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Endogenous Market Thinness and Stock Price Volatility

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Thin equity markets cannot accommodate temporary bulges of buy or sell orders without large price movements. The resulting volatility can induce risk-averse transactors who face transaction costs to desert these markets. Thus thinness and the related price volatility may become joint self-perpetuating features of an equity market, irrespective of the volatility of asset fundamentals. If, however, appropriate incentive schemes are adopted to encourage entry by additional investors, this vicious circle can be broken, eventually shifting the market to a self-sustaining, superior equilibrium characterized by a higher number of transactors, lower price volatility and larger supply of the asset.

1. INTRODUCTION

A number of empirical studies (Cohen *et al.* (1976), Telser and Higimbotham (1977), Pagano (1986), Tauchen and Pitts (1983)) have found that thin speculative markets are *ceteris paribus* more volatile than deep ones. A plausible explanation for this finding is that thin markets are generally characterized by small numbers of transactors per unit time, and thus their prices are more sensitive to the impact of individual traders' demand shocks. Conversely, in deep markets, transactors are so many that the uncorrelated demand shocks experienced by individual traders tend to offset each other and leave market prices largely unaffected. While this suggests a rationale for the observed relationship between market size and price volatility, it does so by taking market size as the exogenous factor. However, the volatility of a speculative market may feed back on its size, in the sense that the high liquidation risk implied by very volatile prices can induce potential entrants to keep out of the market.

This paper shows that, in a stock market with transaction costs, this interaction between thinness and volatility can produce multiple steady-state equilibria, some characterized by low trade and high volatility, and others by high trade and low volatility. Whether the market will settle in one equilibrium or another depends entirely on the expectations held by economic agents (Sections 2 and 3). The existence of these multiple "bootstrap" equilibria can be explained heuristically as follows. Each additional trader generates a positive externality for other (actual or potential) traders by decreasing the riskiness of the stock; lower risk in turn tends to attract more investors, with the effect of raising stock prices and inducing corporations to issue additional equity. We thus have a feedback loop between market size and price volatility—where market size is measured both along the dimension of the number of traders and along that of the total stock of equities. If however investors face transactions costs in the stock market, this positive feedback may fail to be operative: if the volume of trade is expected to be small, investors with relatively high transaction costs will abstain from trading. Thus the market may remain trapped in a low-trade, high-variance Nash equilibrium, and the opinion of outsiders about the thinness and riskiness of that market will be confirmed by the facts. Conversely, if the market had a reputation for being large and stable, even investors facing considerable transactions costs would be willing to trade, and in equilibrium their expectations would be fulfilled. If expectations are formed on the basis of the previous history of the market, its thinness—or depth—will become a self-perpetuating feature.

Interestingly, the equilibria produced by the model have an unambiguous welfare ranking—high-trade equilibria being Pareto-superior to low-trade ones. This leads to the issue of government intervention: are there policies that can shift the stock market to a superior equilibrium? This point is taken up in Section 4, which also analyzes the path of adjustment of the economy between steady states and the level of welfare along this path.

In Section 5 various extensions of the model are analysed. The most important of these extensions is the introduction of imperfectly competitive behaviour on the stock market. In a model where the number of traders can be small, each trader may conceivably perceive himself as having market power, rather than as being a price-taker. If traders are conscious that the price reacts adversely to their orders, they will restrict their trades below the competitive level, and their welfare will be correspondingly lower. Thus entry by other investors now exerts *two* beneficial externalities: beside lowering the volatility of the price at which the stock can be sold, it reduces the adverse price movement that traders experience when they buy it.

Thus the introduction of imperfect competition *reinforces* the beneficial externality that is at the root of the multiplicity of equilibria. In addition, it brings out that there are two distinct channels through which entry by additional traders can be said to make a market more liquid—by reducing the price volatility due to uncorrelated demand shocks and by decreasing the adverse price response to the order flow. The second of these two channels is at the centre of the analysis in the related models by Kyle (1984, 1985, 1986), Admati and Pfleiderer (1988) and Pagano (1988).

The relevance of these issues is far from being purely theoretical. The fact that market thinness may interact perversely with the volatility of the market price and with its sensitivity to the order flow has, for instance, been indicted for the persistent narrowness of continental European equity markets relative to British, U.S. or Japanese standards:

> "One of the main characteristics of the European security markets, and especially the equity markets, is indeed their narrowness, London being the only exception. [...] The consequences are essentially the high degree of disturbances and adverse price movements, which lead to a vicious circle: the narrowness increases the reluctance of institutional investors to enter the market or to operate transactions, and this intensifies the narrowness of the market." (Bertoneche, 1978, p. 344)

One may object that the empirical magnitude of the volatility arising from uncorrelated demand shocks cannot be large, except when the number of traders is extremely low. On empirical grounds, this objection is contradicted by the results of the studies quoted at the start of this section. At a theoretical level, it is based on an excessively literal interpretation of the model's assumption. The analysis can be easily reinterpreted by supposing that demand shocks, rather than being uncorrelated across individual investors, are uncorrelated across social groups (farmers, wage-earners, etc.) and correlated within each of these groups. Then, even if the number of investors is already large, entry by a new group into the market can yield a substantive reduction in the variability of the demand for the stock.

Others may suggest that asymmetric information and insider trading can provide a more convincing explanation of market thinness than the volatility of prices and their sensitivity to the order flow. The idea is that liquidity traders will stay away from markets where insider traders can easily profit at their expense. While intuitively attractive, this idea misses two important points. First, insider traders themselves prefer to stay away from thin markets, since low volume and high price sensitivity to the order flow mean lower profits for them. Second, more insider trading does not necessarily imply lower welfare for other investors.[1]

2. THE MODEL

The model is based on four crucial assumptions. *First*, investors can costlessly borrow and lend at a risk-free interest rate but face a fixed cost to trade equities—a feature intended to capture the lumpy expenses on information acquisition and brokerage services that are often necessary to speculate on the stock market. Prior to their portfolio selection, investors must decide if they want to pay the fixed cost and enter the stock market. The equilibrium number of traders is thus determined endogenously, since only those facing relatively low transaction costs will enter the market, as in Goldsmith (1976) and Mayshar (1979, 1983). For analytical simplicity, the entry and portfolio decisions are assumed to be sequential, rather than simultaneous: in period 0 of their life investors decide on entry, in period 1 they select their portfolio and in period 2 they receive the return on their assets and liquidate them.

Second, individual stock demands contain an agent-specific component: investors place different values on the stock because they have different needs to diversify away the risk arising from their endowments of non-marketable assets, and thus at the same price they demand different amounts of the stock. Assuming that the endowments of non-marketable assets (e_{it}) are independently distributed across investors, these agent-specific disturbances introduce in the aggregate demand for equities a noise term whose variance is inversely related to the number of traders. This feature would obtain also if the investors' valuation of the stock were to differ for reasons other than risk-hedging—for instance, if market participants made independent errors in predicting future dividends. The risk-hedging motive is employed to show that the analysis does not need to rely on non-rational forecasts of future dividends.

