Resale and Bundling in Multi-Object Auctions

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- *What is the effect of resale on bidders' strategies in multi-object auctions?*

- With resale, a losing bidder can purchase the prize from the auction winner

- In **single-object** auctions, resale induces weak bidders to participate and bid aggressively, thus increasing the seller's revenue (Pagnozzi, *RAND* 07)

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  (Wilson, *QJE* 79; Ausubel & Cramton, 98)

  (e.g., FCC auctions, California electricity markets, German GSM auction ...)

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- But **resale** corrects an inefficient allocation ...

⇒ Resale may induce bidders to reduce demand, thus reducing the seller's revenue (Pagnozzi, *AEJ: Micro*)

- **Bundling** is a natural reaction by the seller to the risk of demand reduction, because it forces bidders to win all objects, or none (Anton & Yao, *QJE* 92)

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- Bundling *and* allowing resale are **complement strategies** for the seller:

– revenue is higher with bundling *and* resale than
   without bundling and/or without resale (if bidders are not too asymmetric)

– resale reduces revenue without bundling but
   increases revenue with bundling

– bundling may reduce revenue without resale but
   increases revenue with resale (and by larger amount)

Model

- Uniform-price auction, 2 bidders, 2 (identical) units

	1^{st} unit	2^{nd} unit
Strong	v_S^1	v_S^2
Weak	v_W^1	v_W^2

- Assumptions:
  - Bidders know values, seller does not
  - $v_i^1 \geq v_i^2$ (and $v_S^2 \geq v_W^1$ for today)
  - No weakly dominated strategies
  - In the resale market, W obtains a share α and S obtains a share $(1-\alpha)$ of the gains from trade, $0 < \alpha \leq 1$

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Def. In a *DR equilibrium* bidders bid:

(i) "willingness to pay" for 1st unit (weakly dominant strategy)

(ii) 0 for 2nd unit

$\Rightarrow$ Each bidder wins 1 unit and the seller's revenue is 0

DR without Resale

- Assume resale is not allowed

– i bids v_i^1 for the 1st unit (weakly dominant strategy)

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(i) S can win 2 units at price v_W^1 and obtain $\pi_S\,(\text{No DR}) = v_S^1 + v_S^2 - 2v_W^1$

(ii) S can bid 0 for the 2nd unit and obtain $\pi_S\,(\text{DR}) = v_S^1 - 0$

$\Rightarrow$ There is no DR equilibrium iff:

$$\pi_S\,(\text{No DR}) > \pi_S\,(\text{DR}) \quad \Leftrightarrow \quad \boxed{v_S^2 > 2v_W^1}$$

(i.e., bidders are *asymmetric*)

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- If $v_S^2 < 2v_W^1$, S prefers to win 1 unit at price 0, since outbidding W is costly (DR is the unique Pareto dominant equilibrium)

- With DR, the allocation is inefficient and bidders would like to trade ...

“Willingness to Pay” with Resale

	1^{st}	2^{nd}
S	v_S^1	v_S^2
W	v_W^1	v_W^2

$\overset{\text{resale}}{\Rightarrow}$

	1^{st}	2^{nd}
S	$\alpha v_S^1 + (1-\alpha)\, v_W^2$	$\alpha v_S^2 + (1-\alpha)\, v_W^1$
W	$\alpha v_S^2 + (1-\alpha)\, v_W^1$	$\alpha v_S^1 + (1-\alpha)\, v_W^2$

- If W wins 1 unit, he resells it to S at price $v_W^1 + \alpha\left(v_S^2 - v_W^1\right)$

  $\Rightarrow$ In the auction, S is willing to pay $\alpha v_S^2 + (1-\alpha)\, v_W^1$ for the 2^{nd} unit

- If W wins 2 units, he resells the second to S at price $v_W^2 + \alpha\left(v_S^1 - v_W^2\right)$

  $\Rightarrow$ In the auction, S is willing to pay $\alpha v_S^1 + (1-\alpha)\, v_W^2$ for the 1^{st} unit

