Competitive Markets with Endogenous Health Risks*

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Abstract

We study a general equilibrium model where agents' preferences, productivity and labor endowments depend on their health status, and occupational choices affect individual health distributions. Efficiency typically requires ex ante identical agents to obtain different expected utilities if assigned to different occupations. Under mild assumptions, workers with relatively less safe jobs must get higher expected utilities whether health mainly affects production capabilities or health enhancing consumption activities, including treatments, are sufficiently effective. The converse obtains if health mainly affects preferences and consumption capabilities. Competitive equilibria are first-best if lottery contracts are enforceable, but typically not if only assets with deterministic payoffs are traded. Compensating wage differentials which equalize the utilities of ex ante identical workers in different jobs undermine ex ante efficiency. Finally, we characterize a class of simple deterministic policies based on minimal wages and linear subsidies to health insurance which implement the Pareto optimum. Our welfare analysis shows that public health insurance, or financial incentives to private insurance, are more (resp. less) likely to be Pareto improving if designed to subsidize insurance for health shocks that reduce production (resp. consumption) capabilities, or targeted to low (resp. high) income workers.

Keywords: compensating wage differentials, competitive markets, individual health risks, Pareto efficiency.

JEL Classification: D5, D61, D80, I18.

1 Introduction

The paper studies a simple competitive environment where the aggregate distribution of health within the population is endogenous, and is determined jointly with the allocation of labor and consumption goods. The model has the following key features. First, health affects agents' preferences, productivity

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and their labor endowments, namely their consumption and production capabilities. Second, the health distribution of each worker depends either on his occupational choice or on his health enhancing consumption activities, including treatments, health care services, etc. Third, occupational choices are indivisible: Each worker can choose at most one occupation together with an associated health distribution.[1] These features capture some of the most significant real-life effects of individual health status, and are meant to illustrate the determinants of occupational choices in the presence of endogenous health risks. Indeed, the effects of health on workers' productivity, labor endowment, and preferences are largely documented by the empirical literature (see Rosen, 1986, and Viscusi, 1993, among others), which also suggests that health prospects are heavily determined by wealth and individual lifestyles, even in developed countries. Moreover, occupational choices generally have both direct and indirect effects on individual health prospects. By influencing the likelihood of work-related injuries and diseases, these choices directly affect the distribution of future health states. But, they may also change workers' health risks indirectly by determining their location choices; for instance, by inducing them to locate in less safe areas (i.e., more crime-ridden, more exposed to infectious diseases, or endowed with poorer health facilities). Finally, an important real-world feature of most health risks associated to production activities is that they are diversifiable only to a limited extent. Typically, this is due to labor specialization, leading most workers to choose a single occupation.

We study an environment where each occupation is defined by an indivisible set of tasks and an associated health distribution, so as to encompass both direct and indirect effects of occupational choices; while workers can undertake health enhancing activities, and use competitive markets to insure themselves against health risks and to exchange labor and goods. Within this environment, we illustrate the efficiency trade-offs arising when health shocks affect agents production and consumption capabilities.

By developing a novel characterization of the Pareto frontier, we prove that cross-jobs transfers are typically necessary for efficiency. Our characterization shows that the direction of efficient cross-transfers depends either on the relative magnitude of health effects on labor and consumption choices, or on the effectiveness of medical treatments available to the workers. Specifically, under mild conditions, efficiency requires workers with unsafer jobs to get higher expected utilities and positive transfers if health shocks have important effects either on labor supply, or on health care consumption choices. This happens whenever health affects substantially individual productivity or the disutility of labor, or health care treatments are sufficiently valuable in reducing health losses.

Building upon this characterization, we analyze the efficiency properties of alternative market structures, provide conditions for market efficiency, and show that transfers across health insurance policies can increase ex ante efficiency when lottery contracts are not enforceable and competition forces budget balancing. Thus, our work underscores a new rationale for cross-transfers across insurance policies. Differently from the adverse selection literature stemming from Rothshild and Stiglitz (1976), which has showed that agents' private information on their innate characteristics can create room for Pareto improv-

[1] This assumption is imposed for simplicity; in our setting it is sufficient that a worker cannot choose an arbitrarily large number of jobs and offer a small amount of labor in each of them.

ing transfers across insurance contracts, this paper proves that cross subsidies are necessary for efficiency, independently from private information, when differences in health prospects across agents are determined by their equilibrium choices.[2]

Finally, our policy analysis also provides indications on the welfare effects of policy interventions which are commonly observed in real world health insurance markets. Indeed, financial incentives to private health insurance or public health insurance schemes, representing one of the main pillars of public policies in most developed and developing economies, by their own nature implement subsidies in favour of agents facing worse health prospects. According to our analysis, these subsidies are likely to be welfare improving if they benefit low income individuals and contribute to cover health shocks having strong impact on individuals' production capabilities or on health care needs, but not otherwise.

At a more abstract level, we analyze a set-up where agents (workers) choose among indivisible risky assets (occupations) paying either monetary or non-pecuniary random returns (wages and health, respectively), whereas the latter are only imperfectly transferable (health status cannot be separated from individuals, and can be modified only within certain limits).[3] Hence, our conclusions have a broader scope and, although we shall throughout interpret individual risks as health shocks, can shed light on the determinants of individual choices of indivisible risky assets, such as education, jobs with non pecuniary attributes, clubs' memberships, entrepreneurial activities involving team production, to name only a few.

In particular, the paper is related to a vast literature[4] on health risks and non pecuniary job attributes, focusing on wage premia commanded by risky, or otherwise unpleasant, jobs.[5] Contributions in this literature characterize and estimate competitive wage differentials, under the "equilibrium condition" that ex ante identical workers assigned to different occupations obtain equal utility. This equality is generally derived through a partial equilibrium labor market analysis, or directly imposed as part of the definition of equilibrium. Nonetheless, the conventional wisdom within the literature on non pecuniary job's attributes is that *utility equalizing* wage differentials lead to market efficiency.[6] In contrast with this view, we demonstrate that Pareto optimality typically requires ex ante identical workers to obtain different (expected) utility levels when assigned to different occupations. As a consequence, efficient allocations are not *budget balancing*, and require cross-jobs transfers. Moreover, we show that Pareto optimal (shadow) wages are larger in unsafer occupations, but do not equalize utilities across occupations. These findings hinge upon the imperfect transferability of health[7], which makes consumption and occu-

[2] Most of our analysis is performed by assuming away asymmetric information in order to emphasize that in our environment cross-jobs transfers are welfare improving independently from adverse selection issues. In a final section of the paper, though, we also explain why several qualitative results of the paper continue to hold in a set-up agents hold private information on their preferences or their labor endowment.

[3] Noteworthy, individual risks and occupational indivisibilities are both key ingredients of our model. In a setting with indivisibilities, but state independent preferences and production funcions, the problem at hand would become much more standard and lose much of its appeal.

[4] This literature goes back to Adam Smith (see Evans and Viscusi, 1993, Lucas, 1974, Rosen, 1986, Viscusi, 1990 and 1993, among many others).

[5] This literature formalizes the Smithian idea that "*the whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighborhood, be either perfectly equal or continually tending toward equality*".

[6] See, for instance, the textbooks of Ehrenberg and Smith (2003) and Viscusi et al. (2000).

[7] This imperfect transferability invalidates the separability result between individual consumption and production choices

pational (production) decisions interdependent. More precisely, they rely upon the following optimality argument. Because health risks are specific to occupations, and both preferences and productivity are state-dependent, ex ante identical workers will generally feature different expected utility functions and budget constraints, hence different indirect utility functions, when employed in different sectors. For this reason, the equalization of (expected) marginal (indirect) utilities of contingent goods (income) across agents, which is a standard ex ante efficiency condition, typically prevents either interim efficiency with *equal treatment* (i.e., equalization of the utility of ex ante identical agents assigned to different jobs), or budget balancing. In other words, satisfying ex ante efficiency, does call for an individual not to be indifferent over occupations to which he is assigned with positive probability.

The inconsistency between ex ante efficiency and budget balancing naturally leads to compare equilibria and optima. We first characterize the Pareto frontier of the economy, and identify the main determinants of cross-jobs transfers. After ordering health risks associated to different occupations according to first-order stochastic dominance, the formal analysis relies upon supermodularity arguments, (i.e., upon the relationships of complementarity, resp. substitutability, between health, labor and consumption goods) to demonstrate that the direction of optimal transfers across occupations depends on the impact of health on production choices and health enhancing consumption activities. [8]

The second part of the paper investigates the welfare properties of competitive equilibria and derives the policy implications of our analysis. We study two alternative contractual regimes, one where lottery contracts (i.e., contracts with random payoffs) are enforceable and the other where they are unenforceable, possibly because of limited liability (debt) constraints. In the former, there exist competitive insurance markets to cope with all idiosyncratic risks but only financial contracts with deterministic returns are offered. In the latter regime, agents can also "trade" lottery contracts, i.e., assets with random payoffs. The complete contracts' regime, where lotteries are assumed to be enforceable, turns out to be the natural benchmark for understanding the functioning and the welfare properties of complete competitive markets. The analysis of the case of unenforceable lotteries, though, is warranted by both empirical and theoretical reasons. First, in real markets the use of lottery contracts (or that of other financial instruments making random allocations attainable) does not appear to be very widespread.[9] And, in line with real world circumstances, most of the literature on non-pecuniary job attributes, which is a natural reference point for the problem at hand, has only considered contracts with deterministic payoffs. On a theoretical ground, the use of random contracts may result severely restricted by moral hazard problems, driven by limited liability, *or by* the imperfect verifiability of characteristics and outcomes of the random devices needed for their implementation.

We show that an equilibrium exists in both contractual regimes; typically, however, competitive equi-

which is standard in welfare analysis (see Mas Colell et al., 1995).

[8] Testable implications of different assumptions on preferences for consumption and health are derived by Rey and Rochet (2004), see also on this issue Evans and Viscusi (1990).

[9] Kehoe, Levine and Prescott (2001) show that, if there exists a sufficient number of assets paying units of numeraire in sunspot states of the world, competitive equilibria are first-best efficient. In our setting, however, efficient trades of financial instruments leading to random allocations are typically such that workers must take possibly large *short positions* in the asset markets. This is often impossible in real-life markets also because of incentive problems.

libria are efficient if and only if lottery contracts are enforceable. In the absence of lottery contracts, competition does not deliver ex ante efficiency since equalizing the expected utilities of workers employed in different sectors, creates a wedge between their marginal utilities of expected income. Moreover, understanding the nature of the inefficiencies determined by the missing market problem at hand, permits to go beyond the accomplishment of an efficiency test of competitive equilibria. By relying upon our characterization of Pareto optimal allocations, we identify simple deterministic cross-transfers policies which allow to Pareto improve the market outcome. These policies display two key features: They implement cross-subsidies among insurance contracts designed for workers choosing different occupations, and impose minimum wages aimed at ensuring a natural non-manipulability requirement of the policy scheme. Cross-subsidies equalize the marginal utilities of income of agents assigned to different occupations; while minimum wages prevent wage reductions in sectors where agents are subsidized, and for this reason obtain higher utility levels. Specifically, we characterize two variants of Pareto improving policies. In the first, minimum wages are imposed at the sectorial level, while in the second they are uniformly set across sectors.

These results offer a new efficiency rationale for public health insurance programs or policy schemes providing financial incentives to private health insurance provision, which implement transfers in favor of workers facing higher probabilities of health problems. Albeit derived within an abstract setting for modelling parsimony, they can be also used to evaluate the impact of health insurance policies in countries where minimum wages are commonly adopted, either at the economy level as in US and in many other economies, or at a sectorial level as it often happens in EU. Indeed, our analysis implies that financial incentives to private or public health insurance may be welfare increasing insofar these programs are tailored to subsidize insurance for health shocks which have a strong impact on agents' labor supply or on their consumption of health care services. On the contrary, subsidies to health insurance against the risk of facing shocks that merely affect preferences, by reducing the marginal utility of some consumption goods, generally reduce welfare. Moreover, a necessary conditions for insurance subsidies to workers choosing unsafer occupations to be welfare increasing is that minimum wages regulations are binding at least in some states.

On the normative side, our analysis also suggests that wages differentials for the "dirtiest" an the riskiest jobs within the class of low-skill jobs, which appear to be small according to the empirical literature, may reflect not only some omitted variables (such as low-skill workers' heterogeneity), but also a contractual incompleteness driving an inefficient allocation of workers across sectors. Cross-transfers implemented through health insurance subsidies may potentially correct this inefficiency.

2 Related Literature

In the last decades, several contributions[10] have developed the key insight provided by the seminal contribution of Rothschild and Stiglitz (1976), which show that in the presence of adverse selection cross-

[10] Within the health literature, see Newhouse (1996) and Neudek and Podczeck (1996) among others.

transfers across ex ante different types may be necessary to attain second-best efficiency. The key assumption of the Rothschild and Stiglitz setting is that agents have private information on a variable – the probability of facing a bad state – affecting directly the expected profit made by an intermediary offering him insurance (private information creates a common values problem). The crucial consequence of this fact is that cross-transfers across types are welfare improving whenever the fraction of high-risk agents is sufficiently small. One of the contributions of this paper is to show that cross-transfers aimed at subsidizing insurance against certain risks may be Pareto improving even in the absence of common values problems. Moreover, in our setting cross subsidies among agents of the same types, not across types, lead to Pareto improvements. Thus, the two models provides different, but possibly complementary, efficiency rationales for policies implementing cross-jobs transfers.

Our analysis shares important features with those analyzed in the literature on the optimal design of public health and education policies. The welfare effects of subsidies to education and health insurance are two relevant issues in this literature. Henriet and Rochet (2004) illustrate, within an optimal taxation framework, the role of public health insurance as a redistributive tool. In a close spirit, Diamond and Sheshiniski (1995) analyze the effects on labor supply of health disabilities and retirement benefits. They focus on the optimal structure of disabilities benefits in the presence of asymmetric information. De Fraja (2002), instead, analyzes optimal education policies and uses supermodularity arguments to show that missing capital markets and externalities may lead to elitist (regressive) utilitarian cross-transfers policies. Similarly, our work relies on monotone comparative statics techniques to characterize cross transfers, and shows that optimal policy may increase ex post inequality. However, there are two fundamental differences, among others, between De Fraja's work and ours. First we focus on ex ante efficiency instead of taking the utilitarian viewpoint. Second, we show that, under mild conditions, optimal policies entail positive transfers to the agents who are more likely to be ex post less productive (i.e., incurring in adverse health shocks with larger probabilities).

Concerning the literature on labor and production indivisibilities, Rogerson (1988) is the contribution to which our work is closest. In a framework with production indivisibilities, this article provides an example where random contracts implement transfers across workers, which could be reinterpreted as a form of unemployment insurance. Rogerson's results are derived for a very specific class of preferences, and assuming that a completely indivisible labor supply (agents can either work a fixed amount of time or remain unemployed) generates a positive unemployment rate in equilibrium. In our environment, we prove that random contracts are *almost always* necessary to achieve efficiency through the market, even if the amount of labor a worker must supply within an occupation is perfectly divisible. Moreover, differently from Rogerson, lotteries implement cross-transfers across occupations in our setting. And the determination of the sign of these transfers is at the core of our paper.

Moreover, some examples have been developed in the general equilibrium literature on asymmetric information stemming from Prescott and Townsend (1984) to show that lotteries can be welfare beneficial either in the presence of adverse selection or moral hazard, because of their convexifying effects on incentive constraints.[11] With the exception of Bennardo and Chiappori (2003), and Cole and Prescott (1997), this literature does not focus on the role of ex ante lotteries. More importantly, it is not aimed at characterizing efficient cross-jobs transfers.[12]

Finally, the recent literature on clubs and firms (Cole and Prescott, 1997, Ellickson, Grodal, Scotchmer and Zame, 1999, Makowski and Ostroy, 2005, and Zame, 2007) deals with the complex issue of pricing institutions, firms, and occupations in a general equilibrium setting. An important connection of our paper with this literature is that agents' occupational choices (choices of firms' memberships) directly affect their utility and hence their consumption choices. In our setting, we assume away complementarities between agents working in the same firm, opportunistic behavior, and the related externality problems. These simplifications allow to focus on the characterization of efficiency trade-offs and on the beneficial role of cross-transfers across occupations, which are generated by idiosyncratic uncertainty and indivisible occupations alone, two issues which are not addressed in the club literature. Our conjecture, based on the analysis of the present paper, is that the generic inconsistency between ex ante and interim optimality continues to hold in most of the settings studied in the clubs' and in the asymmetric information literature.