Third, I assume that investors who buy shares hold them for one period only. This provides a simple way to model the inflow and outflow of traders. At each date, a new "generation" of investors appears in the economy. Those who enter the stock market purchase the stock from current shareholders and resell it inelastically to the next generation in the subsequent period. It must be clear that the use of the term "generation" has no implications for the length of the time span between successive trades. In fact, the assumption that shares are resold inelastically after one period means that the time horizon of shareholders is very *short*—they cannot wait for a better price further in the future. When selling, they behave as liquidity traders who bear all the brunt of price

1. See Admati and Pfleiderer (1988). Having a higher number of insiders on a speculative market does not necessarily imply that these traders will be able to extract more profits from liquidity traders, on average. The reason for this ambiguity lies in the fact that a higher number of insiders exerts two opposite effects on their profits: on one hand, it tends to raise them, to the extent that they possess different information, and on the other hand it lowers them, since it increases competition among insiders. Moreover, a correct evaluation of the welfare effects of insider traders should not overlook the fact that they increase the informational efficiency of the market, and thus reduce the risk that liquidity traders will transact at prices far off the fundamental value of the asset.

volatility, rather than as speculators who can condition their orders on the price, so as to sell high and buy low.[2]

Fourth, the stock is issued by a value-maximizing, infinite-lived firm that is entirely equity-financed. The rationale for this assumption is discussed at length at the end of this section. Before that, the assumptions about the behaviour of investors and firm must be spelled out in detail. For ease of reference, they are also summarized in Table I.

Investors' behaviour

Each investor i maximizes a mean-variance utility function in terminal (period 2) wealth w_i. His expected utility $E(U_i)$ can be written as:

$$E(U_i) = E(w_i) - \frac{b}{2}\operatorname{Var}(w_i). \tag{1}$$

In period 0 investor i takes the entry decision, i.e. decides whether in the subsequent period he will pay the fixed cost $f(i)$ and enter the stock market, or will rather confine

TABLE I

Timing of events and actions in the model

Individual i belonging to:	at time $t-1$	at time t	at time $t+1$
generation $t-2$	is in period 1: —observes e_{it-1} —buys debt, and, if he has paid $f(i)$ at $t-2$, buys stock from generation $t-3$ —decides with other shareholders on investment and new share issues I_{t-1}	is in period 2: —liquidates all debt and stock by selling it inelastically to generation $t-1$	
generation $t-1$	is in period 0: —decides about entry in the stock market at time t	is in period 1: —observes e_{it} —buys debt, and if he has paid $f(i)$ at $t-1$, buys stock from generation $t-2$ —decides with other shareholders on investment and new share issues I_t	is in period 2: —liquidates all debt and stock by selling it inelastically to generation t
generation t		is in period 0: —decides about entry in the stock market at time $t+1$	is in period 1: —observes e_{it+1} —buys debt, and if he has paid $f(i)$ at t, buys stock from generation on $t-1$ —decides with other shareholders on investment and new share issues I_{t+1}

2. Obviously, if shareholders had a longer time horizon, speculation would somewhat stabilize the market price. However, stabilization would not be complete unless at least one shareholder had an infinite planning horizon, i.e. were certain of being never a liquidity trader (for a more detailed discussion on this point, see De Long et al., 1987).

his choice to an all-debt portfolio. At this stage, he does not yet have information on his period 1 endowments: the only agent-specific information entering this decision is the size of his fixed cost $f(i)$.

In period 1 the investor chooses his portfolio. If he has previously decided to enter the stock market, he conveys his demand for equities to the market, purchasing them from the previous generation of stockholders at the market-clearing price p_t. At this stage he is obviously conditioning on a complete knowledge of his endowments: he has an endowment of "cash" w and an agent-specific endowment e_{it} of a non-marketable asset yielding a unit return u_{t+1} at time $t+1$ (the total return to i's endowment of this asset is thus $e_{it}u_{t+1}$). The return u_{t+1} is correlated with contemporaneous earnings per share π_{t+1}: $\text{Cov}(u_{t+1}, \pi_{t+1}) \equiv \sigma_{u\pi} \neq 0$. Apart from this crucial correlation, however, the random variables e_{it}, u_{t+1} and π_{t+1} are all i.i.d. over time and mutually independent. The variable e_{it} is also i.i.d. across agents i ($i = 1, \ldots, N_t$) at each date t, with mean μ_e and variance σ_e^2.

In period 2, everyone liquidates his entire portfolio. The entire stock of equities is thus inelastically sold to a new generation of investors entering the stock market.

Firm's behaviour

Equities are issued by an infinite-lived and completely equity-financed firm (the model can be easily restated to accommodate several firms with uncorrelated returns, at the cost of additional notation). Its number of shares, K_t, is standardized by setting it equal to the number of units of physical capital ("machines") that the firm has at the start of the corresponding period. At time t this stock is given, and the firm's choice variable is its current investment (I_t), that is financed by issuing a number of shares equal to the additional machines (thus preserving the above standardization). New machines can be bought at a cost of c per unit, but installing a new machine and issuing the corresponding shares also carries a convex adjustment cost $h(I_t)$ (with $h' > 0$, $h'' > 0$ and $h'(0) = 0$), as often assumed in the investment function literature. For simplicity, capital is assumed not to depreciate, so that all investment is net investment and old machines can be resold at their replacement cost c, irrespective of their age. The profit per period earned by each machine, π_t, has mean[3] μ_π and variance σ_π^2. In each period the firm pays out to shareholders its entire "cash flow", i.e. current profits *minus* investment expenses and adjustment costs: at time $t+1$ the total dividend paid out is thus $\pi_{t+1}K_t - cI_t - h(I_t)$.

At time t, after the market closes, current shareholders choose the size of its investment (and share issue) I_t so as to maximize the current market value of their shares.[4] When investment takes place, the corresponding new shares are distributed on a pro-rata basis to current shareholders.[5] Thus the total return to a share at time $t+1$ is:

$$x_{t+1} = p_{t+1} + \pi_{t+1} + [p_{t+1}I_t - cI_t - h(I_t)]/K_t. \tag{2}$$

In words, (2) says that the return to a share is equal to its resale value, *plus* earnings per share and the resale value of the pro-rata fraction of newly-issued shares, *minus* the cost of investment per share.

3. I am assuming here that the expected profit per unit of capital, μ_π, is a constant, rather than a decreasing function of the size of the firm or of the total stock of capital in the economy. This assumption is warranted if the firm has constant returns to scale and acts as price-taker on the output market.

4. The results of the model would be qualitatively unchanged if one assumed the objective function of the firm to be the maximization of shareholder's welfare (that here does not coincide with the objective of value maximization).

5. Alternatively, one could imagine the firm selling the new issues on the market and distributing the proceeds as dividends. This reinterpretation would leave the formal structure of the problem unaffected.

These assumptions about investment and financing behaviour imply that the supply of equities is price-elastic. This is crucial in the logic of the model, as can be seen by considering the opposite assumption of a fixed supply of equities that is customary in the finance literature: if the stock of equities is given, entry by additional investors—besides reducing the variance of stock returns—permanently increases the market price and thus lowers the expected rate of return. This adjustment in the market price offsets the positive externality flowing from entry, and the basic mechanism of the model becomes inoperative: as a result, with a fixed supply of equities the model turns out to have a unique equilibrium. The more realistic view taken here is instead that a value-maximizing firm takes advantage of abnormally high stock prices to issue new shares and invest, thus causing the price and the expected return per share to go back to their normal values in the long run. This leaves the variance-reducing effect of entry free to trigger the feedback loop between market size and price stability. Obviously an elastic supply of equities can also be rationalized on the basis of different assumptions, as illustrated in Section 5.

