

ASSESSING FINANCIAL CONTAGION IN THE INTERBANK MARKET: MAXIMUM ENTROPY VS. OBSERVED INTERBANK LENDING PATTERNS

Paolo Emilio Mistrulli*

Abstract

Interbank markets allow banks to cope with specific liquidity shocks. At the same time, they may represent a channel for contagion since a bank default may spread to other banks through interbank linkages. This paper analyses how contagion propagates within the Italian interbank market by using a unique data set including actual bilateral exposures. Since information about bilateral exposures was not available to most of previous works, they assumed that banks spread their lending as evenly as possible among all the other banks by maximizing the entropy of interbank linkages. Based on the availability of information on actual bilateral exposures for all the Italian banks, the results obtained by assuming the maximum entropy are compared to those reflecting the observed structure of interbank claims. The comparison indicates that, in line with the thesis prevailing in the literature, the maximum entropy method tends to underrate the extent of contagion. However, this does not hold in general. Under certain circumstances, depending on the structure of the interbank linkages, the recovery rates of interbank exposures and banks' capitalization, the maximum entropy approach overrates the scope for contagion.

Keywords: Interbank market; Financial contagion; Systemic risk; Maximum entropy.

JEL classification: G21, G28.

Contents

1. Introduction.....	7
2. Data and Methodology	10
2.1 Data	10
2.2 The Contagion Mechanism.....	11
3. Is the Italian interbank market vulnerable to financial contagion?.....	14
3.1 The benchmark analysis	14
3.2 The effect of bail-outs within conglomerates	17
4. Maximum entropy versus observed interbank lending patterns	21
4.1 Interbank lending patterns and the theory of financial contagion	21
4.2 The Italian case	28
5. Conclusions.....	34
References	35
Appendix A	38
Appendix B.....	39

* Bank of Italy, Rome, Italy. Email: paoloemilio.mistrulli@bancaditalia.it

1.Introduction¹

Systemic risk has always played a prominent role in the economic policy debate. A widely shared view is that the banking system is an important chain in the propagation of shocks to the entire economy.² Early theoretical works concerning systemic risk in the banking sector have been focused on depositor runs triggered by self-fulfilling expectations (Diamond and Dybvig, 1983) or by signals regarding bank solvency (Chari and Jagannathan, 1988 and Jacklin and Bhattacharya, 1988). More recently, attention has been paid to financial contagion in the interbank market (Rochet and Tirole, 1996). While the interbank market may allow banks to cope with specific liquidity shocks (Bhattacharya and Gale, 1987), interbank relationships may represent a channel for contagion: the economic distress of an insolvent bank may propagate to other banks through interbank linkages.³ Recent theoretical works (Allen and Gale, 2000; Freixas, Parigi and Rochet, 2000) have also highlighted that the propagation of shocks within the interbank market is dependent on the exact pattern of banks' financial linkages.

Many contributions have aimed at measuring the vulnerability of the interbank market to financial contagion.⁴ However these works suffer from two main data limitations. First, detailed data on banks' bilateral exposures are generally not available. In order to circumvent this problem, two approaches have been adopted: a) the severity of financial contagion in the interbank market has been measured by focusing on a specific segment of the market for which actual bilateral exposures were available; in particular Furfine (2003)

¹ I am grateful to Franklin Allen, Ugo Albertazzi, Guglielmo Barone, Marcello Bofondi, Francesco Columba, Xavier Freixas, Eugenio Gaiotti, Leonardo Gambacorta, Andrea Generale, Giorgio Gobbi, Francesca Lotti, Christian Upper and Andreas Worms and two anonymous referees for helpful suggestions. I am greatly indebted to Paolo Angelini for having provided me with very useful comments at different stages of this work. I also like to thank participants in the seminars at the Bank of Italy and at the Universitat Pompeu Fabra, participants in the workshop on "Systemic Risk in the Financial Sector" hosted by Collegium Budapest (Budapest, 2005) and in the 2006 European Meeting of the Econometric Society. The opinions expressed in this paper are mine and in no way involve the responsibility of the Bank of Italy.

² De Bandt and Hartmann (2000) and Kaufman (1994) provide a detailed discussion on the definition of systemic risk.

³ A different channel for contagion through the interbank market could be related to signals or self-fulfilling expectations triggering interbank deposits withdrawals.

⁴ See Upper (2006) and *Appendix A* for details on the literature about financial contagion.

has investigated contagion in the Federal Reserve's large value transfer system (Fedwire) which, however, represents a small fraction of the U.S. total interbank exposures (according to Furfine himself, approximately 14 percent); b) other contributions have taken into account the whole interbank market. However, since detailed information on banks' bilateral exposures is not available, these works have had to assume a specific pattern of interbank bilateral exposures. In particular, it is possible to distinguish between two different cases: *i*) interbank bilateral exposures are obtained on the base of aggregated interbank assets and liabilities by maximizing the entropy⁵ of the matrix of bilateral claims (e.g., Upper and Worms, 2004) or *ii*) when information about large bilateral exposures information is available, the entropy maximization method is applied only to those elements of the matrix of bilateral exposures which are not known (Blavarg and Nimander, 2002; Wells, 2002; Degryse and Nguyen, 2006; van Lelyveld and Liedorp, 2006). The main limitation of this approach is that it assumes a market structure which might be quite different from the actual one. Indeed, the maximization of the entropy rules out lending relationships in the interbank market⁶ since it assumes that each bank lends to all the other banks in the market.

A second data limitation concerns the structure of the banking system. The existing literature has typically overlooked that banks are often affiliated with a conglomerate even if this may, quite obviously, affect the resilience of the banking system to systemic shocks.

In this paper financial contagion in the interbank market is investigated by using a unique data set including detailed information on each actual bilateral exposure for all the Italian banks. Thus, the analysis deals with all kinds of interbank financial linkages, besides derivatives and shares, for every Italian bank and for each bank-to-bank relationship. According to the previous literature, the paper uses simulation techniques to measure the severity of financial contagion.

⁵ See Appendix B for details.

⁶ Cocco *et al.* (2003) find evidence supporting the existence of lending relationships in the interbank market for overnight funds.

The paper contributes to the literature in two main respects. Firstly, it provides a measure of the vulnerability to financial contagion of the Italian interbank market,⁷ thus enriching the evidence so far available for most of the industrialized countries. However, the analysis differs from those made for other countries since it is based on actual bilateral exposures instead of maximum entropy exposures. Furthermore, by taking into account the affiliation of banks with a conglomerate it provides evidence about the link between financial conglomeration and banking system stability. The paper shows that even if the Italian interbank is conducive to financial contagion it hardly triggers a systemic crisis. Banks conglomeration tends to improve the resilience of the banking system to shocks. However, in some cases, since conglomeration induces a strong interdependency among affiliated banks, the extent of contagion may be even wider if banks' affiliation is taken into account with respect to the case in which it is ignored.

Secondly, it shows that the measure of the extent of financial contagion depends on the pattern of interbank linkages. To this end, the paper applies the maximum entropy method, it re-runs the simulation exercises and then it compares them with the results based on actual bilateral exposures. The comparison indicates that, in line with the thesis prevailing in the literature, the maximum entropy method leads to an undervaluation of the extent of contagion. However, this does not hold in general. Under certain circumstances, depending on the structure of the interbank linkages, the recovery rates of interbank exposures and banks' capitalization, the maximum entropy approach implies an overvaluation of the scope for contagion.

These results have two kinds of consequences. On the empirical ground, it indicates that in some circumstances, depending on the structure of the interbank market, the maximum entropy approach may not be very reliable in order to assess the severity of financial contagion. Furthermore, since it does not provide a lower bound to contagion vulnerability, it may be reliable neither for assessing if the interbank market is conducive to contagion or not. On the theoretical one, following the Allen and Gale (2000)'s taxonomy, the comparison between maximum entropy and observed exposures may be interpreted as a

⁷ The paper by Angelini et al. (1996) have also analyzed the risk of contagion in the Italian interbank market. However, that work takes into account only the netting system segment of the interbank market.

comparison between complete and incomplete markets. In this respect, the results obtained in this paper show that complete markets are not always less conducive to contagion than other markets, as Allen and Gale (2000) stated.

