepreneur \( j \) floats his company, and 0 otherwise.

For non-entrepreneurs, the problem instead reduces to maximizing (1) subject to

\[
\begin{align*}
\tilde{w}_{ij} &= \sum_{i \in \mathcal{I}^*} (\tilde{R}_i - R_{ij})k_{ij} + Rw_0 \quad \text{for } j = M + 1, \ldots, M + N, \quad (2b)
\end{align*}
\]

where the set \( \mathcal{I}^* \) results from the equilibrium decisions about going public. Thus, if the expectations of entrepreneur \( j \) are rational, \( \mathcal{I}^* \) equals \( \mathcal{I} \) if \( I_j = 0 \) and equals \( \mathcal{I} \cup j \) if \( I_j = 1 \).

As we know, on the assumption of normally distributed returns these problems can be restated as that of maximizing the mean–variance utility function:

\[
\begin{align*}
\max_{\{\tilde{w}_{ij}\}} E[U(\tilde{w}_{ij})] &= E(\tilde{w}_{ij}) - \frac{b}{2} \text{Var}(\tilde{w}_{ij}),
\end{align*}
\]

a monotone transformation of the expected utility in (1). Substituting from
eqs. (2a) and (2b), we can then rewrite the maximization problem of entrepreneurs as

$$\text{Max } E[U(\hat{w}_{1j})] = \sum_{i \in \mathcal{I}} (\mu_i - Rp_i) k_{ij}$$

$$+ \left[ (\mu_j - Rp_j) k_{jj} + Rp_j K_j - c \right] I_j + \mu_j K_j (1 - I_j)$$

$$- \frac{b}{2} \left( \sum_{i \in \mathcal{I}} \sigma_i^2 k_{ij}^2 + \sigma_j^2 k_{jj}^2 I_j + \sigma_j^2 K_j^2 (1 - I_j) \right)$$

for $j = 1, \ldots, M,$ \hfill (3a)

and that of ordinary investors as

$$\text{Max } E[U(\hat{w}_{1j})] = \sum_{i \in \mathcal{I}^*} (\mu_i - Rp_i) k_{ij} + R\omega_0$$

$$- \frac{b}{2} \sum_{i \in \mathcal{I}^*} \sigma_i^2 k_{ij}^2 \text{ for } j = M + 1, \ldots, M + N. \hfill (3b)$$

For both problems, the first-order condition with respect to $k_{ij}$ yields the same solution:

$$k_{ij} = \frac{\mu_i - Rp_i}{b \sigma_i^2}, \quad j = 1, \ldots, M + N. \hfill (4)$$

Note that for the moment each entrepreneur is assumed to be a price-taker on the stock market: that is, if he goes public he does not exploit the monopoly power stemming from sole ownership of an asset that is not substitutable with other traded assets (owing to the uncorrelated nature of the stock returns). At the end of this section, we shall analyze the consequences of relaxing this assumption.

If firm $j$ is floated on the market, its shares trade at the equilibrium price

$$p_j = \frac{1}{R} \left( \mu_j - b \sigma_j^2 \frac{K_j}{M + N} \right), \hfill (5)$$

and the equity that the initial owner retains in his former company, $k_{jj}$, is given by (4).

Now consider the choice about going public. In this choice the owner takes the corresponding decisions by other entrepreneurs as given, and anticipates that his portfolio choices will be made optimally, according to eq. (4). To analyze the flotation decision, it is convenient to work with the
transformed expected utility function $E[U(\cdot)]$ in (3a). The equilibrium value of $j$'s expected utility ($j = 1, \ldots, M$) is obtained substituting eqs. (4) and (5) in function (3a). If his company stays private ($I_j = 0$), he gets

$$E[U(\tilde{w}_{1j}) | j \text{ private}] = b \sum_{i \in J} \left( \frac{K_i \sigma_i}{M + N} \right)^2 + \mu_j K_j - \frac{b}{2} K_j^2 \sigma_j^2,$$  

(6)

whereas if he decides that his company should go public ($I_j = 1$), his expected utility is

$$E[U(\tilde{w}_{1j}) | j \text{ public}] = b \left[ \sum_{i \in J} \left( \frac{K_i \sigma_i}{M + N} \right)^2 + \left( \frac{K_j \sigma_j}{M + N} \right)^2 \right] + \left( \mu_j - b \sigma_j^2 \frac{K_j}{M + N} \right) K_j - c,$$  

(7)

where the penultimate term captures the income effect of the sale of firm $j$'s shares at the equilibrium price $p_j$ in (5), and the last term is the fixed cost of flotation.

