REPUTATIONAL BIDDING*

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Abstract

We consider auctions where bidders care about the reputational effects of their bidding behavior since each bidder is in an environment where bids act as signals. However, not all auctions - Dutch auctions being a simple example - allow for the disclosure of all bids. Accordingly, we focus our analysis on alternative disclosure rules that capture most of the standard possibilities. In this setting, we characterize symmetric and monotone equilibria for first- and second-price auctions and show that what matters for expected revenues is not the price mechanism but the type of information about bids available *after* the auction. We show that bidders distort their bidding compared to the case where bids have no role as signals, in a way that reflects the informational environment. We then proceed to rank our disclosure rules according to their ability to deliver user-value efficient equilibria or, conditional on achieving the latter, the revenues they imply. We find that these rankings can conflict in that a given disclosure rule can be better than another in terms of guaranteeing user-value efficiency but be worse in terms of expected revenue to the seller. In addition, we find that under certain conditions, full disclosure of bids may be bettered by less disclosure in terms of either user-value efficiency or revenues. We also find that first- and second-price sealed-bid auctions with disclosure of price are not equivalent even if actual types are independent and identically distributed. In fact, these can be ranked depending on the type of reputational effects.

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1. Introduction

It is often the case that auctions are conducted in an environment where bidding behavior has some implications for the bidders' future economic activity. One important instance of this is where bidders care about public perceptions about their own type at the end of the auction, as revealed by their bidding behavior.

This can happen in many different scenarios. For example, consider the case where large corporate firms bid to acquire a technological start-up with a new product. Bidding firms know that the level of interest in this acquisition, as signalled by their bidding behavior, will tell the stockmarket something about the overall managerial strategy pursued by the bidders and the management's competence. Another example is one of art auctions. Here experts give advice to collectors about the value of objects for sale. In some cases, the expert and the collector are the same person, in others, experts work for a collector and get rewarded with a fixed percentage of their client's realized net gains from the auction. In either case, experts care about their reputation either because they want recognition from their peers or because it will affect their future rewards as experts.

These examples have the common feature that bidding behavior does not just determine the chances of winning the auction and the price paid by the winner, but also sends signals that will influence the future utility of bidders (or, equivalently, the experts that work for them on percentage contracts). In other words, bidders care about the information on their bidding behavior that is transmitted to interested parties after the conclusion of the auction, regardless of whether they win the auction or not. It follows therefore that one should expect bidding behavior to be affected by how much of the bidding process will be publicly disclosed at the end of the auction.

We consider auctions where a single, indivisible object is for sale and where the standard assumptions of independence, risk neutrality, no budget constraints and symmetry of the independent private-values paradigm (IPVP) apply.[1] To this paradigm, however, we add reputational concerns for bidders, focusing on the case where a bidder's reputational returns only depend on

[1] As we shall discuss later in the paper, user-values do not necessarily need to be private and independent in our analysis, but reference to the IPVP paradigm is useful to highlight our results.

her perceived type. We emphasize, however, that even with this assumption, a crucial difference with a standard signaling game is that beliefs about a bidder's type might not just depend on her bidding behavior, but also on that of others. For example, if the bidders are participating in an auction where only the winner's bid is disclosed, such as a Dutch auction, they know that their bid will be disclosed only if they win the auction. So, even if all bidders play a fully separating equilibrium by using strictly monotone bidding strategies, for losers the information that will be revealed is simply that their valuation is lower than the winner's.

In the paper, we focus on five different disclosure rules. For each of these rules, the identity of the winner and of the bidders whose bids are revealed are always disclosed. We have disclosure rule T, where all the bids are revealed; disclosure rule S where none of the bids are disclosed; disclosure rule D where only the winning bid is disclosed as in Dutch auctions; disclosure rule P where only the highest losing bid is disclosed, as in a second-price sealed-bid auction where the price is disclosed; and finally, disclosure rule E where only the losing bids are revealed. These disclosure rules cover most of the realistic cases one can imagine.

We also restrict attention to first- and second-price sealed-bid auctions. Note though that in our context a first-price sealed-bid D auction is equivalent to a Dutch auction.[2] Our general analysis begins by characterizing bidding functions in pure strategy Perfect Bayesian Nash Equilibria where bidding strategies are symmetric and monotone, whenever such equilibria exist.

We show that these bidding functions are analogous to the ones in the absence of reputational incentives, after using what we call the bidders' *effective valuations* in the place of their (expected) user-values. These effective valuations are directly related to user-values, but take into account the reputational effects and hence they are dependent on the disclosure rule. We can show that whenever symmetric and monotone bidding functions exist, the expected revenue of any auction is simply the expected value of the effective valuation for the second-highest bidder. A direct consequence is that two auctions with different price mechanisms, but the same disclosure rule will be revenue equivalent.

[2]However, a second-price sealed-bid E auction is not equivalent to an English auction. The reason is that even in an IPVP model, an ascending auction introduces common value considerations during bidding as information that is generated during the auction affects the beliefs about the type of the highest competitor, which, in turn, affects reputational returns, as it will be discussed later. We leave the investigation of ascending auctions for future research.

Given that effective valuations depend on the disclosure rule and the type of reputational incentives, we then proceed on investigating whether reputational concerns provide a possible explanation for why often bids seem to be "too high". For example, can reputational effects explain why shareholders often obtain little benefits or even losses from acquisitions (see Bradley et al. [3], among others, for evidence on this)? We show that in our framework, *for any disclosure rule*, bidders will over- or under- bid depending on their reputational incentives: firms may consciously decide to bid too much for a firm if the stock market is expected to reward the acquisition, while bidders in an art auction may wish their bidding to conform with their peers' priors.

We then turn to studying specific properties of the different disclosure rules. In particular, we show that the different disclosure rules under scrutiny matter both in terms of their ability to guarantee that symmetric monotone equilibria exist (and, thus efficiency in user-values) and in terms of expected revenues for the seller. This is important for various reasons. First, it emphasizes that if we add reputational concerns to the IPVP, revenue equivalence can no longer be expected to hold for two auctions with the same price mechanism if the disclosure rule is different. In our setting, disclosure rules and not price mechanisms impact on the revenue properties of different auctions. Second, it clarifies that a profit-maximizing seller and a government selling some of its assets wishing to maximize user-value efficiency should deploy different disclosure rules when reputational bidding is a concern. Third, it provides further understanding on the role of more information by investigating the implications for auction outcomes of various disclosure rules.[3]

In this context, we rank the different disclosure rules in terms of their ability to guarantee user-value efficiency and in terms of expected revenues to the seller. The main mechanism behind these rankings is that different disclosure rules, by determining whose bids are going to be revealed at the end of the auction, affect the reputational incentives of bidders with different user-values in different ways. The following consequences of these rankings are particularly worth mentioning:

1. Depending on whether reputational returns are increasing or decreasing in perceived type, disclosure rules that perform well in terms of guaranteeing the existence of symmetric

[3] For related literature on the value of transparency see Prat [21], Maier and Ottaviani [14] and references therein.

monotone equilibria, tend to perform badly in terms of expected revenues to the seller and vice-versa. This suggests an interesting trade-off between maximizing revenues for the seller and achieving user-value efficiency across disclosure rules.

2. In some contexts, such as cases where reputational incentives are decreasing in perceived type, full revelation of all bids is dominated both in terms of user-value efficiency and generated revenues by other disclosure rules.

3. First-price sealed-bid auctions where only the price is disclosed and second-price sealed-bid auctions where only the price is disclosed, utilitize different disclosure rules. The former is a D auction, while the latter is a P auction. We show that their expected revenues differ and can be ranked depending on reputational incentives. This is of particular interest given the fact that price-disclosure is a common practice and that in the standard framework revenue equivalence obtains.

There is a literature that deals with cases where reputational effects distort bidding behavior: Goeree [5], Haile [6], Das Varma [4], Salmon and Wilson [22] and Katzman and Rhodes-Kropf [10].[4] There are however two main differences with this literature. First, in most of these papers it is only the T disclosure rule that is investigated; the focus is on the comparison of various price mechanisms. The only exception that we know is Katzman and Rhodes-Kropf [10], where second-price E auctions are also explored. However, these authors highlight that, for their set-up, revenue comparisons between T and E auctions, with the same price mechanism, do not lead to unambiguous results. In our context, however, we can rank them.[5] The second important difference is that our set-up can accommodate the case where bidding has reputational effects regardless of whether a bidder has won or lost the auction, which is more natural in a reputational context.[6]

[4] See also Molnar and Virag [18] who study the revenue-maximizing selling mechanism, that includes also the type of information that a seller releases at the end of the mechanism, when buyers compete for the right to enter an "outside" market.

[5] Note that in some of these papers it is in fact D auctions that are analyzed, but, because of particular assumptions they make, D auctions are strategically equivalent to T auctions.

[6] A related theme appears also in the literature on sequential auctions when bidders with correlated types do not have a unit-demand (see Ortega-Reichert [20], Weber [23] and Hausch [7]). These models deal with the case of two bidders, and Weber [23] discusses also the case of bidders being initially asymmetrically informed. More importantly for our purposes, all these models deal with the case of all initial bids being disclosed prior to the sale of the remaining units, like our T disclosure rule. We, however, investigate a wide range of disclosure rules. We

Our main contribution to the literature on auctions is therefore to isolate the implications of pure reputational concerns (which are relevant in many real life situations) on bidding behavior, and center the analysis on the revenue and user-value efficiency properties of *alternative* disclosure rules. We see also our analysis as belonging to a new trend that seeks to study auctions in more complex environments. Several have been emphasizing for some time the importance of understanding the surrounding environment (Milgrom [16]). For example, Klemperer [11] and [12] observes that collusion, entry issues and political issues are crucial to the performance of actual auctions. Maldoom [13] emphasizes how agency relationship between experts, management and shareholders might have an impact on the outcomes. In this paper, we introduce reputational effects and show how they can critically change the analysis even in the simplest auction environment.

The organization of the paper is as follows. Section 2 introduces the model. Section 3 characterizes bidding functions and discusses expected revenues for given disclosure rules. Section 4 focuses on a comparative analysis of disclosure rules. There, to develop further understanding of reputational incentives, we also explore cases where bidders have more general utility functions. Section 5 summarizes and discusses future research. Most of our proofs and our figure are relegated to an appendix.

2. The Model

To simplify exposition and facilitate focus, in this section we describe our model in terms of the auction environment. In Appendix B, we discuss in more detail how this environment could arise as part of a signaling game where bidders are the senders and bids are the signals. There, we also emphasize some novel aspects of the signaling game in question in relation to the standard sender-receiver games.

There are $N \geq 2$ risk neutral bidders who take part in a standard sealed-bid auction for a single, indivisible object. Bidder i is characterized by a type $\nu_i \in [\underline{\nu}, \overline{\nu}] = \mathcal{V}$. The nature of this type will be discussed shortly. Here note only that this attribute is related to the actual user-value of bidder i from consuming the object. Specifically, the user-value x_i of bidder i is related

also focus on the case of ex ante identical bidders. We leave the extension of our set-up to the case of asymmetric bidders for future research.

to this attribute through a function $\nu = V(x)$. Assume that $V(x)$ is twice differentiable, strictly monotone, bounded and with bounded derivatives. Let $[\underline{x}, \overline{x}] = \mathcal{X}$ be the associated ordered set of bidder i's user-values. Both user-values and types are private and i.i.d. across bidders. Given the monotonicity of $V(x)$, we will use the terms user-value and type interchangeably whenever there is no risk of confusion. Let F be the cdf over $\mathcal{X}$. Assume that it is twice differentiable with $f \equiv F' > 0$. Let F_V be the (derived) cdf over $\mathcal{V}$, with $f_V \equiv F_V' > 0$. Note that, whenever $V_x \neq 0$,

$$f_V(\nu) = \frac{f\left(V^{-1}(\nu)\right)}{\left|V_x\left(V^{-1}(\nu)\right)\right|}.$$

Let $\mathbf{x} \equiv (x_1, ..., x_N)$ and $\mathbf{x}_{-i} \equiv (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$. Also, let $y = \max_{j \neq i} \{x_j\}$ be the highest type amongst i's competitors, which is distributed according to $G \equiv F^{N-1}$. Furthermore, $y_2 = \max_{j \neq i} (\{x_j\} / y)$ is the second highest type amongst i's competitors. Define $L(y_2|y) \equiv \Pr(Y_2 \leq y_2 | Y = y)$. For all these variables, we use capital letters whenever we wish to emphasize they are to be interpreted as random variables and not realizations. We describe bids with the notation $\mathbf{b} = (b_1, ..., b_N)$ where b_i denotes the bid of bidder $i = 1, ..., N$.

Up to now this is a standard auction environment. In our set up, however, actual bids, apart from determining the allocation of the object and the price paid, play an informational role despite the fact that user-values for the single object are private and independent. This can arise naturally, among others, in environments where each and every bidder interacts independently with a third party after the conclusion of the auction and the bidders' user-values are correlated with some private characteristics of bidders which are payoff-relevant for these post-auction interactions:

- Recall, for instance, from the introduction the example of a corporate take-over. Here x_i represents the value i attaches to the take-over target in this particular auction. In particular, this will be a function of factors such as the bidder firm's strategic position or the management's ability to integrate the new assets. These latter factors are what the stock market cares about, which means that the stock market's beliefs about x_i influence the bidding firm's stock value. In this setting, $\nu_i = V(x_i)$ represents the stock market returns for bidder i if its perceived user-value is x_i. In other words, how much bidder i values the take-over influences the valuation that the stock market puts on i itself.

- As another interpretation, suppose that bidders are experts in estimating unknown user-values, and bid for themselves for an object the value of which will become known in the future (such as a work of art).[7] Suppose also that any bidder i is interested in establishing a good reputation for expertise in estimating the (unknown) user-values of objects that may become available in the future. This might be because, for example, the bidder is a collector who is interested in her recognition as an expert with his peers. Alternatively, the bidder might actually be offering his expertise in a perfectly competitive market for experts. In the latter case, if an expert is hired when his expected (by the market) expertise is at least as high as the market wage, then zero profits on the part of employers implies that the wage received by any bidder is equal to his expected (by the market) ability as an expert. More formally, suppose that expert i takes part in an auction for an experience good and that her expertise is unknown to everyone. In this case, x_i denotes his estimation about the (unknown) user-value. Moreover, $V(x_i)$ denotes his expected (by the market) ability as an expert if the market knew his estimation x_i. If the latter was true, the market wage would be $\nu_i = V(x_i)$. An increasing (resp. decreasing) $V(x_i)$ would capture the case of priors in the market for experts being such that higher (resp. lower) value-estimates are associated with better experts.