The remainder of the paper is organized as follows. The next section puts forth the methodology and data. The third section deals with the vulnerability to financial contagion of the Italian interbank market. Section 4 investigates how the maximum entropy method affects the severity of financial contagion by comparing, for the Italian case, the results obtained on the base of the maximum entropy matrix of bilateral exposures with those obtained by using the observed matrix. The final section concludes and summarizes the main findings.

2.Data and Methodology

2.1 Data

Data on gross bilateral interbank exposures are obtained from the Bank of Italy Supervisory Reports database. Since January 1989 all Italian banks, including their foreign branches, submit to the Bank Italy their end-of-month bilateral exposures to all other banks, foreign intermediaries included. Data are referred to all kinds of interbank assets except for shares. The database allows for a distinction among different claims (CDs, checking accounts, repos, other loans, subordinated and unsubordinated bonds) classified according to their maturity (up to 18 months, over 18 months), currency of denomination and counterpart nationality (Italy vs. rest of the world).

At the end of 2003, interbank exposures accounted for a relevant share of the total assets of Italian banks. The total amount of loans to banks and of securities issued by banks held by the Italian banking system was 24 per cent of the total assets at the end of 2003, it was 37 per cent for the banks in the euro area as a whole. More importantly, from the perspective of financial contagion, interbank exposures were over 3 times the total amount of capital and reserves in Italy, 6 times in the euro area. If one considers only the domestic segment of the market, interbank assets were 18 per cent of total assets in Italy, 21 per cent

in the euro area. The interbank asset-to-capital ratio was respectively 2.4 and 3.5 for the Italian banking system and the euro area banks.

2.2 The Contagion Mechanism

All banks raising funds in the interbank market are let fail one at a time. Then the losses suffered by those banks lending to the failed bank are computed. If the amount of losses is greater than lenders' tier-1 capital (i.e. capital and reserves) then lenders default. The simulation is then iterated by verifying if those banks which fail after the first iteration make other banks fail as well. At each iteration those banks which failed in the previous one are dropped from the set of banks which may be affected by contagion. The simulation continues until at least a bank default occurs.

More formally, let B be the set of banks and denote by x_{ij} the funds bank $j \in B$ borrows by bank $i \in B$ where $x_{ij} \geq 0 \quad \forall (i, j) \in B \times B$ and $x_{ii} = 0 \quad \forall i \in B$, by $c_i > 0$ the initial capital endowment of bank i and by $\alpha \in [0, 1]$ the rate of loss (i.e., the incidence of losses due to contagion out of the interbank exposure). Finally, let denote by $z \in B$ the first bank which defaults because of some idiosyncratic shock and define $D_z^n \subseteq B$ and $S_z^n \subseteq B$ as the set of banks, respectively, defaulted and survived at the n -th step of the contagion path initiated by bank z , as follows:

$$(1) \quad D_z^n = \left\{ k \in B : c_{k,z}^n \leq 0 \mid c_{k,z}^{n-1} > 0 \right\} \quad S_z^n = \left\{ k \in B : c_{k,z}^n > 0 \right\}, \quad \forall n \geq 1$$

where $c_{k,z}^n$, which is the capital of bank k at the n -th step of contagion initiated by bank z , is equal to:

$$(2) \quad c_{k,z}^n = c_{k,z}^{n-1} - \alpha \sum_{j \in D_z^{n-1}} x_{kj}, \quad \forall n \geq 1 \text{ and } \forall k \neq z$$

Then the contagion path is the following:

Failed Banks

$$D_{\zeta}^0 = \{ \zeta \}$$

$$D_{\zeta}^1 = \left\{ k \in S_{\zeta}^0 : c_{k,\zeta}^1 = (c_k - \alpha x_{k\zeta}) \leq 0 \right\}$$

$$D_{\zeta}^2 = \left\{ k \in S_{\zeta}^1 : c_{k,\zeta}^2 = \left(c_{k,\zeta}^1 - \alpha \sum_{j \in D_{\zeta}^1} x_{kj} \right) \leq 0 \right\}$$

...

$$D_{\zeta}^n = \left\{ k \in S_{\zeta}^{n-1} : c_{k,\zeta}^n = \left(c_{k,\zeta}^{n-1} - \alpha \sum_{j \in D_{\zeta}^{n-1}} x_{kj} \right) \leq 0 \right\}$$

...

$$D_{\zeta}^N = \{\emptyset\}$$

Survived Banks

$$S_{\zeta}^0 = \{ k \in B \}$$

$$S_{\zeta}^1 = \left\{ k \in S_{\zeta}^0 : c_{k,\zeta}^1 = (c_k - \alpha x_{k\zeta}) > 0 \right\}$$

$$S_{\zeta}^2 = \left\{ k \in S_{\zeta}^1 : c_{k,\zeta}^2 = \left(c_{k,\zeta}^1 - \alpha \sum_{j \in D_{\zeta}^1} x_{kj} \right) > 0 \right\}$$

$$S_{\zeta}^n = \left\{ k \in S_{\zeta}^{n-1} : c_{k,\zeta}^n = \left(c_{k,\zeta}^{n-1} - \alpha \sum_{j \in D_{\zeta}^{n-1}} x_{kj} \right) > 0 \right\}$$

$$S_{\zeta}^N = \left\{ k \in S_{\zeta}^{N-1} : c_{k,\zeta}^N = \left(c_{k,\zeta}^{N-1} - \alpha \sum_{j \in D_{\zeta}^{N-1}} x_{kj} \right) > 0 \right\}$$

and the process stops after N iterations when no default occurs.

Before commenting on some shortcomings of the contagion mechanism it is useful to emphasize two characteristics of the process. First, default condition (1) refers to interbank gross exposures instead of net ones. The main reason for this is that bilateral netting is not feasible because when banks fail, their liabilities are pooled and satisfied according to their seniority. Also, this kind of procedure usually lasts many years. Second, simulations are run on unconsolidated interbank bilateral exposures. An alternative contagion mechanism could be based on consolidated exposures. However, consolidating interbank exposures would amount to assume that, in the case a bank affiliated with a group fails, internal exposures (i.e., interbank exposures among banks affiliated to the same parent company) are senior to the others. On the other hand, one can define an alternative contagion mechanism such that banks affiliated to a group may be bailed out by their parent company if the consolidated tier-1 capital is larger than the losses suffered from contagion (see section 3.2 below).

The contagion mechanism described above suffer from some drawbacks, in particular:

- i) loss rates are constant across banks and through different stages of financial contagion propagation and they are also not dependent on loan contract covenants;

- ii)* the analysis focuses on contagion only among banks operating in Italy;
- iii)* the contagion mechanism concentrates on a specific channel for contagion ignoring any other source which could interact with the propagation of contagion within the interbank market.⁸
- iv)* banks are not allowed to issue shares in order to compensate for the losses they suffer from the failure of some interbank market counterpart;
- v)* creditor runs are ruled out.

As far as loss rates are concerned (*i*), bank failures are rarely observed and then it is difficult to estimate a loss rate for interbank exposures.⁹ For this reason, this paper follows the approach adopted by previous works (e.g., Furfine, 2003; Upper and Worms, 2004) of measuring the risk of contagion at different loss rates between 0 and 1. It is also assumed that the loss rate does not depend on the banks involved. While this assumption is not quite realistic, it is not easy to endogenize loss rates. The risk could be introducing even increasingly unrealistic parameters in simulations.

The analysis focuses on contagion among banks operating in Italy (*ii*) because otherwise it would be not possible to analyze all “second-round” effects. Actually, as far as external interbank exposures are regarded, interbank linkages among foreign banks are not known. This implies that is not possible to verify if, say, the failure of a foreign bank (bank *i*) triggers indirectly the failure of a bank operating in Italy (bank *k*) by causing first the failure of another foreign bank (bank *j*) which has borrowed money from the former (bank *i*).

