Multiple-Bank Lending, Creditor Rights and Information Sharing

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Abstract: Multiple bank lending creates an incentive to overborrow and default. When creditor rights are poorly protected and collateral value is volatile, this incentive leads to rationing and non-competitive interest rates. If banks share information about past debts via credit reporting systems, the incentive to overborrow is mitigated: interest and default rates decrease; credit access improves if the value of collateral is not very volatile, but worsens otherwise. If credit reporting also allows banks to condition loans on clients' subsequent debts, rationing disappears and interest rates drop to the competitive level. These predictions square with the findings of recent empirical studies.

Keywords: multiple-bank lending, rationing, information sharing, creditor rights, seniority.

JEL classification: D73, K21, K42, L51.

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1 Introduction

In most countries, firms tend to borrow from several banks: this applies to more than 85 percent of European firms (Ongena and Smith, 2000), with even small and medium-sized firms patronizing several lenders (Detragiache, Garella and Guiso, 2000, and Farinha and Santos, 2002). This pattern is also found in the United States, as shown by Petersen and Rajan (1994). These authors document that "borrowing from multiple lenders increases the price and reduces the availability of credit" (p. 3). We argue that multiple bank lending can have these adverse effects because it induces both borrowers and lenders to behave opportunistically, whenever creditor rights are not well protected and the value of collateral is volatile.

When people can borrow from several banks and are protected by limited liability, they have the incentive to overborrow: each additional dollar of debt that a borrower takes raises his default probability vis-à-vis all lenders, thus reducing his total expected repayment. Moreover, lenders themselves may behave opportunistically, offering extra credit to their competitors' customers while protecting their own claims via high interest rates. And customers may wish avail themselves of this extra credit to undertake larger and less efficient projects at their lenders' expenses. To shield themselves against the contractual externalities created by both types of opportunistic behavior, lenders may ration credit and increase interest rates.[1]

Our paper brings out the implications of these externalities for credit market equilibrium, and shows that they may be mitigated by information sharing (or credit reporting) among lenders, via private credit bureaus or public credit registries. We show that, with no information sharing and poor creditor right protection, banks will deny credit to some applicants, and borrowers will default strategically when their collateral value is depressed. If the value of collateral is not too volatile, information sharing improves credit market performance, by reducing interest as well as default rates and by eliminating rationing. If instead the value of collateral is very volatile, information sharing may have the opposite effect, inducing the credit market to freeze.

In our model, customers can borrow from several banks, and can invest in a small project or in a larger but less profitable one, whose returns cannot be entirely seized by creditors. The fraction that cannot be seized depends on the degree of creditor protection. Borrowers' collateral

[1] In principle, multiple-bank lending may also have beneficial effects by allowing banks to achieve better risk sharing and thereby offer cheaper loans. For simplicity, in the analysis we abstarct from this aspect.

is risky, so that they may default if its value happens to be low. Lenders cannot observe which project is actually carried out by borrowers, so that they face a common-agency problem.[2]

Depending on the severity of this agency problem, three equilibrium outcomes can emerge. First, when creditor rights are well protected, all entrepreneurs get loans at the competitive interest rate and undertake the small and efficient project.

Second, at intermediate levels of creditor protection, two types of equilibria may emerge. One features rationing and strategic default: only a fraction of credit applicants are funded, and some get loans from several banks. Interest rates are non-competitive, and new lenders do not enter for fear of lending to overindebted entrepreneurs. The other is an equilibrium where loans are granted at non-competitive rates, all entrepreneurs are served by a single bank, and competitors refrain from undercutting it for fear of further opportunistic lending. While the latter parallels the equilibrium in Parlour and Rajan (2001), the rationing equilibrium is novel and inherently related to multi-bank lending, in contrast to that without rationing, which is compatible with exclusive contractual relationships.

Thirdly, if creditor rights are very poorly protected and collateral values are highly volatile, the only equilibria that survive are those with rationing or total market freeze. In this region, credit market segmentation also emerges: if the market does not freeze, different groups of lenders offer credit at different interest rates, possibly at "usurious rates" even higher than the monopoly level.

When banks share information about their clients' outstanding debts, they can condition their loans on the borrowers' contractual history, and thereby better guard against opportunistic lending. Hence information sharing expands the region where lending is offered at competitive rates and efficiency prevails, and eliminates rationing provided entrepreneurs' collateral is not too volatile. But beside this "bright side", credit-reporting systems also have a "dark side" that emerges when the value of borrowers' collateral is very volatile. In this case, lenders have a strong incentive to bet on the appreciation of collateral by providing extra loans to low-debt customers of other banks. Credit-reporting systems may facilitate such opportunistic behavior, allowing lenders to target more easily low-debt customers, and thus further exacerbate rationing.

[2] Bernheim and Whinston (1986a, 1986b) offer the first general tratment of this class of models. Kahn and Mookherjee (1998) specialize the analysis to the case of insurance contracts, but consider a model with sequential offers. Segal and Whinston (2003), Bisin and Guaitoli (2004) and Martimort (2004) consider a more general contracting space by introducing latent contracts and menus.

However, in this area lenders never share information unless forced to do so; hence, information sharing may be socially detrimental only if imposed by regulation.

In most of the paper, defaulted claims are assumed to be liquidated pro rata, and information sharing concerns past obligations. However, we also extend the model to the case where information sharing allows seniority-based liquidation, and to the case it even allows banks to monitor the subsequent indebtedness of their clients. In both instances, the benefits of information sharing are amplified.[3]

Taken together, our model produces three main testable implications. First, absent information sharing, rationing can emerge if collateral values are volatile and credit protection is poor; this rationing is associated with high interest and default rates, consistent with the evidence from developing countries (Mookherjee et al. 2000). For extreme values of collateral volatility, the model predicts that the credit market becomes endogenously segmented, with some lenders charging usurious rates and experiencing very frequent defaults, and others charging more moderate rates and facing less frequent defaults – a pattern often observed in rural areas of developing countries.

Second, we show that when banks share information about past debts (not merely about delinquencies), they end up reducing default and interest rates, particularly for borrowers that are informationally opaque and have risky collateral. These predictions square with an expanding body of evidence, based on cross-country aggregate data (Djankov, McLiesh and Shleifer, 2007, Jappelli and Pagano, 2002, Pagano and Jappelli, 1993) and on microeconomic data (Brown, Jappelli and Pagano, 2008; Galindo and Miller, 2001; Doblas-Madrid and Minetti, 2010, Chen and Degryse, 2009; de Janvry, McIntosh and Sadoulet, 2009).

Third, information sharing about past debts is predicted to increase access to credit by eliminating rationing, for moderate levels of creditor protection and collateral volatility. However, the model also predicts that information sharing may exacerbate rationing in situations where creditor rights are poorly protected and collateral values very uncertain, such as some developing countries or more generally at times of great turbulence like financial crises, consistently with the study by Erzberg, Liberti and Paravisini (2008) on the extension of Argentine credit

[3] In particular, when information sharing also concerns subsequent debts of current clients, it leads to full efficiency, being effectively equivalent to exclusivity. A comparison between exclusive and non-exclusive lending is provided by Bisin and Guaitoli (2004) and Attar, Campioni and Piaser (2006), among others.

reporting coverage.

On the whole, our analysis explains why credit bureaus and registries so often pool data about past debts and report clients' total indebtedness to banks, rather than just reporting past delinquencies and borrowers' characteristics. This activity by credit-reporting systems only makes sense in the context of multiple-bank lending. Hence, this paper complements earlier models of information sharing in credit markets, which invariably assume exclusive lending. These models show that sharing data on defaults and customers' characteristics enables banks to lend more safely, overcoming adverse selection (Pagano and Jappelli, 1993), or promoting borrowers' effort to repay loans (Padilla and Pagano, 1997 and 2000).[4]

Finally, our paper also relates to the vast literature on the determinants of credit rationing (e.g., Stiglitz and Weiss (1981), Besanko and Thakor (1985) and Bester (1987), among others), which all share a common feature: rationing arises because the interest rate charged by banks is "too low" to enable the credit market to clear but no bank attempts to raise it, fearing to worsen the pool of loan applicants. In contrast, in our model banks react to the danger of opportunistic lending both by rationing and by raising their rates above the competitive level, in some cases even beyond the monopoly level. Another distinctive feature of the credit rationing due to multi-bank lending is that it is more likely to arise when collateral value is volatile, which instead is inconsequential in the Stiglitz-Weiss (1981) and in the Holmstrom-Tirole (1997) model.

The paper is structured as follows. Section 2 lays out the model. Section 3 introduces the notions of incentive compatibility relevant for the characterization of equilibria. Section 4 presents the equilibria in the regime with no information sharing. Section 5 analyzes how equilibria change when banks can condition lending on their customers' past indebtedness. Section 6 extends the analysis to the case in which banks can condition also on customers' subsequent borrowing and that in which repayments in case of default are according to seniority, under information sharing. Section 7 concludes. Proofs are in the Appendix.

[4] In a sequential common agency game with adverse selection Calzolari and Pavan, 2006, also analyze the conditions under which information sharing between principals may enhance efficiency.

2 The model

We consider a set of banks $B = \{1, 2, 3, ...\}$ that compete by offering credit to a set of risk-neutral entrepreneurs $E = \{1, 2, ..., \bar{e}\}$. The interest rate at which banks raise funds is standardized to zero. Each entrepreneur can undertake a small project or a large one, requiring an investment x or $2x$ respectively. The two projects have non-stochastic revenues y_S and y_L, with $y_L > y_S$, so that the net surplus is $v_S \equiv y_S - x$ or $v_L \equiv y_L - 2x$. Due to decreasing returns, the optimal project is the small one: $\Delta v \equiv v_S - v_L > 0$. Due to limited managerial capacity, each entrepreneur can undertake at most one project.

Entrepreneurs have no resources when projects can be started, hence they must borrow. Banks offer loans for which entrepreneurs can apply sequentially. A credit contract $c_b = (l_b, r_b)$ issued by bank $b \in B$ consists of a loan l_b and a repayment r_b.

The contractual environment is shaped by the following assumptions:

(A1) *Hidden action.* Lenders cannot verify the actual size of the borrower's project: an entrepreneur with a loan of size x can borrow an additional x and undertake the large project.

(A2) *Limited enforcement.* Borrowers are protected by limited liability and can appropriate a fraction $\phi \in (0, 1]$ from the revenue of the large project, which cannot be seized by lenders in case of default.

(A3) *Uncertain future wealth.* After investing, each entrepreneur receives a stochastic endowment $\widetilde{w}$ that equals either $\overline{w}+\sigma$ or $\overline{w}-\sigma$ with equal probability. We normalize its expected value $\overline{w}$ to 1 and assume that its standard deviation σ lies in the interval $[0, 1]$.

(A4) *Costly state verification*: The realization of future wealth $\widetilde{w}$ is unverifiable except in case of default.

Assumptions (A1) and (A2), together with multiple-bank lending, create a moral hazard problem: after borrowing an amount x, the entrepreneur may want to borrow an additional x and undertake the large project, so as to appropriate a share ϕ of its revenue. This can damage lenders, since the large project yields less than the small one and its return can be partially appropriated by the entrepreneur. The fact that the entrepreneur can divert resources from the large project, but not from the small one, captures the idea that the bank's monitoring is less effective as the project become larger and more complex. This assumption considerably simplifies the analysis without affecting any of its qualitative results.

Assumption (A3) captures uncertainty about the future value of the entrepreneur's personal assets (e.g., his house) or about the firm's profits. This uncertainty is a novel ingredient relative to the setting of Parlour and Rajan (2001): as we shall see, it creates scope for opportunistic lending that is not present in their model. Most of our novel results are traceable to this new assumption, which deeply changes the nature of banking competition. Assumption (A4) rules out financing contracts contingent on future wealth, and implies pure debt financing: verifying borrowers' wealth is so costly as to be worthwhile only if default occurs.[5]

2.1 Information-sharing regimes

We will study two alternative regimes of communication between banks:

- under *no information sharing*, banks lending to the same borrower cannot verify either the total indebtedness of the borrower or their own seniority;
- under *information sharing*, banks can verify the total past indebtedness of their customers and credit applicants.

This capture precisely a common feature of credit reporting systems, which allow lenders to interrogate credit bureaus or registers about the exposure of prospective clients only upon receiving a loan application. This is done to prevent banks from exploiting information-sharing systems as a marketing device.

In Section 6 we consider a more extensive form of information sharing, whereby banks can request credit reports also after the loan application stage, in order to monitor subsequent changes in clients' exposure. This enables lenders to use covenants, so as to make repayments contingent on subsequent borrowing.

All information-sharing regimes are initially analyzed under the assumption that defaulted debts are liquidated *pro rata*, as often occurs for unsecured lending. In Section 5, we consider the case in which information-sharing arrangements allow seniority-based repayment.

[5] This assumption is common to many contributions in the literature, for instance Bizer and DeMarzo (1992) and Bisin and Rampini (2006). It also rules out insurance contracts with which entrepreneurs can hedge against their wealth risk.

2.2 The game

We represent market interactions as a game in which entrepreneurs visit banks and apply for credit sequentially (time line in Figure 1). Each bank b can offer a loan contract $c_b = (l_b, r_b)$. The uncertainty about entrepreneurs' endowments is resolved at the final stage $\bar{\tau}$. To capture the idea that *all* banks are exposed to the increase in default risk due to future borrowing, there is no last lending stage: formally, contracting between 0 and $\bar{\tau}$ features an infinite number of stages in which banks post loan offers, entrepreneurs apply for credit and banks decide whether to grant it. For convenience, an entrepreneur can always opt out of a contract signed at stage τ by returning the corresponding loan to the lender at no cost before the investment stage $\bar{\tau}+1$. In the investment stage at $\bar{\tau}+1$ entrepreneurs decide which investment project to undertake, if any. If the loans granted by banks exceed the desired scale of investment, the excess credit is returned to their lenders.[6]

[Insert Figure 1]

Banks cannot verify which investment project entrepreneurs undertake with their funding, but can require that any loan that they grant is either invested in one of the two projects or kept as a bank deposit at the market interest rate. The return on the investment project chosen and the final value of wealth $\widetilde{w}$ are realized at the final stage $\bar{\tau}+2$, where loans are repaid in full or the borrower defaults.