3 The Economy

Demography, consumption goods and preferences: We consider an economy where a continuum of measure 1 of ex ante identical consumers-workers produce two consumption goods.[13] Agents face health individual risks that may affect their preferences, endowments and productivity. The set of possible health states, $\Theta = \{\theta_1, ..., \theta_N\}$, with $\theta_{n+1} \geq \theta_n$ for all n, is assumed to be finite, and $\theta \in \Theta$ represents a typical health state.[14] In the economy there are three goods, leisure and two produced consumption goods, whose quantities are x_L, x_1 and x_2, respectively. Agents have an endowment $e \in \Re^2_+$ of produced goods which is constant across individual states, and an amount L of time which is allocated between work, l, and leisure, x_L. The maximal fraction of time that each agent can devote to work, $L(\theta)$, may depend on his health state; and $L(\cdot)$ is weakly increasing in θ.[15] Agents' preferences are state (health) dependent and are represented by the utility function $U(x, \theta) : \Re^2_+ \times [0, L] \to \Re$. Formally, $U(x, \theta)$ is n

[11] See Arnott and Stiglitz (1986), Bennardo and Chiappori (2003), Cole (1990), Cole-Prescott (1997), Garrett (1995), Kehoe, Levine and Prescott (2002).

[12] See however Bennardo (2005) for related results on cross-jobs transfers in a multicommodity production economy with moral hazard.

[13] As we explain at the end of the paper, absent asymmetric information, all our results immediately generalize to an environment with several types of agents and many consumption goods (such an environment is studied in the working paper Bennardo and Piccolo, 2007. This paper presents a representaive consumer setting with two consumption goods solely for pedagogical reasons. The effects of the introduction of asymmetric information are discussed in the last section of the paper.

[14] Introducing aggregate uncertainty in our set-up does not involve any analytical complication. All the results of the paper extend to this more general case, provided that the number of aggregate states is finite.

[15] This assumption is intended to capture real-life situations where a worker can perform with an appropriate quality standard a labor activity only for a limited amount of time. And the length of this time interval depends on his health state. For instance aircraft pilots, in order to guarantee appropriate safety standards, cannot fly more than a pre-specified number of hours per week. Similarly, a driver, a sportsman or a miner, who typically suffer of overuse syndromes cannot safely perform certain risky activities more than a certain number of hours in a year.

times differentiable, strictly concave in x, weakly increasing in θ and it satisfies the following regularity conditions: $D_cU(x,\theta) > K$ as $x_c \to 0$, with $K > 0$ sufficiently large, and $D_cU(x,\theta) < k$, as $x_c \to \infty$, with $k > 0$ and small, for all $c = 1, 2$.

Our formulation of preferences allows for some commodities to have health enhancing effects. To capture in a transparent way this feature one can use the utility representation $U(x,\theta) \equiv \hat{U}(x, \rho(x,\theta))$, where the function $\rho(x,\theta)$ represents the actual health conditions of an individual as determined by his initial health status chosen by the nature, θ, and his consumption choices, x. Throughout, we shall use such a representation whenever convenient.

Technologies and uncertainty: Competitive firms produce goods by employing workers, and labor is the only production factor. Firms can hire positive measures of agents, while each worker can supply labor in at most one firm, as specialization prevents workers from performing different jobs. There is only one type of occupation within each sector. The productivity of each single worker is measurable and may depend on his health. Precisely, a worker who is employed in *sector* t, with $t = 1, 2$, and supplies l^t units of labor, produces $y^t(\theta) = a^t(\theta)l^t$ units of commodity t in the health state θ, with $a^t(.)$ weakly increasing in θ.

The distribution of health of an agent working in *sector* t is $\langle p^t, \Theta \rangle$, with $p^t = (p^t(\theta_1), ..., p^t(\theta_N))$; health shocks are identically and independently distributed across workers in the same occupation, and independently distributed across sectors.

Timing: The economy lasts two periods, $\tau = 0, 1$; at $\tau = 0$, agents trade in financial and labor markets. At $\tau = 1$, health shocks are realized; subsequently agents supply labor, and consumption goods are traded and consumed. The space of enforceable contracts will be defined in Section 5. For notational simplicity, we restrict attention to economies where all agents work in equilibrium, and use the following notation. The state contingent consumption vector of an agent employed in *sector* t is denoted by $x^t(\theta)$, with $x^t = (x^t(\theta))_{\theta\in\Theta}$, and $x = (x^t)_{t\in T}$. The vector of state contingent labor for an agent occupied in *sector* t is $l^t = \{l^t(\theta)\}_{\theta\in\Theta}$. Finally, α^t is the fraction of workers employed in sector t, $\alpha = (\alpha^t)_{t\in T}$, with $\sum_{t\in T} \alpha^t = 1$, represents the assignment of workers to production sectors.

4 Ex ante and Interim Pareto Optimality

Ex ante Pareto Optimality: Let $u^t(x^t) = \sum_{\theta\in\Theta} p^t(\theta) U(x^t(\theta), \theta)$ be the expected utility of an agent assigned to sector t and $\bar{x}^t_c = \sum_{\theta\in\Theta} p^t(\theta) x^t_c(\theta)$ his expected consumption of good c. Denote T and C, (with $T = C = \{1, 2\}$) the sets of production sectors and production goods, respectively. By the law of large numbers, a *feasible allocation* of consumption goods and workers, $\langle x, \alpha \rangle$, is defined by the following constraints:

$$\sum_{t\in T} \alpha^t \bar{x}^t_c \le e_c + \alpha^c y^c, \; \forall \; c \in C, \tag{1}$$

(2) $$l^t(\theta) + x_L^t(\theta) = L;\ l^t(\theta) \le L(\theta);\ \forall\, \theta \in \Theta,\ t \in T,$$

(3) $$\sum_{t\in T} \alpha^t = 1.$$

where $y^t = \sum_{\theta\in\Theta} p^t(\theta)\, a^t(\theta) l^t(\theta)$ for all t. Denote F the set of feasible allocations.

A (ex-ante) symmetric Pareto optimum then maximizes $\sum_{t\in T} \alpha^t u^t(x^t)$, subject to $\langle x, \alpha\rangle \in F$. Symmetry implies that all agents face the same probability of being assigned to each occupation; we do not impose, however, the requirement that agents obtain the same expected utility if assigned to different occupations. Such a condition typically holds in the optima of convex economies; in our setting, though, there is no reason to impose it as part of the definition of first-best allocations.

Finally, our definition of Pareto optimum rules out the possibility that an agent obtains a random consumption vector in the optimum conditionally on being assigned to a given occupation. Risk-aversion makes this assumption unrestrictive.[16]

Interim Pareto Optimality: The following definition of interim Pareto optimality will play a central role in the welfare analysis of equilibria with unenforceable lottery contracts.

An interim optimal allocation with equal treatment maximizes $\sum_{t\in T} \alpha^t u^t(x^t)$ subject to $\langle x, \alpha\rangle \in F$, and to the additional constraints $u^1(x^1) - u^2(x^2) = 0$, whenever $\alpha \in (0,1)^2$, which imposes that workers assigned to different sectors obtain the same utility.

5 Competitive Equilibria

We shall now define competitive equilibria by assuming that there exist markets for all consumption goods, as well as financial markets for insuring *all* risks through assets with *deterministic* payoffs. We study either the case where only deterministic contracts (assets with random payoffs) are *enforceable* or that in which agents can also sign lottery contracts. Considering both these cases is useful to fully understand either the beneficial role that random contracts may play in our economy, or the effects of a somewhat natural market friction that may prevent their use.

5.1 Competitive Equilibrium with Deterministic Contracts

The notion of competitive equilibrium with deterministic contracts is standard. Following the approach taken in several contributions of the literature on individual risks (see Malinvaud, 1973, among others), we assume that securities paying in individual states, which allow to insure individual health risks, can be

[16] Since health shocks are independently distributed across workers in the same sector, for any random allocation such that the *sector t* workers receive a non degenerate lottery $(x_1^t, ...x_K^t, p_1, ..., p_K)$, there exists another feasible and Pareto superior allocation where *sector t* workers obtain $\hat{x}^t = \sum_{k=1}^{K} p_k x_k^t$.

offered by competing risk-neutral intermediaries to positive fractions of the population, so that individual uncertainty is washed out at the intermediary level.[17] Production firms supply consumption goods and demand labor at linear prices; for expositional reasons only we do not consider the possibility that these firms provide health insurance to their workers, by paying wages different from their productivity in some states. Finally, agents can work for at most one production firm and trade in consumption, labor and financial markets at linear prices.

Let h_θ^t be a security paying to an agent employed in the t-th sector one unit of numeraire in his individual health state θ, and zero otherwise. Denote z_θ^t and $\hat{z}_\theta^t$ the units of h_θ^t purchased by an agent employed in *sector t*, and the per capita units of this security offered in the market, respectively. Finally, define by $\phi^t(\theta)$ the unit price of h_θ^t, $w^t(\theta)$ the state contingent wage of workers in the t-th occupation, with $w^t = (w^t(\theta))_{\theta\in\Theta}$[18] and $q \in \Re_+^C$ a generic vector of spot prices, respectively.[19]

Because of labor supply indivisibilities, it is expositionally convenient[20] to consider the possibility that workers choose their occupation by using mixed strategies. Let $\varphi \in \Delta^T$ denote a probability vector according to which a worker mixes on occupation, the workers' expected utility is then $\sum_{t\in T} u^t(x^t)\varphi^t$, while their second and first period budget constraints, are respectively defined by:

$$(4) \qquad \sum_{c\in C} q_c(x_c^t(\theta) - e_c) = w^t(\theta)\, l^t(\theta) + z^t(\theta), \quad \forall\, \theta \in \Theta,\ t \in T,$$

and

$$(5) \qquad \sum_{\theta\in\Theta} z^t(\theta)\, \phi^t(\theta) \le 0; \quad l^t(\theta) \le L(\theta);\ \forall\, \theta \in \Theta,\ t \in T.$$

Production firms and financial intermediaries(per-worker) expected profits, respectively defined by:

$$(6) \qquad \sum_{\theta\in\Theta} p^t(\theta)\, l^t(\theta) \left(q_t a^t(\theta) - w^t(\theta)\right), \quad t \in T.$$

and

$$(7) \qquad \sum_{\theta\in\Theta} (\phi^t(\theta) - p^t(\theta))\hat{z}^t(\theta), \quad \forall\, t \in T.$$

Since, by the law of large numbers, φ^t is also the fraction of agents who are employed in sector t in equilibrium, *a competitive equilibrium with deterministic contracts* is thus an allocation $(x^{t*}, \varphi^{t*})_{t\in T}$,

[17] As it is conventional, intermediaries' risk-neutrality can be justified by the law of large numbers.

[18] The introduction of individual risks in a competitive setting requires assets' payoffs to be contingent on individual shocks; this point has been clarified in the seminal contribution of Malinvaud (1973).

[19] In the absence of aggregate uncertainty, spot market prices are independent from the realizations of individual shocks, as these shocks wash-out in the aggregate.

[20] It should be clear in the following that for any given equilibrium in mixed strategies, there exists a payoff equivalent equilibrium with pure strategies.

a collection of vectors $(\widehat{z}^{t*}, z^{t*})_{t \in T}$ and a vector of state contingent prices $(q, \phi^t, w^t)_{t \in T}$ satisfying the following conditions.

(i) *Agents maximize expected utility subject to (4) and 5;*

(ii) *Production firms and intermediaries maximize expected profits in (6) and (7);*

(iii) *Consumption, labor and financial markets clear:*

$$\sum_{t \in T} \varphi^{t*} \bar{x}_c^{t*} = e_c + \varphi^{c*} y^c, \quad \forall\, c \in C, \tag{8}$$

$$x_L^{t*}(\theta) = L - l^{t*}(\theta); \qquad z^t(\theta) = \widehat{z}^{t*}(\theta); \quad \forall\, \theta \in \Theta,\ t \in T. \tag{9}$$

5.2 Competitive Equilibrium with Lottery Contracts

We now introduce lottery contracts, by assuming that agents can buy lotteries (assets with random payoffs) from financial intermediaries before making any other market trade. Following Arnott and Stiglitz (1987), these lotteries will be referred to as *ex ante random contracts.* Formally, a lottery contract, $\mathcal{C} = ((\gamma, G), \rho(\gamma, G))$, is defined by: *(i)* a finite distribution (γ, G) with probabilities $\gamma = (\gamma^1, ..., \gamma^M) \in \Delta^M$ and payoff support $G = (g^1, ..., g^M) \in \Re^M$, with M finite; and *(ii)* a price $\rho(\gamma, G) \in \Re$. The interpretation is that an agent signing $\mathcal{C}$ pays the price $\rho(\gamma, G)$ to the intermediary, and obtains the right to receive the payoff g^m with probability γ^m. A random devise, whose characteristics are publicly verifiable, is used to determine the contractual obligations of the parties signing $\mathcal{C}$. Such a device chooses an *artificial state of the world* by selecting a positive integer $m \in \{1, ..., M\}$ with probability γ^m. Subsequently, the intermediary pays (receives) g^m to the agent whenever the integer m is selected. The expected profit an intermediary earns from $\mathcal{C}$ is $\rho(\gamma, G) - \sum_{m \in M} \gamma^m g^m$.

A general formulation of the competitive equilibrium in the space of random allocations would require all possible lottery contracts (an infinite set) to be priced in equilibrium (see Rustichini and Siconolfi, 2003) and should take into account the possibility that an agent signs several lottery contracts. In order to avoid the technical difficulties arising in working with an infinite dimensional commodity space, as well as a more complex notation, we impose the following unrestrictive assumptions: (i) only the set of fair lottery contracts with payoff support of dimension $M = T$ are offered in the market[21]; (ii) each agent can sign at most one lottery contract; and (iii) an agent will offer labor in *sector* t if and only if he receives the t-th payoff of his lottery contract.

A standard arbitrage argument justifies (i). Assumption (ii) is unrestrictive since any finite distribution of net payoffs obtainable by means of N fair lottery contracts can also be achieved through a single fair

[21] Consistently with the definition of lottery contracts, some or even all of its payoffs may be zero.

contract;[22] moreover, by risk aversion, it is always individually optimal to choose a contract with at most $M = T$ payoffs (elements of the payoff vector g), different from zero. Intuitively, this is because a risk averse agent, conditionally on being assigned to a given production sector, will always prefer the certain payoff, $\hat{g}$, to a non-degenerate lottery, (γ, G), with an expected payoff equal to $\hat{g}$.[23] Finally, (iii) amounts to be a convenient notational convention once (ii) is imposed.

A competitive (Walrasian) equilibrium with lottery contracts is then an allocation $(\tilde{x}^t, \tilde{\gamma}^t)_{t\in T}$, a collection of vectors $(\widehat{z}^t, \tilde{z}^t)_{t\in T}$, a vector of lottery contracts $\mathcal{C}$, and a vector of prices $(\tilde{q}, \tilde{\phi}^t, \tilde{w}^t)_{t\in T}$ satisfying the following conditions:

(i) *Agents maximize expected utility within their budget sets:*

$$(\tilde{x}^t, \tilde{z}^t, \mathcal{C})_{t\in T} \in \arg\max_{\mathcal{C}\in\Gamma} \sum_{t\in T} \gamma^t u^t(x^t) \tag{10}$$

$$s.t. \quad \sum_{c\in C} q_c(x_c^t(\theta) - e_c) = w^t(\theta)\, l^t(\theta) + z^t(\theta) + g^t - \rho(\gamma, G), \quad \forall\, \theta \in \Theta,\ t \in T, \tag{11}$$

$$\sum_{\theta\in\Theta} z^t(\theta)\, \phi^t(\theta) \leq 0,\ \forall\, t \in T, \tag{12}$$

where

$$\Gamma = \left\{ ((\gamma, G), \rho(\gamma, G)) : \rho(\gamma, G) = \sum_{t\in T} \gamma^t g^t \right\},$$

is the set of all fair lottery contracts.

