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VERTICAL SEPARATION WITH PRIVATE CONTRACTS*

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We consider a manufacturer's incentive to sell through an independent retailer, rather than directly to final consumers, when contracts with retailers cannot be observed by competitors. If retailers conjecture that identical competing manufacturers always offer identical contracts (*symmetric beliefs*), manufacturers choose vertical separation in equilibrium. Even with private contracts, vertically separated manufacturers reduce competition and increase profits by inducing less aggressive behaviour by retailers in the final market. Manufacturers' profits may be higher with private than with public contracts. Our results hold both with price and with quantity competition and do not hinge on retailers' beliefs being perfectly symmetric. We also discuss various justifications for symmetric beliefs, including incomplete information.

Can competing manufacturers obtain higher profits by delegating retail decisions to independent agents, rather than selling directly to final consumers? Manufacturers jointly benefit from high retail prices but, when they sell directly to final consumers, competition among them results in low prices and profits. However, a manufacturer can induce an independent retailer to sell at higher prices, by charging a wholesale price higher than marginal cost. And credibly committing to doing so has a 'strategic effect' on rival retailers, who react by selling at higher prices themselves, thus reducing downstream competition (Vickers, 1985; Bonanno and Vickers, 1988; Rey and Stiglitz, 1995).

This insight hinges on the assumption that contracts between manufacturers and retailers are observed by competitors (i.e. public): when contracts are private (or, alternatively, when publicly announced contracts can be secretly renegotiated), a manufacturer's wholesale price cannot affect the strategy of a rival retailer. Therefore, it is often argued that delegation has no strategic effect because manufacturers always charge a wholesale price equal to marginal cost – a *neutrality result* (Coughlan and Wernerfelt, 1989; Katz, 1991; Caillaud and Rey, 1995).

We show that the neutrality result rests on a specific assumption about retailers' conjectures on their competitors' contracts – i.e. *passive beliefs* – and that the equilibrium changes when alternative, but equally reasonable, assumptions are considered. The point is that, with private contracts, a retailer's strategy depends on his conjecture about the wholesale price paid by rival retailers and this conjecture may depend on the contract offered to the retailer. Hence, even if vertical separation cannot directly affect the strategies of rival retailers, it can still affect a retailer's conjecture about his rivals' input cost (as well as the retailer's own input cost).

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If retailers conjecture that identical manufacturers always choose the same wholesale price (*symmetric beliefs*), vertical separation by all manufacturers arises in equilibrium and increases manufacturers' profits. Hence, even when contracts with retailers cannot be observed by outsiders, vertical separation reduces competition by inducing less aggressive behaviour by retailers in the final market. This result holds both with price and with quantity competition.[1]

In models with a single principal and multiple agents, the typical assumption is that agents have passive beliefs (e.g. Cremer and Riordan, 1987; Horn and Wolinsky, 1988; Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Laffont and Martimort, 2000). In contrast to a situation with symmetric beliefs, a retailer who has passive beliefs and is offered a wholesale price different from the one he expects in equilibrium does not revise his beliefs about the offers made to rival retailers. In this case, vertical separation affects neither the strategies of rival retailers, nor a retailer's conjectures about these strategies. Hence, vertically separated manufacturers act as if they were integrated with their retailers and always charge a wholesale price equal to marginal cost – the neutrality result.

When there are competing manufacturers, however, the assumption of passive beliefs is not necessarily the most natural one. If a manufacturer has an incentive to offer a contract different from the one that the retailer expects, then why should another identical manufacturer not have an incentive to do the same? Arguably, a natural first assumption is that retailers perceive deviations as symmetric and conjecture that identical manufacturers always offer the same contract. An alternative interpretation of symmetric beliefs is that retailers are naive, or have 'bounded rationality', and simply believe that the strategy adopted by a rival manufacturer is always identical to the strategy adopted by the manufacturer with whom they are contracting.[2] In this case, symmetric beliefs are a 'rule of thumb' adopted by retailers. Or retailers may be completely uninformed about some private, and (partly) common, characteristic of manufacturers – e.g. the manufacturers' production cost – and so be unable to determine the manufacturers' equilibrium contract.[3]

Hart and Tirole (1990) and McAfee and Schwartz (1994) consider symmetric beliefs in a model with a single (monopolistic) manufacturer and two independent and competing retailers. They show that, with private contracts, the manufacturer's profit depends on retailers' beliefs and is higher with symmetric than with passive beliefs. However, they also argue that the assumption of passive beliefs is the most natural one in their model, because if the manufacturer offers a contract different from the equilibrium one to a retailer, she has no incentive to offer the same contract to the

[1] By contrast, when contracts are public, vertical separation increases profits with price competition but reduces profits with quantity competition.

[2] Symmetric beliefs are much simpler than passive ones for retailers, in the following sense. With passive beliefs, a retailer must compute manufacturers' equilibrium contracts, given retailers' optimal strategies, in order to make a conjecture about his opponent's input cost. By contrast, with symmetric beliefs a retailer simply bases this conjecture on the manufacturer's offer, thus trusting her ability to choose the best contract. So a retailer only needs to compute his own best strategy, given his input cost. Therefore, the assumption of symmetric beliefs appears more natural when retailers face computational or cognitive constraints.

[3] In Section 6, we show that symmetric beliefs arise in a Hotelling model in which manufacturers are privately informed about their costs of production, and these costs are correlated and have full support. Symmetric beliefs also arise when retailers have diffuse prior about manufacturers' cost, or about a shock affecting this cost. See also Lucas (1972) on monetary misperception.

other retailer. By contrast, we believe that symmetric beliefs are especially appealing with identical competing vertical chains, because if one of the manufacturers has an incentive to offer a contract different from the equilibrium one, then a rival manufacturer should have an incentive to do the same. So it is natural for a retailer who receives an unexpected offer to conjecture that the same reason that induced his manufacturer to deviate also induced rival manufacturers to make an identical deviation.

To explore the effects of beliefs, we analyse a delegation game with unobservable contracts. First manufacturers publicly choose whether to sell through independent retailers or not. Second, vertically separated manufacturers offer two-part tariffs to retailers. Finally, price competition takes place in the retail market. We compare equilibria with passive and symmetric beliefs. In contrast to the neutrality result with passive beliefs, with symmetric beliefs delegation is a weakly dominant strategy. Specifically, there are two equilibria with symmetric beliefs: one where all manufacturers delegate, and the other where all manufacturers integrate. But the equilibrium where manufacturers sell through independent retailers both Pareto dominates (from the manufacturers' point of view) and risk dominates the one where manufacturers integrate.

The reason for our result is that, if retailers conjecture that other retailers are offered their same contract, vertical separation generates a 'belief effect': the wholesale price charged by a manufacturer affects the retailer's beliefs about the contract offered to competing retailers and, hence, about the retail price charged by the latter. Therefore, by increasing wholesale prices, manufacturers manage to soften downstream competition, because retailers who pay high wholesale prices expect competitors to pay high wholesale prices as well, and respond by charging higher retail prices in equilibrium.[4] Manufacturers can then charge a higher franchise fee and obtain higher profits.

Hence, even with private contracts, manufacturers have an incentive to sell through independent retailers, when retailers have symmetric beliefs.[5] By doing so, manufacturers manage to coordinate implicitly on high wholesale prices, since a manufacturer who charges a lower wholesale price reduces the franchise fee that the retailer is willing to pay.

Our result that manufacturers choose vertical separation even with private contracts does not hinge on retailers having *exactly* symmetric beliefs. Indeed, the belief effect that we have described arises as long as a retailer who is offered a contract different from the equilibrium one assigns a positive probability, which can be arbitrarily small, to a rival retailer being offered the same contract. As with symmetric beliefs, manufacturers can then obtain a strictly higher profit by selling through independent retailers, because they can induce them to sell at high prices. Moreover, although symmetric beliefs are especially compelling when manufacturers are identical, we also

[4] With public contracts, the strategic effect of a high wholesale price is to induce competitors to charge high prices. By contrast, with private contracts and symmetric beliefs, the effect of offering a high wholesale price is to induce a retailer to believe that his competitors pay high wholesale prices, so that the retailer charges a high retail price and expects high profits.

[5] Kockesen (2007) analyses an extensive form game in which principals can sign private contracts with 'passive' agents (who only receive lump sum transfers) and shows that principals obtain higher profit with delegation. Delegation has a commitment value because principals can induce agents to play a 'minmax strategy' if rival principals do not delegate, regardless of the other agents' action.

show that (partly) symmetric beliefs arise in equilibrium in a model where asymmetric manufacturers are privately informed about their marginal costs of production, and these costs are correlated. Our qualitative results on vertical separation hold if manufacturers are not too asymmetric. The reason is that a belief effect arises when costs are correlated: a retailer uses the wholesale price offered by his own manufacturer to infer information about the marginal costs of other manufacturers and, hence, his competitors' wholesale prices.

We also compare manufacturers' profit with private and public contracts. Since each retailer can observe other retailers' contracts, when those are public, and choose the preferred retail price based on them, it may be expected that manufacturers always obtain higher profits with public contracts. However, this is not necessarily the case. Although with private contracts a manufacturers can only affect the strategy of their own retailer, they can still charge a higher franchise fee by choosing a high wholesale price. But since rival retailers do not respond by increasing their prices, a high wholesale price also reduces the quantity sold by the retailer, thus lowering the manufacturer's wholesale revenue. On balance, a manufacturer obtains lower profit with private contracts when the strategic effect is not too strong – i.e. when a retailer does not increase his price too much in response to an observable increase of a rival's price.[6]

Information sharing among firms is usually considered anti-competitive (Homchick and Singer, 1994). Our results, however, suggest that, if retailers have symmetric beliefs, manufacturers may agree to keep information about wholesale prices private, precisely when public contracts would enhance consumer welfare by reducing retail prices. Hence, allowing retailers to obtain information about their rivals' wholesale prices may actually increase competition.

Although we consider price competition in our main model, we obtain similar results with quantity competition: with symmetric beliefs, manufacturers selling through independent retailers obtain higher profits because of the belief effect of high wholesale prices. Moreover, with quantity competition, since a retailer buys the manufacturer's good before observing the realised market price (and hence before observing the quantity sold by competing retailers), manufacturers manage to obtain the monopoly profit jointly. By contrast, the strategic effect of public contracts harms manufacturers with quantity competition, because it induces them to charge lower wholesale prices (Fershtman and Judd, 1987). So manufacturers always prefer private contracts, rather than public ones, when retailers' choice variables are strategic substitutes.

Our results depend on manufacturers' ability to charge franchise fees before retailers observe the realised demand, when manufacturers can affect the retailers' beliefs about the competitors' choices. A manufacturer can then charge a high franchise fee by choosing a high wholesale price, even if other manufacturers do not choose high wholesale prices. This is consistent with the observation that, in real-world contractual relationships, franchise and royalty fees are usually paid *ex ante* and do not depend on the quantity sold by retailers.

[6] By contrast, when competing retailers contract with a single monopolistic manufacturer, the manufacturer's profits with public contracts are always higher than those with private contracts (both with passive and with symmetric beliefs). In fact, the commitment value of public contracts allow the manufacturer to obtain the monopoly profit (Rey and Tirole, 2007).

Besides providing a new rationale for delegation, our results have implications for a wider range of economic situations involving competing vertical chains. First, they suggest that various types of vertical restraints may soften downstream competition with private contracts and symmetric beliefs. For instance, even with unobservable contracts, exclusive territories may be used to reduce interbrand competition and raise manufacturers' profits. Second, in relation to the literature on the strategic design of managerial incentives (e.g. Fershtman and Judd, 1987; Sklivas, 1987), our model suggests that incentive schemes different from profit maximisation may have a strategic role even when these schemes are private.

The rest of the article is organised as follows. Section 1 presents the model. After discussing the case of passive beliefs in Section 2, in Section 3 we consider symmetric beliefs. Specifically, we first analyse prices and profits when: all manufacturers are vertically integrated; all manufacturers are vertically separated; and a vertically integrated manufacturer competes against a vertically separated one. Then, in Section 3, we characterise the equilibrium choice of organisational structure by manufacturers. Section 4 describes an example with linear demand function. In Section 5 we show that our results hold with a more general class of retailers' beliefs and in Section 6 we show how symmetric beliefs arise with incomplete information, and asymmetric manufacturers. Sections 7 and 8 compare private and public contracts and discuss quantity competition. Finally, Section 9 concludes. All proofs are in the Appendix.

1. The Model

1.1. *Players and Environment*

There are two competing vertical structures, with two (female) manufacturers, M_1 and M_2, that produce substitute goods, and two (male) exclusive retailers, R_1 and R_2.[7] In the downstream market, firms compete by choosing retail prices. (We consider quantity competition in Section 8.) Manufacturers publicly choose their organisational structure: *vertical integration* or *vertical separation*. If M_i is vertically integrated, they choose the retail price and sell directly to final consumers; if M_i is vertical separated, they sell through retailer R_i, who independently chooses the retail price.

The retail price of the good produced by M_i is p_i, and the (twice continuously differentiable) demand function for this good in the downstream market is $D^i(p_i, p_j)$, with $i, j = 1, 2$ and $i \neq j$. We assume that $D^i(p, q) = D^j(p, q)$ for all prices p and q – i.e. demand functions are symmetric. All firms have constant returns to scale and manufacturers' marginal cost of production is normalised to zero.