The difference between (7) and (6) proves to be

$$\Delta E[U(\tilde{w}_{1j})] = b \left( \frac{K_j \sigma_j}{M + N} \right)^2 (M + N - 1)^2 - c.$$  

(8)

Since $M + N > 1$, the first term is positive. Thus, if there were no flotation costs ($c = 0$), it would always pay to list one's firm on the market, sell it off and reallocate one's portfolio freely. Once flotation costs are introduced, this need no longer be true: if $c$ is high enough, expression (8) turns negative. The risk-sharing gains from going public are increasing in the size of the company, $K_j$, and in its riskiness, $\sigma_j^2$.

In this baseline model the decision is unaffected by the entrepreneur's expectations concerning the corresponding decisions of the other entrepreneurs: the gain from floating one's own firm is invariant with respect to the number and identity of the firms in the set $J$. As we shall see, in the presence of capital market imperfections this result no longer holds. But even in the present version of the model, decentralized decision-making may restrict the number of public companies below its most efficient level.

It is easy to show that, when the company of entrepreneur $j$ goes public, the expected utility of any other market participant $i \neq j$ (whether $i$ is an entrepreneur or not) rises by

$^7$Nor would it hold if the returns to the shares of different companies were correlated.
From expressions (8) and (9), one obtains the change in the expected overall welfare of the economy from the flotation of company \( j \):

\[
\Delta E[U(\tilde{w}_{1i})] = \frac{b}{2} \left( \frac{K_i \sigma_i}{M + N} \right)^2, \quad \forall i \neq j.
\]  

(9)

The social gain is larger than that of individual \( j \) alone – expression (10) is larger than expression (8) – because the addition of a new traded company augments the consumer surplus that all other investors gain by bearing firm \( j \)'s risk. Thus we have an interval in which floating a company is not advantageous for its owner – expression (8) is negative – but is beneficial for society as a whole – expression (10) is positive:

\[
\sum_{i=1}^{M+N} \Delta E[U(\tilde{w}_{1i})] = \frac{b}{2} \left( \frac{K_i \sigma_i}{M + N} \right)^2 (M + N)(M + N - 1) - c.
\]  

(10)

If the parameters of the model fall in the region defined by (11), firm \( j \) should be publicly traded but its owner will have no incentive to float it on the market. In other words, the decentralized outcome is inefficient. The size of this inefficient region shrinks to zero as the total number of investors, \( N \), becomes large, so that the market grows deep and its risk bearing ability increases. Conversely, the interval of inefficiency increases if the costs captured by \( c \) are purely private costs, rather than social costs – so that they are not to be netted out from the change in social welfare in eq. (10). Then the condition for inefficiency reduces only to the left-hand inequality in (11): if any of the \( M \) companies is unlisted, the decentralized solution is inefficient, as the costs of flotation are socially irrelevant. This is the case if these costs consist in the danger of losing control rather than legal and brokerage fees: the loss of control by the initial owner need not imply any loss for society (the private benefits of control being transferred to the new owner).

So far, it has been assumed that when a company goes public, its owner acts as a price-taker, ignoring the impact that the size of the initial public offer will have on the price. But in fact when returns are uncorrelated, the price of each stock depends only on that company's supply of shares to the market. In this situation, one would expect the owner to behave as a monopolist in deciding how many shares to put on the market. The appendix presents a modified version of the model where the price-taker assumption is relaxed. It turns out that if entrepreneur \( j \) behaves strategically in floating his company's shares, the change in his expected utility associated with the decision to go public is
\[
\Delta \mathbb{E}[U(\tilde{w}_{1j})] = \frac{b}{2} \left( \frac{K_j \sigma_j}{M + N + 1} \right)^2 \left[ (M + N)^2 - 1 \right] - c,
\]
(12)

The only difference with eq. (8), the corresponding expression under price-taking, is that, other things being equal, the change in expected utility is larger. Taking account of the adverse price relation, the entrepreneur holds the volume of shares offered to the public below the competitive level and accordingly ends up gaining more from his initial public offer.