(ii) *Production firms and intermediaries maximize (6)-(7), respectively.*[24]

(iii) *Consumption, financial and labor markets clear:*

$$\sum_{t\in T} \widetilde{\gamma}^t \widetilde{\overline{x}}_c^t = e_c + \widetilde{\gamma}^c \widetilde{y}^c, \quad \forall\, c \in C \tag{13}$$

$$\tilde{x}_L^t(\theta) = L - \tilde{l}^t(\theta); \qquad \tilde{z}^t(\theta) = \widehat{z}^t(\theta); \qquad \forall\, \theta \in \Theta,\ t \in T. \tag{14}$$

[22] Precisely, such a contract is defined by a vector of probabilities and a vector of payoffs which are linear combinations of the probabilities and the payoffs of the N fair lottery contracts.

[23] More precisely, it is never optimal for a risk averse agent to choose a lottery contract such that: (i) he receives the payoffs g^m and $g^{m'}$, with $g^m \neq g^{m'}$, with positive probabilities γ^m and $\gamma^{m'}$ respectively, and (ii) he chooses to work in *sector t* either when he receives g^m or $g^{m'}$. By convexity, indeed, there exists another fair contract, say $\mathcal{C}'$, which pays $\gamma^m g^m + \gamma^{m'} g^{m'}$ with probability $\gamma^m + \gamma^{m'}$, which, conditionally on working in *sector t*, is strictly preferred to $\mathcal{C}$.

[24] This is exactly as in the competitive equilibrium with deterministic contracts.

6 Pareto Optimal Allocations

This section characterizes first-best allocations. Let $\eta \in \Re^C$ be the vector of Lagrange multipliers to the feasibility constraints. When both consumption goods are produced in the optimum, the first-order conditions with respect to $(x^t(\theta), x_L^t(\theta), \alpha)$ of the (ex ante) Pareto program are:

$$D_c U(x^t(\theta), \theta) - \eta_c = 0, \quad \forall\, c \in C,\ \theta \in \Theta,\ t \in T, \tag{15}$$

$$U_{x_L}(x^t(\theta), \theta) - \eta_t a^t(\theta) \leq 0, \quad \forall\, \theta \in \Theta,\ t \in T, \tag{16}$$

$$u^1(x^1) - u^2(x^2) - (Z^1 - Z^2) = 0, \tag{17}$$

where (15) and (16) hold with equality whenever $x_c^t(\theta) > 0$ and $x_L^t(\theta) > 0$[25]; and where:

$$Z^t = \sum_{\theta \in \Theta} p^t(\theta) \left(\sum_{c \in C} \eta_c \left(x_c^t(\theta) - e_c \right) - \eta_t a^t(\theta)\, l^t(\theta) \right), \quad \forall\, t \in T,$$

is the difference between the value of the consumption of a worker employed in *sector* t and that of the sum of his endowment and his production, both evaluated at the vector of shadow prices η. In other words, Z^t represents the value of the net transfer received by an agent assigned to *sector* t in the optimum.

As standard, (15) and (16) imply the equality of marginal rates of substitution between state contingent commodities. The first-order condition with respect to α in equation (17) is less standard, and plays a crucial role in our analysis. It indicates that the difference in expected utilities across occupations, $\Delta u = u^1(x^1) - u^2(x^2)$, is equal to the difference between the values of the net transfers received (paid if negative) by workers in each sector, which is $\Delta Z = Z^1 - Z^2$. Noteworthy, ex ante identical workers assigned to different occupations will get the same utility only if $\Delta Z = 0$, and ex ante and interim optima coincide.

The next proposition shows that Δu typically differs from zero at the solution of the system of equations (15)-(17) defining the optimum, whenever the health distributions $\langle p^1, \Theta \rangle$ and $\langle p^2, \Theta \rangle$ are different and both consumption goods must be produced in the optimum, implying that interim efficiency is generally incompatible with ex ante efficiency. This is, indeed, the distinguishing feature of our environment.

Proposition 1 *The Pareto optimum is unique. Moreover, the subset of economies such that ex ante and interim Pareto optima are disjoint is generic, if $p^1(\theta) \neq p^2(\theta)$ for at least an health state θ and $||e||$ is small enough.*

The proof of this proposition, as well as all the subsequent ones, is provided in the Appendix; it begins by defining the set of possible economies in which the genericity result is proven as the product of the

[25] For simplicity we neglect the case where $x_L^t(\theta) = L$ in stating the first-order conditions. Implicitly, we assumed $D_{x_L}U$ sufficiently small at $x_L^t(\theta) = L$.

sets of admissible utility functions, technologies and endowments.

Intuitively, ex ante efficiency mandates the equalization of marginal rates of substitution across all workers. Moreover, any pair of ex ante identical workers with different occupations generally feature different expected utility functions and technological constraints, since health distributions are occupation specific. As a consequence, the equalization of the marginal utilities of contingent goods (*margins*) prevents that of the expected utilities (*levels*).

Notewhorty, by checking the first-order conditions one can easily realize that the inconsistency between ex ante and interim efficiency stated in Proposition 1 holds only under the assumption that health distributions are endogenous; occupational indivisibilities per se do not deliver this result. Moreover, imposing the aggregate endowment to be small, is needed only to guarantee that both goods are produced in equilibrium. Hence, this is clearly a sufficient but not necessary condition for establishing the above result.

Proposition 1 has two simple but important corollaries. First, ex ante efficiency typically requires transfers of resources across workers assigned to different occupations, since (17) implies that budget balancing obtains only when utility levels are equalized. Moreover, theses transfers are implemented through a random allocation of workers across occupations. Second, compensating wage differentials which equate (expected) utilities of workers assigned to different sectors undermine first-best efficiency.

The next proposition shows that the Pareto shadow wages, $\eta_t^P a^t(\theta)$, associated to technologies inducing unsafer health distributions, in the sense of First-Order Stochastic Dominance (FOSD), are relatively higher in the optimum whenever $a^t(\theta) = a(\theta)$ for all θ and t.[26] This an important result since it unveils that efficiency still commands higher wages to be paid in riskier occupations, even though wage differentials do not equalize utilities.

Definition: *For any pair of health distributions,* $\langle p^1, \Theta \rangle$ *and* $\langle p^2, \Theta \rangle$*, we shall say that* $\langle p^1, \Theta \rangle$ *FOSD* $\langle p^2, \Theta \rangle$ *if* $\sum_{\theta \leq \theta_n} p^1(\theta) \leq \sum_{\theta \leq \theta_n} p^2(\theta), \forall\, \theta_n \in \Theta$*, with at least one strict inequality.*

Essentially, assuming that $\langle p^1, \Theta \rangle$ *FOSD* $\langle p^2, \Theta \rangle$ implies that the likelihood of better health states is higher in occupation 1 relative to occupation 2.

Proposition 2 *Assume* $a^t(\theta) = a(\theta)$ *for all* θ *and* t*, if* $\langle p^1, \Theta \rangle$ *FOSD* $\langle p^2, \Theta \rangle$ *then* $\eta_1^P < \eta_2^P$ *for all Pareto optima such that* $\alpha^P \in (0,1)^2$*.*

While its proof is not immediate, the proposition has a clear intuition. Were shadow wages equal across the two sectors, the marginal utilities of consuming the goods produced in those sectors would also be equal for all agents in all states. It would then be welfare enhancing to move a fraction of workers from the riskier to the safer sector, and to marginally increase (resp. decrease) the consumption of the good produced in the safer (resp. riskier) sector for all agents. This would indeed leave unaltered the utility agents can get from consumption, but at the same time, would increase their ex ante expected health.

[26] Assuming that $a^t(\theta)$ is invariant across sectors is an innocuous normalization whenever $a^t(\theta_1) = 0$ for all t (i.e., whenever workers are unproductive in the worst health state).

6.1 Ex-Ante Efficiency and Optimal Cross-Jobs Transfers

In this section we study how the effects of health shocks on preferences on one side, and endowments and productivity on the other side, contribute to determine optimal cross-jobs transfers, as well as the differences between the utilities obtained by workers assigned to different technologies. For this purpose, we shall assume that occupations differ in their *health riskiness*: We impose that the health distributions associated to different occupations are ordered by the FOSD criterion, and that *occupation 1* is safer than *occupation 2*. Moreover, for the sake of tractability, throughout the section we shall also assume that preferences are separable with respect to consumption goods and labor, that they are supermodular with respect to consumption goods and display increasing differences in labor and health. More formally, the analysis will be developed under the following assumptions:

(A1) $a^t(\theta) = a(\theta)$ for all t;

(A2) Preferences are represented by the utility function $U(x,\theta) = f(x_1, x_2, \theta) - \psi(l, \theta)$, which satisfies all the assumptions in Section 2 and has also bounded derivatives;

(A3) $f(., \theta)$ is supermodular in x for all θ, and $\psi_{l\theta} \leq 0$ for all θ.

Assuming that $a^t(\theta)$ is invariant across sectors is an innocuous normalization whenever $a^t(\theta_1) = 0$ for all t (i.e., whenever workers are unproductive in the worst health state), as we already pointed in the previous section. **A2** allows us to clearly distinguish the effects of health status on the utility of produced consumption goods from that on the disutility of labor.[27] Indeed, $f(x_1, x_2, \theta)$ and $\psi(l, \theta)$ represent the utility of consumption commodities and the disutility of labor, $l = L - x_L$, respectively, and may both depend on θ. Moreover, the assumption of finite derivatives is almost unrestrictive since finite bounds are allowed to be arbitrarily large. Finally, as for **A3**, while supermodularity, i.e., $f_{12} > 0$, is a simplifying assumption,[28] while imposing $\psi_{l\theta} \leq 0$ seems a natural restriction as health is typically an input for production.

The super (sub) modularity properties of utility and production functions, as determined by complementarity (substitutability) relationships between health, consumption goods and labor, will be the fundamental ingredients of our analysis. Noteworthy **A3** does not impose any restriction on the sign of the cross-derivatives $U_{c\theta}$. In fact this sign depends on the relative magnitude of two effects, which may generally go in opposite directions given the specific nature of health services. First, the marginal utility of most consumption activities increases in better health status, since health can be used as an input for these consumption activities: At worst, a better health should neither increase nor reduce the

[27] Separability is imposed only for the sake of tractability. Most of our results extend immediately to the case where consumption goods affect the labor disutility ψ, and these effects are bounded relatively to the health effects on marginal utility of consumption goods and leisure.

[28] This assumption implies that consumption goods are complements, i.e., $f_{12} > 0$, and it can be easily relaxed. In fact, in order to derive our characterization results, we only need f_{12} to be not too negative.

marginal utility of consuming any good. Thus, were this the only channel through which health impacts consumption choices, one should have $U_{c\theta} \geq 0$ for all c. However, whenever health "can be produced" or, more precisely, whenever agents can influence their health by devoting resources to medical treatments or health enhancing consumption activities, a possibly counterbalancing effect may arise. In the rest of the analysis we will indeed show that in this case the agents' marginal utility of medical treatments, or more generally health enhancing consumption activities, may well be larger in bad health states and hence one may have $U_{c\theta} < 0$ at least for some consumption goods.

For clarity only, in the next sections we shall consider separately how health effects on pure consumption choices, treatments' decisions and labor choices determine the properties of optimal allocations and transfers. We study first a pure consumption economy where neither health nor consumption goods can be "produced", so that adverse health shocks only affect preferences by reducing the marginal utility of consumption. We then turn to the case where agents can undertake health enhancing consumption activities. Finally, we consider the case where agents produce consumption goods and analyze the health effects on agents' labor choices, including the effects of health on disutility of labor, labor endowment, and productivity.

Distinguishing these cases will be key for our analysis. First, indeed, from a pedagogical viewpoint, it allows us to provide a sharp characterization of the "transfers" across occupations that one should actually observe in efficient, possibly regulated, markets. Second, in the real world, one typically observes either instances where individual consumption patterns including choice of health enhancing consumption activities or treatments have significant effects on agents' actual health conditions, or situations where these effects are relatively negligible. Finally, it may be argued that the occupations that most workers can apply for in real markets can roughly be grouped in two broad categories: The ones for which good physical or mental health are necessary and important prerequisites, and those requiring only minimal health requisites to be performed satisfactorily. Intuitively, health effects on production determine the sign of cross transfers for jobs of the former type; while health effects on pure consumption decisions and on health enhancing activities should be relevant otherwise.

6.1.1 Health Effects on Pure Consumption Choices

In order to focus on the properties of Pareto optima in pure consumption economies throughout we shall impose $a(\theta) = l^t(\theta) = 0$ for all θ and $t = 1, 2$. Moreover, we assume that agents cannot modify their health conditions by devoting resources to treatments or other consumption activities, that is, $\rho(x, \theta)$ does not vary with x.[29] As we explained above, under these assumptions health is simply an input for consumption activities, for this reason it is natural to assume $U_{c\theta}(x, \theta) > 0$ for all c. Indeed, it is actually self-evident that the marginal utility of certain goods, including physical activities, and all kind of leisure and consumption activities which require social interactions, and, for instance, do not take place "in

[29] These assumptions allow to characterize optimal allocations and transfers for situations where the productivity of health henancing activities is relatively low and health shocks play a relatively minor role in production decisions.

house", decreases in worst health states.[30]

As the next proposition illustrates, the effect of health on the (marginal) utility of consumption and the direct health effect on well being, as measured by $U_\theta(x,\theta)$, determine optimal cross transfers and the sign of (expected) utility differential.

Since the first-order conditions with respect to α of the Pareto program imply $\Delta u \geq 0$ (resp. $<$) if and only if $\Delta Z \geq 0$ (resp. $<$), from hereafter we shall only study the sign of Δu.

Proposition 3 *If U has increasing differences in (x,θ) then $\Delta u^P > 0$ in the optimum.*

Risk-averse workers assigned to different occupations must obtain the same consumption in each individual health state (i.e., $x^1(\theta) = x^2(\theta) = x^P(\theta)$ for all θ). As consumption goods and health are complements, optimality also imposes agents' consumption to be larger in better health states; and for this reason $U\left(x^P(\theta),\theta\right)$ is increasing in θ. Furthermore, since workers using less risky technologies are more likely to experience better health states, they obtain larger utility levels with larger probabilities. Thus, they also obtain a larger expected utility.

6.1.2 Health Effects on Loss Reduction Activities

We now consider the case where agents can "produce" health by devoting resources to medical treatments or health-enhancing consumption activities. Formally, this amounts to impose $\rho_c(x,\theta) \geq 0$, with strict inequality for at least one good. In particular, by differentiating $\hat{U}$ one obtains:

$$\hat{U}_{c\theta} = \hat{U}_{c\rho}\rho_\theta + \hat{U}_\rho \rho_{c\theta} + \hat{U}_{\rho\rho}\rho_c\rho_\theta.$$

Importantly for our analysis, one immediately sees that $U_{c\theta}$ is negative for $\rho_{c\theta} < 0$ and sufficiently small (i.e., large in absolute value).[31]

Assuming $\rho_{c\theta} < 0$ is completely natural in most real-world situations involving health enhancing consumption activities, and especially medical treatments. Consider, for instance, the choice of consuming a generic treatment c. Since the treatment is effective only in *relatively bad* health states, by its own nature, $\rho_{c\theta}$ must be negative for at least a subset of Θ. Moreover, it must also be large in absolute value in that subset, if the treatment is (marginally) very effective. In addition, imposing that $\rho_{c\theta}$ does not change sign in all its range does not seem very restrictive, and becomes even more appropriate whenever c can be interpreted as a total amount of certain types of medical expenses, i.e., a composite good, as it is often convenient for both theoretical and practical purposes.

[30] Even if theoretically possible, situations where better health reduces the marginal utility of consuming some goods seem really awkward. It is completely more natural to explain the increase in the consumption of certain goods in bad health states as a consequence of the reduction in the marginal utility of other goods.

[31] The first term of this sum represents the effect of θ on the marginal utility of consumption activities, which should be positive as discussed before; the second term represents the effect of health-enhancing consumption activities on $\rho(x,\theta)$; while the third addendum captures a second-order effect which reinforces that of health-enhancing consumption activities.

Similar considerations hold with regard to health enhancing consumption activities, ranging from those aimed at satisfying nutritional and housing needs to physical activities. As an example, consider the case of a worker, living in a low-income country which experiences high diffusion rates of a contagious chronic disease. Contracting the disease generally impairs his consumption and working aptitudes and reduces his utility. However, the more adequately this worker can satisfy his basic consumption and housing needs, the smaller should be the effects of the disease on his actual health conditions. Making this assumption amounts imposing $\rho_{c\theta} < 0$.