1.2. *Contracts*

With vertical separation, M_i offers a two-part tariff contract $C_i = (w_i, T_i)$ to R_i, specifying a wholesale price $w_i \in \mathbb{R}_+$ and a franchise fee $T_i \in \mathbb{R}$. If R_i accepts the contract, he pays T_i, chooses the retail price and then pays w_i for each unit sold in the

[7] R_1 and R_2 can alternatively be interpreted as buyers of an intermediate good, that they transform into a final good through a fixed-coefficient technology.

downstream market. R_i's outside option is normalised to zero. We assume that contracts are private, so that a retailer cannot observe the contract offered to his competitor. This assumption captures the idea that manufacturers lack commitment power, because they can recontract and/or offer secret discounts.

1.3. *Timing*

The timing of the game is as follows:

- *Period* 1. Manufacturers simultaneously and publicly choose their organisational structure.
- *Period* 2. A vertically separated manufacturer offers a contract to her exclusive retailer. If the retailer accepts it, he pays the franchise fee and sells the manufacturer's good in period 3.
- *Period* 3. Firms – i.e. integrated manufacturers, or retailers of vertically separated manufacturers – simultaneously choose retail prices in the downstream market. Retailers of separated manufacturers pay the wholesale price for the quantity they acquire, after observing the realised demand.

1.4. *Equilibrium Concept*

A manufacturer's strategy specifies the choice of organisational structure and, depending on this choice, specifies either the contract offered in period 2 or the retail price charged in period 3. A retailer's strategy specifies an acceptance decision in period 2 and the retail price chosen in period 3, contingent on the contract offered by the manufacturer.

Our model has complete information but unobservable actions, and our solution concept is *weak* perfect Bayesian equilibrium (PBE) (Mas-Colell *et al.*, 1995), that imposes no restriction on beliefs off the equilibrium path. We investigate how the equilibrium of the model depends on the choice of retailers' off-the-equilibrium-path beliefs.

We consider three types of beliefs:

- *Passive beliefs:* When a retailer is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the rival retailer. Hence, given an equilibrium with wholesale prices w_1^* and w_2^*, if R_i is offered a wholesale price different from w_i^*, he believes that R_j is still offered w_j^*.
- *Symmetric beliefs:* Each retailer believes that his competitor is always offered a contract equal to the contract offered by his own manufacturer. Hence, if R_i is offered a wholesale price w_i, he believes that R_j is offered the same wholesale price w_i.[8]
- *Mixed beliefs:* Given an equilibrium with wholesale prices w_1^* and w_2^*, if R_i is offered a wholesale price $w_i \neq w_i^*$, he believes that, with probability α, R_j is

[8] With symmetric beliefs it is only possible to have symmetric equilibria, in which both manufacturers offer the same wholesale price, if they are vertically separated.

offered the same wholesale price w_i and, with probability $(1 - \alpha)$, R_j is offered the equilibrium wholesale price w_j^*.

We first focus on passive and symmetric beliefs. In Section 5, we consider mixed beliefs and show that our qualitative results hold as long as retailers' beliefs are not *exactly* passive – i.e. as long as $\alpha \neq 0$.[9]

An equilibrium with symmetric beliefs does not satisfy the 'no signalling what you don't know' condition required in the definition by Fudenberg and Tirole (1991, p. 332) of a perfect Bayesian equilibrium for multi-stage games with observable actions and *incomplete information*.[10] However, in Section 6, we show that symmetric beliefs are the equilibrium beliefs in a perfect Bayesian equilibrium (satisfying the 'no signalling what you don't know' condition) of a model in which manufacturers are privately informed about their correlated marginal costs. We also allow manufacturers to be asymmetric, and we show that our qualitative results on vertical separation hold if the asymmetry between manufacturers is not too large.

1.5. *Assumptions*

Let $\Pi_i(p_i, p_j) = D^i(p_i, p_j)(p_i - w_i)$ be R_i's profit in period 3, and $\pi_i(p_i, p_j) = D^i(p_i, p_j)p_i$ be M_i's profit when R_i's participation constrain is binding. We make the following assumptions, that are standard in the literature (Bonanno and Vickers, 1988; Rey and Stiglitz, 1995).[11]

[**A1.**] Demand for the good produced by M_i is decreasing and concave in p_i, and satisfies the Inada conditions.
[**A2.**] Goods are substitutes and own price effects are larger than cross price effects.
[**A3.**] Retail prices are strategic complements.
[**A4.**] The functions $\partial\Pi_i(p, p)/\partial p_i$ and $\partial\pi_i(p, p)/\partial p_i$ are downward sloping and the conditions $\partial\Pi_i(p, p)/\partial p_i = 0$ and $\partial\pi_i(p, p)/\partial p_i = 0$ have unique solutions (Vives, 2000, p. 157).

2. Passive Beliefs

With passive beliefs, when a retailer receives an offer different from the one he expects in equilibrium, he does not revise his beliefs about the offer made to the rival retailer.

[9] Of course, there are also other possible types of beliefs. For example, with a single manufacturer and multiple retailers, *wary beliefs* have been proposed by McAfee and Schwartz (1994), and reformulated by Rey and Vergé (2004). A retailer who has wary beliefs expects the manufacturer to offer a rival retailer the contract that is a best response to his own contract. Wary beliefs are arguably more plausible when retailers are sophisticated, while we consider symmetric beliefs especially reasonable with naive retailers, because of their simplicity. Moreover, with price competition, wary beliefs produce a belief effect similar to the one we describe for symmetric beliefs (Rey and Vergé, 2004), because a retailer who is offered a wholesale price higher than he expected knows that the manufacturer's best strategy given this price is to offer a relatively high wholesale price to a rival retailer too.

[10] Roughly, this condition requires that beliefs about a player depend only on the action of that player. With unobservable actions, this is a natural condition when strategies are independent. With competing vertical structures, however, there are many situations in which manufacturers' strategies may be correlated, for example because of correlated costs shocks (see Section 6).

[11] See the Appendix for a formal statement of the Assumptions.

In this case, each manufacturer chooses a wholesale price equal to zero, regardless of the contract and the organisational structure chosen by the competitor.

To see this, suppose that both manufacturers are vertically separated, and denote by p_j the price chosen by R_j in equilibrium in period 3. Because of passive beliefs, R_i's beliefs about R_j's price do not depend on w_i. Hence, R_i's reaction function is[12]

$$p_i(p_j, w_i) \in \arg\max_{p_i}[D^i(p_i, p_j)(p_i - w_i) - T_i].$$

As the franchise fee T_i is a fixed cost, this program yields the standard first-order condition equalising R_i's marginal revenue to marginal cost (i.e. zero)

$$\frac{\partial D^i[p_i(p_j, w_i), p_j]}{\partial p_i}[p_i(p_j, w_i) - w_i] + D^i[p_i(p_j, w_i), p_j] \equiv 0. \tag{1}$$

In period 2, M_i offers the contract that maximises his profit, subject to R_i's participation constraint and given R_j's price.[13] Since the franchise fee is chosen to satisfy R_i's participation constraint as an equality, M_i's problem is

$$\max_{w_i} D^i[p_i(p_j, w_i), p_j]\ p_i(p_j, w_i).$$

Differentiating M_i's objective function and using (1),

$$\begin{aligned} &\frac{\partial p_i(p_j, w_i)}{\partial w_i}\left\{\frac{\partial D^i[p_i(p_j, w_i), p_j]}{\partial p_i} p_i(p_j, w_i) + D^i[p_i(p_j, w_i), p_j]\right\} \\ &\quad = \frac{\partial p_i(p_j, w_i)}{\partial w_i}\frac{\partial D^i[p_i(p_j, w_i), p_j]}{\partial p_i} w_i \le 0. \end{aligned} \tag{2}$$

LEMMA 1. *With passive beliefs, if manufacturers choose vertical separation in period* 1, *in the unique equilibrium wholesale prices are equal to zero.*

As a retailer's choice is unaffected by unobserved changes in the rival's wholesale price, each manufacturer acts as if integrated with the retailer and charges a wholesale price equal to marginal cost.[14] The next Proposition states the well-known *neutrality result* that, with private contracts and passive beliefs, vertical separation has no strategic effect (Katz, 1991).

PROPOSITION 1. *With passive beliefs, in any PBE the retail price* p^e *solves*

$$\frac{\partial D^i(p^e, p^e)}{\partial p_i} p^e + D^i(p^e, p^e) = 0. \tag{3}$$

Any combination of organisational structures is part of a PBE and yields the same manufacturers' profit.

[12] We use the convention of denoting by $g[h(x)]$ the composite function $(g \circ h)(x)$, and by $f\{g[h(x)]\}$ the composite function $(f \circ (g \circ h))(x)$.

[13] Formally, M_i chooses w_i and T_i to maximise $D^i[p_i(p_j, w_i), p_j]w_i + T_i$, subject to the constraint that $T_i \le D^i[p_i(p_j, w_i), p_j][p_i(p_j, w_i) - w_i]$.

[14] As observed by McAfee and Schwartz (1994), this result does not hinge on the nature of downstream production (fixed versus variable proportions) or of downstream competition (strategic substitutes or strategic complements).

Hence, with passive beliefs, manufacturers have no incentive to sell through retailers.[15] The neutrality result, however, does not hold when agents have symmetric beliefs.

3. Symmetric Beliefs

Assume now that retailers have symmetric beliefs – i.e. a retailer always believes that his competitor receives the same offer as he does (Hart and Tirole, 1990; McAfee and Schwartz, 1994). Hence, when a retailer receives from a manufacturer an offer different from what he expects in equilibrium, he believes that the competing manufacturer has also deviated from equilibrium by making the same offer. Of course, in equilibrium retailers' beliefs must be consistent with manufacturers' strategies.

In games of competing hierarchies, it is usually assumed that beliefs are passive. There seems to be no compelling reason, however, to rule out symmetric beliefs *a priori*, especially when upstream manufacturers are symmetric.[16] Why should a retailer who receives an unexpected, off-the-equilibrium, offer believe that a rival manufacturer is still offering the equilibrium contract? If one manufacturer has an incentive to offer a different contract, another identical manufacturer should have an incentive to do the same. Arguably, it is reasonable to assume that retailers expect deviations to be symmetric and conjecture that identical manufacturers always offer identical contracts.

Alternatively, symmetric beliefs capture the idea that retailers are naive or have bounded rationality and so use the simplest conjecture that the strategy adopted by a rival manufacturer is always identical to the strategy adopted by the manufacturer with whom they are contracting. Retailers may find it too costly, or too difficult, to compute the manufacturers' equilibrium contracts, based on the retailers' optimal strategies (which is required with passive beliefs) and simply prefer to infer the equilibrium contract from the manufacturer's actual offer. Hence, symmetric beliefs may be a rule of thumb adopted by retailers.

Or retailers may be completely uninformed about some common characteristic of manufacturers that affects their choice of contract – e.g. their production cost – and so be unable to determine the equilibrium contract.[17] We develop this interpretation in Section 6, where we show that (partly) symmetric beliefs arise in the separating equilibrium of a Hotelling model in which manufacturers are privately informed about their costs of production and these costs are correlated.

[15] Katz (1991) shows that this neutrality result does not hold with agency constraints and that vertical separation may have a commitment effect when manufacturers and retailers have conflicting preferences.

[16] When one (monopolistic) manufacturer contracts with two independent and competing retailers, it is usually argued that symmetric beliefs are unappealing, since the manufacturer's preferred contract with one retailer generally differs from the contract accepted by the other retailer (the 'opportunism problem' in vertical contracting). Moreover, it is argued, since the two retailers represent two separate markets, when the manufacturer changes the offer to one retailer, he has no incentive to also change the offer to the other retailer (e.g. Rey and Tirole, 2007). This criticism is much less compelling in games of competing hierarchies, where a manufacturer may have an incentive to deviate from an equilibrium candidate in order to increase his profit at the expense of the competing manufacturer, but not to harm his own retailer. So if one manufacturer wants to offer a different contract, the other manufacturer should want to do the same.

[17] White (2007) analyses the effect of private information in a model with a single monopolistic manufacturer and two retailers.

When both manufacturers are vertically integrated, beliefs are irrelevant because no contract is offered. Hence, manufacturers choose the retail price that solves condition (4). In the next two subsections, we first analyse the case in which both manufacturers are vertically separated and then the asymmetric case in which one manufacturer is vertically separated, while the other is not.

3.1. *Vertical Separation*

Suppose that both manufacturers choose vertical separation in period 1. First notice that the equilibrium with passive beliefs characterised in Lemma 1 is not an equilibrium with symmetric beliefs.

LEMMA 2. *If both manufacturers choose vertical separation, with symmetric beliefs there is no PBE in which wholesale prices are equal to zero.*

With passive beliefs, manufacturers cannot coordinate to charge a positive wholesale price because each manufacturer has an incentive to undercut it secretly, in order to induce his retailer to choose a lower retail price and obtain higher profit. With symmetric beliefs, however, this incentive is weakened because, if a manufacturer reduces the wholesale price, his retailer conjectures that the other manufacturer is doing the same. Hence, the retailer expects to obtain lower profit and is willing to pay a lower franchise fee.

Let

$$\hat{p}(w_i) \in \arg\max_{pi} D^i[p_i, \hat{p}(w_i)](p_i - w_i)$$

define the price chosen by R_i to maximise his expected profit in period 3, when he is offered the wholesale price w_i and conjectures that R_j pays the same wholesale price w_i and, hence, chooses $\hat{p}(w_i)$ too (since demand functions are symmetric). The first-order condition for R_i's maximisation problem, which is necessary and sufficient under Assumptions A1–A4, is

$$\frac{\partial D^i[\hat{p}(w_i), \hat{p}(w_i)]}{\partial p_i}[\hat{p}(w_i) - w_i] + D^i[\hat{p}(w_i), \hat{p}(w_i)] \equiv 0. \tag{4}$$

Therefore, when a retailer is offered the wholesale price w_i, he chooses a retail price equal to $\hat{p}(w_i)$ and expects his rival to choose the same retail price.

