Financial Connections and Systemic Risk*

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Abstract

We develop a model where financial institutions form strategic connections through overlapping portfolio exposures weighing the benefits of risk diversification against the costs of due-diligence. We study the effects of different network structures for systemic risk and welfare depending on whether financial institutions issue long or short term debt. Clustered networks where banks hold very similar portfolios are compared with unclustered networks where they hold less correlated portfolios. The network structure plays a role only in the case of short term financing, when investors condition their debt rollover decision on a signal revealing potential future bank defaults. We show that, depending on the size of costs banks incur when they default, the arrival of a negative signal can lead to early liquidation in a clustered network but not in an unclustered one so the latter can be superior. But if such a signal leads to early liquidation in both networks then the clustered network can be superior.

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1 Introduction

Understanding the nature of systemic risk is key to understanding the occurrence and propagation of financial crises. There are at least three types of systemic risk. The first is a common asset shock such as a fall in real estate or stock market prices. Herring and Wachter (2001) analyze a number of financial crises and find that in many a collapse of commercial real estate values is the basic cause. The second type of systemic risk is the danger of contagion where the failure of one financial institution leads to the failure of another and so on (see, for example, Allen and Gale, 2000; and Freixas, Parigi and Rochet, 2000). This type of systemic risk is often used by central banks as the justification for intervening and bailing out institutions that are "too big to fail".

In this paper, we consider a third type of systemic risk. Individual financial institutions have an incentive to diversify and acquire exposure to a large range of risks. However, the more that they do this, the more correlated their portfolios become. The failure of one is then likely to coincide with the failure of many so that systemic risk is significant. In our framework financial institutions form connections in order to transfer risk. In addition to selling loans outright, banks and other lenders are active in the markets for syndicated loans, collateralized loan obligations, credit default swaps and other credit derivative products. The main benefits of this credit risk transfer are diversification and a reduction in the costs of raising external capital for loan intermediation. While diversification spreads risks among many institutions, it also creates correlations in the returns of their portfolios. What is initially a benefit for each individual bank, can become a cost for the system as a whole.

Banks invest in projects with a stochastic return. To finance projects, banks borrow funds from investors. In exchange, the investors receive a debt contract that specifies a certain return for their investment. Initially, we consider the case where the banks borrow long term. Banks have an incentive to diversify and transfer the risk of their project to other institutions in order to increase the probability of repaying the investors and thus reduce the promised return. If its portfolio yields a high enough return, the bank pays the investors and retains any excess return. If the return of the portfolio is not sufficient to repay the investors, the bank defaults, incurs bankruptcy costs and its remaining assets are divided among investors. Clearly, diversification decreases the probability of default and increases the expected profits of a bank because of the reduced bankruptcy costs. However, banks incur a cost for each additional project they include in their portfolio. This cost can be interpreted as a due-diligence effort banks exert when they acquire new projects from other banks. In equilibrium, the number of connections is determined by this trade-off between the advantages of diversification and due-diligence costs.

For concreteness, we focus on the case of six banks and compare equilibria with clustered and unclustered networks where the optimal number of connections is two. In a clustered network there are two groups of three banks each. The banks within each group hold identical portfolios. In an unclustered network banks form a circle. As a result their returns are less correlated than in the clustered network. We show that with long term finance, both types of network are equivalent in the sense that the number of defaults and expected costs of default are the same and so ex ante welfare is the same.

Although they are equivalent when banks use long term financing, the two networks present important differences, which play a role when banks use a different financing structure. Defaults are more correlated in the clustered network than in the unclustered one. Given that banks hold the same portfolios within each of the two groups in the clustered network, defaults always occur for at least three banks. By contrast, in the unclustered network given that banks' portfolios are not perfectly correlated, defaults affect a lower number of banks in many states. However, the number of states in which default occurs is higher in the unclustered network than in the clustered network.

Long term finance is often expensive. For this reason, many financial institutions use short term finance to fund themselves. The difference with this kind of financing is that its maturity is shorter than that of the assets and thus banks need to roll it over frequently. The roll over occurs when investors expect at each date a return at least equal to their opportunity cost. In calculating their expected return, investors will then make use of all available information about the future solvency of the banks. In particular, we suppose that at the intermediate date a signal arrives indicating whether at least one bank will default at the final date (bad news) or none will do so (good news). We show that the effect of this signal on investors' decision depends on the structure of the network and on the amount that investors obtain in case their bank defaults net of the bankruptcy costs.

We show that when bankruptcy costs are low, short term debt is always rolled over in both the clustered and unclustered network and the two networks are equivalent. The situation is essentially the same as the case where there is long term debt. Differently, for intermediate values of bankruptcy costs, debt is rolled over in the unclustered network but not in the clustered one when the bad signal is realized. As a result there is early liquidation of projects in the clustered network with consequent deadweight costs. The reason is that since there are two groups of three banks in a clustered network, a signal of at least one bankruptcy indicates there will be three or more. Thus the probability of default conditional on the bad signal is high. This means that it may not be possible for any bank to roll over their short term debt. In contrast, in an unclustered network the portfolios held by banks are diverse. In this case a signal that at least one bank is about to fail indicates a lower probability of the occurrence of a rash of bank failures. Banks may therefore be able to roll over their debt where in the same situation in a clustered network they could not. In such cases, the unclustered network is superior from a welfare perspective because it avoids the costs of early liquidation.

However, since in an unclustered network default occurs in more states, they are not always superior. When bankruptcy costs are sufficiently high, investors do not roll over their debt in either of the two networks and banks are always liquidated when the bad signal occurs. In that case the unclustered network leads to more early liquidations and thus higher costs, and it is inferior to the clustered network.

To sum up, the welfare properties of different network structures depend on the reaction of investors to the arrival of information concerning banks' future default. The key trade off between the clustered and the unclustered structure in our framework derives from the different correlations of banks' portfolio risk in the two networks. Banks have identical portfolios in each of the two groups when they are clustered, while they have diverse portfolios when they are unclustered. This implies a higher probability of receiving a bad signal in the unclustered network and, vice versa, a higher probability of default conditional on receiving the bad signal in the clustered network. The consequence is that the clustered network more often entails early liquidation than the unclustered network but, once early liquidation occurs in both networks, the unclustered structure entails greater inefficient contagion, thus leading to lower welfare.

Our paper is related to several strands of literature. Concerning the effects of diversification on banks' portfolio risk, Wagner (2009) shows in the context of a model with two banks that diversification can increase the likelihood of systemic crises and thus be undesirable. This feature is also present in our model. However, we focus more generally on banks' choice to form links with others, and we analyze the characteristics of different network structures −that is different degrees of diversification− in terms of fragility, contagion risk and welfare when banks finance themselves with either long or short term debt.

In relation to the risks deriving from short term financing, Acharya, Gale and Yorulmazer (2009) explain market freezes in the presence of roll over risk. Similarly, He and Xiong (2009) show that roll over risk leads to dynamic bank runs. Concerning liquidity risk more generally, Diamond and Rajan (2009) find that liquidity dry-ups can arise from the fear of fire sales; while Bolton, Santos and Scheinkman (2009) look at maturity mismatch and its impact on liquidity demand. These studies provide crucial insights using a representative bank/agent framework. However, they do not consider network formation, which is the focus of our paper.

The current crisis has pointed to the importance of having a broader view of financial systems and of analyzing externalities among banks as a basis also to reform regulation. A few papers look at frameworks with more banks and analyze the risk of contagion and more generally systemic risk (Allen and Babus, 2009, provide a survey on contagion in financial networks). For example, Boyson, Stahel and Stulz (2008) provide evidence on such externalities within the hedge fund sector, while Adrian and Brunnermeier (2009) and Danielsson, Shin and Zigrand (2009) point out that basing regulation on individually optimal risk management may not be enough. Our paper fits into this strand of literature in that it analyzes how different network structures entail different fragility and contagion risk, thus implying also different welfare effects.

The paper proceeds as follow. In Section 2 we lay the basic model with long term financing and solve it in Section 3. In Section 4 we introduce short term financing and analyze its effects on the equilibrium outcomes. Section 5 concludes.

2 The basic model with long term financing

Consider for the moment a three-date ($t = 0, 1, 2$) economy with many banks, denoted by $i = 1, ..., N$, and a continuum of small, risk-neutral investors. Each bank i has access at date 0 to an investment project which yields a stochastic return $\theta_i = \{R_H, R_L\}$ at date 2 with probability p and $(1 - p)$, respectively. We assume $R_H > R_L$. The returns of the projects are independently distributed across banks.

Banks raise one unit of funds each from the investors at date 0 and offer them, in exchange, a long term debt contract that specifies a return r at date 2. Investors have access to a long term risk-free asset that offers a per-period return of r_F, and finance the banks only if they expect to at least obtain their opportunity cost. Under the assumption that $R_H \geq r_F^2 > R_L$, a bank can pay r only when the project yields a high return. When the project yields a low return R_L, the bank defaults, it liquidates the project and distributes the returns among the investors. Liquidating a project involves some costs so that investors receive only a fraction $\alpha < 1$ of the project return. The remaining fraction $(1 - \alpha)$ represents the bankruptcy costs that are lost in the liquidation. Then, investors will finance the banks only if their participation constraint as given by

$$pr + (1 - p)\alpha R_L \geq r_F$$

is satisfied. In the case a project succeeds, banks acquire the surplus $(R_H - r)$. Otherwise, they receive 0.