The analysis of a more comprehensive contagion mechanism (*iii*) would certainly adds to the realism of the analysis. On the other hand, by making the contagion process more complex it might be more difficult to isolate the contribution of each channel for contagion.

⁸ For example, exposures from the payment and settlement system may provide alternative channels for contagion. Furthermore, loans to non-banks may amplify the effect of a bank failure by causing liquidity shortages and the insolvency for some borrower which, in turn, may make the banking system suffer by other loan losses.

⁹ Some authors have tried to estimate the loss rates for the U.S. banking system. However, their estimates are very different: James (1991) has found a loss rate of 40 percent while Kaufman (1994) estimate is 5 percent.

Some of the shortcomings mentioned above would be solved if more detailed information were available (*i-iii*). On the other hand, others (*iv-v*) seem to be more difficult to tackle with as they would require a theoretical foundation that, to my best knowledge, is not available yet.

3. Is the Italian interbank market vulnerable to financial contagion?

The aim of this section is to assess whether the Italian interbank market is vulnerable to financial contagion. The analysis is divided into two parts. The first one (subsection 3.1), by following the literature on financial contagion in the interbank market, addresses the issue of financial contagion by assuming that banks cannot react to a shock by, for example, raising capital to compensate for the losses suffered from the failure of their interbank counterparts. In the second part (subsection 3.2), parent banks are allowed to recapitalize their affiliates by redistributing capital resources within the conglomerate.

3.1 The benchmark analysis

Table 1 reports simulation results referred to December 2003.¹⁰ Different measures of financial contagion propagation have been computed: *i*) the number of banks whose default causes at least one bank failure by contagion (*contagious banks*); *ii*) the number of banks which fail by contagion and their total assets as a percentage of the Italian banking system's total assets (*banks failing by contagion*); *iii*) the number and the total assets, as a percentage of the Italian banking system's total assets, of those banks which never fails independently of the bank failing by some idiosyncratic shock (*contagion-proof banks*).

Overall, the evidence indicates that financial contagion may occur in the Italian banking system. This result does not hold in general but depends on the bank which fails at the initial stage. In fact, Table 1 shows that, even for high loss rates, only a limited number of banks are contagious: the number of contagious banks ranges between 16 and 67, depending on the loss rate, compared to the 789 Italian banks operating at the end of 2003. It

¹⁰ It is worthwhile noting that results are not affected by an end-of-year effect. Simulations referred to other months of 2003 (March, June and September) have produced similar results.

is also worthwhile noting that contagious banks are not necessarily large ones. Some small bank may be contagious in particular for high loss rates. On the other hand, even if the loss rate is equal to one, not all large banks (22 at the end of 2003) are able to make other banks fail.

Since many banks are not contagious the propagation of contagion is quite limited on average both in terms of the number of banks failed and their share of the banking system total assets.

Furthermore, as far as contagion-proof banks are concerned (i.e., banks which, for a given loss rate and independently of the bank which initiates the contagion process, never fail) the results indicate that a large part of the banking system is immune to contagion. Even in the worst case (i.e., when the loss rate is equal to 1) banks immune to contagion represent a relevant share of the banking system (529 out of 789 banks; about 60 per cent of banking system total assets).

All in all, these results suggest that, even if contagion may occur in the Italian interbank market, there is little scope for financial contagion in Italy. However, one may be interested in other characteristics of the distribution of the extent of contagion other than mean values. Following previous works on financial contagion, the maximum impact of financial contagion (*worst scenario*) has been also reported Table 1. This is the result of the failure of the bank which produces the maximum contagion impact. While an unlikely event, it makes sense to consider it in an analysis whose aim is to quantify risks and their consequences. In this case, the number of banks failing by contagion ranges between 4 and 116 and is greater than 50 for a loss rate greater or equal to 0.7; the share of total assets of banks failing by contagion ranges between 0.5 and 15.9 percent and is greater than 10 percent for a loss rate greater than or equal to 0.7. The effects of contagion are highly concentrated. Up to a loss rate equal to 0.7, financial contagion affects mainly large banks. Only if loss rates exceed that threshold small banks account for a significative share of the overall extent of contagion. Large banks play in general a relevant role in the contagion mechanism, not only as banks initiating the process (i.e., contagious banks) but also as banks affected by contagion. Among large banks (22 at the end of 2003), most of them are contagious for a loss rate greater than 0.1.

Table 1

The Severity of Financial Contagion
(December 2003)

Loss Rate	Banks Failing by Contagion										Contagion-Proof Banks (2)			Contagious Banks (3)		
	Total Assets (as a percentage of banking system total assets)					Number of Banks					Total Assets (as a percentage of banking system total assets)	Number of Banks	of which: Large Banks (1)	Total Assets (as a percentage of banking system total assets)	Number of Banks	of which: Large Banks (1)
	Worst Scenario		Mean Values		Std.Dev. (all banks)	Worst Scenario		Mean Values		Std.Dev. (all banks)						
	All Banks	Large Banks (1)	All Banks	Large Banks (1)		All Banks	Large Banks (1)	All Banks	Large Banks (1)							
0.1	0.466	0.203	0.004	0.000	0.036	4	1	0.029	0.001	0.219	98.6	755	21	25.2	16	5
0.2	3.467	3.467	0.013	0.007	0.149	4	2	0.056	0.006	0.314	91.9	743	18	41.5	28	12
0.3	7.422	7.339	0.028	0.018	0.326	7	3	0.107	0.010	0.606	84.9	721	17	49.2	35	14
0.4	7.870	7.339	0.037	0.019	0.363	12	3	0.159	0.011	0.890	80.1	694	16	54.3	39	16
0.5	7.906	7.339	0.054	0.031	0.485	21	3	0.243	0.017	1.435	77.5	674	15	58.9	45	17
0.6	7.906	7.339	0.072	0.039	0.544	28	3	0.341	0.022	2.025	73.8	655	14	60.1	51	17
0.7	8.280	7.339	0.092	0.052	0.645	53	4	0.486	0.038	3.286	72.0	623	13	60.2	52	17
0.8	15.207	11.136	0.115	0.068	0.917	79	6	0.621	0.044	4.784	64.5	589	11	60.5	55	17
0.9	15.637	11.136	0.138	0.078	1.029	98	6	0.823	0.050	6.190	59.7	552	10	62.2	57	18
1.0	15.878	11.136	0.154	0.087	1.086	116	6	0.983	0.054	7.314	59.4	529	10	63.5	67	18

(1) Large banks are defined as those banks whose total assets are at least equal to 20 billions of euro. (2) Banks which are never affected by contagion. (3) Banks which make at least a bank fail.

Large banks are relatively less important in the propagation of contagion: for a loss rate lower than 0.9, most of them are immune to contagion. Only in the worst scenario case, they account for a large share of the impact of contagion when it is measured in terms of total assets.

Overall these results suggest that for loss rates up to 0.7 the spreading of contagion is very limited. For loss rates greater than 0.7, financial contagion becomes relevant while it seems unable to trigger a systemic crisis.

3.2 The effect of bail-outs within conglomerates

The results obtained in section 3.1 are based on the hypothesis that banks can not react to any shock by, for example, raising capital to compensate for the losses suffered from the failure of their interbank counterparts. While this assumption may be quite suitable for stand-alone banks it seems less appropriate for banks which are affiliated with a conglomerate. It seems quite reasonable indeed to assume that parent companies may recapitalize those affiliated banks which would otherwise fail by, at least, redistributing capital resources among subsidiaries. On one hand, this would improve the resilience of the banking system to financial contagion since contagion losses are shared among all banks belonging to a group. On the other, banks which are not financially linked (directly or indirectly) to the bank which fails in the first step may suffer from losses due to financial contagion if other affiliates are financially linked to the bank which triggers the contagion process.

In this section we analyze how results change by assuming that banks can be bailed out by other banks belonging to the same conglomerate. Simulations are still based on unconsolidated interbank exposures and run in the same way as before except for assuming a slightly different default condition. In particular, that condition is now referred to the consolidated capital since it is assumed that losses are now shared among banks affiliated with a certain conglomerate.¹¹

¹¹ For stand-alone banks previous conditions hold.

