Overbidding to Harm Competitors: Sequential Auctions with Budget Constraints

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Abstract

Questo saggio analizza aste sequenziali nelle quali i potenzilai acquirenti sono soggetti a vincoli di bilancio, eliminando la consueta ipotesi che essi non possano offrire prezzi superiori al proprio budget. In equilibrio, un potenziale acquirente può scegliere di offrire più del proprio budget in una prima asta, al fine di aumentare il prezzo pagato da un acquirente rivale e renderlo più debole in un'asta successiva. Vengono discussi esempi di questo comportamento strategico nelle aste per licenze di telefonia cellulare in Europa e negli Stati Uniti. Il venditore non ha incentivo ad escludere da un'asta un potenziale acquirente che offre prezzi superiri al proprio budget, perchè questo comportamento accresce il suo reddito totale.

We analyze sequential ascending auctions with budget constrained bidders and relax the standard hypothesis that bidders cannot bid above their budget. In equilibrium, a bidder may choose to overbid — i.e., bid above his budget — in an early auction in order to deplete his rival and hence face a weaker competitor in a later auction. We discuss examples of this strategic behavior from US and European mobile-phone license auctions. Even if it reduces competition in later auctions, allowing bidders to overbid increases the total seller's revenue. So a seller has no incentive to exclude from the auction a bidder who is overbidding [JEL Code: D44].

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1. - Introduction

In the real world, unlike what is often assumed in auction theory, a bidder's payoff in one auction may be affected by the price paid by his rivals in other auctions. This may be due, for instance, to the presence of budget constraints.

Benoît and Krishna (1999) show that, when multiple objects are auctioned and bidders' face budget constraints, a bidder may bid especially aggressively for some of the objects on sale in order to raise the price paid by a rival and deplete his budget. This makes his rival a weaker competitor for the remaining objects and, hence, it allows the bidder to win them at lower prices. In fact, a bidder may gain advantage in one market by raising a rival's cost in another market. We show that this strategy may even induce a bidder to bid a price higher than his total budget, if he is sufficiently confident that his rival will eventually outbid him.

According to Salant (1997) — a member of the bidding team of GTE, a bidder in the 1995 FCC simultaneous ascending auction for mobile-phone licenses in the US — one of GTE's objectives was to "push the price up in nontarget markets [...] to induce rivals to spend more money on those markets."[1] GTE had a particular interest in acquiring a license in Atlanta, but chose not to bid for that license until late in the auction, in order to wait until its competitors had committed a large part of their budgets to other markets. Salant (1997) argues that:

> "a bidder might wish to remain active on markets that are of secondary importance until late in the auction, and only near the end of the auction switch to bidding on primary markets. [...] Where rivals' budget constraints are likely to impinge, there can be strategic advantages from doing so, in that a bidder can make rivals spend more on some markets, leaving them with less to spend in other markets."

Salant (1997) adds that, due to the activity rule that forced bidders to be active in sufficiently many auctions,

> "to preserve our options, we felt that it would be prudent to bid, at times, for more licenses than we really wanted to acquire and to have at stake more money than we were authorized to spend."

[1] For an analysis of the 1995 FCC mobile-phone license auctions, see Cramton P. (1997). See also Klemperer P. - Pagnozzi M. (2004).

As this example shows, in multi-object (simultaneous or sequential) ascending auctions, a bidder may want to "overbid" — i.e., bid prices higher than his budget — in order to raise the price paid by rival bidders for some of the objects on sale, even if this implies the risk of winning an object he does not really want to acquire and/or paying too high a price for it.

A more recent example of this strategy can be found in the 2000 auction for "third generation" (3G) mobile-phone licenses in the UK, where Vodafone outbid BT and won the largest license on sale.[2] After the auction, BT claimed to have bid more than the price it was actually willing to pay for the largest license in order to force Vodafone to pay a higher price for that license. Since it was known that Vodafone strongly desired to acquire the largest license, BT was confident that Vodafone would have topped its bids. So BT was able to raise its rival's costs without risking acquiring the license at a price higher than its valuation.[3] (See *The Financial Times*, 28 April 2000, and Maldoom, 2005.)

The UK 3G auction was part of a series of sequential auctions: after the UK, many other EU countries auctioned 3G licenses with (almost) the same bidders competing in all the auctions.[4] Therefore, by forcing Vodafone to pay a higher price for a license in the UK, BT made Vodafone a weaker competitor in the subsequent 3G auctions and gained an advantage in these auctions.

In this paper we analyze sequential common-value ascending auctions and relax the standard hypothesis that bidders cannot bid above their budget (e.g., Benoît and Krishna, 1999). We show that, under both complete and incomplete information, for a wide range of parameters, it is indeed an equilibrium for a bidder to bid above his budget in an early auction, in order to deplete his rival's budget and reduce competition in a later auction.

Therefore, in sequential auctions some bidders may be willing to bid more than their budget. But should the seller allow bidders to overbid? A bidder who bids above his budget in the first of two sequential auctions has two contrasting effects on the seller's revenue. On the one hand, he reduces competition in the second auction and forces his rival to bid less aggressively, which reduces the seller's revenue. On the other hand, he also bids more

[2] For an analysis of the UK 3G auction, see Binmore K. - Klemperer P. (2002) and Klemperer P. (2004).

[3] If a bidder overbids but is uncertain about the price his rival is willing to pay, he has to trade off the risk of winning even when he does not really want to, since he is bidding just to raise his rival's cost, with the advantage obtained by reducing his rival's residual budget.