Given that projects are risky and returns are independently distributed, banks can reduce their default risk through diversification. In particular, we suppose that each bank exchanges shares of its own project with ℓ_i other banks and that connections are bilateral. That is, bank i exchanges a share of its project with bank j if and only if bank j exchanges a share of its project with bank i. When this happens, there is a link between banks i and j denoted as ℓ_{ij}. This implies that each bank i ends up with a portfolio of $n_i = \ell_i + 1$ projects with a return equal to

$$X_i = \frac{\theta_{i_1} + \theta_{i_2} + ... + \theta_{i_{n_i}}}{n_i}.$$

Exchanging shares of projects with other banks entails a cost c per link. This can be interpreted as a due diligence cost. The idea is that banks know their own project, but they do not know that of the other banks. Thus they need to exert a costly effort to check that the projects of the banks they want to form links with are bona fide as well.

The exchange of project shares creates linkages between banks. When they diversify, banks are connected through a network of overlapping portfolios. Each bank has shares of n_i independently distributed projects in its portfolio, but banks' portfolios are now correlated. The degree of correlation depends on the number of project exchanges or links ℓ_i that each bank has with other banks and on the structure of interconnections among banks for a given number of links. As will be described more in detail in the next section, the structure of a network of overlapping portfolios depends ultimately on the outcome of the link formation process among banks.

In what follows, we investigate the formation of financial networks and evaluate the welfare implications of different network structures.

3 Network formation

In order to diversify, banks exchange shares of their own projects with other banks. This creates links among banks. The collection of all links that banks form can be described as a network g. We model how banks make portfolio investment decisions as a network formation game. To find the network structures that emerge in equilibrium, we first derive the participation constraint of the investors and banks' profits when banks hold a portfolio of $n_i = \ell_i + 1$ projects and have ℓ_i links with other banks.

Denote as $r_{n_i}(g)$ the interest rate that banks promise investors in a network g where banks have ℓ_i links and n_i projects. Investors at bank i receive r_{n_i} at date 2 when the return of bank i's portfolio, X_i, is $X_i > r_{n_i}$, while they receive the return of the portfolio net of bankruptcy costs when $X_i < r_{n_i}$. The participation constraint of the investors is then given by

$$r_{n_i}(g) \Pr(X_i \geq r_{n_i}(g)) + \alpha E(X_i | X_i < r_{n_i}(g)) \Pr(X_i < r_{n_i}(g)) \geq r_F^2 \tag{3.1}$$

where $E(X_i|X_i < r_{n_i}(g)) = \sum_x x \Pr(X_i = x | X_i < r_{n_i}(g))$. The equilibrium $r_{n_i}(g)$ is the minimum interest rate that satisfies (3.1) with equality while minimizing the probability of bank default $\Pr(X_i < r_{n_i}(g))$. Banks retain the surplus $X_i - r_{n_i}(g)$ whenever $X_i \geq r_{n_i}(g)$. Given that diversification increases the probability $\Pr(X_i \geq r_{n_i}(g))$ that investors receive their promised return, banks can offer a lower rate of return when they exchange projects.

The expected profit $\pi_i(g)$ of a bank i in a network g of overlapping portfolios is

$$\pi_i(g) = \Pr(X_i \geq r_{n_i}(g)) \left[E(X_i | X_i \geq r_{n_i}(g)) - r_{n_i}(g) \right] - c\ell_i \tag{3.2}$$

where $E(X_i|X_i \geq r_{n_i}(g)) = \sum_x x \Pr(X_i = x | X_i \geq r_{n_i}(g))$. Substituting the equilibrium interest rate $r_{n_i}(g)$ into eq. (3.2), we can write the expected profit of a bank as

$$\pi_i(g) = E(X_i) - r_F^2 - (1-\alpha) \Pr(X_i < r_{n_i}(g)) E(X_i | X_i < r_{n_i}(g)) - c\ell_i. \tag{3.3}$$

Banks' expected profits increase with the degree of diversification since greater diversification reduces their default probability and allows them to pay a lower interest rate to investors. Greater diversification entails, however, also greater total due diligence costs. Moreover, banks' expected profits decrease with the size of the long term risk free interest rate r_F and of the bankruptcy cost $(1 - \alpha)$.

Banks choose to form connections in order to maximize their expected profits. In particular, they choose the number of banks ℓ_i with which to form links. The equilibrium choice of all banks determine the structure of the network g. The formation of a link ℓ_{ij} requires the consent of both banks i and j, while any bank i has the discretion to unilaterally terminate links in which it is involved. A network g emerges in equilibrium if it satisfies the notion of *pairwise stability* as introduced by Jackson and Wolinski (1996). Formally, this implies that a network g is pairwise stable if

(i) for any pair of banks i and j that are linked in the network g, none of them has an incentive to unilaterally sever their link ℓ_{ij}. That is, the expected profit each of them receives from deviating to the network $(g - \ell_{ij})$ is smaller than the expected profit that each of them obtains in the network g $(\pi_i(g - \ell_{ij}) \leq \pi_i(g)$ and $\pi_j(g - \ell_{ij}) \leq \pi_j(g))$;

(ii) for any two banks i and j that are not linked in the network g, at least one of them has no incentive to form the link ℓ_{ij}. That is, the expected profit that at least one of them receives from deviating to the network $(g + \ell_{ij})$ is smaller than the expected profit that it obtains in the network g (if $\pi_i(g + \ell_{ij}) > \pi_i(g)$ then $\pi_j(g + \ell_{ij}) < \pi_j(g)$ or vice-versa).

The following proposition describes the equilibrium network.

Proposition 1 *Consider a network g^* of overlapping portfolios that is pairwise stable. In symmetric pairwise stable networks g^*, all banks have ℓ^* links, where ℓ^* is the number of exchanged projects that maximizes the expected profit of each bank:* $\ell^* = [\arg\max_{n_i} \pi_i(g)] - 1$.

Proof. The proof is provided in the Appendix. ■

The optimal number of links ℓ^* trades off the benefit of greater diversification in terms of lower bank default probability and lower equilibrium interest rate with the cost of higher total due diligence cost. Given an optimal ℓ^*, there can be multiple equilibrium networks g^*. Each equilibrium network entails different pairwise correlations between banks' portfolios. However, each bank's portfolio is formed by $\ell^* + 1$ independently distributed projects and yields the same expected return $E(X_i)$.

For any network structure, we can represent the total welfare per bank as the sum of a representative bank i's expected profit and its investors' expected returns as given by

$$W(g) = [E(X_i|X_i \geq r(g)) - r(g)] \Pr(X_i \geq r(g)) - \ell_i c$$
$$+[r(g) \Pr(X_i \geq r(g)) + \alpha E(X_i|X_i < r(g) \Pr(X_i < r(g))],$$

which simplifies to

$$W(g) = E(X_i) - (1 - \alpha)E(X_i|X_i < r(g)) \Pr(X_i < r(g)) - \ell_i c. \tag{3.4}$$

Expression (3.4) indicates that in the case of long term financing welfare is just equal to the sum of the expected return of each bank's portfolio net of bankruptcy costs in case of default and total due diligence costs.

The following proposition characterizes the total welfare in the equilibrium networks.

Proposition 2 *The total number of bank defaults and total welfare is the same in any symmetric equilibrium network where all banks have ℓ^* links.*

Proof. The proof is provided in the Appendix. ∎

The intuition behind Proposition 2 is that each bank's portfolios in any symmetric equilibrium network has the same expected return $E(X_i)$ and the same default probability for any given $r(g)$. The implication is that, when banks finance themselves with long term debt, the pairwise correlations among portfolios do not play a role in determining total welfare.

In what follows we illustrate the propositions for a particular set of parameters: The number of banks is $N = 6$, the returns of each project are $R_H = 10$ and $R_L = 2$ with the success probability being $p = 0.5$, the two period risk free interest is $r_F^2 = 3.4$, and the bankruptcy costs are $(1 - \alpha) = 0.8$. For this set of parameters we show that the optimal number of links in a symmetric equilibrium is $\ell^* = 2$ for $c \in (0.1, 0.2)$. For $\ell^* = 2$, there are two equilibrium networks g^* as shown in Figure 1. In the first network, that we define as clustered, banks are connected in two clusters, of three banks each. Banks hold identical portfolios in each cluster, but the two clusters are independent of each other. In the second network, denoted as unclustered, banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks hold identical portfolios. We then denote $g^* = C, U$ to indicate the clustered and unclustered networks, respectively.

In both networks, each bank i has a portfolio of 3 independent projects. This implies that in either network the return of each bank's portfolio can take the following values with the associated probabilities:

$$X_i = \begin{cases} \frac{3R_H}{3} = 10, \text{ with prob. } \frac{1}{8} \\ \frac{2R_H + R_L}{3} = \frac{22}{3}, \text{ with prob. } \frac{3}{8} \\ \frac{R_H + 2R_L}{3} = \frac{14}{3}, \text{ with prob. } \frac{3}{8} \\ \frac{3R_L}{3} = 2, \text{ with prob. } \frac{1}{8} \end{cases} \tag{3.5}$$

so that $E(X_i) = 6$.

The equilibrium interest rate, which we now denote for simplicity as $r(g^*)$ where $g^* = C, U$, is the same in both networks and it equals 3.829. It can be easily checked that $r(g^*) = 3.829$ is the solution to (3.1) given that each bank's probability of default is $\Pr(X_i < r(g^*)) = \frac{1}{8}$ and $E(X_i | X_i < r(g^*)) = 2$. From (3.3), bank expected profit is then $\pi_i(g^*) = 2.4 - 2c$ in either network.