Resale and DR

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- **Resale** makes bidders more "symmetric":
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  - S's willingness to pay is lower (due to option to buy)

  and S can buy in the resale market after reducing demand

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$\Rightarrow$ Bidders always prefer to reduce demand

$$\text{e.g.,} \quad \text{let } b_W^1 \text{ be } W\text{'s bid for first unit,} \quad \pi_S\,(\text{DR}) > \pi_S\,(\text{No DR}) \quad \Leftrightarrow$$

$$v_S^1 + \underbrace{(1-\alpha)\left(v_S^2 - v_W^1\right)}_{S\text{'s resale surplus}} > v_S^1 + v_S^2 - 2b_W^1 \quad \Leftrightarrow \quad b_W^1 > \frac{1}{2}\left[\alpha v_S^2 + (1-\alpha)\, v_W^1\right]$$

- **Lemma**: *With resale, DR is the unique Pareto dominant equilibrium*

Seller's Revenue

- The seller's revenue is $\begin{cases} 0 \text{ with DR} \\ > 0 \text{ without DR} \end{cases}$

– **With resale** there is always DR

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⇒ **Proposition 1**: *In a multi-unit uniform-price auction, allowing resale reduces the seller's revenue*

– W bids more aggressively with resale (like in a single-unit auction) but this induces S to reduce demand and lowers revenue

Bundling

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⇒ Bundling makes it impossible for bidders to profitably reduce demand *but* it makes the auction price depend on the lowest valuation of the losing bidder

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- If resale is not allowed, the seller's revenue is:

(a) With bundling: $\Pi^B_{NR} = v^1_W + v^2_W$

(b) Without bundling: $\Pi^{NB}_{NR} = \begin{cases} 2v^1_W & \text{if } v^2_S > 2v^1_W \text{ (No DR)} \\ 0 & \text{if } v^2_S < 2v^1_W \text{ (DR)} \end{cases}$

⇒ Without resale, bundling increases the seller's revenue iff $v^2_S < 2v^1_W$

Bundling (Cont.)

- If resale is allowed, the seller's revenue is:

(*a*) With bundling: $\Pi_R^B = \alpha \left(v_S^1 + v_S^2\right) + (1 - \alpha) \left(v_W^1 + v_W^2\right) \equiv$ resale price

(*b*) Without bundling: $\Pi_R^{NB} = 0$ (due to DR)

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  (b) Without bundling: $\Pi_R^{NB} = 0$ (due to DR)

$\Rightarrow$ **Proposition 2**: *With resale, bundling increases the seller's revenue*

– DR always takes place with resale, and bundling eliminates DR

Bundling + Resale

- Bundling and allowing resale are **complement strategies** for the seller

- The seller's revenue with bundling and resale is:

$$\Pi_R^B = \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right)$$

e.g., in the resale market S can buy the 1$^{\text{st}}$ unit for $\alpha v_S^1 + (1-\alpha)\, v_W^2$
and the 2$^{\text{nd}}$ unit for $\alpha v_S^2 + (1-\alpha)\, v_W^1$,
so she bids the sum of these prices in the auction

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- **Prop. 3**: *Bundling and allowing resale yields a higher seller's revenue than*

  (i) *not bundling and allowing resale*
  (ii) *bundling and not allowing resale*
  (iii) *not bundling and not allowing resale, if bidders are not too asymmetric*

(i) $\Pi_R^B > \Pi_R^{NB}$ by Proposition 2
(ii) $\Pi_R^B > \Pi_{NR}^B$ like in single-object auctions

Bundling + Resale (Cont.)

$$(iii) \quad \Pi_R^B = \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right) \quad > \quad \Pi_{NR}^{NB} = \begin{cases} 2v_W^1 & \text{if } v_S^2 > 2v_W^1 \\ 0 & \text{if } v_S^2 < 2v_W^1 \end{cases}$$

$$\Leftrightarrow \quad 2v_W^1 > v_S^2 \quad \text{or} \quad \alpha > \frac{v_W^1 - v_W^2}{v_S^1 + v_S^2 - v_W^1 - v_W^2}$$

(i.e., W's value *or* W's share of the resale surplus is not too low compared to S's)

Bundling + Resale (Cont.)