At every stage $\tau \in \mathbb{N}$ between 0 and $\bar{\tau}$, the sequence of events is as follows: (*i*) the bank with the corresponding index ($b = \tau$) posts a contract c_τ;[7] (*ii*) all entrepreneurs can visit it and apply for c_τ; (*iii*) bank $b = \tau$ accepts or rejects applications.[8]

At $\tau \in \mathbb{N}$, the action of a generic entrepreneur e is $a_e(\tau) = 1$ if he files a loan application, or $a_e(\tau) = 0$ if he does not. The action of bank $b = \tau$ is a vector $(c_\tau, \beta(\tau))$ including the posted contract c_τ and a sequence of replies $\beta(\tau) = (\beta_1(\tau), ..., \beta_{\bar{e}}(\tau))$ to each applicant, where

[6] The opportunity to return unused credit at no cost reflects the idea that the excess loans are made available for a very short period (between $\bar{\tau}$ and $\bar{\tau}+1$). The role of this assumption is to guarantee effective competition between lenders: it enables entrepreneurs to apply for many loans and eventually accept only the cheapest ones.

[7] The case in which a bank does not post any offer is captured by the convention that it offers the null contract $c_\emptyset \equiv (l_\emptyset = 0, r_\emptyset = 0)$.

[8] The assumption that each bank can offer only at a specific moment is made just to simplify the description of its strategy. Allowing each bank to offer loan contracts at different stages would not change our results.

$\beta_e(\tau) = yes$ denotes an acceptance of e's application and $\beta_e(\tau) = no$ a refusal. Entrepreneur e and bank $b = \tau$ sign a contract at stage τ if and only if $a_e(\tau) = 1$ and $\beta_e(\tau) = yes$.

The "history" h_e^τ known to entrepreneur e and the "history" h_b^τ known to bank b at each date τ is what each has observed up to that date, respectively. In our setting, the description of the past histories plays a crucial role: different information sharing regimes are formally captured by different assumptions on the histories observable by banks.

Without information sharing, each entrepreneur knows his own applications and outcomes; each bank $b = \tau$ the applications received, its own acceptance decisions and the contracts $M(\tau) = (c_1, c_2, ..., c_{\tau-1})$ offered by banks before stage τ.[9]

Under information sharing, the bank receiving a loan application at stage t observes the entrepreneur's past indebtedness, that is, both the total nominal value of his loans and his total pledged repayment. We denote his total nominal loan value and pledged repayment at t by

$$D(h^t) = \sum_{\tau \leqslant t} l_\tau, \qquad R(h^t) = \sum_{\tau \leqslant t} r_\tau,$$

respectively. When information is shared, at stage τ bank b knows the history $h_b^\tau = \{M(\tau), R(\tau)\}$.

The history of market interactions up to $\overline{\tau}$ is denoted by $h^{\overline{\tau}} = \left\{h_e^{\overline{\tau}}, h_b^{\overline{\tau}}\right\}_{e \in E, b \in B}$, and the set of possible histories by $H^{\overline{\tau}}$. Players' payoffs depend on the realized history $h^{\overline{\tau}}$. However, the actual payoff of each entrepreneur only depends on his final indebtedness $R(h^{\overline{\tau}})$, irrespective of the precise trading sequence $h^{\overline{\tau}}$ that generated that indebtedness.

The final payoff accruing to an entrepreneur with project $n \in \{S, L\}$ and final wealth $\widetilde{w}$ is

$$p_n(h^{\overline{\tau}}, \widetilde{w}) = \phi_n y_n + \max\left\{0, (1-\phi_n)y_n + \widetilde{w} - R(h^{\overline{\tau}})\right\},$$

where the second term reflects the fact that default occurs if the realized value of pledgeable wealth falls short of the total pledged repayment, i.e. $(1-\phi_n)y_n + \widetilde{w} < R(h^{\overline{\tau}})$. This occurs for any history $h^{\overline{\tau}}$ such that agreed contracts require a total repayment $R(h^{\overline{\tau}})$. Note that by

[9]Formally, at each date $\tau \in \mathbb{N}$, entrepreneur e knows the history $h_e^\tau = \{M(\tau); a_e^\tau, \beta_e^\tau\}$ where $a_e^\tau = \{a_e(1), a_e(2), ..., a_e(\tau-1)\}$ is the sequence of his applications and $\beta_e^\tau = \{\beta_e(1), \beta_e(2), ..., \beta_e(\tau-1)\}$ is the corresponding sequence of acceptance decisions by banks. Similarly, for each $\tau \in \mathbb{N}$, the bank lending at stage τ knows the history $h_b^\tau = \{M(\tau), a(\tau)\}$, where $a(\tau) = (a_1(\tau), a_2(\tau), ..., a_{\overline{e}}(\tau))$ are the loan applications received at τ. If instead bank b is inactive up to τ (which happens if $b < \tau$), it only knows the set of loan offers by all banks, that is, $h_b^\tau = \{M(\tau)\}$.

assumption $\phi_S = 0$ and $\phi_L = \phi$, because the entrepreneur can extract private benefits only from the large project. Since the two realizations of $\widetilde{w}$ are equally likely, the expected value of the entrepreneur's payoff is

$$\mathrm{E}_{\widetilde{w}}[p_n(h^{\overline{\tau}}, \widetilde{w})] = \frac{1}{2}[p_n(h^{\overline{\tau}}, \overline{w} - \sigma) + p_n(h^{\overline{\tau}}, \overline{w} + \sigma)].$$

Finally, the profit that bank b expects from lending to an entrepreneur who undertakes a project of size $n \in \{S, L\}$ and has a wealth realization $\widetilde{w}$ (given the contractual history $h^{\overline{\tau}}$) is:

$$\pi_b^n(h^{\overline{\tau}}) = \mathrm{E}_{\widetilde{w}}[r_b^n(h^{\overline{\tau}}, \widetilde{w}) - l_b],$$

where $r_b^n(h^{\overline{\tau}}, \widetilde{w})$ represents the actual repayment of this entrepreneur. The size of this actual repayment depends on whether the entrepreneur repays the loan or defaults. If the entrepreneur repays fully, then $r_b^n(h^{\overline{\tau}}, \widetilde{w}) = r$. Otherwise, under pro-rata repayment, his pledgeable wealth is allotted to banks proportionally to their loans, each bank being entitled to (no less than) a share $l_b/D(h^{\overline{\tau}})$ of the debtor's wealth. For a subset B' of banks, the pledged repayment r_b may be less than this allotted amount: these banks then only receive r_b. Once all such banks are repaid in full, the remaining wealth is divided among the set B'' of remaining creditors proportionally to their loan values, that is,

$$r_b^n(h^{\overline{\tau}}, \widetilde{w}) = \frac{l_b}{\sum_{\tau \in B'} l_\tau}[(1 - \phi_n)y_n + \widetilde{w} - \sum_{\tau \in B''} r_\tau].$$

Since by assumption entrepreneurs can never gain from borrowing more than $2x$, we shall only characterize equilibria where entrepreneurs borrow either x or $2x$ and sign contracts with at most two banks: this is with no loss of generality. In such equilibria, entrepreneurs' strategies are described either by a single loan contract or by a pair of contracts. Accordingly, we shall denote the expected utility of an entrepreneur signing a single contract $c = (x, r)$ by $u(c, c_\emptyset)$, and that of an entrepreneur signing contracts $c = (x, r)$ and $c' = (x, r')$ by $u(c, c')$. Symmetrically, we denote the expected profit of a bank offering contract c by $\pi(c, c_\emptyset)$ when the borrower takes no other loan, and by $\pi(c, c')$ where he also signs contract c' with another bank.[10]

[10]The formal definitions for these expressions are provided at the beginning of the Appendix.

An entrepreneur's strategy is: a sequence of loan requests, the choice of a project's size, and a repayment/default decision, each contingent on previous observable history. A bank's strategy is a sequence of contingent loans' proposals and acceptances. Since with no information sharing each bank does not observe the actions previously taken by its current loan applicants, the game is one of imperfect information, so that the solution concept is Perfect Bayesian Equilibrium (PBE). Instead, when banks share information, the game is one of perfect information and therefore we look for Subgame Perfect Nash Equilibria (SPNE), because in this regime banks know all relevant information about the past.

Finally, in order to simplify the analysis and focus on the most interesting and novel equilibria, we assume that the large project is not financially viable, that is, it does not generate enough pledgeable income for banks to fund it:

(A5) *The large project is not financially viable:* $(1-\phi)y_L - 2x + 1 < 0$.

This hypothesis only rules out equilibria with overinvestment as in Bizer and DeMarzo (1992), where all entrepreneurs manage to fund the large project. Even if the large project is financially unviable, entrepreneurs may still want to carry it out solely to extract private benefits at the expense of some lenders. It is precisely for this reason that banks must worry that their offers might lead to opportunistic behavior, as explained in the next section.

3 Incentive compatibility

In our setting, efficiency requires all entrepreneurs to undertake the small project. This outcome would be ensured by exclusive lending, as banks could costlessly prevent their customers from borrowing from other lenders. But in our model exclusivity clauses are not enforceable: once a borrower has received a loan to fund the small project, he may borrow more and switch to the large project, so as to appropriate a fraction ϕ of its revenue. This inefficient behavior can take two forms. First, an entrepreneur can take two small loans, each issued by a different bank intending to fund a single small project, and undertake the large project. Alternatively, a bank may deliberately design a loan contract to fund an entrepreneur who already obtained a small loan and allow him to undertake the large project at the expenses of the first lender. For a market outcome to be efficient, neither form of opportunistic behavior must be worthwhile.

In other words, efficiency requires one of the following two incentive compatibility conditions to hold. First, each entrepreneur should not wish to apply for more than one loan, among those offered in the market. Second, whenever a given contract is offered, no bank should be able to make money by issuing another contract so as to allow the entrepreneur to undertake the large project. The first is a requirement on unilateral deviations by the entrepreneur, while the second restricts bilateral deviations by a bank and an entrepreneur. Hence, we shall refer to the two conditions as "individual" and "joint" incentive compatibility, respectively.

Our equilibrium characterization will thus rely upon the following two definitions:

Definition 1 (Individual incentive compatibility) *Given the set C of contracts offered on the market, the contract $c = (x, r) \in C$ is individually incentive compatible within C if*

$$u\left(c, c_{\emptyset}\right) \geq \max_{c' \in C} u(c, c'), \tag{1}$$

possibly with $r' = r$, where $c_{\emptyset} = (0, 0)$ is the null contract.

Condition (1) states that the utility of taking only loan c exceeds that of taking it together with another available loan c'. The set C satisfies individual incentive compatibility if (1) holds for all $c \in C$. Instead, joint deviations by entrepreneurs and outside banks are prevented by:

Definition 2 (Joint incentive compatibility) *The contract c is jointly incentive compatible if there exists no other contract $\hat{c}$ such that*

$$\pi(\hat{c}, c) \geq 0, \tag{2}$$

and

$$u(c, \hat{c}) > u(c, c_{\emptyset}). \tag{3}$$

Our equilibrium characterization will rely on these two conditions. The individual incentive constraint must hold in any efficient equilibrium where several banks offer loan contracts and accept applications. For example, a zero-profit equilibrium where many banks offer the perfectly competitive contract $c^{PC} = (x, x)$ can exist only if $u(c^{PC}, c_{\emptyset}) \geq u(c^{PC}, c^{PC})$. If this individual incentive constraint is met, so is the joint incentive constraint: the latter requires

$u(c^{PC}, c_\emptyset) \geq u(c^{PC}, c')$, where c' is offered by the outside bank at a higher interest rate than c^{PC}, to compensate for default risk. Otherwise, an efficient equilibrium can exist if only a single bank is active in the market and the contract it offers meets the joint incentive compatibility constraint, ensuring that outside banks will not provide further lending at its expense.

Finally, in the equilibrium analysis developed in the following sections we impose the following tie-breaking condition:

(A6) *Tie-breaking*: in any PBE a bank τ prefers to lend whenever it is indifferent between lending via a jointly incentive compatible contract and not lending.

This assumption rules out uninteresting equilibria in which banks earn profits by lending at non-competitive rates and their competitors do not undercut them in the belief that their offers would themselves be subsequently undercut.

4 Equilibria with no information sharing

Two different types of equilibria can emerge in the absence of information sharing, depending on parameters: efficient equilibria, where all entrepreneurs implement the small project, possibly borrowing at non-competitive rates, and inefficient rationing equilibria, where only a fraction of entrepreneurs is funded, possibly at usurious rates. Rationing emerges whenever opportunistic lending is not deterred by banks' equilibrium behavior, so that some entrepreneurs get credit from several banks and default strategically.

To identify the regions in which these different equilibria arise, we focus on two key parameters: ϕ, the fraction of revenues that can be appropriated from the large project, and σ, the riskiness of entrepreneurs' wealth. On the one hand, poor creditor protection (a large ϕ) heightens the entrepreneurs' temptation to overborrow and select the large project. On the other, wealth volatility (a high σ) lures outside banks into inducing overborrowing, since limited liability allows them to shift the implied extra default risk onto the initial lender. In short, while higher values of ϕ increase borrowers' individual incentives to behave opportunistically, higher values of σ increase outside banks' gains from opportunistic lending, and therefore raises their incentives to behave opportunistically jointly with entrepreneurs.

In Figure 2, we characterize the equilibrium outcomes for different regions in the plane (σ, ϕ). The admissible parameter space is the square $[0,1]^2$, as both ϕ and σ range between 0 and 1.

[Insert Figure 2]

In region A, where either ϕ or σ is low, the efficient project is funded at the competitive rate: the negative externality due to multi-bank lending is absent, since the individual incentive constraint holds when several banks offer the competitive loan contract c^{PC}.

In region B, where either ϕ or σ is larger than in region A, there is an efficient equilibrium where only one bank is active and earns positive profits as in Parlour-Rajan (2001), since the fear of opportunistic lending prevents entry. In this region, the individual incentive constraint is never satisfied with repayments lower than the monopoly level y_S. However, the joint incentive constraint holds if the inside bank charges non-competitive rates high enough to discourage opportunistic lending by outside banks. On the one hand, the violation of the individual incentive constraint prevents undercutting: no outside bank lends at a rate lower than the equilibrium one, fearing that its customers would then seek additional lending and implement the large project. On the other hand, as the joint incentive constraint is met at the equilibrium rate, no outside bank can induce the entrepreneur to switch to the large project. Hence, the inside bank can safely charge a non-competitive rate: it is precisely by charging a high enough rate that it reduces the joint surplus available to outside banks and entrepreneurs, deterring opportunistic lending.

