Should Speculators Be Welcomed in Auctions?

Marco Pagnozzi

Università di Napoli Federico II and *CSEF*

pagnozzi@unina.it

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  speculators (who have no use value for auction prize)
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Or should the seller discourage them, because value captured by speculators must come from someone else's payoff – possibly the seller's?" (Milgrom, 2004)

... it depends on bidders' relative valuations:

(i) if bidders' valuations are *clustered*,
resale and speculators *increase* the seller's revenue

(ii) if bidders' valuations are *dispersed*,
resale and speculators *reduce* the seller's revenue

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- **Single-object** ascending auction (with full information):
  - 1 bidder (B) with value 8
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- With equal sharing of resale surplus, if S wins, he resells at price $\frac{1}{2}(8+0) = 4$

$$\Rightarrow \begin{cases} S \text{ is willing to bid up to 4 in the auction} \\ B \text{ is willing to bid up to 4 in the auction} \end{cases}$$

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⇒ Multiple equilibria, but resale is not robust to an (arbitrarily small) resale cost

Related Literature

- Resale can take place in **single-object** auctions:
  - if some bidders do not participate in the auction (Milgrom, 1987; Bikhchandani & Huang *RFS*, 1989 ...)
  - if bidders' valuations change after the auction (Haile *GEB*, 2003; *JET*, 2003)
  - in 1^{st}-price asymmetric auctions with uncertainty (Gupta & Lebrun *EL*, 1999; Hafalir & Krishna *AER*, 2007 ...)
  - if speculators induce bidders to bid 0 by bidding "aggressively" (Garratt & Tröger *Econometrica*, 2006)
  - if the auction price affects bargaining in the resale market (Pagnozzi *RAND*, 2007)

- But resale arises much more naturally in **multi-object** auctions ...

– *Can speculators win an auction?*

– *Why should a high-value bidder let speculators win?*

- In **single-object** auctions, it is unclear why a high-value bidder should prefer to buy in the resale market

- In **multi-object** auctions, bidders often bid less than value for marginal units to reduce the auction price (*Demand Reduction* ≡ DR)

  (Wilson, 1979; Ausubel & Cramton, 1998)

  (e.g., FCC auctions, German GSM auction, California electricity markets ...)

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$\Rightarrow$ A high-value bidder may *strictly* prefer to let speculators win some objects in order to keep the auction price low for the objects she wins
(and then buy from speculators in the resale market)

Example (**Cont**.)

- **Multi-object** uniform-price auction**:**
  - 2 units, 1 bidder (B), 1 speculator (S)
  - 2 highest bids win and pay 3$^{\text{rd}}$-highest bid

	1$^{\text{st}}$ unit	2$^{\text{nd}}$ unit
B	8	8
S	0	0

$\overset{\text{resale}}{\Rightarrow}$

	1$^{\text{st}}$ unit	2$^{\text{nd}}$ unit
B	4	4
S	4	4

- As before, B can win the 2 units in the auction at price 4 ... but ...

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- With DR $\left\{ \begin{array}{l} B \text{ bids } (4;\ 0) \\ S \text{ bids } (4;\ 0) \end{array} \right\}$, B and S win one unit each and S resells:

$$\pi_B = \underbrace{8-0}_{\text{auction profit}} + \underbrace{8-4}_{\text{resale surplus}}$$

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$\Rightarrow \left\{ \begin{array}{l} \text{Resale is (the Pareto dominant for } B \text{ and } S\text{) equilibrium} \\ S \text{ wins and the seller's revenue is } 0 \end{array} \right.$

Example (**Cont.**)

- Resale is an equilibrium even with:

  (i) Different sharing of resale surplus, and even a take-or-leave offer by S

  (ii) Not fully efficient resale market (e.g. if $0 < \Pr[\text{no resale}] \leq \frac{1}{3}$)

  (iii) (Not too large) resale cost (i.e. if $c < \frac{1}{3} \cdot B$'s value)

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e.g.(ii)**:** Assume with prob. $\frac{1}{4}$ resale does not take place

$\Rightarrow$ In the resale market B and S obtain expected surplus $\frac{3}{4} \cdot 4 = 3$