In situations like these, any publicly disclosed information at the end of the auction about submitted bids and the outcome of the auction can and will be used by the third party to estimate the types of the bidders with whom it interacts. The kind of *post-auction publicly* available information about bidders' behavior *during* the auction may therefore be important for future interactions and, thereby, for bidding strategies. We capture this here in a very simple way by defining $E_\mu V_i$ as the "transfer/wage" from the third party to bidder i after the end of the auction when the third-party's beliefs about the bidder's type $\nu_i = V(x_i)$ are represented by a cdf μ.

Let us assume hereafter that at the end of the auction the identity of the winner ι and a subset $d(\mathbf{b})$ of submitted bids $\mathbf{b}$ with the identities of the corresponding bidders are publicly

[7] Alternatively, one can think of the bidders as bidding on behalf of their principal on a contract that rewards them for winning at a fixed proportion of the true user-value net of price paid, with the user-value being revealed after the auction to both the principal and the bidder, but not to everyone else.

disclosed, where $d(.)$ is an exogenously given (bid-)disclosure rule. The specific rules we focus on are discussed shortly. Given submitted bids $\mathbf{b}$, payoffs for bidder i, *gross of the price paid* in the auction, are as follows:

$$\begin{aligned} u^w(x_i, d(\mathbf{b})) &= x_i + E_\mu[V_i \mid \iota = i, d(\mathbf{b})] \\ u^l(x_i, d(\mathbf{b})) &= E_\mu[V_i \mid \iota \neq i, d(\mathbf{b})] \end{aligned}$$

where the superscript $o = w, l$ captures the fact that payoffs, gross of price paid, depend on whether a bidder has won ($o = w$) or not ($o = l$) the auction. Accordingly, given the disclosure rule $d(.)$, bids determine the allocation of the object and the price paid as well as the "reputational return" $E_\mu[V_i \mid \iota, d(\mathbf{b})]$. The latter is the transfer from the third party to bidder i given that (by assumption) the *only* information which is available to the third party at the end of the auction is $\{\iota, d\}$, where d denotes, with some abuse of notation, the disclosed bids (and the identities of the corresponding bidders).

Before we proceed to the analysis of this auction environment, we should emphasize that our additively separable formulation allows us to separate bidders' incentives between reputational incentives and those that derive from the auction itself. In addition, our formulation of preferences could easily be generalized to

$$\begin{aligned} u^w(x_i, d(\mathbf{b})) &= U(\mathbf{x}) + H(E_\mu[V_i \mid \iota = i, d(\mathbf{b})]) \\ u^l(x_i, d(\mathbf{b})) &= H(E_\mu[V_i \mid \iota \neq i, d(\mathbf{b})]) \end{aligned}$$

insofar the (expected) user-value $U(\mathbf{x})$ is increasing in types (e.g. in the private estimates of the common user-value) and H is a strictly increasing function of the transfer from the third party. Many of our results would hold for this more general formulation, which we discuss further in section **??**, but we stick to our original formulation for simplicity. What matters here is 1) the additive separability between the (expected) user-value and reputational returns and 2) the fact that there is no common value element to reputational concerns; that is, i only cares about beliefs on her user-value x_i, not on other bidders' user-values. Note that the reputational return and thereby gross payoffs are the same for every bidder with user-value x. This symmetry assumption facilitates comparisons with the standard case of symmetric auctions in the absence of reputational effects.

Given the postulated ex ante symmetry of bidders, we will focus, throughout the paper, on symmetric and monotone (Perfect) Bayesian Nash Equilibria (PBNE) in pure strategies of the auctions under consideration. We will refer to these simply as PBNE, with the understanding that we always refer to symmetric and monotone pure strategy equilibria. Restricting attention to PBNE follows the usual practice in the literature when the auction is symmetric as in our setup. A PBNE will be represented by a strictly increasing bidding function $\beta(x)$, with its inverse be denoted by β^{-1}.

In any PBNE the third party (which interacts with bidder i) will choose her transfer given her beliefs about the bidder's type. These beliefs, and thereby the reputational returns and the corresponding gross payoff of bidder, will depend on the bidding function $\beta(x)$ and the information that is publicly available at the end of the auction (and hence on the outcome of the auction). We define

$$E_F[V(X_i) \quad | \quad X_i < y] \equiv Z(y),$$
$$E_F[V(X_i) \quad | \quad X_i > y] \equiv T(y).$$

We distinguish disclosures rules with a superscript ϕ on $d(\mathbf{b})$. With some abuse of notation, we will also refer to a disclosure rule $d^{\phi}(\mathbf{b})$ as rule ϕ. Given this notation, we also define, for a PBNE β, a function $v^{\phi o}(y, z_i)$ as the reputational return for bidder i given disclosure rule ϕ, a bid $b_i = \beta(z_i)$ and that the highest competing bid is $\beta(y)$ (and so $o = w$ if $z_i > y$, while $o = l$ if $z_i < y$).

We will restrict attention to the following disclosure rules. We reiterate that for all these rules, the identities of the winner and of the bidders whose bids are revealed are always disclosed.

- T auctions. These are auctions where all bids are publicly revealed. Any sealed-bid auction where all bids (and the identities of the corresponding bidders) are disclosed at the end of the auction is a T auction. In such auctions, the reputational return in a PBNE of a bidder who bids $b_i \in \beta(X)$ is

$$E_\mu[V_i \mid \iota, d^T(\mathbf{b})] = V(\beta^{-1}(b_i)) = v^{Tw}(y, \beta^{-1}(b_i)) = v^{Tl}(y, \beta^{-1}(b_i)),$$

regardless of whether the bidder has won or lost the auction.

- D auctions. These are auctions where the only publicly disclosed bid is that of the winner. Dutch auctions are necessarily D auctions, but sealed-bid auctions where only the winner and the winning bid are announced by the auctioneer are also D auctions. Thus in our context, a Dutch auction and a first-price sealed-bid D auction are strategically equivalent. In such auctions, with the other bidders bidding according to β, the reputational return in a PBNE of bidder i who bids $b_i \in \beta(X)$ is

$$E_\mu[V_i \mid \iota = i, d^D(\mathbf{b})] = V(\beta^{-1}(b_i)) = v^{Dw}(y, \beta^{-1}(b_i)),$$

for a winner (i.e. $b_i > \beta(y)$), and

$$E_\mu[V_i \mid \iota \neq i, d^D(\mathbf{b})] = Z(y) = v^{Dl}(y, \beta^{-1}(b_i)),$$

for a loser (i.e. $b_i < \beta(y)$).

- E auctions. These are auctions where all bids, except the winner's, are publicly revealed. English auctions where bidding stops when the next-to-last bidder withdraws are necessarily E auctions. In E auctions, with the other bidders bidding according to β, the reputational return in a PBNE of bidder i who bids $b_i \in \beta(X)$ is

$$E_\mu[V_i \mid \iota = i, d^E(\mathbf{b})] = T(y) = v^{Ew}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E_\mu[V_i \mid \iota \neq i, d^E(\mathbf{b})] = V(\beta^{-1}(b_i)) = v^{El}(y, \beta^{-1}(b_i)),$$

for a loser.

- P auctions. These are auctions where the only publicly disclosed bid is the second-highest bid. A second-price sealed-bid auction where the price is disclosed, alongside the identities of the two highest bidders, is a P auction. Clearly, if there are only two bidders, these auctions are equivalent to E auctions. For this reason when we will be referring to P auctions hereafter we will be assuming that $N > 2$. In such auctions, with the other bidders bidding according to β, the reputational return in a PBNE of bidder i who bids $b_i \in \beta(X)$ is

$$E_\mu[V_i \mid \iota = i, d^P(\mathbf{b})] = T(y) = v^{Pw}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E_\mu[V_i \mid \iota \neq i, d^P(\mathbf{b})] = V(\beta^{-1}(b_i))L\left(\beta^{-1}(b_i)|y\right) + \int_{\beta^{-1}(b_i)}^{\overline{x}} Z(s)dL\left(s|y\right) = v^{Pl}(y, \beta^{-1}(b_i)),$$

for a loser.

- S auctions. These are auctions where no bid is publicly disclosed.[8] In such auctions, with the other bidders bidding according to β, the reputational return in a PBNE of bidder i who bids $b_i \in \beta(X)$ is

$$E_\mu[V_i \mid \iota = i, d^S(\mathbf{b})] = E_G[T(Y)] = v^{Sw}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E_\mu[V_i \mid \iota \neq i, d^S(\mathbf{b})] = E_G[Z(Y)] = v^{Sl}(y, \beta^{-1}(b_i)),$$

for a loser.

Simple inspection shows that for all our disclosure rules, the functions $v^{\phi o}$ are twice differentiable in each argument. The above reputational returns in a PBNE, $v^{\phi o}(y, \beta^{-1}(b_i))$, capture the beliefs of the third party about the type of bidder i. These beliefs follow Bayes rule, wherever it is possible, given the publicly available information at the end of the auction and the expected equilibrium bidding behavior of the bidders. The most important aspect to underline here is that these beliefs do not necessarily depend on b_i alone, but potentially on all other bids, as the disclosure rule may imply that b_i is not revealed. For example, with reference to the D auctions case described above, a losing bidder i's bid will not be known to the third party, but the latter does have the information that i's bid was lower than the winning bidder's bid. In general, note that in the D, P and E auctions, a bidder does not know ex-ante whether her bid will be disclosed: the disclosed bids will depend on the relative ranking of the submitted bids.

We restrict attention to PBNE that are supported by the following off-the-equilibrium-path beliefs, which are compatible with the Universal Divinity refinement introduced by Banks and Sobel [2][9]

[8] Clearly, for T, D and E auctions the information on the identity of the winner is redundant as it can be recovered from the available information on bids and their corresponding bidders, but for P and S auctions, it is not redundant.

[9] The proof is relatively straightforward and is available upon request.

Assumption A (Beliefs) Let $\beta(\bullet)$ be a symmetric and monotone bidding strategy in a PBNE of the auctions under consideration. We assume that in any such equilibrium, any bid lower than $\beta(\underline{x})$ is believed to come from type $x_i = \underline{x}$ and any bid higher than $\beta(\overline{x})$ is believed to come from type $x_i = \overline{x}$. Further, if there is a bid b and a type $\widehat{x}$ such that $b \in \left(\lim_{x_i \to \widehat{x}^-} \beta(x_i), \lim_{x_i \to \widehat{x}^+} \beta(x_i)\right)$ then b is believed to come from type $x_i = \widehat{x}$.

The focus on PBNE with off-the-equilibrium-path beliefs that satisfy Assumption A, allows us to associate to each vector of bids **b** a corresponding vector **z** of types.[10] Given our focus on such equilibria, it will be convenient to work with the vector **z** rather than with **b**. We can also think of z_i as the report to the auctioneer by bidder i of her type (via her bid). We will therefore refer to z_i as the *announcement* of bidder i (to the auctioneer).

In our set up, given one of the disclosure rules ϕ described above, we therefore have that if bidder i announces z_i, the other bidders announce their true types and it is commonly expected that every bidder deploys the bidding function β, then the expected payoff for bidder i - gross of price paid - is equal to

$$\int_{\underline{x}}^{z_i} (x_i + v^{\phi w}(s, z_i))dG(s) + \int_{z_i}^{\bar{x}} v^{\phi l}(s, z_i)dG(s).$$

It would help understanding to note that in the standard model where there are no reputational returns we would have $v^{\phi l} = v^{\phi w} \equiv 0$. Thus, in our framework, announcements/bids affect both the chances of winning (and the price paid) and the payoffs conditional on winning or losing the auction.

We are ready to investigate the properties of PBNE.

3. Bidding Functions

We begin with an assumption and an important definition:

Assumption B (Lower Bound Condition) For $N > 2$, there exists a positive scalar M such that

$$\left| \lim_{x_i \to \underline{x}^+} \frac{V_x(x_i)}{g(x_i)} \right| \leq M$$

[10] That is, with z_i such that $\beta^{-1}(b_i) = z_i$ if $b_i \in \beta(X_i)$, $z_i = \underline{x}$ if $b_i < \beta(\underline{x})$, $z_i = \overline{x}$ if $b_i > \beta(\overline{x})$ and $z_i = \widehat{x}$ if $b_i \in (\lim_{x \to \widehat{x}^-} \beta(x), \lim_{x \to \widehat{x}^+} \beta(x))$.

This condition is automatically satisfied, given our assumption that $f > 0$, for the case $N = 2$. For $N > 2$, it is necessary condition for the existence of a PBNE as it will become clear shortly.[11]

Definition 1 *Let*

$$\Psi^{\phi}(x_i, z_i) \equiv$$
$$\equiv x_i + v^{\phi w}(z_i, z_i) - v^{\phi l}(z_i, z_i) + \frac{1}{g(z_i)} \left\{ \int_{\underline{x}}^{z_i} v_z^{\phi w}(s, z_i) dG(s) + \int_{z_i}^{\overline{x}} v_z^{\phi l}(s, z_i) dG(s) \right\}.$$

We then have that $\psi^{\phi}(x_i) \equiv \Psi^{\phi}(x_i, x_i)$ *is the* effective valuation *for bidder* i *with type* x_i *who faces a disclosure rule* ϕ.

Assumption B and our assumptions about $V(x)$ guarantee that effective valuations are well-defined, bounded and differentiable.

$\Psi^{\phi}(x_i, z_i)$ is the net welfare gain to bidder of type x_i from winning (gross of payments) relative to the increase in the probability of winning, after increasing the announcement marginally over z_i, given that the disclosure rule is ϕ. An effective valuation is such net welfare gain when $z_i = x_i$. It will help when we discuss specific disclosure rules to point out here that the component $v^{\phi w} - v^{\phi l}$ captures the reputational gain (or loss) to the bidder from winning the auction, while the rest captures the additional reputational gain (or loss) from having the user-value revealed. In what follows we will sometimes refer to $\widetilde{\psi}^{\phi}(x_i) \equiv \psi^{\phi}(x_i) - x_i$ as the reputational component of the effective valuation. The following Lemma will be used extensively in proving the forthcoming results.

Lemma If $V_x(x) > 0$ for any $x > \underline{x}$, then $Z_x(x) > 0$ and $T_x(x) > 0$, and vice versa.