Conversely, the investing public stands to benefit less from the initial public offer. The overall gain of society when company \( j \) goes public is

\[
\sum_{i=1}^{N+M} \Delta \mathbb{E}[U(\tilde{w}_{1i})] = \frac{b}{2} \left( \frac{K_j \sigma_j}{M + N + 1} \right)^2 (M + N - 1)(M + N + 2) - c,
\]
(13)

which is smaller than expression (10), the social gain under price taking. As a result, the region in which floating a company is socially beneficial but privately unattractive becomes smaller than under competitive behaviour: the relevant inequality is

\[
\frac{M + N - 1}{M + N + 1} < \frac{2c}{bK_j^2 \sigma_j^2} < \frac{M + N - 1}{M + N + 1} \left( 1 + \frac{1}{M + N + 1} \right),
\]
(14)

which defines a smaller interval than its competitive analogue in (11). So the presence of monopolistic behaviour brings private incentives more into line with social ones regarding the decision to float; but given the decision to float, the social gains are smaller because entrepreneurs do not put the optimal amount of shares on the market.

3.2. Capital market imperfections and multiple equilibria

To this point, capital markets have been assumed to be frictionless, except for the existence of flotation costs: everyone could borrow on perfect credit markets, and trade shares without paying any transaction cost. We have seen that, in this setting, the decision to go public by one entrepreneur affects the welfare of other entrepreneurs (and of ordinary investors), but not their decision to go public. This is because the decision to float company \( j \) does not affect the potential demand for the shares of other, yet unlisted companies.

In the presence of capital market imperfections, this is no longer necessarily true: the very fact that a company goes public can raise the demand for other companies' shares, and thus induce other unlisted companies to go
In the first, the credit market is imperfect. One cannot borrow to buy equities. Faced with binding liquidity constraints and with all his wealth tied up in the 'family business', an entrepreneur can diversify only if he floats part of his shares. If he does, he increases the potential demand for unlisted companies' shares by investing on the stock market, and thus raises their incentive to go public; in addition, he increases the scope for diversification by floating his company's shares, and thereby provides further incentive to other entrepreneurs to go public and diversify. Thus, each entrepreneur's decision to go public is correlated with that of the others.

In the second example, the stock market is imperfect: ordinary investors face a fixed transaction cost to buy equities, which may be the cost of learning how to evaluate shares or to find the right broker. As a result, they invest in the stock market only if they can buy a sufficiently large and well diversified bundle of shares. When a new company is floated, the increase in the supply and variety of shares attracts more investors to the stock market. This raises share prices and thus the incentive to go public for other private companies. Again, flotation decisions are correlated in equilibrium.*

3.2.1. Borrowing constraints

If an entrepreneur can borrow on a perfect credit market, his portfolio decision is independent of the choice about floating his company's shares. If he cannot borrow, the two decisions are no longer independent: with no liquid wealth, the entrepreneur cannot diversify unless his company goes public. As a result, the number of market participants increases one-for-one with the number of publicly traded companies: if T companies go public, the number of market participants is N + T (N ordinary investors plus T entrepreneurs). It is assumed that, by floating his company's shares, an entrepreneur does not need to borrow in order to diversify his portfolio: the

*Another factor that may create interdependence between flotation decisions is the correlation between stock returns, a possibility not considered in the model above. If this correlation is positive, it tends to generate strategic substitutability in the decisions to go public, with effects that run opposite to those studied in this section: the flotation of a company depresses the prices of other shares, and thus discourages other unlisted companies from going public. The strength of this effect depends on the size of the positive correlation between stock returns. In the extreme case in which returns are perfectly correlated (so that all the stocks are perfect substitutes in investors' portfolios) one can show that an increase in the number of listed companies always discourages further entry, in both the models of this section. As a result, both models feature a unique equilibrium (the derivations for this case are available from the author upon request). The reverse obviously occurs if stock returns are negatively correlated, a feature that reinforces the strategic complementarity between flotation decisions highlighted in the text.
revenue from floating his shares exceeds the value of his desired portfolio plus the flotation cost of his own company.\(^9\)

For convenience, the model is made symmetric, assuming that all companies are of the same size \(K\), and that their returns per share have the same mean \(\mu\) and variance \(\sigma^2\). As a result, the shares of any public company trade at the same equilibrium price \(p\). Initially, entrepreneurs whose companies go public are assumed to be price takers when tendering their shares to the public; later we shall see how the results are affected by monopolistic behaviour.