Let denote by C_s the set of banks affiliated with conglomerate s and by k_s the capital of conglomerate s . Let bank $z \in B$ default. The capital of conglomerate s at the n -th contagion path initiated by bank z is equal to:

$$(3) \quad k_{s,z}^n = k_{s,z}^{n-1} - \alpha \sum_{i \in C_s, j \in D_z^{n-1}} x_{ij}, \quad \forall n \geq 1$$

and:

$$(4) \quad D_z^n = \left\{ i \in C_s : k_{s,z}^n \leq 0 \mid k_{s,z}^{n-1} > 0 \right\}, \quad \forall n \geq 1$$

It is worthwhile highlighting some important differences with respect to previous simulations: the default of bank z affects bank $i \in C_s$ solvency not only because of the losses bank i faces since it lends money to bank z but also because of the losses all other banks affiliated with conglomerate s suffer since they lend to bank z . In fact, all the losses suffered by the banks affiliated with a conglomerate s affect the consolidated capital k_s . This reflects the hypothesis that, at each stage of the contagion process, banks can be bailed out by other banks affiliated with the same conglomerate if the consolidated capital is greater than the overall amount of losses.

When conglomerates are considered another issue arises: since interdependencies among banks affiliated with a conglomerate, apart from those related to financial interlinkages, may be quite relevant it could be the case that a shock that hits one bank would hit all banks of the same conglomerate. Thus, in this section, the impact of financial contagion is computed under two different hypothesis: *i*) the initial shock that makes banks fail at the first stage is bank specific (a bank fails); or is *ii*) conglomerate specific (i.e., a conglomerate as a whole fails).

Table 2 shows the results of simulations obtained for bank specific shocks. The comparison with those results based on individual capital (Table 1) indicates that the effect of contagion is on average smaller if bail-outs within conglomerates are allowed. This reflects the fact that some relatively small banks can be bailed out by their parent company. Thus, one might conclude that banking conglomerates may lower the impact of financial contagion.

However, this result does not hold in general. In some cases, the severity of contagion may be even larger. The results for the *worst scenario* indicate that, under the hypothesis of

bail-outs within conglomerates, financial contagion, for loss rates ranging between 0.4 and 0.7, is significantly more dangerous in terms of total assets affected with respect to the benchmark case. Similarly, the total assets of contagion-proof banks account for a lower share of banking system total assets, if loss rates are above 0.3, compared to benchmark case. This means that in the latter case, the impact of contagion tends to be more concentrated than in the former.

On the contrary, the maximum impact measured in terms of the number of banks failed by contagion is almost always lower than the one computed when bail-outs are not allowed. The same holds for contagious banks.

This evidence reflects the fact that, conglomerate bail-outs have two opposing effects on the severity of contagion: on one hand, they imply a subsidiaries' capital position strengthening, on the other, they add another channel for contagion since banks in a conglomerate suffer not only from the losses due to their interbank linkages but also from losses due to all the other conglomerate affiliates interbank exposures. The first effect is relatively more relevant for small banks which may be more easily bailed-out by the conglomerate they are affiliated with. This explains why the number of banks failed tends to be lower, even in the worst scenario case, in comparison with the case in which bail-outs are not allowed. On the contrary, this effect is limited in terms of the total assets affected by contagion since those banks which may be bailed out are typically small.

All in all, the severity of contagion when bail-outs are allowed may be even greater in comparison with the benchmark case. On the other hand, given that the number of banks involved in the contagion process, both contagious and contagion affected banks, is lower it could be easier to manage a systemic crisis in this case.

Apart from bail-outs, bank conglomerates may deteriorate the resilience of the interbank market to financial contagion since they also raise the interdependence among banks. Table 3 shows that, if at the first stage of simulations a conglomerate as a whole fails instead of a single bank, the impact of contagion is greater.

Table 2

Conglomerates and the Severity of Financial Contagion: Bank Specific Shocks
(December 2003)

Loss Rate	Banks Failing by Contagion										Contagion-Proof Banks			Contagious Banks		
	Total Assets (as a percentage of banking system total assets)					Number of Banks					Total Assets (as a percentage of banking system total assets)	Number of Banks	of which: Large Banks (1)	Total Assets (as a percentage of banking system total assets)	Number of Banks	of which: Large Banks (1)
	Worst Scenario		Mean Values		Std.Dev. (all banks)	Worst Scenario		Mean Values		Std.Dev. (all banks)						
	All Banks	Large Banks	All Banks	Large Banks		All Banks	Large Banks	All Banks	Large Banks							
0.1	0.466	0.000	0.003	0.000	0.035	4	0	0.022	0.000	0.203	98.2	760	22	19.7	11	4
0.2	0.466	0.000	0.004	0.000	0.036	4	0	0.024	0.000	0.210	98.2	759	22	23.0	13	5
0.3	0.475	0.000	0.004	0.000	0.036	5	0	0.042	0.000	0.317	98.1	749	22	31.0	18	8
0.4	12.767	11.830	0.049	0.037	0.698	19	5	0.145	0.019	1.236	78.1	712	14	35.9	23	10
0.5	12.767	11.830	0.063	0.046	0.762	19	5	0.199	0.023	1.486	67.5	682	11	41.0	26	11
0.6	12.786	11.830	0.075	0.055	0.817	20	5	0.243	0.027	1.689	67.4	670	11	47.7	31	13
0.7	12.786	11.830	0.079	0.055	0.834	28	5	0.301	0.027	2.086	65.4	643	11	47.7	31	13
0.8	12.786	11.830	0.099	0.069	0.924	43	5	0.381	0.034	2.553	50.8	599	6	48.6	35	13
0.9	12.786	11.830	0.106	0.069	0.937	58	5	0.469	0.034	3.117	49.0	566	6	53.9	41	15
1.0	12.827	11.830	0.110	0.069	0.947	73	5	0.538	0.034	3.637	46.4	534	6	56.2	49	15

(1) Large banks are defined as those banks whose total assets are at least equal to 20 billions of euro. (2) Banks which are never affected by contagion. (3) Banks which make at least a bank fail.

4. Maximum entropy versus observed interbank lending patterns

This section compares the results obtained in the previous section with those that would be obtained if the matrix of bilateral exposures were not known and, by following previous contributions, the maximum entropy method (ME) would have been adopted.¹² This comparison is relevant from two different points of view. Firstly, it sheds some light on the reliability of the ME approach for assessing whether the interbank market is vulnerable to financial contagion. Secondly, since the ME matrix represents, according to Allen and Gale (2000)'s taxonomy, a *complete* market, the comparison between the results obtained by using the actual matrix of bilateral exposures with those based on the ME matrix may provide some evidence supporting the hypothesis that the pattern of interbank linkages affects the severity of financial contagion.

Section 4.1, after having briefly recalled the main results of Allen and Gale (2000) model, shows that, in contrast with Allen and Gale (2000), complete markets are not necessarily less conducive to contagion than incomplete markets. To do this, some simple numerical examples are considered. Section 4.2 applies the same approach to the Italian interbank market. The results support the analysis of section 4.1 indicating that in the Italian case the ME method, under certain circumstances, may overrate the vulnerability to contagion.

4.1 Interbank lending patterns and the theory of financial contagion

Recent theoretical contributions (Allen and Gale, 2000; Freixas, Parigi and Rochet, 2000) have highlighted that the propagation of shocks within the interbank market is dependent on the exact pattern of banks' financial linkages. Allen and Gale (2000) identify two key structural characteristics of the market which affect financial contagion: market *completeness* and market *interconnectedness*. A market is *complete* if each bank lends to all the others.

¹² See *Appendix B* for details.