Proof. Differentiating $T(x)$ we have

$$T_x(x) = \frac{f(x)}{1 - F(x)} [T(x) - V(x)].$$

Differentiating $Z(x)$ we have

$$Z_x(x) = \frac{f(x)}{F(x)} [V(x) - Z(x)].$$

The lemma then follows directly from the fact that if $V_x(x) > 0$ for any $x > \underline{x}$, then $T(x) > V(x) > Z(x)$, and vice versa.■

[11] Indeed, it is of some interest to note that for fixed $V(x)$ and $f(x)$, if this condition is satisfied for N bidders, then it is satisfied for $N - 1$ bidders. The reverse, clearly, is not necessarily true.

For our chosen disclosure rules, we then have that[12]

$$
\begin{aligned}
\psi^T(x_i) &= x_i + \frac{V_x(x_i)}{g(x_i)}, \\
\psi^D(x_i) &= x_i + V(x_i) - Z(x_i) + \frac{G(x_i)}{g(x_i)} V_x(x_i), \\
\psi^E(x_i) &= x_i + T(x_i) - V(x_i) + \frac{1-G(x_i)}{g(x_i)} V_x(x_i), \\
\psi^P(x_i) &= x_i + T(x_i) - V(x_i) \\
&\quad + \frac{1-F(x_i)}{f(x_i)} \left[V_x(x_i) + (N-2) Z_x(x_i) \right] \\
\psi^S(x_i) &= x_i + E_G[T(Y)] - E_G[Z(Y)]
\end{aligned}
$$

It will help intuition to note the following. First, as one should expect, effective valuations collapse to user-values in the absence of reputational concerns, where $V(x_i) \equiv 0$. Second, in the benchmark case where reputational concerns exist but user-values are known to everyone at the end of the auction we have, with some abuse of notation, $v^{\phi o}(y, z_i) \equiv V(x_i)$ for any y and z_i. In this case, $\psi^\phi(x_i) = x_i$, as in the standard case of private and independent values. In other words, as intuition would suggest, if user-values are publicly revealed anyway at the end of the auction, there are no incentives to distort bidding in order to signal. It follows that, in the case of reputational bidding, $\widetilde{\psi}^\phi(x_i)$ captures the reputational effect on bidders' effective valuations. Third, $\frac{V_x(x_i)}{g(x_i)}$ captures, in a monotone and symmetric equilibrium, the reputational gain (or loss) relative to the increase in the likelihood of winning the auction from increasing marginally the perception of the third party about bidder $i's$ type by means of increasing bidder $i's$ bid marginally. This relative gain is relevant only when one's bid is revealed. So, by the nature of the disclosure rules under scrutiny here, this relative gain/loss is always relevant for $T-$auctions, never relevant for $S-$auctions, while it is only relevant for (some) losers in E or $P-$auctions and for the winners of $D-$auctions. Fourth, when some bids may not be revealed, the event of winning itself carries a reputational net gain (or loss). This is captured by $V(x_i) - Z(x_i)$ for $D-$auctions, by $T(x_i) - V(x_i)$ for E and $P-$auctions, and by $E_G[T(Y)] - E_G[Z(Y)]$ for $S-$auctions. Finally, it follows directly that, with monotone $V(x)$, the various relative reputational gains/losses discussed above have the same sign as V_x.

[12]Calculating effective valuations given our definitions of $v^{\phi w}(y, z_i)$ and $v^{\phi l}(y, z_i)$ is immediate except for the case $\phi = P$ which is detailed in the appendix.

Our first result characterizes bidding functions for first-price (FP) and second-price (SP) sealed-bid auctions. We reemphasize, however, that in our setting, Dutch auctions are strategically equivalent to a FP sealed-bid auction with disclosure rule D. From now on, we slightly abuse notation by denoting $\lim_{x_i \to \underline{x}^+} \psi^{\phi}(x_i)$ with $\psi^{\phi}(\underline{x})$.[13]

Proposition 1 *Assume **A** and **B** hold, $\psi^{\phi}(\underline{x}) \geq 0$ and that $\psi^{\phi}(x_i)$ is strictly increasing.*

1. *The PBNE in second-price sealed-bid auctions with a disclosure rule ϕ, $\beta^{SP-\phi}$ is given by*

$$\beta^{SP-\phi}(x_i) = \psi^{\phi}(x_i),\ x \in [\underline{x}, \bar{x}]$$

2. *The PBNE in first-price sealed-bid auctions with a disclosure rule ϕ, $\beta^{FP-\phi}$ is given by*

$$\begin{array}{c} \beta^{FP-\phi}(x_i) = E_G\left[\psi^{\phi}(Y)|Y < x_i\right],\ x \in (\underline{x}, \bar{x}] \\ \text{and}\ 0 \leq \beta^{FP-\phi}(\underline{x}) \leq \psi^{\phi}(\underline{x}) \end{array}.$$

Proof Follows familiar steps. For completeness the proof is in appendix B.■

It is immediate to see the similarity between this result and the bidding functions for the basic, private and independent values set-up. The only difference is that instead of bidding according to x_i bidders use their *effective valuations* $\psi^{\phi}(x_i)$.

Two issues arise from the proposition above. The first is that $\psi^{\phi}(x_i)$ or $E_G\left[\psi^{\phi}(Y)|Y < x_i\right]$ are not guaranteed to be strictly increasing. The second is whether, conditional on symmetric and monotone bidding functions being well defined in each case, revenue equivalence between price mechanisms still obtains in our set-up.

We take up the first issue again in the next section, but with respect to the second issue, we can show that indeed revenue equivalence applies whenever PBNE exist:

Proposition 2 *Consider a disclosure rule ϕ and any $m-$price sealed-bid auction, $m = FP, SP$, such that symmetric and monotone bidding functions exist for the given disclosure rule. Then, the auction provides an expected revenue to the seller equal to*

$$E_{F_2^{(N)}}\left[\psi^{\phi}\left(Y_2^{(N)}\right)\right]$$

[13]For $N = 2$, $\psi^{\phi}(\underline{x})$ is necessarily well defined, given our assumptions, in particular, that $f(\underline{x}) > 0$. However, for $N > 2$, we might have that $g(\underline{x}) = (N-1)F(\underline{x})^{N-2} f(\underline{x}) = 0$. It is in those cases that $\psi^{\phi}(\underline{x})$ should be interpreted as $\lim_{x_i \to \underline{x}^+} \psi^{\phi}(x_i)$.

where $F_2^{(N)}$ *is the cdf of the random variable* $Y_2^{(N)}$ *that represents the second-highest type amongst all bidders.*

Proof Follows familiar steps. For completeness the proof is in appendix B.■

One crucial aspect of proposition 2 is that effective valuations depend on the publicly available information at the end of the auction represented by the disclosure rule ϕ. So, auctions with the same price mechanism but different disclosure rules will in general have different expected revenues. Below, we investigate issues of existence and revenue ranking that deal specifically with the disclosure rules S, T, D, P or E we identified above.

4. Comparing Disclosure Rules

4.1. Existence of PBNE

We begin with some preliminary observations. The first observation is that $E_G\left[\psi^{\phi}(Y)|Y < x_i\right]$ is strictly increasing whenever $\psi^{\phi}(x_i)$ is strictly increasing[14]; in this sense first-price auctions are more likely to have a PBNE than second-price auctions. Given this observation and Proposition 2 we focus hereafter on second-price auctions. Our results below can then be interpreted as characterizing the minimum conditions for existence of a PBNE under any of the price mechanisms under scrutiny.

A second observation is that when $V(x)$ is strictly increasing, PBNE are the only equilibria that can guarantee efficiency. The reason is that only in these equilibria does the highest type always win the auction and, in this case, the highest type is also the type that values winning the most.[15]

The final observation is that simple inspection of the relevant effective valuations shows that a sufficient condition for $\psi^{\phi}(\underline{x}) \geq 0$, whatever the disclosure rule, and given assumption B, is that $\underline{x}$ is sufficiently high. Assume hereafter that this is true. Conditional on these conditions

[14]We have that

$$\begin{aligned} &\frac{d\left(E_G\left[\psi^{\phi}(Y)|Y < x_i\right]\right)}{dx_i} \\ = &\frac{g\left(x_i\right)}{G\left(x_i\right)}\left(\psi^{\phi}(x_i) - E_G\left[\psi^{\phi}(Y)|Y < x_i\right]\right) \end{aligned}$$

which is then positive for all x_i if $\psi^{\phi}(x_i)$ is increasing in x_i (but the reverse is not necessarily true).

[15]If $V(x)$ is strictly decreasing, then all we can argue is that PBNE are the only equilibria that allocate the object to the bidder with the highest user-value/type, not necessarily the bidder who values winning the most.

being satisfied, then one can focus on conditions for the putative bidding functions to be strictly increasing. We can now have our first result of this section:

Proposition 3 *Whenever* $V_x > 0$ *for all* $x \in (\underline{x}, \bar{x})$, *then* $\psi^{\phi}(x) > x$ *for all* $x \in (\underline{x}, \bar{x})$, *and* $\psi^{\phi}(\underline{x}) \geq \underline{x}$ *and* $\psi^{\phi}(\bar{x}) \geq \bar{x}$, *for all disclosure rules* $\phi \in \{S, T, D, E, P\}$. *Conversely if* $V_x < 0$ *for all* $x \in (\underline{x}, \bar{x})$.

Proof. It follows directly from the definitions of effective valuations and $T(x)$ and $Z(x)$ above.■

That is, if the reputational returns when one's bid is revealed in a symmetric and monotone equilibrium, $V(x_i)$, are strictly increasing (resp. decreasing) in x_i, we have that with any of our disclosure rules here, there is almost everywhere overbidding (resp. underbidding) due to the positive (resp. negative) reputational effect.[16]

Bradley et al. [3], among others, provide evidence that shareholders often obtain little benefits or even losses from acquisitions. Several explanations have been put forward for this, some of which (see, for example, Mørck et al. [19]) identify managerial incentives as a possible source of overbidding. Our setting provides a further possible explanation along these lines, which puts emphasis on the reputational incentives that managers might face.

The figure below shows the reputational component of the equilibrium SP bidding functions for our given disclosure rules in a parameterization that guarantees existence: F is uniform on $[0, 1]$, $V(x_i) = \frac{1}{10} x_i^4$ and $N = 4$:[17]

[FIGURE HERE]

The figure emphasizes the overbidding result when $V_x > 0$ since all components are strictly positive for $x \in (\underline{x}, \bar{x})$. We will refer to the figure below when we focus on ranking disclosure rules according to their propensity to generate strictly increasing second-price bidding functions.

We then have the following results regarding the relative user-value efficiency properties of the disclosure rules under scrutiny:

[16] We say *almost* everywhere, because the only cases when there is no over/under-bidding for a type x_i are when (i) $x_i = \underline{x}$, $\phi = T$ and $\lim_{x_i \to \underline{x}^+} \frac{V_x(x_i)}{g(x_i)} = 0$, or (ii) $x_i = \underline{x}$ and $\phi = D$ or (iii) $x_i = \overline{x}$ and $\phi = E, P$.

[17] In other words, the figure shows $\widetilde{\psi}^{\phi}(x_i)$ for different disclosure rules. For $\phi = P$ and $\phi = E$, this reputational component is decreasing at some point but even in these cases, the whole bidding function $\psi^{\phi}(x_i)$ is strictly increasing.

Proposition 4 *I* $\psi^S(x_i)$ *is strictly increasing.*

II With f_V *being log-concave:*

a. *whenever* $V_x > 0$ *for all* $x \in (\underline{x}, \bar{x})$, $\psi^D(x_i)$ *is strictly increasing if* $\psi^T(x_i)$ *is strictly increasing, and*

b. *whenever* $V_x < 0$ *for all* $x \in (\underline{x}, \bar{x})$, $\psi^T(x_i)$ *is strictly increasing if* $\psi^D(x_i)$ *is strictly increasing.*

III With f_V *being log-concave: Then*

a. *whenever* $V_x > 0$ *for all* $x \in (\underline{x}, \bar{x})$, $\psi^T(x_i)$ *is strictly increasing if* $\psi^E(x_i)$ *is strictly increasing, and*

b. *whenever* $V_x < 0$ *for all* $x \in (\underline{x}, \bar{x})$, $\psi^E(x_i)$ *is strictly increasing if* $\psi^T(x_i)$ *is strictly increasing.*

Proof. See appendix.■

The first result in the proposition is very easy to interpret because when the disclosure rule is $\phi = S$ then:

$$\begin{aligned} \psi^S(x_i) &= x_i + \lambda, \\ \text{where } \lambda &\equiv E_G[T(Y) - Z(Y)] \end{aligned}$$

In other words, we have the bidding function from the standard independent private values framework with the addition of a constant, which reflects the net reputational gain from winning the auction with a secret bid. The latter captures that with the $\phi = S$ disclosure rule, only the identity of the winner is revealed, but no actual bids. Whether this additional term is positive or negative depends on whether being of a higher type is beneficial or detrimental vis-a-vis the reputational returns. Thus, for sufficiently high value of $\underline{x}$, so that $\psi^S(\underline{x}) \geq 0$, a PBNE is guaranteed to exist for this disclosure rule.

Proposition 4 II and III allow us to provide a ranking between the disclosure rules $\phi = D, T, E$ in terms of their likelihood of generating a PBNE whenever f_V is log-concave (which is satisfied by most common distributions).[18] Under this log-concavity property, the proposition tells us that if $V(x)$ is strictly increasing then whenever a strictly increasing second-price bidding function exists when the disclosure rule is $\phi = T$, we are guaranteed that the corresponding second-price bidding function with disclosure rule $\phi = D$ also exists (see part IIa). In that sense, the disclosure rule $\phi = D$ is more likely to be efficient than the disclosure rule $\phi = T$. The intuition is easy to grasp by referring to the figure, where we have that reputational effects are increasing in x_i. With disclosure rule $\phi = D$, reputational effects grow stronger as the type increases, because it is the high types that have a greater chance of winning and therefore to have their type revealed. With disclosure rule $\phi = T$, on the other hand, reputational effects are more evenly distributed across types. So, in the former case the bidding function is "steeper". The converse ranking is obtained when V is strictly decreasing, for exactly the opposite reasons (see part IIb): with $\phi = D$, high types have a significantly higher reason to underbid than low types.

A relationship between T and E disclosure rules that echoes the one we just described between the D and T disclosure rules also obtains (see parts IIIa and IIIb) and again the intuition follows in a similar way: $\phi = E$ auctions provide reputational incentives that are relatively stronger for low types than in the $\phi = T$, so that if V is strictly decreasing (resp. increasing) low types have a relatively higher incentive to underbid (resp. overbid).