Interestingly, region B also features rationing equilibria, where several banks are active and only some entrepreneurs receive credit. Since the individual incentive constraint is violated, in this equilibrium borrowers seek multiple loans attempting to fund the large project. Credit rationing, however, reduces default risk by lowering the fraction of entrepreneurs able to over-borrow, and therefore allows some lending activity. Equilibrium obtains when interest rates and rationing ensure that no inactive bank is willing to enter the market, for fear of attracting too many applicants already laden with debt.

In region C, where both ϕ and σ are highest, neither the individual nor the joint incentive constraint is satisfied at rates lower or equal to the monopolistic repayment $r = y_S$. Differently from region B, here defaulting entrepreneurs can expropriate such a large fraction ϕ of the large project's revenue that they wish to take multiple loans at any terms in order to fund the large project, so that it is impossible to deter opportunistic lending even by charging the monopoly rate. As a consequence, there is no equilibrium in which all entrepreneurs are funded, and credit rationing becomes the only way to limit the scope for opportunistic lending. Moreover, very large values of ϕ also imply that opportunistic lenders, who fund the large project to induce strategic default, need to charge usurious rates (above the monopoly level) to avoid losses.

4.1 Equilibrium characterization

In this section, we formally characterize the equilibria described intuitively so far.

4.1.1 Region A

In region A, all entrepreneurs invest in the efficient project and pledge the competitive repayment $r^{PC} = x$, and several banks are active. For this equilibrium to exist, the individual incentive constraint must hold when the contract $c^{PC} = (l = x, r = x)$ is offered by multiple banks, i.e.

$$y_S - r + 1 \geqslant \phi y_L + \mathrm{E}_{\widetilde{w}} \left[\max \left\{ 0, (1-\phi) y_L + \widetilde{w} - r - r' \right\} \right], \tag{4}$$

for $r = r' = x$. The expression on the left-hand side of (4) is the borrower's payoff from executing the small project and repaying r, while the right-hand side is the payoff that he would get by switching to the large project: the revenue that he grabs from the project plus any residual wealth after repaying its creditors. Substituting $r = r' = x$ into (4) yields the boundary:

$$\phi \leq \underline{\phi}(\sigma) = \frac{1 + v_S}{y_L} + \min \left\{ 0, \frac{\Delta v - \sigma}{y_L} \right\}, \tag{5}$$

where we have used the definition $\Delta v \equiv v_S - v_L$.

As explained in Section 3, if the competitive contract $c^{PC} = (x, x)$ satisfies the individual incentive compatibility constraint, it also satisfies joint incentive compatibility. Therefore, the condition $\phi \leq \underline{\phi}(\sigma)$ guarantees that in Region A there are no profitable deviations from the perfectly competitive contract.

The magnitude of this region is inversely related to the excess value generated by the small project, Δv: the greater this difference, the weaker the temptation to switch to the large project.

The function $\underline{\phi}(\sigma)$ is decreasing in σ, because when their wealth is riskier, entrepreneurs gain more by overborrowing and defaulting in the bad state. Thus, efficiency always prevails for Δv sufficiently large and σ sufficiently small. To summarize:

Proposition 1 *In region A there is a unique equilibrium in which all entrepreneurs undertake the efficient project and borrow at competitive rates. This region is non-empty, and its area is increasing in Δv and decreasing in σ.*

Even though for simplicity we prove this proposition with reference to the case where each entrepreneur borrows x from one bank, in this region competitive equilibrium is perfectly compatible with multiple bank lending: if an entrepreneur does not wish to take extra lending after borrowing x from a single bank, he will not wish to do so either after borrowing x/N from N banks at the same rate. In other words, in region A the model does not pin down the number of banks lending to a given entrepreneur, an indeterminacy that is standard in competitive analysis. This underscores that multiple-bank lending does not *per se* lead to rationing, but does only in conjunction with low creditor protection and volatile collateral values, as shown below.

4.1.2 Region B

In region B there are two possible types of equilibria: efficient non-competitive equilibria with only one active bank; and inefficient equilibria where several active banks lend and ration credit, and some entrepreneurs undertake the large project.

Consider first the efficient non-competitive equilibrium candidate. The corresponding equilibrium contract must satisfy two conditions: first, it must not be vulnerable to opportunistic lending by competing banks; second, it must not be vulnerable to undercutting.

We first show that in the whole region B there is a set of contracts that are invulnerable to opportunistic lending, i.e. satisfy joint incentive compatibility. According to Definition 2, contract c is jointly incentive compatible if there is no contract c' yielding positive profits to an entrant, when the entrepreneur signs it in conjunction with contract c. *A fortiori*, this is guaranteed if the entrant loses money, whichever rate he charges. This is indeed the case if the entrant induces default in both states, because the large project is not viable. Hence, the maximal repayment r' that the entrant can charge to an entrepreneur who already took a loan with repayment r is the pledgeable income in the good state, net of the repayment r to the inside bank: $r' = (1-\phi)\, y_L + 1 + \sigma - r$. Hence, contract c deters opportunistic entry if the profit to an entrant who offers credit at r' is non-positive:

$$\pi(c', c) = \frac{1}{2}\frac{(1-\phi)\, y_L + 1 - \sigma}{2} + \frac{1}{2}\underbrace{[(1-\phi)\, y_L + 1 + \sigma - r]}_{=r'} - x \leq 0. \tag{6}$$

As $\pi(c', c)$ is decreasing in the rate r of the inside bank, the latter can always deter entry via a high enough repayment, provided condition (6) holds at least when the inside bank charge the monopoly repayment $r = y_S$. Therefore, inequality (6) evaluated at $r = y_S$ defines the frontier of the region where at least the monopoly contract $c^M = (x, y_S)$ is jointly incentive compatible:

$$\phi \geqslant \phi_m(\sigma) = \frac{1 + v_L}{y_L} + \max\left\{0, \frac{\sigma - 2v_S}{3y_L}\right\}. \tag{7}$$

Figure 2 shows that the function $\phi_m(\sigma)$ is weakly increasing in σ, since greater collateral volatility increases the profits of opportunistic entrants: this must be compensated by a higher ϕ, i.e. worse creditor protection, which tends to lower the entrants' profits.

However, even when condition (7) is not met, joint incentive compatibility may still hold if entrepreneurs are not willing to take the loans that an opportunistic entrant can profitably offer them. Formally, this is the case when the entrepreneur's utility from accepting only the inside bank's contract c exceeds that from taking also the zero-profit contract c'' offered by the entrant:

$$u(c, c_\emptyset) = y_S - r + 1 \geq u(c, c'') = \phi y_L + \frac{1}{2}\max\left\{0, (1-\phi)y_L + 1 + \sigma - r - r''\right\}, \tag{8}$$

where the repayment r'' satisfies the outside bank's zero-profit condition:[11]

$$\pi(c'', c) = \frac{1}{2}\left\{\frac{(1-\phi)y_L + 1 - \sigma}{2} + r''\right\} - x = 0. \tag{9}$$

Since the entrepreneur's gain from taking only the contract of the inside bank is maximal when this contract requires the competitive repayment $r = x$, joint incentive compatibility holds if (8)

[11] The zero-profit repayment that solves this condition never induces default in the good state, since below the frontier $\phi_m(\sigma)$ the entrepreneur's own wealth in the good state is by construction large enough to guarantee that he can fully repay r'' even when the incumbent charges the monopoly rate.

and (9) are both satisfied for $c = c^{PC}$, namely if

$$\phi \leq \underline{\phi}'(\sigma) = \min\left\{\frac{v_S + 1}{y_L}, \frac{1 + 4\Delta v + v_L - \sigma}{y_L}\right\}.$$

To summarize, joint incentive compatibility is satisfied either if $\phi \geq \phi_m(\sigma)$ or if $\phi \leq \underline{\phi}'(\sigma)$. Graphically, we must be above the weakly increasing locus $\phi_m(\sigma)$ or below the weakly decreasing $\underline{\phi}'(\sigma)$ function.

Region B is the area where joint incentive compatibility holds but the competitive equilibrium does not exist because the individual incentive constraint is violated, i.e. $\phi > \underline{\phi}(\sigma)$. Intuitively, the fact that some contracts satisfy joint incentive compatibility implies that the inside bank succeeds in protecting itself against opportunistic lending. However, for such non-competitive contracts to survive in equilibrium, they must also be immune to undercutting. We show that for some of these contracts this is indeed the case, because undercutters are themselves exposed to the danger of opportunistic lending or to the risk that their loan is taken together with that of the incumbent:

Proposition 2 *In region B, there is a continuum of non-competitive equilibria (each with a different repayment $r > x$) where a single bank is active, earns positive profits and funds the efficient project by offering the loan x to all entrepreneurs. This region is non-empty if $\Delta v < x/2$.*

This result is reminiscent of that by Parlour and Rajan (2001), with the difference that in their setting joint incentive compatibility holds by assumption, so that undercutting is not restrained by the danger of further opportunistic lending. In this sense, in our model the scope for non-competitive behavior by the incumbent is larger.

Region B also features a novel type of equilibrium, where several banks are active and some entrepreneurs are denied credit. These symmetric rationing equilibria share the following features. First, entrepreneurs apply to all active banks, hoping to obtain loans from at least two banks, since individual incentive compatibility is violated. Second, banks accept applications randomly, so that in equilibrium some entrepreneurs receive no credit, some manage to obtain a loan of size x and others of size $2x$. An active bank earns $r - x$ on each client who is granted a single loan, and loses money on those who get two loans of size x and default in the bad state. Therefore, each bank's expected profit is decreasing in the number of loans offered by competitors. The fraction of accepted loan applications is such that each bank just breaks even. In region B, this rationing equilibrium is the only one consistent with multiple active banks:

Proposition 3 *In region B there are zero-profit symmetric equilibria with rationing, where only a subset of banks is active and each offers a single loan contract to a fraction α of entrepreneurs.*

In this region there is a continuum of such equilibria, each associated with a different repayment and a corresponding lending probability. For simplicity, we analyze the case where only two banks are active, but it can be shown that a continuum of rationing equilibria exists for any number of active banks. The idea behind all these rationing equilibria is that no bank can profitably deviate from its loan policy by accepting additional applicants, in spite of the presence of rationing: the number of applications accepted in equilibrium by competitors (the fraction α) implies that each bank's prospective customers are too risky to warrant further lending.

To sharpen our empirical predictions, we explore how the equilibrium fraction of funded entrepreneurs, α, varies with the equilibrium repayment and default probability across equilibria:

Proposition 4 *Rationing equilibria where a larger fraction α of entrepreneurs obtains credit feature a higher contractual repayment r and higher default probability.*

Intuitively, the equilibria where more entrepreneurs get credit also feature a higher probability that each of them gets two loans and defaults. To compensate for the resulting increase in losses, each bank requires higher contractual repayments.

4.1.3 Region C

Region C is where moral hazard problems are most severe: the fraction of surplus that borrowers can steal is so large and collateral value so volatile that opportunistic lending may never be deterred, even by charging the monopoly rate. Technically, there is no contract that satisfies joint incentive compatibility. This occurs when $\underline{\phi}'(\sigma) < \phi < \phi_m(\sigma)$. As a result, in this area no equilibrium exists in which all entrepreneurs obtain credit. Here, either there is rationing, in the sense that not all credit applicants obtain a loan, or else the credit market freezes, meaning that no bank posts offers, so that no entrepreneur is funded.

The rationing equilibrium in this region differs from that in region B: at least two different contracts must be offered, one charging a "usurious repayment" r^U above the monopoly level

$r^M = y_S$, and the other requiring the monopoly repayment. The repayment r^U is the maximum that an entrepreneur who already borrowed at the monopolistic rate can pledge without defaulting in the good state. As shown in the Appendix, $r^U = (1 - \phi)\, y_L + 1 + \sigma - y_S > r^M = y_S$.

Proposition 5 *In region C, there are both a competitive equilibrium with rationing and an equilibrium with market freeze. In the rationing equilibrium, banks accept a fraction of loan applications such that each loan application yields zero expected profits. Some banks offer the monopoly contract* $c^M = \left(x, r^M\right)$ *and others the usurious contract* $c^U = (x, r^U)$. *This region is non-empty if and only if* $\Delta v < x/2$.

In the rationing equilibrium, entrepreneurs apply for both the monopoly and the usurious loan: some get no loan, some get one at the monopoly rate, others get both the monopoly and the usurious contract, and the rest take two contracts at the usurious rate. A bank issuing a monopoly loan earns profits on the clients who take no other loan, and makes losses on those who take another loan. A bank lending at usurious rates makes profits on clients who signed the monopoly contract with a competitor, and losses on those with another usurious contract. In equilibrium, usurers receive more applications from entrepreneurs who will eventually default than non-usurers, but charge correspondingly higher rates, as both types of lenders break even. This credit market segmentation is often observed in rural areas of developing countries.

The reason why there must be some banks offering loans at usurious rates is as follows. First, in this region the value of collateral is so volatile. that even the monopolistic contract does not satisfy the joint incentive constraint; second, creditor protection is so poor that an outside bank lending to an entrepreneur who has already accepted a loan at the monopoly rate must charge more that the monopoly rate; third, entrepreneurs are willing to take loans at such a high rate because the usurious loan allows them to appropriate part of the large project's return, while by defaulting they avoid paying these high rates in the bad state.

4.2 Empirical predictions

The model of multiple bank lending developed so far has two main empirical predictions: a novel one regarding the effect of the volatility of collateral value, and another concerning creditor rights protection which is broadly in line with the literature.

The novel testable prediction is that credit rationing due to multi-bank lending only arises when collateral value is sufficiently volatile: as σ increases in Figure 2, we move from competitive equilibrium to an equilibrium with rationing and high interest and default rates. This effect does not arise in single-bank models of credit rationing, such as Holmstrom and Tirole (1988), Stiglitz and Weiss (1981), Williamson (1987) and Longhofer (1997), where increases in the volatility of collateral are neutral. Our finding implies that rationing should be more widespread in countries where real estate prices are more volatile and in industries that feature more unstable secondary market collateral values. It also implies that credit rationing should be more pervasive when the instability of house prices is more pronounced, as in the recent subprime loan crisis and its aftermath.