3. MODEL SOLUTION

To solve for equilibrium values in this model, one has to go through four steps: (i) analyzing the decision problem of individual investors; (ii) computing equilibrium stock prices; (iii) deriving the equilibrium supply of shares by the firm and (iv) pinning down the equilibrium number of traders in the stock market.

(i) *Decision problem of the individual investor*

Consider investor i of generation $t-1$. In period 0 of his life he must decide whether in the subsequent period he should enter the stock market or not. The information set on which he conditions in making this choice (Ω_{it-1}) includes all the information publicly known at time $t-1$ and the size of his fixed transaction cost $f(i)$. He realizes, however, that when he will choose his portfolio in period 1, he will know more than this: his information set (Ω_{it}) will also include knowledge of his endowment of the non-marketable asset e_{it} and of the stock price p_t, and obviously he will exploit all that information in selecting the optimal portfolio.

The investor's problem must thus be solved in two stages, working from the end to the beginning of his planning horizon in a dynamic programming fashion. In the first stage, the investor calculates how many shares of the stock he will demand in period 1 (conditional on Ω_{it}) if he pays the fixed cost $f(i)$ in period 0. In the second stage, he considers instead the following question: given that in period 1 of my life I will select the optimal holdings of equities if I pay the fixed cost $f(i)$, is it worthwhile for me to pay the cost, conditional on the more limited information Ω_{it-1} currently available? In the first stage, the control variable is the demand for shares k_{it}, in the second it is the fixed cost $f(k_{it})$, that takes value 0 if the investor opts for no entry and the value $f(i)$ if he opts for entry.

Formally, the problem that investor i faces when he decides on entry can be written

$$\begin{aligned}&\text{Max}_{f(k_{it})}\, E\{\max_{k_{it}}\,[E(U_i|\Omega_{it})]|\Omega_{it-1}\}\\&\quad=E\left\{\max_{k_{it}}\left[E(w_{it+1}|\Omega_{it})-\frac{b}{2}\text{Var}\,(w_{it+1}|\Omega_{it})\right]\middle|\,\Omega_{it-1}\right\}\end{aligned}\tag{3}$$

subject to:

$$w_{it+1}=e_{it}u_{t+1}+x_{t+1}k_{it}+R[w-p_tk_{it}-f(k_{it})],\tag{4}$$

and

$$f(k_{it})=\begin{cases} f(i) & \text{iff } k_{it}\neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $x_{t+1}\equiv$ total return on a share (as defined by (2)); $k_{it}\equiv$ number of shares held in portfolio by investor i; $R\equiv 1+r\equiv$ return per unit of debt; $f(i)\equiv$ fixed transaction cost for investor i.

The first stage requires then finding the "notional" demand for equities $\hat{k}_{it}$ that maximizes $E(U_i|\Omega_{it})$ subject to (4), setting $f(k_{it})=f(i)$ in (5). The second stage consists of substituting into the maximand in (3) the "notional" demand for equities $\hat{k}_{it}$ computed in the first step, evaluated at the equilibrium price: if the resulting value of the indirect expected utility $E[E(U_i|\Omega_{it})|\Omega_{it-1}]$ as of time $t-1$ is greater than that achieved for the case of no investment in equities, entry into the stock market will be worthwhile. The reason why the problem neatly dichotomizes in this fashion is that the demand for equities $\hat{k}_{it}$ turns out to be independent of the level of initial wealth $w-f(i)$, and thus of the expense sustained to pay for the fixed cost $f(i)$. This is an implication of the particular mean-variance utility function employed here.

Solving the first-stage problem, one gets the "notional" demand for equity of investor i:

$$\hat{k}_{it}=\frac{E(x_{t+1}|\Omega_t)-Rp_t}{b\sigma_x^2}-\frac{\sigma_{ux}}{\sigma_x^2}e_{it}, \tag{6}$$

where

$$\sigma_x^2\equiv \mathrm{Var}\,(x_{t+1}|\Omega_t), \tag{7}$$

$$\sigma_{ux}\equiv \mathrm{Cov}\,(u_{t+1}, x_{t+1}|\Omega_t), \tag{8}$$

and Ω_t is the set of all variables publicly known at time t (inclusive of p_t). Thus σ_x^2 is the variance of the total return to a share, x_{t+1}, and σ_{ux} is the covariance of the return to the non-marketable assets u_{t+1} with the total return to a share x_{t+1}. The reason why the conditioning set in (6), (7) and (8) is Ω_t—rather than Ω_{it}—is that private information about e_{it} is redundant in estimating the moments and comoments of x_{t+1} and u_{t+1}.

The investor will actually convey the demand $\hat{k}_{it}$ to the market if he expects to be better off by paying the fixed cost $f(i)$ and adding $\hat{k}_{it}$ to his portfolio. Thus in his second-stage decision, investor i chooses to pay $f(i)$ and secure entry on the stock market only if:

$$E[E(U_i|\Omega_{it})|\Omega_{it-1}, k_{it}=\hat{k}_{it}]\geqq E[E(U_i|\Omega_{it})|\Omega_{it-1}, k_{it}=0] \tag{9}$$

Using equations (3), (4) and (6), this entry condition reduces just to:

$$E\left[\frac{b\sigma_x^2}{2R}\hat{k}_{it}^2\,\middle|\,\Omega_{it-1}\right]\geqq f(i). \tag{10}$$

To know whether investor i will in fact find it worthwhile to enter the market, one must evaluate the entry rule (10) at equilibrium, i.e. one has to find the value that $\hat{k}_{it}$ and σ_x^2 will take at market-clearing prices. This requires computing the joint conditional distribution of equilibrium prices p_t and p_{t+1}, of earnings per share π_{t+1} and of the return u_{t+1} on non-marketable assets e_{it}—the conditioning set being Ω_{it-1}. The equilibrium prices p_t and p_{t+1}, and thus the moments of this distribution, depend on the number of investors trading on the market at date t, but in turn the decision to enter the market by each investor according to (10) depends on the parameters of the distribution. In other words, the stochastic behaviour of equilibrium prices and the equilibrium set of traders are determined simultaneously. I start by postulating that at time $t-1$ everyone conjectures that at date t there are going to be N_t investors demanding equities on the market—N_t being an arbitrary number; I then solve for equilibrium prices and finally use these values in the entry condition (10) to pin down the equilibrium number of traders N_t in terms of exogenous parameters.