Both the clustered and the unclustered networks are an equilibrium, because no pair of banks, previously not connected, has an incentive to deviate and form a link, and none of the banks find it optimal to sever a link. To see why this is the case, we consider banks' incentives to deviate in the clustered network. Suppose first that a pair of banks, previously not connected, form a link with each other. Each of the deviating banks now has four projects in its portfolio, it offers an equilibrium interest rate equal to 3.6 and has expected profit of $2.5-3c$. This deviation is not profitable as long as $c > 0.1$. Consider now that a bank deviates by severing a link. Then, the bank has two projects in its portfolio, it offers an interest rate equal to 4.4 and has expected profit of $2.2 - c$. This deviation is not beneficial for any $c < 0.2$. Thus, given our set of parameters, the clustered network is an equilibrium for $c \in (0.1, 0.2)$. The same holds for the unclustered network.

4 Short term finance

In the previous sections we have assumed that the maturity of the financing matches the maturity of the assets. In practice, particularly for financial institutions, this is usually not the case. Many banks use large amounts of short term finance. One of the main reasons for this is the term structure of interest rates. Most of the time short term funding is considerably cheaper than long term finance. In this section we consider how the possibility of short term financing affects the analysis. We assume that it is sufficiently cheaper than long term finance for it to be optimal to use it. The per-period short term interest rate is denoted as r_f where $r_f < r_{F.}$

Although short term finance has the advantage of being cheaper, it is not without disadvantages. In particular, if adverse information comes in during the life of the projects at $t = 1$ then this can lead to a refusal by lenders to roll over the debt. This results in early liquidation of the bank, which can be costly. In this section, we consider the effect of a signal at an intermediate date after the start of the projects but before final payments are made. The signal concerns the occurrence of future bank defaults in the economy, and it can have two realizations. One is that no banks will be in distress (good news). The other is that at least one bank will default (bad news). This captures the idea that if there is news of an upcoming bank default, investors will use this information to update their priors and take actions accordingly. In particular, conditional on the realization of the signal, investors decide whether to roll over their short term debt for another period or refuse to do so and force the bank into early liquidation. Differently from the case of long term financing, the structure of the equilibrium network now matters for total welfare. The reason is that the conditional correlations among banks' portfolios do now play a role in determining the probability distribution of the signal and the subsequent decision of the investors to roll over their debt.

We show the role of the conditional correlations between banks' portfolios for total welfare in the context of the clustered and unclustered networks analyzed above. The presence of short term debt and the arrival of the signal at the intermediate date modifies the model as shown in Figure 2. At date 0 banks in network $g = C, U$ raise one unit of funds in exchange for a short term debt contract that promises a return of $r_{01}(g)$ at date 1. At the beginning of date 1 the signal S arrives. The probability distribution of the signal depends on the particular financial network structure g. With probability $q(g)$ the signal reveals the bad news that at least one bank i will be defaulting at date 2. With probability $(1 - q(g))$ it brings the good news that no banks will default at date 2. We denote the two cases with $S = \{B, G\}$, where B and G indicate bad and good news, respectively. Conditional on the realization of the signal, investors decide whether to roll over their short term debt $r_{01}(g)$ in exchange for a total promised return of $r_{12}^{S}(g)$ at date 2.

When $S = B$, investors update the probability $\Pr(X_i \geq r_{12}^{B}(g)|B)$ that their bank will be able to repay them $r_{12}^{B}(g)$ at date 2 and decide to roll over their debt if they are able to recover their total opportunity cost $r_{01}(g)r_f$. Formally, roll over occurs if there exists a total return $r_{12}^{B}(g)$ that satisfies investors' date 1 participation constraint

$$r_{12}^{B}(g)\Pr(X_i \geq r_{12}^{B}(g)|B) + \alpha \Pr(X_i < r_{12}^{B}(g)|B)E(X_i|X_i < r_{12}^{B}(g), B) \geq r_{01}(g)r_f, \quad (4.1)$$

where $E(X_i|X_i < r_{12}^{B}(g), B) = \sum_x x \Pr(X_i = x|X_i < r_{12}^{B}(g), B)$ is the expected return of bank portfolio when the bank defaults at date 2 and $S = B$.

The equilibrium value of $r_{12}^{B}(g)$, if it exists, is the minimum promised total return that satisfies (4.1) with equality and it minimizes the probability of bank default $\Pr(X_i < r_{12}^{B}(g)|B)$. Investors obtain the promised total return $r_{12}^{B}(g)$ and banks retain the surplus $X_i - r_{12}^{B}(g)$ whenever $X_i \geq r_{12}^{B}(g)$. Otherwise, if $X_i < r_{12}^{B}(g)$, at date 2 the bank defaults and receives nothing, while the investors receive the return of bank portfolio X_i net of the bankruptcy costs.

If, on the contrary, constraint (4.1) cannot be satisfied for any $r_{12}^{B}(g) \leq X_i$, investors do not roll over their debt at date 1 and the bank is forced into early liquidation. When liquidated at date 1 the project yields a payoff $\beta \leq r_f < E(\theta_i)$, which accrues to the investors. We do not allow for the possibility of renegotiation of short term debt.

When $S = G$ no banks will default at date 2, investors infer that they will always receive the promised total return $r_{12}^{G}(g)$ at date 2 and therefore always roll over their debt. In equilibrium then, $r_{12}^{G}(g) = r_{01}(g)r_f$.

Short term debt implies that investors have to break even every period. Thus, given their decisions about debt roll over at date 2 and the corresponding payoffs, they are willing to finance the bank initially if and only they are promised at date 0 an interest rate $r_{01}(g)$ that allows them to on average recover at date 1 at least their opportunity cost r_f. In equilibrium, $r_{01}(g)$ will depend on whether the debt is rolled over at date 1. If this is the case when either the bad or the good signal arrives, then investors can always obtain their promised interest rate and $r_{01}(g) = r_f$. Otherwise, when the debt is not rolled over at date 1 after the bad signal $S = B$, the promised interest rate $r_{01}(g)$ at date 0 must compensate investors for the loss they suffer from early liquidation and thus is determined by

$$q(g)\beta + (1 - q(g))r_{01}(g) = r_f. \tag{4.2}$$

The expected profits $\pi_i(g)$ of bank i depend on investors' roll over decision at date 1. When debt is rolled over, banks make positive profits irrespective of the realization of the signal at date 1. Their expected profit is then given by

$$\pi_i(g) = q(g)\Pr(X_i \geq r_{12}^B(g)|B)\left[E(X_i|X_i \geq r_{12}^B(g), B) - r_{12}^B(g)\right] + (1-q(g))\left[E(X_i|X_i \geq r_{12}^G(g), B) - r_{12}^G(g)\right] - 2c,$$

which simplifies to

$$\pi(g) = E(X_i) - r_f^2 - (1-\alpha)\Pr(X_i < r_{12}^B(g))E(X_i|X_i < r_{12}^B(g)) - 2c, \tag{4.3}$$

given that, using (4.1) and $r_{12}^G(g) = r_{01}(g)r_f = r_f^2$ with debt roll over, it follows that

$$q(g)\Pr(X_i \geq r_{12}^B(g)|B) + (1-q(g))r_{12}^G(g) = r_f^2 - \alpha\Pr(X_i < r_{12}^B(g))E(X_i|X_i < r_{12}^B(g)),$$

where $\Pr(X_i < r_{12}^B(g))$ is the probability that a bank's portfolio payoff is lower than the promised total return $r_{12}^B(g)$ at date 2.

When banks are early liquidated at date 1 after the bad signal is received, they make positive profits only when the good signal is received. Thus, their expected profit is

$$\pi(g) = (1-q(g))\left[E(X_i|X_i \geq r_{12}^G(g), B) - r_{12}^G(g)\right] - 2c. \tag{4.4}$$

The crucial difference between the long and the short term financing is that in the latter case network structures matter for the equilibrium interest rates, bank profits and ultimately total welfare. The reason is that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 are a function of the correlation among banks' portfolios, which depend on the network structure. Formally, the probability of receiving at date 1 the bad signal $S = B$ that at least one bank will default at date 2 can be defined as

$$q(g) = 1 - \Pr(\textstyle\bigcap_i(X_i \geq r_{12}^B(g))),$$

where $\Pr(\bigcap_i (X_i \geq r_{12}^B(g)) = \Pr(X_1 \geq r_{12}^B(g), X_2 \geq r_{12}^B(g), ..., X_6 \geq r_{12}^B(g))$ represents the probability that none of the six banks default.

Although from Proposition 2 the total number of bank defaults at date 2 is the same across different network structures for a given number of links, the number of states of the world where at least one default occurs is different across network structures. When banks' portfolio correlation is high, bank defaults are also correlated. This means that defaults always involve more than one bank but occur in fewer states of the world. When the correlation of banks' portfolios is low, defaults are more spread across the different states of the world. This means that there are more default states, but conditional on the bad signal revealing a future default, the probability of a bank defaulting at the subsequent date is lower than when banks' portfolio correlation is high.

These arguments imply that the probability of receiving the bad signal $S = B$ is higher in the unclustered network than in the clustered network as banks' portfolio correlation is higher in the former,

$$q(U) > q(C),$$

whereas the conditional probability of a bank defaulting at the subsequent date is higher in the clustered network,

$$\Pr(X_i < r_{12}^B(C)|B) > \Pr(X_i < r_{12}^B(U)|B). \tag{4.5}$$

This difference has important implications for investors' decision of whether to roll over their debt. Given (4.5), the investors' expected return conditional on $S = B$, $r_{12}^B(g)\Pr(X_i \geq r_{12}^B(g)|B) + \alpha \Pr(X_i < r_{12}^B(g)|B)E(X_i|X_i < r_{12}^B(g), B)$, is higher in the unclustered then in the clustered network for any given $r_{12}^B(g)$. It follows that debt is more likely to be rolled over in the unclustered network because there exists a larger set of parameters for which roll over occurs at date 1. Formally, this means that (4.1) will be satisfied for a larger set of parameters in the unclustered network.

