Sorry Winners

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2000 EU Mobile-Phone License Auctions

- 3G licenses sold by sequential ascending auctions
- *Prices were higher in earlier auctions and much lower in later auctions*:

	Date	Euros per capita
UK	March	650
Netherland	July	170
Germany	July	615
Italy	October	240
Austria	November	100
Switzerland	November	20

⇒ Winners of earlier auctions complained they overpaid based on evidence of lower prices in later auctions and lobbied governments for improved conditions

e.g.: *Germany* and *UK*: firms allowed to share network cost

Italy: licences lengthened from 15 to 20 years

How do rational players overbid?

- Bidders receive:

  (i) **Common-Value Signal**:
  affects value of all bidders alike
  (e.g., demand estimate for mobile phones)

  (ii) **Private-Value Signal**:
  only affects value of bidder who receives it
  (e.g., firm's cost)

- When facing a rival with large private signal, a *rational* bidder may overestimate the prize value and *overpay*

- Overpayment has **political costs** for sellers and **embarrassment costs** for managers of bidding firms

- A **sorry winner** realizes that the prize value is less than the price he paid and regrets winning

How does the order of sale matter?

- A bidder in an auction also obtains information from his opponents' bidding in other auctions

- With **sequential auctions**, prices in later auctions provide information on prize value in earlier auctions
- With **simultaneous auctions**, bidders obtain more information when they can still change their strategy

$\Rightarrow$ Sequential auctions increase risk of overpayment
... but "on aggregate" winners may still make money

- **Information inequality** among bidders affects bidding strategies and seller's revenue

$\Rightarrow$ By reducing information inequality, simultaneous auctions may increase seller's revenue and efficiency

Single-Object Auction

- Ascending auction, 2 bidders: E and I_1

$$\begin{cases} V_E = \underbrace{\theta}_{\text{common signal}} + \underbrace{t_E}_{\text{private signals}} \\ V_1 = \theta + t_1 \end{cases}$$

– $i.i.d.$ signals

– bidder i knows t_i

– I_1 also knows θ

- **Lemma:** *In the unique equilibrium* $\begin{cases} I_1 \textit{ bids up to } \theta + t_1 \\ E \textit{ bids up to } 2 \cdot t_E \end{cases}$

- **Proposition 1**: *If* $t_1 > t_E > \frac{1}{2}(\theta + t_1)$,
  bidder E *wins but obtains negative profit*

$\Rightarrow$ I_1 bids aggressively because of high t_1,
but E rationally expects high θ too

How can a bidder become a sorry winner?

Model

- 2 objects, 2 ascending auctions: A, B
- 3 bidders: $\begin{cases} E \text{ (entrant)} \\ I_1 \text{ and } I_2 \text{ (incumbents)} \end{cases}$
- Valuations:

		Bidder		
		E	I_1	I_2
Auction	A	$\theta + t_E$	$\theta + t_1$	0
	B	$\theta + t_E$	0	$\theta + t_2$

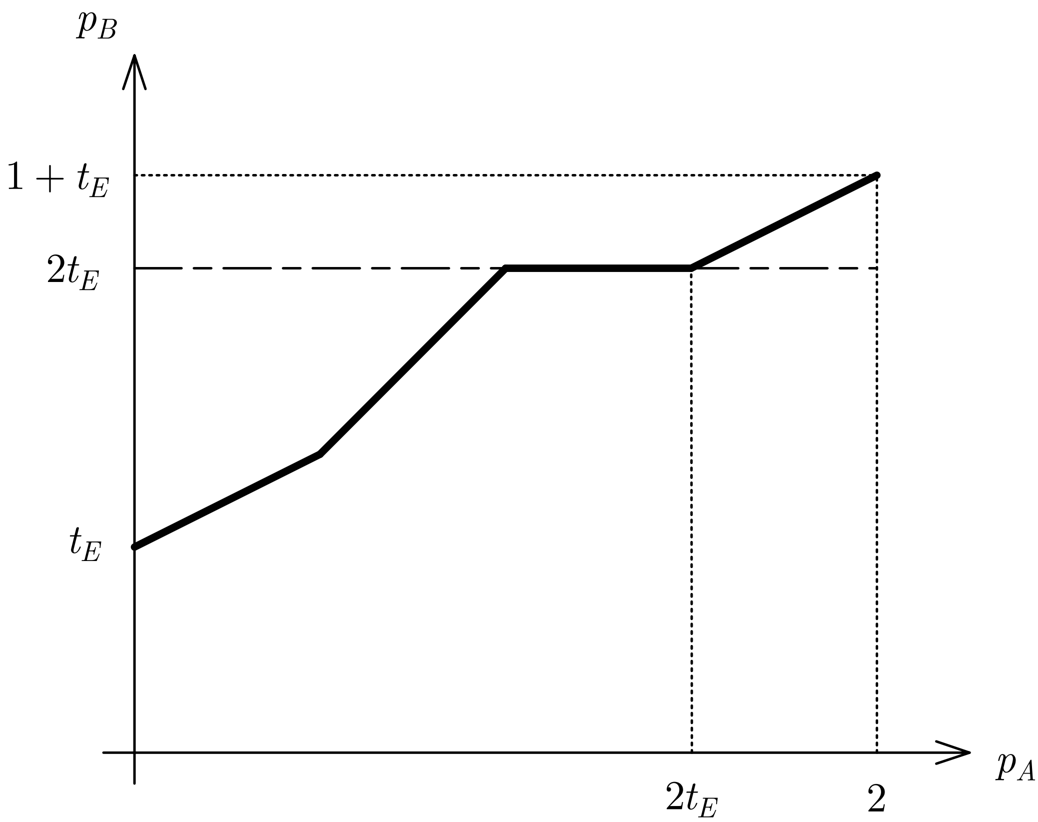
- Assumptions:
  - θ, $t_i \sim U[0,1]$
  - bidder i knows t_i
  - I_1 and I_2 also know θ