Under these assumptions, the expected utility associated with the decision to stay private \((I_j = 0)\) is simply the autarky level:

\[
E[U(\hat{w}_{1j}) | j \text{ private}] = \mu K - \frac{b}{2} K^2 \sigma^2, \tag{15}
\]

while that deriving from going public is

\[
E[U(\hat{w}_{1j}) | j \text{ public}] = \mu K - \frac{b}{2} (2N + T + 1) \left( \frac{K \sigma}{N + T + 1} \right)^2 - c. \tag{16}
\]

Thus the change in expected utility is

\[
\Delta E[U(\hat{w}_{1j})] = \frac{b}{2} \left[ (N + T)^2 + T \right] \left( \frac{K \sigma}{N + T + 1} \right)^2 - c, \tag{17}
\]

which is increasing in \(T\), the number of other companies that entrepreneur \(j\) expects to be publicly traded, and in \(N\), the number of ordinary investors (non-entrepreneurs) who participate to the stock market.

If the expression in (17) is positive, company \(j\) will go public. Due to the symmetry of the model, all the \(M\) companies will go public \((T + 1 = M)\). Conversely, if the expression is negative, no company will go public, and the stock market will fail to get off the ground. Denote by \(T^*\) the value of \(T\) for which expression (17) is zero. If the critical value \(T^* \in (0, M]\), there are two equilibria: if each entrepreneur expects the number of other publicly traded companies, \(T\), to be lower than \(T^*\), autarky will result, and the only rational expectation is \(T = 0\); vice versa, if \(T\) is expected to exceed \(T^*\), the rational

\(^9\)Suppose that \(T + 1\) entrepreneurs list their companies on the market. Owing to the symmetry assumption (see below in the text), in equilibrium each of them will hold \(K/(N + T + 1)\) shares of each listed company. His desired portfolio is worth \((T + 1)pK/(N + T + 1)\), where \(p\) is the price of a share, while his revenue from selling his company on the market, net of the flotation cost, is \(pK - c\). For the borrowing constraint not to be binding, one needs to assume that the inequality \(NpK/(N + T + 1) > c\), where \(p = (1/R)[\mu - ba^2K/(N + T + 1)]\), holds for all values of \(T\). This is ensured if the inequality holds for \(T + 1 = M\) (assuming that \(\mu - ba^2K > 0\), that is, shares are positively valued in autarky).
Fig. 3. The critical number of traded companies \( T^c \) as a function of the number of ordinary investors \( N \).

Expectations outcome is the stock market equilibrium with maximal number of listings, \( T = M \). Clearly the latter equilibrium is Pareto superior. It is not difficult to modify the model so as to produce other equilibria as well, yielding intermediate sizes (for instance by allowing for companies of different size classes).

Fig. 3 plots the critical value \( T^c \) as a function of \( N \), the number of ordinary investors expected to trade on the stock market. \( T^c \) is a decreasing and concave function of \( N \): if the number of ordinary investors rises, the minimum number of company listings needed for the stock market to take off declines more than proportionately. The region of the parameter space below the \( T^c \) locus corresponds to autarky and that above it to the stock market equilibrium. When \( N \) reaches or exceeds the value for which \( T^c = 0 \), i.e. \( N \geq 1/(\sigma K \sqrt{b/2c} - 1) \), it becomes worthwhile to float one's company even if nobody else does, so that the autarkic equilibrium disappears and only the stock market equilibrium is left. Thus, if initially the economy is at the autarkic equilibrium, an institutional innovation – such as the introduction of mutual funds or a fiscal incentive – can shift it to the superior equilibrium by raising the number of ordinary investors who trade on the stock market beyond this critical point. Moreover, the autarky region shrinks (the \( T^c \) locus moves closer to the origin) when the risk to be borne \( (\sigma^2) \) and risk aversion \( (b) \) increase, or the flotation cost \( (c) \) decreases, i.e. when the gains from risk sharing increase relative to the cost of flotation.
With minor changes, one can also assess how privatizations effected via public sales of government-owned companies would affect the boundary between the two regions. If the government tenders to the public all the shares in $S$ companies of size $K$ each and does not reinvest the proceeds on the stock market, eq. (17) must be rewritten as

$$\Delta E[U(\hat{w}_{1j})] = \frac{b}{2} \left[ (N + T)^2 + S + T \right] \left( \frac{K\sigma}{N + T + 1} \right)^2 - c.$$  

(17')

Thus if a program of privatizations is undertaken, the net gain to going public increases: as $S$ increases from 0 to a small positive number in eq. (17'), the $T'$ locus shrinks inward, so that the autarky region shrinks. In this model, privatizations effected via the stock exchange can promote the transition to a superior equilibrium.

