# Markets for Information: Of Inefficient Firewalls and Efficient Monopolies [*]

Antonio Cabrales[†] Piero Gottardi[‡]

April 4, 2007

## Abstract

In this paper we build a formal model for environments where information is costly and where it can be used by potential competitors. The model allows to understand how such a market is organized, and whether it is efficient (ex-post and ex-ante). There is an object for sale, whose type is unknown. The buyers get utility from only one, randomly chosen, variety of the object. The buyers can find out the type of the object for sale by paying a cost. Each buyer has to choose first whether or not to explore the object and, if informed, whether to sell a report on his information, and at which price. After the information is sold, all the buyers participate in a second price auction for the object. We characterize the equilibria and welfare properties for a variety of setups. Information sold may be homogeneous or heterogeneous among buyers, and the seller of information may be a potential competitor, the owner of the good, or a disinterested third party. The results show that disinterested third parties (firewalls) may lead to inefficiencies and monopolies may achieve the efficient outcome.

---

[*] We gratefully acknowledge the financial help from Barcelona Economics CREA, Fundación BBVA, the Spanish Ministry of Science and Technology under grants SEJ2006-11665-C02-00 and , the Italian Ministry for University and Research under grant 2005135328. This project was started when the first author was visiting the University of Venice, which we wish to thank for financial and logistic support. The usual disclaimer applies.

[†] Departament d'Economia i Empresa, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Email: `antonio.cabrales@upf.edu.` http://www.econ.upf.edu/~cabrales

[‡] Dipartimento di Scienze Economiche, Universita' di Venezia, Fond.ta S. Giobbe, Cannaregio 873, 30121 Venezia, Italy. Email: `gottardi@unive.it.` http://venus.unive.it/gottardi/

# 1 Introduction

It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. For example, one can see financial analysts providing news over market events, or soccer coaches pondering on the abilities of (theirs or others') team players. Similar situations arise in the housing market, where we find agents searching and then supplying information over housing properties on sale in the market; in the labor market, where managers often discuss the performance of talented employees in their sector. This is somewhat surprising since the information supplied often has a rival nature. As an example of this rivalry, taken from the private equity market, the following quote from The Economist is illuminating: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own."[1] This reveals a fundamental conflict arising in information markets. The financial trader mentioned above may prefer to be the first to use the discovery of a particularly important event, and the soccer coach could be a potential competitor in a bidding war for a specially talented player. As a consequence, the provider of information may not be trusted to make truthful reports over the information he acquired: the analyst can say that his sources indicate that clinical trials for the wonder drug are going well, when in fact they are flunking. There is, in fact, a serious concern by regulators about the objectivity and the conflicts of interest faced by financial analysts.[2] This concern is particularly important when analysts are not independent, but employed by brokers, who are interested in trading the underlying stocks, as in the case of sell-side analysts, or even by the companies object of study, as in the case of fee-based analysts.

At the same time, information may be quite costly to acquire. Getting to know that the clinical trial of a particularly promising drug are going well (and thus the stock price of the company doing the research) may require nontrivial effort for the analyst. Similarly, finding out that a young left-footed striker currently playing in a second division Argentinian team is likely to be a star also requires time and money. These costs, together with the fact that information is of common interest, generate a clear incentive for the formation of a market for information, where the agents who acquired information can provide reports over

---

[1] The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] See, e.g. the recent report of the Forum Group (2003) created by the European Comission to deal with this problem.

it, possibly in exchange for the payment of a price, to the other agents. The possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

The purpose of this paper is to present a model where we can analyze these issues and which allows us to provide an answer to some questions. For example, we would like to know when information is acquired, and if so, whether it is transmitted to other agents, i.e. whether a market for information forms or rather breaks down because of the problems mentioned above. Also, if there is indeed a market for information, how is it organized. That is, who and how many traders sell information, who and how many traders buy it, and hence how competitive is the market, as well as which type of information is traded. As in the previous motivating examples, we will consider the case where the truthfulness of the information transmitted is not verifiable or contractible. So another natural question concerns the veracity of the information that is transmitted in the market, which is obviously a crucial condition for the market not to break down.

Efficiency issues are also important. Is the market efficient? Also, is the level of investment in information acquisition efficient? Or is there any scope for regulatory restrictions in order to improve welfare? This is important, because in the wake of recent scandals in financial markets, the regulatory bodies of many countries have strengthened requirements on information dissemination in various markets. One typical recommendation is to separate who provides information on a market from who trades in it ("firewalls").[3] We would like to provide a formal basis to analyze and assess these issues.

We consider a market where a single, indivisible unit of a commodity is up for sale. The market is organized as a (second price) auction, where several potential buyers can participate. The good comes in different possible varieties, and each buyer only likes one randomly (i.i.d.) chosen variety. In addition to buyers, there is the seller, who initially owns the object and has no utility for it, and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. Besides the market for the commodity there is another market where information in traded: any agent who acquired information can set a price at which he sells a report over his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk"

---

[3] For example, in the report of the European Comission Forum Group (2003), we read: "Conflict avoidance, prevention and management: Analysts' firms should have in place systems and controls to identify and avoid, prevent or manage personal and corporate conflicts of interest.

Disclosure: Conflicts of interest, whether corporate or personal, should be prominently disclosed."

messages.

We will provide a complete characterization of the equilibria of such game. We find that, when information costs are not too high, information is acquired in equilibrium and in that case it is also sold. That is,. the market for information is active. However, the information sold in that market can be noisy: when the provider of information is the seller of the commodity, he tends to hype the value of the good he declares for the agents who purchase information, while when he is a potential buyer, he tends to depress it.

Typically, only one trader acquires information in equilibrium, hence the market for information is a monopoly. Information is either sold at a positive price such that all the uninformed buyers except one purchase it or, when the cost of acquisition of information is low, at a zero price so that all uninformed buyers purchase it. To understand this, notice that the seller of information may benefit even by transmitting information for free as this allows him to manipulate the behavior of uninformed traders in the auction (for instance, so as to reduce the price he may have to pay to win the object in the auction) and hence to increase the amount of surplus he can appropriate in the auction.

We also show that, if information is acquired at all, the commodity is allocated efficiently in the auction, i.e. to the agent who values it most. But the level of investment in information acquisition is not efficient, in particular there is typically underinvestment. Interestingly, this inefficiency is present no matter what is the identity of the agent acquiring and selling information, i.e. not only when this is a potential buyer or the seller, but also when he is an agent not interested in buying the object. Actually, in the last case the inefficiency is more severe. Hence restricting the access to the market for selling information only to uninterested traders (as in the case “firewalls”), while improving the truthfulness of the information transmitted, makes the inefficiency of the overall market outcome worse in the set-up considered.

On the other hand, an efficient outcome can be attained if different types of reports, of different quality, can be sold in the market (or equivalently if we permit the resale of information). We show, in fact, that this allows the information provider to appropriate all the increase in traders’ surplus generated by the information acquisition and dissemination. When a single type of report is traded, part of these rents accrue either to the buyer choosing to remain uninformed, or to the seller of the object, who are free riding, thus generating inefficiently low information acquisition incentives. At the same time, when differentiated information is sold in the market, entry in such market is often profitable, so that, unless entry is restricted by some regulation, we will have multiple providers, which is also inefficient because of the duplication in investment costs. Information markets are, in a sense, natural monopolies.

In most of the paper, we consider the case where the uncertainty over the characteristics of the commodity up for sale concerns the different possible varieties of the good, over which buyers have different preferences. We thus have a situation of horizontal differentiation. At the end of the paper we also introduce an element of vertical differentiation, by allowing for the possibility that the commodity can also be of high and low quality, and all buyers prefer, at least weakly, high to low quality. In that case, the degree of truthfulness of the reports transmitted deteriorates, both when the provider is also a potential buyer or when he is the seller of the commodity. Hence, if quality is sufficiently important for buyers, we could have here a case where separation between traders and providers of information would be welfare improving.

This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information and their 'owners' is endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the availability and distribution of information which is acquired and transmitted in that market for the properties of the equilibria in the market for the commodity. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of each potential buyer, which is only privately known to him.

There is also a rather large empirical literature which studies the behavior of financial analysts, and in particular the presence of biases in their reports, and its effects for the performance of asset markets; e.g., Womack (1996), Michaely and Womack (1999), Barber et al. (2001), Agrawal and Chen (2006), Bradshaw, Richardson and Sloan (2003), Jegadeesh et al. (2004).

On the other hand there is much less in terms of theoretical work on markets for information. A good part of the attention has been given to the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem of untruthful reports as well as from the problem of information acquisition. Admati and Pfleiderer (1986, 1990) look at the case where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales.[4] When traders are strategic, information transmission may then also provide a strategic advantage to participants, as pointed out by Vives (1990) in a general oligopoly framework, and Fishman and Hagerty (1995) in the case of financial markets.

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2006). This is in line with our results for the case in which the information provider is the seller of the object and there is also vertical differentiation of the information.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders repeatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992) and Ottaviani and Sorensen (2006).

# 2 Model

There is one object for sale, initially owned by an agent, indicated as the seller of the object, who has no utility for it. The object is of uncertain quality: it comes in $K$ different types, all with the same ex-ante probability. Let the true type of the object be $v \in \mathcal{K} = \{1, 2, ..., K\}$. There are then $N$ potential buyers, agents who may be interested in purchasing the object. We denote buyer $i \in \mathcal{N} = \{1, ..., N\}$ by $B_i$; such buyer only cares (has positive utility) for one particular variety in $\mathcal{K}$ indicated as $\theta_i$. The variables $\{\theta_i\}_{i \in \mathcal{N}}$ and $v$ are all i.i.d over $\mathcal{K}$, thus for all $i, j \in \mathcal{N}$, $\theta_i$ is uncorrelated with $\theta_j$ and $v$; all elements of $\mathcal{K}$ have then the same probability, $1/K$. The object is allocated to buyers via a second price auction.

**Information structure.** The realization of $\theta_i$ is private information of individual $i$. On

---

[4] A similar point is made in Milgrom (1981) who explores how to inform others, if providing too good information can hurt the provider.

the other hand, the type of the object for sale is not known to any trader. Before the auction takes place, anybody can acquire, by paying a cost $c$, a signal over the type of the object, which we assume to be perfectly informative. If a trader acquires such signal he can in turn 'sell information' to other traders.

The utility of buyer $B_i$ can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i}$$

In this definition $t_{B_i}$ is the sum of the net monetary payments made by $B_i$ to the seller to gain possession of the object and/or to the other traders to purchase or sell information to them. $I_v$ is an indicator variable that takes the value 1 if $B_i$ gains the object and $v = \theta_i$ and 0 otherwise. Finally $I_e$ is another indicator that takes the value 1 if $B_i$ decides to acquire the signal over the type of the object, and 0 otherwise.

In this paper we are interested in situations where the information sold is not verifiable, i.e. the seller of information sends a report, which is pure 'cheap talk', over the signal he received. Since we abstract from reputational concerns (a one period model is considered), information markets can only exist if there is an element of non-rivalry in the information transmitted. Hence our focus on environments where buyers need not be interested in the same type of the good. Alternatively, one could reinterpret the specification of the model as capturing situations where the agent who sells information is not always able to profit directly from the information acquired. For example, leveraging the information may require the possession of complementary assets or skills, which he may lack. While an element of horizontal differentiation is necessary for a market for information to develop, such a market is compatible with the presence of vertical differentiation as well. In section 6 we extend the model to examine the effects of vertical differentiation in our setup.

We discuss first the case where information is acquired and transmitted by potential buyers of the object. We discuss in section 4 the case where information can also be sold by other traders, as the seller of the object and/or agents not interested in trading the object. Furthermore, to begin with we assume that each seller of information sells one, identical report to all buyers, at a single price; we refer to this situation as no differentiation of the quality of information. In section 5 we discuss the case where different kinds of reports may be sold by the same trader, at different prices.

**Timing of the game.** Is then as follows.

1. First, each potential buyer decides whether or not to acquire the signal over the type

of the object. The cost of the signal is $c$. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.

2. Any potential buyer who has chosen to acquire information, before learning the realization of the (perfectly informative) signal over the quality of the object, can sign a contract with other buyers for the sale of information. The contract prescribes that the seller of information will send a report over the signal, after receiving it, to all the traders who purchased information, in exchange for the payment of a price $p$, set by the seller of information.

3. Each of the buyers who did not choose to acquire information in stage 1. decides whether or not to purchase information from any of the traders selling information (he may choose to purchase information from more than one seller). Each of these buyers has then a final chance, after the market for information closes, to acquire the signal at a cost of $c$.

4. All agents who paid the cost $c$ of information acquisition learn the realization of the signal. Sellers of information then issue a report to buyers.

5. A second price auction takes place among all the buyers for allocating the object.

Since offers to sell information are made by the agents who acquired information in stage 2, before observing the realization of the signal, no signaling issue can arise at this contracting stage. Also, the feature that at stage 3 agents have a final chance to acquire information, but no opportunity to resell that information reflects the fact that acquiring information requires a simpler technology and takes less time than organizing a market for selling reports over it. This last opportunity of direct information acquisition, after contract offers have been made, in turn, limits the ability of the seller of information to corner the market and extract surplus from buyers.