The model also predicts that improving creditor protection – lowering ϕ in Figure 2 – tends to reduce credit rationing and raise competition. If borrowers' wealth is not very volatile (low σ), strengthening creditor rights shifts the economy from region B to region A, thereby improving access to credit and lowering default rates. If instead in region B the market features a non-competitive equilibrium, shifting to region A implies more intense banking competition and lower interest rates. If borrowers' wealth is very volatile (high σ), the same reform may shift the economy from region C to B, implying the transition from rationing to a non-competitive equilibrium where all entrepreneurs are served. In summary, the model predicts that creditor-friendly reforms increase the availability of credit, as in the above-mentioned models of credit rationing, and reduces default and interest rates by increasing banking competition, as in Parlour and Rajan (2001).

These predictions are consistent with cross-country data and with U.S. data on interstate differences in bankruptcy law. La Porta et al. (1997) and Djankov et al. (2007) show that the breadth of credit markets is positively correlated across countries with measures of creditor rights protection. Along the same lines, Gropp, Scholz and White (1997) find that households living in states with comparatively high exemptions are more likely to be turned down for credit, borrow less and pay higher interest rates; and White (2006) shows that debt forgiveness in bankruptcy harms future borrowers by reducing credit availability and raising interest rates.

Figure 3 shows that both of the two key parameters discussed so far vary considerably across countries. The figure plots on the vertical axis the standard deviation of real house price changes between 1970 and 2006, as a proxy of collateral volatility σ, and on the horizontal axis an inverse measure of creditor rights protection, as a proxy of the fraction ϕ of revenues that cannot be pledged to creditors.[12] According to the model's prediction, countries such as Finland, Italy and Spain are those where multiple bank lending is most likely to lead to credit rationing, high defaults and interest rates, since they feature both comparatively large collateral volatility and poor creditor right protection. The opposite applies to countries with excellent creditor protection such as the U.K. and New Zealand, or low collateral volatility such as Germany.

[**Insert Figure 3**]

5 Equilibria with information sharing

We now turn to the case where banks share information on entrepreneurs' borrowing histories, and in particular on their total exposure. This form of information sharing, which is widespread in credit markets, helps banks to guard against the risk of default, by conditioning loan offers on the applicants' financial exposure.

As in the previous section, we continue to assume that creditors recover on a *pro-rata* basis in case of default. Natural enough in markets with no information sharing, where no lender knows customers' past indebtedness, such an assumption may be questionable when information sharing makes each lender aware of his seniority. Nevertheless, pro-rata liquidation occurs even in countries where credit bureaus are widespread. For instance, in the U.S. consumer credit market, the assets of defaulting borrowers (above a minimum threshold) are liquidated pro-rata under Chapter 7 (Parlour and Rajan, 2001). Retaining *pro-rata* liquidation also facilitates the comparison with the no information sharing case analyzed so far. The extension to seniority-based liquidation is left to Section 6.2.

The effects of information sharing are illustrated in Figure 4. Comparing it with Figure 2 shows that information sharing changes the equilibrium configuration by shifting the boundaries between regions from the dashed lines (the boundaries in Figure 2) to the solid ones. Moreover, information sharing eliminates rationing equilibria.

[12] The choice of countries is dictated by the availability of comparable data for real house prices. The Bank of International Settlements provides such data for the 18 countries in the figure. Creditor rights protection is drawn from Djankov et al. (2007). Since the latter ranges between 0 and 4, the inverse measure plotted in the figure equals 4 minus the Djankov et al. indicator.

[Insert Figure 4]

Specifically, the efficient and competitive region expands, from region A in Figure 2 to A' in Figure 4. Now in the area between the dashed line $\underline{\phi}(\sigma)$ and the solid line $\underline{\phi}'(\sigma)$, borrowers eager to switch to the large project can no longer obtain an additional loan at the competitive rate, because banks can refuse lending to borrowers who have already taken a loan and because outside banks cannot profit from opportunistic lending (joint incentive compatibility being satisfied in this area). As they no longer fear entrepreneurs playing them one against another, banks are now willing to offer loans of size x at the competitive rate in equilibrium.

Moreover, in area A' all non-competitive equilibria disappear, because information sharing allows outside lenders to safely undercut incumbents: starting from a non-competitive equilibrium candidate, any bank can now offer a better rate to any entrepreneur who is not yet indebted. This highlights the pro-competitive effect of information sharing.

A second effect of information sharing is that rationing equilibria disappear in area B. To see why, recall that absent information sharing, in region B some entrepreneurs take two loans at a rate above the competitive level and default. With information sharing, instead, banks can identify any entrepreneur who has not yet received credit and target him with a loan offer. In doing so, they can be confident that no competing bank will grant a second loan, anticipating that the double borrower would default and inflict losses on both lenders.

The two effects just described – expansion of the competitive and efficient region, and removal of credit rationing – underscore the positive side of information sharing, its tendency to enhance efficiency by mitigating the contractual externalities of non-exclusivity. To summarize:

Proposition 6 *Under information sharing, the region with a unique, efficient and competitive equilibrium expands from A to A'. This region is non-empty and for $\Delta v > x/2$ coincides with the whole parameter space $[0,1]^2$. In region $B' \subset B$ there is a unique non-competitive equilibrium with no rationing. The corresponding equilibrium repayment is the lowest among all the equilibrium repayments that obtain in region B without information sharing.*

Information sharing may also have a dark side, however. This emerges in region C' (which coincides with region C in Figure 2). Here too, rationing disappears, but information sharing induces a unique equilibrium with market freeze. Indeed, in this region neither the individual nor the joint incentive constraints are met, so that already funded entrepreneurs are willing to take additional loans at the expenses of the non-usurious lenders, and outside banks are willing to offer them credit (as they expect to recover their money at the expense of the non-usurious lenders). Absent information sharing, even usurers must worry about the risk of lending to a customer already indebted with another usurer: since the large project is not viable, in this area two usurers dealing with the same client lose money. In equilibrium, this limits lending at usurious rates. With information sharing, instead, usurers can easily target all clients not indebted with other usurers. In so doing, they make lending unprofitable for any bank charging lower rates, and thereby cause the entire loan market to freeze :

Proposition 7 *In region C' there is a unique equilibrium with market freeze.*

It may seem paradoxical that in region C' information sharing reduces efficiency even though it mitigates contractual externalities. The point, however, is that in this region contractual externalities between usurers were beneficial in the absence of information sharing: banks lending at usurious rates had to worry about customers playing them one against the other, which kept them from competing too aggressively against non-usurious lenders. Information sharing dispenses them from this concern, but their more aggressive lending strategy kills off the market.

5.1 Empirical predictions: effects of information sharing

Our results offer a number of testable predictions on how information sharing about past indebtedness should affect credit market performance. First, information sharing unambiguously reduces default and interest rates, and more so in countries with worse creditor protection and riskier collateral or, within a given country, for informationally opaque and riskier borrowers. Second, eliminating rationing should result in smaller individual loans. Third, when lenders spontaneously share information about past debts, credit availability invariably increases; if instead information sharing is mandated, it increases credit supply if the variability of collateral is not too large but reduces it if poor creditor protection is coupled with high uncertainty on the value of borrowers' collateral. This "dark side" of credit reporting may be relevant in some developing countries, where potential borrowers are farmers with very risky wealth, while lenders often charge usurious rates. In such environments, information sharing would enhance the usurers' ability to target clients, and so disrupt the viability of lending at non-usurious rates.

An expanding empirical literature, based on cross-country aggregate data (Djankov, McLiesh and Shleifer, 2007, Jappelli and Pagano, 2002, Pagano and Jappelli, 1993) and on firm-level data (Brown, Jappelli and Pagano, 2008; Galindo and Miller, 2001) has showed that information sharing is associated more lending and/or lower delinquencies. In particular, Doblas-Madrid and Minetti (2010), who explore contract-level data from a major U.S. credit bureau, find that as lenders enter the bureau, they experience a decline in borrowers' delinquencies, and more so for informationally opaque and riskier clients. They also find that access to the bureau induces creditors to grant smaller individual loans, in line with our model's prediction. Chen and Degryse (2009), who analyze household lending by a major Chinese bank, find that the bank grants a larger credit line to borrowers for whom it receives extra information from other financial institutions, and that its lending decisions are affected by data about lending by other banks, as assumed in our model, rather than about past delinquencies. A randomized experiment on a Guatemalan microfinance lender who gradually started using a credit bureau, conducted by de Janvry, McIntosh and Sadoulet (2009), leads to broadly similar results: recourse to the credit bureau allows increased volume and efficiency of lending, with no increase in defaults.

In terms of our analysis the expansion of lending associated with information sharing may be interpreted as an indication that in most instances information sharing reduces incentives for opportunistic lending, just as the improvement in legal protection of creditors discussed in Section 4.2. The "substitutability" relationship between information sharing and creditor protection is consistent with the evidence of Djankov et al. (2007) and Brown et al. (2008). Indeed, turning back to Figure 3, it is precisely in some of the countries with weaker creditor protection (France) or higher collateral volatility (Italy and Spain) that public credit registers provide to all financial institutions the amount (as well as the maturity) of all the loans granted to each borrower.[13]

Finally, the "dark side" of information sharing identified by our analysis may help to interpret the evidence in Herzberg, Liberti and Paravisini (2008), that the extension of Argentina's public credit register to loans below the $200,000 threshold in 1998 resulted in lower lending and higher default rates, for firms that borrowed from multiple lenders. This evidence accords with the effect of the introduction of information sharing in our rationing equilibrium when the

[13]This information is drawn from Miller (2003). The only other countries in Figure 2 where public credit register disseminate loan information are Belgium and Germany, but in the latter it only applies to very large loans.

uncertainty about collateral value is very high and creditor rights poorly protected. Both of these prerequisites apply in the case at hand: not only Argentina scores quite low on creditor protection according to the Djankov et al. (2007) indicator, and the 1998 extension in the credit register took place soon before Argentina plunged in the worst crisis of its postwar history.

6 Extensions

So far, our analysis has proceeded under two simplifying assumptions: first, in the information sharing regime banks can only use retrospective information on their credit applicants' indebtedness; second, liquidation of defaulted loans is *pro rata*. Now, we show that when either assumption is relaxed, the beneficial effects of information sharing are amplified.

6.1 Full information sharing and loan covenants

The information sharing system described in Section 5 does not allow banks to monitor the subsequent credit exposure of its customers. However, one may envisage a situation in which banks use a credit register to check exposures even after lending. This regime, that we label "full information sharing", is equivalent to a situation where exclusive contracts are enforceable: banks can impose loan covenants that force early liquidation and repayment if total indebtedness exceeds a specified threshold before the investment is made. In this case, it is immediate that there is a unique efficient and competitive equilibrium for all parameter values.

However, in reality covenants are costly to enforce; moreover, lenders may become aware of their violation after the investment stage. In both cases, full information sharing can be shown to be equivalent to the regime described in Section 5.

6.2 Information sharing and seniority

To this point, defaulted debts were assumed to be liquidated *pro rata*. But the creation of a credit reporting mechanism facilitates the seniority ranking of creditors, thus allowing the enforcement of seniority-based liquidation in case of default. For instance, the land registries used to record mortgage claims – the ancestors of modern credit reporting systems – served the dual purpose of enabling lenders to verify the residual collateral of credit applicants and of documenting the seniority of their claims (Hoffman et al., 1998, 1999 and 2001). Accordingly, we now explore what happens in a credit market with information sharing if the liquidation rule shifts from pro-rata to seniority. As we shall see, information sharing benefits more credit market performance when it is used jointly with seniority-based (rather then pro-rata) liquidation.

Seniority-based liquidation changes the definition of bank τ's expected profit

$$\pi_\tau^n(h^{\overline{\tau}}, \widetilde{w}) = \mathrm{E}_{\widetilde{w}}[r_\tau^n(h^{\overline{\tau}}, \widetilde{w}) - l_\tau].$$

Now in case of default the actual repayment $r_\tau^n(h^{\overline{\tau}}, \widetilde{w})$ to bank τ equals $(1-\phi_n)y_n + \widetilde{w} - R^\tau(h^{\overline{\tau}})$, i.e. the debtor's pledgeable wealth minus the repayments to senior creditors, $R^\tau(h^{\overline{\tau}}) = \sum_{\tau'<\tau} r_{\tau'}$, if positive. For instance, with only two lenders $R^2(h^{\overline{\tau}}) = r_1$, where r_1 is the repayment to the senior bank 1. Clearly, the junior bank 2 appropriates a smaller share of the borrower's assets than under *pro rata* liquidation, which weakens the temptation to lend opportunistically.

As a result, the locus $\underline{\phi}'(\sigma)$ of Figure 4, below which all entrepreneurs invest in the efficient project and pledge the competitive repayment x, is replaced by $\underline{\phi}''(\sigma)$ in Figure 5. As before, this locus is defined by the incentive compatibility condition (8), which remains unaffected, and the zero-profit condition $\mathrm{E}_{\widetilde{w}}[\pi(r', \widetilde{w})] = 0$ of the junior bank, which now must take into account its weaker seniority rights. Precisely for this reason, the junior bank must now charge more to break even, so that the entrepreneur's incentive to seek additional credit weakens: opportunistic lending will occur for larger values of ϕ or/and σ, so that the locus $\underline{\phi}''(\sigma)$ lies above $\underline{\phi}'(\sigma)$.[14] Therefore, the competitive and efficient region expands from region A' in Figure 4 to A'' in Figure 5. Part of this expansion comes at the expense of non-competitive region B', which shrinks to region B'', and part at the expense of the market freeze region from C' to C''.

[Insert Figure 5]

Moreover, the $\phi_m(\sigma)$ locus of Figure 4 is replaced by the lower locus $\phi_m'(\sigma)$ in Figure 5. This is because seniority-based liquidation offers better protection against opportunistic lending to the claim of a senior bank offering a small loan, so that non-competitive equilibria occur in a larger region of the parameter space $[0,1]^2$. So region B', where banks charge non-competitive rates, expands at the expense of region C'.