In what follows, we analyze the equilibrium with short term financing in the context of our example. In particular, we conduct comparative statics on the bankruptcy cost $(1-\alpha)$ and we analyze investors' roll over decision and ultimately total welfare in the two network structures.

We start with the case of intermediate bankruptcy costs where short term debt is rolled over in the unclustered but not in the clustered network after a bad signal indicating at least one default. We show that in this case, the unclustered network entails a higher total welfare than the clustered network as it avoids the costs when banks are early liquidated.

We then analyze the case where bankruptcy costs are high. We show that banks are early liquidated in both networks conditional on the arrival of the bad signal, and that the unclustered network now leads to a lower total welfare than the clustered network. The reason is that the unclustered network more often involves early liquidation because the bad signal arrives with a higher probability.

Finally, we show that for low values of the bankruptcy cost, there is no early liquidation in either network and then the two are equivalent in terms of welfare as in the case when there is long term debt.

4.1 Case 1: Liquidation in the clustered network; continuation in the unclustered network $(1-\alpha=0.8)$

We consider our usual example with $(1-\alpha)=0.8$, where now $r_f^2=3.2$.

4.1.1 Expected Utility of Investors

To show that debt is not rolled over at date 1 when the bad signal is received in the **clustered** network, we need to show that condition (4.1) cannot be satisfied for any feasible value of $r_{12}^B(C)$, that is, for all $r_{12}^B(C)\in[3.2,10]$. For this, it is sufficient to show that the expected total return of the investors is smaller than $r_f^2=3.2$. This guarantees that (4.1) is not satisfied since $r_{01}(g)\geq r_f$ must hold for investors to finance the bank at date 0. We consider the various intervals of bank portfolio returns from (3.5) in turn.

A bank can pay a total return $r_{12}^{B}(C) \in (22/3, 10]$ if its portfolio yields a return of 10, while it defaults whenever $X_i \leq \frac{22}{3}$. It can be seen from Table 1 that the only state in which there are no defaults is when all 6 projects pay 10. There are 63 states where there is at least one bank defaulting: in 49 of them there are 6 banks defaulting, and in 14 of them 3 banks default and 3 banks succeed. Hence the total number of banks succeeding in the 63 states is 14×3 while the total number of banks is 63×6. This implies that the probability that investors receive $r_{12}^{B}(C)$ conditional on $S = B$ is

$$\Pr(X_i \geq r_{12}^{B}(C)|B) = \frac{14 \times 3}{63 \times 6} = \frac{7}{63}.$$

Similarly, the probability of a bank defaulting conditional on $S = B$ is

$$\Pr(X_i < r_{12}^{B}(C)|B) = \frac{49 \times 6 + 14 \times 3}{63 \times 6} = \frac{56}{63},$$

since there are $49 \times 6 + 14 \times 3$ banks failing in the 63 states out of a total of 63×6. Out of the 56 states where default occurs with $r_{12}^{B}(C) \in (22/3, 10]$, in 24 a bank's portfolio pays off 22/3, in 24 it pays off 14/3 and in 8 it pays off 2. In all these states, investors receive the portfolio return of their bank net of the bankruptcy costs. From (4.1) their expected return between dates 1 and 2 is then equal to

$$\begin{aligned} r_{12}^{B}(C)\Pr(X_i &\geq r_{12}^{B}(C)|B) + \alpha \Pr(X_i < r_{12}^{B}(C)|B) E(X_i | X_i < r_{12}^{B}(C), B) \\ &= r_{12}^{2}(C)\frac{7}{63} + 0.2\frac{56}{63}\left(\frac{22}{3}\frac{24}{56} + \frac{14}{3}\frac{24}{56} + 2\frac{8}{56}\right). \end{aligned}$$

Since $r_{12}^{B}(C) \leq 10$, investors' expected return can be at most

$$10\frac{7}{63} + 0.9650 = 2.0762 < 3.2.$$

This implies that (4.1) cannot satisfied for any $r_{12}^{B}(C) \in (22/3, 10]$.

Next we consider the interval $r_{12}^B(C) \in (14/3, 22/3]$. As can be seen from Table 1, the number of states where there is at least one bank defaulting is 48: in 16 states there are 6 banks defaulting; in 24 states there are 3 banks defaulting and 3 banks having 22/3; in 8 states there are 3 banks defaulting and 3 banks having 10. In this case we have

$$\begin{cases} \Pr(X_i \geq r_{12}^B(C)|B) = \frac{24\times3+8\times3}{48\times6} = \frac{16}{48}, \\ \Pr(X_i < r_{12}^B(C)|B) = \frac{16\times6+24\times3+8\times3}{48\times6} = \frac{32}{48}. \end{cases}$$

Out of the 32 states where default occurs with $r_{12}^B(C) \in (14/3, 22/3]$, in 24 a bank's portfolio returns 14/3, in 8 it returns 2. Thus, the expected return of the investors is equal to

$$\begin{aligned} r_{12}^B(C)\Pr(X_i &\geq r_{12}^B(C)|B) + \alpha \Pr(X_i < r_{12}^B(C)|B)E(X_i|X_i < r_{12}^B(C), B) \\ &= r_{12}^B(C)\frac{16}{48} + 0.2\frac{32}{48}(\frac{14}{3}\frac{24}{32} + 2\frac{8}{32}), \end{aligned}$$

which is at most equal to 2.9778 when $r_{12}^B(C) = 22/3$. Since this is smaller than $r_f^2 = 3.2$, condition (4.1) cannot be satisfied for any $r_{12}^B(C) \in (14/3, 22/3]$.

Finally, given that $r_{12}^B(C) \geq r_f^2$ must hold, we consider the interval $r_{12}^B(C) \in [3.2, 14/3]$. From Table 1 the number of states when there is at least one default is 15: in 1 state there are 6 banks defaulting; in 6 states there are 3 banks defaulting and 3 banks having 22/3; in 6 states there are 3 banks defaulting and 3 banks having 14/3; and in 2 states there are 3 banks defaulting and 3 banks having 10. Hence we have

$$\begin{cases} \Pr(X_i \geq r_{12}^B(C)|B) = \frac{6\times3+6\times3+2\times3}{15\times6} = \frac{42}{90} = \frac{7}{15}, \\ \Pr(X_i < r_{12}^B(C)|B) = \frac{1\times6+6\times3+6\times3+2\times3}{15\times6} = \frac{48}{90} = \frac{8}{15}. \end{cases}$$

The expected return of the investors becomes

$$\begin{aligned} r_{12}^{B}(C)\Pr(X_i &\geq r_{12}^{B}(C)|B) + \alpha \Pr(X_i < r_{12}^{B}(C)|B) E(X_i|X_i < r_{12}^{B}(C), B) \\ &= r_{12}^{B}(C)\frac{42}{90} + 0.2 \times 2\frac{48}{90}, \end{aligned}$$

which is at most equal to $2.391 < 3.2$ for $r_{12}^{B}(C) = 14/3$. It follows that constraint (4.1) cannot be satisfied for any $r_{12}^{B}(C) \in [3.2, 10]$ and thus the debt is not rolled over in the clustered network.

We next consider the **unclustered** network and find the equilibrium total return as the smallest value of $r_{12}^{B}(U)$ that gives investors an expected return conditional on $S = B$ of at least $r_f^2 = 3.2$. We look for a solution in the interval $r_{12}^{B}(U) \in \left[3.2, \frac{14}{3}\right]$. As can be seen from Table 2, for any $r_{12}^{B}(U)$ in this interval there are 25 default states and 39 no-default states. In the 25 default states there are a total of 48 defaulting banks, all with portfolio return of 2, out of $25 \times 6 = 150$ total banks. Hence $\Pr(X_i < r_{12}^{B}(U)|B) = \frac{48}{150}$ and $\Pr(X_i \geq r_{12}^{B}(U)|B) = \frac{102}{150}$ and the equilibrium solution $r_{12}^{B}(U) = 4.518$ satisfies

$$r_{12}^{B}(U)\frac{102}{150} + 0.2 \times 2\frac{48}{150} = 3.2.$$

Thus we have shown that when $\alpha = 0.2$ there is early liquidation in the clustered network but not in the unclustered network when the bad signal is received. In contrast, as illustrated above, when the good signal is received, debt is rolled over at date 1 in either networks and $r_{12}^{G}(g) = r_{01}(g)r_f$.

Given these investors' decisions, the interest rate $r_{01}(g)$ that investors are promised at date 0 is different in the two networks. Since banks are early liquidated in the clustered network when the bad signal is received, $r_{01}(C)$ must satisfy (4.2) and, taking $\beta = 0.98r_f$, is given by

$$r_{01}(C) = \frac{r_f - q(C)\beta}{1 - q(C)} = \frac{\sqrt{3.2} - (15/64) \times 0.98\sqrt{3.2}}{49/64} = 1.799.$$

In contrast, since debt is always rolled over in the unclustered network, $r_{01}(U) = r_f = \sqrt{3.2} = 1.789$. This also implies that in equilibrium $r_{12}^G(C) = 3.219$ and $r_{12}^G(U) = 3.2$.

4.1.2 Expected profit of banks and total welfare

We now turn to banks' expected profit starting with the **clustered** network. In this case, banks make a positive profit only when the good signal is received. Thus, from (4.4), their expected profit is

$$\begin{aligned}\pi_i(C) = (1-q(C))\left[E(X_i|X_i \geq r_{12}^G(C), B) - r_{12}^G(C)\right] - 2c = \\ = \frac{49}{64}\left[\frac{7}{49}\frac{30}{3} + \frac{21}{49}\frac{22}{3} + \frac{21}{49}\frac{14}{3} - 3.219\right] - 2c \\ = 2.566 - 2c.\end{aligned}$$

The total welfare in the clustered network aggregates the welfare of banks and of investors. Since investors always obtain their opportunity cost r_f^2 over the two periods, their welfare is constant. Thus, for simplicity, we simply use the bank's expected profit as an indicator of total welfare.