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(i.e., W's value *or* W's share of the resale surplus is not too low compared to S's)

– Resale induces W to bid more aggressively
and bundling prevents S from reacting by reducing demand

– This always increases revenue if there is DR without resale (i.e., $2v_W^1 > v_S^2$)

– If there is no DR without resale (i.e., $2v_W^1 < v_S^2$),
bundling makes the auction price depends on W's lowest value,
but resale increases W's values if α is sufficiently high (e.g., $\alpha = \frac{1}{2}$)

Bundling + Resale (Cont.)

- Moreover, each strategy increases the effect of the other:

– The effect of resale on revenue is stronger with bundling than without it:

$$\underbrace{\Pi_R^B - \Pi_{NR}^B}_{>0} \quad > \quad \underbrace{\Pi_R^{NB} - \Pi_{NR}^{NB}}_{\leq 0}$$

Bundling + Resale (Cont.)

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– The effect of resale on revenue is stronger with bundling than without it:

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– The effect of bundling on revenue is stronger with resale than without it:

$$\Pi_R^B - \Pi_R^{NB} = \alpha \left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right) - 0 \quad >$$

$$> \quad \Pi_{NR}^B - \Pi_{NR}^{NB} = \begin{cases} v_W^1 + v_W^2 - 0 & \text{if } v_S^2 > 2v_W^1 \\ v_W^1 + v_W^2 - 2v_W^1 & \text{if } v_S^2 < 2v_W^1 \end{cases}$$

Inefficient Resale Market

- Assume bidders are unable to trade with probability $(1-p)>0$

	1^{st}	2^{nd}
S	v_S^1	v_S^2
W	v_W^1	v_W^2

$\overset{\text{resale}}{\Rightarrow}$

	1^{st}	2^{nd}
S	$v_S^1 - p(1-\alpha)\left(v_S^1 - v_W^2\right)$	$v_S^2 - p(1-\alpha)\left(v_S^2 - v_W^1\right)$
W	$v_W^1 + p\alpha\left(v_S^2 - v_W^1\right)$	$v_W^2 + p\alpha\left(v_S^1 - v_W^2\right)$

– Compared to $p=1$, W is willing to pay a *lower* price and S a *higher* price but there is still a DR equilibrium if p is not too low

Inefficient Resale Market

- Assume bidders are unable to trade with probability $(1-p) > 0$

	1^{st}	2^{nd}
S	v_S^1	v_S^2
W	v_W^1	v_W^2

resale $\Rightarrow$

	1^{st}	2^{nd}
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W	$v_W^1 + p\alpha\left(v_S^2 - v_W^1\right)$	$v_W^2 + p\alpha\left(v_S^1 - v_W^2\right)$

– Compared to $p = 1$, W is willing to pay a *lower* price and S a *higher* price but there is still a DR equilibrium if p is not too low

- Without resale there is no DR equilibrium if $v_S^2 > 2v_W^1$

$\Rightarrow$ **Proposition 4:** *If* $v_S^2 > 2v_W^1$ *and* $p > \frac{v_S^2 - 2v_W^1}{(1+\alpha)(v_S^2 + v_W^1)}$, *allowing resale reduces efficiency with probability* $(1-p)$

– The possibility of resale increases efficiency ex post,
but it may induce demand reduction even if bidders may be unable to trade

Summary

- The possibility of resale affects bidders' strategies

- In multi-object auctions, resale increases bidders' incentives to reduce demand (by increasing the willingness to pay of low-value bidders and reducing the willingness to pay of high-value bidders)
- So resale may *reduce* both revenue and efficiency

- Allowing resale *and* bundling (often) *increase* the seller's revenue