$$\Rightarrow \begin{cases} I_1 \textit{ bids up to } \theta + t_1 \\ I_2 \textit{ bids up to } \theta + t_2 \end{cases}$$

Sequential Auctions (A-B)

- **Lemma:** *If bidder E wins A at p_A, he drops out of B at*

$$p_B = t_1 + \mathbb{E}[\theta \,|\theta + t_1 = p_A,\ \theta + t_2 = p_B]$$



$$\Rightarrow \begin{cases} \text{When } p_A < 2t_E,\ E \text{ bids less aggressively for } B \\ \text{When } p_A > 2t_E,\ E \text{ bids more aggressively for } B \end{cases}$$

(than in single-object auction)

- Winning A at **low price** is **bad news** about θ

Overbidding

- Raising price in auction A provides information about θ

- Assume current price is $2t_E$ and E bids up to $2t_E + \varepsilon$

$\Rightarrow$ Bidder E risks winning A at $2t_E + \varepsilon$
but then can bid better in auction B

- **Potential loss** of order ε:

$$(2t_E + \varepsilon) - \mathbb{E}\left[V_E \left| \theta + t_1 = 2t_E + \varepsilon\right.\right] = \tfrac{\varepsilon}{2}$$

- **Potential gain** of order higher than ε:
  - E learns that $\theta + t_1 = 2t_E + \varepsilon$ (rather than $\theta + t_1 > 2t_E$) and reduces bid for B (by "large" amount)

$\Rightarrow$ **Lemma:** *In the first auction,*
bidder E bids strictly more than $2t_E$

Sorry Winner

- Information obtained in auction B may make bidder E realize he overpaid in auction A

- **Proposition 2: Sorry Winner in Expectation**
  In sequential auctions, after winning auction B, bidder E regrets winning auction A iff

$$\mathbb{E}\left[V_E \left| p^A, p^B \right.\right] = t_E + \mathbb{E}\left[\theta \left| p^A, p^B \right.\right] < p^A$$

- **Proposition 3: Sorry Winner with Certainty**
  In sequential auctions, after winning auction B, bidder E is certain he lost money in auction A iff

$$\max\{V_E\} = t_E + \min\left\{p^A, p^B\right\} < p^A$$

$\Rightarrow$ Winning B at price $p^B < p^A$ is bad news about θ, and may make E a sorry winner in auction A

Sorry Winner on Aggregate?

- Suppose bidder E wins auction A at price $p_A < 2t_E$ and then wins auction B at price $p_B \leq p_A$
- Bidder E's total expected profit is:

$$\begin{aligned}\mathbb{E}\left[\pi_E\right] &= 2\left(\mathbb{E}\left[\theta \left| p_A,\ p_B\right.\right] + t_E\right) - p_A - p_B \\ &\geq 2\left(\tfrac{1}{2}p_B + t_E\right) - p_A - p_B > 0\end{aligned}$$

$\Rightarrow$ Aggregating across both auctions, E is not sorry winner

- A low price in auction B is bad news about value in auction A, but is good news about profit in auction B

e.g.: Winners may regret winning in UK and Germany, but prices in later auction are so low that, they may still make profit on aggregate