(ii) *Equilibrium stock prices*

Assuming thus that at time t there are N_t investors on the demand side of the market facing a total supply of shares K_t, equilibrium requires:

$$\sum_{i=1}^{N_t} \hat{k}_{it} = K_t. \tag{11}$$

This, using the expression for $\hat{k}_{it}$ in (6), implies the following expression for the equilibrium price p_t:

$$\begin{aligned} p_t &= \frac{1}{R}\left[E(x_{t+1}|\Omega_t) - b\left(\frac{\sigma_x^2 K_t}{N_t} + \sigma_{ux}\bar{e}_t\right)\right] \\ &= \frac{1}{R}\left[E(p_{t+1} + \pi_{t+1} + (p_{t+1}I_t - cI_t - h(I_t))/K_t|\Omega_t) - b\left(\frac{\sigma_x^2 K_t}{N_t} + \sigma_{ux}\bar{e}_t\right)\right] \end{aligned} \tag{12}$$

where $\bar{e}_t = \sum_{i=1}^{N_t} e_{it}/N_t$, i.e. denotes the sample mean of the individual endowment shocks of the N_t traders, and the definition of x_{t+1} from (2) has been used in the second step. Now consider leading equation (12) to obtain an expression for the stock price at time $t+1$, p_{t+1}: conditional on Ω_t, the only stochastic term in p_{t+1} is the term involving $\bar{e}_{t+1}$, i.e. the sample mean of endowment shocks e_{it+1}. Now recall that the e_{it+1}'s are i.i.d. and uncorrelated with earnings per share π_{t+1} and with the return on the non-marketable asset u_{t+1} (the only other two stochastic variables in the model). Thus, conditioning on Ω_t, the stock price p_{t+1} is an i.i.d. process, and is uncorrelated with π_{t+1} and u_{t+1}. This has two implications. First, the conditional covariance between the total stock return x_t in (2) and the return on non-marketable assets u_{t+1} only arises from the earnings component of stock returns, π_{t+1}:

$$\sigma_{ux} = \sigma_{u\pi}. \tag{13}$$

Second, the conditional variance of the stock price at time $t+1$, σ_p^2, is a decreasing function of the number of traders at time $t+1$, N_{t+1}:

$$\sigma_p^2 = \alpha\frac{\sigma_e^2}{N_{t+1}} \quad \text{where} \quad \alpha \equiv \left(\frac{b\sigma_{u\pi}}{R}\right)^2. \tag{14}$$

As a result, also the conditional variance of the overall return on equities at time $t+1$, σ_x^2, turns out to be decreasing in the number of traders N_{t+1}:

$$\sigma_x^2 = \sigma_p^2(1+I_t/K_t)^2 + \sigma_\pi^2 = \alpha\frac{\sigma_e^2}{N_{t+1}}(1+I_t/K_t)^2 + \sigma_\pi^2. \tag{15}$$

In steady state $I_t = 0$, so that (15) reduces to $\sigma_x^2 = \sigma_p^2 + \sigma_\pi^2$.

(iii) *Equilibrium supply of the stock*

We can now turn to the determination of the stock of equities. After the market closes at time t, the firm's N_t shareholders decide on investment and new share issues I_t. Since

$K_{t+1} \equiv K_t + I_t$, we can alternatively think of K_{t+1}, the capital stock and share supply at $t+1$, as their control variable. Their agreed objective is the maximization of the value of the firm $V_t = p_t K_t$. Using expression (12) for p_t, the decision problem of the firm is:

$$\text{Max}_{K_{t+1}}\, p_t K_t = \frac{1}{R}\left[E(p_{t+1}K_{t+1} + \pi_{t+1}K_t \,|\, \Omega_t) - (cI_t + h(I_t)) - b\left(\frac{\sigma_x^2 K_t^2}{N_t} + \sigma_{u\pi}K_t \bar{e}_t\right)\right]. \tag{16}$$

Substituting for σ_x^2 from (15), using the identity $I_t \equiv K_{t+1} - K_t$ and differentiating with respect to K_{t+1}, one finds the Euler equation that characterizes the optimal programme for the firm:[6]

$$E\left(\pi_{t+1} + c + h'(I_{t+1}) - 2b\sigma_\pi^2 \frac{K_{t+1}}{N_{t+1}} - b\sigma_{u\pi}\bar{e}_{t+1} \,\middle|\, \Omega_t \right) = R\left(c + h'(I_t) + 2b\sigma_p^2 \frac{K_{t+1}}{N_t}\right). \tag{17}$$

Equation (17) can be used both to analyze the supply of shares in steady state and its dynamics out of steady states. Here I will concentrate on steady-state equilibria, leaving dynamics for the next section. In steady state $I_t = I_{t+1} = 0$ and $N_t = N_{t+1}$, so that (17) yields:

$$K_t^* = \frac{\mu_\pi - rc - b\sigma_{u\pi}\mu_e}{2b(\sigma_\pi^2 + R\sigma_p^2)} N_t = \frac{\mu_\pi - rc - b\sigma_{u\pi}\mu_e}{2b(\sigma_\pi^2 + R\alpha\sigma_e^2/N_t)} N_t, \tag{18}$$

recalling that $E(\pi_{t+1}|\Omega_t) = \mu_\pi$, $E(\bar{e}_{t+1}|\Omega_t) = \mu_e$ and $h'(0) = 0$, and making use of expression (14) for the variance of share prices σ_p^2 in the second step. The steady state supply of the stock K_t^* is increasing in the expected dividends per share μ_π and decreasing in the rental cost of capital rc, in the risk-aversion parameter b, in the covariance with non-marketable assets and in the own return variance. It is also increasing in the number of shareholders N_t, that appears in (18) both directly and indirectly via its negative effect on the variance of the share price σ_p^2.

Solving forward the pricing equation (12) and evaluating it at the steady state level of the supply of shares in (18), one finds the corresponding value of the price p_t:

$$p_t^* = \frac{1}{r}\left(\mu_\pi - b\sigma_{u\pi}\mu_e - b(\sigma_\pi^2 + \sigma_p^2)\frac{K_t^*}{N_t}\right) - \frac{b\sigma_{u\pi}}{R}(\bar{e}_t - \mu_e). \tag{19}$$

The value of a share in steady state is thus the present discounted value of future expected dividends *minus* future risk premia (the risk premium at each future date depending both on the covariance with the return to the non-marketable asset and on the own-variance of the stock) *plus* a zero-mean i.i.d. disturbance whose variance decreases in the number of stock traders N_t (from (14)).

(iv) *Equilibrium number of traders*

At this point the only endogenous variable that has not yet been pinned down in terms of exogenous parameters is the number of traders N_t. But now we are equipped to solve for its equilibrium value: to do so, we must turn back to the entry condition (10) and evaluate it at steady-state equilibrium. Using expression (6) for the demand for equity $\hat{k}_{it}$ and the market-clearing condition (12) in the entry decision rule (10), we find that in equilibrium investor i will enter the stock market if:

$$E\left\{ \frac{b\sigma_x^2}{2R}\left[\frac{K_t}{N_t} - \frac{\sigma_{u\pi}}{\sigma_x^2}(e_{it} - \bar{e}_t)\right]^2 \,\middle|\, \Omega_{it-1}\right\} \gtreqless f(i). \tag{20}$$

6. The expectation operator is not really necessary before I_t and K_{t+1}, because at t they are known with certainty. The number of shareholders N_{t+1}, on the other hand, is assumed to be non-stochastic, and turns out to be so in equilibrium.