	1st unit	2nd unit
B	8	8
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	1st unit	2nd unit
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$$\pi_B = \underbrace{8 - 0}_{\text{auction profit}} + \underbrace{\tfrac{3}{4}(8 - 4)}_{\text{resale surplus}} = 11$$

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UK 3.4GHz Auction (June 2003)

- Simultaneous Ascending Auction for 15 licenses for broadband wireless services
  - **PCCW** (a Hong-Kong telecom company) was the highest-value bidder and was expected to win 15 licenses
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- As soon as PCCW, RS and PH were the only bidders left, PCCW reduced demand to 13 licenses to end the auction
- PCCW's failure to win all licenses was described as
  - "a surprise, ... a gaffe"
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- PCCW's failure to win all licenses was described as
  - "a surprise, ... a gaffe"
  - "a costly mistake that may cost the chance of offering a nationwide service"

But was it really a mistake?

- By March 2004, PCCW took over RS and PH and obtained all licenses

Model

- Uniform-price auction for k (identical) units

  (k highest bids win and pay $(k+1)^{\text{th}}$-highest bid)

$$\begin{cases} n \text{ bidders with values } v_1 > v_2 > ... > v_n & \text{(flat demand)} \\ k-n \text{ speculators} & \text{(more speculators cannot make profit)} \end{cases}$$

	1st unit	2nd unit	$\cdots$	k^{th} unit
B_1	v_1	v_1	$\cdots$	v_1
B_2	v_2	v_2	$\cdots$	v_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
B_n	v_n	v_n	$\cdots$	v_n
S_1	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
S_{k-n}	0	0	$\cdots$	0

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S_{k-n}	0	0	$\cdots$	0

- Assumptions:

(i) B_i and S_i know values, seller does not – e.g., Wilson 1979

(ii) No weakly dominated strategy

(iii) In the resale market, a unit can be traded only once and players equally share the gains from trade (Nash bargaining)

Bargaining for Resale

– *At what price does* S_i *resell to* B_1*?*

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- Our qualitative results hold with many alternative assumptions on bargaining:

1. Equal sharing of the gains fro trade: $\frac{1}{2}v_1$
2. Take-it-or-leave-it offer: v_1
3. "Multi-parties" bargaining: $\frac{1}{2}(v_1 + v_2)$

   (when a unit can be sold more than once)
4. Unequal bargaining power: $\alpha_S \cdot v_1, \quad 0 < \alpha_S \leq 1$

   ...

   (Assumptions 2 and 3 favour speculators and reinforce our results)

"Valuations" with Resale

- If S_i wins, he resells to B_1 for $\frac{1}{2}v_1$
- If B_i, $i \neq 1$, wins, he resells to B_1 for $\frac{1}{2}(v_1 + v_i)$

$\rightarrow$ With resale, S_i's and B_i's "valuation" is higher for the option to resell

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(Zero-Price) Demand Reduction Equilibrium

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Def.: In a **DR equilibrium** each player bids $\begin{cases} \text{her "valuation" for the } 1^{\text{st}} \text{ unit} \\ 0 \text{ for all other units} \end{cases}$

$$\rightarrow \begin{cases} B_1 \text{ bids } \left(\frac{1}{2}(v_1 + v_2);\ 0; \ldots;\ 0\right) \\ B_i \text{ bids } \left(\frac{1}{2}(v_1 + v_i);\ 0; \ldots;\ 0\right), \quad i \neq 1 \\ S_i \text{ bids } \left(\frac{1}{2}v_1;\ 0; \ldots;\ 0\right) \end{cases}$$

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$\Rightarrow$ Each player wins 1 unit at price 0 and $B_2, ..., B_n, S_1, ..., S_{k-n}$ resell:

$$\pi_S^* = \underbrace{\tfrac{1}{2}v_1}_{\text{resale price for } S_i} \qquad \pi_i^* = \underbrace{\tfrac{1}{2}(v_1+v_i)}_{\text{resale price for } B_i}, \quad i \neq 1$$

$$\pi_1^* = k \cdot v_1 - \sum_{i=2}^{n} \underbrace{\tfrac{1}{2}(v_1+v_i)}_{\text{resale price from } B_i} - (k-n) \cdot \underbrace{\tfrac{1}{2}v_1}_{\text{resale from } S_i}$$