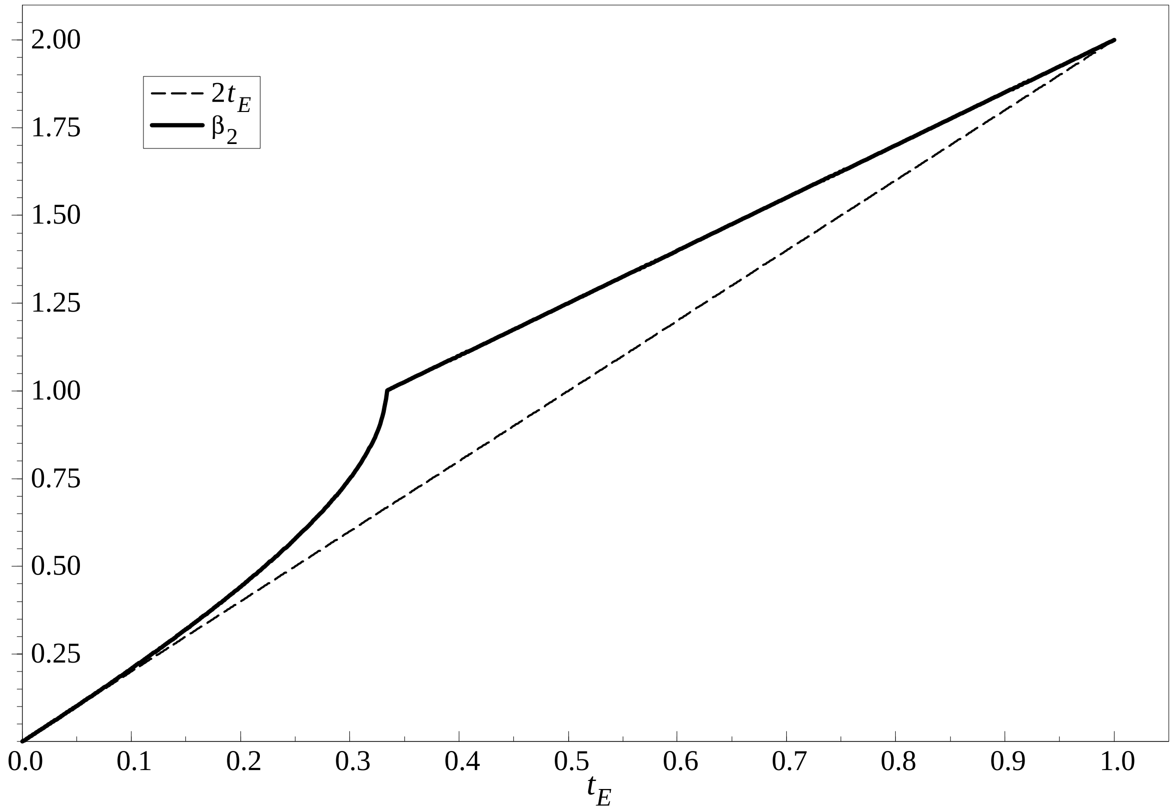
(But if E "overbids" in auction A and then wins auction B at high price (or loses), he obtains negative total profit)

Simultaneous Auctions

(Prices rise simultaneously and continuously)

- **Lemma:** *If* I_1 *and* I_2 *are active, bidder* E *drops out of both auctions at*

$$p^* = t_E + \mathbb{E}\left[\theta \,|\theta + t_i = p^*,\ \theta + t_j > p^*\right]$$