Table 3

Conglomerates and the Severity of Financial Contagion: Conglomerate Specific Shocks
(December 2003)

Loss Rate	Banks Failing by Contagion										Contagion-Proof Banks			Contagious Conglomerates		
	Total Assets (as a percentage of banking system total assets)					Number of Banks					Total Assets (as a percentage of banking system total assets)	Number of Banks	of which: Large Banks (1)	Total Assets (as a percentage of banking system total assets)	Number of Conglomerates	of which: Large Conglomerates (1)
	Worst Scenario		Mean Values		Std.Dev. (all banks)	Worst Scenario		Mean Values		Std.Dev. (all banks)						
	All Banks	Large Banks (1)	All Banks	Large Banks (1)		All Banks	Large Banks (1)	All Banks	Large Banks (1)							
0.1	0.720	0.000	0.006	0.000	0.050	6	0	0.038	0.000	0.351	97.4	755	22	38.1	11	5
0.2	12.703	11.830	0.042	0.033	0.630	14	5	0.083	0.014	0.808	74.6	732	14	61.1	14	7
0.3	12.751	11.830	0.048	0.036	0.647	17	5	0.122	0.020	1.046	70.5	710	11	65.7	17	8
0.4	12.767	11.830	0.085	0.060	0.837	19	5	0.191	0.031	1.401	49.2	675	5	68.9	19	9
0.5	12.767	11.830	0.092	0.060	0.847	19	5	0.251	0.031	1.698	44.5	644	5	71.2	21	9
0.6	12.786	11.830	0.096	0.060	0.852	20	5	0.291	0.031	1.834	42.7	627	5	72.4	23	9
0.7	12.786	11.830	0.099	0.060	0.862	28	5	0.354	0.031	2.212	41.1	598	5	72.8	24	9
0.8	12.786	11.830	0.122	0.077	1.010	43	5	0.442	0.032	2.929	29.2	559	4	73.1	26	9
0.9	12.786	11.830	0.126	0.077	1.024	58	5	0.522	0.032	3.522	28.4	530	4	77.4	29	10
1.0	12.827	11.830	0.127	0.077	1.030	73	5	0.574	0.032	4.050	28.0	508	4	78.3	34	10

(1) Large banks are defined as those banks whose total assets are at least equal to 20 billions of euro.

On the other hand, a market is *perfectly interconnected* if each bank is financially linked to all the others, regardless of the kind of linkage which may be both direct (i.e., each bank lends to all the others) and indirect (the link that a bank could indirectly establish with another if a third one is linked to both of them).

Those authors point out that the more *complete* the market the less the severity of financial contagion and the more *interconnected* the market the more is the severity of contagion. Thus, there is a trade-off in terms of the propagation of contagion between *completeness* and *interconnectedness*. These two concepts are strictly linked. A *complete* market is a specific case of a *perfectly interconnected* market where banks are financially linked to all the others only by direct exposures. Otherwise, *incomplete* markets may be differently *interconnected*: they may be both perfectly or partially *interconnected* depending on the pattern of financial linkages. In general, markets which are *incomplete* and not perfectly *interconnected* may differ both in terms of *completeness* and *interconnectedness*.

The comparison between ME and observed interbank patterns may be interpreted as the comparison between complete and incomplete markets with the notable qualification that Allen and Gale (2000) model does not allow for banks' heterogeneity. On the contrary, the ME method assumes that banks spread their lending as evenly as possible consistently with their aggregate interbank assets and liabilities which may, in turn, differ quite a lot among banks.

Let ignore for a while the issue of banks' heterogeneity that it will be dealt with later. According to Allen and Gale (2000) taxonomy, the comparison between actual and ME bilateral exposures may lead to the following cases:

- (a) the (actual) market is complete;
- (b) the (actual) market is incomplete but perfectly interconnected;
- (c) the (actual) market is incomplete but not perfectly connected;

In the first case, the ME matrix of bilateral exposures and the observed ones coincide. Thus, the measure of the impact of financial contagion is not affected by the ME hypothesis. On the other hand, if the market is incomplete but perfectly interconnected (case b), ME leads to an undervaluation of financial contagion. To show this consider Fig.1 which

represents the ME matrix of bilateral exposures under the assumption that, for each bank, total interbank liabilities and total interbank assets are equal.

Fig. 1

A complete market (maximum entropy)

	<i>Bank A</i>	<i>Bank B</i>	<i>Bank C</i>	<i>Bank D</i>	Total Assets
<i>Bank A</i>	0	2k	2k	2k	6k
<i>Bank B</i>	2k	0	2k	2k	6k
<i>Bank C</i>	2k	2k	0	2k	6k
<i>Bank D</i>	2k	2k	2k	0	6k
Total Liabilities	6k	6k	6k	6k	24k

This assumption is needed only to simplify the maximization of the entropy. It does not imply any loss of generality since contagion is based on gross exposures. Let assume also that bank capital is equal to k and that a bank fails if losses exceed its capital and reserves. Then, if bank A fails due to a specific shock¹³ it is easy to verify that there is no contagion unless the loss rate is greater than 0.5 (i.e, losses are greater than k). In that case, all banks fail by contagion (Table 4).

Now we compare these results with those obtained for case b) (Fig.2).¹⁴ It is easy to verify that all banks in the market fail if the market is incomplete and perfectly connected unless the loss rate is lower than 0.2 (1/6 exactly). This confirms that, if the actual market is incomplete but perfectly connected, the maximum entropy method tends to underrate the extent of contagion (in this specific case when the loss rate ranges between 0.2 and 0.5).

This is due to a typical effect of risk diversification. In fact, while complete markets and incomplete but perfectly connected markets do not differ in terms of the number of banks which may have potentially been affected by contagion (since from each bank a shock may propagate to all the others in both cases), complete markets allow banks to better diversify

¹³ In absence of banks' heterogeneity the identity of the bank which fails due to some idiosyncratic shock is not relevant.

¹⁴ This corresponds to the incomplete market structure depicted in Allen and Gale (2000)'s fig.1.

credit risk with respect to incomplete ones, given that each bank directly lends to all the other banks instead of holding the financial assets issued by a limited number of counterparts.

Table 4

FINANCIAL CONTAGION: COMPLETE VERSUS INCOMPLETE MARKETS
(number of banks failing by contagion)

<i>Loss Rate</i>	<i>Symmetric case</i>			<i>Asymmetric case</i>	
	Complete markets (fig.1)	Incomplete but perfectly connected market (fig. 2)	Incomplete and disconnected market (fig. 3)	Complete markets (fig.4)	Incomplete and disconnected market (fig. 4)
0.1	0	0	0	0	0
0.2	0	3	1	0	1
0.3	0	3	1	0	1
0.4	0	3	1	3	1
0.5	0	3	1	3	1
0.6	3	3	1	3	1
0.7	3	3	1	3	1
0.8	3	3	1	3	1
0.9	3	3	1	3	1
1.0	3	3	1	3	1

Fig. 2

An incomplete but perfectly connected market

	<i>Bank A</i>	<i>Bank B</i>	<i>Bank C</i>	<i>Bank D</i>	Total Assets
<i>Bank A</i>	0	6k	0	0	6k
<i>Bank B</i>	0	0	6k	0	6k
<i>Bank C</i>	0	0	0	6k	6k
<i>Bank D</i>	6k	0	0	0	6k
Total Liabilities	6k	6k	6k	6k	24k

Consider now case c) as it has been depicted in Fig.3¹⁵ In this case it is straightforward to verify that the maximum entropy method does not affect the measure of contagion if loss rates are lower than 0.2 (1/6), it underrates the scope for contagion if loss rates range between 0.2 and 0.5 and it overrates it for loss rates greater than 0.5.

The reason why, in spite of a lower market concentration, complete markets may be, *ceteris paribus*, even less resilient to financial contagion than incomplete ones is that, if the size of the interbank market is sufficiently large with respect to bank capital, there could be limited scope for risk diversification, especially for large loss rates, from spreading interbank exposures over a great number of counterparts. It is easy to verify that if the total amount of interbank assets held by each bank were less than $3k$, complete markets would be totally immune to contagion thus representing, in line with Allen and Gale (2000), the most resilient market structure. For larger interbank markets, the sole effect of diversification is that market interconnectedness rises, thus favouring the propagation of contagion.