**Message subgame.** We consider the case where the set of messages available to the seller of information is:

$$\mathcal{M} = \{1, 2, ..., K\},$$

i.e. it coincides with the set $\mathcal{K}$ of possible types of the object: the report sent by the seller is then given by one of the $K$ possible types of the object. Moreover, the message structure is augmented so as to involve two phases:

a. Each uninformed buyer who has chosen to purchase information sends first a report over his type to the trader who is selling the information (the set of available messages for an uninformed buyer $B_i$ is again the set of possible types of the buyer, given by $\mathcal{K}$). Such report is observed only by the seller of information, not by the other buyers.

b. Subsequently, the trader who is selling information sends a report over it to all the buyers who purchased information from him. The price paid and the report received is the same for all such buyers.

It will be clear from the analysis in the next Section that the presence of the first phase, by providing the seller of information with some information over the buyers' preferences allows to enhance his revenue and, furthermore, that there is no essential loss of generality in restricting attention to direct messages.[5]

We will consider the case where the number $K$ of possible types of the object is strictly greater than the number $N$ of potential buyers:

$$K > N,$$

or the competition among buyers for the object is not very intense. The main difference between the properties of the equilibria in this case and when $K \leq N$ are discussed in Section 3.1.[6]

# 3 Equilibrium and welfare

Since information transmission needs not be truthful, the game (and in particular the message subgame, from stage 4) has many equilibria, as is common in other kinds of "cheap talk" games (see e.g. Crawford and Sobel 1982). We focus our attention on the equilibria where the degree of truthfulness in agents' reporting - and hence the revenue of the seller of information - is maximal.

We will show that an equilibrium always exists where uninformed buyers truthfully report their type to the seller of information and this one adopts the following reporting strategy

---

[5] We will show in particular that our analysis and results extend to the case where the set of messages is expanded, to include the possibility of sending no report; see the discussion in Section 3.

[6] See in particular footnote 8.

(both in and out of equilibrium):

$$m_i = \begin{cases} v, \text{ if } \ v \neq \theta_i \\ y, \text{ with probability } \frac{1}{K-N(B_i)}, \\ \text{for all } y \neq \theta_j \ \forall \, B_j \in \mathcal{N}(B_i), \text{ if } \ v = \theta_i \end{cases} \tag{1}$$

where $B_i$ denotes the buyer selling information, $m_i$ is the report issued by him, $\mathcal{N}(B_i)$ the set given by $B_i$ plus all the buyers purchasing information from $B_i$ and $N(B_i)$ the number of distinct realizations of $\theta_j$ across all buyers $B_j \in \mathcal{N}(B_i)$. Therefore, trader $B_i$ tells the truth about the quality of the object as long as the true quality of the object does not coincide with his own type (i.e. with the type he likes). On the other hand, when the two coincide, the seller of information, $B_i$, faces a conflict of interest as he wishes to get the good and at the lowest possible price. Thus, he will not tell the truth and will send a message which is a randomization over all the types which are different from the type of the seller as well as of all the buyers of information. One could interpret this message as telling the buyers of information that the object is not appropriate for any of them.

It should be clear, also from the following analysis, that the reporting strategy described in (1) entails the maximal degree of information transmission at an equilibrium: if the seller's reports are informative and hence affect buyers' beliefs, we should expect him to wish to deceive buyers when he is interested in getting the object. An alternative form of deception in this event would entail sending a completely uninformative message, i.e. one that does not change the priors of the buyers (which can be interpreted as providing buyers with no advice, or report). This could be done, with the set of available messages given by $\mathcal{M}$, through a randomization among all the messages in $\mathcal{M}$. Alternatively, it could be achieved even more effectively if the set of possible messages is enlarged to allow also for the possibility of sending an empty message, a blank report. It can be shown, though, that in all the equilibria with these alternative forms of deception the utility of the seller of information is lower than in the one we consider (with reporting strategy (1)). Notice that, unlike the one discussed in this paragraph, the form of deception we consider requires the presence of phase a. of the message game in order for the seller to acquire information about the buyers' preferences. One could argue that, in many situations, the advisors indeed have information of that kind and use it in their reports.

An additional source of multiplicity of equilibria comes from traders' behavior in the auction, in the final stage of the game. We show that there is always an equilibrium of the auction where the bid of each trader equals his expected value of the object, conditional on winning the auction and focus our attention on such equilibrium ('with truthful bidding').

Other equilibria may exist, but are typically non robust to trembles and we will ignore them in what follows.

We will characterize the perfect bayesian equilibria of the game described in the previous Section and evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, $c$). Given the selection of equilibria in the message and auction subgames specified above (quite natural, we would like to argue, given our purposes), we will show that the overall equilibrium is, for almost all parameter values, unique:

THEOREM **1** *For all* $c \geq 0$ *there exists a perfect bayesian equilibrium of the game with no differentiation of the quality of information sold where sellers of information adopt the truthful reporting strategy in (1), buyers of information truthfully report their type to sellers, and participants in the auction adopt a truthful bidding strategy:*

1. *If* $c \geq c^I \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, *no buyer chooses to acquire information; the object is then gained by one, randomly chosen, buyer, at a price* $1/K$.
2. *If* $c^I \geq c \geq c^0 \equiv \frac{1}{K^2}\left(\frac{1}{N-2}\right)$, *one buyer acquires information and sells a report over it to all the other buyers except one, for a price* $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$; *the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to* $1/K$ *(when either the seller of information, or only one buyer of information likes the object),* $0$ *(when neither the seller nor any buyer of information likes the object) and* $1$ *otherwise.*
3. *If* $c^0 \geq c$, *one buyer acquires information and sells a report over it to all the other buyers, for a price* $p = 0$; *the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to* $0$ *(when the seller of information, or exactly one of the other buyers, who all purchase information, likes the object), and* $1$ *otherwise.*

*This is the unique equilibrium with truthful reporting and bidding strategies, with the only exception of a subset of region 2., given by* $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c \geq c^0$, *where another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price* $p = 0$; *the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of* $0$ *if nobody else likes it, and* $1$ *otherwise.*

Let us summarize the content of the proposition. When information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is truthful while in others is a lie. Information is sold for a low enough price so that all buyers, or all buyers except one, purchase it. The market for information is typically a monopoly. Furthermore, the seller of information always gets the object when he likes it; when he does not like it, the object goes to one of the buyers of information who likes it, if such buyer exists and otherwise, in case 2. goes to the buyer not purchasing information, while in case 1. to a random buyer who purchased information.

Remark 1 *If the seller of information were to post the price after having learnt the signal realization, the price posted would have signaling value, as the seller would want to post a different price after having learnt that he likes or does like the object.*

*Suppose next the buyers of information post the price, after the seller of information has learnt the realization of the signal. Then we claim the equilibria would have similar though not exactly identical features to the above. In particular, for* $c$ *low we have an equilibrium where all buyers of information set a price equal to their true maximal willingness to pay for every possible number of buyers of information. The seller of information then chooses to sell information to all of them at a zero price. Thus the same outcome as the equilibrium in region 3. of Theorem. On the other hand, for* $c$ *relatively high, we have an equilibrium where all the buyers of information set a price equal to their true maximal willingness to pay for information for every possible number of buyers of information except the total number of buyers of information less one (e.g.* $N-2$ *with a monopolist seller), for which value they set a price equal to zero. The optimal response of the seller of information is then to sell informaion at a positive price to all the buyers of information less two. In this case the payoff of all buyers of information is zero.*

*Thus unlike in Theorem 1 above, the information is sold at a positive price not to* $N-2$ *traders in that case but to* $N-3$*. This is because if all buyers of information report their true willingness to pay also for* $N-2$ *buyers of information, then the seller would want to sell to* $N-2$ *and each buyer of information would have an incentive to post a lower price so as to be the one not buying information, i.e. remaining uninformed, which is the only one with a positive payoff in this situation.*

Figure 1 summarizes the result.

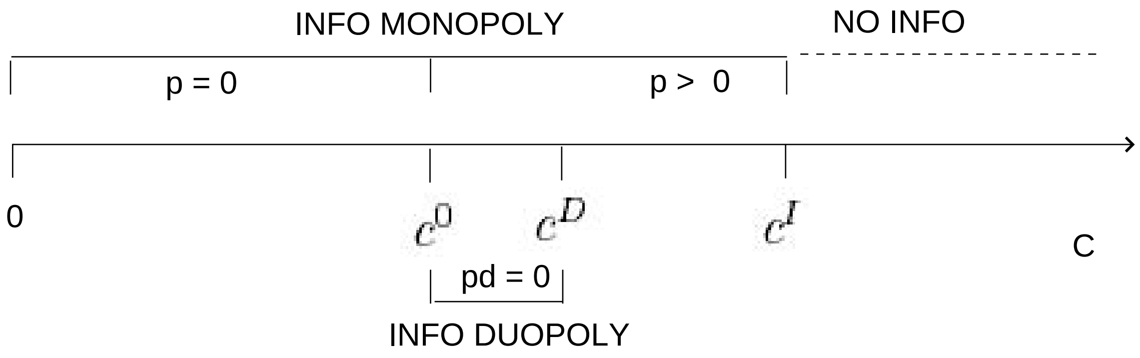

Figure 1: Equilibrium with homogeneous messages

## 3.1 Proof for Theorem 1

We provide a complete proof of the Theorem in Appendix A. First, the consistent beliefs associated with the reporting strategy in equation (1) are determined. Then we study traders' strategies in each stage of the game, and establish their optimality given the beliefs. In the following subsections we provide a sketch of the crucial parts of the argument together with the intuitions for it.

**Beliefs** With the message structure in (1) there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer $B_j$, who receives a report from a single informed buyer, say buyer $B_i$, using Bayes' rule in all cases:

- When buyer $B_j$ receives from $B_i$ a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$.
- On the other hand, when buyer $B_j$ receives a message different from his type, this may happen for two reasons: either the message is truthful, and then $B_j$ does not like the object, or it is not truthful, as the sender likes the object and is randomizing over types different from his own and the types of buyers of information, in which case there is a positive probability $B_j$ may like the object. As we will see later, the precise value of buyers' beliefs in this case does not affect their strategies and hence the outcome of the auction.

Finally, the buyers who did not acquire information directly, nor indirectly by purchasing it in the market, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/K$, $\Pr(v \neq$

$\theta_j) = (K-1)/K$. The beliefs of a buyer who is purchasing two (or more) distinct reports from two (or more) informed buyers are similar.

### Stage 5: Behavior in the auction

Given the beliefs of the buyers who purchased information from a single informed buyer we can show that a ‘truthful bidding strategy’ (where a trader’s bid equals his expected value of the object, conditional on winning the auction) is always optimal if all other traders adopt such strategy:

- When a buyer receives a message indicating that the object is of the type he likes, his belief - as argued above - is that with probability 1 he likes the object. His optimal bid when the other bidders adopt ’truthful bidding strategies’ is then equal to 1, i.e. to his posterior beliefs about the valuation of the object, and is then also truthful.
- On the other hand, a message different from the type a buyer likes is received in two alternative events. The first one is that the buyer likes the object, but so does the sender of the message. In that case he cannot win the object in the auction with a positive surplus, i.e. at a price lower than his valuation. This is because the seller of information is better informed and, if he adopts a truthful bidding strategy, will make a higher bid, equal to 1. The other possibility is that the buyer does not like the object, in which case any positive bid, if successful, would yield him a negative surplus. Hence the optimal bid of a buyer of information, when receiving a message different from his type, is zero.

Notice that, even though we are in a second price auction the buyer’s bid is not always equal to his posterior belief over the value of the object. This is due to the affiliation of the information of traders induced by the sender’s reporting strategy. The information conveyed by winning the auction should also be taken into account. We see in fact that the buyer’s optimal bid, zero after receiving a message different from his type, equals his expected value of the object conditional on winning the auction; thus in such case too it is a truthful bidding strategy.

In contrast, the buyers who do not purchase any information do not suffer from an affiliation problem (there is no relevant information for them in the event of winning the auction). Thus, the optimal bid in their case is: $\Pr(v = \theta_j) = 1/K$.

How important is the restriction to ’truthful bidding strategies’? We claim such restriction only bites when there is a single trader who is not acquiring information nor purchasing it from other buyers, i.e. who chooses to remain uninformed. In this case any strictly positive bid lower than $1/K$ by the uninformed buyer gives him the same payoff as a bid of $1/K$; this may in turn affect the other traders' optimal bidding strategies and generate other equilibria. On the other hand, when more than one trader are not purchasing information, or all traders purchase information, the only equilibrium in the auction game is the one with truthful bidding strategies. We can argue therefore that the equilibria where not all bidders follow a truthful bidding strategy, if they exist, are non robust to trembles concerning the decision of purchasing information.

The bidding behavior of an uninformed buyer receiving more than one report from different informed buyers is analogous to the one described above. When his posterior belief over the value of the object equals one, he will bid one, and will bid zero otherwise (when the messages received are not fully informative).

