Selection of boundedly rational firms and the allocation of resources

Gilles Saint-Paul

The ‘as if’ argument

- The ‘as if’ argument rests on an analogy between profits and fitness.
- Firms that are more profitable are assumed to drive less profitable firms out of business
- By growing, they bid factor prices up to the point where profits=0, imposing losses on less efficient firms.

A critique

- In the nature, fitter agents replicate faster and get more resources.
- In the economy, profitability =/= replicability: a less profitable firm may get more resources by bidding for them in the market
- Conversely, firms that get more resources by bidding more are likely to be irrational (the winner's curse)

Example I: Auctions

- Mobile phone operators are competing for frequencies.
- Those who most overestimate the value get awarded the frequencies.
- That results in a misallocation of resources if these operators are at the same time not the most efficient.

Example II: the allocation of credit

- Entrepreneurs have different projects and compete for funds.
- They observe their return with an error.
- Those who most overestimate their returns are willing to borrow at a higher interest rate
- If making a large mistake is correlated with having a low true return, funds are channelled to the wrong projects

Example III: Mispricing

- Firms compete to sell the same good, they have the same costs, and there are constant returns to scale.
- A (possibly tiny) fraction of firms "gets it wrong" and charges below cost.
- The firm which gets it wrong most gets the whole market--and experiences large losses.
- Thus, profit maximizers do not matter: the allocation of resources is determined by the firm that has made the largest downward mistake in setting its price.

The importance of the financial sector

- In these examples, activity goes to loss-making agents
- These have to be eliminated by some institutional mechanism, which is itself the by-product of evolution
- However, what a good mechanism is is not obvious

Selection is two-sided

- The lowest priced firm attracts most customers
- The highest rate of return firm attracts most funds
- The resulting allocation of resources is the outcome of these two competing selection processes

Deviation from competitive equilibrium is fueled by two effects

- A pecuniary externality: giving more funds to firm A allows it to attract more customers, thus reducing customers to firm B
- A learning externality: one cannot learn the rate of return of firms that are not operating.

A simple model of mispricing

- *N* firms, all producing a single homogenous good.
- Two other goods, "cash" and an input good.
- Cash is the numéraire
- The price of the input good is normalized to one, which is just a choice of measurement units.

Firms

- Firm i has a cost c_i and a price p_i .
- Firms are ranked by increasing prices: $p_j > p_i$, $j > i$.
- Thus, the production function of firm i is

$$y_i = x_i / c_i,$$

CIA constraint

- Inputs must be bought in advance
- Cash is needed to buy inputs
- It is provided by savers

$$x_i \leq k_i.$$

Rationing

- The demand curve is $D(p)$
- Customers get the lowest possible price
- Rationing is proportional
- Demand for i is $\gamma_i D(p_i)$
- Output is $y_i = \min(\gamma_i D(p_i), k_i / c_i)$.
- If $\gamma_i D(p_i) > k_i / c_i$, $\gamma_j = 0$, $j > i$.
- If $\gamma_i D(p_i) < k_i / c_i$,

$$\gamma_{i+1} = \gamma_i \left(1 - \frac{k_i / c_i}{\gamma_i D(p_i)}\right)$$

PROPOSITION 1 – Let $i_{\max} = \min\{i, \sum_{j=1}^{i-1} \frac{k_j}{c_j D(p_j)} \geq 1\}$, *which by (2) exists. For any given allocation of cash* $\{k_1, ..., k_N\}$, *the allocation of output is* $y_i = k_i/c_i$ *for all* $i < i_{\max}$,

$$y_{i_{\max}} = \left(1 - \sum_{j=1}^{i_{\max}-1} \frac{k_j}{c_j D(p_j)}\right) D(p_{i_{\max}}),$$

and $y_i = 0$, *for all* $i > i_{\max}$.

Savers

- Continuum of savers of mass 1 with total cash K
- We assume the cash constraint is not binding for the market as a whole:
- $K \geq \max_i c_i D(p_i)$

The ex-post allocation of cash

- Savers maximize end-of-period cash
- A saver λ lends $k_i(\lambda)$ to firm *i.*
- Firm *i* ends with $k'_i = k_i + p_i y_i - c_i y_i$ units of cash
- Cash is rebated proportionally to the 'shareholders'; saver λ gets

$$k'_i(\lambda) = \frac{k'_i}{k_i} k_i(\lambda).$$

A benchmark: the optimum

- First best: pick the lowest cost firm and force it charge at marginal cost
- Second best: a firm gets positive cash only if the surplus it generates, if it had the whole market, is maximum.