In general, for N homogenous banks the interbank assets-to-capital ratio has to be at least equal to $(N - 1)$ to make the failure of a bank propagate within the market. However, actual interbank markets are not so large. On the other hand, according to the evidence provided by those authors that have resorted to the maximum entropy method, even complete markets are vulnerable to financial contagion. The reason for this result is that, notwithstanding markets are assumed to be complete, the ME does not rule out banks' heterogeneity and then, as shown below, the condition for contagion propagation becomes less stringent.

To show how the resilience of the market may deteriorate when banks' heterogeneity is allowed, let consider the interbank market depicted in Fig. 4 which is equal to the one represented in fig.3 besides the fact that banks differ in terms of interbank asset-to-capital ratio. In this case, the identity of the bank that fails due to some idiosyncratic shock is not irrelevant anymore. The maximum impact of financial contagion is obtained when bank A defaults (i.e., the bank which is better capitalized than the others). For a loss rate greater than

¹⁵ This corresponds to the disconnected incomplete market structure depicted in Allen and Gale (2000)'s fig.3. This market is made of two totally disconnected segments: to one of them belong bank A and B, to the other bank C and D.

0.1 the failure of bank *A* makes only bank *B* fail by contagion according to the observed matrix (panel a).

Fig. 3

An incomplete and disconnected market

	<i>Bank A</i>	<i>Bank B</i>	<i>Bank C</i>	<i>Bank D</i>	Total Assets
<i>Bank A</i>	0	6k	0	0	6k
<i>Bank B</i>	6k	0	0	0	6k
<i>Bank C</i>	0	0	0	6k	6k
<i>Bank D</i>	0	0	6k	0	6k
Total Liabilities	6k	6k	6k	6k	24k

On the other hand, bank *A* makes all other banks fail unless the loss rate is lower than 0.3. Thus, the two matrices produce the same result up to a loss rate equal to 0.2, the maximum entropy method underrates the severity of contagion for loss rates ranging between 0.1 and 0.3 and it overrates the extent of contagion if the loss rate is greater than 0.3. This means that, by allowing for some heterogeneity in banks' capitalization, the maximum impact of financial contagion tends to be overstated for lower loss rates with respect to the case in which banks are homogenous (loss rates greater than 0.5). It may be also verified that in this case, even if each bank held an amount of interbank assets lower than $3k$, complete markets would not be immune to contagion as they were in the symmetric case (Fig. 1).

The main conclusion of this section is that apart, from the case in which the actual market is complete or it is incomplete and perfectly connected, ME may involve, for high loss rates, an overstatement in the severity of financial contagion. This may also happen more easily when banks are heterogeneous.¹⁶

This eventual effect seems to be quite realistic since there are at least three reasons why actual interbank markets are, in general, incomplete and to some extent disconnected. First, the existence of informational asymmetries or, in general, transaction costs may prevent

¹⁶ This result is in line with Iori, Jafarey and Padilla (2006).

banks from exploiting too much their capacity of diversifying interbank counterparts. Second, for those banks which are affiliated with conglomerates the multiple money centre structure seems to be suitable to exploit economies to scale in the liquidity management. Finally, it is common to many countries that banks differ in their size quite a lot and then the assumption that the interbank market is made of homogenous banks seems unrealistic.

4.2 The Italian case

Some characteristics of the Italian interbank market suggest that it is far from being a complete market and then that the adoption of the ME method may significantly affect the assessment of financial contagion. Indeed, the Italian interbank market fits quite well in the multiple money centre structure described in Freixas, Parigi and Rochet (2000), in which some banks trade with a bank (the money centre) while they do not trade with each other: at the end of 2003, more than two thirds of interbank claims were traded among banks belonging to the same group. As far as interconnectedness is concerned, banks were on average financially linked, both directly and indirectly through other banks, to one half of the total number of counterparts.¹⁷

The comparison of the severity of financial contagion in the Italian interbank market by using, alternatively, the maximum entropy (ME) matrix and the observed matrix of bilateral exposures (Fig. 5-9) confirms that ME may provide a quite different measure of financial contagion.

Fig. 5 supports the hypothesis that complete markets are more resilient to contagion than incomplete ones, at least in terms of contagious banks. In this case, one can argue that a fewer number of banks have to be monitored in order to avoid the propagation of contagion within the banking system if the market were complete.

¹⁷ The computation of a measure of interconnectedness for the Italian interbank market has been referred only to those financial linkages such that contagion is at least possible (i.e., such that for a loss rate equal to 1, interbank exposures exceeds bank capital).

Fig. 4

a) *observed matrix of bilateral exposures*

An incomplete and disconnected market

	<i>Bank A</i>	<i>Bank B</i>	<i>Bank C</i>	<i>Bank D</i>	Total Assets
<i>Bank A</i>	0	6k	0	0	6k
<i>Bank B</i>	6k	0	0	0	6k
<i>Bank C</i>	0	0	0	6k	6k
<i>Bank D</i>	0	0	6k	0	6k
Total Liabilities	6k	6k	6k	6k	24k
Capital and Reserves	2k	2/3k	2/3k	2/3k	4k

b) *ME matrix of bilateral exposures*

A complete market (maximum entropy)

	<i>Bank A</i>	<i>Bank B</i>	<i>Bank C</i>	<i>Bank D</i>	Total Assets
<i>Bank A</i>	0	2k	2k	2k	6k
<i>Bank B</i>	2k	0	2k	2k	6k
<i>Bank C</i>	2k	2k	0	2k	6k
<i>Bank D</i>	2k	2k	2k	0	6k
Total Liabilities	6k	6k	6k	6k	24k
Capital and Reserves	2k	2/3k	2/3k	2/3k	4k

Fig. 6 and 7 indicate that the ME method leads, on average, to an undervaluation of the severity of financial contagion, both in terms of the number of banks and of the total assets affected by contagion, if the loss rate is not above 0.8 and 0.9, respectively. Similarly according to Fig. 8 and 9, ME underrates the maximum impact of financial contagion, measured by the maximum number of banks failed and the maximum share of banking system's total assets affected by contagion, for loss rates not greater than 0.7.

Fig. 5

Number of Contagious Banks
(December 2003)

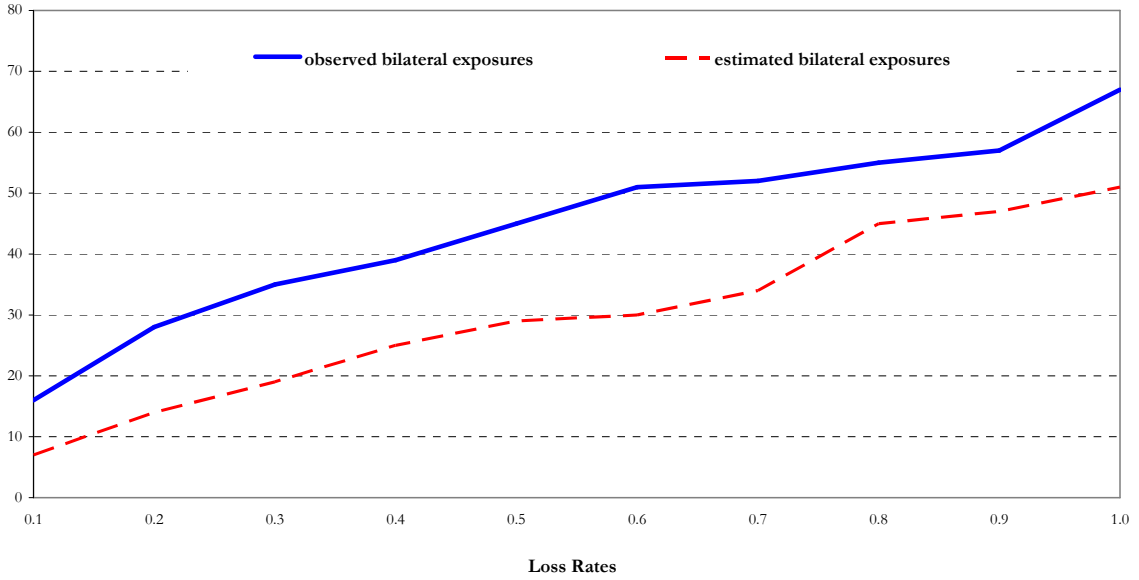


Fig. 6

Banks Failing by Contagion
Number of banks affected by contagion - mean values
(December 2003)