Since the e_{it}'s are i.i.d. for all t, the expectation in (20) can be rewritten as:

$$\frac{b}{2R}\left[\left(\frac{K_t}{N_t}\right)^2 \sigma_x^2+\frac{\sigma_{u\pi}^2\sigma_e^2}{\sigma_x^2}\left(1-\frac{1}{N_t}\right)\right]\geqq f(i). \tag{20'}$$

Let the index i be chosen so that investors are ranked in order of non-decreasing transaction costs, i.e. so that $f(i+1)\geqq f(i)$, $\forall i$. The equilibrium number of investors N_t is obviously defined by the marginal investor, i.e. by the highest-cost investor for whom the (weak) inequality (20′) holds. One can in fact distinguish two possible cases: that condition (20′) holds with equality for the marginal investor N_t (he is just indifferent between entry and no-entry), or that it holds with inequality and reverses sign for the (N_t+1)-th investor (if a discontinuity occurs in the transaction cost distribution $f(i)$ at that point). Both cases are captured by the equilibrium condition:

$$f(N_t)\leqq\frac{b}{2R}\left[\left(\frac{K_t}{N_t}\right)^2 \sigma_x^2+\frac{\sigma_{u\pi}^2\sigma_e^2}{\sigma_x^2}\left(1-\frac{1}{N_t}\right)\right]<f(N_2+1). \tag{21}$$

This condition still involves two endogenous variables: the number of traders N_t and the supply of shares to the market, K_t. We can however evaluate it at the steady state by setting the supply of shares K_t equal to K_t^*, and solve for the equilibrium number of stockholders in the steady state, N_t^*, in terms of model parameters only:

$$f(N_t^*)\leqq\Psi(N_t^*)<f(N_t^*+1), \tag{22}$$

where

$$\Psi(N_t^*)=\frac{b}{2R(\sigma_\pi^2+\alpha\sigma_e^2/N_t^*)}\left[\left(\frac{\mu_\pi-rc-b\sigma_{u\pi}\mu_e}{2b}\,\frac{1}{1+\dfrac{r}{1+N_t^*\sigma_\pi^2/\alpha\sigma_e^2}}\right)^2+\sigma_{u\pi}^2\sigma_e^2\left(1-\frac{1}{N_t^*}\right)\right].$$

The function $\Psi(\cdot)$ is monotonically increasing in N_t^*, the steady-state number of shareholders, with a finite asymptote for $N_t^*\to\infty$. To interpret this result, notice that $R[\Psi(i)-f(i)]$ is the increase in expected utility that investor i derives from entry, evaluated at steady-state equilibrium: when $\Psi(i)$ exceeds the fixed cost $f(i)$, investor i expects to get a surplus from entry into the stock market. For the marginal investor N_t^* the expected surplus $R[\Psi(N_t^*)-f(N_t^*)]$ is positive or zero, while for the next potential entrant, N_t^*+1, it is negative. The fact that $\Psi(N_t^*)$ is increasing in N_t^* thus means that the larger the number of investors who enter the stock market, the higher is the expected surplus from entry for the marginal investor—and so is, *a fortiori*, the expected surplus of all inframarginal investors. The number of traders N_t^* appears in $\Psi(N_t^*)$ three times: the first and second entry of N_t^* capture the fact that a large number of stock traders increases expected utility by reducing the variance of the market price. The third entry of N_t^* has a more subtle interpretation: a higher N_t^*, by making the stock price more stable, also reduces the covariance of the price with the demand of each investor, implying that the adverse price movement in response to each investor's demand will be less pronounced. Also this effect tends to increase the liquidity of the market and raises the expected utility of market participants.

(v) *Multiple Pareto-ranked equilibria*

It is precisely the fact that the expected utility is increasing in N_t^* that creates the potential for multiple steady-state equilibria: as both the $\Psi(\cdot)$ function and the $f(\cdot)$ function are increasing in N_t^* condition (22) can be satisfied for several values of N_t^*. Figure 1 illustrates the two ways in which this outcome may occur: the left-hand panel 1(a) shows the case of investors bunching in discrete cost classes (a stepwise $f(i)$), and the right-hand panel 1(b) portrays the case of a continuous distribution of transaction costs across investors. The starred values on the abscissa denote the number of investors corresponding to alternative equilibria. In both panels the entry and exit process promotes stability only around even-numbered equilibria (E_2 and E_4), which in Figure 1(a) are placed at points of discontinuity of the $f(i)$ locus, and in Figure 1(b) at the intersections where the $f(i)$ function is steeper than $\Psi(i)$. To the right of these points, in fact, $\Psi(i) < f(i)$, the marginal investor expects a negative surplus from trade in the stock market, and has an incentive to exit the market; to the left of those points, $\Psi(i) > f(i)$ for potential entrants, and these have an incentive to enter the market (see Section 4 for a more complete analysis of dynamics). Conversely, in the neighbourhood of the odd-numbered equilibria—E_1 and E_3—entry and exit behaviour is destabilizing.

All these steady-state equilibria are characterized by *rational conjectures*, i.e. by conjectures that are consistent with actual outcomes and are mutually consistent across agents. Consider for instance the equilibrium with N_2^* shareholders: if everyone believes that investors with transaction costs larger than $f(N_2^*)$ are not going to enter the stock market, it is in the best interest of these investors not to enter the market—if one of them departed from this behaviour, he would be worse off. Similarly, the firm refrains from issuing additional shares, since this would reduce its market value, given the conjectures about entry and thus about demand for shares. Thus the equilibrium outcome is consistent with the expectations that each agent holds about the behaviour of all the others.

It has already been shown that equilibria with a relatively high number of traders are characterized by lower *volatility* of stock prices and returns. In addition, they also feature a larger total *supply of equities* K_t^* (see (18)) and a higher *social welfare* level. More precisely, each of these equilibria is Pareto-superior to all those with a lower number of traders. Consider in fact any two steady state equilibria: all the additional entrants in the higher-trade equilibrium must be better off, or they would not enter the stock market (they could still achieve the no-entry welfare level, if they wanted); on the other hand, also inframarginal traders are better off as N_t^* increases, since in equilibrium the expected

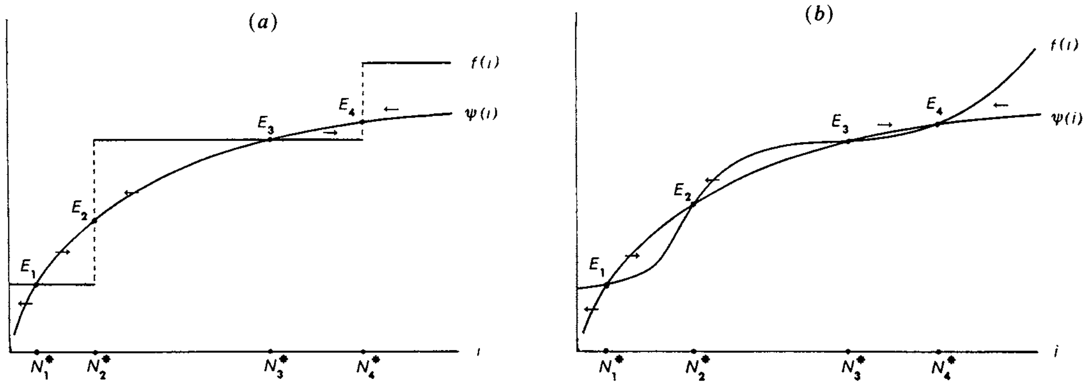

FIGURE 1

utility gain from entry for investor i is $R[\Psi(i)-f(N_t^*)]$—an increasing function of N_t^*. This welfare ranking of equilibria immediately suggests that there is some scope for government intervention. To this I turn in the next section.