A very interesting implication of the above proposition is:

Corollary 1 *Full disclosure is dominated, in terms of guaranteeing the allocation of the object to the bidder with the highest user-value/type by non-disclosure of bids and, if f_V is log-concave, by non-disclosure of each and every winning (resp. losing) bid when overbidding (resp. underbidding) takes place.*

Finally, we note that a relative ranking in terms of monotonicity of effective valuations between P and E, T, D auctions that depends on intuitive properties of our primitives is difficult to obtain.

[18] Actually, for given subsets of these properties, log-concavity of f_V is stronger than what is needed. For example, if f_V is strictly decreasing, then by Corollary 1 in Bagnoli and Bergstrom [1] we have that IIa and IIIb obtain while if f_V is strictly increasing, then by Corollary 3 in Bagnoli and Bergstrom [1] we have that IIb and IIIa obtain.

4.2. Revenue Comparisons

Assuming a PBNE exists for all our disclosure rules, we now turn to (expected) revenue comparisons. Denote by $ER(\phi)$ the expected revenue associated with a specific disclosure rule ϕ. The result below allows us to compare revenue properties:

Proposition 5 *Assume the relevant symmetric and monotone PBNE exist. Then:*

I $ER(T) = ER(E)$

II $ER(D) - ER(S)$ *is positive (resp. zero, negative) if* f_V *is strictly increasing (resp. constant, decreasing)*

III *Whenever* $V_x > 0$ *for all* $x \in (\underline{x}, \bar{x})$ *then*

a.

$$\begin{aligned} ER(T) - ER(D) &> 0 \\ ER(E) - ER(P) &> 0 \\ ER(P) - ER(D) &> 0 \end{aligned}$$

b. *For large enough* N, $ER(T) - ER(S) > 0$.

IV *Whenever* $V_x < 0$ *for all* $\in (\underline{x}, \bar{x})$ *then*

a.

$$\begin{aligned} ER(T) - ER(D) &< 0 \\ ER(E) - ER(P) &< 0 \\ ER(P) - ER(D) &< 0 \end{aligned}$$

b. *For large enough* N, $ER(T) - ER(S) < 0$.

Proof See Appendix.[19]■

[19] In the appendix, we actually prove the stronger result that

$$\begin{aligned} E_G\left[\psi^T(Y)\,|Y < x_i\right] - E_G\left[\psi^D(Y)\,|Y < x_i\right] &> 0 \\ E_G\left[\psi^E(Y)\,|Y < x_i\right] - E_G\left[\psi^P(Y)\,|Y < x_i\right] &> 0 \end{aligned}$$

Proposition 5 provides a simple and comprehensive set of conditions for analyzing the revenue properties of T, D, E, P and S auctions. In the first instance, point I of the proposition tells us that revenue equivalence applies to T and E auctions. To see how this might come about, note, after some straightforward manipulations, that

$$\psi^E(x_i) = \psi^T(x_i) + T(x_i) - V(x_i) - \frac{G(x_i)}{g(x_i)} V_x(x_i).$$

So, if $V(x)$ is strictly increasing, then for $x_i = \underline{x}$ the bidding function for rule $\phi = E$ is above that for the rule $\phi = T$, while for $x_i = \overline{x}$ the reverse occurs. This is clearly visible in figure above. The intuition is that with disclosure rule $\phi = E$, bidders with low valuations have a stronger incentive to overbid, in a PBNE. The reason is that such bidders are more likely to lose and $\phi = E$ implies disclosure of losing bids. Conversely, high types have a much higher chance of winning in a PBNE. Thus, they do not have an incentive to overbid by much, as their high type is likely to not be disclosed. This logic does not apply to auctions with $\phi = T$ because, in this case, all types are revealed in a PBNE. Our result, then, shows that the additional incentives to overbid for low types and the more muted incentives to overbid for high types that one finds with disclosure rule E cancel out in expectation. Similarly, if $V(x)$ is strictly decreasing.

Point II focuses on the comparison between the expected revenues of $\phi = D$ and $\phi = S$ auctions. We have that auctions with disclosure rule $\phi = D$ provide a higher expected revenue than auctions with disclosure rule $\phi = S$, whenever high values of the random variable V are more likely, and vice versa. The intuition here is that with the disclosure rule $\phi = S$ reputational incentives are uniform across types because bids are not disclosed and hence, at the margin, there is no signaling incentive to distort bidding. Disclosure rule $\phi = D$, instead, creates incentives to distort bidding for signaling purposes but does so proportionally more for high types who are more likely to win and hence more likely to have their bids disclosed. That higher values of V are more likely means that there is relatively more overbidding from high types when there in overbidding and relatively less underbidding from high types when there is underbidding. In such cases, we should expect more expected revenues from the disclosure rule $\phi = D$ than from $\phi = S$.

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whenever $V_x > 0$ and vice versa. This clearly implies the expected revenue results described in the proposition. However, the result is in effect stronger when first-price auctions are considered. The reason is that the above implies also a ranking for revenues ex post.

Points IIIa. and IVa. address the comparisons in terms of expected revenues that depend on the monotonicity of V. We begin by discussing the difference in expected revenues between $\phi = T$ and $\phi = D$ auctions on the one hand, and between $\phi = E$ and $\phi = P$ auctions on the other. Our results tell us that when reputational effects are increasing in the perceived type then T or E auctions induce higher expected revenues than D or P auctions, respectively; if reputational effects are decreasing in the perceived type the opposite results hold. There is a symmetry in the relationship between T and D auctions on the one hand and E and P auctions on the other: the difference between T and D auctions is that the latter reveals the winner's bid while the former reveals the winner's bid and the bids of all the lower bidders. Similarly, the difference between E and P auctions is that the latter reveals the highest loser's bid while the former reveals the highest loser's bid and the bids of all the lower bidders. Given this symmetry the simple intuition that comes from our results is that disclosing all lower bids in addition to the highest disclosed bid increases reputational incentives (whether they are increasing or decreasing in the perceived type) because lower types now face a higher chance of their bids being revealed.

With regards to the comparison between $\phi = P$ and $\phi = D$, the results suggest that reputational incentives are stronger revenue-wise in P than in D auctions. This follows from the fact that low types have a higher chance of being the highest loser than the winner while the difference between the probability of being the winner or the highest losers is not so significant for high types. Thus, low types have, compared to high types, proportionately higher incentives to distort their bids in P than in D auctions.

Points IIIb. and IVb. tell us that if $V(x)$ is strictly increasing, then reputational effects with the disclosure rule $\phi = T$ (and hence $\phi = E$) will eventually dominate revenue-wise those with disclosure rule $\phi = S$, and vice versa. To get an intuition for this result, consider the ex-ante expected payment for each bidder in T auctions that is due to reputational concerns. Such expected payment is constant in N because in a symmetric and monotone equilibrium their bidding reveals their type and that is the same information regardless of how many bidders there are. Thus, expected revenues due to reputational concerns are simply a constant multiplied by N, and so are unbounded in N.[20] With S auctions, instead, expected reputational revenues

[20] Of course, if V is strictly decreasing, such expected payment from reputational concerns is negative.

are equivalent to the difference between the reputational returns from winning and those from losing, averaged over the highest opponent's expected type. For a given highest opponent's type, this difference is independent of the number of bidders, which only affects the expectation of what that type will be. Therefore, expected revenues are bounded from above (below) if V is strictly increasing (decreasing) by the maximum difference between the reputational returns from winning and those from losing evaluated at the highest opponent's type.

A very interesting implication of the proposition above, given the much celebrated revenue equivalence of first- and second-price sealed-bid auctions when there are no reputational incentives, is that:

Corollary 2 *Consider a first-price and a second-price sealed-bid auction where only the price, the corresponding bidder and winner are disclosed. Whenever* $V_x > 0$ *(resp.* $V_x < 0$*) for all* $x \in (\underline{x}, \bar{x})$ *the second-price auction generates more (less) expected revenues than the first-price auction.*

Proof Directly from (a) the fact that the second-price auction is de facto a P auction and the first-price auction is de facto a D auction, (b) the revenue equivalence of FP-D and SP-D, and (c) the comparison of P and D auctions in parts IIIa and IVa of the above proposition.■

Further points deserve to be made that compare our results in proposition 4 with those above:

- The result in proposition 5 IIIa. and IVa. above provides a striking contrast with that in proposition 4 because it may reverse the implicit ranking we had in that proposition: if $V_x > 0$, we are more likely to have a PBNE when the disclosure rule is $\phi = D$ than when the disclosure rule is $\phi = T$, but, conditional on existence of PBNE in both cases, expected revenues will be higher with $\phi = T$. Thus, a government aiming for an efficient in terms of user-values allocation of an asset it owns and sells might prefer $\phi = D$ while a profit-maximizing seller might prefer $\phi = T$.

- It is interesting that the linkage principle - obtained for single-object auctions by Milgrom and Weber [17] - has been broadly interpreted as implying that more public information raises prices and revenue. Several of our results above, however, could be interpreted as a failure of such interpretation of the linkage principle in environments where user-values are independent but reputational effects are in place.[21] This is, in particular, emphasized by:

Corollary 3 *Suppose that underbidding takes place. Full disclosure is dominated in terms of expected revenues by non-disclosure of bids if the number of bidders is sufficiently high, and by the disclosure rules* $\phi = D, P$.

Proof Directly from part I and the relevant comparisons in part IV of the above proposition.■

Corollaries 1 and 3, in turn, imply the following interesting result in terms of optimality of full-disclosure:

Corollary 4 *Suppose that underbidding takes place. Then, if the number of bidders is high enough, full disclosure is dominated by secrecy both in terms of expected revenues and guaranteeing that the object is sold to the bidder with the highest type/user-value.*

This result provides another instance in which transparency may not be optimal, in particular if there is underbidding. So, for instance, when out of fashion items or items perceived as "guilty pleasures" are sold, S auctions will dominate T auctions both in terms of expected revenues (when there are sufficiently many bidders) and guaranteeing that the object will be sold to the bidder with the highest user-value. Holmström [8] has shown, in a principal-agent model, that more information improves the principal's inference about the agent's effort and this, in turn, results in a Pareto improvement. Prat [21] shows that more information is not necessarily optimal when the principal cannot commit ex-ante to an incentive scheme. Transparency may also not be beneficial in a common agency model where the decentralized incentive schemes offered by the principals are strategic substitutes (Maier and Ottaviani [14]). In our case, transparency is not optimal, neither in terms of expected revenues nor in terms of allocating the object to the bidder with the highest user-value, because of the reputational effects on bidders' behavior when low types are associated with high reputational returns.

To develop further understanding of the above results, it is helpful to consider generalizing the bidder's utility function to allow for the possibility of interdependent user-values and to allow

[21] For a similar argument in multi-unit sequential auction with unit-demands and interdependent types/signals see Mezzetti et al. [15].

for the case where reputational effects enter the bidder's utility function in a non-linear way. In other words, suppose that

$$\begin{aligned} u^{w}(\mathbf{x}, d(\mathbf{b})) &= U(\mathbf{x}) + H(E_{\mu}[V_i \mid \iota = i, d(\mathbf{b})]) \\ u^{l}(\mathbf{x}, d(\mathbf{b})) &= H(E_{\mu}[V_i \mid \iota \neq i, d(\mathbf{b})]) \end{aligned}$$

where H is a strictly increasing and differentiable function while $U(\mathbf{x})$ is standard from the Milgrom-Weber [17] set up. Our setting can easily accommodate the latter as long as one restricts attention to non-ascending auctions, as we do in this paper. Some results change qualitatively to the same extent that they would change even without reputational concerns: revenue equivalence between first- and second-price auctions for a given disclosure rule would no longer obtain. However, our main results concerning the relative rankings of disclosure rules with respect to seller revenue and achieving a PBNE remain unaffected.

If reputational effects do enter the bidder's utility function in a non-linear way then our results change more significantly. Here we focus on our revenue results. Details are available in appendix B, but we have that:

1. E auctions expected revenue dominate T auctions whenever H is strictly concave and vice-versa.
2. Relative rankings for T versus D, T versus S and E versus P continue to obtain.
3. P auctions expected revenue dominate D auctions whenever H is concave and V strictly increasing, while the reverse applies whenever H is convex and V strictly decreasing.

The first of these results is particularly interesting because it reveals that with a fairly standard specification of preferences, E auctions actually revenue dominate auctions where all bids are disclosed *even* when V is increasing. Also, the second and third set of results imply that certain rankings (but not all) can be generalized.

The intuition for all of this is that if H is concave, then it will amplify reputational incentives for low (resp. high) types relative to high (resp. low) types when V is increasing (resp. decreasing). Conversely if H is convex. Recall that in E auctions, low types face relatively higher reputational incentives than high types, while the opposite is true for D auctions, and something in between applies to T auctions. Compared to the linear case, when V is strictly increasing, concavity of H puts relatively more weight on the reputational incentives of low types, hence making E auctions generate higher revenues than T auctions, while the difference in revenue between the latter and D auctions is further increased. Again, conversely, if V is strictly decreasing. For S versus T auctions, it is still true that the reputational incentives for the latter will dominate for large enough N for the same reasons as in the linear case, although the shape of H might change the number of bidders at which this occurs. Finally, analyzing the effect of the shape of the H function on P auctions is more complicated. If N is small, the effect is similar to that on E auctions, while if N is large, the effect is similar to that on D.[22] In any case, these effects will leave the ranking between E and P unaffected while our comparison between P and D auctions still applies in the particularly salient case where V is strictly increasing and H is concave.

5. Conclusions

This paper is a step towards the study of auction theory when bidders have reputational concerns. We show how disclosure rules and not price mechanisms are crucial in this context and discuss the relative implications of using different disclosure rules for guaranteeing existence of symmetric and monotone equilibria and for maximizing the seller's expected revenue.

The analysis we have presented here is just a first step towards understanding the effects of reputational incentives on bidding behavior. Research in progress is concerned with extending the results presented here. First of all, external incentives distort bidding behavior in a world where the revenue equivalence theorem would otherwise apply. One would want to investigate how these incentives would affect revenue ranking results in the case of multi-object auctions and/or bidders who are asymmetrically informed initially. Also, one could ask how external incentives and disclosure rules affect the decision to participate in the auction. Furthermore, an interesting extension of our model would be to investigate the case where reputational returns depend on the relative reputation of the bidders (i.e. on the perceived types of other bidders as well). Finally, it would be interesting to investigate the case where bidders are agents for principals (say managers bidding on behalf of shareholders) and whether a) explicit incentives

[22] If $N = 2$, then P reduces to E, while as N increases the type of information disclosed by P is more and more similar to that disclosed by D as opposed to that disclosed by E.

can help to reduce the extent of the problem and b) credible communication between bidders and shareholders is possible.