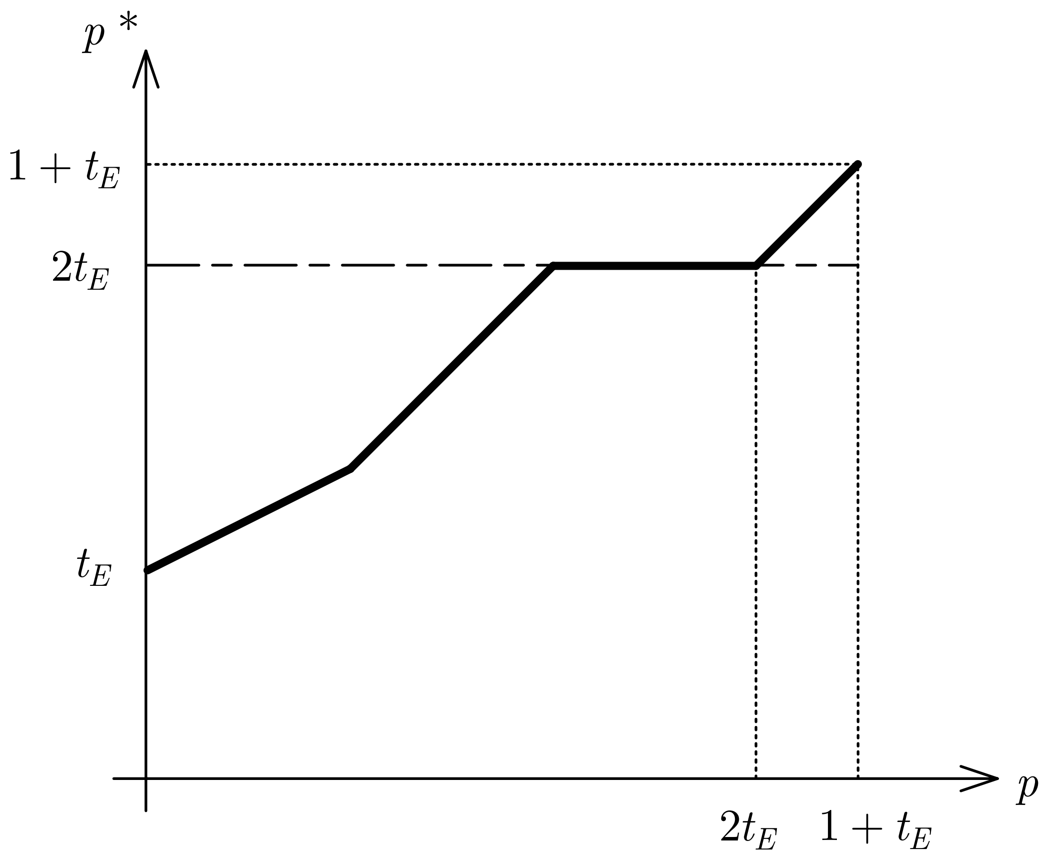


$\Rightarrow$ E can bid hgher (than in single-object auction), because observing prices in both auctions reveals information about θ and reduces his "winner's curse"

- **Lemma:** (i) *If bidder E wins one auction at $p < 2t_E$, he drops out of the other auction at*

$$p^* = t_E + \mathbb{E}\left[\theta \,|\theta + t_i = p,\ \theta + t_j = p^*\right]$$

(ii) *If bidder E wins one auction at $p \geq 2t_E$, he drops out of the other auction immediately*



$\Rightarrow$ After winning one auction at **low price**
E bids less aggressively in other auction
After winning one auction at **high price**
E drops out immediately of other auction

No Sorry Winner in Simultaneous Auctions

- E's bidding strategies depend on all information revealed by his opponents' bidding in both auctions

$\Rightarrow$ E learns whether $\theta + t_i$ is low in one auction when he can still modify bidding in other auction

- **Proposition 4:** *In simultaneous auctions bidder E is never a sorry winner*

- The outcome of one auction never induces bidder E to regret winning other auction

$\Rightarrow$ Auction process does not **endogenously** provide evidence of overpayment

Seller's Revenue

- With **information inequality** among bidders:
  - **simultaneous auctions** reveal more information about θ and allow E to bid aggressively
  - **sequential auctions** induce E to bid aggressively in auction A to acquire information

$\Rightarrow$ First effect prevails when E has no incentive to "overbid" in first of 2 sequential auctions

- **Proposition 5:** *If E's value for B is independent of θ, expected seller's revenue and efficiency is higher in simultaneous auctions than in sequential auctions*

- Information inequality induce less informed bidder to bid more cautiously to avoid winner's curse
- Simultaneous auctions reduce information inequality more than sequential auctions

Ascending vs Second-Price Auctions

- **Lemma:** *Expected seller's revenue is higher in simultaneous ascending auctions than in simultaneous second-price auctions*

- Simultaneous auctions reduce information inequality among bidders more than second-price auctions

Conclusions

- Bidding in one auction provides information relevant for other auctions
- *Rational* bidders can become sorry winners

- **Sequential auctions** may provide proof of overpayment (e.g., European 3G auctions)

  but sorry winners may still have positive aggregate profit

- **Simultaneous auctions**:

(i) reduce risk of sorry winners and increase efficiency (by revealing more information during bidding)

(ii) may increase revenue (by reducing information inequality among bidders)

(With explicit embarrassment cost, firms would bid even less aggressively in sequential auctions)