In period 2, a manufacturer offers the contract that maximises his profit subject to the retailer's participation constraint, given the retailer's beliefs and the price charged by the competitor.

LEMMA 3. *With symmetric beliefs, if both manufacturers choose vertical separation, in period* 2 *they offer the wholesale price*

$$w^* \in \arg\max_{w_i}\{D^i[\hat{p}(w_i), \hat{p}(w^*)]w_i + D^i[\hat{p}(w_i), \hat{p}(w_i)][\hat{p}(w_i) - w_i]\}.$$

Notice that, while M_i takes the competitor's retail price as given (since he expects M_j to offer w^* and R_j to choose $\hat{p}(w^*)$ in equilibrium), R_i's beliefs about the competitor's retail price depend on w_i. Since R_i believes that R_j also chooses $\hat{p}(w_i)$, he is willing to pay a franchise fee that is not higher than his profit when both retailers choose $\hat{p}(w_i)$. Therefore, the wholesale price chosen by a manufacturer affects the franchise fee also through its effect on the retailer's conjecture about the competitor's retail price.

By the 'envelope theorem' – i.e. using condition (4) – the first-order condition of M_i's problem is

$$\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}\frac{\partial \hat{p}(w^*)}{\partial w_i}w^* + D^i[\hat{p}(w^*),\hat{p}(w^*)]$$
$$+\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_j}\frac{\partial \hat{p}(w^*)}{\partial w_i}[\hat{p}(w^*) - w^*] - D^i[\hat{p}(w^*),\hat{p}(w^*)] \equiv 0. \tag{5}$$

A change in the wholesale price has two effects. First, w_i affects the wholesale revenue – $D^i[\hat{p}(w_i),\ \hat{p}(w^*)]w_i$ – as reflected by the first two terms in condition (5): a higher w_i increases the wholesale revenue for a given demand but it also reduces demand because it increases the retail price $\hat{p}(w_i)$. Second, w_i has a 'belief effect' because it affects R_i's expected profit – $D^i[\hat{p}(w_i),\ \hat{p}(w_i)][\hat{p}(w_i) - w_i]$ – and, hence, the franchise fee that he is willing to pay, as reflected by the last two terms in condition (5): a higher w_i increases R_i's input cost, which reduces R_i's expected profit, but it also induces R_i to believe that R_j charges a higher retail price, which increases R_i's expected profit.[18]

Simplifying (5), we have

$$\underbrace{\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}w^*}_{<0} + \underbrace{\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_j}[\hat{p}(w^*) - w^*]}_{>0} = 0, \tag{6}$$

where the second term captures the 'belief effect'.

Denote the (equilibrium) price elasticity of demand by $\varepsilon^i_i[\hat{p}(w^*)]$ and the (equilibrium) cross price elasticity of demand by $\varepsilon^i_j[\hat{p}(w^*)]$.[19]

Proposition 2. *When retailers have symmetric beliefs and both manufacturers choose vertical separation in period* 1*:*

- *Given a wholesale price w_i, in period 3 R_i chooses the retail price $\hat{p}(w_i)$ defined by the first-order condition* (4).
- *In period* 2*, there is a symmetric PBE where both manufacturers offer the contract $C^* = (w^*, T^*)$ such that*

$$\frac{\hat{p}(w^*) - w^*}{\hat{p}(w^*)} \equiv \frac{\varepsilon^i_i[\hat{p}(w^*)] - 1}{\varepsilon^i_j[\hat{p}(w^*)]}, \tag{7}$$

[18] Out of the equilibrium, by choosing an appropriately high wholesale price, a manufacturer can 'fool' the retailer into believing that the other retailer is choosing any high retail price. Of course, the benefit of this must be weighed against the reduction in demand caused by a high wholesale price.

[19] Formally, $\varepsilon^i_i[\hat{p}(w^*)] = -\partial \log D^i[\hat{p}(w^*),\hat{p}(w^*)]/\partial \log p_i$ and $\varepsilon^i_j[\hat{p}(w^*)] = \partial \log D^i[\hat{p}(w^*),\hat{p}(w^*)]/\partial \log p_j$.

and
$$T^* = D^i[\hat{p}(w^*), \hat{p}(w^*)][\hat{p}(w^*) - w^*]. \tag{8}$$

- *M_i's profit is $D^i[\hat{p}(w^*), \hat{p}(w^*)]\hat{p}(w^*)$.*

With symmetric beliefs, separated manufacturers charge higher wholesale prices (than integrated manufacturers, or separated ones with passive beliefs) to reduce competition among retailers. Indeed, when a retailer is offered a high wholesale price, he believes that the competing retailer receives the same offer and chooses a high retail price. Hence, he expects high profit and is willing to pay a high franchise fee.

Notice that, when the demand elasticities are constant, w^* is decreasing in ε_i^i because, if ε_i^i is large, M_i wants R_i to charge a relatively low retail price (to prevent a large reduction in demand).[20] Moreover, w^* is increasing in ε_j^i. The reason is that, if ε_j^i is large, R_i expects a relatively large demand when he is offered a high wholesale price (since he expects his competitor to choose a high retail price), and pays a high franchise fee.[21]

3.2. *Asymmetric Vertical Structures*

Suppose now that, in period 1, M_i chooses to sell his product through a retailer while M_j does not. In this case, M_i has no incentive to increase his wholesale price, because R_i knows that his competitor's input cost is zero (since M_j is integrated), regardless of the wholesale price offered by M_i. In other words, M_i cannot affect R_i's beliefs in order to obtain a higher franchise fee. Hence, M_i offers a wholesale price equal to his marginal cost, acting as an integrated manufacturer.

LEMMA 4. *When one manufacturer is vertically integrated while the other is vertically separated, the separated manufacturer charges a wholesale price equal to zero. In period 3, there is a unique equilibrium in which both goods are sold at the retail price p^{e} such that*
$$\frac{\partial D^i(p^{\mathrm{e}}, p^{\mathrm{e}})}{\partial p_i} p^{\mathrm{e}} + D^i(p^{\mathrm{e}}, p^{\mathrm{e}}) = 0.$$

Notice that the equilibrium retail price is equal to the one with two integrated manufacturers, or with passive beliefs (see Proposition 1). Hence, the profit obtained by a vertically separated manufacturer competing against an integrated manufacturer is equal to the profit of an integrated manufacturer.

3.3. *Equilibrium*

Consider the choice of organisational structure by manufacturers. We first compare the retail price when both manufacturers choose separation with the retail price when at least one manufacturer chooses integration.

LEMMA 5. *The equilibrium retail price with two vertically separated manufacturers is higher than the equilibrium retail price with at least one vertically integrated manufacturer – i.e. $p^* \equiv \hat{p}(w^*) > p^{\mathrm{e}}$.*

[20] Rearranging (7), $w^*/\hat{p}(w^*) \equiv 1 - [(\varepsilon_i^i - 1)/\varepsilon_j^i]$.
[21] This is consistent with the evidence discussed in Lafontaine and Slade (1997), who show that retail prices of delegated outlets are higher when the cross-price elasticity of demand is large relative to the own-price elasticity, and when reaction functions are steep. They also show that delegation is more likely in these cases.

In contrast to an integrated manufacturer, a manufacturer selling through a retailer has an incentive to offer a strictly positive wholesale price, in order to induce the retailer to believe that his competitor also pays a positive wholesale price and, hence, chooses a high retail price. The retailer is then willing to sell at a high retail price. Therefore, both wholesale and retail prices are higher when manufacturers sell through retailers.

Since manufacturers extract the whole surplus from retailers, manufactures' profits when they choose integration (I) or separation (S) are given by

		M_2	
		I	S
M_1	I	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$
	S	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$	$\pi_1(p^*,p^*)$, $\pi_2(p^*,p^*)$

where $\pi_i(p, p) = D^i(p, p)p$.

Proposition 3. *With symmetric beliefs, there are two equilibria: one where both manufacturers choose vertical integration and one where both manufacturers choose vertical separation in period* 1. *The equilibrium where both manufacturers choose separation Pareto dominates (from manufacturers' point of view) and risk dominates the one where they both choose integration.*

In the proof of Proposition 3, we show that $\pi_i(p^*, p^*) > \pi_i(p^e, p^e)$, since $p^* > p^e$ by Lemma 5 and both prices are lower than the price that maximises the function $\pi_i(p, p)$ (which is strictly concave by Assumption A4). Therefore, vertical separation is also a weakly dominant strategy for manufacturers. The intuition is that, when one manufacturer is vertically separated, the other manufacturer prefers to choose vertical separation too, in order to commit not to undercut her competitor, when her competitor charges a high wholesale price.[22]

Hence, we expect both manufacturers to sell through independent retailers, when those retailers have symmetric beliefs. By choosing vertical separation and charging high wholesale prices, manufacturers induce retailers to sell at high retail prices, thus reducing competition and increasing profit.

4. The Linear Example

We analyse a simple example with linear inverse demand function $P_i(q_i, q_j) = a - bq_i - dq_j$, where q_i is the quantity produced by M_i that is sold in the retail market. This is a natural and often analysed demand function (e.g. Vives, 2000). We assume that $a > 0$ and $b > d \geq 0$, so that inverting the system of inverse demand functions yields direct demand functions

[22] Notice that, if manufacturers choose their organisational structures sequentially rather than simultaneously, there is a unique subgame perfect equilibrium in which both manufacturers choose vertical separation (even with private contracts).

$$D^i(p_i, p_j) = \frac{a(b-d) - bp_i + dp_j}{b^2 - d^2}, \quad i = 1, 2.$$

The parameter d reflects the degree of substitutability among products.

First consider passive beliefs. Vertically separated manufacturers charge a wholesale price equal to zero and, by condition (3), the unique equilibrium retail price is

$$p^{\mathrm{e}} = \frac{a(b-d)}{2b-d}.$$

Manufacturers' profit is

$$\pi^{\mathrm{e}} = \frac{a^2 b(b-d)}{(2b-d)^2(b+d)}.$$

Now consider symmetric beliefs. Using (4) and (6), when both manufacturers choose vertical separation the unique equilibrium wholesale price is

$$w^* = \frac{ad(b-d)}{2b^2 - d^2} > w^{\mathrm{e}} = 0,$$

and the unique equilibrium retail price is

$$p^* = \frac{a(b^2 - d^2)}{2b^2 - d^2}.$$

Therefore, in the Pareto dominant equilibrium, a manufacturer's profit is

$$\pi^* = \frac{a^2 b^2 (b-d)}{(2b^2 - d^2)^2}.$$

As expected by Proposition 3, $p^* > p^{\mathrm{e}}$ and $\pi^* > \pi^{\mathrm{e}}$: retail prices and manufacturers' profits are higher when they are vertically separated than when they are integrated. Clearly, prices and profits with separation and integration are equal when $d = 0$, since products are independent. Moreover, the difference between prices and profits with separation and integration tends to zero as $b \to d$, because products become closer substitutes and manufacturers competing *à la* Bertrand make zero profit.

5. Mixed Beliefs

If retailers have symmetric rather than passive beliefs, manufacturers are not indifferent between vertical separation and vertical integration. Passive and symmetric beliefs, however, may be considered extreme assumptions. It is worth asking how robust the neutrality result of passive beliefs is to a small change in retailers' beliefs. To answer this question we consider mixed beliefs, a more general class of beliefs that includes passive and symmetric beliefs as special cases (when $\alpha = 0$ and $\alpha = 1$, respectively). For $\alpha \in (0, 1)$, mixed beliefs capture the idea that, after being offered a contract different from the equilibrium one, a retailer is uncertain about the contract offered to the rival retailer and assigns a positive probability α, which can be arbitrarily small, to the other manufacturer offering the same contract, rather than the equilibrium one.

Consider a symmetric equilibrium with wholesale price w_α^* and retail price p_α^*. With mixed beliefs, if R_i is offered a wholesale price $w_i \neq w_\alpha^*$, he believes that, with probability α, R_j is offered the same wholesale price w_i while, with probability $(1-\alpha)$, R_j is offered the equilibrium wholesale price w_α^* and therefore chooses the equilibrium retail price p_α^*. Hence, R_i's objective function is

$$(p_i - w_i)\{(1-\alpha)D^i(p_i, p_\alpha^*) + \alpha D^i[p_i, \tilde{p}_j(w_i)]\}, \tag{9}$$

where $\tilde{p}_j(w_i)$ is the retail price that R_i expects R_j to choose when R_j is offered w_i. In this case, R_j has exactly the same beliefs as R_i when he is offered the wholesale price w_i, and therefore has the same objective function (9).