In the **unclustered** network, banks always make profits irrespective of the realization of the signal. From (4.3), their expected profit is then equal to

$$\begin{aligned}\pi_i(U) &= E(X_i) - r_f^2 - (1-\alpha)\Pr(X_i < r_{12}^B(U))E(X_i|X_i < r_{12}^B(U)) - 2c \\ &= 6 - 3.2 - 0.8\frac{1}{8}2 - 2c \\ &= 2.6 - 2c,\end{aligned}$$

where, from (3.5) it follows that $\Pr(X_i < r_{12}^B(U)) = \Pr(X_i < 4.518) = \frac{1}{8}$.

We have then the following result.

Proposition 3 *In the case of bankruptcy costs* $(1-\alpha) = 0.8$, *the total welfare in the unclustered network is higher than in the clustered network.*

4.1.3 The optimality of the clustered and unclustered networks with $\ell^* = 2$

Now that we have determined the equilibrium interest rates and banks' profits taking the network structure as given, we show that there is a range of values for the due diligence cost c in which both the clustered and unclustered network are supported in equilibrium. Specifically, we look for the parameter values for c where each network is pairwise stable by looking at whether any pair of banks i and j has an incentive to form an additional link and whether any bank i has the discretion to unilaterally terminate links in which it is involved.

We start with the **clustered** network. To show that no pair of banks has an incentive to form a link, we take banks 1 and 4 and calculate their potential benefit from deviating. If deviating, each of the two banks has $\ell = 3$ and a portfolio consisting of four projects. As before, their expected profit depends on investors' rollover decisions at date 1. If they roll over their debt, from (4.3) the expected profit $\pi(C + \ell_{14})$ of each deviating bank $i = 1, 4$ is

$$\begin{aligned}\pi_i(C+\ell_{14}) &= E(X_i) - r_f^2 - (1-\alpha)\Pr(X_i < r_{12}^B(C+\ell_{14}))E(X_i|X_i < r_{12}^B(C+\ell_{14})) - 3c \\ &= 2.7 - 3c.\end{aligned}$$

If, instead, debt is not rolled over, from (4.4) the expected profit of each deviating bank is

$$\begin{aligned}\pi_i(C+\ell_{14}) &= (1-q(C+\ell_{14}))\left[E(X_i|X_i \geq r_{12}^G(C+\ell_{14}), B) - r_{12}^G(C+\ell_{14})\right] - 3c \\ &= 2.566 - 3c,\end{aligned}$$

where $q(C + \ell_{14}) = q(C) = \frac{15}{64}$ and $r_{12}^G(C + \ell_{14}) = 3.219$. Thus, a sufficient condition for banks not to find it optimal to deviate is $\pi_i(C + \ell_{14}) = 2.7 - 3c \leq \pi_i(C) = 2.566 - 2c$. This requires c to be larger than 0.134.

To show next that each individual bank i has no incentive to sever a link, we take bank

1 and calculate its potential benefit from deleting the link with bank 3. If investors roll over their debt, from (4.3) the expected profit $\pi_1(C-\ell_{13})$ of bank 1 is now

$$\begin{aligned}\pi_1(C-\ell_{13}) &= E(X_1)-r_f^2-(1-\alpha)\Pr(X_1<r_{12}^B(C-\ell_{13}))E(X_1|X_1<r_{12}^B(C-\ell_{13}))-c\\ &= 2.4-c.\end{aligned}$$

If, instead, debt is not rolled over, from (4.4) the expected profit of bank 1 is

$$\begin{aligned}\pi_1(C-\ell_{13}) &= (1-q(C-\ell_{13}))\left[E(X_1|X_1\geq r_{12}^G(C-\ell_{13}),B)-r_{12}^G(C-\ell_{13})\right]-c\\ &= 2.377-c,\end{aligned}$$

where $r_{12}^G(C-\ell_{13})=3.253$ and $q(C-\ell_{13})=q(C)+\frac{14}{64}=\frac{29}{64}$ as there are now 7 additional states where only bank 1 defaults and 7 states where only bank 3 defaults. Thus, a sufficient condition for banks not to sever links is that $\pi_1(C-\ell_{13})$ when the debt is rolled over is no greater than the expected profit $\pi(C)$ in the clustered network C. That is, $\pi_1(C-\ell_{13})=2.4-c\leq\pi_i(C)=2.566-2c$. This holds for values of c smaller than 0.166. All this implies that the clustered network is supported in equilibrium for $c\in(0.134,0.166)$.

We now turn to analyze the incentives for banks to deviate in the **unclustered** network. As above, we first analyze the expected profit of banks 1 and 4 when they form a new link with each other. If debt is rolled over, from (4.3) each deviating bank $i=1,4$ has expected profit $\pi_i(U+\ell_{14})$ equal to

$$\begin{aligned}\pi_i(U+\ell_{14}) &= E(X_i)-r_f^2-(1-\alpha)\Pr(X_i<r_{12}^B(U+\ell_{14}))E(X_i|X_i<r_{12}^B(U+\ell_{14}))-3c\\ &= 2.7-3c.\end{aligned}$$

If, on the contrary, debt is not rolled over, then from (4.4) the expected profit $\pi_i(U+\ell_{14})$

of each deviating bank is

$$\begin{aligned}\pi_i(U+\ell_{14}) &= (1-q(U+\ell_{14}))\left[E(X_i|X_i \geq r_{12}^G(U+\ell_{14}),B) - r_{12}^G(U+\ell_{14})\right] - 3c \\ &= 2.333 - 3c,\end{aligned}$$

where $r_{12}^G(U+\ell_{14}) = 3.235$ and $q(U+\ell_{14}) = q(U) - \frac{2}{64} = \frac{23}{64}$ as there are 2 states less where either bank 1 or bank 4 defaults. Thus, a sufficient condition for banks not to find it optimal to deviate is $\pi_i(U+\ell_{14}) = 2.7 - 3c \leq \pi_i(U) = 2.6 - 2c$. This requires $c > 0.1$.

To show next that each individual bank i has no incentive to sever a link in the unclustered network, we take bank 1 and calculate its potential benefit from deleting the link with bank 6. If investors roll over their debt, from (4.3) the expected profit $\pi_1(U-\ell_{16})$ of bank 1 is now

$$\begin{aligned}\pi_1(U-\ell_{16}) &= E(X_1) - r_f^2 - (1-\alpha)\Pr(X_1 < r_{12}^B(U-\ell_{16}))E(X_1|X_1 < r_{12}^B(U-\ell_{16})) - c \\ &= 2.4 - c.\end{aligned}$$

If, instead, debt is not rolled over, from (4.4) the expected profit of bank 1 is

$$\begin{aligned}\pi_1(U-\ell_{16}) &= (1-q(U-\ell_{16}))\left[E(X_1|X_1 \geq r_{12}^G(U-\ell_{16}),B) - r_{12}^G(U-\ell_{16})\right] - c \\ &= 2.404 - c,\end{aligned}$$

where $r_{12}^G(C-\ell_{13}) = 3.247$ and $q(U-\ell_{16}) = q(U) + \frac{2}{64} = \frac{27}{64}$ as there is now 1 additional state where bank 1 defaults and 1 additional state where bank 6 defaults. Thus, a sufficient condition for banks not to sever a link is that $\pi_1(U-\ell_{16})$ when the debt is rolled over is no greater than the expected profit $\pi(U)$ in the unclustered network C. That is, $\pi_1(U-\ell_{16}) = 2.404 - c \leq \pi_i(U) = 2.6 - 2c$. This holds for values of $c < 0.196$. All this implies that the clustered network is supported in equilibrium for $c \in (0.1, 0.196)$.

To conclude, both the clustered and the unclustered network are an equilibrium for

$c \in (0.134, 0.166)$.

4.2 Case 2: Early liquidation in both networks $(1 - \alpha = 1)$

When bankruptcy costs are sufficiently high, it will not be possible in either network to satisfy investors' participation constraint when the bad signal is received at date 1. As in Case 1, we start with investors' rollover decision and we then calculate banks' expected profits. Similarly to Case 1, it can be shown that both networks are still supported in equilibrium.

4.2.1 Expected utility of investors

From Case 1, where $1 - \alpha = 0.8$, investors do not roll over their debt in the **clustered** network because there is no total return $r_{12}^B(C)$ that guarantees them expected returns of at least r_f^2. Given that with $1 - \alpha = 1$ investors' expected return are even lower, they still do not roll over their debt and banks are liquidated at date 1.

To show that also in the **unclustered** network debt is not rolled over at date 1 when the bad signal has arrived, it is again sufficient to show that there does not exist a total return $r_{12}^B(U) \in [3.2, 10]$ that guarantees investors an expected return at date 1 equal to at least r_f^2. As in Case 1, we consider the various intervals of bank portfolio returns from (3.5) in turn.