Next proposition considers the simplest situation where both goods have a positive and strong effect on agents' actual health, so that $U_{c\theta} < 0$ for all c. We shall turn subsequently to the more general case where one good is complement with health while the other one (i.e., the treatment) is substitute.

Proposition 4 *If U has decreasing differences in (x, θ) there exists a pair of real numbers (k, K) with $k < K < 0$ such that:*

(i) $\Delta u^P < 0$ whenever $U_{c\theta}/U_\theta < k$ for at least one good c;

(ii) $\Delta u^P > 0$ whenever $U_{c\theta}/U_\theta > K$ for all $c = 1, 2$.

The economic intuition behind this result follows from the fact that the optimal consumption allocation $x^P(\theta)$ is smaller in better health states when consumption goods and health are substitutes. If such an effect is sufficiently large to compensate the impact of U_θ, the function $U\left(x^P(\theta), \theta\right)$ is decreasing in θ. Thus, workers using riskier technologies obtain a larger utility at the optimum. The converse is true otherwise.

The results of Proposition (4) can be easily extended by using a continuity argument to the case where, besides consuming a treatment (or devoting resources to an health enhancing activity), agents can also consume another good whose impact on health is negligible. It then remains uncovered the case where one good is substitute with health, the other is complement, and none of these "cross-marginal" effects is negligible relative to the other. In such a case, one can easily verify that the direction of optimal transfers depends on both the magnitude of cross derivatives, and on the marginal utility of the two goods in the optimum (which, in turn, is affected by initial endowments). The main issue hence becomes whether one can use a synthetic measure, having empirical correlates, to determine which effect prevails. We now show that the cross derivative of the indirect utility with respect to income and health provides such a synthetic measure. Let $V(q, I(q), \theta) \equiv \max_{x \in \Re_+} \{U(x, \theta) \ s.t., \ qx \leq I(q)\}$ be the state dependent indirect utility associated to the vector of prices q and total wealth $I(q)$.

Corollary 5 *Assume $V_{I\theta}$ has constant sign for all q, I, and θ. Then, the following properties hold:*

(i) *if $V_{I\theta} > -k$ with k positive and sufficiently small, $\Delta u^P > 0$;*

(ii) *if $V_{I\theta} < -K$ with K positive and sufficiently large, $\Delta u^P < 0$.*

Efficiency requires workers assigned to riskier jobs to get lower expected utilities if health enhancing consumption activities have relatively negligible effects on health, while the converse will often be true otherwise.[32]

The proof follows from straightforward comparative statics and it is left to the reader.[33] It simply consists in verifying that $U\left(x^P(\theta),\theta\right)$ is increasing (resp. decreasing) whenever $V_{I\theta}$ is sufficiently large and positive (resp. negative), thereby health and income are *strong* complements (resp. substitutes). By FOSD the slope $U\left(x^P(\theta),\theta\right)$ implies the sign of Δu^P, as for previous propositions.

6.1.3 Health Effects on Labor Choices

This section illustrates the health effects on labor choices. We begin by considering the case where health conditions affect labor endowment. This case is the simplest to analyze and permits to illustrate, in its simplest form, a key effect for the determination of optimal cross-jobs transfers. We shall then turn to study the more complex trade-offs which arise whenever health affects labor supply indirectly by influencing also agents' disutility of labor and productivity and hence labor choices.

Health Effects on Labor Endowment: We shall now consider the case in which health has no effect on preferences and productivity (formally, $f_\theta(.) = 0$ for $c = 1, 2$, $\psi_\theta = 0$, $a(\theta) = a$ for all θ) and only determines workers labor endowment. These simplifying assumptions capture, in a simple way the consequences of health risks which can only be limited by imposing bounds on the maximal time of labor supplied per unit of time. For instance, this is the case of physicians, scientists or manual workers exposed to radiations and toxic materials, as well as drivers or pilots, whose maximal amount of labor is often determined by health considerations.

It will also be convenient to stick to the assumption of inelastic labor supply, such that $l^t(\theta) = L(\theta)$ for all θ and t. Imposing this condition, does not change the qualitative results of the analysis of this section, since health shocks are assumed relevant only to the extent that they impose binding constraints in some individual state on the maximal time that an agent can work. Therefore, assuming that labor supply is inelastic only amounts to impose that those constraints are binding in all individual states. Thus, the effects of health shocks on labor supply simply result magnified if labor supply is completely unaffected by state contingent shadow wages.

The next proposition demonstrates that agents employed in the sector yielding the worst health distribution (i.e., in sector 2) obtain a larger utility (and a positive transfer) in the optimum.

Proposition 6 *Assume $L(\theta_n) \geq L(\theta_{n-1})$ with strict inequality for at least one n, then $\Delta u^P < 0$.*

The proof, which is left to the reader, follows exactly the same steps as those of Propositions 3 and 4. Intuitively, agents work more and obtain a lower utility in better health states, where their

[32] Note, however, that if one restricts attention to the case where no health henancing activity provides utility directly (as it is the case for medical treatments) Δu^P is negative only if $\rho\left(x^P(\theta),\theta\right)$ is decreasing at least in some subset of Θ. This is not anymore true, though, as soon as some health enhancing consumption activities increase directly agents utility.

[33] Note that $U_{c\theta}(x,\theta) > 0$ (resp. < 0) for all c implies $V_{I\theta} > 0$ (resp. < 0), hence Proposition 5 generalizes the results stated in Propositions 3 and 4.

labor endowment and hence their labor supply, are larger. Optimality requires them to have the same consumption in all states,as a consequence workers in safer occupations, which supply more labor on average, obtain a lower expected utility.

Health Effects on Disutility of Labor and Individual Productivity: We now study the more general case where health risks mainly affect agents' disutility of labor or their individual productivity and labor supply is not inelastic.[34] In this framework health has both a *direct* and an *indirect* effect on labor choices, and these effects influence the sign of the optimal utility differentials in opposite directions.

In particular, bad health shocks exert a direct (quantity) effect on labor choices, since efficiency requires agents in each sector to work more in better health states, in which the disutility of labor is lower or the individual productivity is higher, and better health states are more likely in the safer sector. This effect has the same nature as that determining the sign of optimal transfers when health affects labor endowment and may lead to positive transfers in favor of workers using less safe technologies, who are the ones working less in average. However, at the optimum labor supply is also influenced by shadow wages, and Pareto optimality imposes compensating wage differentials in favor of the riskier occupation, as established in Proposition 2. This indirect price effect which goes in the opposite direction than the quantity effect described above, since agents in the sector with lower shadow wages work less in the optimum. Analyzing the trade-off between these two effects is the main objective of this section. In particular, we shall show that under mild restrictions on preferences, the quantity effect dominates the price effect and hence determines the optimal utility differential.

The next proposition makes a first step in this direction, by showing that workers in the riskier sector obtain a positive transfers whenever health has a sufficiently strong impact on marginal disutility of labor. For the sake of simplicity, in proving the result we shall impose the standard Inada condition $\psi_l(0, \theta) = 0$ for all θ.

Proposition 7 *Assume $\psi_{lll}(l, \theta) > 0$ for all (l, θ), then $\Delta u^P < 0$ (resp. $>$) if $|\psi_{l\theta}|$ is sufficiently large (resp. small) relatively to $|\psi_\theta|$ for all (l, θ).*

Imposing $\psi_{lll} > 0$ is common in applications.[35] Intuitively, this assumption guarantees that the labor schedules for the two occupations are strictly concave in the shadow wage, while $\psi_{l\theta}$ large ensures that the disutility of labor becomes relatively small for sufficiently good health states. As a consequence $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ cannot be too large neither under bad health states, where the disutility of labor is large and hence actual labor supply is small, nor in favorable health state, where agents labor supply become less responsive to the wage as the marginal disutility of labor is large. Therefore, workers in the riskier sector must obtain a higher utility if the health effect on the marginal disutility of labor is

[34] As we shall formally show, this direct effect is qualitatively analogous to the effects of health on endowments. For the sake of readability, throughout we shall then assume that the Pareto program has only internal solutions and labor contingent endowments are health independent, i.e., $L(\theta) = L$ for all θ, so that the health effect on endowment does not appear.

[35] This restriction is typically imposed in the agency literature, as well as in the (theoretical and applied) literatures studying the effects of multiple risks.

sufficiently large.[36]

Finally, by using similar but somewhat more involved arguments one can obtain an analogous result for the case where health only affects productivity, i.e., $\psi_\theta(l,\theta) = 0$ for all (l,θ) and $\partial a(\theta)/\partial\theta \neq 0$, (see Bennardo and Piccolo, 2007). For brevity, here we shall only consider the case where the distribution of health in the safer sector is concentrated around the healthiest state θ_N. We will show that in this case, which is often studied in the applied labor literature, optimal transfers and utility differentials can be derived by imposing very mild conditions on the sensitivity of labor with respect to the shadow wage.

Denote $w_\theta = a(\theta)\eta$ and let $l(w_\theta,\theta)$ be the contingent labor supply schedule implicitly defined by optimality conditions; and let $\zeta_{l,w} = dl(w_\theta,\theta)/dw_\theta)/(l(w_\theta,\theta)/w_\theta)$ be the sensitivity of the optimal labor schedule with respect to the shadow wage. In the next proposition we show that workers in the less safe sector obtain an higher utility in the optimum whenever $\zeta_{l,w}$ is non-decreasing in the shadow wage, that is when agents who are already "working a lot" react less to wage increases.

Proposition 8 *Assume $p^1(\theta_N) = 1$, then $\Delta u^P < 0$ (resp. $\geq$) if $\partial\zeta_{l,w}/\partial w_\theta < 0$ (resp. $\geq$) for all θ.*

Finally, the next table summarizes the results obtained throughout the section, by illustrating the direction of the optimal cross-job transfers as determined by the health effects on consumption and production capabilities.

Table 1	**Small effects of health on productivity or endowment of labor**	**Large effects of health on productivity or endowment of labor**
Small effects of health enhancing consumption activities on health	Positive transfers to workers in the safe sector	Positive transfers to workers in the unsafe sector
Large effects of health enhancing consumption activities on health	Positive transfers to workers in the unsafe sector	Positive transfers to workers in the unsafe sector

As the table shows, transfers go in favour of workers in the unsafe sector in all cases expect that in which health has negligible effects on health enhancing and production activities.

7 Characterization of Competitive Equilibria

In this section, we characterize competitive equilibria for economies with either enforceable or unenforceable lottery contracts. We begin by stating an existence result.

[36] It is worth to notice that this result holds more generally irrespective of the sign of ψ_{lll} if $\sigma_\psi = \psi_{ll}/\psi_l$ is large enough (see the working paper version Bennardo and Piccolo, 2007). This is a necessary condition for the elasticity of labor with respect to the wage to be small. There exists a quite large empirical literature (see the seminal contribution of Abowd and Card 1989, among others) indicating that this elasticity is not large in actual labor markets, and may even be quite close to zero.

Proposition 9 *A competitive equilibrium exists either with or without enforceable lottery contracts.*

The proof of this result follows standard argument.

Next proposition shows that occupations associated to riskier health distributions command compensating wage differentials in both contractual regimes. Moreover security prices are fair and state contingent wages are equal to the value of the corresponding individual productivity (which in turn is determined by equilibrium commodity prices) either with or without lotteries. Commodities prices and in turn the welfare properties of competitive equilibria are, however, different in the two regimes. In particular, full efficiency obtains only in economies where lottery contracts are enforceable; while equilibria are interim efficient allocations with equal treatment if only deterministic contracts can be enforced.

Proposition 10 *The following properties hold:*

(i) Under enforceability of lottery contracts, the competitive equilibrium is first-best efficient .

(ii) Under unenforceability of lottery contracts, the competitive equilibrium allocation is interim efficient with equal treatment.

(iii) In both contractual regimes, $\phi^t(\theta) = g^t p^t(\theta)$ *for some* $g^t \in \Re_+$ *and* $w^t(\theta) = q_t a^t(\theta)$*. Moreover,* $w^1(\theta) < w^2(\theta)$ *if* $\langle p^1, \Theta \rangle$ *FOSD* $\langle p^2, \Theta \rangle$ *and strictly positive measures of agents are assigned to both sectors*

Parts (i) and (ii) of the proposition result from the logic of the first welfare theorem applied to the space of random and deterministic allocations, respectively. The first two statements appearing in Part (iii), follow from the linearity of the intermediaries and production firms maximization programs; while the inequality $w^1(\theta) < w^2(\theta)$ indicates that *compensating wage differentials* are paid to riskier occupations, and is directly implied by the first-order conditions of the agents' optimization programs.

Finally, it follows as a corollary from the proposition above that competitive equilibria with deterministic contracts generically fail to be first-best. Differently, in any equilibrium with lottery contracts in which positive measures of agents are employed in both sectors $u^t(x^t) - u^{t'}(x^{t'}) \geq 0$ (resp. $<$) if and only if the value of the consumption of sector t workers exceeds (respectively falls short of) the value of their resources and their expected wage. By using lottery contracts, indeed, wealth is optimally transferred across occupations in such a way that agents obtaining the higher (resp. lower) expected utility get a positive (resp. negative) transfer.

8 Cross-Subsidization Insurance Policies and Welfare

The main insight of all our efficiency analysis is that transfers across agents occupied in different jobs may typically increase welfare. A natural question to address is then whether unenforceability of lottery contracts, which prevents the market from implementing these transfers, creates room for public policy interventions. Addressing this issue requires to consider how specific cross-subsidization policies may affect labor demand and supply, or change the profit prospects of firms and intermediaries.

Within our environment a policy scheme that simply transfers resources across sectors, if not appropriately designed, could merely result in an increase of the profits earned by firms active in the subsidized sector. These firms indeed might well find it profitable to lower wages in response to the provision of the subsidy to their workers. More formally, a policy intervention that implements cross-transfers in a competitive economy will generally affect workers' labor supply, thus leading to market clearing wages adjustments. Hence, the welfare effects of transfers will in general depend on the existence of (or the possibility of designing) institutional mechanisms deterring wage adjustments that go in the direction of restoring *equal treatment* across occupations.

In real economies the most widespread mechanisms playing this role are minimum wages regulations. De facto, minimum wages do exist in most real world economies and affect large segments of the labor market. Indeed, independently from the reasons leading to their implementation, minimum wages prevent wage reductions, and hence full adjustments to transfers policies.[37] Their specific implementation rules (including the size of wage floors, the criteria of access, whether they are set at the economy level or at sectorial level, etc.), however, determine which fraction of the population is ensured against wage reductions, under what contingencies, and finally what type of labor supply distortions they may generate.

To study the effects of transfers policies under minimum wages regulations we need to suitably modify the previous definition of competitive equilibrium. This is because one needs to take into account that minimum wages may lead to labor supply rationing. More precisely, since minimum wages can generally make some occupations more attractive than others, we need to specify a rule according to which workers are assigned to occupations. We shall assume that whenever workers receive a larger utility in *sector* t than in *sector* t', the probability that each of them is assigned to *sector* t in equilibrium is equal to α^t, which is the measure of workers assigned to *sector* t. This workers' assignment rule can be seen as the outcome of a *decentralized* job search process where: in a first stage, workers simultaneously apply for occupations; subsequently, applications are randomly selected whenever the number of workers applying for a job is larger than the number of posted vacancies, and firms offer jobs to workers; while in a third stage, workers, who have possibly received more than one offer, choose which occupation to join. Noteworthy, while this type of assignment mechanism introduces endogenous uncertainty in the assignment of workers to jobs, the transfers policies we consider are completely deterministic, and hence their implementation does not rely on any random device.

Finally, throughout we shall assume that, in any equilibrium with transfers, all commodity as well as asset markets clear at "Walrasian" prices without rationing (i.e., exactly as in the absence of transfers),

[37] The large majority of European countries operate statutory or collectively determined minimum wage rates. In most but not all countries minimal wages provide a standard of living that is close to subsistence levels. Denmark, Finland, Germany, Italy and Sweden do not operate national minimum rates, but have minimum rates set through sectorial collective agreements that cover a high proportion of the working population. Minimum wages exist also in the United States as well as in many other countries in the world. Overall minimum wages set at national level appear to be a "binding constraint" for most of the so called dirty jobs, which mostly include the unsafe occupations.

and that firms' labor demand is not rationed as well. The motivation for the clearing rule of consumption and assets markets is the usual one: namely, were firms or agents *not* rationed, they would have an incentive to manipulate prevailing prices.[38] The same type of argument justifies the assumption that labor demand is never rationed in the equilibrium.