### Stage 4: Behavior in the message game

We show first that the reporting strategy we postulated for the seller of information is indeed optimal for such trader. We then verify that each buyer of information is willing to report truthfully his type to the seller of information. A key element in the argument is the fact that, by changing the message strategy neither the seller nor any buyer of information can affect the outcome of the auction in his favor.

**Seller:** There are two possible deviations which need to be considered to verify the optimality of the reporting strategy of the seller of information. When he likes the object, he may announce a type corresponding to one of the traders purchasing information from him. As a consequence, the bid of that trader will be higher and equal to 1, and the seller will end up paying more for the object than if he had followed the equilibrium message strategy, so he never wants to make such deviation. Furthermore, when the seller of information does not like the object, he may deviate by announcing a type different from the true one. But that only changes the outcome in the auction, which has no effect on the seller's utility in this case since he is not interested in the object. So the seller can never gain with such a deviation.

**Buyer:** A deviation by a buyer of information consists in reporting something different from his true type. This has no consequence when the seller of information does not like the object, because in that case the seller reports the truth, no matter what are the reports he receives from the buyers of information. On the other hand, when the seller of information likes the object a buyer's lie may change the seller's report; in particular, the seller could announce the buyer's type (if different from the type of the object). In this second case the seller of information always bids 1, hence the buyer still gains no surplus. In the first situation, since the receipt of a report by the seller equal to the buyer's type is now less informative, the buyer's utility is never higher and possibly lower. It thus follows that a buyer of information cannot gain, and may actually loose, by misreporting his type.

**Stage 3: Purchase of information.**

In this stage each uninformed buyer has to choose whether to purchase information from one - or more - of the informed buyers who are selling information, at the price posted by them, or alternatively to acquire the information directly (at the cost $c$), or do nothing of the two.

**Stage 2: Sale of information.**

This is the key stage of the game, where the market for information opens and each trader who at stage 1 has chosen to acquire information posts a price at which he is willing to sell a report over it to any other buyer. The price is set at the level which maximizes the utility of the seller of information, i.e. his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response strategies of buyers to the prices posted in the next stage. The main elements of the analysis are summarized below.

The **maximal willingness to pay for information** of an uninformed buyer is given by the increase in the buyer's payoff in the auction if he purchases information and hence becomes indirectly informed, relative to the best of his two alternatives: acquire information directly at the cost $c$, or remain uninformed. The buyer's expected payoff in the auction is in turn determined by the probability that a buyer of information gains the object with a positive surplus, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. The probability of this event is $1/K\,[(K-1)/K]^{M-1}$, where $M$ is the total number of buyers who acquired information directly or indirectly, and is clearly higher the lower the number of agents who are purchasing information. The price paid to win the object in the auction, given the bidding strategies described in the previous steps, is 0 if $M = N$, i.e. if no buyer remains uninformed, and $1/K$ otherwise.

If an uninformed buyer does not purchase information but acquires it directly in the next stage of the game his payoff is same, less the cost $c$. This implies that, for the sale of information to take place, its price $p$ cannot be higher than $c$.[7] On the other hand, if the buyer remains uninformed his payoff is zero if $M < N$, i.e. if there are other uninformed buyers, while if $M = N$ it is positive and equal to $1/K\,[(K-1)/K]^{M-1}$. By comparing this term with the ones above it follows that a buyer is only willing to pay a positive price for information when not all other buyers are either directly or indirectly informed. In this case, the maximal price he is willing to pay is strictly decreasing in $M$. Thus the demand for information faced by a seller is strictly decreasing in its price $p$, for $p < c$. In fact, it should be clear from this argument that, overall

$$p = \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{M-1}\right\}$$

When there is a **monopolist seller of information**, he always sets the price so as to extract all the surplus from the number of buyers he wishes to attract, i.e. equal to their maximal willingness to pay. The issue is then to determine the number of buyers of information which maximizes the seller's payoff. In the Appendix, we show the following

CLAIM **1** *When $p < c$, the* ***revenue from the sale of information*** *for a monopolists seller of information is maximized when the price $p$ is set low enough that all uninformed buyers, except one, purchase information, i.e. $M = N - 1$.*[8]

But the second component in the seller's payoff is given by his **payoff in the auction.** Given the reporting strategies described above, the sale of information does not affect the fact that the seller always gains the object whenever he likes it. Hence its only possible effect is on the price at which the seller gains the object in the auction, via its influence on buyers' bids. As already argued in the case of uninformed buyers, when the price is low enough that all uninformed buyers purchase information from the seller ($M = N$), he gains the object at a zero price, otherwise he has to pay $1/K$ to get the object (the same as with no sale of information).

---

[7] Also, as long as $p \leq c$ no buyer will choose to acquire information directly in stage 3 of the game.

[8] When $K \leq N$ the main change in the analysis concerns this result as the maximal revenue from the sale of information may be attained by leaving more than one buyer uninformed. The intuition is that when $K \leq N$, competition among buyers is more intense, so that at some point adding an extra informed buyer increases more the probability that at least two informed buyers like the object than the probability that only one informed buyer likes it.

This leads us to the **crucial tradeoff** for the monopolist seller. He has in fact to choose between the price which maximizes his revenue from the sale of information (and induces all uninformed buyers except one to purchase information) and the lower (zero as we said) price which maximizes his payoff in the auction, attracting all buyers. Then, as we show in the Appendix:

CLAIM **2** *When* $c < c^0$ *the seller's payoff is higher if he gives the information away, setting* $p = 0$*, to guarantee that the auction price is low. On the other hand, for* $c > c^0$ *the price at which information can be sold is sufficiently high that the seller's payoff is greater with* $p > 0$*.*

With an **information oligopoly,** the equilibrium **price of information is zero.** Note first that, when there are two or more sellers of information, the additional benefit for an uninformed buyer of purchasing a second report is always zero. This follows from the fact that purchasing information from one seller allows to gain a positive surplus in the auction only when the buyer likes the object and nobody else who is informed, either directly or indirectly, likes the object. Purchasing information also from a second seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth), however in such event no positive surplus can be gained since the seller of information who likes the object bids one. Furthermore, the benefit for a buyer of purchasing one report is essentially the same as when there is a monopolist seller; in particular, it is positive only if not all the other uninformed buyer have purchased information, i.e. at least one report.

Given that each buyer is willing to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers between the providers of information, with at least one buyer not purchasing information. But then, each of the sellers has an incentive to undercut. By lowering his price the seller retains all those already buying from him and manages to steal the buyers from the other sellers of information, which entails a discrete jump not only in his revenue from the sale of information but also in his payoff in the auction. The latter is in fact positive (and equal to $1 - 1/K$ if at least one trader is not purchasing information) when he likes the object and neither the other sellers of information nor any other buyer that is purchasing information from the *other* sellers, like the object. Hence the probability that a seller has a positive surplus increases with the number of buyers who purchase information only from him. Since such incentive to undercut persists as long as the posted prices for information are positive, the only possible equilibrium is where all sellers post a zero price for information, and all uninformed buyers purchase information from every seller. In this situation, the sellers of information have the same payoff as the buyers of information, less the cost of acquiring information, thus their overall payoff is lower.

### Stage 1: Information acquisition

Claim **3** *When the cost of acquiring information is so high* ($c \geq c^I$) *that it exceeds the maximal gains that a monopolist seller of information can get from the sale of information* ($(N-2)1/K[(K-1)/K]^{N-1}$) *plus the gains from obtaining the object in the auction* ($1/K[(K-1)/K]$)*, no buyer chooses to acquire information. On the other hand, when* $c \leq c^I$ *one buyer always acquires information.*

When $c \leq c^I$, as argued above in the discussion of Step 2, such buyer will always choose to sell information. Furthermore, the entry in the market for the sale of information by a second buyer is never strictly profitable, as the payoff of an informed duopolist is less or equal than the payoff the trader would get by not acquiring information directly but rather by purchasing a report from the other informed buyer. We show in the Appendix that

Claim **4** *For a range of intermediate values of c,* $c^D \geq c \geq c^0$,[9] *the payoff is exactly equal in the two situations, in which case there are two equilibria, one with a monopolist seller of information and the other with two sellers of information. Outside this range there is a unique equilibrium with a monopolist seller.*

## 3.2 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. Given the assumptions made on traders' utilities, welfare can be simply evaluated by considering the sum of the payoffs of all buyers and the seller. In particular we are interested in comparing the equilibria to the Pareto efficient allocations, i.e. to the allocations which could be attained by a planner who (knows the buyers' types but) is also uninformed about the type of the object and may acquire information, at the same cost $c$, over it.

Notice first that, if information is acquired by some buyer, the resulting equilibrium allocation is always ex post efficient, in the sense that the object always goes to a buyer who likes it the most. Hence the only possible source of inefficiency may lie in the information

---

[9] For $c$ in this interval, the monopolist sells information at a price $p = c$.

acquisition decision: is that also efficient at equilibrium, or rather is there overinvestment, or underinvestment in information? Evidently, the equilibrium with two buyers both acquiring information, which obtains when $c^D \geq c \geq c^0$, is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, an equilibrium where only one buyer acquires information, hence without any wasteful duplication, was shown to exist for all $c \leq c^I$.

When is information acquisition efficient? If information is acquired, the object can always be allocated to a buyer who likes it, in the event where such buyer exists. In that case the total surplus of traders from the object equals one, while it is zero otherwise. Total welfare is then obtained by subtracting the cost of information:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{K-1}{K}\right)^N - c.$$

On the other hand, if information is not acquired the total surplus is one only if the agent who receives the object (and, with no information, this agent can only be randomly chosen) happens to like it. Thus total welfare is then in this case:

$$W_0 = \frac{1}{K}.$$

By comparing $W_0$ and $W_1$ we find that it is socially efficient for information acquisition to take place if, and only if, $1 - ((K-1)/K)^N - c \geq 1/K$, or:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \geq c. \tag{2}$$

This threshold can then be compared to the one found in Theorem 1, $c^{\text{inf}}$, for information acquisition to take place in equilibrium:

**Proposition 1** *In equilibrium there is a less than efficient level of investment in information. In particular, for values of $c$ lying in the following, nonempty interval:*

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{3}$$

*no information is acquired in equilibrium, though it would be socially efficient to acquire it.*

Thus there is a range of values of $c$ for which acquiring information is efficient but in equilibrium the gains from information acquisition are too low so that nobody chooses to become informed. To understand the reasons of this result, it is useful to examine first the distribution of welfare gains and losses across agents when we compare the situation where no information is acquired to the situation with a monopolist seller of information, in particular when $c$ is close to its threshold value $c^I$ ($c^I \geq c \geq c^0$).

**Who gains and who loses from information acquisition** Let $B_1$ denote the buyer who acquires information directly and then sells it, as a monopolist, and $B_N$ the single buyer who remains uninformed. $B_1$ clearly gains as his payoff goes from 0 (in the situation where no information is acquired) to a strictly positive level (except when $c = c^I$); so does $B_N$, whose payoff[10]

$$\pi_{B_N} = 1/K\left[(K-1)/K\right]^{N-1} \tag{4}$$

is strictly positive. On the other hand, the payoff of the remaining buyers, who acquire information indirectly by purchasing a report in the market, is unchanged at zero.

What about the seller of the object? His payoff, in the region under consideration ($c^I \geq c \geq c^0$) is given by

$$\begin{gathered}\left[\left(\tfrac{K-1}{K}\right)\left(1-\left(\tfrac{K-1}{K}\right)^{N-2}-(N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-3}\right)\right]+\tfrac{1}{K}\left[\tfrac{1}{K}+(N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2}\right]\\ =\left(\tfrac{K-1}{K}\right)\left(1-\left(\tfrac{K-1}{K}\right)^{N-2}\right)-(N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1}+\tfrac{1}{K}\tfrac{1}{K}\end{gathered}$$

As claimed in 2. of Theorem 1, the price at which the object is gained in the auction can be 1, $1/K$ or 0. The terms in square brackets above are then the probabilities of the price being, respectively, 1 and $1/K$. The difference in the revenue of the seller of the object between this case and the one where nobody is informed (where the price in the auction is always $1/K$) is then:

$$\Delta\pi_S = \left(1-\frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]-\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \tag{5}$$

which is positive if, and only if:

$$1 > \left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right) \tag{6}$$

As we show in section 4.2 below, this inequality is satisfied for some, but not all values of $K$ and $N$.