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6. Proofs

Effective Valuation for P auctions We have:

$$\begin{aligned} v^{Pw}(y,z_i) &= T(y) \\ v^{Pl}(y,z_i) &= \int_{\underline{x}}^{z_i} V(z_i)dL(y_2|y) + \int_{z_i}^{\overline{x}} Z(y_2)\,dL(y_2|y) \end{aligned}$$

where

$$l(y_2|y) = \begin{cases} \frac{(N-2)F(y_2)^{N-3}f(y_2)}{F(y)^{N-2}} & if \quad \underline{x} \le y_2 \le y \\ 0 & if \quad \overline{x} \ge y_2 > y \end{cases}$$

$$L(y_2|y) = \begin{cases} \frac{F(y_2)^{N-2}}{F(y)^{N-2}} & if \quad \underline{x} \le y_2 \le y \\ 1 & if \quad \overline{x} \ge y_2 > y \end{cases}.$$

Given the above

$$\begin{aligned} v_z^{Pw}(y,z_i) &= 0 \\ v_z^{Pl}(y,z_i) &= L(z_i|y)\,V_x(z_i) + V(z_i)\,l(z_i|y) - Z(z_i)\,l(z_i|y) \end{aligned}$$

so that

$$\begin{aligned} \Psi^P(x_i,x_i) \equiv \psi^P(x_i) &= x_i + T(x_i) - V(x_i)L(x_i|x_i) - \int_{x_i}^{\overline{x}} Z(y_2)dL(y_2|x_i) \\ &+ \frac{V_x(x_i)}{g(x_i)}\int_{x_i}^{\overline{x}} L(x_i|t)\,dG(t) + \frac{[V(x_i)-Z(x_i)]}{g(x_i)}\int_{x_i}^{\overline{x}} l(x_i|t)\,dG(t) \end{aligned}$$

But

$$L(x_i|x_i) = 1$$
$$\text{and} \int_{x_i}^{\overline{x}} Z(y_2)dL(y_2|x_i) = 0.$$

Also

$$\begin{aligned} \int_{x_i}^{\overline{x}} L(x_i|t)\,dG(t) &= \int_{x_i}^{\overline{x}} \frac{F(x_i)^{N-2}}{F(t)^{N-2}} dF(t)^{N-1} \\ &= (N-1)\,F(x_i)^{N-2}(1-F(x_i)) \\ &= \frac{(1-F(x_i))\,g(x_i)}{f(x_i)} \end{aligned}$$

and

$$\begin{aligned} \int_{x_i}^{\overline{x}} l(x_i|t)\,dG(t) &= \int_{x_i}^{\overline{x}} \frac{(N-2)F(x_i)^{N-3}f(x_i)}{F(t)^{N-2}} dF(t)^{N-1} \\ &= (N-2)\frac{g(x_i)}{F(x_i)}(1-F(x_i)). \end{aligned}$$

Using these results we have

$$\psi^P(x_i) = x_i + T(x_i) - V(x_i) + \frac{[V_x(x_i) + (N-2)Z_x(x_i)]}{f(x_i)}(1 - F(x_i))$$

where we have used

$$Z_x(x_i) = \frac{f(x_i)}{F(x_i)}[V(x_i) - Z(x_i)]$$

from Lemma in the main text.■

Proof of Proposition 4:

I. This is trivial.

II and III

Recall that

$$\begin{aligned}
\psi^T(x) &= x + \frac{V_x(x)}{g(x)} \\
\psi^D(x) &= x + V(x) - Z(x) + G(x)\frac{V_x(x)}{g(x)} \\
\psi^E(x) &= x + T(x) - V(x) + (1 - G(x))\frac{V_x(x)}{g(x)}
\end{aligned}$$

Thus,

$$\begin{aligned}
\psi_x^D(x) &= 1 - G(x) + V_x(x) - Z_x(x) + g(x)\widetilde{\psi}^T(x) + G(x)\psi_x^T(x) \\
\psi_x^E(x) &= G(x) + T_x(x) - V_x(x) - g(x)\widetilde{\psi}^T(x) + (1 - G(x))\psi_x^T(x)
\end{aligned}$$

Clearly, then, if $V_x > 0$ (and hence $\widetilde{\psi}^T(x) > 0$) and $V_x(x) > Z_x(x)$, then $\psi_x^T(x) > 0$ implies that $\psi_x^D(x) > 0$. Moreover, if $\psi_x^E(x) > 0$ and $V_x(x) > T_x(x)$, then $\psi_x^T(x) > 0$. Similarly, if $V_x < 0$ (and hence $\widetilde{\psi}^T(x) < 0$) and $V_x(x) < T_x(x)$, then $\psi_x^T(x) > 0$ implies that $\psi_x^E(x) > 0$. Finally, if $\psi_x^D(x) > 0$ and $V_x(x) < Z_x(x)$, then $\psi_x^T(x) > 0$.

To complete the proof define first

$$\begin{aligned}
Z_V(\nu) &= E_{F_V}[V|V \leq \nu] \\
T_V(\nu) &= E_{F_V}[V|V \geq \nu],
\end{aligned}$$

and note that if $V_x > 0$ then $Z_V(\nu) = E[V(X) \mid X \leq V^{-1}(\nu)]$ and $T_V(\nu) = E[V(X) \mid X \geq V^{-1}(\nu)]$, while if $V_x < 0$ then $Z_V(\nu) = E[V(X) \mid X \geq V^{-1}(\nu)]$ and $T_V(\nu) = E[V(X) \mid X \leq$

$V^{-1}(\nu)]$. It follows, after recalling the Lemma, that (a) if $V_x > 0$, then $V_x(x) - Z_x(x)$ has the sign of $\frac{d}{d\nu}(\nu - Z_V(\nu))$, while $V_x(x) - T_x(x)$ has the sign of $\frac{d}{d\nu}(\nu - T_V(\nu))$, evaluated at $\nu = V(x)$ and (b) if $V_x < 0$, then $V_x(x) - Z_x(x)$ has the sign of $\frac{d}{d\nu}(T_V(\nu) - \nu)$, while $V_x(x) - T_x(x)$ has the sign of $\frac{d}{d\nu}(Z_V(\nu) - \nu)$, evaluated at $\nu = V(x)$. Finally, note that (a) $\frac{d}{d\nu}(\nu - Z_V(\nu))$ be positive is equivalent to the condition that $\int_{V(\underline{x})}^{\nu} F_V(s)\,ds$ is log-concave (Bagnoli and Bergstrom [1], Lemma 1), (b) $\frac{d}{d\nu}(\nu - T_V(\nu))$ be positive is equivalent to the condition that $\int_{\nu}^{V(\overline{x})} (1 - F_V(s))\,ds$ is log-concave (Bagnoli and Bergstrom [1], Lemma 2), and (c) $\int_{V(\underline{x})}^{\nu} F_V(s)\,ds$ and $\int_{\nu}^{V(\overline{x})} (1 - F_V(s))\,ds$ are log-concave whenever f_V is log-concave (Bagnoli and Bergstrom [1], Theorems 1 and 3).■

Proof of Proposition 5: Given our bidding functions are separable between a non-reputational component and a reputational component, and given that the former is the same across disclosure rules, we can restrict attention to the reputational component of expected revenue for each disclosure rule. This is defined as $\widetilde{ER}(\phi)$ and for T auctions we have

$$\widetilde{ER}(T) = N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \frac{V_x(y)}{g(y)} dG(y)\, dF(x_i),$$

for D auctions

$$\widetilde{ER}(D) = N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \left[V(y) - Z(y) + \frac{G(y)}{g(y)} V_x(y) \right] dG(y)\, dF(x_i),$$

for E auctions

$$\widetilde{ER}(E) = N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \left[T(y) - V(y) + \frac{1 - G(y)}{g(y)} V_x(y) \right] dG(y)\, dF(x_i),$$

for P auctions

$$\widetilde{ER}(P) = N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \left[T(y) - V(y) + \frac{1 - F(y)}{f(y)} \left(V_x(y) + (N-2) Z_x(y) \right) \right] dG(y)\, dF(x_i),$$

and, finally, for S auctions

$$\begin{aligned}
\widetilde{ER}(S) &= N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \int_{\underline{x}}^{\overline{x}} (T(y) - Z(y))\, dG(y)\, dG(y)\, dF(x_i) \\
&= \left(\int_{\underline{x}}^{\overline{x}} (T(y) - Z(y))\, dG(y) \right) \int_{\underline{x}}^{\overline{x}} N G(x_i)\, dF(x_i) \\
&= \int_{\underline{x}}^{\overline{x}} (T(y) - Z(y))\, dG(y) \int_{\underline{x}}^{\overline{x}} dF^N(x_i) \\
&= \int_{\underline{x}}^{\overline{x}} (T(y) - Z(y))\, dG(y).
\end{aligned}$$

I. We have that

$$\widetilde{ER}(T) - \widetilde{ER}(E) = N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \left[V(y) - T(y) + \frac{G(y)}{g(y)} V_x(y) \right] dG(y) \, dF(x_i)$$

We begin by noting that

$$\begin{aligned}
& \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \frac{G(y)}{g(y)} V_x(y) \, dG(y) \, dF(x_i) \\
= & \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \int_{\underline{x}}^{y} \frac{1}{g(y)} V_x(y) \, dG(s) \, dG(y) \, dF(x_i) \\
= & \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} \int_{s}^{x_i} V_x(y) \, dy dG(s) \, dF(x_i) \\
= & \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} [V(x_i) - V(s)] \, dG(s) \, dF(x_i) \\
= & \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} [V(x_i) - V(y)] \, dG(y) \, dF(x_i).
\end{aligned}$$

So,

$$\begin{aligned}
\widetilde{ER}(T) - \widetilde{ER}(E) &= N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x_i} [V(x_i) - T(y)] \, dG(y) \, dF(x_i) \\
= N \int_{\underline{x}}^{\overline{x}} V(x_i) \, G(x_i) \, dF(x_i) &- N \int_{\underline{x}}^{\overline{x}} \left(\frac{1}{1 - F(x_i)} \int_{x_i}^{\overline{x}} V(s) \, dF(s) \right) (1 - F(x_i)) \, dG(x_i) \\
= N \int_{\underline{x}}^{\overline{x}} V(x_i) \, G(x_i) \, dF(x_i) &- N \int_{\underline{x}}^{\overline{x}} \int_{x_i}^{\overline{x}} V(s) \, dF(s) \, dG(x_i) \\
= N \int_{\underline{x}}^{\overline{x}} V(x_i) \, G(x_i) \, dF(x_i) &- N \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{s} V(s) \, dG(x_i) \, dF(s) = 0
\end{aligned}$$

as desired.

II. We will need first to prove the following: if $V(\bullet)$ is strictly increasing or decreasing, then $T(x) - Z(x)$ has the opposite monotonicity of f_V. To prove this, note first from Jewitt [9] that $E_F[X|X \geq x] - E_F[X|X < x]$ has the opposite monotonicity of f_X. Note now that if $V(\bullet)$ is strictly increasing, with $\nu \equiv V(x)$, then

$$\begin{aligned}
& T(x) - Z(x) \\
= & E_F[V(X) | X \geq x] - E_F[V(X) | X \leq x] \\
= & E_F[V(X) | V(X) \geq \nu] - E_F[V(X) | V(X) \leq \nu] \\
= & E_{F_V}[V | V \geq \nu] - E_{F_V}[V | V \leq \nu] \\
\equiv & T_V(\nu) - Z_V(\nu)
\end{aligned}$$

Thus, $T(x) - Z(x)$ has the opposite monotonicity of f_V. Conversely, if $V(\bullet)$ is strictly decreasing then

$$T(x) - Z(x) = Z_V(\nu) - T_V(\nu)$$

but then

$$\frac{d\left[T(x) - Z(x)\right]}{dx} = \frac{d\left(Z_V(\nu) - T_V(\nu)\right)}{d\nu}\frac{d\nu}{dx}$$

Since $\frac{d\nu}{dx} < 0$ by assumption, we have then that the monotonicity of $T(x) - Z(x)$ has the same sign as the monotonicity of $T_V(\nu) - Z_V(\nu)$ and thus the opposite monotonicity of f_V.

Next, we compare expected revenues for disclosure rules $\phi = D$ versus $\phi = S$. We know from I. that $\widetilde{ER}(E) = \widetilde{ER}(T)$. This means that

$$\begin{aligned}
\widetilde{ER}(D) &= \widetilde{ER}(D) + \widetilde{ER}(E) - \widetilde{ER}(T) \\
&= N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[T(y) - Z(y)\right]dG(y)\,dF(x_i) \\
&= \int_{\underline{x}}^{\overline{x}}\left[T(y) - Z(y)\right]N(1 - F(y))\,dG(y) \\
&= \int_{\underline{x}}^{\overline{x}}\left[T(y) - Z(y)\right]dF_2^{(N)}(y),
\end{aligned}$$

while, recall that

$$\widetilde{ER}(S) = \int_{\underline{x}}^{\overline{x}}\left[T(y) - Z(y)\right]dG(y).$$

So, we have that

$$\begin{aligned}
\widetilde{ER}(D) - \widetilde{ER}(S) &= \int_{\underline{x}}^{\overline{x}}\left[T(y) - Z(y)\right]\left(f_2^{(N)}(y) - f_1^{(N-1)}(y)\right)dy \\
&= \int_{\underline{x}}^{\overline{x}}\left(F_1^{(N-1)}(y) - F_2^{(N)}(y)\right)\left(T_x(y) - Z_x(y)\right)dy.
\end{aligned}$$

But

$$\begin{aligned}
& F_1^{(N-1)}(y) - F_2^{(N)}(y) \\
&= F^{N-1}(y) - NF^{N-1}(y) + (N-1)F^N(y) \\
&= (N-1)\left(F^N(y) - F^{N-1}(y)\right) < 0 \text{ a.e.}
\end{aligned}$$

So, we have, after recalling our result above on the properties of $T_x(y) - Z_x(y)$ that

$$\widetilde{ER}(D) - \widetilde{ER}(S) \begin{array}{lll} > 0 & if & f_V \text{ increasing} \\ = 0 & if & f_V \text{ uniform} \\ < 0 & if & f_V \text{ decreasing} \end{array}.$$

IIIa and IVa.