[4] The EU chose not to regulate the timing of the sale of 3G licenses which resulted, *de facto*, in the use of sequential auctions. This may not have been the best choice. Pagnozzi M. (2007c) shows that simultaneous auctions reduce the risk of winner's regret and litigation compared to sequential auctions. And with information inequality among bidders, simultaneous auctions may also yield a higher seller's revenue. Moreover, simultaneous auctions may increase efficiency by allowing bidders to choose the combination of licenses to acquire while observing for which licenses their opponents are bidding, and before committing their budget to any of the licenses on sale.

aggressively in the first auction and raises the price paid by his rival, which increases the seller's revenue. We show that the second effect prevails on the first one, and hence that allowing bidders to overbid increases the seller's total revenue in the two auctions.[5]

Our analysis also suggests that, in order not to discourage overbidding, a seller should commit not to impose a fine on an auction winner whose budget turns out to be lower than the auction price. Moreover, because overbidding favors the bidders with the lower budgets who are disadvantaged in the auction with respect to bidders with higher budgets, allowing overbidding may induce a more competitive market structure after the auction. For example, in auctions for mobile-phone licenses, thanks to overbidding "weak" bidders with low budgets may win some of the licenses on sale, that would otherwise all be assigned to "strong" bidders with large budgets.[6]

Therefore, even if a bidder observes his rival bidding above his budget, it is difficult to prevent him from doing so, because the seller has no incentive to exclude a bidder who is overbidding from the auction. And a threat by the first bidder of dropping out of the auction to induce his rival to win at a price higher than his budget is not credible either, because the rival knows that the bidder will not drop out at a price at which he prefers to win the current auction, rather than a later one.[7]

The rest of the paper is organized as follows. After a review of the literature, Section 2 introduces the model. Section 3 presents the results of the paper by means of examples. Section 4 generalizes the insights of the examples and Section 5 analyzes the seller's revenue. A simple example of incomplete information about a bidder's budget is discussed in Section 6. The last section concludes. The Appendix contains some details of the analysis.

1.1. - Related Literature

The analysis of Benoît and Krishna (1999) is more closely related to the present paper. Benoît and Krishna (1999) consider multiple-object auctions with budget constrained bidders and complete information, under the assumption that bidders are not allowed to bid prices higher than their budgets. They show that: (*i*) in a sequence of English auctions, it is always optimal for the seller to sell the more valuable object first; and (*ii*) a sequential auction

[5] By contrast, Brusco S. - Lopomo P. (2007a) show how, with incomplete information, the possible presence of budget constraints can reduce the seller's revenue in simultaneous ascending auction, because bidders can pretend to be constrained, even if they are not.

[6] Pagnozzi M. (2007a, 2007b) shows how a "weak" (i.e., low-value) bidder may win against a "strong" (i.e., high-value) bidder even in a single-object ascending auction, both with pure private values (thanks to the possibility of resale) and with common-value elements.

[7] More details are in Section 4.

yields a higher seller's revenue than a simultaneous ascending one.[8] Pitchick and Shotter (1988) provide an experimental test for the results of Benoît and Krishna (1999) and show that budget constraints actually affect bidders' behavior, and bidders attempt to exploit the constraints of their opponents.

Benoît and Krishna (1999) and Pitchick and Shotter (1988) show that, with budget constraints, it may be advantageous for a bidder to bid aggressively on one object in order to raise the price paid by his rival and deplete his budget. However, both papers assume a bidder's aggressiveness is limited by his budget. We build on the analysis of Benoît and Krishna (1999), but we allow bidders to bid above their budget. We show that it is often an equilibrium for bidders to bid a price higher than their budget, if they are allowed to do so.

Pitchick (2004) introduces incomplete information in a two-bidder, two-object, sealed-bid sequential auction and proves that, when bidders are privately informed about their budget, the sequence of sale affects the seller's revenue. Brusco and Lopomo (2007a, 2007b) analyze simultaneous ascending auctions with budget constraints and incomplete information.

Various papers analyze single-object auctions with budget constrained bidders. Che and Gale (1998) assume bidders are privately informed about their budgets and show that first-price auctions yield a higher expected seller's revenue than second-price auctions, because in second-price auctions bids are higher and, hence, the budget constraint is more likely to bind. Zheng (1999) analyzes first-price auctions and assumes bidders have to pay a borrowing cost to bid above their budget and are allowed to default on their bids. Fang and Parreiras (2002, 2003) analyze single-object second-price auctions with budget constrained bidders.

2. - The Model

Consider two sequential ascending auctions for two objects, A and B, with two risk-neutral bidders, 1 and 2. In an ascending auction the price is raised continuously by the auctioneer, and bidders who wish to be active at the current price depress a button. When a bidder releases the button, he is withdrawn from the auction and cannot become active again. The price level and the number of active bidders are continuously displayed, and the auction ends when only one active bidder is left.[9]

We assume that object A is at least as valuable as object B and that object A is sold first. This assumption is consistent with the empirical evidence (see, for instance, Beggs

[8] Benoit J.-P. - Krishna V. (1999) also show that budget constraints arise endogenously and that, in a two-bidder sequential auction, if budgets are private information there is no symmetric equilibrium bidding function which is increasing in budget.

[9] All our results also hold for second-price auctions.

and Graddy, 1997, and Cramton, 1997), and the maximization of the seller's revenue (see Benoît and Krishna, 1999). Let V^k be the value of object k, $k = A, B$, which is the same for both bidders. So we assume that the objects have pure common values, and hence that the outcome of the auction is always efficient, regardless of the objects' allocation among bidders. We also exclude the presence of externalities, so that the value of obtaining both objects is $V^{AB} = V^A + V^B$.

Let w_i be the budget of bidder i, $i = 1, 2$. Without loss of generality, we assume that $w_1 \geq w_2$. To make the model interesting, we assume that $w_2 \geq V^B$, so that both bidders have enough budget to pay for at least one of the objects. We also assume that $w_1 > V^A - V^B$. This assumption simplifies the analysis, but our results do not hinge on it. Values and budgets are common knowledge among bidders. Finally, we assume that the objects' values and the bidders' budget are common knowledge among bidders. We will relax this last assumption in Section 6.

Define by p_k the price paid by the winning bidder for object k, $k = A, B$. The total price paid for the two objects is $p_{AB} = p_A + p_B$. As anticipated, we allow bidders to bid above their budgets. However, a bidder cannot pay a price higher than his budget, and reneging on a bid is not allowed. Therefore, if a bidder wins any of the auctions at a price higher than his budget and hence is unable to pay the auction price, the seller does not award him the object and the bidder obtains no profit. We assume that the seller imposes no fine to a bidder who wins at a price higher than his budget and, to simplify the analysis, that the seller does not resell an object if the auction winner is unable to pay the auction price.[10] In Section 5, we show that it is indeed optimal for the seller not to impose a fine.