**The source of the inefficiency** From the ex post efficiency of the equilibrium allocations with a monopolist seller of information it follows that the sum of the changes in the welfare of all traders between the equilibrium with and without information acquisition equals the difference between the levels of maximal total welfare in these two situations, $W_1 - W_0$. Since, as we said, the payoff of the indirectly informed buyers is zero in both situations (when $c$ is

---

[10] See equation (18) in the Appendix.

close to $c^I$), the change in total welfare equals the change in the payoff of the seller of the object $\Delta\pi_S$ plus the payoff of the buyer who acquires and sells information and the payoff of the buyer who remains uninformed:

$$W_1 - W_0 = \Delta\pi_S + \pi_{B_1} + \pi_{B_N}. \tag{7}$$

Thus underinvestment in information obtains whenever $\pi_{B_N} + \Delta\pi_S > 0$. The analysis of the distribution of the welfare changes across agents in the previous section allows us to gain some further understanding of the source of the inefficiency result we obtained. From (4) and (5) we get:

$$\pi_{B_N} + \Delta\pi_S = \left(1 - \frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right] > 0, \tag{8}$$

which is equal to the first term of expression (5) and is strictly positive. The term describes the gains accruing to the seller when at least two indirectly informed buyers happen to like the object, so their bids raise to 1 the price at which the object is won in the auction. We refer to such term as *rent dissipation* by indirectly informed buyers since these are rents generated by the information acquisition that the buyer who makes the investment in information will not appropriate, and instead go the seller of the good.

As argued in the previous section, the term $\pi_{B_N}$ is strictly positive. This shows that also the uninformed buyer appropriates some informational rents, by successfully free riding on the information acquisition of all other buyers, which allows him to get the object at a zero price when nobody else likes it. We indicate then this term as *free riding*. Note that it is exactly equal to the second, negative, term in the expression (5) for $\Delta\pi_S$, which reflects the fact that the free riding happens entirely at the expense of the seller of the object. It entails so a pure transfer of surplus from the seller to $B_N$, and hence not undermine incentives for efficient information acquisition.

What does undermine such incentives, and is reflected in equation (8), is thus only the rent dissipation. We will show in the section 5 that the seller of information may indeed be able to solve this problem by selling information of different quality to different buyers, thus avoiding the "ties" that generate the extra rents for the seller of the object. But before doing that, we show that having an "independent" seller of information (i.e. somebody commonly known to have no interest in trading the object) is not a solution for the inefficiency we have discovered.

# 4 Who should sell information ?

Does the inefficiency we found depend on the fact that information is sold by a trader who is also interested in purchasing the object? We examine here the efficiency properties of equilibria when other types of traders can be the providers of information.

## 4.1 A disinterested trader

A common proposal for solving inefficiencies in information gathering/transmission in markets is the separation between information providers and traders. We model this by introducing a new type of agents, who do not own the object nor have any utility for it and hence have no interest or are not allowed to participate in the market where the object is traded. To keep the comparison clean, we assume that also this type of disinterested traders has to pay a cost $c$ to acquire the information.

First of all, one should note that a disinterested trader is never present as a seller of information in equilibria where there are two or more sellers of information. Such trader in fact only gains from the revenue obtained from the sale of information, and the price of information when there are two or more sellers is always zero, by the argument given in the previous section, so that the revenue from the sale of information is less than the cost of information acquisition. Thus, we can have an equilibrium where information is sold by a disinterested trader only if he is the monopolist provider of information and, furthermore, in that case information is always sold at a strictly positive price.

The reporting strategy of a disinterested trader is clearly different from the one of an interested trader since the first one never has an interest in lying over the type of the object. The optimal reporting strategy with maximal degree of truthfulness for the disinterested trader is to always tell the truth. Hence the quality of the information sold is clearly higher. Does this imply the equilibrium with a disinterested trader as provider of information has better efficiency properties? We will show that the answer to such question is negative, the efficiency properties are actually worse in this case, as the incentives to information acquisition are weaker, hence the problem of the inefficient underinvestment in information acquisition is more severe.

Proposition **2** *When information can be sold only by disinterested traders, in equilibrium there is again a less than efficient level of investment in information. Furthermore, the interval of values of $c$ for which information is not acquired in equilibrium though it is* *socially efficient to acquire it is*

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

*which is larger than the one we found in Proposition 1 when information is sold by potential buyers.*

The intuition for this result is not too hard to understand. The disinterested trader has one additional customer than a potential buyer as he can sell information at a positive price to $N-1$ rather than $N-2$ buyers. Since the price at which information is sold is the same, this means an extra gain from the sale of information equal to $1/K\,[(K-1)/K]^{N-1}$. On the other hand, a disinterested trader does not gain any surplus in the auction, so he loses, with respect to a potential buyer, the surplus this one gets in the auction, which is $(K-1)\,/\,(K)^2$. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the disinterested trader.

## 4.2 The seller of the good

We examine next whether the owner of the good could do better as a provider of information, in terms of efficiency of the equilibrium, than a potential buyer. We will consider the case where the owner of the good does not ask buyers to report their type. This is, in fact, the best for him, as we will show later. If the seller has no information over the buyers' types, he is willing to report truthfully the type of the object, like the uninterested trader.

Proposition **3** *When only the owner of the good can be seller of information, in equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, nonempty interval:*

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right) \leq c \leq \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right) \tag{9}$$

*information acquisition is socially efficient, but does not take place in equilibrium.*

This result can also be fairly easily understood in the light of the discussion in section 3.2. As when information is sold by a potential buyer of the good, whenever information is acquired the equilibrium allocations are always ex-post efficient. Hence, the sum of the increases in the payoff of all traders between the situation without and with information acquisition equals the change in total welfare, $W_1 - W_0$. Since the payoff of buyers who purchase information is zero in both cases (for $c$ close to its threshold $c^I$), the change in total welfare equals the change in the payoff of the seller, $\Delta\pi_S^S$, plus the payoff of the buyer who remains uninformed ($B_N$).[11] That is:

$$W_1 - W_0 = \pi_{B_N}^S + \Delta\pi_S^S. \tag{10}$$

Thus, underinvestment obtains whenever $\pi_{B_N}^S > 0$. Since $\pi_{B_N}^S = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, i.e. has the same value as when information is sold by a potential buyer, the result follows. Notice that the source of the inefficiency is not here the *rent dissipation* by the indirectly informed buyers, as we found in section 3.2 when the information provider is a buyer, but rather the informational *free riding* of the uninformed buyer.

Another natural question is when information acquisition is more efficient if carried out by the owner of the good rather than by a potential buyer. Comparing the threshold for a potential buyer to acquire information derived in theorem 1, $c^I$, with the one obtained in proposition 3 for the owner of the good, we find that the inefficiency is more severe with the owner of the good as information provider when:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} > \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2}\right),$$

or equivalently,

$$\left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right) > 1 \tag{11}$$

We characterize in the following result when this condition holds:[12]

Lemma **1** *When $K - N$ is sufficiently large, or $K < 6$, the buyer is more efficient than the seller as provider of information. The opposite holds when $K - N$ is sufficiently small and $K \geq 6$.*

Thus, for $N$ sufficiently smaller than $K$, the buyer is more efficient than the seller. This is a consequence of the fact that, when $N$ is much smaller than $K$, the probability that two agents wish the same type of object becomes small. Hence, the *rent dissipation*, the source of inefficiency for the buyer, is small.

Before concluding this section we establish the following claim made earlier:

---

[11] Evidently the incentives to acquire information are highest at the equilibria with a monopolist seller of information, so we concentrate on those.

[12] Note that (11) is exactly the reverse inequality of (6), which describes when $\Delta\pi_S > 0$. By comparing (7) and (10) we see in fact that a buyer is more efficient than the seller as provider of information, or (11) holds, when $\pi_{B_N^S}$ is bigger than $\Delta\pi_S + \pi_{B_N}$; that is, since $\pi_{B_N^S} = \pi_{B_N}$, when $\Delta\pi_S < 0$.

Proposition 4 *If the owner of the good asks buyers to report their type before sending his own report, his equilibrium payoff will be lower.*

We show in the proof of such claim in the Appendix that if the owner of the good knows the buyers' type, he is no longer willing to report always the truth over the type of the object. Whenever none of the buyers happen to like the object the seller will lie and report one of the buyers' types. Similarly, if only one of the buyers like the object but at least two other buyers like a different type of the object, i.e. say $v = \theta_1$ but $\theta_2 = \theta_3 \neq \theta_1$, the seller prefer to report $\theta_2$ rather than the truth. The reason is that such lies allow the seller to increase the price in the auction. In all other circumstances, the owner of the good reports the true type of the object, so that his report still contains some, though imperfect, information over the true type of the object. Hence the market for information will still be active and buyers are still willing to pay a positive price for the seller's report. We also show that given the above reporting strategy of the seller, if there is an uninformed buyer, he never gets the object, so that there is no free riding by this agent and buyers are willing to pay a positive price also when all of them purchase information.

At the same time, given the above reporting strategy of the seller, the equilibrium allocation will not be always ex post efficient, as sometimes the object will go to a buyer who does not like it even when there is another buyer who likes it (this will happen in particular when only one buyer likes the object while there are at least two other buyers with the same type: in that even as we said the seller prefer to report their type to extract a higher price in the auction). This inefficiency also adversely affects the buyers' willingness ot pay for information and hence the revenue from the sale of information, while the lower informational content of the reports sold adversely affects the seller's revenue from the auction. We show that altogether such negative effects prevail so that the expected payoff of the owner of the good is indeed higher if he does not know the buyers' types (in that even the seller tells the truth, the object is efficiently allocated and the seller extracts all the rent from all the N-1 buyers who purchase information, he only fails, as we said, to appropriate the free riding rent of the uninformed buyer).

We also show that, given the above strategy of the seller, buyers still prefer to report thruthfully their type (by lying, the informational content of a 'good' report, equal to a buyer's type, is unaffected but the buyer will receive such report less often and only when competition from higher bueyrs is fiercer, so that his payoff is also lower.

Lying about the type of the good in order to increase the demand of the object is akin to the "hyping" by securities' analysts that so worries the authors of the report of the

European Comission Forum Group (2003). Notice that this happens in spite of the fact that the information transmitted here is not about quality, but about the type of the good for sale. It will occur 'a fortiori' when information about quality is transmitted as well. See our discussion of an extension of the model with information on quality in section 6.

### 4.3 Who will sell information?

So far, we considered situations where only one type of trader has the 'licence' to sell information. We discuss here briefly what happens when all three types of traders (potential buyers, disinterested traders and owner of the good) can acquire and sell information. Evidently, for $c$ close to the threshold values for information acquisition to take place in the three cases considered, there will still be an equilibrium where only type of trader is the monopolist seller of information. Furthermore, for any $c$ lower than some threshold (which we show is higher than $c^0$ whenever there is one trader selling information, either a potential buyer or an uninterested trader), the seller always wants to also acquire and sell information, i.e. to enter the market for information. Then the disinterested trader does not want to enter, while for intermediate value of $c$ the potential buyer may still want to enter. As a consequence, for $c$ low we only have equilibria where the seller of the object sells information, for an intermediate range of values of $c$, we have both a monopoly equilibrium where only the seller of the object sells information and a duopoly equilibrium where both the seller of the object and a potential buyer sells information, and for $c$ higher, then we have three types of equilibria with a monopolist seller of information, which is either the seller of the object, a potential buyer or a disinterested trader.

## 5 Differentiation of the Information Sold

### 5.1 A monopolist seller

We show next that a way to have efficient information gathering, as well as allocational efficiency, at equilibrium is to allow informed traders to sell different kinds of reports over their information, at different prices. Hence there will be differentiation of the information which is transmitted, and the extent of such differentiation is optimally chosen by the seller. In this subsection we assume the seller of information is always a monopolist, i.e. entry to the market for the sale of information is restricted to a single trader. We discuss in the following subsection the consequences of eliminating this restriction and allowing for free entry in this market (as in the previous sections).

To see why the differentiation of information may be useful for the seller of information and allow to increase his profits, recall that, when a single type of report is sold, the price a buyer is willing to pay for information depends on the number $J$ of other buyers who choose to remain uninformed. Purchasing information is indeed valuable for the buyer because it gives him an informational advantage over the uninformed buyers; the larger is $J$ the more valuable information is. This informational advantage manifests itself in a priority over uninformed buyers in obtaining the good when the buyer wants it. With a single type of report, there are up to three priority levels. First, the directly informed buyer. Then, all the indirectly informed buyers (which share the same priority level). Finally, the uninformed buyers, if any. We now show that by differentiating the reports, a seller of information (as long as he is a potential buyer or a disinterested trader) can arrange the set of indirectly informed buyers into several distinct priority levels, and by so doing can increase his revenues. As a consequence, this improves the incentives for efficient information acquisition, by reducing the *rent dissipation* which we showed in section 3.2 was at the root of inefficiency in information acquistion.

Informed traders can choose the number of types of reports and the price at which they are willing to sell them. Facing the menu of reports on offer, each buyer chooses which ones to buy. We will consider in particular the case where information can be vertically differentiated, or the different types of reports offered for sale can always be arranged in a hierarchy, of reports of decreasing quality, or informativeness. Let $L$ denote the number of different types of reports sold. The hierarchy of the qualities of the different reports is modelled by assuming that buyers purchasing a report of type $l$, $l \in \{1, .., L\}$ observe all the messages $m_j$, $j = l + 1, .., L$. The information provided by the reports has then a nested structure, in the sense that receiving report $i$ conveys no additional information to the one in report $l > i$, while the reverse is not true. We consider again the case where the set of possible messages available to the sellers of information for any $l$ is the set of direct messages, $m_l \in \mathcal{M} = \{1, 2, ..., K\}$.