[14] It can be shown that the $\underline{\phi}''(\sigma)$ locus is decreasing in σ and has the same slope $1/y_L$ as the locus $\underline{\phi}'(\sigma)$ in Figure 3.

Summarizing, these shifts enlarge the region where the small and efficient project can be funded (from $A' \cup B'$ to $A'' \cup B''$), and also the region where it can be funded at competitive terms (from A' to A''). Both of these shifts compress the region where information sharing triggers market freeze, from C' to C'': seniority-based liquidation, by better protecting senior creditors, leaves less scope for usurious lending, which is responsible for market freeze – a result that echoes Bisin and Rampini (2005).

7 Concluding remarks

When people can borrow from several banks, lending by each bank increases the customer's default risk. We show that the strength of this contractual externality depends on creditor rights protection and on the volatility of collateral values. When creditor rights are well protected, the externality is absent or tenuous, so that banks can lend at competitive rates without fearing that their customers will take additional loans. When creditor protection is in an intermediate range, this externality generates equilibria with non-competitive rates and possibly credit denial to some applicants (rationing). When the value of collateral is sufficiently volatile, the equilibrium always involves rationing and even usurious rates by some lenders.

For moderate levels of creditor protection and collateral volatility, information sharing about past debts mitigates these contractual externalities by allowing banks to condition their loans on the borrower's contractual history, so to guard themselves against opportunistic lending by competitors. As a result, it increases access to credit by eliminating rationing. However, in situations where creditor rights are poorly protected and collateral values very uncertain, information sharing may exacerbate rationing and even induce market freezes: this may be relevant for some developing countries or more generally at times of great turbulence, like financial crises.

Our model has three main testable predictions. First, credit rationing should be tighter, and interest and default rates larger when collateral is risky and creditor rights are poorly protected. Second, information sharing about past debts should reduce default and interest rates. Third, information sharing should increase credit access, unless collateral values are very volatile. These three predictions are largely consistent with the existing empirical evidence.

Appendix: Proofs

Throughout the proofs, we characterize credit market equilibria by assuming that any active bank offers only loans of size x. This restriction is inconsequential, because for any Perfect Bayesian Equilibrium (PBE) where the entrepreneur e borrows $l \notin \{0, x\}$ there is another equilibrium where $l \in \{0, x\}$ and is payoff-equivalent for the entrepreneurs. For instance, a competitive equilibrium where two banks lend $x/2$ each is payoff-equivalent to one in which a single bank lends x at competitive terms. This is because entrepreneurs have no incentive to borrow a total sum that differs from x or $2x$, not being able to steal extra funding not needed to carry out one of the two projects. Hence, they will find it optimal to borrow at most $2x$, even if they may apply to many banks, given the sequential nature of the negotiation process.

Consistent with the notation introduced in Section 2, the utility that an entrepreneur obtains if he signs both contracts $c = (x, r)$ and $c' = (x, r')$ is

$$u(c, c') = \phi y_L + \mathrm{E}_{\widetilde{w}} \left[\max \left\{ 0, (1-\phi) y_L + \widetilde{w} - r - r' \right\} \right],$$

and the expected profit of a bank offering contract c to an entrepreneur who also signs c' with another bank is

$$\pi(c, c') = \mathrm{E}_{\widetilde{w}} \left[\min \left\{ r, r(\widetilde{w}) \right\} \right] - x,$$

with $\widetilde{w} \in \{\overline{w} - \sigma, \overline{w} + \sigma\}$, and

$$\begin{aligned} r(\widetilde{w}) &= r \text{ if } (1-\phi) y_L + \widetilde{w} - r' - r > 0 \\ &= \max \left\{ \frac{(1-\phi) y_L + \widetilde{w}}{2}, (1-\phi) y_L + \widetilde{w} - r' \right\} \text{ otherwise.} \end{aligned}$$

We denote by $c^{PC} = (x, x)$ and $c^M = (x, y_S)$ the contracts requiring the perfectly competitive and the monopolistic repayments, respectively. Moreover, we refer throughout to $C(h_\tau)$ as the set of contracts issued up to stage τ, while $\mu_\tau \in [0, 1]$ is the fraction of applications for c_τ that bank τ would accept, according to competitors' equilibrium beliefs, after issuing c_τ. We will assume that all players have common beliefs.

Finally, to simplify the description of entrepreneurs' strategies, we assume without loss of generality that each entrepreneur applies for all contracts. Note that this is a weakly dominant strategy since entrepreneurs can opt out of a contract at any stage before $\overline{\tau}$.

The following lemma will be used to prove Proposition 1. The first part of the lemma states that the entrepreneur may benefit from taking two loans only if he defaults in the bad state. The second part identifies the region in which the entrepreneur takes two loans with the same rate, if available, i.e. the region where individual incentive compatibility does not hold.

Lemma 1 *The following properties hold:*

(i) Consider any pair of contracts $c = (x, r)$ and $c' = (x, r')$, with $r' \geq r \geq x$. Then $u(c, c') > u(c, c_\emptyset)$ only if $(1-\phi) y_L + 1 - \sigma - r - r' < 0$.

(*ii*) *Consider any contract* $c = (x, r)$ *such that* $r \in [x, y_S]$ *and* $2r > (1-\phi)y_L + 1 - \sigma$. *Then* $u(c, c) > u(c, c_\emptyset)$ *(resp.* $\leq$*) if* $\phi > \underline{\phi}(\sigma)$ *(resp.* $\leq$*), with:*

$$\underline{\phi}(\sigma) = \min\left\{\frac{y_S + 1 - r}{y_L}, \frac{1 + \Delta v + v_S - \sigma}{y_L}\right\}.$$

Proof. The proof of part (*i*) follows immediately from the fact that the NPV of the small project exceeds that of the large one. As for part (*ii*), straightforward calculations imply that $\underline{\phi}(\sigma)$ solves

$$u(c, c) - u(c, c_\emptyset) = \phi y_L + \mathrm{E}_{\widetilde{w}}[\max\{0, (1-\phi)\, y_L + \widetilde{w} - 2r\}] - (y_S + 1 - r) =$$

$$= \phi y_L + \frac{1}{2}\max\{0, (1-\phi)\, y_L + 1 + \sigma - 2r\} - (y_S + 1 - r) = 0.$$

This implies the result, since $u(c, c) - u(c, c_\emptyset)$ is increasing in ϕ. ■

Proof of Proposition 1. We first prove existence and then uniqueness.

Existence. Consider the candidate equilibrium where (*i*) all banks issue the contract c^{PC} and extend credit to all applicants for any possible $C(h_\tau)$; (*ii*) each entrepreneur takes a loan x by randomizing with equal probability among the banks offering his most preferred contract, and undertakes the small efficient project; (*iii*) at each stage τ, banks' beliefs are that their competitors accept all applications, i.e. $\mu_\tau = 1$ for any possible c_τ and $C(h_\tau)$.

Conditions (*i*)-(*iii*) identify a PBE. Indeed, entrepreneurs' strategies are sequentially rational given (*ii*) and (*iii*), since the individual incentive constraint (IIC thereafter) is satisfied for $\phi \leq \underline{\phi}(\sigma)$ by Lemma 1, so that $u(c^{PC}, c_\emptyset) \geq u(c^{PC}, c^{PC})$. Moreover, banks' strategies are sequentially rational (given the common beliefs on competitors' acceptance policies) since no bank can earn positive profit by offering $c' = (x', r') \neq c^{PC}$. Indeed, the IIC condition guarantees that no entrepreneur will ever sign a contract $c' = (x', r')$ such that $r' > x$ under the beliefs (*iii*).

Uniqueness. We must show that for $\phi \leq \underline{\phi}(\sigma)$ there exists no equilibrium where a contract $c = (x, r)$, with $r > x$, is taken by any entrepreneur. The condition $\phi \leq \underline{\phi}(\sigma)$, together with the continuity of the entrepreneurs' expected utility, implies that $u(c'', c_\emptyset) > u(c'', c)$, with r'' sufficiently close to x, and $r > r''$. As a consequence, Assumption **A6** guarantees that if c^{PC} is not offered, any bank can profitably deviate by offering c''. Indeed, this contract makes positive profits if accepted by any entrepreneur. Therefore, a necessary condition for contracts charging non-competitive rates to be signed in equilibrium by some entrepreneurs is that all banks earn positive profits. But if so, then by assumption **A6** some bank will undercut its competitors. hence, this cannot be an equilibrium.

Finally, region A is non-empty since $\underline{\phi}(\sigma) > 0$ at $\sigma = 1$, so that $\underline{\phi}(\sigma) > 0$ for all σ. ■

Proof of Proposition 2. Let us first introduce a definition and some new notation. Let us denote by JIC the set of contracts that satisfy joint incentive compatibility, and by K the subset of contracts in JIC that cannot be undercut by any other contract in JIC, that is:

$$K \equiv \{c = (x, r) : r \leq y_S,\ c \in JIC,\ u(\widetilde{c}, c) \geq u(\widetilde{c}, c_\emptyset),\ \forall\, \widetilde{c} \in JIC\}.$$

This subset is non-empty, as it trivially contains the contract with the lowest repayment in JIC. We shall denote by $c^* = (x, r^*)$ and $c^{**} = (x, r^{**})$, where $r^{**} > x$, the contracts with the minimal and maximal repayment in K, respectively. Note that by definition $c^* = c^{PC}$ whenever c^{PC} is jointly incentive compatible, which is true for $\phi \leqslant \underline{\phi}'(\sigma)$.

We shall prove that in region B there is a PBE where only one bank is active, offers c^{**} and accepts all applications for this contract. By using the same logic, one can show that, for any $c \in K$, in region B there exists a PBE where only one bank offers c and accepts all applications for this contract, while other banks are inactive.

Next, we prove the preliminary result that if bank 1 has issued contract c^{**} and bank 2 tries to undercut it by requiring a repayment below r^*, then there is always a bank 3 that can issue a contract c^d such that both banks 2 and 3 make zero expected profits. Let $\underline{c} = (x, \underline{r})$ be the contract that earns zero profits if an entrepreneur taking this contract and undertaking the large project defaults only in the bad state, i.e. $\pi(\underline{c}, c) = 0$ for $c = (x, r)$, with $r \in ((1-\phi)\, y_L + 1 - \sigma - \underline{r}, (1-\phi)\, y_L + 1 + \sigma - \underline{r}]$ and $r < \underline{r}$. Then:

Lemma 2 *For any $c = (x, r)$ with $x \leq r < r^*$, and $c^d = (x, r^d)$ with $r^d > \underline{r}$, such that $u(c^{PC}, c^d) = u(c^{PC}, c_\emptyset)$, there is a couple $(\tilde{\alpha}, \alpha^d) \in (0,1)^2$ that solves:*

$$\tilde{\alpha}\pi(c^d, c) + (1 - \tilde{\alpha})\pi(c^d, c^{**}) = 0, \tag{A1}$$

$$\alpha^d \pi(c, c^d) + (1 - \alpha^d)\pi(c, c_\emptyset) = 0. \tag{A2}$$

Proof. Equation (A2) holds for some $\alpha^d \in (0,1)$ since $c \notin JIC$ and $r^d > \underline{r}$ imply $\pi(c^d, c) > 0$, while $\pi(c^d, c^{**}) < 0$ because the large project is not viable, and $r^d < r^{**}$. Similarly, $\tilde{\alpha}$ solves (A1) since $\pi(c, c^d) < 0$ because the large project is not viable, and $r^d > r$, while $\pi(c^d, c_\emptyset) > 0$ since $r^d > x$. ■

Now let $\tilde{c}(h_\tau) = (x, \tilde{r}_\tau(h_\tau))$ be the contract with the lowest repayment in $C(h_\tau)$ and let $\widehat{c}\,(h_\tau)$ denote the first contract with a repayment lower than r^*, which is issued before τ. We shall prove that the following strategies and beliefs describe a PBE:

Equilibrium strategies:

$(s1)$ Bank τ issues c^{**} and accepts all applications either when no contract has been issued before τ, or if $\tilde{r}(h_\tau) \geq r^*$ and neither c^{**} nor $\underline{c}$ are offered before τ.

$(s2)$ Bank τ issues $\underline{c}$ and accepts all applications whenever $\tilde{r}(h_\tau) < r^*$, $\underline{c} \notin C(h_\tau)$, $c^{**} \notin C(h_\tau)$.

$(s3)$ Bank τ chooses (c^d, α^d) with α^d solving (A2) for $c = \widehat{c}\,(h_\tau)$, whenever $c^{**} \in C(h_\tau)$, $c^d \notin C\,(h_\tau)$, $c = (x, r) \notin C(h_\tau)$ for all $r \in [r^d, r^{**})$, $\tilde{r}(h_\tau) < r^*$, and either $\underline{c} \notin C\,(h_\tau)$ or $\underline{c}$

is issued after c^{**}. Bank τ issues c^d and accepts all applications if $\tilde{r}(h_\tau) < r^*$, $c^{**} \in C(h_\tau)$, $c^d \notin C(h_\tau)$ and $c \in C(h_\tau)$ with $c = (x, r)$ and $r \in (r^d, r^{**}]$.

($s4$) Bank τ does not issue any contract if some bank has issued $\underline{c}$ or c^{**} or c^d before τ.

We shall assume that if an entrepreneur decides to opt out of some, but not all, the contracts with the same repayment r, he will retain those signed at the earliest stage.

Equilibrium beliefs:

($b1$) If $c_\tau = c^{**}$, then $\mu_\tau = 1$ whenever $\{\underline{c}\} \cup \{c^{**}\} \cap C(h_\tau) = \emptyset$ and $\tilde{r}(h_\tau) > r^*$, while $\mu_\tau = 0$ for any other history.

($b2$) If $c_\tau = c^d$, then $\mu_\tau = \alpha^d$ solving (A2) for $c = \widehat{c}(h_\tau)$, whenever $\tilde{r}(h_\tau) < r^*$, $c^{**} \in C(h_\tau)$, $c = (x, r) \notin C(h_\tau)$ for all $r \in [r^d, r^{**})$, and either $\underline{c} \notin C(h_\tau)$ or $\underline{c}$ is issued after c^{**}. While $\mu_\tau = 1$ if $\tilde{r}(h_\tau) < r^*$, $c^{**} \in C(h_\tau)$ and $c \in C(h_\tau)$ with $c = (x, r)$ and $r \in (r^d, r^{**}]$. Finally, $\mu_\tau = 0$ in all other histories.