Let

$$\hat{p}_\alpha(w_i) \in \arg\max_{p_i}(p_i - w_i)\{(1-\alpha)D^i(p_i, p_\alpha^*) + \alpha D^i[p_i, \tilde{p}_j(w_i)]\} \tag{10}$$

define the retail price chosen by R_i if he is offered the wholesale price w_i. By symmetry of the demand functions, $\hat{p}_\alpha(w_i)$ is also the price chosen by R_j when he is offered w_i and, by definition, $p_\alpha^* = \hat{p}_\alpha(w_\alpha^*)$. Therefore, the first-order condition for (10) is

$$\begin{aligned}&(1-\alpha)D^i[\hat{p}_\alpha(w_i), p_\alpha^*] + \alpha D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]\\ &+ [\hat{p}_\alpha(w_i) - w_i]\left\{(1-\alpha)\frac{\partial D^i[\hat{p}_\alpha(w_i), p_\alpha^*]}{\partial p_i} + \alpha\frac{\partial D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]}{\partial p_i}\right\} \equiv 0.\end{aligned} \tag{11}$$

In a symmetric equilibrium in period 2, M_i offers the contract $C_\alpha^* = (w_\alpha^*, T_\alpha^*)$ such that

$$\begin{aligned}w_\alpha^* \in \arg\max_{w_i}(&D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_\alpha^*)]w_i\\ &+ \{\alpha D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)] + (1-\alpha)D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_\alpha^*)]\}[\hat{p}_\alpha(w_i) - w_i])\end{aligned}$$

and T_α^* satisfies R_i's participation constraint as an equality. Therefore, by the envelope theorem – i.e. using condition (4) – the equilibrium wholesale price w_α^* solves

$$\begin{aligned}&\frac{\partial D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)]}{\partial p_i}\frac{\partial \hat{p}_\alpha(w_\alpha^*)}{\partial w_i}w_\alpha^* + D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)]\\ &+ \alpha\frac{\partial D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)]}{\partial p_j}\frac{\partial \hat{p}_\alpha(w_\alpha^*)}{\partial w_i}[\hat{p}_\alpha(w_\alpha^*) - w_\alpha^*] - D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)] \equiv 0,\end{aligned}$$

$$\Leftrightarrow \quad \frac{\partial D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)]}{\partial p_i}w_\alpha^* + \alpha\frac{\partial D^i[\hat{p}_\alpha(w_\alpha^*), \hat{p}_\alpha(w_\alpha^*)]}{\partial p_j}[\hat{p}_\alpha(w_\alpha^*) - w_\alpha^*] \equiv 0. \tag{12}$$

The second term of condition (12) represents the belief effect. Comparing this with condition (6), the belief effect is weaker with mixed than with symmetric beliefs. Moreover, by inspection: $w^*_\alpha = 0$ for $\alpha = 0$, as with passive beliefs; $w^*_\alpha = w^*$ for $\alpha = 1$, as with symmetric beliefs; and $w^*_\alpha > 0$ for every $\alpha \neq 0$.

PROPOSITION 4. *Assume that retailers have mixed beliefs and* $\alpha \in (0,1)$. *When both manufacturers are vertically separated, each manufacturer offers the wholesale price* w^*_α *defined by* (12), *where* $0 < w^*_\alpha < w^*$, *and each retailer chooses the retail price* $\hat{p}_\alpha(w^*_\alpha)$, *where* $\hat{p}_\alpha(\cdot)$ *is defined by* (11) *and* $p^e < \hat{p}_\alpha(w^*_\alpha) < \hat{p}(w^*)$. *In period* 1, *vertical separation is a weakly dominant strategy for manufacturers, for every* $\alpha \neq 0$.

With mixed beliefs, vertically separated manufacturers charge strictly positive wholesale prices and obtain higher profit than integrated ones, although their profit is not as high as with symmetric beliefs. Therefore, our qualitative results hold as long as, when a manufacturer offers a contract different from the equilibrium one, the retailer is not certain that the other manufacturer is still offering the equilibrium contract and assigns some positive probability to the other manufacturer offering the same contract. An arbitrarily small uncertainty is sufficient to generate the belief effect and allows manufacturers to obtain higher profit by selling through retailers. The neutrality result hinges on retailers' beliefs being *exactly* passive.

6. Uncertainty about Manufacturers' Costs

In this Section, we show that (partly) symmetric (or correlated) beliefs naturally arise in the separating equilibrium of a Hotelling model of differentiated products in which vertically separated manufacturers are privately informed about their marginal costs of production, which are correlated. Manufacturers may be asymmetric, since they may have different marginal costs.

There is a unit mass of consumers uniformly distributed over [0, 1]. Two vertical structures produce a homogeneous good and are located at the extremes of the interval; specifically, manufacturer M_1 and retailer R_1 are located at 0, while manufacturer M_2 and retailer R_2 are located at 1. Each consumer has a valuation v for a single unit of the good. For simplicity, we assume $v \to +\infty$, so that each consumer always buys 1 unit, regardless of the price. The transportation cost paid by a consumer located at $x \in [0,1]$ who buys from R_1 (R_2) is tx^2 ($t(1-x)^2$).

As in our main model, manufacturers offer a two-part tariff contract to retailers: M_i charges R_i a wholesale price $w_i \in \mathbb{R}$ and a fixed fee $T_i \in \mathbb{R}$; and R_i chooses the retail price $p_i \in \mathbb{R}$, $i = 1, 2$. Given retail prices p_1 and p_2, a consumer located at x buys from R_1 if and only if

$$p_1 + tx^2 < p_2 + t(1-x)^2.$$

Therefore, in an interior solution, the demand for the good sold by R_i is

$$D^i(p_i, p_j) = \frac{p_j - p_i + t}{2t}, \quad i,j = 1,2, \quad i \neq j.$$

We assume that, before offering contracts to retailers, each manufacturer i is privately informed about his constant marginal cost of production c_i , which is distributed on $(-\infty,+\infty)$ – i.e. the cost has 'full support' – and has expected value equal to zero. The assumption that c_i has full support is for modelling convenience, since it allows us to characterise a separating equilibrium without needing to specify out-of-equilibrium beliefs for retailers (since the set of equilibrium wholesale prices offered by manufacturers is also unbounded). Even when c_i is distributed on a bounded support, the equilibrium wholesale and retail price functions that we characterise in Proposition 5 are part of an equilibrium with an appropriate choice of out-of-equilibrium beliefs for retailers who are offered an out-of-equilibrium wholesale price (see footnote 27).

We also assume that, with probability $\beta \in [0,1]$ the two manufacturers are identical and have the same marginal cost – i.e. $c_1 = c_2$ – while with probability $(1 - \beta)$ manufacturers' costs are independently and identically distributed. This can be interpreted as a model in which manufacturers face cost shocks that are unobserved by retailers: when $\beta = 1$ manufacturers face a common cost shock; when $\beta = 0$ manufacturers face idiosyncratic cost shocks. This interpretation is similar to Lucas' misperception that arises because agents are uncertain about macroeconomic conditions: an agent who observes changes in the market price of the product he produces does not know whether this is caused by a change in aggregate demand, or by an idiosyncratic change in the demand of his own product (Lucas, 1972).

Manufacturers may have different marginal costs as long as $\beta \neq 1$ but, from retailers' point of view, they are *ex ante* symmetric. However, our results do not hinge on symmetry among manufacturers. In order to show this, in the Appendix we prove that partly symmetric beliefs also arise when manufacturers are asymmetric with probability 1 – i.e. when they have different (but correlated) costs.[23] The reason is that a retailer still uses the wholesale price offered by his own manufacturer to infer the marginal cost of the other manufacturer and, hence, the wholesale price offered to the other retailer.

We consider a symmetric, separating, perfect Bayesian equilibrium that satisfies the 'no signalling what you don't know' condition defined by Fudenberg and Tirole (1991). Because retailers' beliefs must be consistent with manufacturers' strategies in equilibrium, when R_i is offered a wholesale price w_i, he expects that, with probability β, R_j is also offered w_i (since manufacturers have the same cost) and hence charges the same retail price as R_i does.

LEMMA 6. *In the separating perfect Bayesian equilibrium, when R_i is offered a wholesale price w_i, he chooses the retail price*

$$\hat{p}(w_i) = t + w_i + \frac{1-\beta}{2-\beta}(\bar{w} - w_i),$$

where $\bar{w}$ is the (equilibrium) average wholesale price offered by manufacturers.

[23] Specifically, we consider a Hotelling model of differentiated products in which M_1 has marginal cost c, while M_2 has marginal cost $c + k$, and the common part of costs c is private information to manufacturers. We show that, in a separating equilibrium, when R_1 is offered a wholesale price w_1, he expects R_2 to be offered the wholesale price $w_1 + 3k/5$; while when R_2 is offered a wholesale price w_2, he expects R_1 to be offered the wholesale price $w_2 - 3k/5$.

When $\beta = 1$, a retailer learns the rival manufacturer's cost and has no uncertainty about the wholesale price paid by his competitor. Hence, as in a standard Hotelling model with two firms having the same marginal cost w_i, retailers choose a retail price equal to $t + w_i$ and charge a fixed markup that does not depend on the wholesale price. When $\beta < 1$, a retailer is uncertain about the wholesale price paid by his competitor and charges a higher (lower) markup when his wholesale price is lower (higher) than the average wholesale price offered by manufacturers. Finally, when $\beta = 0$, R_i's wholesale price is uninformative about M_j's cost and R_i chooses a retail price equal to $t + (w_i + \bar{w})/2$.

As in our main model, the franchise fee charged by a manufacturer is equal to his retailer's expected profit, given the wholesale price offered. Hence, the manufacturer chooses the wholesale price to maximise the sum of the retailer's expected profit and his expected wholesale revenue, given her expectations about the rival manufacturer's cost.

Proposition 5. *In the separating perfect Bayesian equilibrium of the Hotelling model with uncertainty about manufacturers' costs, M_i offers the wholesale price*

$$w^*(c_i) = t\beta + \frac{2-\beta}{2-\beta^2}c_i, \quad i = 1, 2, \tag{13}$$

and R_i chooses the retail price

$$p^*(c_i) = (1+\beta)t + \frac{1}{2-\beta^2}c_i, \quad i = 1, 2. \tag{14}$$

Notice that, in equilibrium, given a wholesale price w_i, retailer R_i has the following beliefs: with probability β he expects R_j to be offered the same wholesale price $w_j = w_i$; with probability $(1 - \beta)$ he expects R_j to be offered the average equilibrium wholesale price $\mathbb{E}_{c_i}[w^*(c_i)] = t\beta$. If $\beta = 1$ retailers have exactly symmetric beliefs and obtain expected profit equal to $t/2$. Hence, M_i charges a franchisee fee equal to $t/2$ and chooses a wholesale price equal to $t + c_i$.[24] By contrast, if $\beta = 0$ retailers have passive beliefs referred to the 'average' manufacturer – i.e. R_i expects R_j to be offered the average wholesale price equal to $\mathbb{E}(c_j) = 0$. Specifically, when $\beta = 0$, in equilibrium each manufacturer offers a wholesale price equal to his marginal cost, acting as an integrated manufacturer, and each retailer expects his rival to be offered a wholesale price equal to the average cost.[25]

If there is a positive and arbitrarily small probability that manufacturers have the same marginal cost (i.e. $\beta \neq 0$), retailers' equilibrium beliefs are partly symmetric (or correlated), in the sense that R_i's beliefs about R_j's wholesale price depend, at least in part, on R_i's wholesale price. This is similar to the notion of mixed beliefs in our main model with complete information, since in both cases R_i believes that, with some positive probability, R_j is offered the same wholesale price as R_i and, otherwise, R_j receives an offer that is independent of R_i's wholesale price.[26] The equilibrium

[24] M_i chooses w_i to maximise $(w_i - c)(w_j - w_i + t)/2t$.

[25] It can be shown that, when $\beta = 0$, retailers of separated manufacturers choose the same retail price as integrated manufacturers. This is analogous to the result with complete information that, when beliefs are passive, retailers of separated manufacturers choose the same retail price as integrated manufacturers.

[26] The difference is that, with mixed beliefs, R_i assigns a positive probability to R_j paying the equilibrium wholesale price; while with incomplete information, R_i assigns a positive probability to R_j paying the average equilibrium wholesale price.

wholesale price is strictly higher than marginal cost if $\beta \neq 0$ and the equilibrium retail price is increasing in β.[27]

Assume now that $\beta \approx 1$, so that manufacturers are almost symmetric because they have the same marginal cost with probability close to 1. Then, as we have shown, retailers have almost symmetric beliefs in a separating equilibrium. Moreover, as in our model with complete information about the manufacturers' cost, manufacturers choose vertical separation in equilibrium because this induces retailers to choose higher prices than integrated manufacturers.

PROPOSITION 6. *If β is sufficiently close to 1 in the Hotelling model with uncertainty about manufacturers' costs, vertical separation is a strictly dominant strategy for manufacturers.*

When $\beta = 1$ manufacturers have exactly the same marginal cost, say c, as in our main model. By Proposition 5, when manufacturers are vertically separated, $p^*(c) = 2t + c$ and each manufacturer's profit is t. By contrast, as in a standard Hotelling model, with vertical integration the retail price is $t + c$ and each manufacturer's profit is $t/2$. Hence, with two integrated vertical structures retail prices are lower than with two separated vertical structures, yielding lower manufacturers' profit. Moreover, as shown in the proof of Proposition 6, asymmetric vertical structures also yield lower retail prices and manufacturers' profit than separated vertical structures. Therefore, as in our main model, manufacturers prefer vertical separation because it allows them to sell at higher retail price and obtain higher profit.

7. Private *Versus* Public Contracts

We now return to our main model with complete information and symmetric beliefs in order to compare retail prices of vertically separated manufacturers with private and public contracts.

With public contracts, a retailer observes both manufacturers' wholesale prices and chooses the retail price to maximise his profit – $D^i(p_i,p_j)(p_i - w_i)$ – yielding the first-order conditions

$$\frac{\partial D^i(p_i,p_j)}{\partial p_i}(p_i - w_i) + D^i(p_i,p_j) = 0, \quad i = 1,2. \tag{15}$$

These conditions define the function $p_i(w_i,w_j)$.[28]

Since a manufacturer chooses the franchise fee so that the retailer's participation constraint is binding, M_i solves

[27] If $c_i \in [\underline{c}; +\infty)$ and $\beta = 1$, it can be shown that there exists a 'semi-separating' equilibrium with the following characteristics. First, when $c_i \geq \underline{c} + t/5$, the wholesale and retail prices are as defined in Proposition 5. Second, when $c_i < \underline{c} + t/5$, both manufacturers offer a (pooling) wholesale price equal to $\underline{c} + 6t/5$ and retailers choose a retail price equal to $\underline{c} + 11t/5$. Third, if R_i is offered an out-of-equilibrium wholesale price, he believes that M_j has marginal cost $\underline{c}$.