Consider first $r_{12}^B(U) \in (22/3, 10]$. From Table 2, there are 63 states where there is at least one bank defaulting: in 39 of them there are 6 banks defaulting, in 12 of them 5 banks default, in 6 of them 4 banks default, and in 6 there are 3 banks defaulting. Hence, given that the total number of banks is 63×6, the probability that investors receive $r_{12}^B(U)$ conditional on $S = B$ is

$$\Pr(X_i \geq r_{12}^B(U)|B) = \frac{12 \times 1 + 6 \times 2 + 6 \times 3}{63 \times 6} = \frac{7}{63},$$

and the probability of a bank defaulting conditional on $S = B$ is

$$\Pr(X_i < r_{12}^B(U)|B) = \frac{39\times 6 + 12\times 5 + 6\times 4 + 6\times 3}{63\times 6} = \frac{56}{63}.$$

From (4.1) and taking into account that $\alpha = 0$, investors expected return between dates 1 and 2 then equals

$$r_{12}^B(U)\Pr(X_i \geq r_{12}^B(U)|B) = r_{12}^B(U)\frac{7}{63},$$

which is at most equal to $1.111 < 3.2$ for $r_{12}^B(U) = 10$.

The next step is to suppose that banks offer $r_{12}^B(U) \in (14/3, 22/3]$. Then, the number of states where there is at least one default is 54. In this case we have

$$\begin{cases} \Pr(X_i \geq r_{12}^B(U)|B) = \frac{6\times1+6\times2+20\times3+6\times4+6\times5}{54\times6} = \frac{22}{54}, \\ \Pr(X_i < r_{12}^B(U)|B) = \frac{10\times6+6\times5+6\times4+20\times3+6\times2+6\times1}{54\times6} = \frac{32}{54}. \end{cases}$$

The expected return of the investors becomes $r_{12}^B(U)\frac{22}{54}$, which is at most equal to $2.9876 < 3.2$ for $r_{12}^B(U) = 22/3$.

When $r_{12}^B(U) \in (3.2, 14/3]$ the number of states where there is at least one default is 25. In this case we have

$$\begin{cases} \Pr(X_i \geq r_{12}^B(U)|B) = \frac{6\times3+6\times4+12\times5}{25\times6} = \frac{17}{25}, \\ \Pr(X_i < r_{12}^B(U)|B) = \frac{1\times6+6\times3+6\times2+12\times1}{25\times6} = \frac{8}{25}. \end{cases}$$

The expected return of the investors becomes $r_{12}^B(U)\frac{17}{25}$, which is at most equal to $3.1733 < 3.2$ for $r_{12}^B(U) = 14/3$.

It follows that there is no payoff $r_{12}^B(U)$ that a bank in the unclustered network can promise depositors in order not to be early liquidated. Thus, in both networks, banks will be early liquidated. Given this, the date 0 interest rate $r_{01}(g)$ must satisfy (4.2) in either network. Taking again $\beta = 0.98r_f$, $r_{01}(C) = 1.799$ as in Case 1, while $r_{01}(U)$ is

$$r_{01}(U) = \frac{r_f - q(U)\beta}{1 - q(U)} = \frac{\sqrt{3.2} - (25/64)\times 0.98\sqrt{3.2}}{39/64} = 1.811.$$

This also implies that in equilibrium $r_{12}^G(C) = 3.219$ and $r_{12}^G(U) = 3.241$.

4.2.2 Expected profit of banks

We now turn to banks' expected profit. Given that there is always early liquidation when the bad signal is received, in either network banks make a positive profit only when the good signal is received. The expected profit of banks in the **clustered** network is the same as before when $1 - \alpha = 0.8$, that is $\pi_i(C) = 2.566 - 2c$.

Using (4.4) each bank's expected profit of banks in the **unclustered** network is

$$\begin{aligned}\pi_i(U) &= (1 - q(U)) \left[E(X_i|X_i \geq r_{12}^G(U), B) - r_{12}^G(U)\right] - 2c \\ &= \frac{39}{64}\left[\frac{7}{39}\frac{30}{3} + \frac{19}{39}\frac{22}{3} + \frac{13}{39}\frac{14}{3} - 3.241\right] - 2c \\ &= 2.243 - 2c.\end{aligned}$$

Clearly, $\pi_i(U) > \pi_i(U)$ and we have the following result.

Proposition 4 *In the example with $1 - \alpha = 1$, the total welfare in the clustered network is higher than in the unclustered network.*

It can be shown that Proposition 4 holds for a larger set of values of the liquidation value β than that used in the example above. For instance, it can be shown that even when $\beta = r_f$ so that there are no costs from early liquidation, banks' expected profits in the clustered network, $\pi_i(C) = 2.5813 - 2c$, are still greater than those in the unclustered network, $\pi_i(U) = 2.2687 - 2c$.

Interestingly, however, it can also be shown that for very low values of β (e.g. $\beta = 0$), the unclustered network is no longer feasible as banks are not able to offer a return rate $r_{01}(U)$ such that investors can break even. This is because in this case $r_{01}(U) = \frac{r_f}{1-q(U)}$ and $r_{12} = \frac{r_f^2}{1-q(U)}$ and

For any $r_{12}^G(U) \in (3.2, 14/3] \Rightarrow q(U) = 25/64 \Rightarrow \frac{r_f^2}{1-q(U)} = 5.2512 > \frac{14}{3}$.

For any $r_{12}^G(U) \in (14/3, 22/3] \Rightarrow q(U) = 54/64 \Rightarrow \frac{r_f^2}{1-q(U)} = 20.48 > \frac{22}{3}$.

For any $r_{12}^G(U) \in (22/3, 10] \Rightarrow q(U) = 63/64 \Rightarrow \frac{r_f^2}{1-q(U)} = 204.8 > 10$.

This is not the case in the clustered network. Even if $\beta = 0$, banks can offer $r_{12}^G(C) = 4.18$ and compensate investors with at least the risk free interest rate at any date.

4.3 Case 3: Continuation in both networks $(1 - \alpha = 0.03)$

Finally, when bankruptcy costs are sufficiently low, investors' participation constraint at date 1 when the bad signal is received can be satisfied in both networks. The equilibrium total return offered at date 1 is the smallest value of $r_{12}^B(g) \in \left[3.2, \frac{14}{3}\right]$ that gives investors an expected return conditional on $S = B$, $r_{12}^B(g)\Pr(X_i \geq r_{12}^B(g)|B) + \alpha \Pr(X_i < r_{12}^B(g)|B)E(X_i|X_i < r_{12}^B(g), B)$, of at least $r_f^2 = 3.2$. In the **clustered** network, where $\Pr(X_i \geq r_{12}^B(C)|B) = \frac{42}{90}$ and $\Pr(X_i < r_{12}^B(C)|B) = \frac{48}{90}$, $r_{12}^B(C) = 4.64$. In the **unclustered** network, where $\Pr(X_i \geq r_{12}^B(U)|B) = \frac{102}{150}$ and $\Pr(X_i < r_{12}^B(U)|B) = \frac{48}{150}$, $r_{12}^B(U) = 3.8$.

Given that debt is rolled over at date 1 in both networks, banks' expected profits are given by (4.3) in both networks and turn out to be equal to $\pi(C) = \pi(U) = 2.793$. Thus the welfare in both the clustered and unclustered networks is the same. In fact this result can be shown to hold in general. When there is no early liquidation, the output in each state is the same and the expected number of defaults is the same. Thus both cases must be equivalent.

Proposition 5 *For sufficiently low bankruptcy costs such that debt is rolled over at date* 1 *in both networks, the welfare in both the clustered and the unclustered networks is the same.*

5 Concluding remarks

Understanding network formation in the financial system is important for understanding systemic risk. In this paper we have developed a model where financial institutions choose their links to other financial institutions in an optimal way, trading off decreased bankruptcy costs from improved diversification with higher due diligence costs from more interactions. With long term debt this trade-off is relatively straightforward. However, because short term finance is usually cheaper than long term finance, financial institutions often use the former to finance long term assets. This introduces the possibility of interim signals about the possibility of default of financial institutions. Our main finding is to show that the effect of such signals depends on the networks that are formed and the level of bankruptcy costs. For low levels of bankruptcy cost, there is no early liquidation if a signal that at least one default will occur is received. In this case whether the network is clustered or unclustered does not make any difference. However, with intermediate bankruptcy costs network architecture does matter. In a clustered network, bank portfolios are more highly correlated and this means that a signal about default can lead to early liquidation in situations where in an unclustered network short term finance is rolled over. In this case, because of the costs of early liquidation, the clustered network is worse than the unclustered one. If bankruptcy costs are high, then a signal of at least one default can lead to liquidation in both kinds of network. Here the clustered network is superior because the number of states in which default occurs is less and this means the costs associated with early liquidation are less.

An important topic for future research concerns the implication of our analysis for financial regulation. Our results suggest there are no simple conclusions concerning the desirability of particular patterns of links. The desirability of clustered and unclustered networks depends on the bankruptcy and early liquidation costs. In addition it is not immediately clear what policies central banks and governments should adopt to influence financial links.

A Appendix

Proposition 1 *Consider a network g^* of overlapping portfolios that is pairwise stable. In symmetric pairwise stable networks g^*, all banks have ℓ^* links, where ℓ^* is the number of exchanged projects that maximizes the expected profit of each bank: $\ell^* = [\arg\max_{n_i} \pi_i(g)] - 1$.*

Proof. Consider g^* a network of overlapping portfolios that is pairwise stable. Clearly, any two banks i and j that are not linked in the network g^* must have exactly ℓ_i^* and ℓ_j^* links respectively, where $\ell_i^* = \arg\max_{n_i} \pi_i(g^*) - 1$ and $\ell_j^* = \arg\max_{n_j} \pi_j(g^*) - 1$. Otherwise, if at least one of the two banks i and j has more links, then it can benefit from deviating by severing a link. Alternatively, if both i and j have fewer links than ℓ_i^* and ℓ_j^* respectively, they can agree to form a link ℓ_{ij} and benefit from the deviation. In either case, g^* would not be pairwise stable.