Let π_i^k be the profit of bidder i ($i = 1, 2$) when he wins object(s) k ($k = A, B, AB$). The profit of bidder i is equal to:

$$\pi_i^k = \begin{cases} V^k - p_k & \text{if } i \text{ wins object } k \text{ and } p_k \leq w_i \ (k = A, B, AB), \\ 0 & \text{otherwise.} \end{cases}$$

As a tie-breaking rule, we assume that if two bidders offer exactly the same price in one auction, one of the bidder can, if he wants to, renounce the object and allow his opponent to win at the current price.[11] If no bidder renounces, the seller assigns the object randomly. We

[10] In reality, when an auction winner defaults the seller probably re-auctions the object, but only after a significant delay. Notice however that, for our results to hold, we only need to assume that a bidder's valuation is substantially reduced if he waits for the seller to re-auction the object, before acquiring it. This may be because bidders have low discount factors for future profits; or because, in the case of mobile-phone licenses, starting to provide mobile-phone services later than its rivals is a major strategic disadvantage for a firm; or because a new auction attracts other potential buyers.

[11] This tie-breaking rule was suggested by an anonymous referee. It simplifies the analysis because, when a

also assume that a bidder who is exactly indifferent between winning and losing an auction at the current price chooses to win.

We assume that bidders act non-cooperatively and do not play weakly dominated strategies. It follows that, in the last auction, since a bidder has no incentive to bid more than his budget, each bidder bids up to the minimum of the value of the object and his remaining budget. (In the appendix, we discuss how collusion among bidders may affect the outcome of the auction.)

3. - Examples

In this section, we discuss two numerical examples to highlight our main results. In the first example, bidding above one's budget turns out to unprofitable. In the second example, it is profitable in equilibrium for one bidder to offer more than his budget in the first auction, in order to acquire the second object at a lower price. The analysis follows Benoît and Krishna (1999).

EXAMPLE 1: Let $V^A = 50$, $V^B = 40$, $w_1 = 100$ and $w_2 = 80$. If bidder 1 wins the first auction, then no bidder obtains any surplus from the second auction since they are both left with budgets greater than V^B and, hence, object B sells for $p_B = 40$. Therefore, in the first auction it is dominated for bidder 2 to drop out at a price lower than 50 and bidder 1 cannot obtain any surplus. Hence, bidder 1 is better off letting bidder 2 win object A at price 50 and then winning object B at price $w_2 - 50 = 30$, which is bidder 2's remaining budget.[12] Bidder 1 obtains a surplus of 10 while bidder 2 obtains no surplus.

So bidder 1 exploits the budget constraint of his rival by bidding up the price in the first auction, in order to deplete bidder 2's budget and manage to win the second auction at a relatively low price. But bidder 1 does not need to bid above his budget. And, since $w_1 \geq V^{AB}$, bidder 2 cannot profit from bidding above his budget either, because bidder 1 always has a budget higher than V^B in the second auction. However, as the following example shows, this is not always the case.

EXAMPLE 2: Let $V^A = 55$, $V^B = 10$, $w_1 = 50$ and $w_2 = 40$. If bidder 1 loses the first auction at price p, he wins object B at price $\max\{w_2 - p; 0\}$ and obtains a surplus of $V^B -$

bidder wants his opponent to win the auction at the maximum price his opponent is willing to pay, he can drop out exactly at that price, and then renounce the object. See Example 1.

[12] In order to make bidder 2 pay exactly 50 for object A, bidder 1 drops out at 50 together with bidder 2 and then renounces the object. Bidder 2, being indifferent, acquires object A.

$\max\{w_2 - p; 0\}$. If he wins the first auction, he does not obtain any surplus in the second auction, because bidder 2 is left with a budget higher than V^B. Therefore, in the first auction bidder 1 is willing to bid up to p^* such that:

$$\pi_1^A(p^*) = \pi_1^B(p^*) \quad \Leftrightarrow \quad V^A - p^* = V^B - \max\{w_2 - p^*; 0\} \quad \Leftrightarrow \quad p^* = 45.$$

Assume first that bidder 2 cannot bid above his budget. If bidder 2 loses the first auction, he always prefers to bid up to his budget, because his profit in the second auction is (weakly) increasing in the price paid by bidder 1 in the first auction. Therefore, in this case bidder 1 pays 40 for object A and obtains a profit of 15. While bidder 2 obtains no surplus, because in the second auction bidder 1 is left with a budget of $w_1 - p_A = 10$, and hence the price of the second auction goes up to V^B.

However, if bidder 2 is allowed to, he wants to bid above his budget for object A. Indeed, by bidding up to 45 in the first auction and then renouncing to winning, bidder 2 reduces bidder 1's budget for the second auction to 5. As a consequence of this strategy, bidder 2 wins the second auction at price 5 and obtains a profit of 5, while bidder 1's profit is reduced to 10. Hence, bidder 2 has an incentive to bid over his budget in the first auction, in order to reduce bidder 1's budget in the second auction.

Notice that, by bidding up to 45 in the first auction, bidder 2 does not run the risk of winning object A and having to pay more than his budget, since bidder 1 strictly prefers not to drop out of the first auction at a price lower than 45, and this is common knowledge. On the other hand, bidder 2 cannot bid more than 45 in an attempt to reduce bidder 1's budget further, because bidder 1 is not willing to win object A at a price higher than 45.

When bidder 2 does not bid above his budget in the first auction, the seller's revenue is 50. The seller obtains the same revenue even if bidder 2 bids above his budget, because although this strategy reduces the price paid in the second auction by bidder 2, it also increases the price paid in the first auction by bidder 1 by exactly the same amount. In Section 5 we show that this is not always the case, and that the seller's revenue may be affected by whether bidder 2 is allowed to bid above his budget or not.

4. - Overbidding to Harm Competitors

In this section we generalize the examples of Section 3. Notice that, in the second auction, it is a weakly dominant strategy for each bidder to bid up to the maximum between his valuation for the second object and his residual budget. But this is not generally the case in the first auction. We first analyze bidders' strategies when they cannot overbid, then we show under which conditions a bidder may benefit from overbidding in equilibrium in the first auction.