4. POLICY INTERVENTION, DYNAMICS BETWEEN STEADY STATES AND WELFARE ON THE TRANSITION PATH

The analysis of government intervention in this economy would be rather uninteresting if one were to focus only on steady states: it is clear in fact that by forcing additional investors into the market via an appropriate incentive scheme, a benevolent government can eventually shift the economy to a superior steady-state equilibrium. It is more interesting to study the dynamic behaviour of the economy after the adoption of such a scheme, and the welfare impact of the proposed policy along the transition to the new steady state. The analysis also applies, in its general lines, to any innovation that reduces transaction costs for the marginal investor or for the marginal class of investors, like the introduction of mutual funds.

The analysis of dynamics will proceed under the assumption that the process of entry is instantaneous: if entry is perceived to be ex-ante welfare-improving, it takes place with no delay. Moreover, to keep things simple, I will assume that investors are grouped in two discrete cost classes, as shown by the broken line in Figure 2, and that there are four steady-state equilibria. For brevity, I will denote the transaction cost faced by the two groups as f_L and f_H, in ascending order of magnitude (e.g. $f_L=f(i)$ for $i=1,\ldots,N_2^*$).

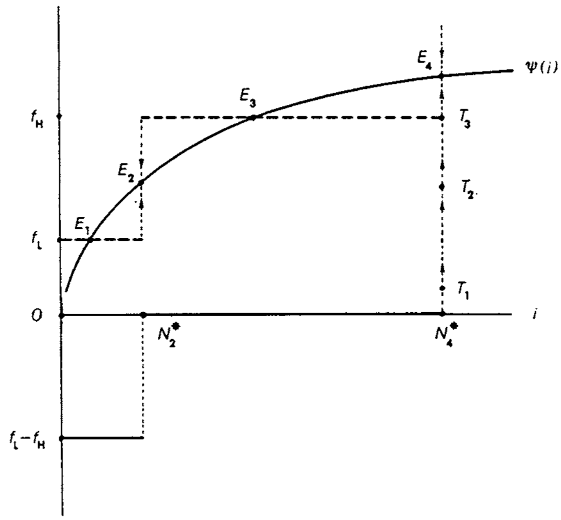

FIGURE 2

Imagine that the economy is initially stuck at the low-level equilibrium E_2, and consider the following policy scheme: at time t the government announces that, from time $t+1$ onwards, it will levy a lump-sum tax of size f_H on anyone who is in period 1 of his life and will give a rebate of the same size to any investor who will trade any amount of equity on the stock market. The rebate changes the set of incentives faced by investors as shown by the solid line in Figure 2: the first group of investors now perceives entry to be associated with a net subsidy f_H-f_L, and the second group perceives after-subsidy transaction costs to be zero. As a result, all of them will want to enter the stock market—starting with the t-th generation that will enter at time $t+1$.[7] Demand for equities is thus permanently raised relative to the previous steady-state level. Supply, however, can respond only gradually over time, due to the adjustment costs involved in installing new capital and issuing the corresponding shares. As a result, the stock price jumps discretely when the new policy is announced (as no expected capital gains must occur between the announcement and the enactment of the new policy), and then gradually declines as new share issues cumulate. Moreover, as soon as the policy scheme goes into effect, the variance of the stock price decreases relative to the initial steady-state level, due to the entry of the $N_4^* - N_2^*$ additional investors: I will denote the initial variance of stock returns by $\sigma_x^2(N_2^*)$, and its new level by $\sigma_x^2(N_4^*)$.

The dynamics of the economy in the transition towards the new steady state E_4 are ruled by the Euler equation (17), that characterizes the optimal pace of investment and share issues by the firm, and thus the optimal path of K_t over time. In fact, in the case at hand equation (17) can be simplified by setting $N_t = N_{t+1} = N_4^*$: this is because after the policy intervention the number of shareholders is going to stay fixed at N_4^* (entry occurs all at once at time $t+1$ as an effect of the incentive scheme). As an example, consider the special case of a quadratic adjustment-cost function, $h(I_t) = \gamma I_t^2$ (γ being a positive constant) where the implied law of motion of K_t can be expressed in explicit form. The Euler equation (17) then specializes to:

$$E(-K_{t+2} + \theta K_{t+1} - RK_t \mid \Omega_t) = \frac{1}{\gamma}(\mu_\pi - b\sigma_{u\pi}\mu_e - rc), \qquad \theta \equiv 1 + R + \frac{2b}{\gamma N_4^*}(\sigma_\pi^2 + R\sigma_p^2), \tag{23}$$

that can be solved to yield:

$$K_t = \lambda K_{t-1} + \frac{\lambda}{(R-\lambda)\gamma}(\mu_\pi - b\sigma_{u\pi}\mu_e - rc), \qquad \lambda \equiv \frac{\theta}{2} - \left(\frac{\theta^2}{4} - R\right)^{1/2}. \tag{24}$$

It is easy to show that $\lambda < 1$, so that the solution for K_t is stable, and that successive increments to K_t are decreasing in γ: the larger adjustment costs, the smaller the level of investment and share issues I_t at each date, and thus the slower the movement of the economy towards its new steady state E_4. As the firm invests and issues new shares, the stock price declines in a perfectly anticipated way. At the new steady state the average shareholdings (K^*/N_4^*) are larger than in the initial steady state (K^*/N_2) and the stock price p^* is higher in expected value (as can be verified by using equations (18) and (19)). How is the welfare of investors affected during the transition? The N_2^* shareholders who happen to be selling their stock at time t are clearly better off: they make an unexpected capital gain due to the jump in stock prices and they also enjoy the benefit of reselling their shares at more stable prices. For subsequent generations, instead, welfare results are more ambiguous: they enjoy greater price stability, but purchase the stock at a higher price, and thus receive a lower expected return as well.

Consider in fact an investor at time $t+j$, i.e. j periods after the policy is announced and denote by ΔU_i his utility surplus from entry (where $i = L$ for low-cost investors, $i = H$ for high-cost ones). For a low-cost investor, under the policy scheme the expected surplus

7. This scheme will obviously work only if the "entrance fee" $f(i)$ is not merely due to informational costs, but also to the fixed costs arising from brokerage fees, from the need of monitoring one's broker, etc. If the cost were purely information-related, an investor could abstain from purchasing the information, buy an infinitesimal amount of equities and claim the rebate from the planner.

from entry is (from (20')):

$$E(\Delta U_L|\text{policy})=\frac{b}{2}\left[\left(\frac{K_{t+j}}{N_4^*}\right)^2\sigma_x^2(N_4^*)+\frac{\sigma_{u\pi}^2\sigma_e^2}{\sigma_x^2(N_4^*)}\left(1-\frac{1}{N_4^*}\right)\right]-Rf_L, \tag{25}$$

whereas, if the policy had not been adopted, his expected surplus from entry would have been:

$$E(\Delta U_L|\text{no policy})=\frac{b}{2}\left[\left(\frac{K_t^*}{N_2^*}\right)^2\sigma_x^2(N_2^*)+\frac{\sigma_{u\pi}^2\sigma_e^2}{\sigma_x^2(N_2^*)}\left(1-\frac{1}{N_2^*}\right)\right]-Rf_L, \tag{26}$$

where K_t^* is the initial steady state number of shares (with no intervention, at $t+j$ the stock of equities would still be K_t^* and the number of shareholders would still be N_2^*). Low-cost investors entering the stock market at time $t+j$ gain from the policy scheme only if expression (25) is larger than expression (26). Simple inspection suggests that the result of this comparison is ambiguous: K_{t+j} is larger than K_t^* because under the policy scheme the firm issues additional shares, the size of this difference depending on the time elapsed since the policy announcement (j) and on the magnitude of the adjustment cost; the number of investors N^*, however, enters the two expressions in a complicated way, both directly and via the return variance σ_x^2. If the stock of equities grows slowly and the overall effect of N^* is negative, the expression in (26) may exceed that in (25) for small values of j, so that soon after the intervention low-cost investors may be worse off; later on (for j large), as the stock of shares grows further, low-cost investors will start gaining from the policy scheme.