We provide the proof by comparing $E_G\left[\psi^{\phi}(Y)|Y<x_i\right]$ across for the relevant rules for T vs D and for E vs. P. This establishes that for FP auctions with V strictly increasing, T and E provide higher revenues than D and P respectively. For the comparison between P and D, on the other hand, our result only applies to expected revenue.

We begin with the comparison between T and D.

$$\begin{aligned} & E_G\left[\psi^T(Y)|Y<x_i\right]-E_G\left[\psi^D(Y)|Y<x_i\right] \\ = & \frac{1}{G(x_i)}\int_{\underline{x}}^{x_i}\left[Z(y)-V(y)+\frac{1-G(y)}{g(y)}V_x(y)\right]dG(y). \end{aligned}$$

But

$$\begin{aligned} & \int_{\underline{x}}^{x_i}\frac{1-G(y)}{g(y)}V_x(y)\,dG(y) \\ = & \int_{\underline{x}}^{x_i}\int_y^{\overline{x}}\frac{1}{g(y)}V_x(y)\,dG(s)\,dG(y) \\ = & \int_{\underline{x}}^{x_i}\int_{\underline{x}}^{s}V_x(y)\,dydG(s)+\int_{x_i}^{\overline{x}}\int_{\underline{x}}^{x_i}V_x(y)\,dydG(s) \\ = & \int_{\underline{x}}^{x_i}\left[V(s)-V(\underline{x})\right]dG(s)+\int_{x_i}^{\overline{x}}\left[V(x_i)-V(\underline{x})\right]dG(s) \\ = & \int_{\underline{x}}^{x_i}V(y)\,dG(y)+(1-G(x_i))\,V(x_i)-V(\underline{x}), \end{aligned}$$

and so we have

$$\begin{aligned} & \int_{\underline{x}}^{x_i}\left[Z(y)-V(y)+\frac{1-G(y)}{g(y)}V_x(y)\right]dG(y) \\ = & \int_{\underline{x}}^{x_i}Z(y)\,dG(y)+\int_{x_i}^{\overline{x}}V(x_i)\,dG(y)-V(\underline{x}). \end{aligned}$$

Clearly, if $V(\bullet)$ is strictly increasing then $V(x_i)>V(\underline{x})$ and $Z(y)>V(\underline{x})$ for any $x_i,y>\underline{x}$, while if $V(\bullet)$ is strictly decreasing then $V(\underline{x})>V(x_i)$ and $V(\underline{x})>Z(y)$ for any $x_i,y>\underline{x}$, and the result follows directly.

Now consider the comparison between E and P.

$$\begin{aligned} & E_G\left[\psi^E(Y)|Y<x_i\right]-E_G\left[\psi^P(Y)|Y<x_i\right] \\ = & \frac{1}{G(x_i)}\int_{\underline{x}}^{x_i}\left[\frac{1-G(y)}{g(y)}V_x(y)-\frac{1-F(y)}{f(y)}\left[V_x(y)+(N-2)Z_x(y)\right]\right]dG(y). \end{aligned}$$

We already know that

$$\int_{\underline{x}}^{x_i} \frac{1-G(y)}{g(y)} V_x(y)\, dG(y) = \int_{\underline{x}}^{x_i} V(y)\, dG(y) + (1-G(x_i))\, V(x_i) - V(\underline{x})$$

Now, for $N > 2$ we have,

$$\begin{aligned}
& \int_{\underline{x}}^{x_i} \frac{1-F(y)}{f(y)} V_x(y)\, dG(y) \\
= \;& \int_{\underline{x}}^{x_i} \int_{y}^{\overline{x}} \frac{1}{f(y)} V_x(y)\, dF(s)\, dG(y) \\
= \;& (N-1)\left[\int_{\underline{x}}^{x_i} \int_{\underline{x}}^{s} V_x(y)\, F(y)^{N-2}\, dy dF(s) + \int_{x_i}^{\overline{x}} \int_{\underline{x}}^{x_i} V_x(y)\, F(y)^{N-2}\, dy dF(s)\right] \\
= \;& (N-1)\left[\begin{array}{c} \int_{\underline{x}}^{x_i}\left[F(s)^{N-2} V(s) - \int_{\underline{x}}^{s} V(y)\, dF(y)^{N-2}\right] dF(s) \\ + \int_{x_i}^{\overline{x}}\left[F(x_i)^{N-2} V(x_i) - \int_{\underline{x}}^{x_i} V(y)\, dF(y)^{N-2}\right] dF(s) \end{array}\right] \\
= \;& \int_{\underline{x}}^{x_i} V(y)\, dG(y) + (N-1)(1-F(x_i))\left[F(x_i)^{N-2} V(x_i) - \int_{\underline{x}}^{x_i} V(y)\, dF(y)^{N-2}\right] \\
& -(N-1)\int_{\underline{x}}^{x_i} \int_{\underline{x}}^{s} V(y)\, dF(y)^{N-2}\, dF(s) \\
= \;& \int_{\underline{x}}^{x_i} V(y)\, dG(y) + (N-1)(1-F(x_i))\left[F(x_i)^{N-2} V(x_i) - \int_{\underline{x}}^{x_i} V(y)\, dF(y)^{N-2}\right] \\
& -(N-1)\int_{\underline{x}}^{x_i} V(y)\,(F(x_i) - F(y))\, dF(y)^{N-2} \\
= \;& \int_{\underline{x}}^{x_i} V(y)\, dG(y) + (N-1)\left[(1-F(x_i))\, F(x_i)^{N-2} V(x_i) - \int_{\underline{x}}^{x_i} V(y)\,(1-F(y))\, dF(y)^{N-2}\right]
\end{aligned}$$

Finally, from the Lemma,

$$\begin{aligned}
& \int_{\underline{x}}^{x_i} (N-2)\, Z_x(y)\, \frac{1-F(y)}{f(y)} dG(y) \\
= \;& \int_{\underline{x}}^{x_i} (N-2)\,[V(y) - Z(y)]\, \frac{1-F(y)}{F(y)} dG(y) \\
= \;& (N-1)\int_{\underline{x}}^{x_i} [V(y) - Z(y)]\,(1-F(y))\, dF(y)^{N-2}
\end{aligned}$$

which gives us

$$\int_{\underline{x}}^{x_i}\left[\frac{1-G(y)}{g(y)}V_x(y)-\frac{1-F(y)}{f(y)}\left[V_x(y)+(N-2)Z_x(y)\right]\right]dG(y)$$

$$= (1-G(x_i))V(x_i)-(N-1)\left((1-F(x_i))V(x_i)F(x_i)^{N-2}-\int_{\underline{x}}^{x_i}Z(y)(1-F(y))dF(y)^{N-2}\right)-V(\underline{x})$$

$$= V(x_i)\left(1-F(x_i)^{N-1}\right)-V(x_i)\left(F_2^{(N-1)}(x_i)-F(x_i)^{N-1}\right)+\int_{\underline{x}}^{x_i}Z(y)\,dF_2^{(N-1)}(y)-V(\underline{x})$$

$$= V(x_i)\left(1-F_2^{(N-1)}(x_i)\right)+\int_{\underline{x}}^{x_i}Z(y)\,dF_2^{(N-1)}(y)-V(\underline{x})$$

$$= \int_{\underline{x}}^{x_i}Z(y)\,dF_2^{(N-1)}(y)+\int_{x_i}^{\overline{x}}V(x_i)\,dF_2^{(N-1)}(y)-V(\underline{x})$$

The above is positive for $V_x > 0$ since then $V(x) > V(\underline{x})$ and $Z(x) > V(\underline{x})$ for any $x > \underline{x}$. If $V_x < 0$ the above is now negative since $V(x) < V(\underline{x})$ and $Z(x) < V(\underline{x})$ for any $x > \underline{x}$.

Finally, we focus on the comparison between P and D. Recall that

$$\widetilde{ER}(D) = N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[T(y)-Z(y)\right]dG(y)\,dF(x_i),$$

from II above.

Now,

$$\widetilde{ER}(P)-\widetilde{ER}(D) = N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[Z(y)-V(y)+\frac{1-F(y)}{f(y)}\left(V_x(y)+(N-2)Z_x(y)\right)\right]dG(y)\,dF(x_i)$$

Using from earlier that

$$\int_{\underline{x}}^{x_i}\frac{1-F(y)}{f(y)}V_x(y)\,dG(y) =$$

$$\int_{\underline{x}}^{x_i}V(y)\,dG(y)+(N-1)\left[(1-F(x_i))F(x_i)^{N-2}V(x_i)-\int_{\underline{x}}^{x_i}V(y)(1-F(y))\,dF(y)^{N-2}\right],$$

we can manipulate the term

$$N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[Z(y)-V(y)+\frac{1-F(y)}{f(y)}V_x(y)\right]dG(y)\,dF(x_i)$$

into

$$N(N-1)\{\int_{\underline{x}}^{\overline{x}}(1-F(x_i))V(x_i)F(x_i)^{N-2}dF(x_i)$$

$$+\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[Z(y)F^{N-2}(y)-(N-2)V(y)(1-F(y))F(y)^{N-3}\right]dF(y)\,dF(x_i)\}$$

$$=\int_{\underline{x}}^{\overline{x}}V(x_i)\,dF_2^{(N)}(x_i)$$

$$+N(N-1)\int_{\underline{x}}^{\overline{x}}(1-F(y))\left[Z(y)F^{N-2}(y)-(N-2)V(y)(1-F(y))F(y)^{N-3}\right]dF(y)$$

$$=\int_{\underline{x}}^{\overline{x}}[V(s)+Z(s)]\,dF_2^{(N)}(s)-2\int_{\underline{x}}^{\overline{x}}V(s)\,dF_3^{(N)}(s),$$

where the last equality follows from noting that

$$\begin{aligned} f_3^{(N)}(x) &= \frac{N!}{(N-3)!2!}F(x)^{N-3}(1-F(x))^2 f(x) \\ &= \frac{N(N-1)(N-2)}{2}F(x)^{N-3}(1-F(x))^2 f(x) \end{aligned}$$

Now, we focus on the remaining component of expected revenue:

$$N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[\frac{1-F(y)}{f(y)}(N-2)Z_x(y)\right]dG(y)\,dF(x_i)=$$

$$N\int_{\underline{x}}^{\overline{x}}\left[\frac{(1-F(y))^2}{f(y)}(N-2)Z_x(y)\right]dG(y)=$$

$$=\int_{\underline{x}}^{\overline{x}}N(N-1)(N-2)F(s)^{N-3}(1-F(s))^2 f(s)[V(s)-Z(s)]\,ds$$

$$=2\int_{\underline{x}}^{\overline{x}}[V(s)-Z(s)]\,dF_3^{(N)}(s),$$

where for the second equality we use the Lemma. Thus,

$$\widetilde{ER}(P)-\widetilde{ER}(D)$$

$$=\int_{\underline{x}}^{\overline{x}}[V(s)+Z(s)]\,dF_2^{(N)}(s)-2\int_{\underline{x}}^{\overline{x}}Z(s)\,dF_3^{(N)}(s)$$

$$>2\int_{\underline{x}}^{\overline{x}}Z(s)\,dF_2^{(N)}(s)-2\int_{\underline{x}}^{\overline{x}}Z(s)\,dF_3^{(N)}(s)$$

$$=2\int_{\underline{x}}^{\overline{x}}\left(F_3^{(N)}(s)-F_2^{(N)}(s)\right)Z_x(s)\,ds>0$$

with the first inequality above being true if $V_x>0$ and hence (from the Lemma) $Z_x>0$. The last equality follows from integration by parts. The argument is symmetric if $V_x<0$

IIIb. and IVb. We know that

$$\widetilde{ER}(T) = N \int_{\underline{x}}^{\overline{x}} [V(x_i) - V(\underline{x})] \, dF(x_i).$$

This is clearly positive and unboundedly increasing in N, if $V(\bullet)$ is strictly increasing. Conversely, if $V(\bullet)$ is strictly decreasing. Also,

$$\widetilde{ER}(S) = \int_{\underline{x}}^{\overline{x}} [T(y) - Z(y)] \, dF^{N-1}(y).$$

If $V(\bullet)$ is strictly increasing, then $T(y) > Z(y)$ almost everywhere, and hence $\widetilde{ER}(S)$ is bounded from above by $\max_y\{T(y) - Z(y)\}$. Conversely, if $V(\bullet)$ is strictly decreasing $\widetilde{ER}(S)$ is bounded from below by $\min_y\{T(y) - Z(y)\}$. The proof follows immediately.■

7. Figure

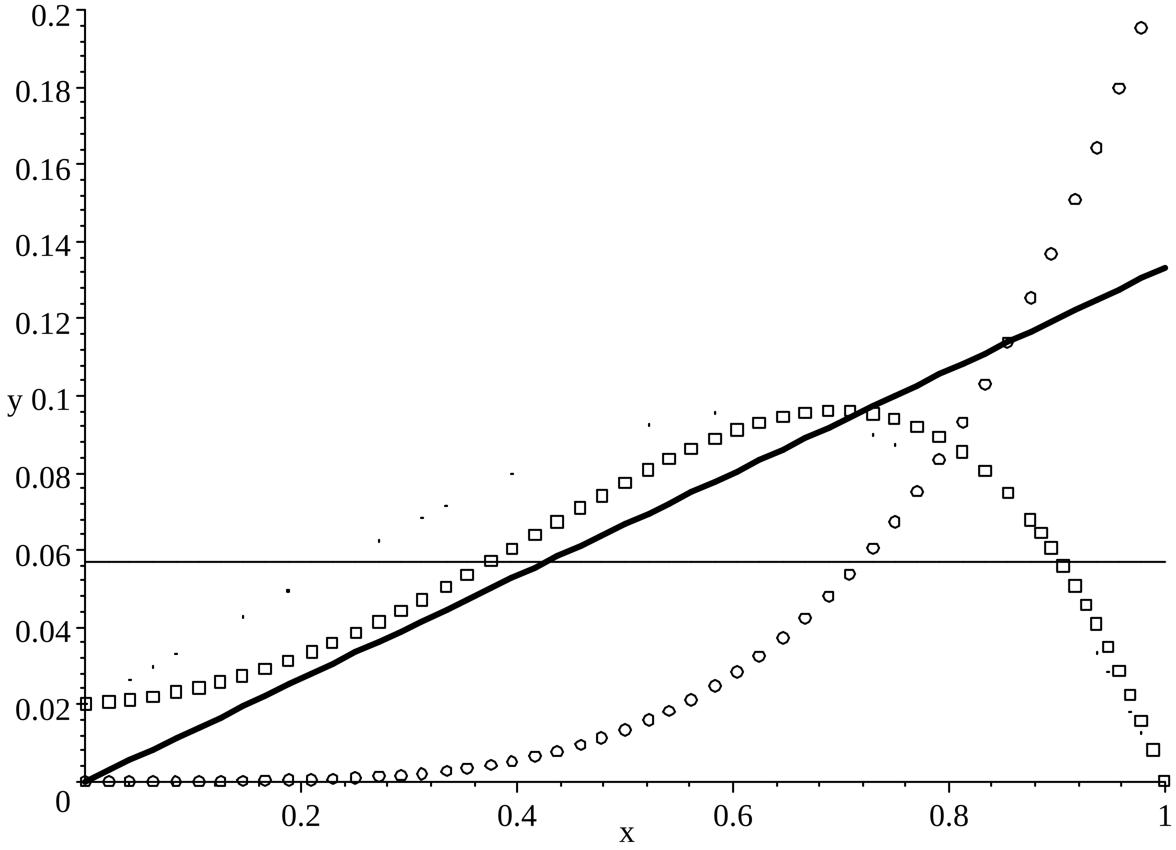

Figure. $\widetilde{\psi}^{\phi}(x)$ for our parametrization. The solid thick line is the case $\phi = T$, the solid thin line is $\phi = S$, the circles represent $\phi = D$, the dots $\phi = E$ and the boxes $\phi = P$.