We restrict the analysis to the case $w_1 < V^{AB}$. The reason is that, when $w_1 \geq V^{AB}$ (as in Example 1), bidder 2 knows that he cannot obtain any surplus from the second auction, because bidder 1 never bids more than V^A in the first auction, and hence he is always left with a budget greater than V^B in the second auction. Therefore, when $w_1 \geq V^{AB}$, it is not profitable for bidder 2 to bid above his budget.[13]

Assume first that bidders cannot bid above their budget. If bidder 1 wins the first auction, he obtains no surplus in the second auction because $w_2 \geq V^B$; hence, bidder 2 bids up to V^B in the second auction. If bidder 1 loses the first auction at price p, he can win the second auction at price $\max\{w_2 - p; 0\}$. Hence, in the first auction, the maximum price bidder 1 is willing to pay is p^* such that:

$$\begin{aligned} & \pi_1^A(p^*) = \pi_1^B(p^*) \\ \Leftrightarrow \quad & V^A - p^* = V^B - \max\{w_2 - p^*; 0\} \\ \Leftrightarrow \quad & p^* = \begin{cases} \frac{1}{2}\left(V^A - V^B + w_2\right) & \text{if } w_2 \geq V^A - V^B, \\ V^A - V^B & \text{if } w_2 < V^A - V^B. \end{cases} \end{aligned}$$

Firstly, consider the case $w_2 \geq V^A - V^B$, so that bidder 2's budget is higher than p^*. This implies that $w_1 \geq V^A - V^B$ too. Therefore, by an argument similar to the one for bidder 1, the maximum price that bidder 2 is willing to pay in the first auction is $\frac{1}{2}\left(V^A - V^B + w_1\right) > p^*$. And bidder 2 can always bid up to $w_2 > p^*$. So bidder 2 is willing to outbid bidder 1 for object A, and bidder 1 bids up to:

$$p'_A = \min\left\{\tfrac{1}{2}\left(V^A - V^B + w_1\right); w_2\right\},$$

and then renounces the object, in order to reduce bidder 2's budget for the second auction as much as possible.[14]

So bidder 2 wins the first auction and obtains a profit of:

$$\begin{aligned} \pi_2^* &= V^A - p'_A \\ &= \max\left\{\tfrac{1}{2}\left(V^A + V^B - w_1\right); V^A - w_2\right\}. \end{aligned}$$

[13] A complete analysis of the case $w_1 \geq V^{AB}$ is contained in the Appendix.

[14] Notice that $\frac{1}{2}\left(V^A - V^B + w_1\right) < w_1$; hence, bidder 1 does not bid more than his budget. Moreover, $\frac{1}{2}\left(V^A - V^B + w_1\right) < V^A$ (since $w_1 < V^{AB}$); hence, bidder 2 is happy to pay $\frac{1}{2}\left(V^A - V^B + w_1\right)$ in the first auction.

While bidder 1 wins object B (at price $w_2 - p'_A = \max\left\{w_2 - \frac{1}{2}\left(V^A - V^B + w_1\right); 0\right\}$) and obtains a profit of:

$$\begin{aligned} \pi_1^* &= V^B - \left(w_2 - p'_A\right) \\ &= \min\left\{\tfrac{1}{2}\left(V^A + V^B + w_1\right) - w_2; V^B\right\}. \end{aligned}$$

Clearly, no bidder can benefit from bidding above his budget in this case.

Secondly, consider the case $w_2 < V^A - V^B = p^*$. In this case, bidder 2 cannot outbid bidder 1 in the first auction, because his budget is lower than the maximum price bidder 1 is willing to pay. Hence, (when overbidding is not allowed) bidder 2 bids up to his budget in the first auction, and bidder 1 wins object A at price $p''_A = w_2$, obtaining a profit of:

$$\pi_1^{**} = V^A - w_2.$$

In the second auction, bidder 1 bids up to $w_1 - p''_A$. Therefore, bidder 2 wins object B and obtains a positive surplus if and only if $V^B > w_1 - p''_A$.[15] So bidder 2's profit is equal to:

$$\begin{aligned} \pi_2^{**} &= \max\left\{V^B - \left(w_1 - p''_A\right); 0\right\} \\ &= \max\left\{V^B - w_1 + w_2; 0\right\}. \end{aligned}$$

The next lemma summarizes our discussion.

LEMMA 1: *When overbidding is not allowed, in the first auction:* (*i*) *if* $w_2 \geq V^A - V^B$*, it is an equilibrium for both bidders to bid up to* $\min\left\{\frac{1}{2}\left(V^A - V^B + w_1\right); w_2\right\}$, and *and for bidder 1 to renounce the object;* (*ii*) *if* $w_2 < V^A - V^B$*, it is an equilibrium for bidder 1 to bid up to* $V^A - V^B$*, and for bidder 2 to bid up to* w_2*.*

Table 1 summarizes bidders' profits when bidders are not allowed to overbid.[16]

Table 1: Bidders' profit if they cannot bid above their budget and $w_1 < V^{AB}$

	$w_2 < V^A - V^B$	$w_2 \geq V^A - V^B$
bidder 1's profit	$V^A - w_2$	$\max\left\{\frac{1}{2}\left(V^A + V^B + w_1\right) - w_2; V^B\right\}$
bidder 2's profit	$\max\left\{V^B - w_1 + w_2; 0\right\}$	$\max\left\{\frac{1}{2}\left(V^A + V^B - w_1\right); V^A - w_2\right\}$

[15] Recall that, by assumption, $w_2 > V^B$. Hence, after losing the first auction, bidder 2 bids up to V^B in the second auction.

[16] It can be shown that bidder 1, having a larger budget, always obtains a higher profit than bidder 2.

Now assume that bidders are allowed to bid more than their budget. When $w_2 < V^A - V^B$, bidder 2 benefits from driving up the price in the first auction, because his profit in the second auction is increasing in the price paid by bidder 1 for object A. Hence, if he is allowed to, bidder 2 wants to bid up to the maximum price bidder 1 is willing to pay for object A — i.e., $V^A - V^B$ — and then renounce the object.[17] So in the first auction bidder 2 bids a price he is not actually willing to pay in order to raise his rival's cost.