Below we formally define the set of feasible policy interventions, which consists of state contingent transfers to firms and workers, and minimum wages. We shall not exclude the possibility that workers receive health contingent minimum wages, since describing policies based on this kind regulation may have some normative interest. The most appealing results of this section however will focus on the case of health independent minimum wages.

Let $s^t(\theta)$ be the (possibly negative) monetary transfer[39] received by an agent who signs a health insurance contract designed for *sector* t and experiences the state θ. Moreover, denote $f^t(\theta)$ the monetary transfer received by a *sector* t firm for each worker in state θ which it employes. Finally, let $\hat{w}^t(\theta)$ the minimum state contingent wage that firms must pay to workers employed in *sector* t who experiences the health state θ.

A policy scheme, $\wp = (s, f, \hat{w})$, consists of a vector $s = (s^t(\theta))_{t\in T}^{\theta\in\Theta}$ of subsidies to the workers; a vector $f = (f^t(\theta))_{t\in T,}^{\theta\in\Theta}$ of transfers to production firms and a vector $w = (\hat{w}^t(\theta))_{t\in T}^{\theta\in\Theta}$ of state contingent minimum wages.

Feasible policy schemes must satisfy the budget-balancing condition :

$$\wp \in \mathcal{P} \equiv \left\{ \wp : \sum_{t\in T} \varphi^t \left(\sum_{\theta\in\Theta} p^t(\theta) \left(s^t(\theta) + f^t(\theta) \right) \right) = 0 \right\},$$

where φ^t represents the measure of workers who are effectively assigned to *sector* t in an equilibrium with transfers.

A rational expectation equilibrium with transfers and minimum wages, is formally defined by the following conditions:

(i) Consumers' choose $\left(x^t, \varphi^t, z^t\right)_{t\in T}$ by maximizing $\sum_{t\in T} u^t(x^t)\varphi^t$ subject to the budget constraints:

$$\sum_{c\in C} q_c(x_c^t(\theta) - e_c) = w^t(\theta)\left(L - x_L^t(\theta)\right) + z^t(\theta) + s^t(\theta),\ \forall\ \theta \in \Theta,\ t \in T,$$

$$\sum_{\theta\in\Theta} z^t(\theta)\,\phi^t(\theta) \leq 0,\ \forall\ t \in T,$$

and to the set of *rationing* constraints:

$$\varphi^t \leq \alpha^t,\ \forall\ t \in T,$$

[38] See, for instance, Mas Colell et al.(pp. 315, 1995) for a justification of the Walrasian equilibrium notion along these lines.
[39] We will use monetary transfer as a synonimus of "transfer in units of numeraire".

indicating that an agent who offers labor in *sector* t will be assigned to that sector with probability lower or equal to α^t, which is the measure of workers effectively assigned to *sector* t in the equilibrium;

(ii) Production firms' labor demand, l^t, maximizes $\sum_{\theta\in\Theta} p^t(\theta)\left(l^t(\theta)\left(q_t a^t(\theta) - w^t(\theta)\right) + f^t(\theta)\right)$; while intermediaries assets' supply, $\hat{z}^t$, maximizes the same expected profits as in the competitive equilibrium with deterministic contracts (i.e., conditions (7));

(iii) Minimum wages constraints, $w^t(\theta) \geq \hat{w}^t(\theta)$ for all θ and t, are satisfied;

(iv) Feasibility conditions hold including budget balancing for the policymaker.

The next proposition shows that all Pareto optimal allocations can be implemented as equilibria with transfers, provided that agents' health states are public information. Optimal policy schemes generally hinge on state and sector contingent minimum wages.

Let $Z^t(\eta^P, \theta) = \sum_{c\in C} \eta_c^P (x_c^{tP}(\theta) - e_c) - \eta_t^P l^{tP}(\theta) a^t(\theta)$ denote the differences between the value of the consumption of a worker employed in *sector* t experiencing the health state θ and that of the sum of his endowment and production, evaluated at the shadow price vector η^P, respectively.

Proposition 11 *The Pareto optimum can be implemented as an equilibrium with transfers by policy schemes such that* $s^t(\theta) = Z^t(\eta^P, \theta)$, $\hat{w}^t(\theta) = \eta_t^P a^t(\theta)$, *and* $f^t(\theta) = 0$ *for all* θ *and* t.

Monetary transfers equalize, at the Pareto optimal shadow prices, the marginal utilities of contingent wealth across occupations. Minimum health-contingent wages prevent firms from manipulating the transfers' scheme by undercutting wages in the sectors where workers obtain higher utility levels and labor supply is rationed.[40] Moreover, since minimum wages can be either contingent on the health status or sector specific, one does not need to impose any kind of transfer to production firms for zero profit conditions to be satisfied.

Proposition 11 provides a normative benchmark, but relies on a quite restrictive assumption, namely that minimum wages contingent on workers' health status can be enforced. Indeed, in order to design mechanisms with these features the policymaker should have access to reliable information concerning workers' individual health as well as fine details on the specific technologies adopted by firms. For this reason, in the rest of the section we shall investigate under what conditions policy schemes based on uniform minimum wages allow to obtain Pareto improvements. We shall focus on the case in which labor supply choices are affected in a dichotomic fashion by health shocks on productivity, which determine individual wages. Indeed, if labor supply is a dichotomic function of wage, introducing uncertainty in

[40] A decentralization result similar to the one stated in the previous proposition can be proved if one considers alternative policy schemes based on (possibly negative) *non-linear* subsidies to health-enhancing consumption activities. The logic of the proof remains the same as the one of the previous proposition since *non-linear* subsidies to the purchase of health services turn out to be substantially equivalent to cross subsidies. However, it is noteworthy that these subsidies must necessarily be non-linear for the implementation of Pareto optima. This is because *linear* consumption subsidies would distort individual consumption choices, thereby preventing the equalization of marginal rates of substitution to relative prices.

workers' consumption by paying state contingent wages may be suboptimal because of risk aversion; and for this reason wages uniform across states turn out to be optimal. In practice, technological reasons often make plausible this behavioral assumption, at least as an approximation: For many jobs, workers can either satisfactorily perform the required set of tasks defining a given job or they cannot do it at all. Hence wages, and in turn labor supply choices, are approximately dichotomic. Moreover, workers dichotomic responses may also be driven by agents' preferences for consumption and leisure which, in general, make labor supply quite unresponsive to small wage's (productivity) changes, and lead workers to supply an approximately constant number of hours when wages and health are above a certain threshold, and to not work otherwise.[41]

We shall say that a labor supply schedule, $l^t(\theta)$, is dichotomic if for any positive wage there exists a health state $\theta^* \in \Theta$ such that $l^t(\theta) = 0$ for all $\theta < \theta^*$ and $l^t(\theta) = L(\theta)$ for all $\theta \geq \theta^*$. It is completely straightforward to verify that in equilibrium this is the case whenever the marginal disutility of labor is sufficiently large in bad health realizations while it is small enough in good states, or when agents' productivity is sufficiently small in bad states and it is large enough in good states. It is convenient to formally state these conditions in the next lemma,

Lemma 12 *Equilibrium labor supply is dichotomic if one of the following properties holds:*

(i) $\psi_l(l, \theta) > K$, *with* K *sufficiently large, for all* $\theta \leq \theta^*$, *and* $\psi_l(l, \theta) < k$, *with* k *sufficiently small, for all* $\theta > \theta^*$;

(ii) $a^t(\theta) < k$ *for all* $\theta \leq \theta^*$, *with* k *small enough for all* t, *and* $a^t(\theta) > K$ *for all* $\theta > \theta^*$, *with* K *large enough, for all* t.

Next proposition shows that policy schemes, based on uniform minimum wages, allow to implement efficient allocations as equilibria either when health affects preferences for consumption goods and workers' disutility of labor or in the case where heath affects productivity, and labor supply is dichotomic.

Proposition 13 *Assume that* $a^t(\theta) = a$ *for all* θ *and* t *or that labor supply is dichotomic, then there exists either a policy scheme imposing sectorial minimum wages or one in which minimum wages are uniformly set, which allow to restore ex ante efficiency. Both schemes provide health insurance subsidies to the workers. Under sectorial minimum wages no subsidies (taxes) on firms are necessary to reach the optimum. Policy schemes implementing uniform minimum wages require production firms adopting unsafer technologies to be taxed or, equivalently, to subsidize health insurance purchases of their workers.*

The reason why firms need not be taxed (subsidized) under sectorial minimum wages is that these wages can be appropriately set for zero profits conditions to hold. Differently, under uniform minimum

[41] There are certainly important exceptions to this type of behavior: labor supply of relatively wealthy educated agents in their old age may be sensitive to their productivity, and effects of health on productivity are more important for jobs requiring high intensity of human capital. However, these exceptions do not seem at all pervasive, at least for low skill workers.

wages firms in the unsafer sector where relative prices are larger must necessarily obtain a positive profit unless they are taxed or forced to contribute to the health insurance of their workers. But, were these firms able to make positive profits, their workers would not receive enough resources for their marginal utilities of expected wealth to be equal to that of the workers occupied in the safer sector. And this explains why taking profits away from these firms is necessary to obtain full efficiency. In this respect, it is also worthwhile to observe that in many countries, employers do actually contribute to fund health insurance programs covering workers. Indeed, firm's based group insurance and employers' contributions to the public system providing health insurance to their workers, are widespread both in Europe and US.

Finally, observe that the assumption of dichotomic labor supply schedule is needed only to obtain fully efficient equilibria with transfers and uniform minimum wages. Indeed, a continuity argument allows to show that Pareto improving policy schemes relying on uniform minimum wages do exist also if the elasticity of labor supply is sufficiently small even if labor supply is not dichotomic.

These kind of policy schemes are common in real life: In many countries, employers do actually contribute to fund health insurance programs covering workers. For instance, firm's based group insurance and employers' contributions to the public system providing health insurance to their workers, are widespread both in Europe and US.

Proposition 13 may serve two purposes. Beyond suggesting how welfare improving policies should be designed to remedy the market failure identified in this paper, it also provides new guidelines to assess the effects of public policy interventions that one often observes in the health insurance sector. By their own nature, indeed, health policies generally implement transfers in favor of agents choosing less safe course of actions. In this respect, the results of this section, together with the characterization developed in Section 6, imply that health care and health insurance public policies are more likely to be effective under the following circumstances:

- Health care services that subsidized workers have access to are sufficiently effective, and/or subsidies are tailored to subsidize insurance against health risks which affect substantially production capabilities;
- Production firms contribute to finance directly or indirectly part of the insurance provision of their workers;
- Subsidies are targeted to workers for which minimum wage regulations are more likely to be binding.

On the other hand, our findings also imply that cross subsidies targeted to agents for which minimum wages are not binding constraints, or financial incentives to purchase insurance against shocks which manly affect agents' consumption capabilities, are welfare reducing. All these remarks apply more directly to labor markets where the statistical distribution of health is to a large extent determined by technological factors and observable agents' characteristics such as age or initial health status, and where workers' private information on the distribution of health is not a fundamental issue.

9 Extensions

In the previous sections we made several simplifying assumptions. We have developed the analysis in a simple representative consumer economy with two occupations and without informational asymmetries. We assumed away prevention activities and we did not explicitly consider the possibility of non-linear prices and markets for rights. In this section we explain how the model can be extended along these directions.

Ex ante Heterogenous Agents, Many Commodities and Asymmetric Information: Introducing ex ante heterogenous agents or allowing for a large finite number of consumption goods does not change any of the results of the previous sections provided that: (i) the number of active sectors in the optimum is larger than the number of the agents' types, and (ii) the information about these types is public.[42] Indeed, all the results of the competitive analysis still hold true provided that condition (i) above holds so that workers with the same ex ante characteristics (types) are occupied in different sectors in equilibrium. Moreover, absent adverse selection issues, the same kind of Pareto improving cross subsides characterized in Section 8 can be implemented on a type-by-type basis. Extending the analysis to second-best environments where workers are privately informed is unsurprisingly more complex. As it is well known, whether agents' private characteristics have only *private* or *common* components, changes the nature of the asymmetric information problem. In the case of private values, which is commonly studied in the optimal taxation literature, agents' private information concerns only variables such as their preferences or their endowment, which do not influence directly the payoffs of their contractual counterparts, which is not the case in a common values environment.

Analyzing, and even defining, competitive equilibria may be problematic under common values as showed by Rothshild and Stiglitz (1976) and several subsequent contributions. Differently, asymmetric information of the private values type can be easily introduced in our set-up. In particular, one can show that the equilibrium analysis does not present major differences with respect to that of this paper. The policy analysis instead becomes more complicated since appropriate incentive constraints must be introduced in the definition of the equilibrium with transfers. Still, as showed in Bennardo and Piccolo (2008), one can find robust examples in which agents are privately informed on their labor endowment or their disutility of labor and simple deterministic policies based on minimum wages regulation and type independent cross job transfers can still allow to Pareto improve upon the competitive allocations. Ex ante efficiency, though, cannot be fully restored. The key point is that, when the policymaker is unable to induce truthful information revelation by offering type contingent schemes, polices that redistribute inside types necessarily involve transfers across types. In this case, transfers across types prevent ex ante efficiency simply because the equalization of the marginal utilities of workers of the same type assigned to different occupations is incompatible with the policymaker's budget balancing condition.

Prevention Activities: Introducing prevention behavior in our set-up is a natural extension which

[42] See the working paper Bennardo and Piccolo, 2007.

requires some carefulness. Prevention is naturally described by an interim investment which allows to obtain, at a positive cost, a first-order stochastic shift of the health distributions associated to a given occupations.

If a pair of health distributions are initially ordered by the FOSD criterion, prevention activities may determine three possible scenarios. In the *first*, prevention technologies are such that the ordering of the two health distributions is preserved after prevention is undertaken. This is the case, for instance, whenever prevention activities are *very* costly, or have a similar impact on the two health distributions. In the *second scenario*, the ordering of the two distributions is reversed after prevention activities are performed. This occurs if prevention is relatively much more effective under the riskier health distribution. Finally, there also exists a *third* possible scenario where, once prevention activities are undertaken, health distributions cannot be anymore ordered by the FOSD criterion. As for the *first* case, introducing prevention leaves unaltered the results derived in the paper. In the second case, our analysis continues to apply entirely but must be appropriately reinterpreted. Indeed , one can easily verify that in this case the ranking of the distributions determining optimal cross transfers and utility differentials is the ex-post one (i.e., the one emerging in equilibrium as a result of prevention activities), and not that holding ex ante. Only in the third case our characterization, which relies on the FOSD criterion, becomes of no use as it applies to an empty domain.[43]

Non-linear Contracts and Markets for Rights: Our analysis has been developed under the assumption of linear contracts for the sake of tractability. It is robust to the introduction of non-linear contracts as well as to that of markets for rights.[44] First, enlarging the contract space would substantially leave unchanged the flavour of our main results. Indeed, in the absence of markets for rights, any fully decentralized non-linear pricing mechanism achieving ex ante efficiency must necessarily involve a random assignment of workers to sectors. This is just because the solution of the first-best program requires workers to be assigned randomly to the two jobs.

Moreover, the introduction of markets for rights would not restore ex ante efficiency in our set-up, unless the policymaker is able to enforce a random allocation of the rights to offer labor or to hire workers in each sector. The reason being that Pareto improving cross-transfers cannot be implemented by assigning to all agents the same rights to obtain jobs and to consume. Indeed, were all agents of the same type treated symmetrically in the property rights's assignment, they would obtain, by a revealed preferences argument, the same expected utility irrespective of the occupation they choose.

Thus, Pareto improvements can possibly be obtained only by assigning rights to apply for some jobs to a subset of workers (or, equivalently, by limiting firms' rights to hire workers). These mechanisms require a public enforcement, and would amount to introduce quotas, or similar kinds of quantity constraints, by far a more invasive intervention than the one we suggest.

[43] All the results mentioned in this section are formally proved in a more extended version of this paper. Their proofs are available on request.