Bidding

- S_i has no incentive to deviate from the DR equilibrium
  (because to win more units he has to pay at least $\frac{1}{2}v_1$)
- B_1 has no incentive to deviate from the DR equilibrium
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- B_2 does not outbid S_i to win $(k-n+1)$ units iff:

$$\pi_2^* = \underbrace{\tfrac{1}{2}(v_1+v_2)}_{\text{resale price for 1 unit}} > \underbrace{(k-n+1)\cdot\tfrac{1}{2}(v_1+v_2)}_{\text{resale price for } k-n+1 \text{ unit}} - \underbrace{(k-n+1)\cdot\tfrac{1}{2}v_1}_{\text{auction price for } k-n+1 \text{ unit}}$$

$$\Leftrightarrow \quad \boxed{v_1 > (k-n)\,v_2} \qquad (\dagger)$$

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$$\begin{cases} \text{high } v_1 & \Rightarrow \text{high willingness to pay for } S_i \\ \text{low } v_2 & \Rightarrow \text{low resale price for } B_2 \end{cases}$$

→ with high v_1 and low v_2, outbidding speculators is too costly for B_2

Bidding

- B_2 does not outbid B_j, $j > 2$, to win $(k - j + 2)$ units iff:

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→ with high v_1, v_j and low v_2, outbidding B_j is too costly for B_2

- B_2 has the strongest incentive to deviate from the DR equilibrium
  ⇒ If B_2 does not deviate, no other bidder deviates

⇒ There is a DR equilibrium iff (†) and (‡) are satisfied — e.g., if $\boxed{v_1 > (k-2)\,v_2}$

Equilibria

There are 2 types of equilibria:

1. **DR equilibrium**:

   if values are **dispersed** (e.g., if $v_1 > (k-2)\,v_2$)
   bidders reduce demand and accommodate speculators

   $\Rightarrow$ Speculators win and the auction price is 0

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- When it exists, the DR equilibrium is the *Pareto dominant equilibrium* for B_i and S_i (among all equilibria in undominated strategies)

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$\Rightarrow$ Speculators win and the auction price is 0

- When it exists, the DR equilibrium is the *Pareto dominant equilibrium* for B_i and S_i (among all equilibria in undominated strategies)

2. Positive Price equilibrium:

if values are **clustered** (i.e., if (†) and/or (‡) are not satisfied)
B_2 outbids lower-value bidders and/or speculators to win more units
(Multiple equilibria: B_i can win $1, ..., k$ units)

$\Rightarrow$ Speculators lose and the auction price is $\geq \frac{v_1}{2}$

Effect of Resale and Speculators on Seller's Revenue

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(ii) **Demand Reduction Effect:**

- Resale makes DR more profitable for high-value bidders because they buy in the resale market the units they lose in the auction
- Resale makes it more costly for a bidder to deviate from DR and outbid lower-value competitors, because they bid higher when they can resell

Effect of Resale and Speculators on Seller's Revenue

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  - Resale is allowed but speculators are not allowed to participate

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→ We compare 3 scenarios:

		Speculators: Yes	Speculators: No
Resale	Yes	Scenario 1	Scenario 3
	No		Scenario 2

Scenario 1 vs 2: Effect of Resale

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$$\text{seller's revenue is } \Pi_R = \begin{cases} 0 & \text{if } v_1 > 2v_2 \quad \text{(i.e., with DR)} \\ \frac{v_1}{2} & \text{if } v_1 < 2v_2 \quad \text{(i.e., without DR)} \end{cases}$$

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(2) Without resale (and so no speculator):

– B_2 has no incentive to deviate from a DR equilibrium

$\Rightarrow$ There is a DR equilibrium iff B_1 does not want to outbid B_2:

$$\max \pi_1 (\text{DR}) > \pi_1 (\text{No DR}) \quad \Leftrightarrow \quad 3(v_1 - 0) > 4(v_1 - v_2) \quad \Leftrightarrow \quad \boxed{v_1 < 4v_2}$$

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– If bidders' valuations are **clustered**, B_1 prefers to win fewer units at price 0 because outbidding B_2 to win more units is too costly