As a result of this strategy, bidder 2 wins the second auction at a lower price of $p_B = w_1 - V^A + V^B$, which is bidder 1's remaining budget. This is lower than the price bidder 2 would pay without overbidding. So he obtains a profit of:

$$\begin{aligned} \pi_2^{***} &= \max\left\{V^B - p_B; 0\right\} \\ &= \max\left\{V^A - w_1; 0\right\} \geq \pi_2^{**}. \end{aligned}$$

Therefore, by bidding above his budget, bidder 2 strictly increases his profit when $V^A > w_1$. By contrast, when $V^A < w_1$, bidder 2 cannot obtain a positive profit in the second auction, even if he is allowed to overbid. On the other hand, bidder 1's profit is reduced to:

$$\begin{aligned} \pi_1^{***} &= V^A - p_A \\ &= V^B < \pi_1^{**}. \end{aligned}$$

Notice that, even if bidder 2 bids over his budget in the first auction, he does not run the risk of winning object A and having to pay more than his budget, because bidder 1 strictly prefers not to drop out of the first auction at a price lower than $V^A - V^B$ (which is the highest price he would be happy to pay to win the first auction, rather than the second one), and this is common knowledge.[18] So bidder 1 does not drop out of the first auction, even if he realizes that bidder 2 is bidding above his budget.

Summing up, we have the following result.

PROPOSITION 1: *When $V^A - V^B > w_2$ and bidders are allowed to bid above their budget, in the first auction it is an equilibrium for bidder 1 to bid up to $V^A - V^B$ and for bidder 2 to bid up to $V^A - V^B$ and then renounce the object. In this equilibrium, bidder 2 bids above his budget and reduces bidder 1's profit. When $V^A > w_1$, this strategy allows bidder 2 to obtain a strictly higher profit in the second auction (than the profit he would obtain without overbidding).*

[17] Recall that, by assumption, bidder 1's budget is higher than $V^A - V^B$.

[18] Even if the seller re-auctions the object if bidder 2 defaults after winning at a price higher than his budget, bidder 1 does not drop out of the first auction at a price lower than $V^A - V^B$, provided it is sufficiently costly for him to wait for the new auction before acquiring the object. See footnote 10.

So it may be profitable for a bidder to bid above his budget in the first auction to reduce the budget of his competitor in the second one, thus managing to win the second auction at a lower price. Notice that it is only the bidder with the lowest budget who may profit from overbidding.

4.1. - Discussion

How credible is our result in Proposition 1 of bidder 2 bidding more than his budget in the first auction? When $w_2 < V^A - V^B$, bidder 1 knows his rival is bidding above his budget just to deplete him, because bidder 2 has no chance of winning the first auction. Therefore, it could be argued, bidder 1 should cry foul as soon as the price raises above w_2 and ask the auctioneer to exclude bidder 2 from the auction.

Arguably, this is what happened in the year 2000 3G auction in the Netherlands, where Versatel, a relatively weak bidder and new entrant in the mobile-phone market, was competing and bidding aggressively against the five incumbent firms for the five licenses on sale. Telfort, one of the incumbents, sent a letter to Versatel threatening legal action if Versatel continued to bid (Klemperer, 2002).

Telfort's letter, as quoted by Klemperer (2002), claimed that Versatel's weak financial and strategic position meant that the only reason why Versatel was continuing to bid was that it "believes that its bids will always be surpassed by bids of the other participants in the auction," so it "must be that Versatel is attempting to either raise its competitors' costs or to get access to their 2G or future 3G networks." Telfort claimed that such a bidding strategy "constitutes a tort towards Telfort, who will hold Versatel liable for all damages as a result of this." So Telfort argued that Versatel was only bidding to deplete it and not for a genuine interest in winning the auction.

But how should the seller react to an attempt by bidder 1 to have bidder 2 excluded? On the one hand, bidder 2's strategy reduces competition in the second auction and the price paid by bidder 2, which reduces the seller's revenue. On the other hand, the strategy increases competition in the first auction and forces bidder 1 to pay a higher price, which increases the seller's revenue. However, as we will show in Proposition 2, the second effect prevails, and hence the seller does not have any incentive to exclude bidder 2 from the auction, when he bids above his budget.

The seller may also have imperfect monitoring abilities and be unable to verify whether a player is actually bidding above his budget or not. If the auctioneer cannot observe bidder 2's budget, bidder 1 has an incentive to claim that bidder 2 is overbidding, even when he is not actually doing so. Anticipating this incentive, the auctioneer will be even more reluctant to exclude bidder 2 from the auction when bidder 1 asks him to.

Moreover, notice that the possibility of overbidding only helps bidder 2, who is more budget constrained than bidder 1, and allows him to win more often than he would otherwise. Therefore, overbidding may also be desirable for competition policy reasons, because it favors "weak" bidders against "strong" ones and may induce a more competitive market structure after the auction.[19]

Finally, as argued above, bidder 1 cannot profit from dropping out of the first auction and forcing bidder 2 to win at a price higher than his budget. So bidder 1 can arguably take no action against bidder 2, even if he knows he is bidding more money than he has available.[20]

5. - Seller's Revenue

When bidder 2 bids above his budget, he increases the price paid by bidder 1 in the first auction, but reduces the price bidder 2 pays in the second auction. In this section we analyze the net effect of overbidding on the seller's total revenue.

When $w_2 \geq V^A - V^B$ the possibility of overbidding does not affect the seller's revenue, because no bidder has an incentive to bid more than his budget. When $w_2 < V^A - V^B$, if bidder 2 cannot bid above his budget in the first auction, the seller's total revenue in the two auctions is equal to:

$$\begin{aligned} \Pi_{\mathrm{NO}} &= w_2 + \min\left\{w_1 - w_2; V^B\right\} \\ &= \min\left\{w_1; w_2 + V^B\right\}. \end{aligned}$$

On the other hand, if bidder 2 is allowed to bid above his budget in the first auction, the

[19] In our model, the auctions are always efficient because the objects have the same common value for both bidders. By contrast, with private-value elements, favoring a "weak" bidder may reduce efficiency.

[20] In the Netherlands auction, Versatel actually stopped bidding after receiving the threatening letter but complained to the government, arguing that Telfort's letter violated the rule that "a Participant shall refrain from any conduct that could hinder the competition to be created during the Auction Procedure." The government, however, took no action, perhaps because excluding Telfort would have ended the auction immediately, and, if that happened, it might have been hard to impose a meaningful fine on the firm. See also Klemperer P. (2004).