Of course, when c_i is distributed on a bounded support, the separating equilibrium defined in Proposition 5 can also be supported by symmetric out-of-equilibrium beliefs such that, if R_i is offered an out-of-equilibrium wholesale price, he believes that R_j is offered the same wholesale price.

[28] Of course, under our assumptions, $p_i(w_i, w_j)$ is increasing in both w_i and w_j.

$$\max_{w_i} D^i[p_i(w_i, w_j), p_j(w_j, w_i)]p_i(w_i, w_j).$$

Hence, the (symmetric) equilibrium wholesale price w^{**} is defined by the first-order conditions

$$\left[\frac{\partial D^i(\cdot)}{\partial p_i}p_i(\cdot) + D^i(\cdot)\right]\frac{\partial p_i(\cdot)}{\partial w_i} + \frac{\partial D^i(\cdot)}{\partial p_j}\frac{\partial p_j(\cdot)}{\partial w_i}p_i(\cdot) = 0, \quad i = 1, 2 \tag{16}$$

and the equilibrium retail price is $p^{**} = p_i(w^{**}, w^{**})$. The second term in (16) represents the strategic effect: when choosing the wholesale price, M_i anticipates R_j's reaction, and the resulting effect on his own product's demand (Bonanno and Vickers, 1988; Rey and Stiglitz, 1995). Since prices are strategic complements, the strategic effect of an increase in w_i on M_i's profit is positive.

Let $\varphi(p_j|w_i)$ be R_i's reaction function in period 3, given p_j and w_i, defined by condition (15). Then $\partial\varphi(p^{**}|w^{**})/\partial p_j$ is the slope of a retailer's reaction function, in the symmetric equilibrium with wholesale price w^{**}. To simplify the analysis, we assume that there is a unique equilibrium with both public and private contracts.

Proposition 7. *Assume that manufacturers' profits are 'single-peaked' with both private and public contracts. With symmetric beliefs, wholesale prices, retail prices and manufacturers' profits are higher (lower) with private contracts than with public contracts if and only if*

$$\frac{p^{**} - w^{**}}{p^{**}} > (<)\frac{\partial\varphi(p^{**}|w^{**})}{\partial p_j}. \tag{17}$$

Public contracts allow retailers to observe the wholesale price paid by competitors and respond to it, thus creating a strategic effect that facilitates coordination among players. The strategic effect is captured by the right-hand side of condition (17), and its strength depends on retailers' reaction function. On the other hand, with private contracts, manufactures can induce retailers to expect a high price from their competitors and, hence, to pay a high franchise fee, regardless of the wholesale price that competitors actually pay. The belief effect is captured by the left-hand side of condition (17) and its strength depends on retailers' price-cost markup. By condition (17), the strategic effect dominates the belief effect when the retailer's reaction function with public contracts is relatively steep – i.e. when an increase in a retailer's price induces a large increase in the competitor's price with public contracts, which in turn increases the manufacturer's wholesale revenue.

To see the intuition for this result, consider the equilibrium wholesale price with public contracts w^{**}. Does a manufacturer have an incentive to charge a price higher than w^{**} when contracts are private? There are two effects. First, a higher wholesale price induces the retailer to expect higher profit (since he expects the competitor's retail price to be higher). Hence, the retailer is willing to pay a higher franchise fee. Second, however, a higher wholesale price also induces the retailer to choose a higher retail price, while the other retailer still chooses p^{**}. Hence, the first retailer sells a lower quantity, and the manufacturer obtains a lower wholesale revenue. This second, negative, effect is stronger when the slope of the reaction functions at p^{**} is larger, because in this case the increase in the retailer's price is larger, resulting in a larger reduction in demand.

Therefore, although public contracts have a commitment value for competing manufacturers, with symmetric beliefs manufacturers' profit may be higher with private contracts than with public ones and manufacturers may prefer not to share information about their retail contracts with competitors. By contrast, when a single monopolistic manufacturer sells to competing retailers, his profit is maximised by public contracts (e.g. Hart and Tirole, 1990).

In the linear example of Section 4, the unique equilibrium wholesale price with public contracts is

$$w^{**} = \frac{ac^2(b-d)}{b(4b^2 - d^2 - 2bd)},$$

and $w^* > w^{**}$. Hence, wholesale prices are higher with private contracts.[29] Similarly, also retail prices and manufacturers' profits are higher with private contracts.

8. Quantity Competition

Suppose that firms compete by choosing the quantity produced, rather than the retail price. In this case, if a manufacturer chooses vertical separation, in period 3 the retailer first acquires the quantity he chooses to produce, paying the wholesale price, and then sells it to final consumers at the market clearing price.

Let $P(Q)$ be the demand function, where $Q = q_1 + q_2$ is the total quantity produced. We assume that $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$.

8.1. *Private Contracts*

As in the case of price competition, with private contracts and passive beliefs, vertical separation has no strategic effect (see Proposition 1). In the unique equilibrium with vertically separated manufacturers, the wholesale price is equal to zero and each retailer sells the quantity q^e such that

$$P'(2q^e)q^e + P(2q^e) = 0. \tag{18}$$

This is the same quantity produced by each of two integrated manufacturers. Therefore, any combination of organisational structures is a PBE and yields manufacturers' profit equal to $P(2q^e)q^e$.

By contrast, with symmetric beliefs, each manufacturer has an incentive to charge a wholesale price greater than zero when he is vertically separated, in order to induce the retailer to produce a lower quantity and to expect his competitor to do the same. The retailer is then willing to pay a higher franchise fee, because he anticipates higher profits. This confirms the insight of our analysis with price competition: a positive wholesale price reduces competition among retailers when they have symmetric beliefs.

[29] With linear demand, the condition of Proposition 7 is $[(2b + d)(b - d)/2b^2] > 0$.

PROPOSITION 8. *With symmetric beliefs and quantity competition, if both manufacturers choose vertical separation in period 1:*

- *Given a wholesale price w_i, in period 3 R_i produces the quantity $\hat{q}(w_i)$ such that*

$$P'[2\hat{q}(w_i)]\hat{q}(w_i) + P[2\hat{q}(w_i)] - w_i \equiv 0. \tag{19}$$

- *In period 2, there is a symmetric PBE where both manufacturers offer the contract $C^* = (w^*, T^*)$ such that*

$$w^* \equiv -P'[2\hat{q}(w^*)]\hat{q}(w^*) > 0, \tag{20}$$

and

$$T^* = \{P[2\hat{q}(w^*)] - w^*\}\hat{q}(w^*). \tag{21}$$

- *M_i's profit is $P[2\hat{q}(w^*)]\hat{q}(w^*)$ and each retailer produces the quantity $\hat{q}(w^*) < q^e$.*

The quantity produced by a retailer when the manufacturer is vertically separated, $\hat{q}(w^*)$, is equal to half the quantity produced by a monopolist. Hence, vertically separated manufacturers manage to maximise joint profit. Manufacturers obtain higher profit than with price competition because, with quantity competition, a retailer buys from the manufacturer *before* observing the market price and learning the quantity chosen by his competitor (while with price competition, he only buys from the manufacturer after observing the price chosen by his competitor). Hence, a manufacturer can extract the whole total expected surplus from the retailer *ex ante*, via the franchise fee and the wholesale payment.[30]

Consider now manufacturers' choice between vertical separation and integration. A vertically separated manufacturer induces the retailer to produce a lower quantity than a vertically integrated manufacturer because of the 'belief effect'.

PROPOSITION 9. *With symmetric beliefs and quantity competition, there are two equilibria: one where both manufacturers choose vertical integration and one where both manufacturers choose vertical separation in period 1. The equilibrium where both manufacturers choose separation Pareto dominates (from manufacturers' point of view) and risk dominates the one where they both choose integration.*

As in the case of price competition, manufacturers' profits with vertical separation exceed those with vertical integration. Therefore, even with quantity competition, it is a weakly dominant strategy for manufacturers to choose vertical separation in order to reduce competition among retailers, when retailers have symmetric beliefs.

[30] In contrast to price competition, a manufacturer has no incentive to reduce the wholesale price in order to increase the wholesale revenue, when he expects the rival manufacturer to charge a high wholesale price: if M_i reduces the wholesale price, R_i conjectures that M_j also reduced the wholesale price and, hence, he does not produce a much larger quantity.

8.2 *Public Contracts*

In contrast to price competition, with quantity competition and public contracts manufacturers charge *lower* wholesale prices if they are vertically separated (than if they are integrated). The reason is that a lower wholesale price tends to increase the quantity produced by the retailer and, since quantities are strategic substitutes, this induces the competing retailer to respond by reducing his own quantity (Vickers, 1985). *Ceteris paribus*, this strategic effect increases manufacturer's profit. Therefore, both manufacturers have an incentive to choose vertical separation but they obtain lower profits by doing so, since the total quantity produced is higher (Vickers, 1985; Fershtman and Judd, 1987).

Proposition 10. *With symmetric beliefs and quantity competition, wholesale prices and manufacturers' profits are higher with private than with public contracts.*

In the proof of Proposition 10, we show that the equilibrium wholesale price with public contracts is lower than manufacturers' marginal cost (hence, lower than zero). Therefore, retailers produce larger quantities and charge lower prices with public contracts. This reduces manufacturers' profits compared to private contracts.

Our analysis suggests that, when retailers compete by choosing the quantity produced and have symmetric beliefs, manufacturers *always* prefer to agree to maintain contracts private, rather than disclose them to competitors. Indeed, with quantity competition, the strategic effect of public contracts harms manufacturers, while private contracts have a positive belief effect.

9. Conclusions

Manufacturers strictly prefer to sell through independent retailers who have symmetric (or at least not completely passive) beliefs, even if contracts with retailers are private and regardless of the nature of competition in the retail market. The reason is that, by charging high wholesale prices, manufacturers manipulate retailers' beliefs about competitors' strategies, thus reducing competition among retailers and increasing profit. Manufacturers may even prefer to agree to keep contracts private, rather than disclose them to competitors, precisely because private contracts allow manufacturers to affect retailers' beliefs about the contracts offered to competitors.

We have shown that symmetric beliefs naturally arise when manufacturers are privately informed about some characteristics that affect the contracts they offer, and do not require manufacturers to be symmetric. Alternatively, symmetric beliefs may be interpreted as a simple rule of thumb adopted by retailers.

With private contracts, vertical separation can also arise when retailers have private information – see Caillaud *et al.* (1995) for the case of adverse selection, and Katz (1991) for the case of moral hazard. Our analysis, however, shows that vertical separation may arise even without privately informed retailers.

Appendix

Throughout the Appendix we use the following technical assumptions:

[A1.] $\dfrac{\partial D^i(p_i, p_j)}{\partial p_i} < 0$ and $\dfrac{\partial^2 D^i(p_i, p_j)}{\partial p_i^2} \leq 0, \forall p_i, p_j$ and $\lim_{p_i \to \infty} D^i(p_i, p_j) = 0, \forall p_j$.

[**A2.**] $\dfrac{\partial D^i(p_i, p_j)}{\partial p_j} \geq 0, \forall p_i, p_j$. Moreover, $\left|\dfrac{\partial D^i(p_i, p_j)}{\partial p_i}\right| > \dfrac{\partial D^i(p_i, p_j)}{\partial p_j}$.

[**A3.**] $\dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} = \dfrac{\partial D^i(p_i, p_j)}{\partial p_j} + (p_i - w_i)\dfrac{\partial^2 D^i(p_i, p_j)}{\partial p_i \partial p_j} > 0$, for every p_j, w_i and $p_i \geq w_i$.

[**A4.**] Stability: $\dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i^2} + \dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} < 0$, for every p_j, w_i and $p_i \geq w_i$; and $\dfrac{\partial^2 \pi_i(p_i, p_j)}{\partial p_i^2} + \dfrac{\partial^2 \pi_i(p_i, p_j)}{\partial p_i \partial p_j} < 0, \forall p_i, p_j$.

Proof of Lemma 1. The proof follows from (2) and the fact that $[\partial D^i(\cdot)/\partial p_i] < 0$ and $[\partial p_i(\cdot)/\partial w_i] > 0$.

Proof of Proposition 1. When manufacturers are vertically separated, condition (3) follows from Lemma 1 and (1). Under Assumptions A1–A4, this condition is necessary and sufficient for an optimum. Clearly, condition (3) also defines the retail price chosen by an integrated manufacturer. Hence, the equilibrium retail price and the manufacturers' profit do not depend on the organisational structure chosen by manufacturers in period 1.