Now we show that in any pairwise stable network where each bank i has ℓ_i^* where $\ell_i^* = [\arg\max_{n_i} \pi_i(g^*)] - 1$, it must be that $\ell_i^* = \ell_j^* (= \ell^*)$ for any banks i and j. Suppose that there exists a pairwise stable network g^* such that at least three banks i, j, k in this network have links as follows $\ell_i^* \leq \ell_j^* < \ell_k^*$. Note that if at least three such banks exist, it is always possible to select a triple i, j, k such that i, k and j, k are linked ($\ell_{ik} \in g^*$ and $\ell_{jk} \in g^*$) and i, j are not linked ($\ell_{ij} \notin g^*$). Since $\ell_k^* = [\arg\max_{n_k} \pi_k(g^*) - 1]$ it must be that

$$\pi_k(g^*) - \pi_k(g^* - \ell_{ik}) \geq 0$$

which implies from (3.3) that

$$(1-\alpha)\Pr(X_k < r_{n_k}(g^* - \ell_{ik}))E(X_k | X_k < r_{n_k}(g^* - \ell_{ik})) - (1-\alpha)\Pr(X_k < r_{n_k}(g^*))E(X_k | X_k < r_{n_k}(g^*)) > c$$

However, since $X_i = \frac{\theta_{i_1} + \theta_{i_2} + \ldots + \theta_{i_{\ell_i^*}}}{\ell_i^*}$, all projects θ_{i_j} are Bernoulli identically and independently distributed and $\ell_i^* < \ell_k^*$ it follows that

$$(1-\alpha)\Pr(X_i < r_{n_i}(g^*))E(X_i|X_i < r_{n_i}(g^*)) - (1-\alpha)\Pr(X_i < r_{n_i}(g^*+\ell_{ij}))E(X_i|X_i < r_{n_i}(g^*+\ell_{ij})) > c \tag{A.1}$$

Similarly, since $\ell_j^* < \ell_k^*$, it follows that

$$(1-\alpha)\Pr(X_j < r_{n_j}(g^*))E(X_j|X_j < r_{n_j}(g^*)) - (1-\alpha)\Pr(X_j < r_{n_j}(g^*+\ell_{ij}))E(X_j|X_j < r_{n_j}(g^*+\ell_{ij})) > c \tag{A.2}$$

Both (A.1) and (A.2) hold when $\Pr(X_i < r)$ is a decreasing and convex function in the number of links ℓ.

From (A.1) and (A.2) it follows that both i and j have an incentive to agree and form a link ℓ_{ij} which contradicts that the network g^* is stable. ■

Proposition 2 *The total number of bank defaults and total welfare is the same in any symmetric equilibrium network where all banks have ℓ^* links.*

Proof. In an equilibrium network, the aggregate welfare of investors and banks is

$$W(g) = [E(X_i|X_i \geq r(g)) - r(g)]\Pr(X_i \geq r(g)) - \ell_i c +$$
$$+[r(g)\Pr(X_i \geq r(g)) + \alpha E(X_i|X_i < r(g)\Pr(X_i < r(g))],$$

Using the law of total expectations

$$E(X_i) = E(X_i|X_i < r(g)\Pr(X_i < r(g)) + E(X_i|X_i > r(g)\Pr(X_i > r(g))$$

and that the investors always receive their opportunity cost,

$$r(g)\Pr(X_i \geq r(g)) + \alpha E(X_i|X_i < r(g))\Pr(X_i < r(g)) = r_F^2$$

$W(g)$ simplifies to

$$W(g) = E(X_i) - (1-\alpha)E(X_i|X_i < r(g))\Pr(X_i < r(g)) - \ell_i c.$$

To show that the aggregate welfare is the same in all equilibrium networks it is sufficient to show that $\Pr(X_i \geq r(g)) = \Pr(X_i \geq r(g'))$ and $E(X_i|X_i < r(g)) = E(X_i|X_i < r(g'))$ for any g and g' equilibrium networks. This is straightforward, since $X_i = \frac{\theta_{i_1}+\theta_{i_2}+\ldots+\theta_{i_{\ell*}}}{\ell^*}$ and all projects θ_{i_j} are identically and independently distributed. In any equilibrium network X_i is just a permutation of ℓ^* projects. Moreover, since any bank i has ℓ^* links in any equilibrium network, then $r(g) = r(g')$.

Since $\Pr(X_i \geq r(g)) = \Pr(X_i \geq r(g'))$, it also follows that the total number of defaults is the same in any g, g' equilibrium networks. ■

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Table 1: Clustered network

State	Cluster 1					Cluster 2					Total Defaults
	θ_1	θ_2	θ_3	X_i	Defaults (X_i<14/3)	θ_4	θ_5	θ_6	X_i	Defaults (X_i<14/3)	
1	1	10	10	10	0	10	10	10	10	0	0
2	1	10	10	10	0	10	10	2	22/3	0	0
3	1	10	10	10	0	10	2	10	22/3	0	0
4	1	10	10	10	0	2	10	10	22/3	0	0
5	1	10	10	10	0	10	2	2	14/3	0	0
6	1	10	10	10	0	2	10	2	14/3	0	0
7	1	10	10	10	0	2	2	10	14/3	0	0
8	1	10	10	10	0	2	2	2	2	3	3
9	1	10	2	22/3	0	10	10	10	10	0	0
10	1	2	10	22/3	0	10	10	10	10	0	0
11	2	10	10	22/3	0	10	10	10	10	0	0
12	1	10	2	22/3	0	10	10	2	22/3	0	0
13	1	10	2	22/3	0	10	2	10	22/3	0	0
14	1	10	2	22/3	0	2	10	10	22/3	0	0
15	1	2	10	22/3	0	10	10	2	22/3	0	0
16	1	2	10	22/3	0	10	2	10	22/3	0	0
17	1	2	10	22/3	0	2	10	10	22/3	0	0
18	2	10	10	22/3	0	10	10	2	22/3	0	0
19	2	10	10	22/3	0	10	2	10	22/3	0	0
20	2	10	10	22/3	0	2	10	10	22/3	0	0
21	1	10	2	22/3	0	10	2	2	14/3	0	0
22	1	10	2	22/3	0	2	10	2	14/3	0	0
23	1	10	2	22/3	0	2	2	10	14/3	0	0
24	1	2	10	22/3	0	10	2	2	14/3	0	0
25	1	2	10	22/3	0	2	10	2	14/3	0	0
26	1	2	10	22/3	0	2	2	10	14/3	0	0
27	2	10	10	22/3	0	10	2	2	14/3	0	0
28	2	10	10	22/3	0	2	10	2	14/3	0	0
29	2	10	10	22/3	0	2	2	10	14/3	0	0
30	1	10	2	22/3	0	2	2	2	2	3	3
31	1	2	10	22/3	0	2	2	2	2	3	3
32	2	10	10	22/3	0	2	2	2	2	3	3
33	1	2	2	14/3	0	10	10	10	10	0	0
34	2	10	2	14/3	0	10	10	10	10	0	0
35	2	2	10	14/3	0	10	10	10	10	0	0
36	1	2	2	14/3	0	10	10	2	22/3	0	0
37	1	2	2	14/3	0	10	2	10	22/3	0	0
38	1	2	2	14/3	0	2	10	10	22/3	0	0
39	2	10	2	14/3	0	10	10	2	22/3	0	0
40	2	10	2	14/3	0	10	2	10	22/3	0	0
41	2	10	2	14/3	0	2	10	10	22/3	0	0
42	2	2	10	14/3	0	10	10	2	22/3	0	0
43	2	2	10	14/3	0	10	2	10	22/3	0	0
44	2	2	10	14/3	0	2	10	10	22/3	0	0
45	1	2	2	14/3	0	10	2	2	14/3	0	0
46	1	2	2	14/3	0	2	10	2	14/3	0	0
47	1	2	2	14/3	0	2	2	10	14/3	0	0
48	2	10	2	14/3	0	10	2	2	14/3	0	0
49	2	10	2	14/3	0	2	10	2	14/3	0	0
50	2	10	2	14/3	0	2	2	10	14/3	0	0
51	2	2	10	14/3	0	10	2	2	14/3	0	0
52	2	2	10	14/3	0	2	10	2	14/3	0	0
53	2	2	10	14/3	0	2	2	10	14/3	0	0
54	1	2	2	14/3	0	2	2	2	2	3	3
55	2	10	2	14/3	0	2	2	2	2	3	3
56	2	2	10	14/3	0	2	2	2	2	3	3
57	2	2	2	2	3	10	10	10	10	0	3
58	2	2	2	2	3	10	10	2	22/3	0	3
59	2	2	2	2	3	10	2	10	22/3	0	3
61	2	2	2	2	3	2	10	10	22/3	0	3
61	2	2	2	2	3	10	2	2	14/3	0	3
62	2	2	2	2	3	2	10	2	14/3	0	3
63	2	2	2	2	3	2	2	10	14/3	0	3
64	2	2	2	2	3	2	2	2	2	3	6