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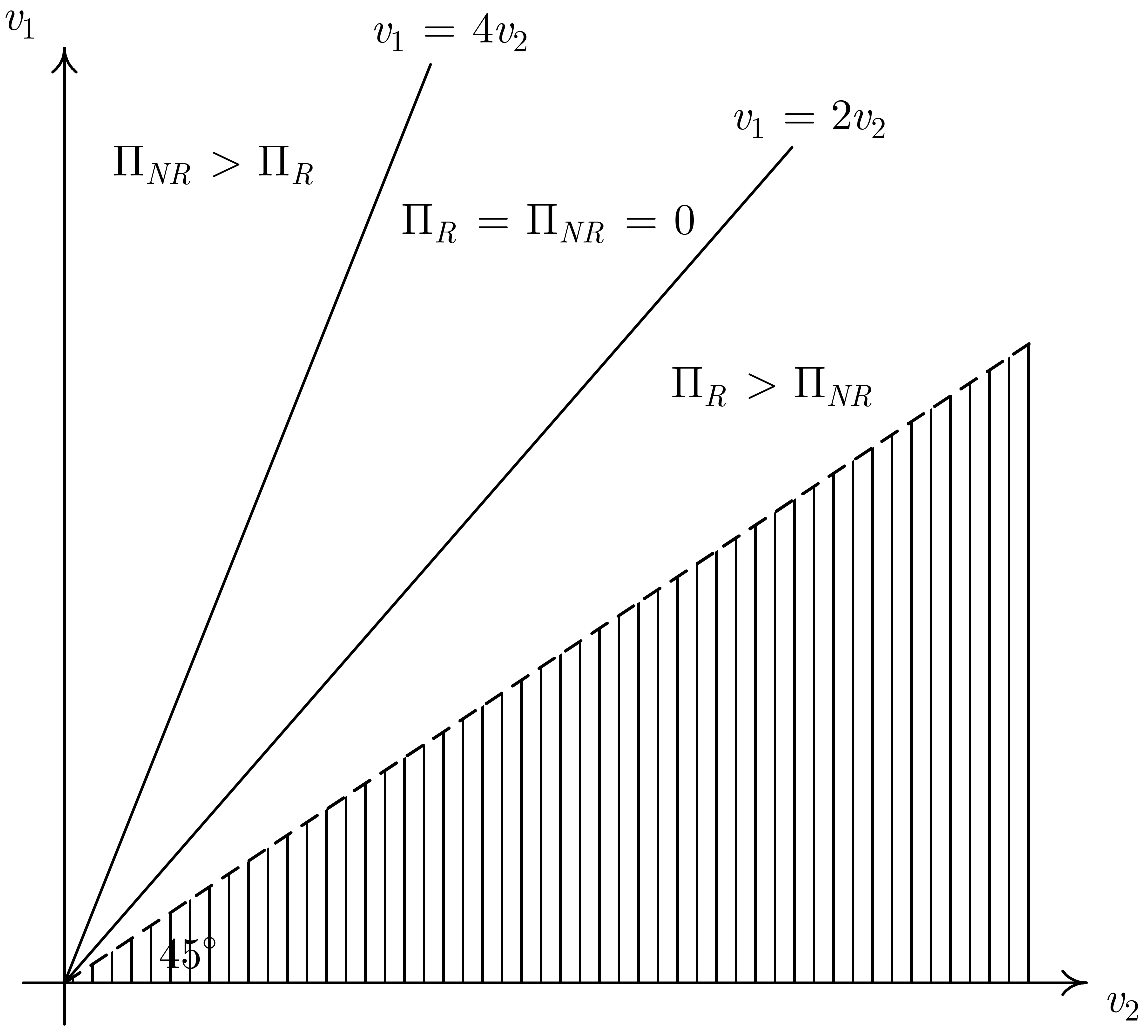

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(Same qualitative results hold with n bidders and k objects)

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- If resale is always allowed:

(1) With speculators: the auction has a DR equilibrium iff:

$$\boxed{v_1 > (k-n)\,v_2} \quad (\dagger) \quad \text{and} \quad \boxed{(k-j+2)\,v_j + v_1 > (k-j+1)\,v_2, \quad j \geq 3} \quad (\ddagger)$$