Proof of Lemma 2. We show that $w_i = w_j = 0$ is not an equilibrium with symmetric beliefs. To see this, suppose that $w_j = 0$. Then M_i solves

$$\max_{w_i}\{D^i[\hat{p}(w_i), \hat{p}(0)]w_i + D[\hat{p}(w_i), \hat{p}(w_i)][\hat{p}(w_i) - w_i]\}.$$

The derivative of the objective function evaluated at $w_i = 0$ is

$$\left.\begin{array}{l}\dfrac{\partial D^i[\hat{p}(w_i), \hat{p}(0)]}{\partial p_i}\dfrac{\partial \hat{p}(w_i)}{\partial w_i}w_i + D^i[\hat{p}(w_i), \hat{p}(0)] - D^i[\hat{p}(w_i), \hat{p}(w_i)] \\ \qquad + \dfrac{\partial D^i[\hat{p}(w_i), \hat{p}(w_i)]}{\partial p_j}\dfrac{\partial \hat{p}(w_i)}{\partial w_i}[\hat{p}(w_i) - w_i]\end{array}\right|_{w_i=0}$$

$$= \frac{\partial D^i[\hat{p}(0), \hat{p}(0)]}{\partial p_j}\frac{\partial \hat{p}(0)}{\partial w_i}\hat{p}(0) > 0.$$

Therefore, when M_j charges a wholesale price equal to 0, it is not a best reply for M_i to choose $w_i = 0$.

Proof of Lemma 3. If M_i expects his rival to offer the wholesale price w^*, he expects R_j to choose price $\hat{p}(w^*)$ (since R_j believes that R_i pays his same wholesale price w^* and demand functions are symmetric). By contrast, given w_i, R_i believes that R_j pays the wholesale price w_i and sells at price $\hat{p}(w_i)$. Hence, M_i's problem is

$$\max_{(w_i,T_i)\in\mathbb{R}^2}\{D^i[\hat{p}(w_i),\hat{p}(w^*)]w_i+T_i:\quad D^i[\hat{p}(w_i),\hat{p}(w_i)][\hat{p}(w_i)-w_i]\geq T_i\}.$$

Finally, in equilibrium, R_i's participation constraint is binding.

Proof of Proposition 2. From (4), it follows that, given wholesale prices w_1 and w_2, retail prices are $p_1=\hat{p}(w_1)$ and $p_2=\hat{p}(w_2)$. Using the implicit function theorem,

$$\frac{\mathrm{d}\hat{p}(w_i)}{\mathrm{d}w_i}=\frac{\dfrac{\partial D^i[\hat{p}(w_i),\hat{p}(w_i)]}{\partial p_i}}{2\dfrac{\partial D^i[\hat{p}(w_i),\hat{p}(w_i)]}{\partial p_i}+\dfrac{\partial D^i[\hat{p}(w_i),\hat{p}(w_i)]}{\partial p_j}+[\hat{p}(w_i)-w_i]\left(\dfrac{\partial^2 D^i[\hat{p}(w_i),\hat{p}(w_i)]}{\partial p_i^2}+\dfrac{\partial^2 D^i[\hat{p}(w_i),\hat{p}(w_i)]}{\partial p_i\partial p_j}\right)}.$$

This is strictly positive by Assumption A4.

Consider a symmetric equilibrium with wholesale contract $C^*=(w^*,\ T^*)$ and retail price $\hat{p}(w^*)$. By (4), which defines the function $\hat{p}(\cdot)$,

$$\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}w^*=\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}\hat{p}(w^*)+D^i[\hat{p}(w^*),\hat{p}(w^*)].$$

Substituting this in (6), that defines w^*, we have

$$\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}\hat{p}(w^*)+D^i[\hat{p}(w^*),\hat{p}(w^*)]+\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_j}[\hat{p}(w^*)-w^*]=0 \tag{A.1}$$

$$\Leftrightarrow\quad \frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}\frac{\hat{p}(w^*)}{D^i[\hat{p}(w^*),\hat{p}(w^*)]}+1+\frac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_j}\frac{\hat{p}(w^*)}{D^i[\hat{p}(w^*),\hat{p}(w^*)]}\frac{\hat{p}(w^*)-w^*}{\hat{p}(w^*)}=0$$

$$\Leftrightarrow\quad \frac{\hat{p}(w^*)-w^*}{\hat{p}(w^*)}=-\frac{1+\dfrac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_i}\dfrac{\hat{p}(w^*)}{D^i[\hat{p}(w^*),\hat{p}(w^*)]}}{\dfrac{\partial D^i[\hat{p}(w^*),\hat{p}(w^*)]}{\partial p_j}\dfrac{\hat{p}(w^*)}{D^i[\hat{p}(w^*),\hat{p}(w^*)]}}\equiv\frac{\varepsilon_i^i[\hat{p}(w^*)]-1}{\varepsilon_j^i[\hat{p}(w^*)]}.$$

To prove the existence of an equilibrium satisfying this condition, notice that:

(*i*) the first-order condition (6) is continuous in w since $\hat{p}(\cdot)$ is continuous and $D^i(\cdot)$ is twice continuously differentiable by assumption;

(*ii*) the derivative of the manufacturer's profit is strictly positive at $w_i=w_j=0$ (see (6));

(*iii*) by (A.1), the derivative of the manufacturer's profit tends to $-\infty$ as w_i and w_j tend to $+\infty$ because $\lim_{w\to+\infty}D^i[\hat{p}(w),\hat{p}(w)]=0$ by Assumption A1 and since $\hat{p}(w)$ is increasing in w, and

$$\lim_{w\to+\infty}\left\{\frac{\partial D^i[\hat{p}(w),\hat{p}(w)]}{\partial p_j}+\frac{\partial D^i[\hat{p}(w),\hat{p}(w)]}{\partial p_i}\right\}\hat{p}(w)<0$$

by Assumption A2.

Finally, M_i extracts the whole retailer's surplus by charging a franchise fee defined by (8), and obtains a profit equal to $\pi_i[\hat{p}(w^*),\ \hat{p}(w^*)]$.

Proof of Lemma 4. Suppose that M_j chooses the retail price p_j. Given a contract $C_i = (w_i, T_i)$, R_i chooses the retail price that solves

$$\max_{p_i}[D^i(p_i, p_j)(p_i - w_i) - T_i].$$

Therefore, R_i's best response function $p_i^{\mathrm{e}}(w_i)$ is defined by the first-order condition

$$\frac{\partial D^i[p_i^{\mathrm{e}}(w_i), p_j]}{\partial p_i}[p_i^{\mathrm{e}}(w_i) - w_i] + D^i[p_i^{\mathrm{e}}(w_i), p_j] \equiv 0.$$

Since the retailer's participation constraint is binding, M_i's wholesale price is

$$w_i^{\mathrm{e}} \in \arg\max_{w_i} D^i[p_i^{\mathrm{e}}(w_i), p_j]p_i^{\mathrm{e}}(w_i).$$

By the 'envelope theorem', the derivative of M_i's objective function is

$$\frac{\partial p_i^{\mathrm{e}}(w_i)}{\partial w_i}\frac{\partial D^i[p_i^{\mathrm{e}}(w_i), p_j]}{\partial p_i}w_i. \tag{A.2}$$

Since $[\partial p_i^{\mathrm{e}}(\cdot)/\partial w_i] > 0$ and $[\partial D^i(\cdot)/\partial p_i] < 0$, this derivative is strictly negative for every $w_i > 0$. Hence, M_i chooses $w_i = 0$.

Given the retail price p_i chosen by R_i, the integrated manufacturer M_j chooses the retail price that satisfies

$$\frac{\partial D^j(p_j, p_i)}{\partial p_j}p_j + D^j(p_j, p_i) = 0. \tag{A.3}$$

Since $w_i = 0$, by (1) R_i also chooses a retail price satisfying condition (A.3). Therefore, under Assumption A4, there is a unique equilibrium in which both R_i and the integrated manufacturer choose the same retail price p^{e}.

Proof of Lemma 5. From Section 3.1, recall that the first order condition for the choice of $p^* \equiv \hat{p}(w^*)$ is

$$\frac{\partial D^i(p^*, p^*)}{\partial p_i}(p^* - w^*) + D^i(p^*, p^*) = 0, \tag{A.4}$$

where w^* is defined by

$$\frac{\partial D^i(p^*, p^*)}{\partial p_i}w^* + \frac{\partial D^i(p^*, p^*)}{\partial p_j}(p^* - w^*) = 0. \tag{A.5}$$

Hence, substituting condition (A.4) in (A.5), p^* must satisfy

$$\frac{\partial D^i(p^*, p^*)}{\partial p_i}p^* + D^i(p^*, p^*) = -\frac{\partial D^i(p^*, p^*)}{\partial p_j}(p^* - w^*). \tag{A.6}$$

Consider the function $\phi(p) \equiv [\partial D^i(p, p)/\partial p_i]p + D^i(p, p)$, which is decreasing by Assumption A4. By Lemma 4, p^{e} is such that $\phi(p^{\mathrm{e}}) = 0$. By condition (A.6), and since $[\partial D^i(\cdot)/\partial p_j] > 0$ and $p^* > w^*$, p^* is such that $\phi(p^*) < 0$. Therefore, it must be that $p^* > p^{\mathrm{e}}$.

Proof of Proposition 3. Consider the function $\Phi(p) \equiv \pi_i(p, p) = D^i(p, p)p$. The profit obtained by an integrated manufacturer competing against another integrated manufacturer is equal to $\Phi(p^{\mathrm{e}})$. When one manufacturer is vertically separated while the other is not, the profit obtained by each manufacturer is also equal to $\Phi(p^e)$ by Lemma 4. Therefore, there exists an equilibrium in which both manufacturers choose vertical integration.

The profit obtained by a vertically separated manufacturer when competing against another vertically separated manufacturer is equal to $\Phi(p^*)$. In order to show that there is also an equilibrium in which both manufactures choose vertical separation, we need to show that $\Phi(p^*) \geq \Phi(p^e)$.

By Assumption A4, $\Phi(p)$ is strictly concave and has a unique maximum. Let

$$p^M \equiv \arg\max_p \Phi(p),$$

so that

$$\Phi'(p^M) = \frac{\partial D^i(p^M, p^M)}{\partial p_i} p^M + \frac{\partial D^i(p^M, p^M)}{\partial p_j} p^M + D^i(p^M, p^M) = 0.$$

Clearly, $\Phi'(p) > 0$ if and only if $p < p^M$.

By condition (A.6), p^* is such that $\Phi'(p^*) = [\partial D^i(p^*, p^*)/\partial p_j]w^*$. Since $[\partial D^i(\cdot)/\partial p_j] > 0$ by assumption, $\Phi'(p^*) > 0$ and, therefore, $p^M > p^*$. Moreover, by Lemma 5, $p^* > p^e$. Summing up, $p^M > p^* > p^e$ and, therefore, $\Phi(p^M) > \Phi(p^*) > \Phi(p^e)$. This also proves that manufacturers obtain higher profits in the equilibrium where they both choose vertical separation than in the equilibrium where they both choose integration. By inspection, the former equilibrium is also risk dominant.

Proof of Proposition 4. By inspection of the first order condition (12), $0 < w^*_\alpha < w^*$ for $\alpha \in (0,1)$. To analyse how the equilibrium retail price changes as w_i changes, apply the implicit function theorem to condition (11) to obtain

$$\frac{d\hat{p}_\alpha(w_i)}{dw_i} = \frac{(1-\alpha)\partial D^i[\hat{p}_\alpha(w_i), p^*_\alpha]/\partial p_i + \alpha \partial D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]/\partial p_i}{\Delta(\alpha, w_i, p^*_\alpha)},$$

where

$$\begin{aligned}\Delta(\alpha, w_i, p^*_\alpha) &= 2(1-\alpha)\frac{\partial D^i[\hat{p}_\alpha(w_i), p^*_\alpha]}{\partial p_i} + 2\alpha\frac{\partial D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]}{\partial p_i} + \alpha\frac{\partial D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]}{\partial p_j}\\ &+[\hat{p}_\alpha(w_i) - w_i]\left((1-\alpha)\frac{\partial D^i[\hat{p}_\alpha(w_i), p^*_\alpha]}{\partial p_i^2} + \alpha\left\{\frac{\partial^2 D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]}{\partial p_i^2} + \frac{\partial^2 D^i[\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)]}{\partial p_i \partial p_j}\right\}\right).\end{aligned}$$

Hence, under Assumptions A1, A2 and A4, $[d\hat{p}_\alpha(w_i)/dw_i] > 0$.

When $w_i = w^*_\alpha = 0$, condition (11) is identical to condition (3) and, hence, $\hat{p}_\alpha(0) = p^e$. In equilibrium, when $w_i = w^*_\alpha$, condition (11) is also identical to condition (4) and, hence, $\hat{p}_\alpha(w_i) = \hat{p}(w_i)$. Therefore, the retail price with vertical separation and mixed beliefs is higher than the retail price with vertical integration – i.e. $\hat{p}_\alpha(w^*_\alpha) > p^e$ for all $\alpha > 0$ – and lower than the retail price with vertical separation and symmetric beliefs – i.e. $\hat{p}_\alpha(w^*_\alpha) < \hat{p}(w^*)$ for all $\alpha < 1$.

The proof that delegation is a weakly dominant strategy for manufacturers, for every $\alpha > 0$, follows the proof of Proposition 3: since equilibrium retail prices when both manufacturers choose vertical separation are higher than equilibrium retail prices when one or more manufacturers choose vertical integration and equilibrium retail prices when only one manufacturer chooses vertical separation are equal to equilibrium retail prices when both manufacturers choose vertical integration, a manufacturer obtains a (weakly) higher profit if she chooses vertical separation. As with symmetric beliefs, for every $\alpha > 0$, there are two equilibria: one where both manufacturers choose vertical separation and one where they both choose vertical integration. But the former equilibrium Pareto dominates and risk dominates the latter equilibrium.