Computing social welfare

$$CS_i = \frac{y_i}{D(p_i)} \int_{p_i}^{+\infty} D(q)dq.$$

$$\begin{aligned} SW &= \sum_{i=1}^{i_{\max}} CS_i + k_i' \\ &= K + \sum_{i=1}^{i_{\max}} y_i \omega_i, \end{aligned}$$

$$\omega_i = \frac{1}{D(p_i)} \int_{p_i}^{+\infty} D(q)dq + p_i - c_i.$$

Characterizing feasible allocations

LEMMA 1 – The two following properties are equivalent

(A) $\{y_1, ..., y_N\} \in S$

(B) $0 \leq y_i \leq D(p_i)\left[1 - \sum_{j=1}^{i-1} \frac{y_j}{D(p_j)}\right]$, *for all* i, *with* $y_N = D(p_N)\left[1 - \sum_{j=1}^{N-1} \frac{y_j}{D(p_j)}\right]$.

$$\sum_{j=1}^{N} \frac{y_j}{D(p_j)} = 1. \qquad (4)$$

Consequently,

LEMMA 2 – Let $s_i = D(p_i)\omega_i = \int_{p_i}^{+\infty} D(q)dq + (p_i - c_i)\,D(p_i)$ *be the total surplus generated by firm i if it had the whole market. The allocation of production that maximizes social welfare is defined by* $y_j = h_j D(p_j)$, *where* h_j *is the set of weights which maximize* $\sum_{i=1}^{N} h_i s_i$, *subject to* $h_i \geq 0$ *and* $\sum_{i=1}^{N} h_i = 1$.

PROPOSITION 2 – An allocation of production $\{y_i\}$, $y_i \geq 0$, *maximizes social welfare if and only if:*

$$\{y_1, ..., y_N\} \in S, \text{ i.e. (4) holds,}$$

and

$$y_i > 0 \Longrightarrow i \in \arg\max_i s_i.$$

Equilibrium

- Equilibrium holds if, holding other savers' allocation constant, a saver cannot profitably reallocate his savings.
- In equilibrium, I pick the firm which maximizes my rate of return $r_i = k_i'/k_i$

PROPOSITION 3 – Consider an allocation of capital $\{k_i(\lambda)\}$*,and the associated allocation of output* $\{y_i\}$ *and end-of-period cash* $\{k_i'\}$*. Let* r_i*, the "rate of return" of firm* i*, be defined as*

$r_i = p_i/c_i,\ i < i_{\max}$

$r_{i_{\max}} = 1 + \gamma_{i_{\max}} D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})/k_{i_{\max}}$

$r_i = 1,\ i > i_{\max}.$

Then the allocation is in equilibrium if and only if

$$k_i > 0 \Longrightarrow i \in \arg\max_i \{r_i\}.$$

Three additional assumptions

$ASSUMPTION\ 1 - i \neq j \Longrightarrow p_i/c_i \neq p_j/c_j.$

$ASSUMPTION\ 2 - \exists i,\ p_i > c_i.$

ASSUMPTION 3 – $\forall i, p_i \neq c_i.$

LEMMA 3 – Assume assumptions 1 and 2 hold; then

(i) in any equilibrium $y_i = 0$ *for at least* $N - 2$ *firms*

(ii) in any equilibrium $k_i = 0$ *if* $y_i = 0$ *or* $p_i < c_i$.

2-firm equilibria are unstable:

DEFINITION – An equilibrium is pairwise stable if and only if

(i) $\forall i, j$ *s.t.* $i \neq j, k_i k_j > 0$,

$$\frac{\partial r_j}{\partial k_i} + \frac{\partial r_i}{\partial k_j} > \frac{\partial r_j}{\partial k_j} + \frac{\partial r_i}{\partial k_i}.$$

(ii) $\forall i, j$ *s.t.* $k_i > k_j = 0$, $r_i > r_j$

LEMMA 4 – An equilibrium such that $k_i > 0$ *for more than one firm is not pairwise stable.*

Why?

- Relocating cash to the inframarginal firm leaves its ROR unchanged
- It reduces cash at the marginal firm, which tends to increase its ROR
- But customers shift from the marginal to the inframarginal firm
- The latter effect dominates, and the marginal firm's ROR falls

PROPOSITION 4 – An allocation of cash $\{k_i\}$ *is a pairwise stable equilibrium if and only if there exists* i *such that*

(i) $k_i = K;\ k_j = 0,\ j \neq i$

(ii) $p_i > c_i$

(iii) $\forall j < i,\ p_j / c_j \leq 1 + \frac{(p_i - c_i) D(p_i)}{K}$.