Table 2: Unclustered network

State	States of the world						Total defaults	No default states			Default states			
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6		X_i=10	X_i=22/3	X_i=14/3	X_i=10	X_i=22/3	X_i=14/3	X_i=2
1	10	10	10	10	10	10	0	6	0	0				
2	10	10	10	10	10	2	0	3	3	0				
3	10	10	10	10	2	10	0	3	3	0				
4	10	10	10	10	2	2	0	2	2	2				
5	10	10	10	2	10	10	0	3	3	0				
6	10	10	10	2	10	2	0	1	4	1				
7	10	10	10	2	2	10	0	2	2	2				
8	10	10	2	10	10	10	0	3	3	0				
9	10	10	2	10	10	2	0	0	6	0				
10	10	10	2	10	2	10	0	1	4	1				
11	10	10	2	10	2	2	0	0	3	3				
12	10	10	2	2	10	10	0	2	2	2				
13	10	10	2	2	10	2	0	0	3	3				
14	10	2	10	10	10	10	0	3	3	0				
15	10	2	10	10	10	2	0	1	4	1				
16	10	2	10	10	2	10	0	0	6	0				
17	10	2	10	10	2	2	0	0	3	3				
18	10	2	10	2	10	10	0	1	4	1				
19	10	2	10	2	10	2	0	0	3	3				
20	10	2	10	2	2	10	0	0	3	3				
21	10	2	2	10	10	10	0	2	2	2				
22	10	2	2	10	10	2	0	0	3	3				
23	10	2	2	10	2	10	0	0	3	3				
24	10	2	2	10	2	2	0	0	0	6				
25	2	10	10	10	10	10	0	3	3	0				
26	2	10	10	10	10	2	0	2	2	2				
27	2	10	10	10	2	10	0	1	4	1				
28	2	10	10	2	10	10	0	0	6	0				
29	2	10	10	2	10	2	0	0	3	3				
30	2	10	10	2	2	10	0	0	3	3				
31	2	10	2	10	10	10	0	1	4	1				
32	2	10	2	10	10	2	0	0	3	3				
33	2	10	2	10	2	10	0	0	3	3				
34	2	10	2	2	10	10	0	0	3	3				
35	2	10	2	2	10	2	0	0	0	6				
36	2	2	10	10	10	10	0	2	2	2				
37	2	2	10	10	2	10	0	0	3	3				
38	2	2	10	2	10	10	0	0	3	3				
39	2	2	10	2	2	10	0	0	0	6				
40	10	10	10	2	2	2	1				1	2	2	1
41	10	10	2	2	2	10	1				1	2	2	1
42	10	2	10	2	2	2	1				0	1	4	1
43	10	2	2	2	10	10	1				1	2	2	1
44	10	2	2	2	10	2	1				0	1	4	1
45	2	10	10	10	2	2	1				1	2	2	1
46	2	10	2	10	2	2	1				0	1	4	1
47	2	10	2	2	2	10	1				0	1	4	1
48	2	2	10	10	10	2	1				1	2	2	1
49	2	2	10	2	10	2	1				0	1	4	1
50	2	2	2	10	10	10	1				1	2	2	1
51	2	2	2	10	2	10	1				0	1	4	1
52	10	10	2	2	2	2	2				0	2	2	2
53	10	2	2	2	2	10	2				0	2	2	2
54	2	10	10	2	2	2	2				0	2	2	2
55	2	2	10	10	2	2	2				0	2	2	2
56	2	2	2	10	10	2	2				0	2	2	2
57	2	2	2	2	10	10	2				0	2	2	2
58	10	2	2	2	2	2	3				0	0	3	3
59	2	10	2	2	2	2	3				0	0	3	3
61	2	2	10	2	2	2	3				0	0	3	3
61	2	2	2	10	2	2	3				0	0	3	3
62	2	2	2	2	10	2	3				0	0	3	3
63	2	2	2	2	2	10	3				0	0	3	3
64	2	2	2	2	2	2	6				0	0	0	6

Number of banks

Number of banks

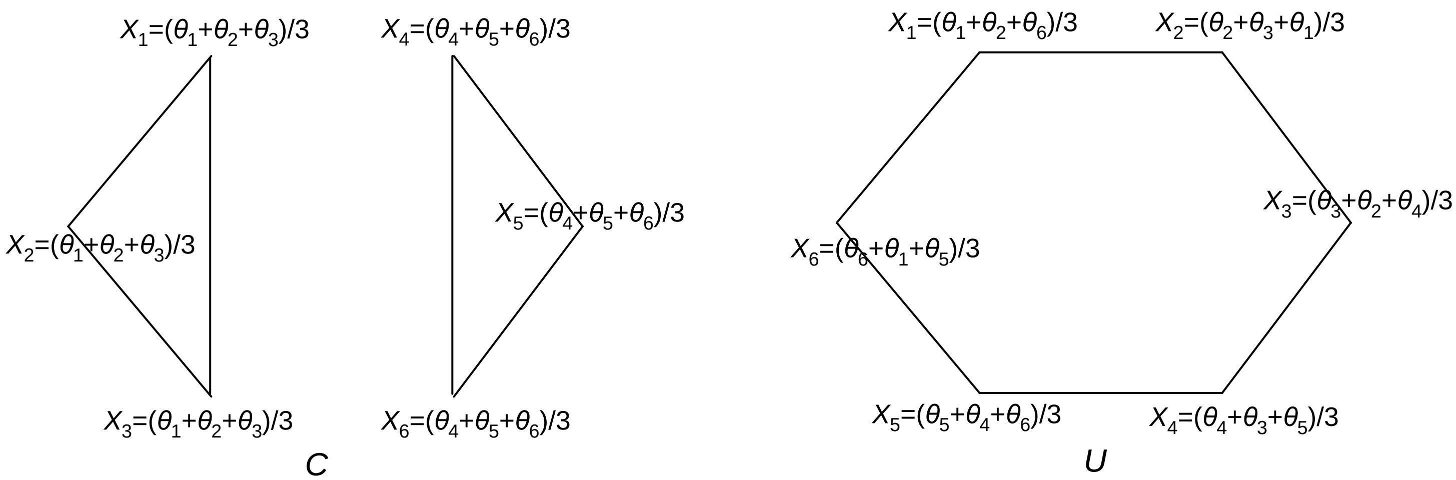

Fig. 1: *C* - clustered network; *U* – unclustered network.

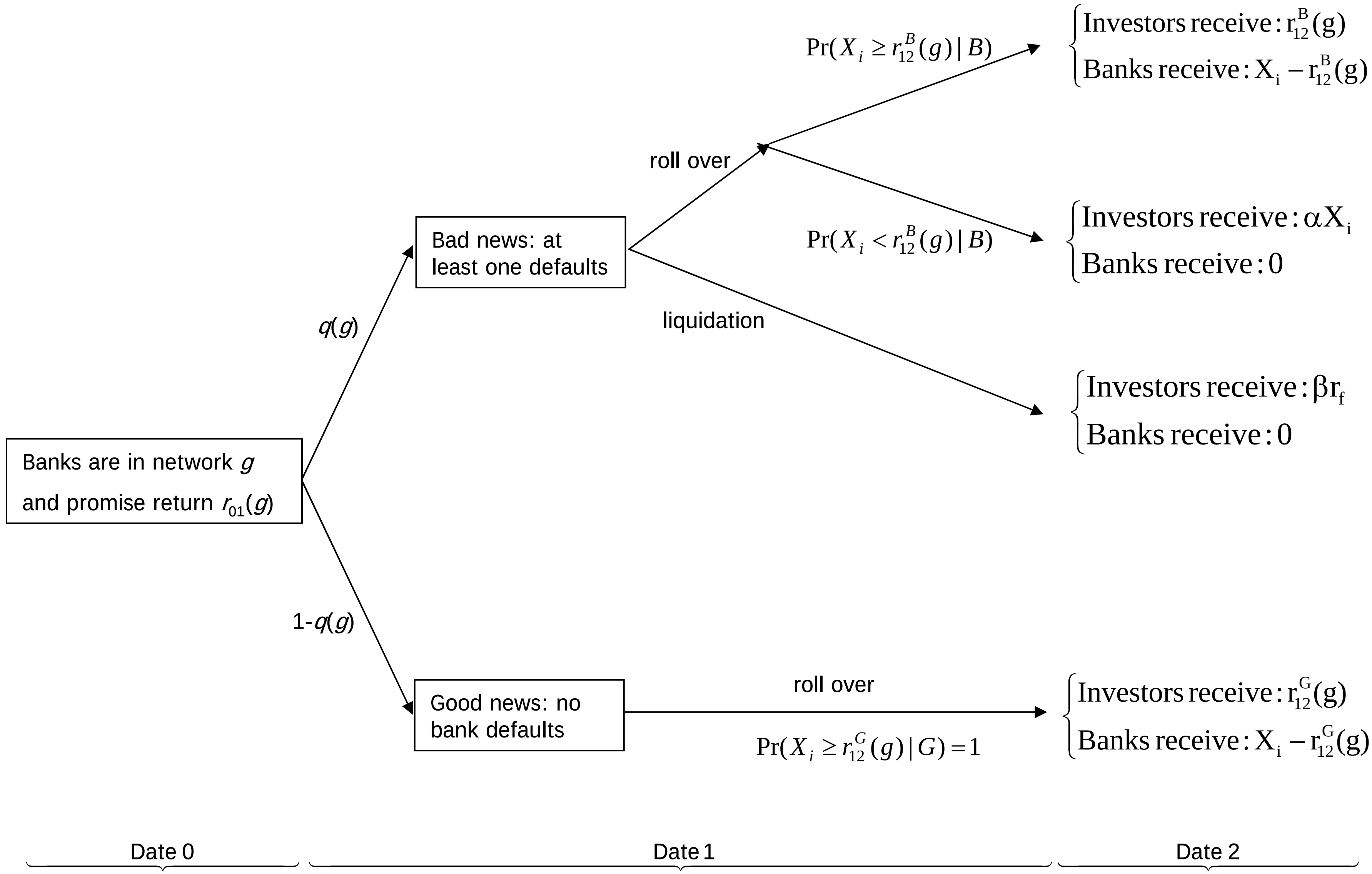

Fig. 2: Timing sequence