Proof of Lemma 6. Consider a symmetric separating equilibrium in which M_i offers a wholesale price defined by the function $w^*(c_i)$ and R_i charges a retail price defined by the function $p^*(c_i)$. The set of wholesale prices that a manufacturer can offer in equilibrium is

$$\Omega = \{w_i : \exists c_i \in (-\infty, +\infty) \text{ such that } w^*(c_i) = w_i\}.$$

Because retailers' beliefs must be consistent with manufacturers' strategies in equilibrium, when R_i is offered a wholesale price $w_i \in \Omega$, he expects that, with probability β, R_j is offered the same wholesale price w_i and, with probability $(1-\beta)$, R_j is offered the average wholesale price $\mathbb{E}_{c_j}[w^*(c_j)]$ and chooses the retail price $\mathbb{E}_{c_j}[p^*(c_j)]$.

Given a wholesale price w_i, denote by $\hat{p}(w_i)$ the price that R_i expects R_j to offer when manufacturers have the same marginal cost – i.e. $\hat{p}(w_i) = p^*[w^{*-1}(w_i)]$. Hence, R_i solves

$$\max_{p_i}\left(\left\{\beta\frac{\hat{p}(w_i)-p_i+t}{2t}+(1-\beta)\frac{\mathbb{E}_{c_j}[p^*(c_j)]-p_i+t}{2t}\right\}(p_i-w_i)\right).$$

The first-order condition for this problem is

$$\beta\hat{p}(w_i)-2p_i+w_i+(1-\beta)\mathbb{E}_{c_j}[p^*(c_j)]+t=0.$$

In equilibrium, R_i chooses price $p_i = \hat{p}(w_i)$. Hence,

$$\hat{p}(w_i)=\frac{1-\beta}{2-\beta}\mathbb{E}_{c_j}[p^*(c_j)]+\frac{t+w_i}{2-\beta}. \tag{A.7}$$

Evaluating $\hat{p}(w_i)$ at the equilibrium wholesale price (i.e. $w_i = w^*(c_i)$), taking expectation with respect to c_i, and using symmetry and the fact that c_1 and c_2 are identically distributed (so that $\mathbb{E}_{c_j}[p^*(c_j)] = \mathbb{E}_{c_i}[p^*(c_i)]$), we obtain the equilibrium expected retail price $\mathbb{E}_{c_i}[p^*(c_i)] = t + \mathbb{E}_{c_i}[w^*(c_i)]$. Substituting this into (A.7),

$$\hat{p}(w_i)=t+\frac{1}{2-\beta}w_i+\frac{1-\beta}{2-\beta}\bar{w}, \tag{A.8}$$

where $\bar{w} \equiv \mathbb{E}_{c_i}[w^*(c_i)] \equiv \mathbb{E}_{c_j}[w^*(c_j)]$.

Proof of Proposition 5. When manufacturers have the same marginal cost, by symmetry equilibrium demand is equal to $\frac{1}{2}$ for each retailer. Hence, given a wholesale price w_i, the transfer that satisfies R_i's participation constraint as equality is

$$T(w_i)=[\hat{p}(w_i)-w_i]\left\{\beta\frac{1}{2}+(1-\beta)\frac{\mathbb{E}_{c_j}[p^*(c_j)]-\hat{p}(w_i)+t}{2t}\right\}, \tag{A.9}$$

where $p^*(c_j)$ is the equilibrium retail price chosen by R_j.

In equilibrium, M_i expects M_j to offer the equilibrium wholesale price $w^*(c_j)$. Therefore, M_i chooses w_i to solve

$$\max_{w_i}\left(T(w_i)+(w_i-c_i)\left\{\beta\frac{\hat{p}[w^*(c_j)]-\hat{p}(w_i)+t}{2t}+(1-\beta)\frac{\mathbb{E}_{c_j}[p^*(c_j)]-\hat{p}(w_i)+t}{2t}\right\}\right).$$

Using (A.8), (A.9), and the fact that $c_i = c_j$ with probability β, M_i's problem is

$$\max_{w_i}\left\{\frac{1}{2t}\left[t+\frac{1-\beta}{2-\beta}(\bar{w}-w_i)\right]^2+\frac{w_i-c_i}{2t(2-\beta)}[\beta w^*(c_i)+(1-\beta)\bar{w}-w_i+t(2-\beta)]\right\}.$$

The first-order condition evaluated at $w_i = w^*(c_i)$ is

$$(1-\beta)\bar{w}+(2-\beta)[t-w^*(c_i)]+c_i-2(1-\beta)\left\{t+\frac{1-\beta}{2-\beta}[\bar{w}-w^*(c_i)]\right\}\equiv 0 \quad \forall c_i. \tag{A.10}$$

Taking expectations with respect to c_i,

$$\frac{\beta}{2(2-\beta)}-\frac{\bar{w}}{2t(2-\beta)}=0 \quad \Leftrightarrow \quad \bar{w}=t\beta.$$

Substituting $\bar{w}$ into condition (A.10), we obtain (13). Hence, $\Omega=(-\infty,+\infty)$ – i.e. every wholesale price can be offered in equilibrium by manufacturers. Finally, substituting $\bar{w}$ and (13) into (A.8), we obtain (14).

Asymmetric Manufacturers

Consider the Hotelling model described in Section 6, in which manufacturers are privately informed about their marginal costs but assume that M_1 has marginal cost $c_1(c)=c$, while M_2 has marginal cost $c_2(c)=c+k$, where $k\neq 0$ measures the degree of asymmetry between manufacturers. Retailers know k but they do not know c, which is distributed on $(-\infty,+\infty)$. We show that partly symmetric beliefs arise in a separating equilibrium.

Consider a separating equilibrium with linear wholesale prices defined by $w_1^*(c_1)=a_1+b_1c$ and $w_2^*(c_2)=a_2+b_2(c+k)$, where a_1, a_2, b_1 and b_2 are scalars. In equilibrium, given a wholesale price w_1, R_1 believes that c is equal to $\tilde{c}(w_1)\equiv[(w_1-a_1)/b_1]$ and hence that R_2 is offered a wholesale price $a_2+b_2\{[(w_1-a_1)/b_1]+k\}$. Similarly, given a wholesale price w_2, R_2 believes that c is equal to $\tilde{c}(w_2)\equiv[(w_2-a_2-b_2k)/b_2]$ and hence that R_1 is offered a wholesale price $a_1+b_1[(w_2-a_2-b_2k)/b_2]$. Let $p_i^*(c)$ be the equilibrium retail price charged by R_i and let $\hat{p}_j(w_i)$ be the price that R_i expects R_j to choose when R_i is offered w_i.

R_i's optimisation program is

$$\max_{p_i}(p_i-w_i)\frac{\hat{p}_j(w_i)-p_i+t}{2t}.$$

In equilibrium, $\hat{p}_j(w_i)\equiv p_j^*[\tilde{c}(w_i)]$. Hence, solving the retailers' optimisation programs,

$$\begin{cases} p_1^*[\tilde{c}(w_1)]=t+\frac{2}{3}w_1+\frac{1}{3}\left[a_2+b_2\left(\frac{w_1-a_1}{b_1}+k\right)\right], \\ p_2^*[\tilde{c}(w_2)]=t+\frac{2}{3}w_2+\frac{1}{3}\left[a_1+b_1\left(\frac{w_2-a_2-b_2k}{b_2}\right)\right]. \end{cases} \tag{A.11}$$

Moreover, using retailers' inferences about c and about their competitors' wholesale prices (based on their own wholesale prices), retailers' beliefs about their competitors' retail prices are

$$\begin{cases} \hat{p}_1(w_2)=t+\frac{2}{3}\left[a_1+b_1\left(\frac{w_2-a_2-b_2k}{b_2}\right)\right]+\frac{1}{3}w_2, \\ \hat{p}_2(w_1)=t+\frac{2}{3}\left[a_2+b_2\left(\frac{w_1-a_1}{b_1}+k\right)\right]+\frac{1}{3}w_1. \end{cases} \tag{A.12}$$

M_i's optimisation program is

$$\max_{w_i}\left((w_i-c_i)\frac{p_j^*(c)-p_i^*[\tilde{c}(w_i)]+t}{2t}+\{p_i^*[\tilde{c}(w_i)]-w_i\}\frac{\hat{p}_j(w_i)-p_i^*[\tilde{c}(w_i)]+t}{2t}\right).$$

The first-order conditions for manufacturers' optimisation programs, evaluated at the equilibrium wholesale prices, are

$$(p_i^*\{\tilde{c}[w_i^*(c_i)]\} - w_i^*(c_i))\frac{\partial \hat{p}_j[w_i^*(c_i)]}{\partial w_i} \equiv [w_i^*(c_i) - c_i]\frac{\partial p_i^*\{\tilde{c}[w_i^*(c_i)]\}}{\partial w_i} \quad \forall c_i, \quad i,j = 1,2.$$

Using (A.11) and (A.12), these conditions yield

$$[3t - a_1 - b_1 c + a_2 + b_2(c+k)]\left(1 + \frac{2b_2}{b_1}\right) \equiv 3[a_1 + (b_1 - 1)c]\left(2 + \frac{b_2}{b_1}\right) \quad \forall c, \tag{A.13}$$

and

$$[3t - a_2 - b_2(c+k) + a_1 + b_1 c]\left(1 + \frac{2b_1}{b_2}\right) \equiv 3[a_2 + (b_2 - 1)(c+k)]\left(2 + \frac{b_1}{b_2}\right) \quad \forall c. \tag{A.14}$$

Differentiating (A.13) and (A.14) with respect to c, we obtain the following system of equations

$$\begin{cases} 6b_1 + 3b_2 - 4b_1 b_2 - 7b_1^2 + 2b_2^2 = 0 \\ 3b_1 + 6b_2 - 4b_1 b_2 + 2b_1^2 - 7b_2^2 = 0, \end{cases}$$

that yield $b_1 = b_2 = 1$. Finally, substituting in (A.13) and (A.14), we obtain that $a_1 = t + \frac{1}{5}k$ and $a_2 = t - \frac{1}{5}k$. Hence, there is a separating equilibrium with wholesale prices

$$w_1^*(c_1) = t + c + \frac{1}{5}k$$

and

$$w_2^*(c_2) = t + c + \frac{4}{5}k.$$

Therefore, in equilibrium, when R_1 is offered a wholesale price equal to w_1, he believes that R_2 is offered a wholesale price equal to $w_1 + \frac{3}{5}k$; and, when R_2 is offered a wholesale price equal to w_2, he believes that R_1 is offered a wholesale price equal to $w_2 - \frac{3}{5}k$. So beliefs are partly symmetric (or correlated).

Finally, using (A.10), the equilibrium retail prices are $p_1^*(c) = 2t + \frac{2}{5}k + c$ and $p_2^*(c) = 2t + \frac{3}{5}k + c$.

Proof of Proposition 6. By Proposition 5, M_i's expected profit when both manufacturers are vertically separated is

$$\pi^*(c_i) = \frac{1+\beta}{2t}\left(t - \frac{1-\beta}{2-\beta^2}c_i\right)^2.$$

Assume that the two manufacturers are integrated. Let $p^e(c_i)$ be the equilibrium price function. Since $c_i = c_j$ with probability β, M_i solves

$$\max_{p_i}(p_i - c_i)\left\{\beta\frac{p^e(c_i) - p_i + t}{2t} + (1-\beta)\frac{\mathbb{E}_{c_j}[p^e(c_j)] - p_i + t}{2t}\right\}.$$

The first-order condition evaluated at $p_i = p^e(c_i)$ is

$$t + c_i + (1-\beta)\mathbb{E}_{c_j}[p^e(c_j)] - (2-\beta)p^e(c_i) \equiv 0 \quad \forall c_i.$$

Taking expectations with respect to c_i and using symmetry, the average retail price is $\mathbb{E}_{c_i}[p^e(c_i)] = t$. Substituting back into the first-order condition, $p^e(c_i) = t + [c_i/(2-\beta)]$. Therefore, M_i's expected profit is

$$\pi^e(c_i) = \frac{1}{2t}\left(t - \frac{1-\beta}{2-\beta}c_i\right)^2.$$

Notice that, for $\beta \approx 1$,

$$\pi^*(c_i) - \pi^e(c_i) \approx \frac{t}{2} - (1-\beta)\left(c_i + \frac{t}{2}\right).$$

This difference is strictly positive in expectation (with respect to c_i). Hence, when the asymmetry between manufacturers is small, their expected profits are higher when they are both vertically separated than when they are both integrated.

Assume now that there are two asymmetric vertical structures: M_i sells directly to final consumers, while M_j sells through R_j. Consider a separating equilibrium in which, for every c_j, M_j offers a wholesale price $w(c_j)$. Define the set of wholesale prices that M_j can offer in equilibrium by

$$\Omega' = \{w_j : \exists c_j \in (-\infty, +\infty) \text{ such that } w(c_j) = w_j\}.$$

Because R_j's beliefs must be consistent with M_j's strategy in equilibrium, when R_j is offered a wholesale price $w_j \in \Omega'$, he expects that M_i's marginal cost is $w^{-1}(w_j)$ with probability β and $\mathbb{E}_{c_j}[w(c_j)] \equiv \bar{w}$ with probability $(1-\beta)$.