If entrepreneurs behave monopolistically in deciding the number of shares to be offered, the results are qualitatively similar. The change in expected utility associated with going public is again increasing in $T$ (see the appendix for derivations):

$$\Delta E[U(\hat{w}_{1j})] = \frac{b}{2} \left[ N + 3T + (N + T)^2 \right] \left( \frac{K\sigma}{N + T + 2} \right)^2 - c.$$  

(18)

This expression can be shown to be smaller than its analogue (17), obtained on the assumption that the entrepreneur is a price taker. Thus the introduction of monopoly power in the flotation decision reduces the private gain from going public, which contrasts with the case in which there are no borrowing constraints (see section 3.1, above). The difference has an intuitively persuasive logic. In the setting of section 3.1 (perfect credit markets), the fact that each entrepreneur, acting monopolistically, withholds part of his shares does not affect the others' decision on floating. But when there are borrowing constraints this feedback comes into play. In this case, the attractiveness of floating one's company depends on the opportunities to invest in other companies' shares; if their owners restrict their offering to keep their price above the competitive level, such investment will be less attractive; this diminishes the gain from floating one's own company and diversifying. As a result, $T'$, the critical number of listed companies, is larger than in the competitive case, for any given value of $N$.

3.2.2. Transaction costs on the stock market

When trading shares is costly, the number of ordinary investors is no longer exogenous, since it results from the entry decisions of these investors.
Let $n \in [0, N]$ denote the number of ordinary investors who decide to be active on the stock market, and $f$ denote the fixed transaction cost that they pay to operate on the exchange. Note that $f$ is an 'entry fee' to the stock market as a whole and not a cost paid to trade a single stock: it can be thought of as the cost of acquiring some familiarity with investing in stocks or finding a trustworthy broker. For the sake of simplicity – as well as realism – entrepreneurs are assumed not to face transaction costs on the stock market. In addition, they are not subject to borrowing constraints as in the previous subsection. Thus all $M$ entrepreneurs will trade stocks.

The problem of the $h$th ordinary investor thus is

$$\text{Max } E[U(\tilde{w}_{1h})] = \left( \sum_{i \in \mathcal{G}} (\mu_i - Rp_i)k_{ih} - b \sum_{i \in \mathcal{G}} \sigma_i^2 k_{ih}^2 - f \right) J_h + R\omega_0$$

for $h = M + 1, \ldots, M + N$, \hspace{1cm} (19)

where the indicator function $J_h$ equals 1 if $h$ enters the stock market, and 0 otherwise. As for the flotation choice of entrepreneurs, the entry choice of investors is assumed to be taken before their portfolio decision and conditioning on optimal behaviour in the portfolio choice. If investor $h$ pays the fixed cost $f$, the equilibrium value of his expected utility is easily shown to be

$$E[U(\tilde{w}_{1j}) | j \text{ enters}] = \frac{b}{2} T \left( \frac{K \sigma}{M + n} \right)^2 - f - R\omega_0$$

for $j = M + 1, \ldots, M + N$, \hspace{1cm} (20)

whereas if he does not enter, it is equal to $R\omega_0$. Thus the net gain from entry for ordinary investors is given by the first two terms of eq. (20) – an expression decreasing in $n$. Setting it equal to zero, one obtains the value of $n$ for which ordinary investors are indifferent between trading on the stock market and abstaining from it:

$$n = K \sigma \sqrt{b T / 2f} - M.$$ \hspace{1cm} (21)

The equilibrium number of active investors is the largest integer smaller than expression (21), if $n > 0$ – and equals 0 if $n < 0$. Thus the equilibrium number of investors is increasing in $T$: the larger the number of companies listed on

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10To the extent that the cost $f$ arises from the need to acquire information about the stocks listed on the exchange, the information must be non-transferable (it cannot be passed over to other agents) and generic (useful to appraise all the share issues listed on the exchange), as in Gale (1992), to which the model in this subsection is closely related.
the stock exchange, the greater the number of ordinary investors who in equilibrium will be attracted to the stock market.