($b3$) If $c_\tau = \underline{c}$, then $\mu_\tau = 1$ if $\tilde{r}(h_\tau) < r^*$ and $c^{**} \notin C(h_\tau)$. Otherwise, $\mu_\tau = 0$.

($b4$) If $c_\tau = (x, r_\tau)$ with $r_\tau \geq r^*$ and $r_\tau \neq \{r^d, r^{**}, \underline{r}\}$, then $\mu_\tau = 0$ for any possible h_τ.

($b5$) If $c_\tau = (x, r_\tau)$ with $r_\tau < r^*$, then $\mu_\tau = \tilde{\alpha}$, where $\tilde{\alpha}$ solves (A1) if $r_{\tau'} > r^*$ for all $\tau' < \tau$ or $c_\tau = \widehat{c}(h_\tau)$. Otherwise $\mu_\tau = 0$.

According to the above strategies only bank 1 is active, issues c^{**} and accepts all applications. We shall now show that these strategies maximize bank τ's expected profit in any continuation game, by considering in turn the cases where $\widetilde{r}(h_\tau) \geq r^*$ and $\widetilde{r}(h_\tau) < r^*$.

Case 1: $r_{\tau'} \geq r^*$ for all $\tau' < \tau$.

We prove that in this case it is optimal for bank τ to play according to ($s1$): issue c^{**} and accept all applications if only if no previous bank has issued c^{**}.

Let start by assuming $c^{**} \in C(h_\tau)$. Bank τ does not obtain profits from any policy (c_τ, α_τ), with $r_\tau > r^*$ and $\alpha_\tau > 0$. This is because ($s1$) implies $c_{\tau''} = c^{**}$ and $\alpha_{\tau''} = 1$ for some $\tau'' < \tau$, while $u(c^{**}, c_\tau) > u(c_\tau, c_\emptyset)$ since $r_\tau > r^*$, so that an entrepreneur taking the loan c_τ will also take c^{**}, thereby inflicting losses to bank τ. Moreover, bank τ would earn zero expected profit by issuing $c_\tau = (x, r_\tau)$, with $r_\tau < r^{**}$, and choosing $\alpha_\tau > 0$. Were this contract issued, according to ($s2$) bank $\tau + 1$ would choose (c^d, α^d), with α^d satisfying (A2). Thus, for all τ, bank τ will issue $c_\emptyset$, as prescribed by the equilibrium strategy. Moreover, issuing $c_\emptyset$ is sequentially rational for bank τ, since a bank moving after τ chooses $\left(c^d, \alpha^d\right)$ after observing that $c = (x, r)$ if $r < r^*$ is issued. Similarly, choosing $\left(c^d, \alpha^d\right)$ is sequentially rational at stage $\tau + 1$ since, were c^d not issued, bank $\tau + 2$ would choose $\left(c^d, \alpha^d\right)$.

Consider next the case where $c^{**} \notin C(h_\tau)$. Given ($b1$), $\alpha_{\tau'} = 0$ for all $\tau' < \tau$. Moreover, ($s1$) implies $c_{\tau+m} = c_\emptyset$ for all $m > 0$ if $c_\tau = (x, r^{**})$, while $c_{\tau+1} = c^{**}$ if $c_\tau = (x, r)$, with $r \neq r^{**}$. Thus, the policy $(\alpha = 1, c^{**})$, maximizes bank τ's expected profit given its beliefs. Finally, ($b1$) also implies that all banks remain inactive in the continuation game starting at stage $\tau + 1$ after bank τ issued c^{**}. Hence, sequential rationality is satisfied.

Case 2: $r_{\tau'} < r^*$ for some $\tau' < \tau$.

We prove that in this case it is optimal for bank τ to play according to ($s2$-$s4$): issue either $\underline{c}$ or c^d depending on previous histories, or remain inactive.

Consider first the case in which $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$ and $c^{**} \notin C(h_\tau)$. In this case according to ($s3$) bank τ issues $\underline{c}$ and accepts all applications. This strategy maximizes bank τ's (expected) profits, because according to equilibrium beliefs there will be no acceptances for $c_{\tau'} = (x, r_{\tau'})$ if $c^{**} \notin C(h_\tau)$, while according to equilibrium strategies bank $\tau+1$ issues $\underline{c}$ and accepts all applications if bank τ has not issued $\underline{c}$, and remains inactive otherwise. Therefore, bank τ earns $\pi(\underline{c}, c_\emptyset)$ by offering $\underline{c}$ and $\pi(c, \underline{c})$ if it offers $c = (x, r)$. But $\pi(c, \underline{c}) < \pi(\underline{c}, c_\emptyset)$ for any $r > x$ since the large project is not viable, so that $\underline{c}$ is bank τ's optimal choice. Finally, sequential rationality is satisfied as banks play a Nash equilibrium in any continuation game starting at $\widehat{\tau} > \tau$: any bank moving after τ optimally choose to remain inactive if bank τ issued $\underline{c}$, while it strictly prefers to issue $\underline{c}$ and accept all applications otherwise. This is true because $\underline{c}$ is the only contract that yields positive profits to bank $\tau' > \tau$ according to ($b3$).

Consider now the case in which $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$, $c^{**} \in C(h_\tau)$ and bank τ's equilibrium strategy prescribes to issue c^d and to accept the fraction α^d of applications solving (A2) for $c = c_{\tau''} = (x, r_{\tau''})$. Deviating from this strategy is not profitable for the following reasons. First, requiring a repayment below $r = r^d$ would entail expected losses for bank τ by (A1). Second, if bank τ offers $r > r^d$, bank $\tau+1$ will choose $(c = c^d, \alpha = 1)$ according to ($b2$). Then bank τ will make losses because it will attract only entrepreneurs who already signed jointly incentive compatible contracts. Moreover, equilibrium strategies satisfies sequential rationality. Indeed, in all histories where $r_{\tau''} > r^*$ for all $\tau'' < \tau$, with $\tau'' \neq \tau'$, $c^{**} \in C(h_\tau)$, and a contract with a repayment larger than r^d is issued, bank $\tau+1$ makes zero profit in expectation, according to ($b5$), by choosing $(c^d, \alpha_{\tau+1} \in [0,1])$ while it makes losses, according to ($s3$), if it issues any contract different from c^d and accepts a positive fraction of applications. By the same logic, according to ($s3$), no bank can profitably offer loans, after any history in which $r_{\tau'} < r^{**}$ for some $\tau' < \tau$, and c^d and c^{**} are both issued.

Finally, consider the case where several banks charge less than r^* before stage τ. Bank τ then believes that only the first mover, among those charging less than r^*, accepts applications. The same logic used above can be then used to prove sequential rationality at all τ. ■

Proof of Proposition 3. We now show that, for any $c \in K$, in region B there exists a symmetric PBE such that two banks are active, each issues contract c and rations the applications by choosing randomly among applicants. The proof is developed in two steps.

Step 1. We first consider c^{**}, the contract with the highest repayment in K. Suppose that there is PBE with the following features: (i) only two banks are active, say τ and τ', each offers $c^{**} = (x, r^{**})$ and accepts a fraction α of its applications; (ii) entrepreneurs who borrow x undertake the small project, those who manage to borrow $2x$ undertake the large one.

The expected per-client profit of each active bank is

$$\Pi(\alpha) = (1-\alpha)\pi(c^{**}, c_\emptyset) + \alpha\pi(c^{**}, c^{**}).$$

The first term of $\Pi(\alpha)$ is bank τ's expected profit if bank τ' does not accept the entrepreneur's application, while the second is bank τ's expected profit if bank τ' accepts this application. Hence, a first step to prove the existence of a PBE satisfying (*i*)-(*ii*) is to show that $\Pi(\alpha) = 0$ has a solution $\alpha \in (0,1)$. Since $\Pi(\cdot)$ is continuous and monotone in α, $\Pi(0) > 0$ because $y_S \geq r^{**} > x$, and $\Pi(1) < 0$ because the large project is unviable and $\pi(c^{**}, c^{**}) < 0$, then equation $\Pi(\alpha) = 0$ has a unique internal solution:

$$\alpha^{**} = \frac{\pi(c^{**}, c_\emptyset)}{\pi(c^{**}, c_\emptyset) - \pi(c^{**}, c^{**})}.$$

Step 2. We now show that there exists a zero-profit PBE with the features described in step 1.

Equilibrium strategies:

Formally, bank τ's equilibrium strategy is defined by five properties ($s1'$-$s5'$). Property ($s1'$) is a modified version of ($s1$) in the proof of Proposition 2: bank τ issues c^{**} and accepts applications with probability α^{**} when no contract has been issued before τ, or if $\tilde{r}(h_\tau) \geq r^*$, $c^{**} \notin C(h^\tau)$ and $\underline{c} \notin C(h^\tau)$. Otherwise, it issues $c_\emptyset$.

Properties ($s2'$) and ($s3'$) are identical to ($s2$) and ($s3$) stated in the proof of Proposition 2. Instead, ($s4'$) is stated below:

($s4'$) Bank τ issues $c_\emptyset$ if: (a) two other banks already issued contract c^{**} before τ; or (b) at least one bank issued c^{**} and another issued c^d; or (c) some bank issued $\underline{c}$, and $\tilde{r}(h_\tau) < r^*$.

Equilibrium beliefs:

Equilibrium beliefs satisfy five properties ($b1'$-$b5'$). Properties ($b3'$) and ($b4'$) are the same as ($b3$) and ($b4$) stated in the proof of Proposition 2. Instead, ($b1'$), ($b2'$) and ($b5'$) are:

($b1'$) If $c_\tau = c^{**}$, then $\mu_\tau = \alpha^{**}$ if $c^{**} \notin C(h_\tau)$ and $\widetilde{r}(h_\tau) > r^*$, or if only one bank before τ issued c^{**} and $\widetilde{r}(h_\tau) > r^*$. Instead, $\mu_\tau = 0$ for any other history.

($b2'$) If $c_\tau = (x, r_\tau)$, with $r_\tau < r^*$, then $\mu_\tau = \widetilde{\alpha}$, where $\widetilde{\alpha}$ solves

$$\alpha^{**}(1-\widetilde{\alpha})\pi(c^d, c^{**}) + \widetilde{\alpha}\pi(c^d, c) + (1-\widetilde{\alpha})(1-\alpha^{**})\pi(c^d, c_\emptyset) = 0, \tag{A3}$$

when $c_\tau = \widehat{c}(h_\tau)$, $c^d \in C(h_\tau)$ and only one contract c^{**} was issued before τ. Instead, $\mu_\tau = \widehat{\alpha}$ solving

$$2\alpha^{**}(1-\widehat{\alpha})\pi(c^d, c^{**}) + \widehat{\alpha}\pi(c^d, c) + (1-\widehat{\alpha})(1-\alpha^{**})^2\pi(c^d, c_\emptyset) = 0, \tag{A4}$$

when $c_\tau = \widehat{c}(h_\tau)$, $c^d \in C(h_\tau)$ and two banks issued c^{**} before τ. Otherwise, $\mu_\tau = 0$.

($b5'$) If $c_\tau = c^d$, then $\mu_\tau = \alpha^d$ if $\widetilde{r}(h_\tau) < r^*$, $c^{**} \in C(h_\tau)$, $c^d \notin C(h_\tau)$, and either $\underline{c} \notin C(h_\tau)$ or $\underline{c}$ was issued after c^{**}. Otherwise, $\mu_\tau = 0$.

Note that $\widetilde{\alpha} \in (0,1)$ because $\pi\left(c^d, c^{**}\right) < 0$, since c^{**} and c^d are both contained in K and $\underline{r} < r^d < r^{**}$. By the same token, also $\widehat{\alpha} \in (0,1)$.

Going through the same logical steps used to prove Proposition 2, the above strategies and beliefs can be shown to identify a PBE. These steps are omitted for brevity. These arguments can also be extended to show that for any $c \in K$, there exists a rationing equilibrium where two banks are active and symmetrically offer c by choosing randomly among applicants. ■

Proof of Proposition 4. All rationing equilibria satisfy the following zero-profit condition:

$$\alpha\pi\left(c, c\right) + (1-\alpha)\pi\left(c, c_{\emptyset}\right) = 0,$$

where $c = (x, r)$ with $r \in (r^{*}, r^{**})$. Total differentiation with respect to α and r yields:

$$\frac{d\alpha}{dr} = -\frac{\alpha\frac{d\pi(c,c)}{dr} + (1-\alpha)\frac{d\pi(c,c_{\emptyset})}{dr}}{\pi\left(c, c\right) - \pi\left(c, c_{\emptyset}\right)},$$

where the numerator is positive because a higher repayment (weakly) increases profits in all states, and the denominator is negative because $\pi\left(c, c\right) < 0$ and $\pi\left(c, c_{\emptyset}\right) > 0$ for $r \in K$. Hence, $d\alpha/dr > 0$. To complete the proof, note that if applicants who obtain two loans default in both states, the default probability is $\delta = \alpha^{2}/\left[2\alpha(1-\alpha) + \alpha^{2}\right]$, while it is $\delta/2$ if they default in the bad state only. Since $d\delta/d\alpha > 0$, in both cases the default probability is increasing in α. ■

Proof of Proposition 5. We start by showing that in the region under consideration there is a zero-profit PBE where one bank issues the monopoly contract c^{M} and accepts a fraction $\alpha^{M} < 1$ of applicants, two banks issue the usurious contract $c^{U} = (x, r^{U})$ with $r^{U} > y_{S}$ and accept a fraction of applicants $\alpha^{U} < 1$, while the remaining banks stay inactive. In this equilibrium, each entrepreneur applies to all active banks and, when indifferent between the loans offered by two of them, randomizes with equal probability. For convention, we shall assume that bank 1 issues c^{M} and bank 2 and 3 issue c^{U}. The zero-profit conditions corresponding to this outcome are:

$$\Pi_{1} = \alpha^{M}\left[\left(1-\alpha^{U}\right)^{2}\pi(c^{M}, c_{\emptyset}) + (2(1-\alpha^{U})\alpha^{U} + (\alpha^{U})^{2})\pi(c^{M}, c^{U})\right] = 0, \tag{A5}$$

$$\Pi_{2} = \Pi_{3} = \alpha^{U}\left[\left(\alpha^{M}\left(1-\alpha^{U}\right) + \frac{\alpha^{M}\alpha^{U}}{2}\right)\pi\left(c^{U}, c^{M}\right) + \left(1-\alpha^{M}\right)\alpha^{U}\pi\left(c^{U}, c^{U}\right)\right] = 0, \tag{A6}$$

where the left-hand-side of (A5) is the expected profit of bank 1, earning $\pi(c^{M}, c_{\emptyset}) > 0$ on the fraction $(1-\alpha^{U})^{2}$ of clients whose applications for c^{U} are refused and $\pi(c^{M}, c^{U}) < 0$ on the fraction $(2(1-\alpha^{U})\alpha^{U} + \left(\alpha^{U}\right)^{2})$ of clients who obtain at least one acceptance for c^{U}. Similarly, the right-hand side of (A6) is the expected profit of bank 2 and 3, which earn $\pi\left(c^{U}, c^{M}\right) > 0$ on the fraction $\alpha^{M}(\left(1-\alpha^{U}\right) + (\alpha^{U}/2))$ of clients who successfully apply for c^{M} and $\pi\left(c^{U}, c^{U}\right) < 0$ on the fraction $\left(1-\alpha^{M}\right)\alpha^{U}$ of clients who sign at least another usurious contract. It is straightforward to verify that (A5) and (A6) have a unique solution $(\alpha^{M*}, \alpha^{U*}) \in (0,1)^{2}$.