Let $p_i(c_i)$ be the retail price charged by M_i in equilibrium, and $p_j(w_j)$ be the retail price charged by R_j in equilibrium. When he is offered the wholesale price $w_j \in \Omega'$, R_j expects M_i to choose the retail price that solves

$$\max_{p_i}\left\{\beta[p_i - w^{-1}(w_j)]\frac{p_j(w_j) - p_i + t}{2t} + (1-\beta)(p_i - c_i)\frac{\mathbb{E}_{c_j}[p_j(c_j)] - p_i + t}{2t}\right\}$$

and R_j chooses the retail price that solves

$$\max_{p_j}(p_j - w_j)\left\{\beta\frac{p_i[w^{-1}(w_j)] - p_j + t}{2t} + (1-\beta)\frac{\mathbb{E}_{c_i}[p_i(c_i)] - p_j + t}{2t}\right\},$$

where $p_i[w^{-1}(w_j)]$ is the retail price that R_j expects M_i to charge with probability β. The first-order conditions of these problems yield expected prices $\mathbb{E}_{c_i}[p_i(c_i)] = t + \frac{1}{3}\bar{w}$ and $\mathbb{E}_{c_j}[p_j(c_j)] = t + \frac{2}{3}\bar{w}$, and hence

$$p_j(w_j) = t + \frac{\bar{w}(2\beta^2 - 2) - 6w_j - 3w^{-1}(w_j)\beta}{3\beta^2 - 12}, \tag{A.15}$$

and

$$p_i[w^{-1}(w_j)] = t + \frac{\bar{w}(\beta^2 + 3\beta - 4) - 3\beta w_j - 6w^{-1}(w_j)}{3\beta^2 - 12}. \tag{A.16}$$

Therefore, M_j solves

$$\max_{w_j}\left(T_j(w_j) + (w_j - c_j)\left\{\beta\frac{p_i(c_j) - p_j(w_j) + t}{2t} + (1-\beta)\frac{\mathbb{E}_{c_i}[p_i(c_i)] - p_j(w_j) + t}{2t}\right\}\right),$$

where, in order to satisfy R_j's participation constraint,

$$T_j(w_j) = [p_j(w_j) - w_j]\left\{\beta\frac{p_i[w^{-1}(w_j)] - p_j(w_j) + t}{2t} + (1-\beta)\frac{\mathbb{E}_{c_i}[p_i(c_i)] - p_j(w_j) + t}{2t}\right\}.$$

Using the envelope theorem (applied to R_j's maximisation problem), the necessary and sufficient first-order condition of this problem evaluated at the equilibrium wholesale price $w_j = w(c_j)$ – i.e. where $p_i\{w^{-1}[w(c_j)]\} = p_i(c_j)$ with probability β – is

$$\beta\frac{\partial p_i[w^{-1}(w_j)]}{\partial w_j}\{p_j[w(c_j)] - w(c_j)\} - \frac{\partial p_j(w_j)}{\partial w_j}[w(c_j) - c_j] \equiv 0 \quad \forall c_j.$$

Using (A.15) and (A.16), we obtain the following differential equation

$$\left[2+\frac{\beta}{\dot{w}(c_j)}\right][w(c_j)-c_j]-\beta\left[\frac{2}{\dot{w}(c_j)}+\beta\right]\{p_j[w(c_j)]-w(c_j)\}=0. \tag{A.17}$$

We consider a linear equilibrium where the wholesale price is a linear function with constant A. Then, $\ddot{w}(c_j) = 0$ and, differentiating (A.17) with respect to c_j and using (A.15),

$$\left[2+\frac{\beta}{\dot{w}(c_j)}\right][\dot{w}(c_j)-1]+\beta\left[\frac{2}{\dot{w}(c_j)}+\beta\right]\left[\frac{\beta+2\dot{w}(c_j)}{\beta^2-4}+\dot{w}(c_j)\right]=0$$

$$\Leftrightarrow \quad \dot{w}(c_j)=\frac{4\beta+\beta^2-2\beta^3-4-\sqrt{16+24\beta^2+16\beta^3-15\beta^4-8\beta^5+2\beta^6+\beta^7}}{\beta^4-8}.$$

Assume now that β is close to 1. Then $\dot{w}(c_j)\approx 1-(1-\beta)\frac{3}{4}$ and, substituting this in (A.17), $A\approx\frac{3}{4}t-(1-\beta)\frac{27}{64}t$. Therefore, for $\beta\approx 1$,there is a linear equilibrium where, for every c_j, M_j offers the wholesale price

$$w(c_j)\approx c_j+\frac{3}{4}t,$$

with $\bar{w}\approx\frac{3}{4}t$.

Using (A.15) and (A.16), when $\beta{\approx}1$ equilibrium retail prices are $p_i(c_i)\approx c_i+\frac{5}{4}t$ and $p_j[w(c_j)]\approx c_j+\frac{2}{3}t$ and manufacturers' profits are $\pi_i\approx\frac{25}{32}t$ and $\pi_j\approx\frac{9}{16}t$.[31] Moreover, when $\beta\approx 1$ manufacturers' profit when they are both vertically separated is $\pi^*{\approx}t$, and manufacturers' profit when they are both vertically integrated is $\pi^e\approx\frac{1}{2}t$. Summing up, if $\beta{\approx}1$, manufacturers' profits are approximately

		M_2			
		I		S	
M_1	I	$\frac{1}{2}t$	$\frac{1}{2}t$	$\frac{25}{32}t$	$\frac{9}{16}t$
	S	$\frac{9}{16}t$	$\frac{25}{32}t$	t	t

By inspection, separation is a strictly dominant strategy for manufacturers.

Proof of Proposition 7. Let $\psi(p^{**})\equiv[\partial\varphi(p^{**}|w^{**})/\partial p_i]$. From (15) and the implicit function theorem,

$$\psi(p^{**})=-\frac{[\partial^2 D^i(p^{**},p^{**})/\partial p_i\partial p_j](p^{**}-w^{**})+[\partial D^i(p^{**},p^{**})/\partial p_j]}{[\partial^2 D^i(p^{**},p^{**})/\partial^2 p_i](p^{**}-w^{**})+2[\partial D^i(p^{**},p^{**})/\partial p_i]}.$$

By Assumption A4, $0\leq\psi(p^{**})\leq 1$. Moreover, it can be shown that $[\partial p_j(w^{**},w^{**})/\partial w_i]/[\partial p_i(w^{**},w^{**})/\partial w_i]=\psi(p^{**})$. Therefore, dividing (16) by $[\partial p_i(w^{**},w^{**})/\partial w_i]$ yields

$$\frac{\partial D^i(p^{**},p^{**})}{\partial p_i}p^{**}+D^i(p^{**},p^{**})=-\frac{\partial D^i(p^{**},p^{**})}{\partial p_j}\psi(p^{**})p^{**}. \tag{A.18}$$

Consider now private contracts. From (A.6), the derivative of M_i's objective function with symmetric beliefs, evaluated at $p_i=p_j=p^{**}$ and $w_i=w_j=w^{**}$, is

[31] With asymmetric vertical structures, an integrated manufacturer obtains higher profit than a separated manufacturer because the separated manufacturer charges a wholesale price higher than marginal cost to her retailer, who then chooses a retail price higher than the integrated manufacturer.

$$\frac{\partial D^i(p^{**},p^{**})}{\partial p_i}p^{**}+D^i(p^{**},p^{**})+\frac{\partial D^i(p^{**},p^{**})}{\partial p_j}(p^{**}-w^{**}). \tag{A.19}$$

Substituting (A.18) in (A.19), we have

$$-\frac{\partial D^i(p^{**},p^{**})}{\partial p_j}\psi(p^{**})p^{**}+\frac{\partial D^i(p^{**},p^{**})}{\partial p_j}(p^{**}-w^{**})=\frac{\partial D^i(p^{**},p^{**})}{\partial p_j}\{[1-\psi(p^{**})]p^{**}-w^{**}\}. \tag{A.20}$$

Uniqueness of the equilibrium with private contracts implies that $p^* > p$ if and only if the derivative of the manufacturer's objective function evaluated at p is greater than zero. (By the assumptions on $D^i(\cdot)$, this derivative is continuous.) Therefore, since $[\partial D^i(\cdot)/\partial p_j] > 0$, $p^* > p^{**}$ if and only if (A.20) is positive – i.e.

$$\{[1-\psi(p^{**})]p^{**}-w^{**}\}>0 \quad \Leftrightarrow \quad \frac{p^{**}-w^{**}}{p^{**}}>\psi(p^{**}).$$

Clearly, $p^* < p^{**}$ if and only if $[(p^{**}-w^{**})/p^{**}] < \psi(p^{**})$. Finally, it is easy to show that the same condition also ranks wholesale prices and manufacturers' profits with private and public contracts.

Notice that manufacturers' profits are single-peaked with private contracts if

$$\frac{\mathrm{d}}{\mathrm{d}p}\left[\frac{\partial D^i(p,p)}{\partial p_i}p+D^i(p,p)+\frac{\partial D^i(p,p)}{\partial p_j}(p-w)\right]<0, \quad \forall w\leq p.$$

See Rey and Stiglitz (1995) for conditions that guarantee that manufacturers' profits are single-peaked with public contracts.

Proof of Proposition 8. With symmetric beliefs, if M_i offers the wholesale price w_i, R_i conjectures that: M_j offered w_i to R_j, and R_j believes that M_i offered w_i to R_i. Hence, since for every w_i R_i expects R_j to choose his same quantity, R_i chooses $\hat{q}(w_i)$ such that

$$\hat{q}(w_i)\in\arg\max_{q_i}\{P[q_i+\hat{q}(w_i)]-w_i\}q_i.$$

This implies condition (19).

Consider a symmetric equilibrium in which both manufacturers charge a franchise fee T^* defined by (21). Then each manufacturer chooses the wholesale price

$$w^*\in\arg\max_{w_i}(\hat{q}(w_i)w_i+\{P[2\hat{q}(w_i)]-w_i\}\hat{q}(w_i)).$$

Using the envelope theorem, the first-order condition of this problem is

$$w^*+P'[2\hat{q}(w^*)]\hat{q}(w^*)=0.$$

This implies (20). Moreover, $w^* > 0$. Equation (21) holds because manufacturers extract the whole retailers' surplus through the franchise fee. Finally, comparing $\hat{q}(w^*)$ defined by (19) with (18), it follows that $\hat{q}(w^*) < q^e$.

Proof of Proposition 9. The proof follows the same logic of the proof of Proposition 3. Indeed, it is straightforward to show that: when both manufacturers choose integration, their marginal cost is zero by assumption; when one manufacturer chooses integration while the other chooses separation, since the integrated manufacturer's marginal cost is zero, the separated manufacturer charges a wholesale price equal to zero. In both cases, each retailer produces the quantity q^e defined by condition (18). Hence, there is an equilibrium where both manufacturers choose integration.

To prove that there is also an equilibrium where both manufacturers choose separation, and that this equilibrium Pareto dominates (and also risk dominates) the equilibrium where both manufacturers choose integration, we show that manufacturers' profits with separation – i.e. $P[2\hat{q}(w^*)]\hat{q}(w^*)$ – are larger than manufacturers' profits with integration – i.e. $P(2q^e)q^e$. Let $\theta(q)=P(2q)q$. The function $\theta(q)$ is strictly concave by the assumption on $P(\cdot)$, and has a unique maximum at q^* such that

$$2P'(2q^*)q^* + P(2q^*) = 0.$$

By (19) and (20) it follows that

$$2P'[2\hat{q}(w^*)]\hat{q}(w^*) + P[2\hat{q}(w^*)] = 0.$$

Hence, $\hat{q}(w^*)$ maximises $\theta(q)$, and $P[2\hat{q}(w^*)]\hat{q}(w^*) > P(2q)q$ for every $q \neq \hat{q}(w^*)$. Notice that $2\hat{q}(w^*)$ is the quantity produced by a monopolist.

Proof of Proposition 10. First consider public contracts. Given manufacturers' contracts, equilibrium quantities are determined by the first-order conditions

$$P'(q_i + q_j)q_i + P(q_i + q_j) - w_i = 0, \quad i = 1, 2. \tag{A.21}$$

These conditions define the quantities $q_1(w_1,w_2)$ and $q_2(w_2,w_1)$ produced by the two retailers, as a function of the wholesale prices.

Hence, M_i solves

$$\max_{w_i}(q_i(w_i, w_j)w_i + \{P[q_i(w_i, w_j) + q_j(w_j, w_i)] - w_i\}q_i(w_i, w_j)).$$

Using the envelope theorem, the first-order condition is

$$\frac{\partial q_i(w_i, w_j)}{\partial w_i}w_i + P'[q_i(w_i, w_j) + q_j(w_j, w_i)]\frac{\partial q_j(w_j, w_i)}{\partial w_i}q_i(w_i, w_j) = 0.$$

Therefore, a symmetric equilibrium with wholesale price w^{**} and quantity $q_i(w^{**},w^{**})=q^{**}$ is characterised by

$$w^{**} = -P'(2q^{**})\frac{[\partial q_j(w^{**}, w^{**})/\partial w_i]}{[\partial q_i(w^{**}, w^{**})/\partial w_i]}q^{**}.$$

It is easy to verify that $[\partial q_i(\cdot)/\partial w_i]<0$ and $[\partial q_j(\cdot)/\partial w_i]>0$, so that $w^{**}<0$.

As shown by Vickers (1985) and Fershtman and Judd (1987), regardless of the organisational structure chosen by the competitor, with public contracts each manufacturer obtains a higher profit with vertical separation than with integration. Hence, manufacturers choose vertical separation with public contracts.

Now consider private contracts. Since $w^* > 0$, comparing (18) and (A.21), it follows that the quantity produced by retailers is lower with private contracts than with public contracts – i.e. $\hat{q}(w^*) < q^{**}$. Finally, manufacturers' profits with private contracts – i.e. $P[2\hat{q}(w^*)]\hat{q}(w^*)$ – are higher than with public contracts – i.e. $P(2q^{**})q^{**}$ – since the function $\theta(q)=P(2q)q$ has a unique maximum at $\hat{q}(w^*)$ (see the proof of Proposition 9).

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