Notice that, from the point of view of the Netherlands government, the auction was a single-object one. So the government had little incentive to exclude one of the bidders. On the other hand, in sequential auctions, excluding a bidder in an auction may increase competition in a later auction and, hence, may increase the seller's revenue. Therefore, the Netherlands government may have neglected the potential benefit for other EU countries of excluding Telfort from its own auction.

seller's total revenue is equal to:

$$\begin{aligned}\Pi_{\mathrm{O}} &= V^A - V^B + \min\left\{w_1 - V^A + V^B; V^B\right\} \\ &= \min\left\{w_1; V^A\right\}.\end{aligned}$$

To identify the effect of overbidding on the seller's revenue, we compare Π_{O} with Π_{NO}.

PROPOSITION 2: *Allowing bidder 2 to overbid — i.e., to offer a price higher than his budget — (weakly) increases the seller's total revenue.*

PROOF: First assume that $w_2 + V^B < w$, so that $\Pi_{\mathrm{NO}} = w_2 + V^B$. If $w_1 < V^A$, then $\Pi_{\mathrm{O}} = w_1 > \Pi_{\mathrm{NO}}$. If, on the other hand, $V^A < w_1$, then (because $V^A - V^B > w_2$) $\Pi_{\mathrm{O}} = V^A > \Pi_{\mathrm{NO}}$. Therefore, in this case the seller's revenue is strictly higher if bidder 2 is allowed to bid above his budget.

Second assume that $w_1 < V^B + w_2$, so that $\Pi_{\mathrm{NO}} = w_1$. It follows that $w_1 < V^A$ (because $V^B + w_2 < V^A$), and hence $\Pi_{\mathrm{O}} = w_1 = \Pi_{\mathrm{NO}}$. Therefore, in this case the seller's revenue does not depend on whether bidder 2 is allowed to bid above his budget or not.

Summing up, when $w_2 < V^A - V^B$,

$$\Pi_{\mathrm{O}} = \begin{cases} V^A > w_2 + V^B = \Pi_{\mathrm{NO}} & \text{when } V^A < w_1, \\ w_1 > w_2 + V^B = \Pi_{\mathrm{NO}} & \text{when } V^B + w_2 < w_1 < V^A, \\ w_1 = \Pi_{\mathrm{NO}} & \text{when } w_1 < w_2 + V^B. \end{cases}$$

In all other cases, the seller's revenue is unaffected by whether bidders are allowed to bid above their budgets or not. ■

Therefore, when bidders can bid above their budget, even if bidder 2 manages to reduce competition in the second auction, he does so by bidding more aggressively in the first auction to harm bidder 1, and the net effect is an increase in the seller's revenue. So allowing bidders to bid above their budget in sequential auctions is a profitable strategy for the seller.[21] And the seller has no incentive to impose a fine on a bidder who wins at a price higher than his budget, because this would risk discouraging overbidding, and hence reducing the price in the first auction.[22]

[21] PAGNOZZI M. (2007a) shows that, in single-object auctions, when winners can resell the object on sale, the seller also benefits from allowing bidders to bid more than their budget, and in fact even more than their valuation.

[22] If the seller imposes a fine to bidder 2 when he wins an auction at a price higher than his budget, bidder 1 may want to let bidder 2 win when he overbids, in order to make him pay the fine and reduce his budget. But this strictly reduces the seller's total revenue, even including the fine. Suppose, for example, that the seller imposes a fine equal to w_2, which is the highest fine bidder 2 can pay, and that bidder 1 drops out of the first auction when $w_2 < V^A - V^B$ and bidder 2 is bidding more than his budget. Then the seller's revenue is only equal to the fine w_2, because bidder 1 faces no competition in the second auction. This is lower than the seller's revenue when bidder 2 overbids and there is no fine. And if the fine simply prevents bidder 2 from overbidding, then the seller's revenue is clearly lower.

6. - Private Information on Budget

In this section, by means of an example, we show that bidding above one's budget can be an equilibrium strategy even when bidders are privately informed about their budget.

We introduce the simplest form of private information and assume that only bidder 2 is privately informed about his budget, while bidder 1's budget is common knowledge. For example, this hypothesis applies to an auction in which bidders are asymmetric: one of the bidders is a big firm whose capitalization and financial situation are well known to competitors, while the other bidder is a new firm whose capitalization and ability to raise funds in financial markets are only known with uncertainty by competitors.

Consider the following example.

Example 3: Let $V^A = 50$, $V^B = 10$ and $w_1 = 40$. Let $w_2 \sim U[30, 50]$. The realization of the random variable w_2 is private information to bidder 2. All other variables are common knowledge among bidders.

Lemma 2: *In Example 3, it is an equilibrium for bidder 1 to bid up to 40 in the first auction and to bid up to 0 in the second auction; and for bidder 2 to bid up to 40 in the first auction and then renounce the object, and to bid up to w_2 in the second auction.*

Proof: See the Appendix. ■

In this example, regardless of his actual budget, in equilibrium bidder 2 always prefers to win the second auction; hence, he drops out at price 40 in the first auction in order to deplete bidder 1's budget and increase his own profit in the second auction. Therefore, if $w_2 \in [30, 40)$, bidder 2 bids up to a price strictly higher than his budget.

As the proof of Lemma 2 shows, bidder 1 is better off bidding up to 40 and winning the first auction, even if he knows that bidder 2 may be bidding more than his budget to harm him. This is because bidder 1 prefers to obtain a profit of 10 in the first auction, rather than drop out of the first auction and risk facing a competitor with a high residual budget in the second auction. Moreover, because bidder 2's strategy is independent of his budget, in this case bidder 1 is never sure whether bidder 2 is bidding above his budget or not.

(In the Appendix, we discuss a second example in which one bidder is privately informed about his budget, but bidders' budgets are not equal in expectation.)

7. - Conclusions

In sequential auctions, it can be an equilibrium for a bidder to bid above his budget in an early auction in order to deplete his rival's budget in a later auction, thus managing to win a later auction at a lower price. We have argued that examples of this type of strategic behavior can be found in the 1995 FCC spectrum auctions and in the European 3G auctions.

Our analysis suggests that the seller should allow bidders to bid above their budget and, in order not to discourage overbidding, he should commit not to charge a high penalty for a winner who is unable to pay for the auction price. In fact, although a bidder who bids above his budget in a first auction reduces competition in a later auction, this is more than compensated by the higher auction price paid by his rival in the first auction. So the seller's total revenue is higher with overbidding.