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- B_1 has no incentive to deviate from DR
- if B_2 does not deviate deviate from DR, no bidder deviates
- B_2 does not deviate from DR by outbidding B_j to win $(n-j+2)$ units iff:

$$\pi_2^* = \underbrace{\tfrac{1}{2}(v_1+v_2)}_{\text{resale price for 1 unit}} > \underbrace{(n-j+2)\cdot\tfrac{1}{2}(v_1+v_2)}_{\text{resale price for } n-j+2 \text{ unit}} - \underbrace{(n-j+2)\cdot\tfrac{1}{2}(v_1+v_j)}_{\text{auction price for } n-j+2 \text{ unit}}$$

$$\Leftrightarrow \quad \boxed{(n-j+2)\, v_j + v_1 > (n-j+1)\, v_2, \quad j \geq 3} \quad \underset{\text{since } k>n}{\Leftarrow} \quad (\ddagger)$$

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$\Rightarrow$ A DR equilibrium is "easier" without speculators:

$$\left\{ \begin{array}{c} v_i \text{ s.t. DR is an equilibrium} \\ \text{with speculators} \end{array} \right\} \subset \left\{ \begin{array}{c} v_i \text{ s.t. DR is equilibrium} \\ \text{without speculators} \end{array} \right\}$$

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- **Proposition:** *When resale cannot be prevented,*
  speculators (weakly) increase the seller's revenue

- Low-value bidders can themselves resell to high-value bidders
  so the *DR Effect* of resale is present even without speculators

  ⇒ Speculators only have a *Competition Effect*

Seller's Strategy: Summary

- *If the seller cannot prevent resale,*
  he should always allow speculators to participate
- *If the seller can prevent resale and knows bidders' relative values, he should:*
  allow resale to attract speculators if bidders' values are clustered,
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$\Rightarrow$ { Bidders accommodating speculators is bad news for the seller
Speculators increase seller's revenue only if they are outbid

Scenario 2 vs 3: Effect of Resale without Speculators

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⇒ **Proposition:** *With no speculator who may participate in the auction, allowing resale reduces the seller's revenue when it induces bidders to reduce demand*

- But without a DR equilibrium, resale may increase the seller's revenue because it may induce bidders (apart from B_1) to bid more aggressively

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(ii) If the resale market is not necessarily efficient (e.g., bidders are unable to trade with positive probability) *allowing resale reduces efficiency* when it induces a DR equilibrium

Inefficient Resale Market

- 2 units, 2 bidders ($v_1 = 10$, $v_2 = 2$), no speculator
- With probability $\frac{1}{4}$ resale does not take place

$\Rightarrow$ In the resale market B_1 and B_2 obtain expected surplus $\frac{3}{4} \cdot \left[\frac{1}{2}(10 - 2)\right] = 3$

	1st unit	2nd unit
B_1	10	10
B_2	2	2

$\overset{\text{resale}}{\Rightarrow}$

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- With DR $\left\{\begin{array}{l} B_1 \text{ bids } (7;\ 0) \\ B_2 \text{ bids } (5;\ 0) \end{array}\right\}$, B_1 obtains $\pi_B = \underbrace{10-0}_{\text{auction profit}} + \underbrace{3}_{\text{resale surplus}} = 13$

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(This holds iff $\Pr[\text{resale}] > \frac{2(v_1 - 2v_2)}{v_1 + v_2}$ and $v_1 > 2v_2$)

Empirical Evidence

- The model's predictions are:

1. Resale price $>$ (multi-object) auction price
(since bidders trade in resale market only after reducing demand in auction)

e.g.: – 1980s New Zealand auctions for import licenses
(McAfee, Tacks & Vincent, *RAND* '99)
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2. Auction price when speculators lose > auction price when speculators win

⇒ Inverse relation between trading volume in the resale market
(by auction speculators) and auction price

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- Speculators are attracted by the possibility of resale
- In *multi-object* auctions, high-value bidders may let speculators win in order to keep the auction price low

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- Without speculators, resale is even more likely to reduce the seller's revenue