Finally, to prove the main result of the proposition, denote by

$$\Pi^{dev}(c) = \alpha^{M}\pi\left(c, c^{M}\right) + (2-\alpha^{U})\alpha^{U}\left(1-\alpha^{M}\right)\pi\left(c, c^{U}\right)$$

the profit per loan that a bank earns if it offers $c = (x, r)$ with $r \in (y_{S}, r^{U})$, if another bank issues c^M and accepts the fraction α^M of applicants, two other banks issue c^U and accept the fraction α^U of applicants, and the remaining banks remain inactive. In the following, we shall prove that there exists a PBE supported by the following strategies and beliefs.

Strategies: Entrepreneur e applies for all contracts; if only one bank, say bank τ, accepts his application, he uses this loan to fund investment if and only $r_\tau \leq y_S$; if two or more of his applications are accepted, entrepreneur e takes two of the loans with the lowest contractual repayments, by randomizing with equal probability between loans with identical repayment.

Banks' strategies have the following features:

($s1$) If $c^M \notin C(h_\tau)$, bank τ issues c^M and accepts the fraction α^{M*} of applications; if $c^M \in C(h_\tau)$ and at most one bank has offered c^U before τ, bank τ issues c^U and accepts the fraction α^{U*} of applications.

($s2$) If $c^M \in C(h_\tau)$, $c^U \in C(h_\tau)$, c^U was issued by at least two banks, bank τ issues c' such that $\Pi^{dev}(c') = 0$ and accepts all applications, if some contract c such that $\Pi^{dev}(c) > 0$ was already issued.

($s3$) In all other histories, bank τ remains inactive.

Beliefs: Each bank believes that any deviating competitor refuses all applications.

Consider first a bank that deviates by issuing $c = (x, r)$ with $r \in (\underline{r}, r^U)$ at some stage τ. According to equilibrium beliefs (μ) and competitors' equilibrium strategies, this bank earns the expected profit per loan $\Pi(c; C(h_t), \mu) = (1 - \alpha^{M*})\pi(c, c')$. This deviation is unprofitable, i.e. $\pi(c, c') < 0$, because: first, by construction $r' > \underline{r}$ (the break-even repayment conditional on no default in the good state); second, an entrepreneur signing $\hat{c} = (x, \hat{r})$ with $\hat{r} > \underline{r}$ from two banks (and undertaking the large project) must default in both states, since otherwise both banks would make zero profits, contradicting the assumption that the large project is unviable, implying $\pi(c', c') < 0$; third, c will also induce default in both states, if taken jointly with c', since $r > r'$, implying $\pi(c, c') < 0$.

Moreover, no bank can profitably offer a rate $r \in (x, y_S) \cup (r^U, \infty)$. Indeed, given competitors' strategies and its own beliefs, any bank offering $c' = (x, r')$ with $r' < y_S$ earns an expected profit per loan $\Pi(c'; C(h_\tau), \mu)$ equal to

$$(1 - \alpha^M)(1 - \alpha^U)^2 \alpha^M \pi(c', c_0) + \alpha^M \pi(c', c^M) + (1 - \alpha^M)(2 - \alpha^U)\alpha^U \pi(c', c^U).$$

One can easily verify that $\Pi(c'; C(h_\tau), \mu) < \Pi_1 = 0$. In addition, given competitors' strategies, a bank offering $c'' = (x, r'')$ with $r'' > r^U$, earns the expected profit per loan:

$$\Pi(c'; C(h_\tau), \mu) = \alpha^M (1 - \alpha^U)^2 \pi(c'', c^M) + (1 - \alpha^M) 2(1 - \alpha^U)\alpha^U \pi(c'', c^U).$$

Again, it is easy to check that $\Pi(c''; C(h_\tau), \mu) < \Pi_2 = 0$. Thus, banks' and entrepreneurs' strategies define a Nash equilibrium. Finally, it is straightforward to verify that no bank can profitably deviate by issuing a contract $c = (x, r)$ with $r \in [y_S, \underline{r})$.

Moreover, banks' strategies are sequentially rational because, after any history, the continuation game starting at τ has a PBE where all banks use equilibrium strategies, given that banks moving after τ hold equilibrium beliefs.

To complete the proof, it remains to be shown that (i) there is no PBE satisfying **A6** in which all funded entrepreneurs undertake the small project and (ii) there is a PBE where no entrepreneur is funded. Both these results are immediate. First, in any equilibrium candidate where all entrepreneurs undertake the small project, a deviating bank would earn a strictly positive profit by issuing $c' = (x, r')$ with $r' = \underline{r} + \varepsilon$ and $\varepsilon > 0$, provided no other bank accepts applications. This implies that **A6** cannot be satisfied in any equilibrium where some entrepreneurs are funded and they all undertake the small project. Second, in the region under consideration there is an equilibrium with market freeze, supported by the following banks' strategies: for any τ, bank τ issues $\underline{c}$ and accepts all applications whenever $\underline{c}$ has not been issued before τ. The details of these proofs are straightforward and therefore are omitted. ■

Remark: under information sharing banks may condition their acceptances to the entrepreneur's past credit history. Taking this into account requires some additional notation: bank τ conditions the acceptance of a loan application to the pre-existing set of contracts $\hat{C}(h_\tau)$, which applicants signed and did not opt out before stage τ. For instance, $\hat{C}(h_\tau) = c_\emptyset$ means that bank τ only accepts applications from entrepreneurs with zero debt. Moreover, let $C_e(h_\tau)$ be the set of contracts that entrepreneur e *actually* signed up to stage $\tau - 1$, and from which he did not opt out before applying to bank τ.

Proof of Proposition 6. We shall first prove existence and then uniqueness.

Existence. We show that, with information sharing, in the region of parameters $A' \cup B'$ there exists a SPNE where only bank 1 is active and issues c^*, all entrepreneurs borrow from this bank and undertake the small project. Showing this is equivalent to proving that in region A' there is a competitive and efficient equilibrium, since from the proof of Proposition 2 $c^* = c^{PC}$ for $\phi \leq \underline{\phi}'(\sigma)$. Consider the following strategies:

(i) Banks' strategies:

$(b1)$ bank 1 issues c^* and accepts all applications for this contract;

$(b2)$ for any history h_τ where all contracts issued at $\tau' < \tau$ charged repayments $r_{\tau'}$ not lower than r^*, bank τ issues c^* and accepts entrepreneur e's application if and only if $C_e(h_{\tau-1}) = \varnothing$ or this entrepreneur opted out of all pre-existing contracts before applying for c_τ;

$(b3)$ for any history h_τ such that some bank has previously offered a contract with repayment $\widetilde{r}(h_\tau)$ below r^*, bank τ issues $\underline{c}$ and accepts applications by entrepreneurs who signed at most one contract requiring a repayment below r^*.

(ii) Entrepreneurs' strategies:

($e1$) for any possible previous history, entrepreneurs apply for all contracts;

($e2$) the entrepreneur e accepts a new contract c_τ and retains a previously accepted contract $\hat{c}$ that bank τ does not require him to drop, if and only if the pair $(c_\tau, \hat{c})$ makes him better off than any other pair of contracts (c', c'') that he already signed. Formally, he opts out of all the previously accepted contracts in $C_e(h_{\tau-1})$ but not in $\hat{C}(h_\tau)$ and applies for contract c_τ if and only if $u(\hat{c}, c_\tau) > u(c', c'')$, for some $\hat{c} \in \hat{C}(h_\tau) \cup \{c_\emptyset\}$ and for all pairs (c', c'') in $C_e(h_{\tau-1}) \cup \{c_\emptyset\}$, provided $\hat{C}(h_\tau) \subseteq C_e(h_{\tau-1})$;

($e3$) if instead the entrepreneur e has not previously signed any contract $\hat{c}$ required by bank τ, i.e. $\hat{C}(h_\tau) \nsubseteq C_e(h_{\tau-1})$, he retains all the previously accepted contracts.

Consider first bank 1's deviations. Bank 1 cannot increase its profits by charging a repayment amount below r^*, since according to equilibrium strategies all entrepreneurs sign c^* with this bank and do not opt out of it. Bank 1 cannot increase its profits by charging a higher repayment either, because it would be successfully undercut: given ($b2$), bank 2 will issue $c_2 = c^*$ and accept all applications of entrepreneurs with no debt at the application stage.

Next, after bank 1 issues c^*, no subsequent bank can profitably deviate. First, no other bank can gain by issuing a contract requiring a repayment larger than r^*, because by ($b2$) entrepreneurs will never find it profitable to sign such a contract. Second, no bank $\tau > 1$ cannot gain by issuing a contract $c_\tau = (x, r_\tau)$, with $r_\tau < r^*$: if it did, according to (b_3) a third bank would offer the zero-profit contract $\underline{c}$, the entrepreneur would be better off by bundling $\underline{c}$ and c_τ, and opting out of c^*. As a result, bank τ would make losses, because the entrepreneur will undertake the unviable project and the bank that offered $\underline{c}$ makes zero profits.

For any history h_τ where $c_{\tau'} = (x, r_{\tau'})$, with $r_{\tau'} \geq r^*$ for all $\tau' < \tau$, bank τ's strategy satisfies perfection since by ($e3$) an entrepreneur will only retain the first one of several identical contracts that he signed. Next, consider an history h_τ such that $\widetilde{r}(h_\tau) < r^*$. Then the strategy of all banks moving after τ prescribes to offer $c_\tau = \underline{c}$ and accept applications from all entrepreneurs who have taken only one loan with repayment lower than r^*. These strategies, given ($e3$), are part of a perfect equilibrium in the subgame starting at τ. By the same logic, subgame perfection holds for all τ.

Finally, consider possible deviations by entrepreneurs. Since banks' strategies condition acceptances only on the total indebtedness of entrepreneurs at the application stage, and not on their entire history of applications and acceptances, and since entrepreneurs can always opt out of previous loan contracts if optimal, the latter are always better off accepting all the available loan offers. Hence, ($e1$) and ($e2$) imply that entrepreneurs strategies are sequentially rational at any τ, given that banks follow their equilibrium strategies from τ on.

Uniqueness. We now show that the SPNE characterized in step 1 is unique in $A' \cup B'$. First, there cannot be a SPNE where some entrepreneurs sign the contract $c = (x, r)$, with $r < r^*$. This is trivial in region A' where $c^* = c^{PC}$. Consider instead region B'. In this region a bank issuing c makes losses in any subgame after c is issued. This is because according to **A6** some subsequent bank will issue either $\underline{c}$ or $c' = (x, \underline{r} + \varepsilon)$ with ε such that $u(c', c) > u(c, c_\emptyset)$, and accept applications by entrepreneurs with contract c. Second, there cannot be a SPNE where some entrepreneurs sign $c = (x, r)$, with $r > r^*$, and undertake the small project: this is because contract c can be undercut by a cheaper and jointly incentive compatible contract. Finally, it is immediate that in region $A' \cup B'$ there cannot exist a SPNE where some entrepreneurs are excluded from credit: since c^* is jointly incentive compatible, the contract charging $r^* < y_S$ is profitable and make the entrepreneur better off than with no borrowing. ■

Proof of Proposition 7. We shall first prove existence and then uniqueness.

Existence. We prove that there is a no-trade SPNE equilibrium where agents' strategies satisfy the following properties.

(i) Banks' strategies:

For any possible previous history, bank τ issues $\underline{c}$ and accepts entrepreneur e's application if and only if this entrepreneur has signed only one contract with repayment lower or equal the monopoly repayment y_S.

(ii) Entrepreneurs' strategies are the same as in the proof of Proposition 6.

We start by proving existence. Consider first banks' deviations. First, suppose that bank τ issues $c_\tau = (x, r_\tau)$ with $r_\tau \in (x, y_S)$ and accepts applications from entrepreneurs with zero total indebtedness at τ (for whom $\hat{C}(h_\tau) = c_\emptyset$); then it will make losses, given competitors' strategies. This is because all entrepreneurs who sign c_τ will also succeed in obtaining $\underline{c}$ from some other bank, and in this region they will actually prefer to take both c_τ and $\underline{c}$ and invest in the large and unviable project. Second,bank τ cannot profitably deviate by issuing any $c_\tau = (x, r_\tau)$ and accepting applications only from entrepreneurs who already signed one or more contracts. This is because, according to the banks' equilibrium strategy, no entrepreneur will be able to sign any contract before c_τ is offered. Thus, bank τ will end up making zero profits. Moreover, banks' strategies are are part of a perfect equilibrium: for all τ, the equilibrium strategy of bank τ maximizes its expected profit for any possible previous history, if the banks moving after τ stick to their equilibrium strategies.

Consider next the entrepreneurs: proving that their strategies are sequentially rational at all stages follows the same logic as in the proof of Proposition 6.