In this subsection we look in particular at the situation where there is a single informed trader, i.e. only one buyer has chosen to pay the cost $c$ to acquire direct information over the quality type of the object up for sale. Let us denote by $B_1$ the informed buyer, and by $B_i$, $i = 2, .., N$ all the uninformed buyers who are purchasing information from $B_1$. We will find his optimal choice concerning the differentiation of information sold and his maximal payoff, thus determining also the maximal level of $c$ such that information acquisition is worthwhile.

In subsection 5.2 we allow for the possibility of entry in the market for information, i.e. that more than a single buyer acquires information and sells suitably differentiated information in the market.

We will find and characterize the equilibria of the subgame starting from the node where a single buyer, $B_1$, has acquired information and is selling differentiated information in the market. We determine first for any given level of $L$ the optimal choice of $B_1$ concerning the prices posted for the different types of reports sold, and the equilibrium strategies in the rest of the subgames (purchase of information, reporting strategies and bids in the auction). We then find the level of $L$ which maximizes the revenue of the seller $B_1$. By comparing the maximal payoff which obtained for $B_1$ we can then see also when information acquisition is worthwhile (when entry is restricted to at most one seller of information).

As in the previous sections, there are two phases in the reporting of messages. First each buyer of information sends a report over his type to the seller of information and this one subsequently sends the messages $m_l, l = 1, .., L$. The buyers of report of type $l$ receive then the messages $(m_L, m_{L-1}, .., m_l)$, for any $l \in \{1, .., L\}$.

We still focus our attention on the equilibria where agents' reporting is characterized by the maximal degree of truthfulness and, at the same time, is consistent with the differentiation of information in $L$ levels (so that the revenue of the seller of information is maximal). In particular, we will show that there is always an equilibrium where the uninformed buyers always report their type and the seller of information adopts the reporting strategy described below.

To this end, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from $B_1$ a report of quality $l$ as the buyers in layer $l$ of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote then the set of buyers in layer $l$ or below (i.e. purchasing reports of type $i \geq l$) and $N_l(B_1)$ the number of different realizations of $\theta_i$ across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1)$ indicates the set of buyers in layer $l-1$. $\mathcal{N}_1(B_1)$ and $N_1(B_1)$ are then similarly defined except for the fact that $\mathcal{N}_1(B_1)$ is augmented by $B_1$, i.e. is the set of buyers who purchased any type of report from $B_1$, plus $B_1$. The reporting strategy of $B_1$ for the messages $m_1, .., m_L$ is then defined recursively as follows:

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \\ y, \text{ with probability } 1/[K - N_1(B_1)], \\ \text{for all } y \neq \theta_j, \;\; B_j \in \mathcal{N}_1(B_1), \text{ if } v = \theta_1 \end{cases} \tag{12}$$

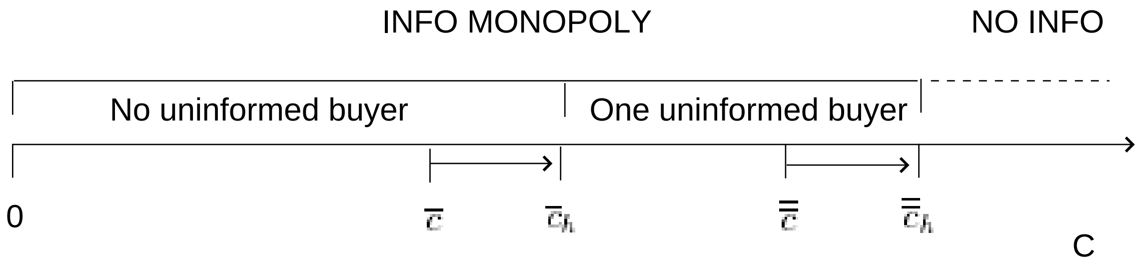

Figure 2: Equilibrium with heterogeneous messages

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } \; m_{l-1} \neq \theta_i \text{ for all i} \in \mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1) \\ y, \text{ with probability } 1/[K - N_l(B_1)], \\ \text{for all } \; y \neq \theta_j, \; B_j \in \mathcal{N}_1(B_1), \text{ if } \; m_{l-1} = \theta_i \text{ for some } B_i \in \mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1) \end{cases} \tag{13}$$

Thus at each layer $l$ the informed trader tells the truth about the quality of the object as long as the true quality of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality. Otherwise, the informed trader randomizes over any value different from the type of any of the agents who purchased information, including his own.

With this message structure and a monopolist, there are a couple of interesting features of the equilibrium.

**Proposition 5** *A monopolist which sells differentiated information under a reporting strategy as in 12 and 13 sells information to all buyers but possibly one. In both cases the optimal distribution is to create as many layers as players with whom he communicates. The condition that determines when is it optimal to leave no buyer uninformed is:*

$$c \leq \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right)\frac{1}{N-2} = \overline{c}_h \tag{14}$$

*The condition that determines when is it optimal for the monopolist to acquire information is:*

$$c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) = \overline{\overline{c}}_h \tag{15}$$

Figure 2 summarizes the result.

The result that the monopolist wishes to create as many layers as buyers with whom he communicates, is the main innovation of the equilibrium with heterogeneous information. The key insight for this result is that for a given buyer any other buyer who is **equally or better informed** than himself lowers his payoff by exactly as much. This means that the price this individual is willing to pay in the auction will not change if anybody who is equally or better informed gets an even better signal. The one who gets ahead of the queue, however, does improve his payoff, and therefore is willing to pay more. The monopolist can thus increase revenue from information by improving the signal of one player (at no cost to him in the auction as long as he still gets the best information). By induction, the best is to completely rank all the buyers with whom he communicates.

Condition (14) reflects a tradeoff for the seller of information that is slightly different from the case of homogeneous information. By maintaining one uninformed buyer, the price in the auction is higher for all the informed buyers. Once this individual is informed, the price of the auction is lower for all other buyers. This benefits the buyer who acquires information directly, and also through an increase in the rents of other buyers and hence on the willingness to pay for information. But, at the same time, when all buyers are informed, the payoff of remaining uninformed (the outside option) increase, and thus reduces the maximum willingness to pay for information. The key difference with respect to the homogeneous case is that through the heterogeneous reports, the provider of information can benefit indirectly from the increased willingness to pay of indirect buyers when all are informed. Because of this, the level of costs at which all buyers become informed in equilibrium is higher with heterogeneous messages (that is, $\overline{c}_h > c^0$).

### 5.1.1 Efficiency and equilibrium revisited

The condition for a monopolist to prefer being informed to not having information is given in equation 15. Since the condition for efficient investment in information is:

$$W_1 \geq W_0 \Longleftrightarrow \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right) - c \geq 0.$$

Since this is equivalent to 15, we immediately have efficient information acquisition by the monopolist in equilibrium.

To summarize: we have shown that the optimal structure of reports is given by a number of reports $L$ equal to the number $N-1$ of potential buyers of information, with prices such that each buyers chooses to purchase one report, and each type of report is purchased by only one agent. We show in particular that the optimal level of prices for such reports is such that $B_1$ can appropriate (via the revenue from the sale of information and the gain in $B_1$'s surplus from the auction with respect to the case where information is not acquired) all the social surplus generated by the acquisition of information, which as shown in (2) is given by

$$W_1 - W_0 = \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) - c.$$

Notice the *free-riding* by the uninformed buyers still occurs in this case. The *rent dissipation*, however, now disappears. This is because the information structure generated completely ranks buyers, and thus there will be no ties in the auction. This immediately implies that there is no underinvestment in information acquisition.

## 5.2 The competitive case

In the previous section, we showed that a monopolist selling information of heterogeneous quality could achieve an efficient outcome in this market. In this section, we show that the possibility to sell heterogeneous information does not guarantee efficiency. In particular, it is likely that when costs of information are not too high, too much information will be acquired. More precisely, we show that more than one individual may acquire information in equilibrium, which in our setup is inefficient.

Before we do that, we should note that the outcome with a monopoly seller of heterogeneous information is always an equilibrium. The reason is that if even if entry profits are potentially very high, if any "entrant" is assumed to send uninformative signals, he may optimally send uninformative signals after entry (out of the equilibrium path). But no uninformed buyer will pay any positive price for such signals. As a consequence he will not buy information.This equilibrium is ruled out since we focus our attention on equilibria where the degree of truthfulness in agents' reporting is maximal.

Similarly, the outcome with the monopolist can always be sustained by assuming that in the case two informed buyers are present, both of them give away the signal truthfully (whenever different from their type) and without discrimination for free. We also rule out this kind of *destructive subgame outcome*. Since the strategies used in this kind subgame equilibrium are weakly dominated, they would be selected against by refinements in the spirit of *forward induction* (Van Damme 1989). Intuitively, sellers in a subgame should ask themselves: "Why would my competitor buy information just to give it away?" and conclude that another kind of equilibrium is likely in the subgame.

For these reasons, we refine the equilibrium and assume that there is no *destructive* *subgame outcome* and that the signaling policy of all buyers of information is like the one pursued by the monopolist in the previous section. More precisely, we assume that any seller of information $B_i$ arranges his buyers in a set of $L_i$ *layers (or levels),* $L_i \geq 1$. This seller of information sends a different message $m_l^i$ to all buyers in each layer $l$, $l = 1, .., L_i$. Buyers in any layer $l$ observe not only the message sent to layer $l$ but also the messages sent to all other layers below $l$, i.e. observe $m_j^i$, $j \geq l$ thus ensuring information to be of (weakly) lower quality for buyers in lower layers.

We now describe the reporting strategy of $B_i$. Let layer 0 be an initial layer where only $B_i$ lies. Also, let $N(B_i)$ denote the number of different realizations of $\theta_j$ across all buyers who buy information from $B_i$, $j = 1, .., N_i$.In particular we assume that all competitors are (correctly) assumed to follow the following signaling policy:[13]

$$m_1^i = \begin{cases} v, \text{ if } v \neq \theta_i \\ y, \text{ with probability } 1/[K - N_1(B_i)]\,, \\ \text{for all } y \neq \theta_j, \;\; j = 1, .., N_i, \text{ if } \; v = \theta_i \end{cases} \tag{16}$$

and, for $l = 2, .., L_i$

$$m_l^i = \begin{cases} m_{l-1}^i, \text{ if } \; m_{l-1}^i \neq \theta_j \text{ for all } \; j \text{ corresponding to } B_j \text{ belonging to layer } l-1 \\ y, \text{ with probability } 1/[N - N_l(B_i)]\,, \\ \text{for all } \; y \neq \theta_j, \;\; j = 1, .., N_i, \text{ if } \; m_{l-1}^i = \theta_j \text{ for some } B_j \text{ belonging to layer } l-1 \end{cases} \tag{17}$$

PROPOSITION **6** *For sufficiently low c, in all "refined" equilibria (i.e. where the signals are as in 16 and 17), and without* destructive subgame outcome, *there are at least two sellers of information.*

REMARK **2** *To understand this result we have to remember what prevented multiple entry in the game with homogeneous information (and with low costs). In that situation the (unique) signal would get its price driven to zero with multiple entrants, and thus entry would only be marginally profitable for at most one extra entrant (in the case of low costs). Now, however, there may be two entrants or more sharing the (now higher) rents from information. A buyer of information would have to buy information from more than one provider to retain his privileged position in the hierarchy of information. That is, the (credible) threat if a buyer did not buy a (redundant) signal from a second (or third) provider is that this provider would*

---

[13] This is the same as in the monopoly case, except that some more notation is needed to distinguish the different potential senders - a superinded for messages and a subindex for $M_i$ and $L_i$.

*then offer a signal of the same quality to another buyer, which would ruin the value of the signal(s) already purchased.*

## 6 Discussion

We have built a model for information markets which allows us to study the equilibrium and welfare properties of such markets in contexts where information is vital and may be of use to potential rivals. The model is versatile, so we have been able to study a number of potential setups. Information transmitted may be homogeneous or heterogeneous, and the seller of information may be a potential buyer of the good, the seller or a "neutral" third party. We have found that in most setups the outcome is ex-post efficient. This was the result of selling through an auction and of almost everybody getting informed. As we discussed, more intense competition for the object ($N > K$) would reverse this result. Obviously, another way in which this would cease being true is if the verification technology for the type of the good was imperfect.

Ex-post efficiency is not, nevertheless, a major problem in our setup. Efficiency in information acquisition is tougher to achieve. We have seen that one potential avenue for success in this respect is for information to be transferred in different qualities. This may give a potential efficiency rationale to the behavior of the likes of Henry Blodget, the noted Merril Lynch analyst who was issuing an "accumulate" recommendation for Excite at Home while in an internal e-mail he was writing "ATHM is such a piece of crap!" This does not mean that heterogeneous information is the solution to all information acquisition problems in our setup. For low costs, it may lead to too much information acquisition as providers join in an excessive race to share the rents from the activity.