However, if each of a series of sequential auctions is run by a different seller, allowing bidders to bid more than their budget can increase the revenue of the sellers who run the first auctions, but reduce the revenue of the sellers who run later auctions. Therefore, if sellers expect bidders to adopt the strategy we have described, each seller should want to run his auction before the others, in order to benefit from the competitive effects of overbidding. In this context, a common strategy for all sellers may be difficult to coordinate. This may partly explain why, in the European 3G auctions, the UK government deliberately chose to auction its license first (Binmore and Klemperer, 2002).

Appendix

A. Analysis of the case $w_1 \geq V^{AB}$

When $w_1 \geq V^{AB}$, bidder 2 knows that he cannot obtain any surplus from the second auction (because bidder 1 never bids more than V^A in the first auction, and hence he is always left with a budget greater than V^B in the second auction). Therefore, in the first auction it is weakly dominated for bidder 2 to drop out at a price lower than $\min\left\{w_2; V^A\right\}$.

If bidder 1 wins the first auction, he pays $\min\left\{w_2; V^A\right\}$ and obtains a total profit equal to:

$$\pi_1^A = \max\left\{V^A - w_2; 0\right\}.$$

If instead bidder 1 loses the first auction, he bids up to a price that leaves bidder 2 with the lowest possible budget for the second auction — i.e., up to $\min\left\{w_2; V^A\right\}$. In this case, bidder 1 wins the second auction and obtains a total profit equal to:

$$\begin{aligned} \pi_1^B &= V^B - \left(w_2 - \min\left\{w_2; V^A\right\}\right) \\ &= \min\left\{V^B; V^A + V^B - w_2\right\}. \end{aligned}$$

Comparing these two profits, we have the following result.

LEMMA 2: *When $w_1 \geq V^{AB}$, bidder 1 prefers to win the first auction if and only if $w_2 < V^A - V^B$.*

PROOF: Bidder 1 prefers to win the first auction if and only if $\pi_1^A > \pi_1^B$. This is the case if and only if: (i) $\pi_1^A = V^A - w_2$, (ii) $\pi_1^B = V^B$, and (iii) $V^A - w_2 > V^B$. Conditions (i) and (ii) require $V^A > w_2$ and condition (iii) requires $V^A - V^B > w_2$. Rearranging yields the result. ■

Notice that bidder 1 never needs to bid above his budget, even when he prefers to win the second auction rather than the first one. And, when $w_1 \geq V^{AB}$, it is not profitable for bidder 2 to bid above his budget in the first auction either, because he can never obtain a positive profit in the second auction anyway.

We have implicitly assumed that bidders do not collude or coordinate their strategies. But when bidder 1 wins the first auction, both bidders are better off if bidder 1 can commit not to bid for object B and, in return, bidder 2 drops out of the first auction at price zero. However, this type of collusive agreement among bidders is probably not sustainable, because in the second auction it is a weakly dominant strategy for each bidder to bid up to the minimum of V^B and his remaining budget. Therefore, bidder 2 should expect bidder 1 to deviate from the collusive agreement after winning the first auction.

Exactly the same logic, with inverted roles for the two bidders, applies to the equilibrium described in Proposition 1.

B. Proof of Lemma 2

Let us assume that there is a pure-strategy equilibrium in which bidder 1 bids up to price p in the first auction. We will find bidder 2's best response as a function of w_2 and p. Then, we will compute the optimal p, given 2's best response and bidder 1's expectation of w_2. Finally, we will check that bidders do not want to deviate, and hence that these strategies are indeed an equilibrium.

First notice that no bidder can obtain positive profit in both auctions because, if a bidder wins object A, then his competitor is left with a budget higher than V^B in the second auction.

If bidder 1 bids up to p in the first auction, then: (i) bidder 2 obtains a total profit of $\pi_2^A = V^A - p$ by winning the first auction; (ii) bidder 2 obtains a total profit of $\pi_2^B = V^B - \max\{w_1 - p; 0\}$ by winning the second auction. Therefore, bidder 2 prefers to win the second auction if and only if:

$$\pi_2^B \geq \pi_2^A \quad \Leftrightarrow \quad 10 - \max\{40 - p; 0\} \geq 50 - p \quad \Leftrightarrow \quad p \geq 40.$$

Therefore, if $p < 40$ and $w_2 \geq p$, bidder 2 prefers to win object A and has enough budget to do so. If $w_2 < p < 40$ or if $p \geq 40$, bidder 2 bids up to p in the first auction in order to reduce bidder 1's budget, and obtains a positive profit in the second auction if and only if $w_2 > w_1 - p$ and $V^B > w_1 - p$.

Bidder 1 chooses p to maximize his expected profit. If $p \geq 40$, he wins the first auction and obtains a profit of $V^A - p$. Hence, his profit is maximized by choosing $p = 40$ and equals 10. If instead $p < 40$, bidder 1 loses the first auction and wins the second one if $w_2 \geq p$, while he wins the first auction if $w_2 < p$. So bidder 1's expected profit is:

$$\begin{aligned}
\mathbb{E}\left[\pi_1 | \, p < 40\right] &= \mathbb{E}\left[\pi_1^B \middle| \, 2 \text{ wins } A \text{ at } p\right] \cdot \Pr\left(2 \text{ wins } A \text{ at } p\right) + \pi_1^A \cdot \Pr\left(2 \text{ loses } A \text{ at } p\right) \\
&= \max\left\{V^B - \left(\mathbb{E}\left[w_2 | \, w_2 \geq p\right] - p\right); 0\right\} \cdot \Pr\left(w_2 \geq p\right) + \left(V^A - p\right) \cdot \Pr\left(w_2 < p\right) \\
&= \max\left\{10 - \tfrac{50+p}{2} + p; 0\right\} \cdot \tfrac{50-p}{20} + (50 - p) \cdot \max\left\{\tfrac{p-30}{20}; 0\right\} \\
&= \begin{cases} 6p - \frac{3}{40}p^2 - \frac{225}{2} & \text{if } 30 < p < 40, \\ 0 & \text{if } p < 30. \end{cases}
\end{aligned}$$

The maximum value of this function is only 7.5.

Hence, bidder 1 prefers to win the first auction choosing $p = 40$, the lowest price at which bidder 2 prefers to drop out and win the second auction. And bidder 2 bids up to 40 in order to increase the price paid by bidder 1 and win object B at price zero. Both bidders obtain a profit of 10.