Uniqueness. Suppose there is a SPNE in which a subset of entrepreneurs is funded. For all banks to at least break even, these entrepreneurs must get a loan of size x and undertake the small project. If so, entrepreneurs' rationality requires that they will accept a contract $c_\tau = (x, r_\tau)$ with $r_\tau \in [x, y_S]$. But in this region any such contract c_τ violates joint incentive compatibility, i.e. $u(c_\tau, \underline{c}) > u(c_\tau, c_\emptyset)$, and since contract $\underline{c}$ entails no losses, it will be issued by some other bank according to **A6**. Hence, contract c_τ makes losses, and no bank will accept applications for it. Hence, c_τ will not be offered in equilibrium. ■

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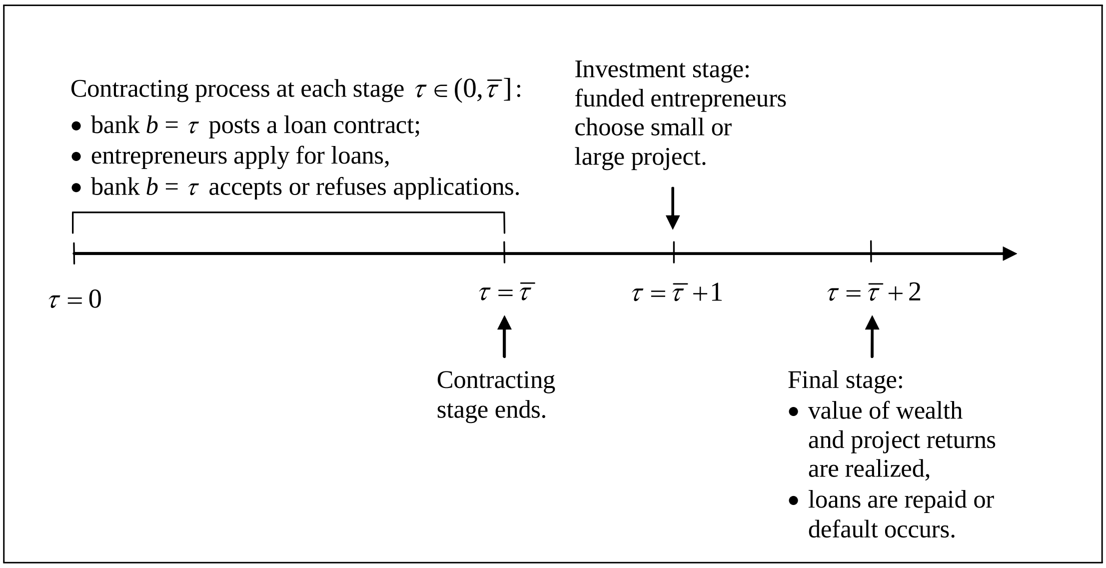

FIGURE 1. TIME LINE

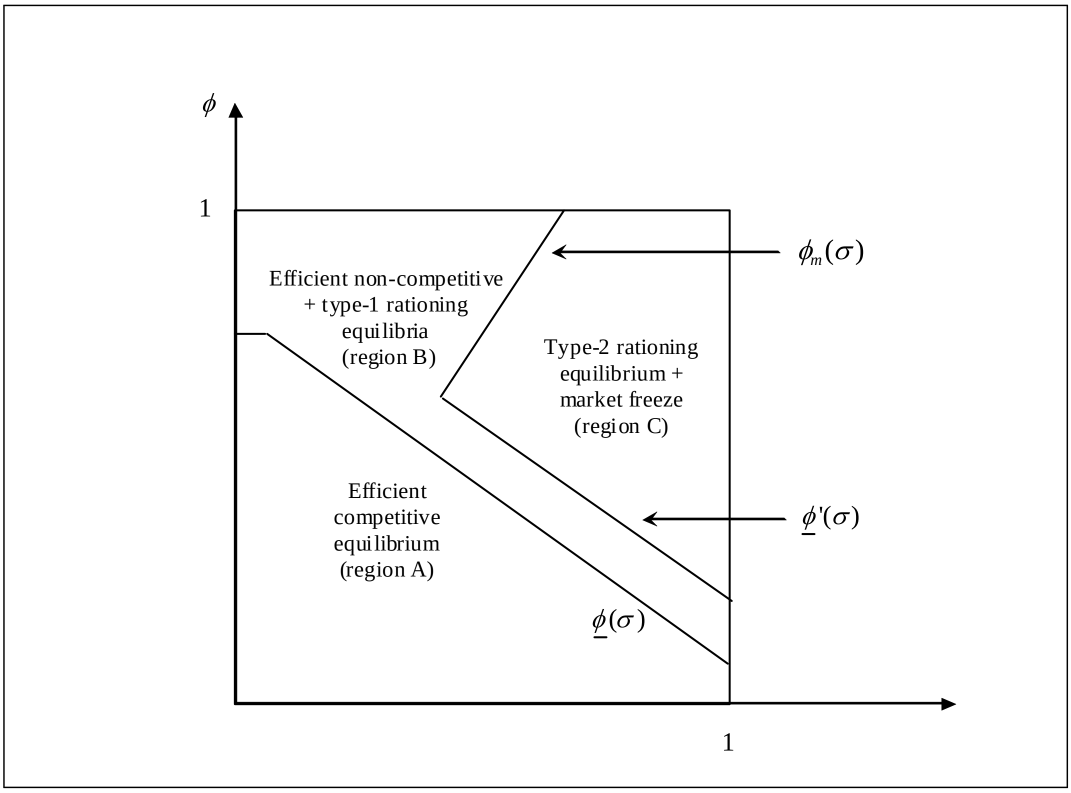

FIGURE 2. EQUILIBRIA WITH NO INFORMATION SHARING

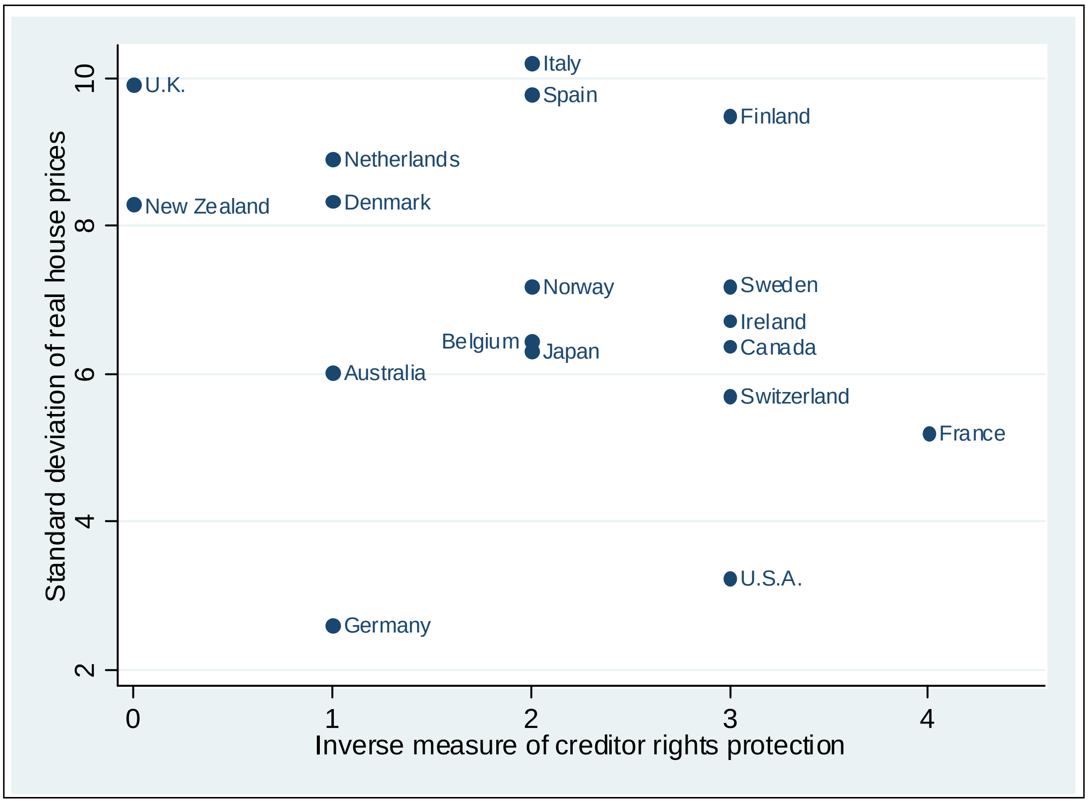

Figure 3. Volatility of collateral and creditor rights

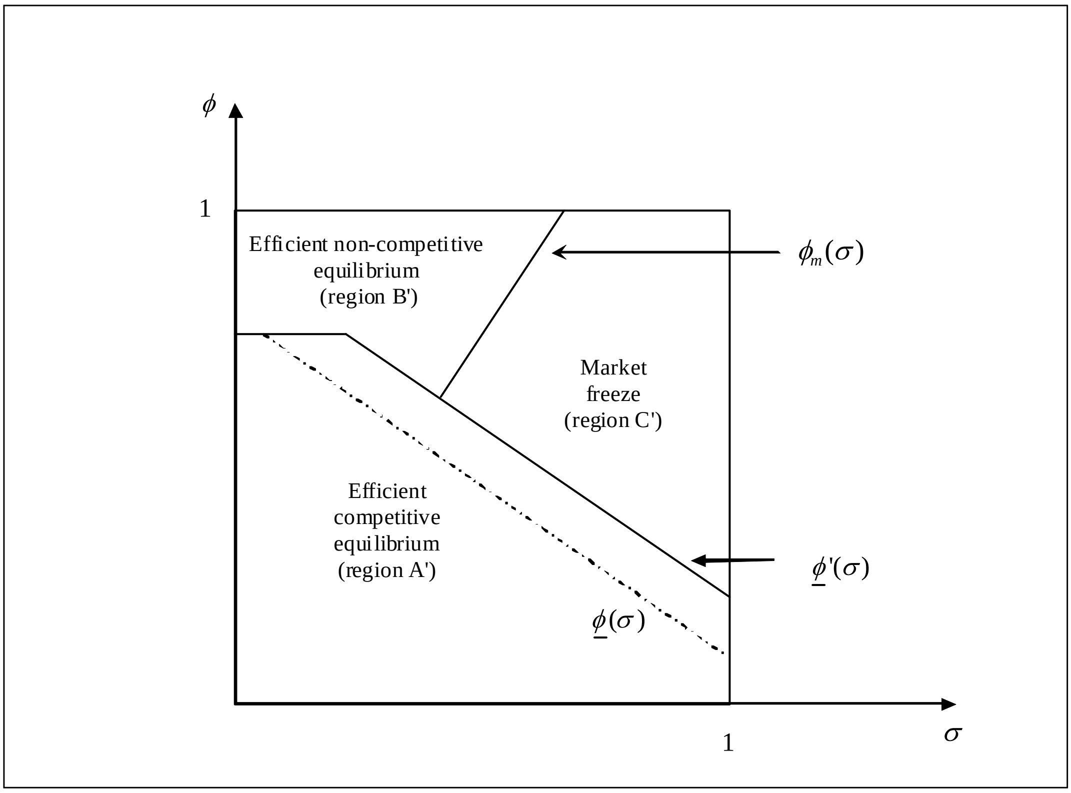

Figure 4. Equilibria with information sharing and pro-rata liquidation

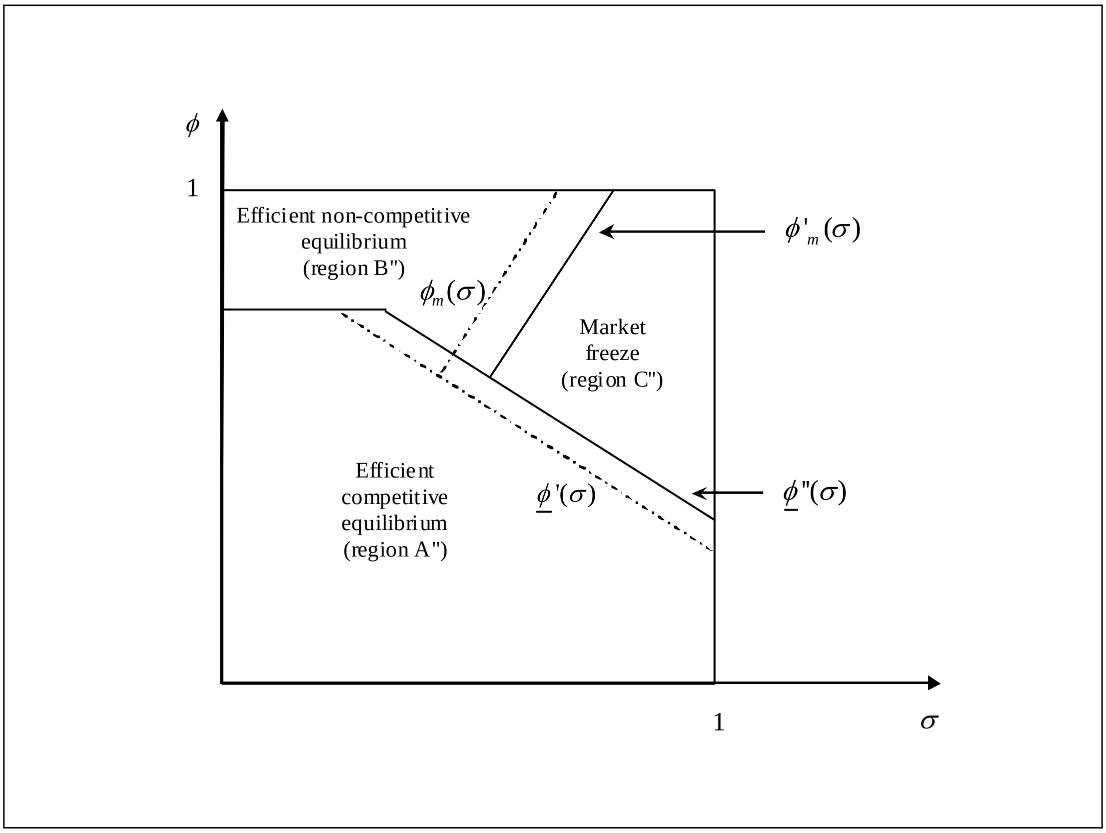

FIGURE 5. EQUILIBRIA WITH INFORMATION SHARING AND SENIORITY-BASED LIQUIDATION