There are several extensions that could be studied within our framework. A possible extension would consider multiple units of the good for sale. Suppose in this case that the directly informed buyers had limited capacity for "enjoying" the good. This could lead to ex-post inefficiency in the case of homogeneous information. The seller of infomation will lie about the type of the object to lower the competition in the auction, as he does there is a single unit. But now, this will lead to some units sold to buyers who did not like them, and thus ex-post inefficiency. Another possible extension concerns an imperfect verification technology for the type of the object. In that case, multiple signals could improve ex-post efficiency with respect to single signals (unlike in our setup).

There is an extension which we can discuss in some more detail. We have seen that, in contrast with a potential buyer of the good, the owner of the good prefers to tell the truth about the object in all cases like the uninterested trader. This is somewhat surprising, since common sense tells us that an owner of a good (and his agents) should also be somewhat untrustworthy as a source of information, albeit for different reasons. A potential buyer of the good has an incentive to tell his competitors that the good is not "right for them." Our model captures well this phenomenon. The owner of the good should have incentives to exagerate how much the buyers like the good. However, in our context, with horizontal differentiation, there is no way to do that, unless the seller knows the types of the buyers, and can use a coincidence of wants to increase auction competition. But there is a more natural way in which the seller can exagerate, which leads to misallocation problems.

Assume that, beyond the types, the good may come in 2 quality levels, $H$ (High) and $L$ (Low). The buyers, as before, like only one randomly chosen variety. Formally, the good has a type $v \in S^{HL} = \left\{1^H, ..., K^H, 1^L, ..., K^L\right\}$. Denote by $S^H = \left\{1^H, ..., K^H\right\}$ the set of High quality goods, $S^L = \left\{1^L, ..., K^L\right\}$ and the set of Low quality goods and by $i = \{i^H, i^L\}$ the set of goods of variety $i$, independent of the quality, for $i \in S = \{1, ..., N\}$.. In addition, the buyers are also of two types: some buyers are sensitive to quality ($Se$) and some of them are insensitive ($In$). An $In$ consumer has a constant valuation of 1 for a good of the type he desires. An $Se$ consumer values a good of the type he desires as $V$, if the good is of $H$ quality; and he values it at 0 if it is of $L$ quality. Let us assume for simplicity that $H$ and $L$ have identical probabilities for each type of good, and that consumers have identical probabilities to be of type $Se$ and $In$. Also, assume that messages are structured so that the message $m \in M$ must consist of a variety $i \in S$ and a quality $J \in \{H, L\}$, so that a generic $m = (i, J)$.

Proposition 7 *For any pair of messages* $m^H = (i, H)$ $m^L = (i, L)$ *from an owner of the good, we must have beliefs such that* $\Pr(v \in S^H | m^H) = \Pr(v \in S^H | m^L) = \Pr(v \in S^H) = \frac{1}{2}$. *Thus, if* $V > \frac{1}{2}$, *for any equilibrium message* $m$ *the* $Se$ *type buyers bid more for the good after hearing* $m$ *than the* $In$ *type buyers. If* $V < \frac{1}{2}$, *for any* $m$ *the* $Se$ *type buyers bid less for the good after hearing* $m$ *than the* $In$ *type buyers.*

The previous proposition shows that there is an inefficient allocation of the good as long as $V \neq \frac{1}{2}$, clearly a knife-edge case. This inefficient allocation also guarantees that not all social value can be appropriated by any seller, and therefore that there will be inefficient information acquisition, at least when $c$ is expensive enough for a "natural monopoly."

# APPENDIX A

### Proof of Theorem 1

**Optimal prices and payoffs for given numbers of sellers and buyers of information**

We identify each possible configuration with a number:

1. We examine first the situation where there is one seller of information (wlog let it be $B_1$), $J \geq 1$ agents do not buy information (let them be $B_{N-J+1}, ..., B_N$) and the remaining $N-J-1$ (let them be $B_2$ to $B_{N-J}$) buyers purchase information from the single seller. From the considerations we formulate in the main text, it is clear that the payoffs for the different players are in this case:

$$\begin{aligned}
\pi_{B_1}^J &= \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-(J+1))p(J)-c \\
\pi_{B_i}^J &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1-\frac{1}{k}\right)-p(J)=\{\pi_C^J,\pi_U^J\}, \text{ for } i=2,...,N-J \\
\pi_{B_N}^J &= ... = \pi_{B_{N-J+1}}^J = 0 \text{ if } J \geq 2 \qquad (18)\\
\pi_{B_N}^J &= \left(\frac{k-1}{k}\right)^{N-1}\frac{1}{k} \text{ if } J=1 \\
\pi_C^J &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1-\frac{1}{k}\right)-c=\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}-c \\
\pi_U^J &= 0
\end{aligned}$$

and the price of information is

$$p(J)=\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}\right\}$$

2. We now examine the configuration where there is again one seller of information (wlog let it be again $B_1$), and $N-1$ (let it be $B_2$ to $B_N$) buyers of information from him. The payoffs are now.

$$\begin{aligned}
\pi_{B_1}^0 &= \frac{1}{k}+(N-1)p(0)-c \\
\pi_{B_i}^0 &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}-p(0)=\{\pi_C^0,\pi_U^0\}=\pi_U^0 \text{ for } i=2,..,N \\
\pi_C^0 &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}-c \\
\pi_U^0 &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \Longrightarrow \pi_U^0>\pi_C^0 \Longrightarrow p(0)=0
\end{aligned}$$

3. Consider the case with $J \geq 2$ sellers of information (wlog let them be $B_1, ..., B_J$), and $N-J$ (let them be $B_{J+1}$ to $B_N$) buyers of information. In this case, as we argued in the main text, (a) each seller posts a zero price for information; (b) each uninformed buyer purchases information from all sellers. The payoffs in this case are:

$$\begin{aligned}\pi_{B_i}^{OL} &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, \; i = J+1, .., N \\ \pi_{B_1}^{OL} &= ... = \pi_{B_J}^{OL} = \pi_{B_i}^{OL} - c\end{aligned}$$

4. We now consider the situation where no buyer acquires information, hence there is no market for information as there is nobody selling it. In this case, the payoff of every buyer is:

$$\pi_{B_i}^{No} = 0 \text{ for all } i = 1, .., N$$

   Since all buyers are uninformed, they all make a bid equal to their expected valuation, $\frac{1}{k}$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the good and hence gets no surplus.

**Proof of Claim 1** We first restrict our attention to the case:

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, \tag{19}$$

under which we have $\max\{\pi_C^1, \pi_U^1\} = \pi_U^1$, hence $p(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. We then show that a deviation $J \geq 2$ is never profitable in this case. Let us examine first the case with $J = 2$ :

$$\begin{aligned}\pi_{B_1}^1 &\geq \pi_{B_1}^2 \iff \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-2)p(1) - c \geq \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-3)p(2) - c \\ &\iff (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-3)\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}, c\right\} \\ &= (N-3)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2},\end{aligned}$$

where the last equality follows from condition (19). The inequality condition we obtained, $(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-3)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}$, can be equivalently rewritten as:

$$\frac{N-2}{N-3} \geq \frac{k}{k-1} \iff 1 + \frac{1}{N-3} \geq 1 + \frac{1}{k-1} \iff k-1 \geq N-3,$$

and is always satisfied since $k > N$. With $J^* > 2$, we have:

$$\begin{aligned}
\pi^1_{B_1} &\geq \pi^{J^*}_{B_1} \\
\iff & \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-2)p(1) - c \geq \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-(J^*+1))p(J^*) - c \\
\iff & (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-(J^*+1)) \min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*}, c\right\} \\
= & (N-(J^*+1))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*}
\end{aligned}$$

Here we use an induction argument. We already showed the inequality above holds for $J = 2$ and we will show that for $J > 2$:

$$(N-(J+1))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \geq (N-(J+2))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}. \tag{20}$$

The above inequality can be rewritten as

$$\frac{(N-(J+1))}{(N-(J+2))} \geq \frac{k}{k-1} \iff 1 + \frac{1}{(N-(J+2))} \geq 1 + \frac{1}{k-1} \iff k-1 \geq N-(J+2)$$

which is always satified.

We verify in what follows that (19) is indeed a necessary condition for any $J \geq 2$ to be optimal, thus establishing, together with the reasoning above that $J \geq 2$ cannot be optimal.

For $J^* \geq 2$ to be optimal, the following must hold:

$$\pi^{J^*}_{B_N} = 0 \geq \max\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*+1} - p(J^*), \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*}\left(1-\frac{1}{k}\right) - c\right\}, \tag{21}$$

which ensures that none of the $N - J^*$ uninformed agents wishes to deviate to either purchase information, or acquire directly information in the final stage at a cost $c$.

1. We show that condition (21) implies (19). Consider first the case where

$$\max\{\pi^{J^*}_C, \pi^{J^*}_U\} = \pi^{J^*}_C, \text{ or } \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*} \geq c,$$

   and $p(J^*) = c$. In this case condition (21) simplifies to $0 \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*+1} - c$, or $c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*+1}$. Since

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*+1} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1},$$

if (21) is satisfied, (19) a fortiori must hold. In the other case, where

$$\max\{\pi_C^{J^*}, \pi_U^{J^*}\} = \pi_U^{J^*} = 0, or \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*} \leq c,$$

since $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J^*} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ (19) must hold (this is true in fact whether or not (21) is satified).

**This completes the proof of Claim 1.**

**Proof of Claim 2** Given Claim 1 we simply need to establish the range of values of $c$ for which

$$\pi_{B_1}^1 \geq \pi_{B_1}^0 \tag{22}$$

The only seller of information prefers setting the price equal to $p(1)$ rather than $p(0)$) if

$$\pi_{B_1}^1 = \frac{1}{k}\left(1 - \frac{1}{k}\right) + (N-2)p(1) - c \geq \pi_{B_1}^0 = \frac{1}{k} - c$$

or equivalently,

$$(N-2)\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} \geq \frac{1}{k} - \frac{1}{k}\left(\frac{k-1}{k}\right) = \frac{1}{k^2}$$

When $\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} = c$, the above condition reduces to:

$$\begin{aligned}(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> (N-2)c \geq \frac{1}{k^2}, \text{ or:} \\ \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> c \geq \frac{1}{N-2}\frac{1}{k^2}\end{aligned}$$

Else, when $\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ the condition is

$$c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \frac{1}{N-2}\frac{1}{k^2}$$

The join of the two conditions is then:

$$c \geq \frac{1}{N-2}\frac{1}{k^2} = c^0 \tag{23}$$

**This completes the proof of Claim 2.**

**Proof of Claim 3** We check the conditions under which no information is gathered in equilibrium. For this, we need:

$$\pi_{B_i}^{No} = 0 \geq \begin{cases} \pi_{B_1}^1 \text{, if } c > \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_1}^0 \text{, if } c < \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_1}^1 \text{ or } \pi_{B_1}^0 \text{, if } c = \frac{1}{N-2}\frac{1}{k^2} \end{cases}.$$

The last two conditions can be equivalently written as:

$$\frac{1}{N-2}\frac{1}{k^2} \geq c \geq \frac{1}{k},$$

which is never satisfied. The first condition can be rewritten as

$$\begin{aligned} c &\geq \max\left\{\frac{1}{N-2}\frac{1}{k^2}, \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\min\left[c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right]\right\} \\ &= \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\min\left[c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right], \end{aligned}$$

which can only be satisfied if $c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, in which case the condition reduces to:

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}. \tag{24}$$

Hence we conclude that when (24) holds, configuration 4. obtains at an equilibrium.

**This completes the proof of Claim 3**

**Proof of Claim 4** To get a duopoly at an equilibrium of the overall game we need:

$$\pi_{B_1}^{OL} \geq \begin{cases} \pi_{B_i}^1 \text{, for } i = 2, .., N-1 \text{, if } c > \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_i}^0 \text{, for } i = 2, .., N \text{, if } c < \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_i}^1(1) \text{ or } \pi_{B_i}^0 \text{, if } c = \frac{1}{N-2}\frac{1}{k^2} \end{cases} \tag{25}$$

$$\pi_{B_i}^{OL} \geq \pi_{B_1}^{OL} \text{, for } i = 3, .., N \tag{26}$$

When $c \geq \frac{1}{N-2}\frac{1}{k^2}$, condition (25) can be rewritten as

$$\begin{aligned} \pi_{B_1}^{OL} &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq \pi_{B_i}^{OL} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - p(1) \\ &= \max\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c, 0\right\}. \end{aligned}$$

which only holds when $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq 0$. On the other hand, if $c < \frac{1}{N-2}\frac{1}{k^2}$ (25) becomes:

$$\pi_{B_1}^{OL} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1},$$

never true.

1. Finally, condition (26) can be written as:

$$\pi^{OL}_{B_1} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$

   always satisfied.

   We conclude that a duopoly obtains as an equilibrium outcome of the overall game when

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq c \geq \frac{1}{N-2}\frac{1}{k^2},$$

   **This completes the proof of Claim** 4

**This completes the analysis of the possible configurations which may arise at equilibrium and hence the proof of Theorem 1**.