Let us check that bidder 1 does not want to deviate. In equilibrium, he learns nothing about 2's budget. If bidder 1 drops out at $\widehat{p} < 40$ in the first auction, then: if $w_2 < \widehat{p}$, he wins object B at price 0; if $w_2 \geq \widehat{p}$, he expects to pay $\mathbb{E}\left[w_2 | \, w_2 \geq \widehat{p}\right] - \widehat{p}$ (i.e., bidder 2's remaining budget) for object B. Hence, bidder 1's expected profit is:[23]

$$\begin{aligned}\mathbb{E}\left[\pi \middle| \widehat{p} < 40\right] &= V^B \cdot \Pr\left(w_2 < \widehat{p}\right) + \max\left\{V^B - \mathbb{E}\left[w_2 \middle| w_2 \geq \widehat{p}\right] + \widehat{p}; 0\right\} \cdot \Pr\left(w_2 \geq \widehat{p}\right) \\ &= \tfrac{5}{2}\widehat{p} - \tfrac{1}{40}\widehat{p}^2 - \tfrac{105}{2},\end{aligned}$$

which (for $\widehat{p} < 40$) is strictly less than the equilibrium profit of 10.[24]

Deviating from equilibrium by bidding more than 40 in the first auction does not increase bidder 1's profit either because, at best, this does not affect bidder 2's strategy (who still drops out at price 40), and hence the price paid by bidder 1. ■

C. Private Information on Budget II

We discuss a second example of a sequential auction with incomplete information, in which bidder 1 has a strictly higher budget than bidder 2 and, in equilibrium, bidder 2 bids above his budget.[25]

Example 4: Let $V^A = 50$, $V^B = 10$ and $w_1 = 50$. Let $w_2 \sim U[30, 50]$. The realization of the random variable w_2 is private information to bidder 2. All other variables are common knowledge among bidders.

Lemma 3: *In Example 4, in the first auction it is an equilibrium for bidder 1 to bid up to* 40*; and for bidder 2 to bid up to* 40 *and then renounce the object, if* $w_2 \leq 40$*, and to bid up to* w_2*, if* $w_2 > 40$*. In the second auction, both bidders bid up to their remaining budget.*

Proof: We proceed as in the proof of Lemma 1. If bidder 1 bids up to price p in the first auction, bidder 2 prefers to win object B if and only if:

$$\pi_2^B \geq \pi_2^A \quad \Leftrightarrow \quad 10 - \max\{50 - p; 0\} \geq 50 - p \quad \Leftrightarrow \quad p \geq 45.$$

Therefore, (i) if $p < 45$ and $w_2 \geq p$, bidder 2 wins the first auction; (ii) if $p < 45$ and $w_2 < p$ or if $p \geq 45$, bidder 2 loses the first auction but bids up to p (in order to reduce bidder 1's budget for the second auction).

Bidder 1 chooses p to maximize his expected profit. When $p \geq 45$, he wins the first auction and obtains $V^A - p$. Hence, his profit is maximized by $p = 45$ and equals to 5. When $p < 45$, he wins the second auction if $w_2 \geq p$, while he wins the first auction if $w_2 < p$. Therefore, bidder 1's expected profit is:

$$\begin{aligned}\mathbb{E}\left[\pi_1 \middle| p < 45\right] &= \mathbb{E}\left[\pi_1^B \middle| 2 \text{ wins } A \text{ at } p\right] \cdot \Pr\left(2 \text{ wins } A \text{ at } p\right) + \pi_1^A \cdot \Pr\left(2 \text{ loses } A \text{ at } p\right) \\ &= \max\left\{V^B - \mathbb{E}\left[w_2 \middle| w_2 \geq p\right] + p; 0\right\} \cdot \Pr\left(w_2 \geq p\right) + \left(V^A - p\right) \cdot \Pr\left(w_2 < p\right) \\ &= \begin{cases} 6p - \frac{3}{40}p^2 - \frac{225}{2} & \text{if } 30 < p < 45, \\ 0 & \text{if } p < 30. \end{cases}\end{aligned}$$

[23] The second equality follows for $\widehat{p} > 30$. Clearly, even if bidder 1 drops out at $\widehat{p} \leq 30$, his profit is lower than 10 because object B's value is only 10 and bidder 2's residual budget would be strictly positive.

[24] This also follows from the fact that, by deviating from equilibrium and losing the first auction, bidder 1 cannot obtain more than 10, because object B's value is only 10.

[25] This example is a modified version of Example 2.

The maximum value of this function is 7.5, when $p = 40$. Hence, bidder 1 prefers to bid up to 40 in the first auction.

At this price, bidder 2 is happy to win the first auction if his budget is sufficiently high to allow him to do so. Therefore, bidder 2 bids up to his budget and wins the first auction if $w_2 > 40$. Otherwise, he bids up to 40 but then renounces the object.

It can also be shown that no bidder has an incentive to deviate from this equilibrium. ■

In this example, bidder 2 always bids up to at least 40 in the first auction. Hence, if $w_2 \in [30, 40)$, bidder 2 bids above his budget in order to deplete bidder 1's budget.[26,27]

Bidder 1 does not always win the first auction. When $w_2 > 40$ (i.e., with probability $\frac{1}{2}$), bidder 2 wins the first auction at price 40, and obtains a profit of 10; while bidder 1 wins the second auction at an expected price of $\mathbb{E}[p_B | w_2 \geq 40] = 5$. When $w_2 < 40$ (i.e., with probability $\frac{1}{2}$), bidder 1 wins the first auction at price 40, and obtains a profit of 10; while bidder 2 obtains no profit, since bidder 1's residual budget in the second auction is equal to the object's value.

Notice that for bidder 1 it is not profitable to bid more than 40 in the first auction, even if bidder 2 may be bidding up to a higher price. In fact, bidder 1 is better off dropping out when the price is equal to 40 because of the possibility that bidder 2's budget is less than 40, in which case bidder 2 renounces the object after the end of the first auction.

[26] However, in this example bidder 2's strategy does not increase his profit, because bidder 1's budget in the second auction is equal to V^B.

[27] Before the first auction ends, bidder 1 cannot infer bidder 2's budget from his bidding behavior. But if bidder 2 wins the first auction, bidder 1 learns that his budget is higher than 40; while if bidder 2 loses, bidder 1 learns that his budget is lower than 40, and hence that he overbid.

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