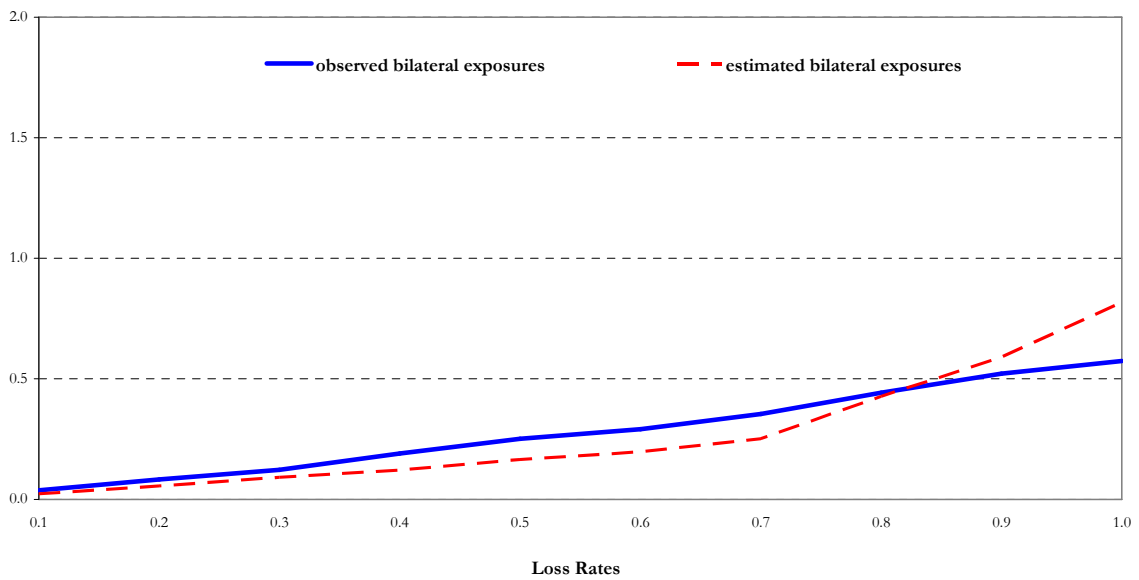


Fig. 7

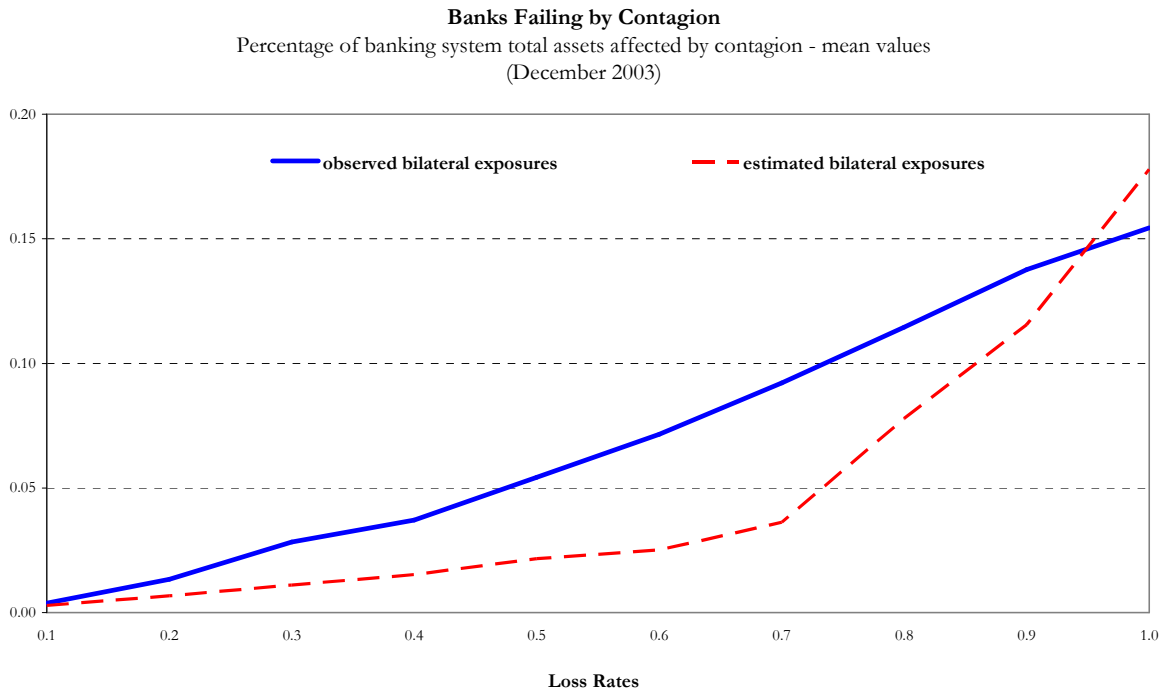


Fig. 8

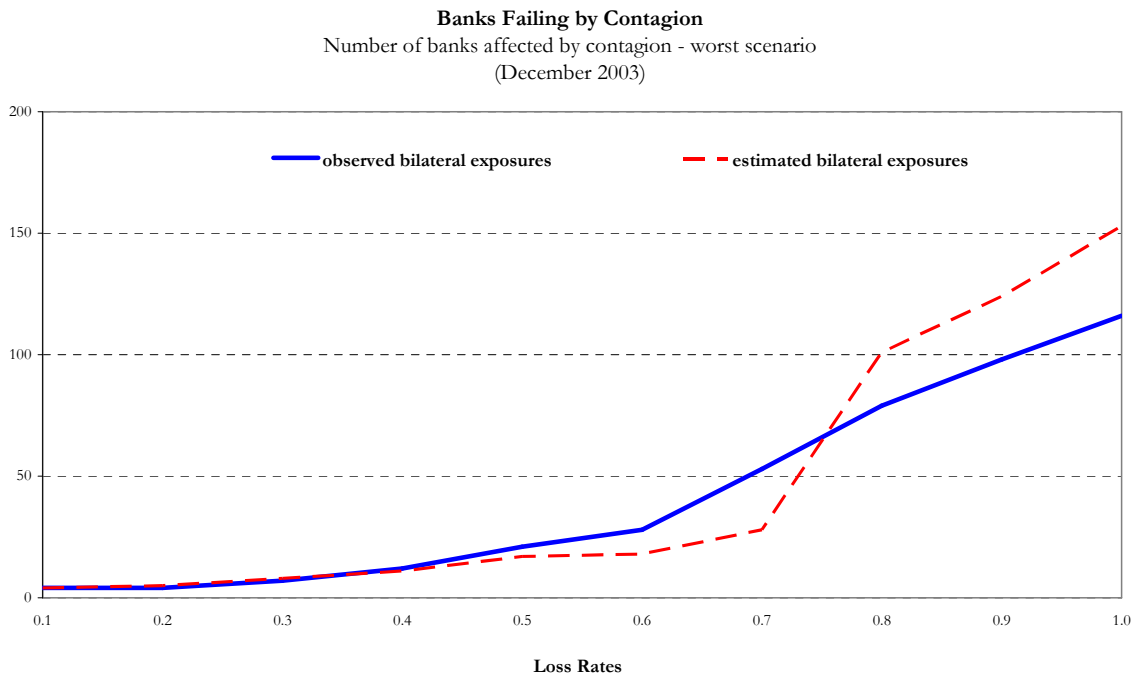
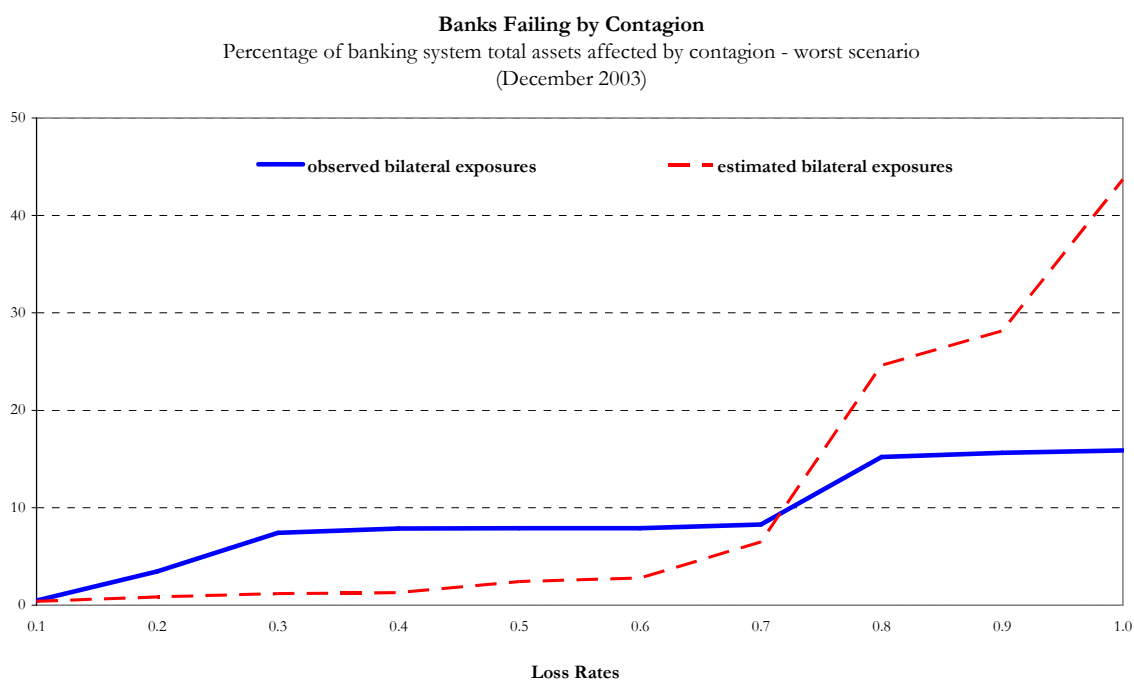