8. Appendix B

8.1. A Signaling Model

There are several ways to think of our setup as a signaling game with the auction as the signaling device. One way, which will be our focus, is that of $N \geq 2$ parallel sender-receiver pairs. In these parallel games, the senders use a standard sealed-bid auction for a single, indivisible object, with a given disclosure rule ϕ, as a costly signaling device. Crucially, however, depending on the disclosure rule, the signal each receiver gets may not just depend only on her sender's bid (recall our discussion of Dutch auctions in Sections 1 and 2).[23]

So, we have a game between N bidders and N receivers, where each and every one bidder is affected by one and only one receiver. We refer to such receiver as "the relevant receiver" (she). Bidder i (he) provides his relevant receiver with some "fixed services", which are not modelled here for expositional clarity. As a return for these services, bidder i receives from his receiver a "wage" or "return/transfer" $\nu_i \in V$. This interaction between bidders and their receivers is superceded by a sealed-bid auction for a single, indivisible object that is distinguished by a disclosure rule ϕ. Payoffs for bidder i, *gross of the price paid* in the auction, are defined as follows:

$$
\begin{aligned}
U^w(x_i, \nu_i) &= x_i + \nu_i \\
U^l(\nu_i) &= \nu_i
\end{aligned}
$$

where, recall, the superscript $o = w, l$ captures the fact that payoffs depend on whether a bidder has won or not the auction.

As for receivers, we assume that the relevant receiver for bidder i has a utility function $\varpi(\nu_i, \theta_i)$.[24] Thus, her action choice only depends on her beliefs about some characteristic θ_i of bidder i. This characteristic can be interpreted as the "quality" of the services the bidder provides

[23] Another possibility, at the other extreme, is that of N senders and just one receiver. As long as, conditional on the receiver knowing the senders' types, the receiver's action that affects a particular sender's utility only depends on that sender's type, our analysis goes through. In a similar manner, we can also think of N senders and $1 < K < N$ receivers, with every receiver interacting separately with at least one and no more than $N - K + 1$ senders. The forthcoming discussion can easily be adjusted, with the cost of some more notation, to capture these cases.

[24] The fact that in this formulation, the relevant receiver only cares about ν_i is for simplicity of notation. We could easily have allowed for more general formulations. What matters is that her choice of ν_i only depends on the private characteristic θ_i of bidder i.

to his receiver. We allow for the case in which $\theta_i = x_i$, of course, but we also wish to allow for the case in which the *auction-relevant* type x_i simply functions as a signal on i's characteristic θ_i that the relevant receiver really cares about. In particular, for the case in which $\theta_i \neq x_i$, we assume that for all i, the θ_i's are (conditionally) i.i.d. Moreover, it is common knowledge that θ_i and x_i are related according to the common cdf $T(\theta_i \mid x_i)$, which is assumed to be twice continuously differentiable with associated density $t(\theta_i \mid x_i)$ and bounded derivatives. We define with

$$V(x_i) \equiv \arg\max_{\nu_i \in V} E_T \left[\varpi\left(\nu_i, \Theta_i\right) \mid x_i\right]$$

the (assumed to be) unique choice of action for i's relevant receiver if she knew x_i.

Assume that $\varpi\left(\nu_i, \theta_i\right)$ is twice continuously differentiable, bounded and with bounded derivatives. Note, then, by the properties of $T(\theta_i \mid x_i)$, that $V(x_i)$ is twice continuously differentiable, bounded and with bounded derivatives. We also assume that the receiver's preferences are such that she has a unique optimal action for any given beliefs she might have over the true value of x_i. Let μ be a cdf that represents the beliefs about x_i of bidder-$i's$ relevant receiver.

An important point to note here is that we are imposing a restriction on the form of utility function $\varpi\left(\nu_i, \theta_i\right)$ by capturing reputational returns with

$$E_\mu \left[V(X_i)\right]$$

rather than with the more general

$$\arg\max_{\nu_i \in V} E_\mu \left[E_T \left[\varpi\left(\nu_i, \Theta_i\right) \mid X_i\right]\right]$$

This restrictions, however, is compatible with many natural examples. If, for instance, $\theta_i = x_i$, and the relevant receiver's payoff from interacting with bidder i is $-(\nu_i - x_i)^2$ then

$$E_\mu \left[V(X_i)\right] = E_\mu(X_i) = \arg\max_{\nu_i \in V} E_\mu \left[-(\nu_i - X_i)^2\right].$$

As an example of the above setup, consider bidders as firms attempting a takeover and the receiver as a potential investor. In this case, θ_i is the underlying attribute for bidder/firm i that the investor cares about such as managerial efficiency. Moreover, ν_i is the level of investment and $T(\theta_i \mid x_i)$ could be the investor's beliefs over bidder i's attribute given that i's user-value for the acquisition is x_i. $V(x_i)$ would be a strictly increasing function whenever the investor perceives a high willingness to pay for the firm for sale as a signal of high managerial ability. This would arise if the higher the acquisition's profitability, the higher the managerial efficiency is likely to be. Conversely, for a strictly decreasing $V(x_i)$.

8.2. Additional Proofs

Proof of Proposition 1

We save on notation by omitting the subscript i throughout. Note first, that given that $\psi^{\phi}(x)$ is strictly increasing, $\beta^{SP-\phi}(x) = \psi^{\phi}(x)$ as well as $\beta^{FP-\phi}(x) = E_G\left[\beta^{SP-\phi}(Y)\,|Y < x\right]$ are strictly increasing in x.

Also, define

$$\overline{\Psi}^{\phi}(x, y, z) = x + v^{\phi w}(y, z) - v^{\phi l}(y, z)$$

and note that

$$W^{m-\phi}(x, z) \equiv \int_{\underline{x}}^{z} \left[\overline{\Psi}^{\phi}(x, y, z) - p^{m-\phi}(y, z)\right] dG(y) + \int_{\underline{x}}^{\overline{x}} v^{\phi l}(y, z) dG(y)$$

is the expected profit of type x from bidding $\beta^{m-\phi}(z)$ where $p^{FP-\phi}(y, z) = \beta^{FP-\phi}(z)$ is the price in a first-price auction and $p^{SP-\phi}(y, z) = \beta^{SP-\phi}(y)$ is the price in a second-price auction. We begin our proof with first-price auctions

Part (a): FP auctions

Suppose now that a symmetric and monotone equilibrium β exists. Note then that in a such an equilibrium

$$\int_{\underline{x}}^{\beta^{-1}(b)} \left[\overline{\Psi}^{\phi}\left(x, y, \beta^{-1}(b)\right) - b\right] dG(y) + \int_{\underline{x}}^{\overline{x}} v^{\phi l}(y, \beta^{-1}(b)) dG(y)$$

is the expected profit of type x from bidding $b \geq 0$, with, by assumption A, $\beta^{-1}(b) \equiv \overline{x}$ if $b \geq \beta(\overline{x})$, $\beta^{-1}(b) \equiv \underline{x}$ if $0 \leq b \leq \beta(\underline{x})$ and $\beta^{-1}(b) \equiv \widehat{x}$ if $b \in \left(\lim_{x_i \to \widehat{x}^-} \beta^{m-\phi}(x_i), \lim_{x_i \to \widehat{x}^+} \beta^{m-\phi}(x_i)\right)$. Moreover, β, being strictly increasing, is almost everywhere differentiable. The first-order condition (FOC) for a maximum of the expected profit of type x is (except in points of non-differentiability of $\beta(.)$)

$$\begin{aligned}&\{(\overline{\Psi}^{\phi}(x,\beta^{-1}(b),\beta^{-1}(b))-b)g(\beta^{-1}(b))+\\&+\int_{\underline{x}}^{\beta^{-1}(b)}v_z^{\phi w}(y,\beta^{-1}(b))dG(y)+\int_{\beta^{-1}(b)}^{\overline{x}}v_z^{\phi l}(y,\beta^{-1}(b))dG(y)\}\frac{1}{\beta'(\beta^{-1}(b))}\\=\;&G(\beta^{-1}(b)).\end{aligned}$$

So, if β is a symmetric and monotone equilibrium, then it must be that $b=\beta(x)$, with $\beta(x)>0$ for any $x>\underline{x}$, and hence

$$\begin{aligned}&[\overline{\Psi}^{\phi}(x,x,x)-\beta(x)]g(x)+\\&+\int_{\underline{x}}^{x}v_z^{\phi w}(y,x)dG(y)+\int_{x}^{\overline{x}}v_z^{\phi l}(y,x)dG(y)\\=\;&\beta'(x)G(x)\end{aligned}$$

almost everywhere in $x\in(\underline{x},\overline{x}]$.

One can easily see that if β is a symmetric and monotone equilibrium, then it must be continuous: if $\hat{x}$ was a jump point then bidding $lim_{x\to\hat{x}^-}\beta(x)$ is preferred to bidding $lim_{x\to\hat{x}^+}\beta(x)$ by bidder of type $\hat{x}$ (resp. $\hat{x}+\varepsilon$, where ε is arbitrarily small) when $lim_{x\to\hat{x}^+}\beta(x)=\beta(\hat{x})$ (resp. when $lim_{x\to\hat{x}^-}\beta(x)=\beta(\hat{x})$); such deviation does not have an effect on the auction's outcome and the reputational return, but leads to lower price upon winning. Note also that in any symmetric and monotone equilibrium, $\beta(\underline{x})G(\underline{x})=0$. Continutity of β, and hence $\beta(x)G(x)$, implies, therefore, that the differential equation

$$\overline{\Psi}^{\phi}(x,x,x)g(x)+\int_{\underline{x}}^{x}v_z^{\phi w}(y,x)dG(y)+\int_{x}^{\overline{x}}v_z^{\phi l}(y,x)dG(y)=\frac{d[\beta(x)G(x)]}{dx},\ x\in(\underline{x},\overline{x}]$$

with the boundary condition $\beta(\underline{x})G(\underline{x})=0$ has unique, for any $x\in(\underline{x},\overline{x}]$, solution the proposed equilibrium, $\beta^{FP-\phi}$.[25]

---

[25] Note

$$\lim_{x\to\underline{x}^+}E_G[\psi^{\phi}(Y)\mid Y\le x]=\psi^{\phi}(\underline{x})$$

Also, note that if $g(\underline{x})>0$ (which is necessarily the case when $N=2$), then it must be that

$$\beta(\underline{x})\ge\psi^{\phi}(\underline{x}).$$

To see this note first that a bid of $\beta(\underline{x})$ loses the auction with certainty and ensures a payoff of

$$\int_{\underline{x}}^{\overline{x}}v^{\phi l}(y,\underline{x})dG(y)$$

But, if $\beta(\underline{x})<\psi^{\phi}(\underline{x})$, then by deviating to $\beta(\underline{x})+\varepsilon$, the bidder with type $x=\underline{x}$ would win with probability

It remains thus to show that $\beta^{FP-\phi}$ is indeed an equilibrium. To this end, note first that, given that competitors deploy $\beta^{FP-\phi}$, any bidder is indifferent over any bid weakly lower than $\beta^{FP-\phi}(\underline{x})$. Also, any bidder strictly prefers $\beta^{FP-\phi}(\overline{x})$ to any higher bid. Recall that $W^{FP-\phi}(x,z)$ is the expected profit of type x from bidding $\beta^{FP-\phi}(z)$. We have that

$$W_2^{FP-\phi}(x,z)=[\overline{\Psi}^{\phi}(x,z,z)-\beta^{FP-\phi}(z)]g(z)-\frac{d\beta^{FP-\phi}(z)}{dz}G(z)+\int_{\underline{x}}^{z}v_z^{\phi w}(y,z)dG(y)+\int_{z}^{\overline{x}}v_z^{\phi l}(y,z)dG(y).$$

So,

$$\begin{aligned}&W_2^{FP-\phi}(x,z)-W_2^{FP-\phi}(z,z)\\ =\;& g(z)[\overline{\Psi}^{\phi}(x,z,z)-\overline{\Psi}^{\phi}(z,z,z)].\end{aligned}$$

Given that $\overline{\Psi}^{\phi}(x,z,z)$ is strictly increasing in x, we have that if $z<x$ then $W_2^{FP-\phi}(x,z)>W_2^{FP-\phi}(z,z)$, and vice versa. That is, $W_2^{FP-\phi}(x,z)$ is increasing in x. Note also that $\beta^{FP-\phi}$ satisfies $W_2^{FP-\phi}(z,z)=0$ for any $z\in(\underline{x},\overline{x}]$. These, in turn, imply that for any z and x such that $\underline{x}<z<x\leq\overline{x}$ we have $W_2^{FP-\phi}(x,z)>0$, while for any z and x such that $\underline{x}\leq x<z\leq\overline{x}$ we have $W_2^{FP-\phi}(x,z)<0$. Note also that for any $x>\underline{x}$ bidding $\beta^{FP-\phi}(\underline{x})$ is not optimal. To see this, note first that from the above we have that $W_2^{FP-\phi}(x,z)\geq 0$ for any $z=\underline{x}+\varepsilon$ where ε is arbitrarily small. So, it will be enough to show that $W^{FP-\phi}(x,\underline{x})\leq\lim_{z\to\underline{x}^+}W^{FP-\phi}(x,z)$. This holds as an equality by continuity of $v^{\phi l}(y,z)$.