Comments

- An equilibrium always exists: take the ‘minimal’ one, i.e. give all cash to lowest priced profitable firm.
- However, other firms may satisfy i ➔ multiple equilibria
- Some monopoly power may be sustained
- Equilibrium converges to minimal as K goes to infinity

Multiplicity

- Here, equilibrium multiplicity comes from the *pecuniary externality.*
- More efficient firms need cash to steal customers from less efficient one;
- But the first unit of cash given to them will yield a lower rate of return than in dominant firm, which has a higher markup.
- However, cash to dominant firm would attract no customer if all cash goes to the more efficient firm.

Efficiency

- As K goes to infinity, most cash is idle, and ROR converges to one.
- Thus, any cash given to a profitable, lower priced firm, would yield a better ROR.
- The only equilibrium is the minimal one.
- If c is constant across firms, and distribution of prices dense enough, minimal equilibrium arbitrarily close to walrasian outcome.

Summary

- Here, all RORs are observable ex-ante
- Selection by shareholders dominates and creates a bias in favor of overpricing (monopoly power)
- Selection by customers generates a pecuniary externality ➔ multiple equilibria
- If c is constant, then most efficient (minimal) equilibrium selected as cash becomes very abundant
- If enough firms, then very close to walrasian outcome.

Entrants

- 1 incumbent, cost c_I, price $p_I >= c_I$
- 1 entrant, known cost c_E, unknown price p
- p distributed with density $f(p)$
- Look for equilibria where the entrant gets k_E and the incumbent $K - k_E$

4 possibilities

- Entrant underbids incumbent and is constrained by cash
- Entrant gets the whole market
- Incumbent underbids and is cash constrained
- Incument gets the whole market

$$r_E(p, k_E) = p/c_E.$$

$$r_I(p, k_E) = 1 + (p_I - c_I)(1 - \frac{k_E}{c_E D(p)})\frac{D(p_I)}{K - k_E}.$$

$$r_E(p, k_E) = 1 + (p - c_E)\frac{D(p)}{k_E},$$

$$r_I(p, k_E) = 1.$$

$$r_E(p, k_E) = 1 + (p - c_E)(1 - \frac{K - k_E}{c_I D(p_I)})\frac{D(p)}{k_E},$$

$$r_I(p, k_E) = \frac{p_I}{c_I}.$$

$$r_E(p, k_E) = 1$$

$$r_I(p, k_E) = 1 + (p_I - c_I)\frac{D(p_I)}{K - k_E}.$$

A pairwise stable equilibrium is such that

(i) $k_E = 0$ and $E(r_I) = \int_{p_{\min}}^{p_{\max}} r_I(p, k_E) f(p) dp > \int_{p_{\min}}^{p_{\max}} r_E(p, k_E) f(p) dp = E(r_E)$, or

(ii) $k_E = K$ and $E(r_I) < E(r_E)$,or

(iii) $0 < k_E < K$, $E(r_I) = E(r_E)$, and $\frac{d}{dk_E}(E(r_I) - E(r_E)) > 0$.

Corner equilibria

PROPOSITION 5 –

(i) $k_E = 0$ *is a pairwise stable equilibrium if and only if*

$$\frac{(p_I - c_I)D(p_I)}{K} > \int_{p_{\min}}^{p_I} \frac{p - c_E}{c_E} f(p)dp$$

(ii) $k_E = K$ *is a pairwise stable equilibrium if and only if*

$$\int_{p_{\min}}^{p_{\max}} \frac{p - c_E}{K} D(p) f(p) dp > \frac{p_I - c_I}{c_I} D(p_I)(1 - F(p_I)).$$

Multiple equilibria

- The two corner equilibria may co-exist
- The usual pecuniary externality is at work
- However, here it takes the form of a *truncation effect:*
- Giving the first unit of cash to entrant is profitable only if it underbids incumbent, but then more likely that it underprices and makes losses

Example: $p_I = c_I$

Entrant gets no cash is an equilibrium iff

$$E(p - c_E \mid p < p_I) < 0,$$

Incumbent gets no cash is an equilibrium iff

$$E((p - c_E)D(p)) > 0.$$

Interior equilibria

- There are also cases where there are interior equilibria
- They are more likely, and corner equilibria are less likely, the more elastic demand
- Demand effects: entrant is likely to sell more if its price is lower
- This pushes down its ROR when unconstrained

Implication for growth

- One may plausibly assume entrants are more productive
- Equilibria where cash goes to entrant « grow faster »
- But they are also « more turbulent »➔Entrants will often fail
- Incumbent equilibria are safe but stagnant (no growth)