Graphically, the magnitude of the surplus without policy (expression (26)) is represented in Figure 2 by the vertical distance between point E_2 and the height of the fixed cost f_L. When the policy is implemented, the number of stockholders rises to N_4^*, the economy moves to a position such as T_1, T_2 or T_3, and starts gradually moving up towards the new steady state E_4. The surplus from entry under the policy (expression (25)) is thus measured by the vertical distance between the new position of the economy above point N_4^* and the height of f_L. Point T_2 is drawn at the same height as the initial steady state surplus E_2: if the economy moves to a point below T_2 (such as T_1), the surplus of low-cost investors is lower than it would be with no policy, and the first generations lose from the policy; if instead it moves to a point about T_2 (such as T_3), they benefit from it.

Obviously, if low-cost investors initially lose, the loss is even larger for high-cost investors. Since without policy intervention high-cost investors would not enter the market, they benefit from it only if being forced into the market by the new set of incentives makes them better off. Formally, they gain only if their surplus from entry

$$E(\Delta U_H|\text{policy})=\frac{b}{2}\left[\left(\frac{K_{t+j}}{N_4^*}\right)^2\sigma_x^2(N_4^*)+\frac{\sigma_{u\pi}^2\sigma_e^2}{\sigma_x^2(N_4^*)}\left(1-\frac{1}{N_4^*}\right)\right]-Rf_H \tag{27}$$

is greater than zero. In Figure 2, this translates into a position above T_3, that is the point where the expected utility from entry is equal to the fixed cost f_H. Thus below T_2 (e.g. at T_1) both classes of investors are worse off, between T_2 and T_3 low-cost investors gain and high-cost ones lose, and above T_3 everyone benefits from the policy.

Intuitively, the reason for this potential ex-ante welfare loss for those investing soon after the adoption of the policy is that, as new investors are forced into the market by the new set of incentives, the market price has to increase to balance the unchanged supply with the increased demand, lowering the expected return on equities and investors' ex-ante welfare. However, as investment and new share issues take place period after period, the stock of equities rises, their price declines and correspondingly investors' welfare rises. In fact, the faster the response of the supply of equities, the earlier point T_3 is reached, i.e. the earlier the policy scheme starts producing beneficial effects for both types of investors (in the limiting case where adjustment costs are zero, the economy jumps to E_4 and everyone is better off right from the start). The timing of this event is important for the policy-maker as well: starting from that moment, the government can withdraw from the scene, removing both tax and subsidy, because entry by high-cost investors will be self-sustaining. From point T_3 the economy will reach the superior steady-state equilibrium E_4 on its own.

In conclusion, while the new steady state is Pareto-superior to the old, there may be losers and gainers along the transition to the new equilibrium: if the supply of equities adjusts slowly to the increased demand, the stock price initially rises, so that investors holding the stock when the policy is announced benefit from a large capital gain, while the first few generations of investors after them suffer a loss from purchasing the stock at high prices. As new shares are issued, the price declines and eventually all investors gain from the policy scheme. If the first generations of investors lose from the policy scheme, they can conceivably compensated by a more complex scheme, involving intertemporal transfers. This suggests that the solution to the co-ordination problem created by the entry externality may be hard to implement via private contractual agreements, and may actually require government intervention.

5. INTRODUCING IMPERFECT COMPETITION AND OTHER EXTENSIONS OF THE MODEL

In this section I test the robustness of the results, by indicating how the model can be extended in various directions, especially to increase the realism of its assumptions. I examine three such extensions: (i) letting investors behave as imperfect competitors on the stock market; (ii) introducing other sources of noise in the stock price; (iii) relaxing the assumption of complete equity-financing.

(i) An assumption implicitly made so far is that investors behave as price-takers on the stock market. Since the chief purpose of the analysis is to study speculative markets with few traders, it is more appropriate to posit that investors take into account the effect of their actions on the market price, and behave as imperfect competitors in conveying their demands to the market. In the Appendix it is shown how the entire analysis can be recast in these terms, solving for a Nash conjectural equilibrium at each date. Upon doing so, one finds that if traders behave as imperfect competitors, in equilibrium they restrict their trades below the competitive level, and their welfare is correspondingly lower. The welfare loss deriving from imperfect competition decreases as the number of traders increases, since that brings the market closer to the perfectly competitive standard. This implies that the introduction of imperfect competition in the model reinforces the beneficial externality from entry. A larger number of traders reduces not only the volatility of the stock price but also its sensitivity to each trader's orders. Since the externality from entry is at the root of the multiplicity of equilibria, imperfectly competitive behaviour strengthens the key result of the analysis. It brings out that both the aspects normally associated with the notion of market liquidity—price stability and insensitivity to the order flow—are related to the volume of trade and, in the presence of transaction costs, can give rise to low-volume traps.

(ii) In the model the variance of the stock price vanishes asymptotically as the number of traders increases. However, in the real world, even in very deep markets, stock prices can be very volatile. This is because in actual markets the variance of stock prices derives from two main factors, i.e. the flow of new information that at any given moment leads investors to revise their expectations of future dividends and the transaction-related noise deriving from the sample variance of investor's demands. The above model has intentionally ignored the first component in order to concentrate only on the second source of variability. But information-related volatility can be incorporated into the model in a relatively simple way without affecting the qualitative conclusions of the model. For instance, assume that there are two stochastic components to earnings per share at time $t+1$, π_{t+1}, one that gets publicly disclosed at time t and another that is revealed at $t+1$, when the dividend is distributed: the stock price at time t will then also respond to the "news" revealed by the realizations of the first of these two components. As a result, the variance of the stock price will reflect the variance of the conditional forecast of next period's dividend *as well as* the transaction-related variance stemming from the uncorrelated demands of traders. As a result, the variance of the stock price would not vanish as the number of traders goes to infinity (as in the model laid out above), but would rather shrink to its information-related component: only transaction-related volatility would disappear.

(iii) The assumption that firms are completely equity-financed is another simplifying assumption that might be questioned on the grounds of realism. In practice, firms listed on the stock market finance themselves issuing debt as well as equities. In fact a shift in asset demands from debt to equity due to the entry of new investors on the stock market, by putting downward pressure on the equity premium, can induce a corresponding shift in the relative supplies of assets issued by corporations, rather than an increase in real capital investment: corporations, perceiving equities to be a cheaper source of finance than debt, could retire debt and issue stock. To model such a response, however, one needs an explicit or implicit model of the optimal debt-equity ratio. While this development is not pursued here, it is in a similar model in Pagano (1985.)[8] One of the results there is that, again, there is scope for multiple equilibria characterized by different stock market size, provided the elasticity of substitution between debt and equity as sources of finance is large enough. One can conjecture that this conclusion would carry over also to this model, if the firm were assumed to respond to changes in the equity premium by varying its debt-equity ratio rather than its capital stock.