The net gain to entrepreneur $j$ from floating his company's shares is instead given by eq. (8), upon replacing the variable $N$ with $n$:

$$\Delta E[U(\hat{w}_{1j})] = \frac{b}{2} \left( \frac{K\sigma}{M+n} \right)^2 (M+n-1)^2 - c. \quad (22)$$

This expression is increasing in $n$: the gain from going public is increasing in the number of ordinary investors trading on the stock exchange. Setting it equal to zero, and solving for $n$, one obtains the number of ordinary investors, $n^c$, that makes entrepreneur $j$ indifferent between going public and staying private. Substituting this into eq. (21), we obtain the number of companies that must be listed on the stock exchange in order to attract $n^c$ investors:

$$T^c = \frac{2f}{b} \left( \frac{1}{K\sigma - \sqrt{2c/b}} \right)^2. \quad (23)$$

If $T^c \in (0, M)$, we have two equilibria, as in the last subsection. This situation is illustrated in fig. 4. If the number of companies expected to go public falls short of $T^c$, too few investors will be attracted to the stock market for any
company to go public, and the stock market will not take off. Vice versa, if people expect \( T > T^c \), all the \( M \) companies will go public. In the figure, these two outcomes occur to the left and the right of the vertical \( T^c \) locus, respectively.

One can easily construct an example with two size classes: \( M_1 \) large companies, with \( K_1 \) shares each, and \( M_s \) small companies, with \( K_s \) shares each. It turns out that the minimum number of listed companies needed to induce large companies to go public, \( T^*_1 \), is smaller than the corresponding number for small companies, \( T^*_s \). If the parameters are such that \( 0 < T^*_1 < M_1 < T^*_s < M_1 + M_s \), the model has three equilibria: autarky (\( T < T^*_1 \)), an equilibrium where only the large companies go public (\( T^*_1 < T < T^*_s \)) and another where also small companies do so (\( T^*_s < T \)). If the economy is at the middle equilibrium and a large enough number of small companies (larger than \( T^*_s - M_1 \)) goes public, the number of ordinary investors grows enough to induce all \( M_s \) small companies to go public. In the absence of such a coordinated move, an institutional innovation that reduces the transaction cost \( f \) for ordinary investors (or a policy of privatizations) can wipe out the intermediate equilibrium, by shifting the \( n \) locus sufficiently to the left: enough investors would be attracted to induce all companies to go public.

4. Conclusions

The point of departure of this paper consists of two highly notable yet often neglected facts: the great disparities in stock market size between countries at approximately the same stage of economic development; and the persistent failure of stock market size and the number of listed companies in some countries to keep pace with economic growth. The thesis of this paper is that externalities among potential market participants can help explain how the stock market can be trapped in a secular stagnation with only a few listed companies.

I propose a simple illustrative model, in which each entrepreneur decides whether or not to float his company's shares on the market by weighing the prospective gains from portfolio diversification after selling off the company against the costs of going public. If he does decide to float his shares, he gives rise to a positive externality on other stock market participants by increasing their risk sharing opportunities. Thus the decentralized outcome may be inefficient, with too few publicly traded companies. Moreover, in the presence of capital market imperfections – borrowing constraints or transaction costs in the stock market – the decisions to go public are interdependent: when a company goes public, the demand for the shares of other companies increases and their price rises, making other entrepreneurs more willing to float their own companies' shares as well. Consequently, depending on each entrepreneur's expectations about the behaviour of others, the
economy may end up at a Pareto-superior equilibrium where all companies are traded, or at others where only a few companies are publicly listed – or possibly none at all. These results parallel those of other models of participation externalities in asset markets [Chatterjee (1988), Pagano (1989a, b), Allen and Gale (1991), Gale (1992)].

This simple model concentrates on the risk-sharing benefits that a stock market yields in a pure exchange model. In a model with production, the multiple equilibria result found in this paper would translate into propositions about the level of output or its growth rate, as in Saint-Paul (1992). A more complete model should also consider that, beside offering better risk-sharing, the stock market processes and disseminates information about companies, which can also raise the growth rate of the economy by improving the allocation of capital across investment projects [see the survey in Pagano (1993)]. Thus, if the stock market is trapped at a low-level equilibrium, the consequences may extend well beyond those considered in this paper.