**Proof of proposition 1** The result follows immediately by comparing the threshold for efficient information acquisition, found in (2), with the threshold found in Theorem 1 for information acquisition not to take place in equilibrium, given by (24). Hence by combining the two, we obtain that, as claimed, when (3) holds, in equilibrium no information will be gathered even though social welfare is maximized when information is acquired.

We also show that the interval of values of $c$ identified in condition (3) is nonempty, i.e.:

$$\begin{aligned}\frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &< \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) \Longleftrightarrow \\ \left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-3}{k}\right) &< 1\end{aligned}$$

Let $x \equiv k - N$, so that the condition $\left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-3}{k}\right) < 1$ can be equivalently written as:

$$\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-3}{N+x}\right) < 1. \tag{27}$$

Since the term on the left hand side approaches one as $x \to \infty$ , it suffices to show that such term is strictly increasing in $x$. Notice that a term is increasing if its logarithm is increasing. Take then the logarithm of the left hand side of (27), we get

$$(N-3)\ln\left(1-\frac{1}{N+x}\right) + \ln\left(1+\frac{N-3}{N+x}\right);$$

differentiating it with respect to $x$ yields:

$$\begin{aligned}(N-3)\frac{1}{(N+x)^2}\frac{1}{\left(1-\frac{1}{N+x}\right)}-\frac{(N-3)}{(N+x)^2}\frac{1}{\left(1+\frac{N-3}{N+x}\right)} &= \frac{(N-3)}{(N+x)^2}\left(\frac{1}{\left(1-\frac{1}{N+x}\right)}-\frac{1}{\left(1+\frac{N-3}{N+x}\right)}\right)\\ &= \frac{(N-3)}{(N+x)^2}\left(\frac{\frac{N-2}{N+x}}{\left(1-\frac{1}{N+x}\right)\left(1+\frac{N-3}{N+x}\right)}\right),\end{aligned}$$

which is strictly positive since $N+x>1$, or $k>1$.

**This completes the the proof of Proposition 1**.

**Proof of proposition 2** The maximal price that buyers are willing to pay for information to a monopolist seller of information when a total number of $N-J$ buyers purchase information is given by $\min\left\{c,(k-1)^{N-J-1}/k^{N-J}\right\}$. By an argument similar to the one that allowed us to show the analogous result in Theorem 1, the revenue of the seller is always higher if information is sold to $N-1$ buyers than to any smaller number of buyers.

Hence, if an equilibrium exists where information is acquired and sold by an uninterested trader, his profits are:

$$\pi_{B_u}=(N-1)\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}-c\geq 0. \tag{28}$$

It is immediate to see from (28) that the highest level of $c$ for which an equilibrium with information acquisition exists is:

$$c=(N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}.$$

On the other hand, the threshold found in (24) for information acquisition to take place at an equilibrium when information is sold by an interested trader is

$$\frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

Hence to prove the result it suffices to show that

$$\begin{aligned}\frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\\ &\iff \frac{1}{k}\left(\frac{k-1}{k}\right)>\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\end{aligned}$$

which is always true.

**This completes the the proof of Proposition 2**

**Proof of proposition 3** The payoffs for the seller of the good when he acquires information depend both on information and auction revenues. In terms of the revenues from information, the maximum willingness to pay for the buyers is their surplus minus their outside option (which is $\max\{\pi_C, \pi_U\}$).

Let $J$ be the number of uninformed buyers. When $J > 0$, the surplus for buyers is positive only when they obtain the good and no other informed buyer likes it. In that case the surplus is $1 - 1/k$ leading to an expression for the surplus:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1-J}\left(1-\frac{1}{k}\right) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}$$

The outside option for $J \geq 1$, we have

$$\pi_C = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c < 0 = \pi_U.$$

Thus the price of information in this case is:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}.$$

And therefore the revenue from information is

$$(N-J)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}. \tag{29}$$

Notice that we showed in equation (20) that this is a decreasing function of $J$, thus revenues from information are decreasing in $J$, for $J > 0$.

When $J = 0$, the surplus for buyers is positive when they obtain the good and no other buyer likes it. In that case the surplus is 1 leading to an expression for the surplus:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}.$$

The outside option for $J = 0$,

$$\pi_C = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c < \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} = \pi_U.$$

So the price of information will be zero.

The revenues at the auction when $J \geq 2$ can only be $1/k$ (if no informed buyer likes the object) or 1 (if more than one informed buyer likes the object). Thus, when $J \geq 2$ the revenues decrease with $J$, as the probability that more than one informed buyer likes the object is decreasing in $J$. Putting this together with the fact that the revenue from information is decreasing with $J$ for $J > 0$, as we showed earlier from equations (29) and (20) tells us that the best possible option for the seller from the set $J \geq 2$ is exactly $J = 2$.

We are thus left to compare the total payoffs for the seller from $J = 0$, $J = 1$ and $J = 2$.

The payoffs when $J = 0$ are simply the revenues from the auction (which in this case are 0 if no buyer, or exactly one, likes the object, and 1 otherwise):

$$1 - \left(\frac{k-1}{k}\right)^{N} - N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

The payoffs when $J = 1$ are the payoffs from information sale (from equation 29) plus the revenues from the auction (which in this case are 0 if no informed buyer likes the object, $1/k$ if exactly one informed buyer likes the object, and 1 otherwise):

$$\begin{aligned}
& (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} + \Bigg(1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\Bigg) + \frac{1}{k}\Bigg((N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N} \\
= \;& (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} + 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \\
= \;& 1 - \left(\frac{k-1}{k}\right)^{N-1}
\end{aligned}$$

The payoffs when $J = 2$ are the payoffs from information sale (from equation 29) plus the revenues from the auction (which in this case are $1/k$ if no informed buyer or exactly one likes the object, and 1 otherwise):

$$\begin{aligned}
& (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} + \Bigg(1 - \left(\frac{k-1}{k}\right)^{N-2} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\Bigg) + \frac{1}{k}\Bigg(\left(\frac{k-1}{k}\right)^{N-2} + (N - \\
= \;& (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} + 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} \\
= \;& 1 - \left(\frac{k-1}{k}\right)^{N-1}
\end{aligned}$$

Thus, the payoffs for the seller are maximized when $J = 1$ or $J = 2$, if

$$1 - \left(\frac{k-1}{k}\right)^{N-1} > 1 - \left(\frac{k-1}{k}\right)^{N} - N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

which is always true since this is equivalent to

$$N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

The result now follows by comparing the threshold for efficient information acquisition, found in (2), with the threshold for information acquisition not to take place in equilibrium. Thus, the threshold for information acquisition in equilibrium is when

$$1-\left(\frac{k-1}{k}\right)^{N-1}-c\geq\frac{1}{k}$$

or

$$\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}\right)\geq c$$

Remember that the threshold for efficient information acquisition is:

$$1-\left(\frac{k-1}{k}\right)^{N}-c\geq\frac{1}{k}$$

or

$$\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right)\geq c$$

and the result follows.

**This completes the the proof of Proposition** 3

**Proof of Lemma 1** Let $x\equiv K-N$, so that condition (11) can be equivalently written as:

$$\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-2}{N+x}\right)>1. \tag{30}$$

Since the term on the left hand side approaches one as $x\to\infty$ , if such term is strictly decreasing in $x$, for $x$ sufficiently large, we can say that condition (30) holds for $K$ sufficiently large. Notice that a function is decreasing if its logarithm is decreasing. Take then the logarithm of the term on the left hand side of (30)

$$(N-3)\ln\left(1-\frac{1}{N+x}\right)+\ln\left(1+\frac{N-2}{N+x}\right)$$

and differentiate it with respect to $x$. We obtain:

$$(N-3)\frac{1}{(N+x)^2}\frac{1}{\left(1-\frac{1}{N+x}\right)}-\frac{(N-2)}{(N+x)^2}\frac{1}{\left(1+\frac{N-2}{N+x}\right)}=\frac{1}{(N+x)^2}\left(\frac{\frac{(N-2)^2-(N+x)}{N+x}}{\left(1-\frac{1}{N+x}\right)\left(1+\frac{N-2}{N+x}\right)}\right)$$

which is strictly negative if and only if $(N-2)^2-N<x$. This implies that the term on the left hand side of (30) is first increasing and then decreasing in $x$. Since, as we said, this term also tends to one as $x$ goes to infinity, it follows that condition (11) necessarily holds

for $x$ large (thus for $K$ sufficiently larger than $N$). Furthermore, if (30) is satisfied for $x = 0$, condition (11) is satisfied for all $x$. To conclude, notice that, when $x = 0$, i.e. $N = K$, (30) can be written as:

$$2\left(\frac{K-1}{K}\right)^{K-2} > 1$$

which is true for $K < 6$ and false for $K \geq 6$.

**Proof of Proposition 4** Denote by $P$ the price paid to gain the object in the auction and by $p$ the price of information. Let then $B_i^{Win}$ be the event in which buyer $B_i$ wins the auction and $\mathbb{E}_S$ the expectation conditional on the information of the seller of the good. The payoff for a buyer of information, when there are $J > 0$ buyers who remain uninformed, is:

$$\begin{aligned}
\pi_{B_i}^J &= \Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - p \\
\text{for } i &= 1, .., N - J.
\end{aligned}$$

When there is a monopolist seller of information its price is then:

$$p = \Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - \max\{\pi_C^J, \pi_U^J\},$$

where $\pi_C^J$ and $\pi_U^J$ are as defined in the proof of theorem 1.

Substituting this expression into the one of the payoff of the seller of the good when he is also the monopolist seller of information and there are $J > 0$ uninformed buyers, in any equilibrium both when the seller has no information over buyers types and when he ask them to report their types, yields:

$$\begin{aligned}
\pi_S^{S,J} &= \mathbb{E}_S(P) + \sum_{i=1}^{N-J}\left(\mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - \mathbb{E}_S\left(\mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win})\right)\right) \\
&\quad -(N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c \\
&= \sum_{i=1}^{N-J}\mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) + \sum_{i=N-J+1}^{N}\mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\
&\quad + \sum_{i=1}^{N-J}\left(\mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - \mathbb{E}_S\left(\mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win})\right)\right) - (N-J)\max\{\pi_C^J(S), \\
&= \sum_{i=N-J+1}^{N}\mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\
&\quad + \sum_{i=1}^{N-J}\mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c
\end{aligned}$$

Look at $c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. Seller's payoff with no information on buyers' types, as shown in Proposition 3?, is, for any $J \geq 1$ :

$$\begin{aligned} \pi_S^{S,J,no\inf o} &= \sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\ &\quad + \Pr(v=\theta_i,\text{ for some } i=1,..,N-J) - c \end{aligned} \tag{32}$$

Note also that whenever $J \geq 2$, $\sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win})$ is independent of whether the seller of information has any information over buyers' types (since $\mathbb{E}_S\left(P|B_i^{Win}\right) = 1/K = \Pr(\theta_i = v)$ for any $i > N-J$), and $\Pr(v=\theta_i$, for some $i = 1,..,N-J)$ is clearly the maximal value of $\sum_{i=1}^{N-J} \mathbb{E}_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right)$ since it says that some (indirectly) informed buyer always gets the objects whenever some informed buyer likes it.

We show next that same is true when $J = 1$. This requires us to derive some properties of the reporting strategies and payoffs at an equilibrium with maximal truthtelling. We will show in particular that at such equilibrium the uninformed never gets the object, $\Pr(B_N^{Win}) = 0$. Hence by comparing (31) and (32) and recalling that at the equilibrium with no information $\sum_{i=1}^{N-J} \mathbb{E}_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right)$ is maximal, the claim follows.

We consider the case where the message structure is similar to Section 2: the seller information first asks buyers of information to report their types and then send his report. We will show that an equilibrium with maximal truthfulness of agents reporting buyers report thruthfully their type, while the seller does not. In particular the seller's reporting strategy is:

$$m_S = \begin{cases} i) \text{ if there is at least one pair } i,j \in (1,..N-1) \text{ such that } \theta_i = \theta_j \text{ (i.e.: there is a 'tie') :} \\ \qquad v \text{ if } v = \theta_i = \theta_j \text{ for some } i,j \in (1,..N-1) \text{ or} \\ \qquad \theta_i \neq v \text{ if } v = \theta_k \text{ for at most one } k \in (1,..,N-1) \\ \\ ii) \text{ if } \theta_k \neq \theta_j \text{ for all } j,k \in (1,..,N-1) \text{ (there are no 'ties'):} \\ \qquad v \text{ if } v = \theta_i \text{ for some } i \in (1,..N-1) \text{ or} \\ \qquad \theta_i \neq v \text{ with probability } 1/(N-1) \text{ if } v \neq \theta_i \text{ for all } i = 1,..,N-1 \end{cases} \tag{33}$$

Given this reporting strategy

$$\begin{aligned} \Pr(\theta_i &= v|m_S = \theta_i) = \\ \Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie})\Pr(\text{there is a tie}) + \\ + \Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there are no ties})\Pr(\text{there are no ties}). \end{aligned}$$

Furthermore, it is clear from the above reporting strategy that both

$$\begin{aligned}\Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}) > 1/K,\\ \Pr(\theta_i &= v|m_S = \theta_i \ \cap \text{ there are no ties}) > 1/K.\end{aligned}$$

From this it follows that the optimal bidding strategy of an indirectly informed buyer is:

$$b_i(m_S = \theta_i) = \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there is a tie}), \tag{34}$$

since in the event in which there are no ties the buyer knows he is the only one buyer to receive a message equal to his type, hence the highest bid among all other buyers is the one of the uninformed buyer, $1/K$. Thus if the buyer knew he were in such event, any bid greater than $1/K$ would be optimal as it yields him the object. On the other hand, in the event where there is a tie, the value of the object for the buyer is $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie). Thus if the buyer kned he were in such event his optimal bid would be precisely $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie). Now the buyer does not know which of these two events is true when he makes his bid. However, a bid equal to $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie) is an optimal bid if the buyer knew which if the two events were true and can be made without knowing this.