Finally, it should be stressed that the basic point made in this paper need not be restricted to equity markets: it could just as well apply, with appropriate changes, to any centralized speculative market, such as futures markets, commodity exchanges or foreign currency markets.

6. CONCLUSIONS

Thin equity markets, being characterized by low numbers of transactors per unit time, are typically unable to absorb temporary bulges of buy or sell orders without wide fluctuations in the market price. Thus market thinness tends to increase the volatility of asset prices and their tendency to react adversely to the orders of traders—two features that are obviously unappealing to investors. Conversely, entry by additional investors, by increasing the depth of the market, exerts a beneficial externality on all other actual and potential market participants.

When this externality combines with transaction costs, individual investors may have no incentive to enter the market although as a group they would benefit from entry. This

8. Chapter 5, pp. 103–130.

creates the potential for multiple equilibria. If few investors decide to enter, the market will be trapped at an equilibrium where prices are volatile and highly sensitive to the order flow. Vice versa, if many of them choose to trade the asset, the resulting equilibrium will feature greater price stability and absorptive capacity.

High-trade equilibria are unambiguously superior to low-trade ones, and it is possible to design incentive schemes capable of shifting the economy from a low-trade to a high-trade equilibrium. Along the transition path, however, welfare losses can occur for those that invest immediately after the policy intervention, if firms adjust slowly the supply of equities to changes in their price.

APPENDIX

Derivations for the case of imperfect competition

Suppose that stock traders behave as imperfect competitors, rather than price-takers. As in the text, one must first work out again the solution to the portfolio problem of investors (the first-stage decision), and then find the implied solution to their entry problem (the second-stage decision). The essential difference obviously comes in the first step: each investor now perceives that his demand for equities, and thus his portfolio choice, affects the market price, i.e. will be maximizing against the perceived demand function of the remaining $N_t - 1$ traders. Since objective functions are quadratic, a symmetric Nash equilibrium exists if conjectures about other traders' responses are linear in price, as in Kyle (1986) and Pagano (1988).

Formally, this translates into adding another constraint to investor i's decision problem. He now maximizes the objective function in (3), subject to (4), (5) and to the conjecture that the residual demand for equities by the other $N_t - 1$ traders is a linear function $A - Bp_t + \eta_t$ (where η_t is a stochastic disturbance), i.e. to a conjecture about market equilibrium:

$$A - Bp_t + \eta_t + k_{it} = K_t. \tag{A1}$$

This relationship shows that investor i perceives the market price p_t to be (linearly) affected by his own demand for equities k_i. A, B and η_t are to be determined by solving for the Nash equilibrium.

From the first-order condition of investor i's portfolio problem, we find his demand for equities:

$$\hat{k}_{it} = \frac{E(x_{t+1} | \Omega_t) - Rp_t - b\sigma_{ux} e_{it}}{R/B + b\sigma_x^2}. \tag{A2}$$

Since each of the other $N_t - 1$ will be solving the same problem under the same conjecture about market equilibrium, we can use (A2) to obtain the residual demand for equities $A - Bp_t + \eta_t$ (simply summing over $j = 1, \ldots, N_t, j \neq i$) and thus obtain a new expression for market equilibrium:

$$\frac{[E(x_{t+1} | \Omega_t) - Rp_t](N_t - 1) - b\sigma_{ux} \sum_{j=1, j\neq i}^{N_t - 1} e_{jt}}{R/B + b\sigma_x^2} = K_t - k_{it}. \tag{A3}$$

The value of A, B and η_t are then found by equating the coefficients of (A3) to those of the initial conjecture (A1):

$$A = \frac{E(x_{t+1} | \Omega_t)}{b\sigma_x^2}(N_t - 2), \qquad B = \frac{R}{b\sigma_x^2}(N_t - 2), \qquad \eta_t = -\frac{\sigma_{ux}}{\sigma_x^2}\frac{N_t - 2}{N_t - 1}\sum_{j=1, j\neq i}^{N_t - 1} e_{jt} \tag{A4}$$

Finally, summing in (A2) over all N_t investors, and using the value of B from (A4), we obtain the Nash equilibrium price:

$$p_t = \frac{1}{R}\left[E(x_{t+1} | \Omega_t) - b\left(\sigma_x^2 \frac{N_t - 1}{N_t - 2}\frac{K_t}{N_t} + \sigma_{ux}\bar{e}_t\right)\right] \quad \text{where } \bar{e}_t \equiv \sum_{i=1}^{N_t} \frac{e_{it}}{N_t}. \tag{A5}$$

that can be compared to expression (12) in the text. The amount of equities that investor i will demand in equilibrium is:

$$\hat{k}_{it} = \frac{K_t}{N_t} - \frac{\sigma_{ux}}{\sigma_x^2}\frac{N_t - 2}{N_t - 1}(e_{it} - \bar{e}_t). \tag{A6}$$

The competitive counterpart of this expression can be obtained by letting $N_t \to \infty$, so that $(N_t - 2)/(N_t - 1) \to 1$. The term $(N_t - 2)/(N_t - 1)$ captures the fact that under imperfect competition each agent perceives himself as affecting the market price adversely and thus restrains his demand or supply of equities below the competitive level. For instance, if $\sigma_{ux} < 0$ (i.e. equities can be used as a hedge against fluctuations in the return to non-marketable asset) and $e_{it} > \bar{e}_t$ (i.e. investor i has an abnormally high endowment of this asset), then i will restrain his demand for equities below the competitive level—and will restrain it by more the smaller N_t (i.e. the larger he perceives to be relative to the market).

Substituting for $\hat{k}_{it}$ from (A6) into the appropriate entry decision rule, one finds that now in equilibrium investor i will enter the stock market if:

$$\frac{N_t}{N_t - 2}\frac{b}{2R}\left[\left(\frac{K_t}{N_t}\right)^2 \sigma_x^2 + \frac{\sigma_{u\pi}^2 \sigma_e^2}{\sigma_x^2}\left(\frac{N_t - 2}{N_t - 1_t}\right)^2\left(1 - \frac{1}{N_t}\right)\right] \geqq f(i). \tag{A7}$$

It is easy to show that in steady state the supply of shares K_t now is:

$$K_t^* = \frac{\mu_\pi - rc - b\sigma_{u\pi}\mu_e}{2b(\sigma_\pi^2 + R\alpha\sigma_e^2/N_t)}\frac{N_t - 2}{N_t - 1}N_t, \tag{A8}$$

The rule (A7) can be evaluated at steady state by setting K_t equal to K_t^*, to find the equilibrium number of traders N_t^* in steady state, i.e. the analogue of condition (22) in the text. This condition is simply:

$$f(N_t^*) \leqq \Psi(N_t^*)\left(1 - \frac{1}{(N_t^* - 1)^2}\right) \leqq f(N_t^* + 1), \tag{A9}$$

where $\Psi(N_t)$ is defined as in (22). The key point is that the middle term is monotonically increasing in N_t^*, just as in condition (22), implying that also in the imperfectly competitive case the expected utility from entry is increasing in the number of market participants and that the mechanism that is at the root of the multiplicity of equilibria is operative. In fact, the elasticity of the surplus from entry with respect to N_t^* is now larger. The reason is that an increase in the number of traders, beside stabilizing the market price, leads each trader to expect a less adverse price response to his demand, bringing the market closer to perfect competition.

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