Appendix: The model with imperfectly competitive behaviour

A.1. The case with perfect credit markets

When he decides to go public, entrepreneur \( j \) must choose how many shares to offer \( (K_j - k_{jj}) \), and correspondingly how many to keep in his portfolio \( (k_{jj}) \). From the first-order conditions of the other \( N - 1 \) investors, he can compute the inverse demand function for his company's shares:

\[
p_j = \frac{1}{R} \left( \mu_j - b \sigma_j^2 \sum_{i=1, i \neq j}^{N+M} \frac{k_{ij}}{M+N-1} \right) = \frac{1}{R} \left( \mu_j - b \sigma_j^2 \frac{K_j - k_{jj}}{M+N-1} \right). \tag{A.1}
\]

Substituting this expression in the expected utility (3a), after setting \( I_j = 1 \), and differentiating with respect to \( k_{jj} \), one finds the optimal size of the initial public offering \( (K_j - k_{jj}) \) and the shares retained by the seller \( (k_{jj}) \):

\[
K_j - k_{jj} = \frac{M + N - 1}{M + N + 1} K_j \quad \text{and} \quad k_{jj} = \frac{2}{M + N + 1} K_j. \tag{A.2}
\]

The corresponding equilibrium price is

\[
p_j = \frac{1}{R} \left( \mu_j - b \sigma_j^2 \frac{K_j}{M+N+1} \right). \tag{A.3}
\]

Due to the symmetry of the model, the price for the shares of all the other \( T \) companies is given by the same expression, simply changing subscripts.
Substituting the expressions (A.2) and (A.3) in eq. (3a), one obtains the equilibrium value of \( j \)'s expected utility, conditional on his company staying private \((I_j=0)\) and going public \((I_j=1)\), respectively:

\[
E[U(\hat{w}_{1j})|j \text{ private}]=\frac{b}{2} \sum_{i \in \mathcal{G}} \left( \frac{K_i \sigma_i}{M+N+1} \right)^2 + \mu_j K_j - \frac{b}{2} K_j^2 \sigma_j^2, \tag{A.4}
\]

\[
E[U(\hat{w}_{1j})|j \text{ public}]=\frac{b}{2} \sum_{i \in \mathcal{G}} \left( \frac{K_i \sigma_i}{M+N+1} \right)^2 + \mu_j K_j - b \frac{K_j^2 \sigma_j^2}{M+N+1} - c. \tag{A.5}
\]

Subtracting the expression in (A.4) from (A.5) yields the change in the expected utility of entrepreneur \( j \) associated with his decision to go public, given by eq. (12) in the text. It is easy to show that, if company \( j \) goes public, each other investor \( i \) \((\neq j)\) gains

\[
\Delta E[U(\hat{w}_{1i})]=\frac{b}{2} \left( \frac{K_i \sigma_i}{M+N+1} \right)^2, \quad i \neq j. \tag{A.6}
\]

The expected welfare change for all other investors is simply this expression multiplied by their number, \( M+N-1 \). Adding the resulting expression to the change in the expected utility of entrepreneur \( j \) in (12), one obtains the change in the overall expected welfare of the economy due to the flotation of company \( j \) [expression (13) in the text].

**A.2. The case with borrowing constraints**

In this case, going through the same steps of the method illustrated above for the case of perfect credit markets, one finds that

\[
K-k_{jj} = \frac{N+T}{N+T+2} K \quad \text{and} \quad k_{jj} = \frac{2}{N+T+2} K. \tag{A.7}
\]

and

\[
p = \frac{1}{R} \left( \mu - b \sigma^2 \frac{K}{N+T+2} \right). \tag{A.8}
\]

The expected utility from staying private is again the autarky level (15), while the expected utility from going public is

\[
E[U(\hat{w}_{1j})|j \text{ public}]=\mu K - \frac{b}{2} (2N+T+4) \left( \frac{K \sigma}{N+T+2} \right)^2 - c. \tag{A.9}
\]
Subtracting the expression in (15) from (A.9), one obtains the net gain that entrepreneur $j$ obtains from floating his shares on the market [eq. (18) in the text].

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