The seller's reporting strategy in (33) and the bidding strategy of the indirectly informed buyers in (34) imply that in equilibrium, in every state, at least one such buyer $i$ will receive a message $m_S = \theta_i$ and hence make a bid $b_i > 1/K$, so that the uninformed buyer never gains the object: $\Pr(B_N^{Win}) = 0$ as claimed.

To complete the argument for the case $J = 1$ it remains to verify that for a buyer, say $B_1$, who purchases information it is optimal to truthfully report his type to the seller (given that all other buyers truthfully report their types and the seller follows the above strategy). If $B_1$ lies about his own type to the seller ($L$), the above reporting strategy of the seller implies that $B_1$ can only receive a report $m_S = \theta_1$ if there exists at least one other buyer $j = 2, .., N-1$ such that $\theta_j = \theta_1$. Hence

$$\begin{aligned}\Pr(\theta_1 &= v|m_S = \theta_1; L) = \Pr(\theta_1 = v|m_S = \theta_1 = \theta_j \text{ some } j; L) &(35)\\ \Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \cap \text{ there is a ’reported’ tie;L}) \Pr(\text{there is a ’reported’ tie}) +\\ + \Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \ \cap \text{ there are no ’reported’ ties;L}) \Pr(\text{there are no ’reported’ ties}).\end{aligned}$$

Furthermore, notice that:

$$\begin{aligned}\Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \cap \text{ there is a ’reported’ tie;L}) = \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there is a tie}), \quad (\\ \Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \ \cap \text{ there are no ’reported’ ties;L}) = \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there are no t}\end{aligned}$$

The optimal bidding strategy of $B_1$ is then $b_1(m_S = \theta_1; L) = \Pr(\theta_1 = v|m_S = \theta_1)$.

On this basis we can compare the payoff of $B_1$ if he reports truthfully his type

$$\begin{aligned} \Pr(\text{there is no tie} \cap m_S &= \theta_1)(b_i(m_S = \theta_1) - 1/K) = \\ \Pr(\text{there is no tie} \cap m_S &= \theta_1)(\Pr(\theta_1 = v|m_S = \theta_1 \cap \text{ there are no ties}) - 1/K) \end{aligned} \tag{37}$$

to the one if he misreports his type:

$$\begin{aligned} \Pr(m_S &= \theta_1 = \theta_j \text{ some } j; L)(\Pr(\theta_1 = v|m_S = \theta_1; L) - b_j(m_S = \theta_j)) = \\ \Pr(m_S &= \theta_1 = \theta_j \text{ some } j; L)(\Pr(\theta_1 = v|m_S = \theta_1; L) - \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there is a tie})) \end{aligned}$$

which using (35) and (36) becomes:

$$\begin{aligned} &= \Pr(m_S = \theta_1 = \theta_j \text{ some } j; L) \Pr(\text{there are no 'reported' ties}) \max\{\Pr(\theta_i = v|m_S = \theta_i \cap \text{ there} \\ -\Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}); 0\}. \end{aligned}$$

Comparing (37) and (38), and noticing that Pr(there are no 'reported' ties) = Pr(there are no ties), $\Pr(m_S = \theta_1 = \theta_j \text{ some } j; L) < \Pr(m_S = \theta_1)$ and recalling that $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie$) > 1/K$ establishes the result.

We are then left with the case where $J = 0$. Note that in such case (if all buyers report truthfully their types)

$$\pi_S^{S,0} = \sum_{i=1}^{N} \mathbb{E}_S \left( \Pr(B_i^{Win}|v = \theta_i) \Pr(v = \theta_i) \right) - c.$$

since, by the previous argument the payoff of a buyer if he were not to purchase information is zero as he would never gain the object: $\pi_U^0(S) = 0$. Given the seller's reporting strategy described above, which implies the only possible misallocation of the object is when only one of the $N$ buyers like the object and there are at least two other buyers of the same type (there is a tie), this expression is equal to

$$= \Pr(v = \theta_i, \text{ for some } i = 1, .., N) - \Pr(v = \theta_i, \text{ for exactly one } i = 1, .., N \text{ and there is a tie}) - c.$$

Recall that the seller's payoff in the equilibrium where the seller has no information over buyers' types (nor ask them to report them) is:

$$\pi_S^{S,1,no\inf o} = \Pr(v = \theta_i, \text{ for some } i = 1, .., N-1) - c.$$

Hence we need to show that

$$\begin{aligned} \Pr(v &= \theta_i, \text{ for some } i = 1, .., N-1) > \\ \Pr(v &= \theta_i, \text{ for some } i = 1, .., N) - \Pr(v = \theta_i, \text{ for exactly one } i = 1, .., N \text{ and there is a tie}), \end{aligned}$$

or

$$\begin{aligned}
\Pr(v &= \theta_i,\ \text{for exactly one } i = 1,..,N \text{ and there is a tie}) > \\
\Pr(v &= \theta_i,\ \text{for some } i = 1,..,N) - \Pr(v = \theta_i,\ \text{for some } i = 1,..,\\
\left(1 - \left(\frac{K-1}{K}\right)^N\right) - \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) &= \\
& \left(\frac{K-1}{K}\right)^{N-1}\left(\frac{1}{K}\right).
\end{aligned}$$

Note that

$$\begin{aligned}
\Pr(v &= \theta_i,\ \text{for exactly one } i = 1,..,N \text{ and there is a tie}) = \\
&= 1 - \Pr(\text{no ties}) - \Pr(\text{tie on } v) \\
&= 1 - \left[\left(\frac{K-1}{K}\right)\cdot ... \left(\frac{K-(N-1)}{K}\right)\right] - \left[1 - \left(\frac{K-1}{K}\right)^N - \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1}\right] \\
&= \left(\frac{K-1}{K}\right)^N + \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} - \left[\left(\frac{K-1}{K}\right)\cdot ... \left(\frac{K-(N-1)}{K}\right)\right] > \\
& \left(\frac{K-1}{K}\right)^N + \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} - \left(\frac{K-1}{K}\right)^{N-1},
\end{aligned}$$

which is clearly greater than $\left(\frac{K-1}{K}\right)^{N-1}\left(\frac{1}{K}\right)$. Furthermore, in this case buyers would still want to report truthfully their type. This follows from the fact that now, with no uninformed traders, the buyers who report truthfully their type make a bid equal to $\Pr(\theta_i = v | m_S = \theta_i)$ and, by essentially the same argument as above, $\Pr(\theta_1 = v | m_S = \theta_1) = \Pr(\theta_1 = v | m_S = \theta_1; L)$, i..e lying does not change the information contained in a report equal to the buyer's own type. Since that, in the even of a lie, only happens if there is at least one other buyer who receives that report, who will then make that bid, the expected payoff by lying is zero.

**Proof of Proposition 7** Suppose first $V > \frac{1}{2}$. If there were a pair of messages $m^H$, and $m^L$ which were sent with positive probability and $\Pr(v \in S^H | m^H) > \Pr(v \in S^H | m^L)$, then the owner of the good would induce a higher bid by from $Se$ buyers by announcing $m^H$, even if the true state is such that $v \in S^L$. The reason is that the bid of $In$ buyers does not change, as they do not care about the quality. In addition the price paid for the information is sunk at the moment when the message is sent, so there is no change in revenues from information sales by changing the announcement from $m^H$ to $m^L$ Thus, the owner of the good would increase profits by announcing $m^H$, and thus $m^L$ would not be sent in equilibrium, which is a contradiction. If for a given type $i \in S$, only $m^H = (i, H)$ is sent in equilibrium (and not

$m^L = (i, L))$, then necessarily $\Pr(v \in S^H | m^H) = \Pr(v \in S^H) = \frac{1}{2}$. Given these beliefs, it is immediate that an $Se$ individual who likes variety $i$ will bid more for the good for any $m$ as their expected valuation is $\frac{V}{2} \Pr(v \in i|m)$ whereas the expected value for an $In$ individual is $\Pr(v \in i|m)$, and since $V > \frac{1}{2}$, then $\frac{V}{2} \Pr(v \in i|m) > \Pr(v \in i|m)$. The proof for $V < \frac{1}{2}$ is similar.

**This completes the the proof of Proposition** 7

**Proof of Proposition 5** Most of this proof (in Appendix B) involves routine computations similar in nature to those of Proposition 1. We simply report here the proof of a crucial lemma that is clearly different from the previous material.

Lemma **2** *The optimal distribution is to create as many layers as remaining players.*

**Proof of lemma 2** To see this notice

1. The price paid at the auction does not change by increasing the number of layers.
2. If an old layer $l$ is split in two $l'$ and $l''$, then the willingness to pay of individuals in old layers $l + k$ does not change as they only care about the number of people in their layer or above, not their distribution.
3. If an old layer $l$ is split in two $l'$ and $l''$, then the willingness to pay of individuals in old layers $l - k$ does not change as they only care about the number of people in their layer or above, and this has not changed.
4. If an old layer $l$ is split in two $l'$ and $l''$, then the willingness to pay of individuals in old layer $l$ who is now in the new lower layer $l''$ does not change as they only care about the number of people in their layer or above, and this has not changed.
5. If an old layer $l$ is split in two $l'$ and $l''$, then the willingness to pay of individuals in old layer $l$ who is now in the new upper layer $l'$ strictly increases as they only care about the number of people in their layer or above, and this is now strictly lower.

**This completes the proof of Lemma 2 and Proposition 5**

**Proof of proposition 6** To show this, we only need to establish that profits from acquiring information are always strictly positive. This is because we showed in the previous section that for low $c$ the monopolist drives surplus to zero for indirectly informed players. More precisely, by equation (44), the condition for there being no unconnected players, and thus $\pi_{B_i} = 0$ for an indirectly connected player, is that $c$ is low. Thus, as long as profits from buying information are positive, there will be incentives to purchase the information.

Suppose first that in the subgame with two informed sellers there is one (call him $B_{i_1}$, and call the other seller $B_{i_2}$) not offering the complete hierarchy. There are two cases to consider:

A If $B_{i_1}$ is not extracting full rents from the indirectly informed players (that is if there is a $B_i$ purchasing information from $B_{i_1}$ such that $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ ), then $B_{i_2}$ can just replicate the signaling structure of $B_{i_1}$and get non zero profit by charging a positive price to $B_i$ (and zero, or even some negative $-\varepsilon$ to all other buyers of information, to be sure they accept the signal). Such $B_i$ would lose his position in the hierarchy when not buying that extra information from $B_{i_2}$. This is so because, if $B_i$ does not buy, there is always at least one individual who used to get the distorted information from $B_{i_1}$ and who now knows the truth when $B_i$ likes the good. Thus $B_i$ will get only $\max\{\pi_u, \pi_c\}$ as payoff.

B If $B_{i_1}$ is extracting full rents (conditional on $L_{i_1} < N - 1$), then if $B_{i_2}$ offers the complete hierarchy, $L_{i_2} = N - 1$, and prices which leave $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ to all the indirectly informed, they will all buy only from $B_{i_2}$ and stop buying the signal from $B_{i_1}$, and thus give him strictly positive profits.

The only alternative is that in the subgame with two informed sellers both $B_{i_1}$ and $B_{i_2}$offer the full hierarchy, so that $L_{i_1} = L_{i_2} = N - 1$. Here we also have two cases:

C If $B_{i_1}$ is extracting full rents, then if $B_{i_2}$ offers the complete hierarchy, $L_{i_2} = N-1$, and prices which leave $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ to all the indirectly informed, they will all buy only from $B_{i_2}$ and stop buying the signal from $B_{i_1}$, and thus give him strictly positive profits.

D If $B_{i_1}$ is not extracting full rents, that is if there is a $B_i$ purchasing information from $B_{i_1}$ such that $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ ), then $B_{i_2}$ can get positive surplus by replicating the hierarchy and asking $p_{l_{i_2}} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N_l} - \max\{\pi_u, \pi_c\} - p_{l_{i_1}}$, that is, the monopoly prices we computed in the last section minus what $B_{i_1}$asks. The indirectly informed buyers $B_i$ have to buy the signal, or risk their own position in the hierarchy, as we argued in case $A$ above.

**This completes the proof of Proposition 6**