[44] This approach has been developed in Bisin and Gottardi (2006).

10 Concluding Remarks

Universal health insurance system are a hallmark of the welfare policies of many developed countries, and are becoming increasingly important in many developing countries. Even in economies where universal health insurance is not entirely publicly funded, policy interventions providing financial incentives to private health insurance provision benefit a large fraction of the population.[45] The most important effects of these interventions is that they implement cross subsidies across individuals facing different health prospects. These transfers policies may have either redistributive or efficiency purposes. Rothschild Stiglitz (1976) showed that transfers across types facing different distributions of individual states can be welfare improving when agents' types are their own private information. The literature on health risks has investigated how these cross transfers can be implemented through public health insurance systems (see Neudek and Podczeck, 1996, among others). This paper provides a new efficiency rationale for cross-transfers among agents taking different indivisible courses of actions (choosing different occupations) which affect their health prospects. We show that the endogeneity of individual health distributions makes - by itself - cross-jobs transfers necessary for efficiency, independently from adverse selection issues. We prove this result by focusing on the specific "cost-benefit trade-offs" involving agents' occupational choices and their consumption and production capabilities.

In our economy, the relative magnitude of health effects on production and consumption choices determines the sign of Pareto optimal utility differentials across workers, as well as that of optimal cross-jobs transfers. Moreover, competitive equilibria implement cross-jobs transfers, and are for this reason ex ante efficient if lottery contracts are enforceable but not otherwise. The unenforceability of lottery contracts, which we argue is a weak form of contractual incompleteness, can thus in principle justify a public intervention subsidizing health insurance provision. By taking advantage of our efficiency characterization, we finally analyze under what circumstances simple policy schemes can improve upon the market allocation.

From a more theoretical perspective, our results suggest that cross-jobs transfers may result necessary for efficiency, in any setting where consumption and production choices are interdependent because of complementarities between consumption and production activities, or owing to asymmetric information. Our conjecture, based on the analysis of the present paper, is that the inconsistency between ex ante and interim optimality, determining the need for cross-transfers, continues to hold in most of the settings studied in the general equilibrium literature on clubs and in the asymmetric information literature. A result in this spirit is obtained by Bennardo (2005), which characterizes optimal transfers in a moral hazard set-up where health effects are not considered, but occupations influence agents' consumption choices via incentive constraints.

[45] For intance in the U.S. subsidies to health insurance are provided either by federal institutions, such as Medicare, that covers about 13 percent of the population, or by combined state-federal programs, such as Medicaid, that covers the 6 percent of the population.

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11 Appendix

Proof of Proposition 1

The proof of the uniqueness part relies on a standard convexity argument. Before proving the genericity result we show the following lemma which will be useful in the reminder of the section. Throughout we shall denote by α^{1P} the fraction of agents that in the optimum are assigned to sector 1.

Lemma 14 *Whenever $D_c U(x,\theta) > K$ as $x_c \to 0$, with $K > 0$ sufficiently large, ex ante efficiency implies $\alpha^{1P} \in (0,1)$.*

Proof. Observe first that if $e = 0$, from the hypothesis that $D_c U(x,\theta) > K$ as $x_c \to 0$, with $K > 0$ sufficiently large, it is immediate to verify that $\alpha^{1P} \in (0,1)$. We now show that $\alpha^{1P} \in (0,1)$ for any e sufficiently close to 0.

Let $V(e)$ denote the value function of the program

$$\max_x \left\{ \sum_{\theta\in\Theta, t=1,2} \alpha^t p^t(\theta)\, U(x^t(\theta), \theta) : \ \langle x, \alpha\rangle \in F(e), \quad \alpha^1 = \alpha^{1P} \right\}, \tag{18}$$

and $\hat{V}(e)$ that of

$$\max_x \left\{ \sum_{\theta\in\Theta, t=1,2} \alpha^t p^t(\theta)\, U(x^t(\theta), \theta) : \ \langle x, \alpha\rangle \in F(e), \quad \alpha^1 = 0 \right\}, \tag{19}$$

where $F(e)$ denotes the map associating to each endowment vector e the set of feasible allocations.

As a preliminary step we show that $V(e)$ and $\hat{V}(e)$ are continuous in $e = 0$. Take any sequence $\{e^k\}_{k=0}^{+\infty}$ such that $e^k \to 0$ as $k \to +\infty$, for $V(e)$ and $\hat{V}(e)$ to be continuous in $e = 0$ it must be true that $V(e^k) \to V(0)$ and $\hat{V}(e^k) \to \hat{V}(0)$ as $k \to +\infty$. To prove this result denote by $\hat{x}^k$ and x^k the solutions of programs (19) and (18), respectively, when the endowment is e^k. Then, observe that by strict concavity of $U(.)$ and convexity of the set of feasible allocations (19) and (18) have unique solutions at each e^k. As $U(.)$ is continuous and $F(e)$ is linear in e it follows immediately that $V(e^k) \to V(0)$ and $\hat{V}(e^k) \to \hat{V}(0)$ as $k \to +\infty$.

We now turn to prove that $\alpha^{1P} \in (0,1)$ for any e sufficiently close to 0. The argument is by contradiction. Suppose that $\alpha^{1P} = 0$ for $||e|| < \varepsilon$, with ε small enough. As both $V(e)$ and $\hat{V}(e)$ are continuous in $e = 0$, one necessarily has $\hat{V}(e) > V(e)$ for all for $||e|| < \varepsilon$ since $\alpha^1 \neq 0$ at $e = 0$. This contradicts

$\alpha^{1P} = 0$ for $||e|| < \varepsilon$, with ε small enough. By the same argument one shows that $\alpha^{1P} \neq 1$ for all e sufficiently close to zero. □

In order to prove the genericity result, we need to introduce some notation. Let $\mathbf{t}= \left(\left\langle p^t, \Theta\right\rangle, a^t\right)_{t\in T}$, be the *sector* t technology available to each worker. And let $\varepsilon = \langle e, \mathbf{t}, U\rangle$ represent a specific economy defined by an aggregate endowment $e \in \Re^C_{++}$, a vector of production technologies $\mathbf{t}$ and a utility function U. The set of possible economies is then defined as $\mathcal{E} = \Re^C_{++} \times \mathcal{T} \times \mathcal{U}$, where $\mathcal{T}$ is the set of all possible technologies, $\mathcal{U}$ is the set of admissible utility functions. To formally define the utility space $\mathcal{U}$, following the literature[46], assume that, beyond all assumptions stated in Section 2, agents' preferences satisfy the following property: a sequence $U_k(x,\theta)$ in $\mathcal{U}$ converges to $U(x,\theta) \in \mathcal{U}$ if and only if $U_k(x,\theta)$, $DU_k(x,\theta)$ and $D^2U_k(x,\theta)$ uniformly converge to $U(x,\theta)$, $DU(x,\theta)$ and $D^2U(x,\theta)$, respectively, for all θ, on any compact subset of $\Re^C_+ \times [0, L]$.[47]

Let $\xi = (x, \alpha, \eta)$ define the vector of variables in the Pareto program. Note that since we have showed in Lemma 14 that for $||e||$ small enough $\alpha^P \in (0,1)^2$ whenever $\lim_{x_c\to 0} D_cU(x,\theta) > K$ with $K > 0$ sufficiently large for all c, a Pareto optimum must solve:

$$\mathcal{F}(\xi,\varepsilon) = \begin{pmatrix} D_cU(x^t,\theta) - \eta_c \quad \forall\, c \in C \\ -U_{x_L}(x^t,\theta) + \eta_t a^t(\theta) \\ u^1(x^1) - u^2(x^2) - (Z^1 - Z^2) \\ \overline{x} - e - y \end{pmatrix}_{\theta\in\Theta, t\in T} = \mathbf{0}.$$

Now for any arbitrary economy $\varepsilon \in \mathcal{E}$, define the extended system of equations

$$\mathcal{G}(\xi,\varepsilon) \equiv (\mathcal{F}(\xi,\varepsilon),\ (u^1(x^1) - u^2(x^2))) = 0,$$

obtained by adding the interim efficiency condition to $\mathcal{F}(.) = 0$. Finally, let $\mathcal{S} = \{\varepsilon \in \mathcal{E} : \mathcal{G}(\xi,\varepsilon) = 0\}$ be the subset of economies for which a solution $\xi(\varepsilon)$ of $\mathcal{G}(.)$ exists, that is the set of economies whose optima are interim efficient,. We will show that the sets of ex ante and interim Pareto optima are generically disjoint, by proving the equivalent statement that the complement of $\mathcal{S}$ is open and dense. The space, $\mathcal{E}$, of economies is infinite dimensional. However, as *density* is a local property, one may restrict attention to a properly defined subset of $\mathcal{E}$. Specifically, we will consider the linear subspace of $\mathcal{U}$ defined as follows. Given an utility function $\hat{U} \in \mathcal{U}$, we shall consider the perturbed utility function $U(x,\theta) = \hat{U}(x,\theta) + \kappa(\theta) + \beta(\theta)(x - x^P(\theta|\bar{\varepsilon}))$ where, for all θ, $\kappa(\theta)$ is a scalar and $\beta(\theta)$ denotes a $(C+1)$ dimensional vector. Assume $|\kappa(\theta_n)|$, $\|\beta(\theta_n)\|$, $|\kappa(\theta_{n+1}) - \kappa(\theta_n)|$ and $\|\beta(\theta_{n+1}) - \beta(\theta_n)\|$ sufficiently small for all n. This class of utility functions clearly satisfies all the assumptions stated in Section 2 and defines a finite dimensional, linear subspace of $\mathcal{U}$. We shall prove density on $\widehat{\mathcal{E}} = \mathcal{E} \times \mathcal{T} \times \widehat{\mathcal{U}}$.

Let $D_{(\xi,\varepsilon)}\mathcal{F}(\xi^P,\varepsilon)$ and $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P,\varepsilon)$, the matrices associated to the Jacobian of $\mathcal{G}(.)$ and $\mathcal{F}(.)$ evaluated

[46] See Geanakoplos and Polemarchakis (1986) and Villanacci et al. (2002) for a detailed discussion.

[47] In words, we assume that $\mathcal{U}$ is endowed with the subspace topology of the C^2 uniform convergence topology on compact sets.

at (ξ^P, ε), respectively. In proving the density result one could proceed in two steps by proving first that $\mathcal{F}(.)$ is differentiable in a neighborhood of the Pareto optimum. For the sake of brevity we shall skip this step; it will be straightforward in the following that if $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P, \varepsilon)$ has full rank, then also $D_{(\xi,\varepsilon)}\mathcal{F}(\xi^P, \varepsilon)$ has full rank.

(i) *Density:* Define $\widehat{\mathcal{S}} = \left\{\varepsilon \in \widehat{\mathcal{E}} : \mathcal{G}(\xi, \varepsilon) = 0\right\}$ and let (ξ^P, ε) a generic point such that $\mathcal{G}(.) = 0$. We now show that the complement of $\widehat{\mathcal{S}}$ is dense by proving that $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P, \varepsilon)$ has full row rank, i.e., $\mathcal{G}(.)$ is transversal to zero.

As a preliminary step we prove that the rank of $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P, \varepsilon)$ is equal to the rank of the following matrix:

$$\mathbf{A} = \begin{pmatrix} \text{equat.\textbackslash variab.} & x & a^1(\theta_n) & e & \kappa(\theta_{n'}) & \beta_{x_L}(\theta_n) \\ \text{FOCs}(x) & \mathbf{H} & \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{D} \\ \text{FOCs}(\alpha) & \mathbf{0} & p^1(\theta_n)l^1(\theta_n) & \mathbf{0} & * & \mathbf{0} \\ \text{FEAs} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ u^1(x^1) - u^2(x^2) = 0 & * & \mathbf{0} & \mathbf{0} & p^1(\theta_{n'}) - p^2(\theta_{n'}) & \mathbf{0} \end{pmatrix}.$$

Where $\mathbf{A}$ is the square matrix obtained by differentiating the extended system $\mathcal{G}(\xi, \varepsilon)$ with respect to $(x, a, e, \kappa(\theta_1), \beta_{x_L}(\theta_n))$.[48] $\mathbf{H}$ denotes the agents' Hessian submatrix; $\mathbf{C} \in \Re^{3\times 2\times N}$ (where 3 is the number of commodities, 2 the number of sectors and N the number of individual states) has all entries equal to zero except for the one corresponding to the first-order condition (FOC) with respect to $x^1_L(\theta_n)$ which is equal to η_1, and where $\mathbf{D} \in \Re^{3\times 2\times N}$ has all null entries except for the element corresponding to FOC with respect to $x^t_L(\theta_n)$, which is equal to 1. Finally, the symbol "$*$" denotes generic submatrices whose rank does not influence that of $\mathbf{A}$.

Simple elementary operations allow to obtain $\mathbf{A}$ from $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P, \varepsilon)$. First, one obtains the null matrices appearing in the rows corresponding to FEAs by summing the columns corresponding to e (multiplied by appropriate scalars) to the ones corresponding to $a^1(\theta_n)$ and $\kappa(\theta_{n'})$, respectively. Second, by using the first-order conditions with respect to x of the Pareto program one shows that the elements corresponding to FOCs(α) and x are zero.

To prove that $\mathbf{A}$ is nonsingular one sums the column corresponding to $\beta_{x_L}(\theta_n)$ multiplied by $-\eta_1$ to that corresponding to $a^1(\theta_n)$, and then sums the resulting vector (multiplied by an appropriate scalar)

[48] Note that in the matrix $\mathbf{A}$ the equations associated to FOCs(x) indicate the first-order conditions of the Pareto program with respect to x, those associated to FOCs(α) indicate the first-order conditions with respect to α and the equations corresponding to FEAs are the feasibility conditions.

to the column corresponding to $k(\theta_{n'})$, thus obtaining :

$$\mathbf{A}' = \begin{pmatrix} \text{equat.\textbackslash variab.} & x & a^1(\theta_n) & e & k(\theta_{n'}) & \beta_{x_L}(\theta_n) \\ \text{FOCs}(x) & \mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D} \\ \text{FOCs}(\alpha) & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \text{FEAs} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ u^1(x^1) - u^2(x^2) = 0 & * & * & \mathbf{0} & p^1(\theta_{n'}) - p^2(\theta_{n'}) & \mathbf{0} \end{pmatrix},$$

where $p^1(\theta) \neq p^2(\theta)$ for at least a health state θ. As the Hessian $\mathbf{H}$ has full rank because preferences are strictly convex, it follows that $\mathbf{A}'$ has full rank, and so does $\mathbf{A}$. Thus $\mathcal{G}(.)$ is transversal to zero and $\widehat{\mathcal{S}}$ is dense. □

(ii) *Openness:* Let $\mathcal{P} = \{(\xi, \varepsilon) : \mathcal{F}(\xi, \varepsilon) = 0\}$, and consider the natural projection $\pi : \mathcal{P} \to \mathcal{E}$, $\pi(\xi, \varepsilon) = \varepsilon$. As proper mappings take closed sets into closed sets, $\mathcal{S}$ is open if the natural projection is *proper*. Hence we need to prove that for any sequence $(\xi_k, \varepsilon_k)_{k=1}^{+\infty}$ such that $\mathcal{F}(\xi_k, \varepsilon_k) = 0$ for all k, and $\varepsilon_k \to \varepsilon$ as $k \to +\infty$, there exists a converging subsequence of $(\xi_k)_{k=1}^{+\infty}$ with limit ξ such that $\mathcal{F}(\xi, \varepsilon) = 0$. To this end, note first that $\{\alpha_k\}_{k=1}^{+\infty}$ must converge, say to α, as it belongs to the compact set $[0,1]^2$. Moreover, $D_c U(x, \theta) > K$ for all $c \in \hat{C}(\theta)$ as $x_c \to 0$,with K large, imply $\{x_k\}_{k=1}^{+\infty} \gg 0$; while since $D_c U(x, \theta) < k$, with k small, as $x_c \to +\infty$, there exists a positive vector G such that $x_k < G$, hence $\{x_k\}_{k=1}^{+\infty}$ must converge, say to x. Given the assumptions on U, $U(x, \theta) \to U(x, \theta)$ implies $DU_k(x, \theta) \to DU(x, \theta)$ uniformly on compact sets for all (x, θ), then this must also hold at $x = x_k$. Finally, from (15)-(17) one gets $\eta_k \to \eta$. □

Proof of Proposition 2

For the sake of brevity we provide the proof only for the case where $U_\theta > 0$ at least in some interval $d\theta$. The proof for the case where $U_\theta = 0$ for all θ and $\partial L(\theta)/\partial\theta > 0$ in some interval $d\theta$ follows exactly the same logic; while the result for the case where $U_\theta = 0$, $\partial L(\theta)/\partial\theta = 0$ and $\partial a^t(\theta)/\partial\theta > 0$ can be obtained through simple algebraic manipulations of first-order conditions of the Pareto program.