$G(\beta^{-1}(\beta(\underline{x})+\varepsilon))>0$ and attain a payoff

$$\begin{aligned}&\int_{\underline{x}}^{\overline{x}}v^{\phi l}(y,\beta^{-1}(\beta(\underline{x})+\varepsilon))dG(y)\\ &+\int_{\underline{x}}^{\beta^{-1}(\beta(\underline{x})+\varepsilon)}(\overline{\Psi}^{\phi}(\underline{x},y,\beta^{-1}(\beta(\underline{x})+\varepsilon))-\beta(\underline{x})-\varepsilon)dG(y)\end{aligned}$$

For small ε this equals

$$\begin{aligned}&\int_{\underline{x}}^{\overline{x}}v^{\phi l}(y,\underline{x})dG(y)+\frac{1}{\beta'(\beta(\underline{x}))}\varepsilon\int_{\underline{x}}^{\overline{x}}v_z^{\phi l}(y,\underline{x})dG(y)\\ &+\frac{1}{\beta'(\beta(\underline{x}))}\varepsilon g(\underline{x})\left[\overline{\Psi}^{\phi}(\underline{x},\underline{x},\underline{x})-\beta(\underline{x})\right]\\ =\;&\int_{\underline{x}}^{\overline{x}}v^{\phi l}(y,\underline{x})dG(y)+\frac{1}{\beta'(\beta(\underline{x}))}\varepsilon g(\underline{x})\left[\psi^{\phi}(\underline{x})-\beta(\underline{x})\right]\\ >\;&\int_{\underline{x}}^{\overline{x}}v^{\phi l}(y,\underline{x})dG(y),\end{aligned}$$

So, $\beta(\underline{x})\geq\psi^{\phi}(\underline{x})$. Monotonicity on the other hand requires $\beta(\underline{x})\leq\psi^{\phi}(\underline{x})$. Thus, when $g(\underline{x})>0$, in a symmetric and monotone equilibrium

$$\beta^{FP-\phi}(\underline{x})=\psi^{\phi}(\underline{x}).$$

Clearly, also, if $g(\underline{x})=0$ (which is necessarily the case if $N>2$ and, due to the requirement imposed by assumption B, $V_x(\underline{x})=0$), then we have that any non-negative bid lower than or equal to $\psi^{\phi}(\underline{x})$ is consistent with PBNE.

Thus, $z = x$ is indeed indeed a global maximum of $W^{FP-\phi}(x,z)$, for any $x \in [\underline{x}, \overline{x}]$, given that competitors deploy $\beta^{FP-\phi}$. Thus, $\beta^{FP-\phi}$ is an equilibrium.

Part (b): SP auctions

Note that if $\beta(.)$ is a symmetric and monotone equilibrium then the expected profit of type x from bidding $b \geq 0$ in such an equilibrium is

$$\int_{\underline{x}}^{\beta^{-1}(b)} [\overline{\Psi}^{\phi}(x, y, \beta^{-1}(b)) - \beta(y)]dG(y) + \int_{\underline{x}}^{\overline{x}} v^{\phi l}(y, \beta^{-1}(b))dG(y)$$

with, by assumption A, $\beta^{-1}(b) = \overline{x}$ if $b \geq \beta(\overline{x})$, $\beta^{-1}(b) = \underline{x}$ if $0 \leq b \leq \beta(\underline{x})$, and, finally, $\beta^{-1}(b) \equiv \widehat{x}$ if $b \in \left(\lim_{x_i \to \widehat{x}^-} \beta^{m-\phi}(x_i), \lim_{x_i \to \widehat{x}^+} \beta^{m-\phi}(x_i)\right)$. The first-order condition (FOC) for a maximum of it is (except in points of non-differentiability of $\beta(.)$)

$$\begin{aligned} & \{(\overline{\Psi}^{\phi}(x, \beta^{-1}(b), \beta^{-1}(b)) - \beta(\beta^{-1}(b)))g(\beta^{-1}(b)) + \\ & + \int_{\underline{x}}^{\beta^{-1}(b)} v_z^{\phi w}(y, \beta^{-1}(b))dG(y) + \int_{\beta^{-1}(b)}^{\overline{x}} v_z^{\phi l}(y, \beta^{-1}(b))dG(y)\} \frac{1}{\beta'(\beta^{-1}(b))} \\ = \ & 0 \end{aligned}$$

If β is a symmetric and monotone equilibrium then it must be that $b = \beta(x)$, with $\beta(x) > 0$ for any $x > \underline{x}$, and thereby,

$$\begin{aligned} & [\overline{\Psi}^{\phi}(x, x, x) - \beta(x)]g(x) + \\ & + \int_{\underline{x}}^{x} v_z^{\phi w}(y, x)dG(y) + \int_{x}^{\overline{x}} v_z^{\phi l}(y, x)dG(y) = 0, \end{aligned}$$

almost everywhere, which implies $\beta(x) = \psi^{\phi}(x)$, for $x \in (\underline{x}, \overline{x}]$. Note now that monotonicity requires also that $\beta(\underline{x}) \leq \psi^{\phi}(\underline{x})$. In addition, a necessary condition for $\beta(\underline{x})$ to be part of a symmetric and monotone equilibrium is that

$$\begin{aligned} & [\overline{\Psi}^{\phi}(\underline{x}, \underline{x}, \underline{x}) - \beta(\underline{x})]g(\underline{x}) + \\ & + \int_{\underline{x}}^{\overline{x}} v_z^{\phi l}(y, \underline{x})dG(y) \leq 0 \implies \beta(\underline{x}) \geq \psi^{\phi}(\underline{x}). \end{aligned}$$

Thus, by continuity of $\psi^{\phi}(x)$, if β is a symmetric and monotone equilibrium then it must be given by $\beta^{SP-\phi}(x)$ for any $x \in [\underline{x}, \overline{x}]$.

Next we show that $\beta^{SP-\phi}$ is indeed an equilibrium. To this end, note, first, that, given that competitors deploy $\beta^{SP-\phi}$, any bidder is indifferent over any bid weakly lower than $\beta^{SP-\phi}(\underline{x})$. Also, any bidder is indifferent over any bid weakly higher than $\beta^{SP-\phi}(\overline{x})$. Recall that $W^{SP-\phi}(x,z)$ is the expected profit of type x from bidding $\beta^{SP-\phi}(z)$. We have that

$$W_2^{SP-\phi}(x,z) = [\overline{\Psi}^{\phi}(x,z,z) - \beta^{SP-\phi}(z)]g(z) + \int_{\underline{x}}^{z} v_z^{\phi w}(y,z)dG(y) + \int_{z}^{\overline{x}} v_z^{\phi l}(y,z)dG(y)$$

So,

$$\begin{aligned} & W_2^{SP-\phi}(x,z) - W_2^{SP-\phi}(z,z) \\ = \; & g(z)[\overline{\Psi}^{\phi}(x,z,z) - \overline{\Psi}^{\phi}(z,z,z)]. \end{aligned}$$

Given that $\overline{\Psi}^{\phi}(x,z,z)$ is strictly increasing in x, we have that if $z < x$ then $W_2^{SP-\phi}(x,z) > W_2^{SP-\phi}(z,z)$, and vice versa. That is, $W_2^{SP-\phi}(x,z)$ is strictly increasing in x. Note also that $\beta^{SP-\phi}$ satisfies $W_2^{SP-\phi}(z,z) = 0$ for any $z \in (\underline{x},\overline{x}]$. These, in turn, imply that for any z and x such that $\underline{x} < z < x \leq \overline{x}$ we have $W_2^{SP-\phi}(x,z) > 0$, while for any z and x such that $\underline{x} \leq x < z < \overline{x}$ we have $W_2^{SP-\phi}(x,z) < 0$. Next, notice that, after using a similar argument to that we used in part (a), for any $x > \underline{x}$ bidding $\beta^{SP-\phi}(\underline{x})$ is not optimal. Thus, $z = x$ is indeed a global maximum of $W^{SP-\phi}(x,z)$ for any $x \in [\underline{x},\overline{x}]$, given that competitors deploy $\beta^{SP-\phi}$. Thus, $\beta^{SP-\phi}$ is an equilibrium.■

Proof of Proposition 2

The expected payoff to bidder i from submitting a bid that corresponds to type z is

$$\begin{aligned} & \int_{\underline{x}}^{z} \overline{\Psi}^{\phi}(x,y,z)dG(y) + \int_{\underline{x}}^{\overline{x}} v^{\phi l}(y,z)dG(y) - \pi^{m-\phi}(z) \\ = \; & \int_{\underline{x}}^{z} \left[x + v^{\phi w}(y,z) - v^{\phi l}(y,z)\right] g(y)\, dy + \int_{\underline{x}}^{\overline{x}} v^{\phi l}(y,z) g(y)\, dy - \pi^{m-\phi}(z) \end{aligned}$$

where $\pi^{m-\phi}\left(z^i\right)$ is the expected payment as a result of an implicit bid z^i in auction $m-\phi$, which is assumed to be a differentiable function. Now, the FOC in maximizing the above is

$$\int_{\underline{x}}^{z} \left[v_z^{\phi w}(y,z) - v_z^{\phi l}(y,z)\right] g(y)\, dy$$
$$+\left[x + v^{\phi w}(z,z) - v^{\phi l}(z,z)\right] g(z)$$
$$+\int_{\underline{x}}^{\overline{x}} v_z^{\phi l}(y,z) g(y)\, dy - \pi_z^{m-\phi}(z) = 0.$$

In a symmetric and monotone equilibrium it is optimal to report $z = x$ and so we have

$$\begin{aligned}\pi_x^{m-\phi}(x) &= \left[x + v^{\phi w}(x,x) - v^{\phi l}(x,x)\right] g(x) \\ &\quad + \int_{\underline{x}}^{x} v_z^{\phi w}(y,x) dG(y) + \int_{x}^{\overline{x}} v_z(y,x) dG(y)\end{aligned}$$

and thus

$$\begin{aligned}\pi^{m-\phi}(x) &= \pi^{m-\phi}(\underline{x}) + \int_{\underline{x}}^{x} \left[y + v^{\phi w}(y,y) - v^{\phi l}(y,y)\right] dG(y) \\ &+ \int_{\underline{x}}^{x} \left[\int_{\underline{x}}^{y} v_z^{\phi w}(s,y) dG(s) + \int_{y}^{\overline{x}} v_z^{\phi l}(s,y) dG(s)\right] dy \\ &= G(x)\, E_G\left[\psi^{\phi}\left(Y_1^{(N-1)}\right) | Y_1^{(N-1)} < x\right],\end{aligned}$$

where we have used that in a monotone equilibrium $\pi^{m-\phi}(\underline{x}) = 0$. Note that this is a differentiable function of x. We then have that expected revenue must be

$$\begin{aligned}& N\int_{\underline{x}}^{\overline{x}} \pi^{m-\phi}(x)\, dF(x) \\ =\; & N\int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{x} \psi^{\phi}(y)\, dG(y)\, dF(x) \\ =\; & N\int_{\underline{x}}^{\overline{x}} (1 - F(y))\, \psi^{\phi}(y)\, g(y)\, dy \\ =\; & E_{F_2^{(N)}}\left[\psi^{\phi}\left(Y_2^{(N)}\right)\right],\end{aligned}$$

as desired.■

Proof of results for alternative preferences

1. It is immediate to show that now

$$\widetilde{ER}(T)-\widetilde{ER}(E)=N\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\left[H(V(y))-H(T(y))+\frac{G(y)}{g(y)}H_V(V(x_i))V_x(y)\right]dG(y)\,dF(x_i)$$

and equally,

$$\begin{aligned}&\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}\frac{G(y)}{g(y)}H_V(V(x_i))V_x(y)\,dG(y)\,dF(x_i)\\ =&\int_{\underline{x}}^{\overline{x}}\int_{\underline{x}}^{x_i}[H(V(x_i))-H(V(y))]\,dG(y)\,dF(x_i)\end{aligned}$$

so that, if H is strictly concave

$$\begin{aligned}\widetilde{ER}(T)-\widetilde{ER}(E) &= E_{F_1^{(N)}}[H(V(x_i))]-E_{F_2^{(N)}}[H(T(x_i))]\\ &= E_{F_1^{(N)}}[H(V(x_i))]-E_{F_2^{(N)}}[H(E_F(V(X)|X>x_i))]\\ &< E_{F_1^{(N)}}[H(V(x_i))]-E_{F_2^{(N)}}[E_F(H(V(X))|X>x_i)]\\ &= 0\end{aligned}$$

where the inequality follows from Jensen's inequality while the last equality follows from the proof above for the linear case (just replace V with $H(V)$). Obviously if H is strictly convex, the inequality reverses.

2. Effective valuations with $H(.)$ are

$$\psi^T(x_i)=x_i+\frac{1}{g(x_i)}H_V(V(x_i))V_x(x_i),$$

$$\psi^D(x_i)=x_i+H(V(x_i))-H(Z(x_i))+\frac{G(x_i)}{g(x_i)}H_V(V(x_i))V_x(x_i),$$

$$\psi^E(x_i)=x_i+H(T(x_i))-H(V(x_i))+\frac{1-G(x_i)}{g(x_i)}H_V(V(x_i))V_x(x_i),$$

$$\psi^P(x_i)=x_i+H(T(x_i))-H(V(x_i))$$

$$+\frac{1-F(x_i)}{f(x_i)}H_V(V(x_i))V_x(x_i)+(N-2)\frac{1-F(x_i)}{F(x_i)}[H(V(x_i))-H(Z(x_i))]$$

$$\psi^S(x_i)=x_i+E_G[H(T(Y))]-E_G[H(Z(Y))]$$

It is easy to see that the proofs that involve T versus D, T versus S and E versus P generalize to this setting.

3. Just note that if H is concave, then by 1. above $\widetilde{ER}(D)+\widetilde{ER}(E)-\widetilde{ER}(T)\geq\widetilde{ER}(D)$. Given that, the proof from the linear case for $V_x>0$ that $\widetilde{ER}(P)>\widetilde{ER}(D)+\widetilde{ER}(E)-$

$\widetilde{ER}(T)$ easily generalizes to this setting. Conversely, if H is convex, then then by 1. above $\widetilde{ER}(D) + \widetilde{ER}(E) - \widetilde{ER}(T) \leq \widetilde{ER}(D)$. Given that, the proof from the linear case for $V_x < 0$ that $\widetilde{ER}(P) < \widetilde{ER}(D) + \widetilde{ER}(E) - \widetilde{ER}(T)$ easily generalizes to this setting.■