Assume without loss of generality that $\langle p^1, \Theta \rangle$ FOSD $\langle p^2, \Theta \rangle$ and that $\alpha^1 > 0$, $\alpha^2 > 0$ and let $\langle x^P, \alpha^P \rangle$ a generic Pareto optimal allocation. Let $\mathbf{\Pi}$ be an $N \times N$ matrix, and denote $\tilde{\pi}(x^{2P}(\theta_n), \theta_m)$ be the element in the n-th row and the m-th column of $\mathbf{\Pi}$ satisfying the following conditions: (i) $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = 0$ for all pairs (n, m) such that $n > m$; (ii) $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_m)$ for all pairs (n, m) with $n = m$, and (iii)

$$\tilde{\pi}(x^{2P}(\theta_n), \theta_m) = \min \left\{ p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l), \theta_m),\ p^2(\theta_n) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_n), \theta_k) \right\}$$

for all (n, m) with $n < m$. As a preliminary step, we show that $\mathbf{\Pi}$ is a stochastic matrix satisfying the following properties:

(I) $\tilde{\pi}(x^{2P}(\theta_n), \theta_m) \geq 0$ and $\sum_{n=1}^{N} \tilde{\pi}(x^{2P}(\theta_n), \theta_m) = p^1(\theta_m)$;

(**II**) $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) \leq p^2(\theta_n)$ for $n = m$;
(**III**) $\sum_{m=1}^{N} \tilde{\pi}(x^{2P}(\theta_n),\theta_m) = p^2(\theta_n)$;
(**IV**) $u^2(x^{2P}) < \sum_{m=1}^{N}\sum_{n=1}^{N} \tilde{\pi}(x^{2P}(\theta_n),\theta_m)U(x^{2P}(\theta_n),\theta_m)$.

Part (I) By construction $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) \geq 0$, for all (n,m). Then (i) implies

$$\sum_{l=1}^{N} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) = \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) + \tilde{\pi}(x^{2P}(\theta_n),\theta_m),$$

and by using (ii) one obtains

$$\sum_{l=1}^{N} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) = \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) + p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) = p^1(\theta_m).$$

Part (II) We can restrict attention to the case of $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) > 0$ for $n = m$. By construction, in this case $\tilde{\pi}(x^{2P}(\theta_l),\theta_m) = p^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_k)$ for all $l < n$. Indeed, were $\tilde{\pi}(x^{2P}(\theta_{\hat{l}}),\theta_m) = p^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_m)$ for some $\hat{l} < n$, it would follow from part (I) that $\tilde{\pi}(x^{2P}(\theta_{\hat{l}+t}),\theta_m) = 0$ for all $t > 0$, contradicting $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) > 0$ for $n = m$. Therefore, for all (n,m) such that $n = m$ one must have $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) = p^1(\theta_m) - \sum_{l=1}^{n-1}\left(p^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_k)\right)$. As for $n = m$, $\sum_{k=1}^{m-1}\sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_k) = \sum_{k=1}^{m-1} p^1(\theta_k)$ by part **(I)**, we obtain $\tilde{\pi}(x^{2P}(\theta_{\hat{l}}),\theta_m) = p^1(\theta_m) - \sum_{l=1}^{n-1} p^2(\theta_l) + \sum_{k=1}^{m-1} p^1(\theta_k)$, which implies $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) < p^2(\theta_n)$ by FOSD.

Part (III) The proof is by induction. We first prove that the equality holds for $n = 1$. Since $\sum_{l=1}^{n-1} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) = 0$ for $n = 1$, by (iii) this amounts to show that $p^1(\theta_m) > p^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_1),\theta_k)$ for some $m < N$. If this were not true, one should have $p^1(\theta_m) \leq p^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_1),\theta_k)$ for all $m \leq N$. However, for $m = N$ this is impossible; indeed by (iii) $\sum_{k=1}^{N-1} \tilde{\pi}(x^{2P}(\theta_1),\theta_k) = \sum_{k=1}^{N-1} p^1(\theta_k)$ whenever $p^1(\theta_m) \leq p^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x^{2P}(\theta_1),\theta_k)$ for all $m \leq N$. Suppose now that $\sum_{k=1}^{N} \tilde{\pi}(x^{2P}(\theta_n),\theta_k) = p^2(\theta_n)$ for $n = 1, 2, ...\bar{n}$, but $\sum_{k=1}^{N} \tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_k) < p_i^2(\theta_{\bar{n}+1})$. In this case, $\sum_{k=1}^{m} \tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_k) < p^2(\theta_{\bar{n}+1})$ for all $m \leq N$ so that by (iii) $\tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_m) = p^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x^{2P}(\theta_l),\theta_m)\ \forall\ m > n$. By summing over m it follows:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_m) = \sum_{m=\bar{n}+1}^{N}\left(p^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x^{2P}(\theta_l),\theta_m)\right),$$

which, in turn, implies:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_m) = 1 - \sum_{m=1}^{\bar{n}} p^1(\theta_m) - \sum_{m=1}^{N}\sum_{l=1}^{\bar{n}} \tilde{\pi}(x^{2P}(\theta_l),\theta_m) + \sum_{m=1}^{\bar{n}}\sum_{l=1}^{\bar{n}} \tilde{\pi}(x^{2P}(\theta_l),\theta_m).$$

As we are assuming for $n = 1, 2, ...\bar{n}$, $\sum_{m=1}^{N} \tilde{\pi}(x^{2P}(\theta_n),\theta_m) = p^2(\theta_n)$, the right-hand-side of this expression is equal to $1 - \sum_{m=1}^{\bar{n}} p^1(\theta_m) - \sum_{n=1}^{\bar{n}} p^2(\theta_n) + \sum_{m=1}^{\bar{n}} p^1(\theta_m) = 1 - \sum_{n=1}^{n} p^2(\theta_n)$. This proves the claim, since $\sum_{m=\bar{n}+1}^{N}\tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_m) = 1- \sum_{n=1}^{n} p^2(\theta_n) > p^2(\theta_{\bar{n}+1})$ contradicts $\sum_{m=\bar{n}+1}^{N}\tilde{\pi}(x^{2P}(\theta_{\bar{n}+1}),\theta_m) \leq p^2(\theta_n)$.

Part (IV) As $\tilde{\pi}(x^{2P}(\theta_n),\theta_m) = 0$ for all (n,m) with $n > m$ and $U_\theta > 0$, for all n, we have:

$$\sum_{m=1}^{N}\tilde{\pi}(x^{2P}(\theta_n),\theta_m)U(x^{2P}(\theta_n),\theta_m) > \sum_{m=1}^{N}\tilde{\pi}(x^{2P}(\theta_n),\theta_m)U(x^{2P}(\theta_n),\theta_n) = p^2(\theta_n)U(x^{2P}(\theta_n),\theta_n)$$

by summing up over n one obtains **(IV)**.

We can now prove that $\eta_1^P < \eta_2^P$. The proof is again by contradiction. Assume first that $\eta_1^P > \eta_2^P$ and that $0 < \alpha^{2P} \leq 1$ (i.e., some workers are assigned to sector 2 in the optimum). Consider an allocation having the following features. A measure $\alpha^1 = \alpha^{1P} + d\alpha$ of workers is assigned to *sector 1* while a measure $\alpha^2 = \alpha^{2P} - d\alpha$ is assigned to *sector 2*, with $d\alpha$ sufficiently small. All workers in *sector* 2 and a measure $\alpha^{1P} - d\alpha$ of workers in *sector 1* obtain x^{2P} and x^{1P}, respectively; a set of measure $d\alpha$ of workers in sector 1 obtain $\tilde{x}^1 = (..., \tilde{x}^1(\theta_m), ...)$, with $\tilde{x}^1(\theta_m) = \sum_{n=}^{N}\tilde{\pi}(x^{2P}(\theta_n),\theta_m)x^{2P}(\theta_n)$, while another set of measure $d\alpha$ workers in sector 1 obtain the allocation $x^{1P} + \varepsilon = (..., x^{1P}(\theta_n) + \varepsilon(\theta_n), ...)$, where, for all n, $\varepsilon(\theta_n)$ is such that $\varepsilon_1(\theta_n) = \varepsilon$, $\varepsilon_2(\theta_n) = -\varepsilon$, with $\varepsilon > 0$ and sufficiently small. By construction, this allocation is feasible; moreover the result proved in **Part (IV)** and strict convexity of preferences imply $\tilde{x}^1 \succ x^{2P}$, while $\eta_1^P > \eta_2^P$ implies $x^{1P} + \varepsilon \succ x^{1P}$. This contradicts the optimality of $\langle x^P, \alpha^P \rangle$; thus, $\eta_1^P \leq \eta_2^P$. Finally, a standard continuity argument implies $\eta_1^P \neq \eta_2^P$. $\square$

Proof of Proposition 3

As a preliminarily step we state, without proving it, the following well known lemma, that we shall use several times subsequently. Let $P^t(\theta_n) = \sum_{\theta \leq \theta_n} p^t(\theta_n)$ for $n \in N$ and $t = 1, 2$,

Lemma 15 *For any map $g : \Theta \to \Re^+$, $\theta \to g(\theta)$, with $dg(\theta_{n+1}) = g(\theta_{n+1}) - g(\theta_n)$, the following identity holds:*

$$\sum_{\theta \in \Theta}\left(p^1(\theta) - p^2(\theta)\right)g(\theta) := \sum_{n \in N}\left(P^2(\theta_n) - P^1(\theta_n)\right)dg(\theta_{n+1}).$$

We can now prove the statement of the proposition. The first-order conditions with respect to x of the Pareto program, together with strict concavity of $U(x,\theta)$ in x imply $x^{1P}(\theta) = x^{2P}(\theta) = x^P(\theta)$ for all θ. Let $x^P : \Theta \to \Re^2_+$, $\theta \to x^P(\theta)$, be the map associating to each $\theta \in \Theta$ the optimal consumption vector $x^P(\theta)$. Assume $\theta_{n+1} - \theta_n = d\theta$ for all n, with $d\theta$ sufficiently small, and let $dU\left(x^P(\theta_{n+1}),\theta_{n+1}\right) = U\left(x^P(\theta_{n+1}),\theta_{n+1}\right) - U\left(x^P(\theta_n),\theta_n\right)$, one then obtains:

$$dU\left(x^P(\theta_{n+1}),\theta_{n+1}\right) \approx \sum_{c=1,2} dx_c^P(\theta_{n+1})\eta_c^P + U_\theta\left(x^P(\theta_{n+1}),\theta_{n+1}\right)d\theta. \tag{20}$$

By Lemma (15), $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $dU\left(x^P(\theta),\theta\right) \gtreqless 0$; hence (20) implies $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $\sum_{c=1,2} dx^P(\theta_{n+1})\eta_c^P + U_\theta\left(x^P(\theta_{n+1}),\theta_{n+1}\right)d\theta \gtreqless 0$. For $d\theta$ small, the first-order conditions of the Pareto program imply $dx_1^P(\theta_{n+1}) \approx (U_{1\theta}|U_{22}|+U_{2\theta}U_{21})/\Lambda)d\theta$ and $dx_2^P(\theta_{n+1}) \approx (U_{2\theta}|U_{11}|+U_{1\theta}U_{12})/\Lambda)d\theta$, where $\Lambda = U_{11}U_{22} - (U_{12})^2$ is strictly positive because of the strict concavity of $U(x,\theta)$ in x. Summing up, we obtain:

$$\sum_{c=1,2} dx_c^P(\theta_{n+1})\eta_c^P \approx U_1 \frac{U_{1\theta}|U_{22}| + U_{2\theta}U_{12}}{\Lambda} d\theta + U_2 \frac{U_{2\theta}|U_{11}| + U_{1\theta}U_{12}}{\Lambda} d\theta, \tag{21}$$

(20) and (21) then imply that $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}(U_1|U_{22}| + U_2U_{12}) + \frac{U_{2\theta}}{\Lambda}(U_2|U_{11}| + U_1U_{12}) + U_\theta \gtreqless 0, \tag{22}$$

equation (22) together with supermodularity in x (i.e., $U_{12} \geq 0$), increasing differences in (x,θ) (i.e., $U_{c\theta} \geq 0$ for $c = 1,2$) and $U_\theta > 0$, imply the result. □

Proof of Proposition 4

As showed in Proposition 3, $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}(U_1|U_{22}| + U_2U_{12}) + \frac{U_{2\theta}}{\Lambda}(U_2|U_{11}| + U_1U_{12}) + U_\theta \gtreqless 0.$$

Since U has increasing differences in (x,θ) (i.e., $U_{c\theta} \leq 0$ for $c = 1,2$), the first two addenda in the above expressions are negative, while the third is positive. Then, it is easy to verify that if $|U_{c\theta}|/U_\theta$ is large enough for at least one c one must have $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) < 0$ and, therefore, $\Delta u^P < 0$. This proves part (i). Similarly, if $|U_{c\theta}|/U_\theta$ small enough for each $c = 1,2$ $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) > 0$, hence $\Delta u^P > 0$. This proves part (ii). □

Proof of Proposition 7

Let $\Delta u^P = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^{2P}(\theta),\theta) - \sum_{\theta\in\Theta} p^1(\theta)\psi(l^{1P}(\theta),\theta)$ and define $\sigma_\psi(l,\theta) \equiv \psi_{ll}(l,\theta)/\psi_l(l,\theta)$. Rearranging by parts we obtain,

$$\Delta u^P = -\sum_{\theta\in\Theta}(p^1(\theta) - p^2(\theta))\psi(l^{1P}(\theta),\theta) + \sum_{\theta\in\Theta} p^2(\theta)(\psi(l^{2P}(\theta),\theta) - \psi(l^{1P}(\theta),\theta)).$$

Now, let η_t^P the value of the Lagrange multiplier, calculated in the optimum, and denote $l_t(\theta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = \eta_t^P$ for $t = 1,2$; and $l(\eta,\theta)$ that defined by $\psi_l(l(\theta),\theta) = \eta$. Finally, let $\Delta P(\theta_n) = P^1(\theta_n) - P^2(\theta_n)$, we have:

$$\Delta u^P \approx \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(l_2(\theta),\theta) \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta)a(\theta) \int_{\eta_1^P}^{\eta_2^P} \frac{1}{\sigma_\psi(l(\eta,\theta),\theta)} d\eta.$$

Showing that $\Delta u^P > 0$ for $|\psi_\theta|$ sufficiently large relative to $|\psi_{l\theta}|$ is straightforward since $\Delta P(\theta_n) \leq 0$ for all θ_n and $\eta_2^P > \eta_1^P$ from Proposition (2). In the following, we prove that $\Delta u^P < 0$ for $|\psi_\theta|$ sufficiently small relative to $|\psi_{l\theta}|$. Let $T(\eta,\theta) := \sigma_{\psi}^{-1}(l(\eta,\theta),\theta) - l(\eta,\theta)$, then $\psi_l(0,\theta) = 0$ for all θ together with $\psi_{ll}(l,\theta) > 0$ imply $T(0,\theta) = 0$ for all θ. As it can be easily checked that $\psi_{lll}(l,\theta) > 0$ implies $\partial T(\eta,\theta)/\partial\eta \leq 0$, it follows that $l(\eta,\theta) \geq \sigma_{\psi}^{-1}(l(\eta,\theta),\theta)$ for all (η,θ).

Given the definition of Δu^P above, the inequality $l(\eta,\theta) \geq \sigma_{\psi}^{-1}(l(\eta,\theta),\theta)$ implies:

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_{\psi}(l_2(\theta),\theta)} + \psi_{\theta}(l_2(\theta),\theta) \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta) d\eta \right). \tag{23}$$

We shall now use the first-order condition with respect to α of the Pareto program, and rewrite the second addendum in the right hand side of (23) in a way which makes it comparable with the first addendum. To this end, let $h(l,\theta) := \psi_l(l,\theta)l$, by adding and subtracting $\sum_{\theta\in\Theta} p^1(\theta)\psi(l^{2P}(\theta),\theta)$ to the left-hand-side and $\sum_{\theta\in\Theta} p^1(\theta)h(l^{2P}(\theta),\theta)$ to the right-hand-side of (17), and using Lemma 15 one gets:

$$\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{d\psi(l_2(\theta),\theta)}{d\theta} \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} \left(\psi_l(l(\theta,\eta),\theta) \frac{\partial l(\theta,\eta)}{\partial\eta} \right) d\eta =$$

$$\sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} \left(h_l(l(\theta,\eta),\theta) \frac{\partial l(\theta,\eta)}{\partial\eta} \right) d\eta + \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{dh(l_2(\theta)\,\theta)}{d\theta} \right) d\theta.$$

By straightforward algebraic manipulations allow we then obtain :

$$\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \psi_{\theta}(l_2(\theta),\theta) d\theta = \sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta) d\eta, \tag{24}$$

Equations (23) and (24) then imply:

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_{\psi}(l_2(\theta),\theta)} + 2\psi_{\theta}(l_2(\theta),\theta) \right) d\theta.$$

Finally, since by FOSD $\Delta P(\theta_n) \leq 0$ with at least one strict inequality, it follows that $\Delta u^P \leq 0$ if $|\psi_{l\theta}| \geq 2\sigma_{\psi}|\psi_{\theta}|$ for all (l,θ). □

Proof of Proposition 8

To begin with, we show the following preliminary lemma which will be useful in proving the proposition. Let $h(l) = \psi'(l)l$, $\sigma_h(l) = h''(l)/h'(l)$, $\Delta h = \sum_{\theta\in\Theta} p^2(\theta)h\left(l^2(\theta)\right) - h(l^1(\theta_N))$, and $\Delta\sigma = (\sigma_{\psi} - \sigma_h)$.

Lemma 16 *(i)* $\partial\zeta_{l,w}/\partial w_{\theta} \gtreqless 0$ *for all* (l,θ) *if and only if* $\sigma_{\psi}(l) \gtreqless \sigma_h(l)$ *for all* l*; (ii)* $\Delta u^P = 0$ *implies* $sign\Delta h = -sign\Delta\sigma$*.*

Proof The proof of part (i) follows from straightforward manipulations of the FOCs of the Pareto program, and is omitted. In order to prove part (ii) denote $l^2(h) = \sum_{\theta\in\Theta} p^2(\theta)h\left(l^2(\theta)\right)$ the certainty equivalent under h of the distribution $\left\langle p^2, (l^2(\theta))_{\theta\in\Theta}\right\rangle$. Since $\hat{x}^{1P} = \hat{x}^{2P}$, $\Delta u^P = 0$ implies

$\psi\left(l^1(\theta_N)\right) = \sum_{\theta\in\Theta} p^2(\theta)\psi\left(l^2(\theta)\right)$; therefore, $l^1(\theta_N)$ is the certainty equivalent under ψ of the distribution $\left\langle p^2, (l^2(\theta))_{\theta\in\Theta}\right\rangle$. Since $h(l)$ is an increasing function, and $l^1(\theta_N) \gtreqless l^2(h)$ whenever $\Delta\sigma \gtreqless 0$, it follows that $\Delta h \lesseqgtr 0$ for $\Delta\sigma \gtreqless 0$. □

We shall now prove the claim of the proposition beginning with the case $\sigma_\psi < \sigma_h$. To this end, we introduce an auxiliary program which maximizes $\sum_{t=1,2} \alpha^t u^t(x^t)$ under the feasibility constraints and the additional constraint:

$$\Delta u = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) \leq 0. \tag{25}$$

The FOCs with respect to $l^t(\theta)$, $t = 1, 2$, and α of this program are:

$$\psi'(l^1(\theta_N)) = \hat{\eta}_1 a(\theta_N) + \frac{\varkappa}{\alpha}\psi'(l^1(\theta_N)), \tag{26}$$

$$\psi'(l^2(\theta)) = \hat{\eta}_2 a(\theta) - \frac{\varkappa}{1-\alpha}\psi'(l^2(\theta)), \quad \forall\, \theta \in \Theta \tag{27}$$

$$\sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) = \hat{\eta}_2 \sum_{\theta\in\Theta} p^2(\theta)a(\theta)l^2(\theta) - \hat{\eta}_1 a(\theta_N)l^1(\theta_N), \tag{28}$$

where $\hat{\eta}_t$ for $t = 1, 2$ are the Lagrangian multipliers associated to the feasibility constraints of the auxiliary program, and $\varkappa$ is the multiplier associated with (25). Substituting (26) and (27) into (28) one gets:

$$\Delta u = \Delta h + \varkappa\left(\frac{\sum_{\theta\in\Theta} p^2(\theta)h(l^2(\theta))}{1-\alpha} + \frac{h(l^1(\theta_N))}{\alpha}\right). \tag{29}$$

We can now verify that $\varkappa = 0$ and that (25) holds as inequality whenever $\Delta\sigma < 0$. This immediately implies $\Delta u^P < 0$ for $\Delta\sigma < 0$.

First we must have $\varkappa = 0$; indeed $\varkappa > 0$ and $\Delta u = 0$ would imply (29) $\Delta h < 0$; but this is impossible, as we showed above that $\Delta h > 0$ whenever $\Delta u = 0$ and $\Delta\sigma < 0$. Moreover, (25) must hold as inequality. Otherwise, one would have $\Delta u = 0$, and hence $\Delta h > 0$ whenever $\Delta\sigma < 0$, which contradicts (29).

Showing that $\Delta\sigma > 0$ implies $\Delta u^P > 0$, and that $\Delta\sigma = 0$ for $\Delta u^P = 0$, requires exactly the same type of argument developed above and is left to the reader. □

Proof of Proposition 9

We begin by showing the existence result in the case where lottery contracts are unenforceable. Consider the *auxiliary* program which maximizes $\sum_{t\in T} u^t(x^t)\varphi^t$ within the compact set defined by the agents' budget constraints (4)-(5) and the additional constraints $x^t(\theta) \in \bar{X} \subset \Re^C \times [0, L]$, $z^t(\theta) \in \bar{Z}$ with $\bar{X}$ and $\bar{Z}$ finite but sufficiently large. A standard risk aversion argument implies that the set of solutions of the workers' maximization program defined in Section 5.2 coincides with that of the auxiliary program for

$\bar{X}$ sufficiently large. As both production and intermediation technologies are linear, equilibrium prices must satisfy: $\phi^t(\theta) = g^t p^t(\theta)$ for some $g^t \in \Re_+$, and $w^t(\theta) = q_t a^t(\theta)$ for $t \in T$ and $\theta \in \Theta$. At these prices assets' supply and labor demands are indeterminate. By using the above equilibrium conditions, and normalizing prices appropriately, the budget correspondence can be rewritten as:

$$B^t(q) = \left\{ (x^t, \varphi^t) : \sum_{\theta \in \Theta, c \in C} p^t(\theta) \, q_c(x_c^t(\theta) - e_c) - q_t \sum_{\theta \in \Theta} p^t(\theta) \, a^t(\theta)(L - x_L^t(\theta)) \leq 0, \ \varphi^t \in \Delta \right\}.$$

Therefore, $B^t(q)$ is continuous for all $q \gg 0$. Let $\zeta^t(q)$ and $\varphi^t(q)$ be respectively the individual demand correspondences for commodities and occupations. The continuity of $B^t(q)$ implies that both $\zeta^t(q)$ and $\varphi^t(q)$ are upper- hemicontinuous for all $q \gg 0$. Though, only $\varphi^t(q)$, but not $\zeta^t(q)$, is convex valued. Since $\zeta^t(q)$ upper- hemicontinuous however, the per capita demand correspondence $\xi^t(q) = \sum_{t \in T} \varphi^t(q) \zeta^t(q)$ is either upper-hemicontinuous or convex valued. Hence, a standard application of the Kakutani Fixed Point Theorem in the commodity space $\Re^L$ implies the existence result.

The existence proof for the case of enforceable lottery contracts is completely analogous. It only requires more carefulness to prove that the feasible set defined by the constraint of program (10)-(12) can be bounded without loss of generality. To show this, suppose by the contrary that the auxiliary program defined by (10)-(12) and the additional constraints $x^t(\theta) \in \bar{X} \subset \Re^C \times [0, L]$, $z^t(\theta) \in \bar{Z}$, $g \in \bar{G}$, with $\bar{X}$, $\bar{Z}$, and $\bar{G}$ sufficiently large, have a boundary solution such that, for some pair (θ', t'), $\hat{x}^{t'}(\theta)$ belongs to the boundary of $\bar{X}$. Since γ belongs to the *2-dimensional* simplex, for (10) to be satisfied, there must necessarily exist at least one t such that $\hat{x}^t(\theta)$ belongs to the interior of $\bar{X}$, for all θ. However, the multiplier associated to the initial period budget constraints must necessarily be positive and equal to $D_{x_1} U(\hat{x}^t(\theta), \theta)$ after prices' normalization. Hence, for any $q \gg 0$, from the first-order optimality conditions one obtains $D_{x_c} U(\hat{x}^{t'}(\theta'), \theta') \geq k D_{x_1} U(\hat{x}^t(\theta), \theta)$ for some finite number k. For this inequality to be satisfied, $\hat{x}^t(\theta') = (\hat{x}_1^{t'}(\theta'), \hat{x}_2^{t'}(\theta'))$ must be contained in the interior of $\bar{X}$, for $\bar{X}$ sufficiently large, since all derivatives of $U(x, \theta)$ are assumed to be finite. This proves that the auxiliary, bounded program and program (10)-(12) have the same set of solutions (for all positive price vectors) for $\bar{X}$ sufficiently large. The rest of the proof follows exactly the same lines as the one for the deterministic case.

Proof of Proposition 10

The proof of the First Welfare Theorem for the case of enforceable lottery contracts is standard, thus is omitted. As for the case of unenforceable lottery contracts, competitive equilibria satisfy the fair treatment condition, so that $u^1(x^1) = u^2(x^2)$ if $(\varphi^1, \varphi^2) >> 0$. Indeed, if $u^t(x^t) > u^{t'}(x^{t'})$, $\varphi^{t'} > 0$ would not be optimal.

Now, consider a competitive equilibrium allocation (x^*, φ^*) such that $\varphi^* >> 0$. Suppose it is not interim efficient, there must exist a feasible allocation $(\widehat{x}, \widehat{\varphi}) \neq (x^*, \varphi^*)$ such that $u^1(\widehat{x}^1) = u^2(\widehat{x}^2)$ whenever

$(\widehat{\varphi}^1, \widehat{\varphi}^2) >> 0$, and $(\widehat{x}, \widehat{\varphi}) \succ (x^*, \varphi^*)$. Then it must be:

$$\sum_{t\in T} \widehat{\varphi}^t \sum_{\theta\in\Theta} p^t(\theta) \sum_{c\in C} q_c^*(\widehat{x}_c^t(\theta) - e_c) > \sum_{t\in T} \widehat{\varphi}^t \sum_{\theta\in\Theta} p^t(\theta) q_t^* a^t(\theta)(L - \widehat{x}_L^t(\theta)).$$

where q_c^* is the equilibrium price of good c.

This inequality can be rewritten as:

$$\sum_{t\in T} \widehat{\varphi}^t \left(\sum_{\theta\in\Theta} p^t(\theta) \sum_{c\in C} q_c^*(\widehat{x}_c^t(\theta) - e_c) - \sum_{\theta\in\Theta} p^t(\theta) q_t^* a^t(\theta)(L - \widehat{x}_L^t(\theta)) \right) > 0$$

which immediately implies that the allocation $(\widehat{x}, \widehat{\varphi})$ violates feasibility.

Finally, as for part (iii), it is immediate to verify that $\phi^t(\theta) = g^t p^t(\theta)$ for some $g^t \in \Re_+$ and $w^t(\theta) = q_t a^t(\theta)$ follow from the linearity of the intermediaries and production firms maximization programs; while $w^1(\theta) < w^2(\theta)$ is directly implied by the first-order conditions of the agents' optimization programs. □

Proof of Proposition 11

Let $\langle \alpha^P, x^P \rangle$ be the Pareto optimal allocation. We now show that there exists an equilibrium implemented by the transfers' policy $\tilde{\wp}$ such that $w^t = \hat{w}^t = \eta_t^P a^t$, $f^t = 0$ and:

$$s^t(\theta) = \sum_{c\in C} (\eta_c^P (x_c^{tP}(\theta) - e_c) - \eta_t^P a^t(\theta) l^{tP}(\theta),$$

where $\varphi^t = \alpha^{tP}$, $x = x^P$, $q_2/q_1 = \eta_2^P/\eta_1^P$, $\phi^t = p^t$, for $t \in T$.

First, $\tilde{\wp}$ is budget balancing by construction. Moreover, it is immediate to verify that $\langle \alpha^P, x^P \rangle$ satisfies as equality all the budget constraints at the prices, wages and subsidies vectors defined above. Hence $\langle \alpha^P, x^P \rangle$ must solve the agents' maximization program. Finally, all the market clearing conditions are satisfied at $\phi^t = p^t$ and $w^t = \hat{w}^t = \eta_t^P a^t$ for all $t \in T$. Indeed, at these prices the supply of all state contingent assets, as well as labor demand, are indeterminate. □

Proof of Proposition 13

Let $\langle \alpha^P, x^P \rangle$ be the Pareto optimal allocation. We show that whenever health only affects preferences for consumption goods and workers' disutility of labor, or labor supply is dichotomic, there exists an equilibrium with transfer decentralizing the ex ante efficient allocation. To begin with assume $a^t(\theta) = a$ for all θ and t.

(i) For the case of *sector contingent minimum wages*, consider the policy scheme $\tilde{\wp}$ such that: $\hat{w}^t(\theta) = \eta_t^P a$ for all t and θ; in each health state θ workers employed in sector t receive an health insurance subsidy equal to

$$Z^t(\theta) = \sum_{c\in C} \eta_c^P (x_c^{tP}(\theta) - e_c) - \eta_t^P a l^{tP}(\theta);$$

production firms are neither taxed nor subsidized, that is $f^t(\theta) = 0$ for all θ and t.

Showing that this policy decentralizes the first-best allocation follows exactly the same lines of the proof of Proposition 11.

(ii) For the case of *uniform minimum wages*, consider a policy scheme $\tilde{\wp}$ defined by the following properties. The uniform minimum wage is $\hat{w} = \eta_1^P a$ for all t and θ; in each state θ, workers employed in sector t receive an health insurance subsidy equal to

$$Z^t(\theta) = \sum_{c \in C} \eta_c^P (x_c^{tP}(\theta) - e_c) - \eta_1^P a l^{tP}(\theta).$$

Production firms in the safe sector are neither taxed nor subsidized, that is $f^1(\theta) = 0$, while production firms in the unsafe sector either pay a tax equal to $f^2(\theta) = -al^{2P}(\theta)\left(\eta_2^P - \eta_1^P\right)$ or subsidize their workers health insurance by paying a fraction $\tau^2(\theta)$ of the subsidy $Z^2(\theta)$, such that $\tau^2(\theta) = al^{2P}(\theta)\left(\eta_2^P - \eta_1^P\right)/Z^2(\theta)$ for each health state θ. By construction, this policy is budget balancing, it implies that $q_2/q_1 = \eta_2^P/\eta_1^P$, $\phi^t(\theta) = p^t(\theta)$ for all t and θ, and satisfies firms zero profit conditions. Moreover, $\langle \alpha^P, x^P \rangle$ satisfies as equality all the budget constraints and all first-order conditions at the prices, wages and subsidies vectors defined above; hence $\langle \alpha^P, x^P \rangle$ solves agents' maximization program in the equilibrium with transfers. Finally, all the market clearing conditions are satisfied at $\phi^t(\theta) = p^t(\theta)$ and $w^t(\theta) = \eta_1^P a$ for all θ. Indeed, at these prices the supply of all state contingent assets, as well as labor demand, are indeterminate since intermediaries and production firms make zero profits.

Proving the result for the case where labor supply is dichotomic requires exactly the same steps and will be thus omitted. □