Fig. 9



To sum up, the evidence suggests that, in most cases, ME tends to underrate the impact of financial contagion. On the contrary, for high loss rates, ME may imply an overvaluation in the severity of contagion.

The Kolmogorov-Smirnov test suggests that, for any loss rate, the ME distribution of the severity of contagion is not statistically different from the actual distribution (Table 5). However, if only contagious banks (i.e., banks which are contagious at least for one of the two distributions) are taken into account, the test rejects the hypothesis of equality if loss rates are at least equal to 0.5 and to 0.8 for, respectively, the distribution of total assets and the number of banks affected by contagion. This evidence indicates that the main reason why the ME approach does not lead to a measure of the severity contagion which statistically differ from the one which would have been obtained on the base of the actual matrix of bilateral exposures is that most of banks are not contagious. On the other hand, if one is interested in quantifying even unlikely events, the recourse to the ME approach would be problematic, at least for “relatively high loss rates”.

Table 5

**A TEST FOR THE EQUALITY OF ME AND OBSERVED FINANCIAL
CONTAGION DISTRIBUTIONS (1)**
(Kolmogorov-Smirnov test; *December 2003*)

		<i>Loss rates</i>									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Total assets affected by contagion											
<i>All banks</i>											
Kolmogorov-Smirnov test		0.013	0.020	0.023	0.019	0.023	0.030	0.026	0.026	0.027	0.026
<i>p-value</i>		1	0.999	0.993	1	0.993	0.911	0.975	0.975	0.959	0.975
<i>Contagious banks</i>											
Kolmogorov-Smirnov test		1	0.429	0.417	0.412	0.400	0.524	0.565	0.483	0.500	0.500
<i>p-value</i>		0.270	0.541	0.2490	0.1120	0.082	0.006	0.001	0.002	0.001	0.000
Number of banks affected by contagion											
<i>All banks</i>											
Kolmogorov-Smirnov test		0.013	0.020	0.023	0.019	0.023	0.030	0.026	0.019	0.021	0.024
<i>p-value</i>		1	0.999	0.993	1	0.993	0.911	0.975	1	0.997	0.986
<i>Contagious banks</i>											
Kolmogorov-Smirnov test		0.5000	0.4286	0.2500	0.1765	0.2000	0.2857	0.3478	0.2759	0.3333	0.4118
<i>p-value</i>		0.964	0.541	0.847	0.954	0.819	0.358	0.124	0.220	0.071	0.006

(1) Distributions are referred to the percentage of the banking system total assets affected by contagion and to the number of banks failing by contagion.

A further complication is that the lowest loss rate such as ME overvalues the severity of financial contagion depends on the characteristics of the market. As it has been shown in section 4.1, that critical value may be also quite low. From the previous discussion, it may be also argued that the lowest loss rate such that ME underrates the extent of contagion is negatively related with the asymmetry among market participants in terms of the amount of bilateral exposures and with the size and the disconnectedness of the interbank market. In other terms, when the interbank market is made of few big players raising funds from many relatively small counterparts, interbank exposures account for a large share of banks' capital and the market is highly segmented, then ME may be of little help in assessing the severity of financial contagion since it would overstate the impact of financial contagion even for low

loss rates. As a consequence, in that situation the ME approach might be little reliable even for assessing if, apart from an accurate measure of the severity of contagion, a danger of financial contagion exists or not.

5. Conclusions

This paper has investigated whether interbank linkages may be conducive to financial contagion. The contribution to the existing literature is twofold. Firstly, it enriches the evidence so far available on financial contagion by providing the first comprehensive analysis for the Italian interbank market. To this aim, in spite of previous contributions, which are based on ME bilateral exposures, the paper is based on a unique data set on actual bilateral exposures. It also has taken into account the effect of banking conglomerates in terms of banking system stability. This aspect has been ignored by previous works. Secondly, the paper compares the results obtained by using ME bilateral exposures with those obtained on the base of actual bilateral exposures. This allows to show how the ME method widely used in the literature may affect the analysis of financial contagion.

The main results are the following: the Italian interbank market is conducive to financial contagion. However, even for high loss rates, the default of banks raising funds in the interbank market hardly triggers a systemic crisis. Only in some extreme cases the severity of financial contagion seems relevant. Simulations also indicates that, by allowing conglomerates to recapitalize their affiliates which otherwise would fail, the resilience to financial contagion of the banking system tends to improve. However, in some cases the fact that losses are shared among banks affiliated to a conglomerate, which means that another channel for contagion is added, banking stability may even worsen.

The paper also indicates that the maximum entropy procedure, widely adopted in previous contributions, may involve an overvaluation of the severity of contagion. This contrasts with the common view that complete markets are more resilient to financial contagion. Under some circumstances, depending on the size of the interbank market, the presence of large players, and the loss rate, complete markets may be even more conducive to contagion than incomplete ones. In that case, the benefits from diversifying the interbank

exposures are more than counterbalanced by the costs due the fact that each bank establishes a high number of financial linkages, thus making a domino effect viable.

References

- Allen F. and Gale D. (2000), "Financial Contagion", *Journal of Political Economy*, 108, 1-34.
- Angelini P., Maresca G. and Russo D. (1996), "Systemic Risk in the Netting System", *Journal of Banking and Finance*, 20, 853-886.
- Bech M. L., Madsen B. and Natorp L. (2002), "Systemic Risk in the Danish Interbank Netting System", Danmarks Nationalbank Working Papers, n.8.
- Bhattacharya S. and Gale D. (1987), "Preference shocks, liquidity and central bank policy. " In *New Approaches to Monetary Economics*, edited by Barnett and Singleton, Cambridge University Press, 1987.
- Blåvarg M. and Nimander P. (2002), "Interbank Exposures and Systemic Risk", Sveriges Riksbank, *Economic Review*, 2, 19-45.
- Censor, Y. and Zenios, S.A. (1997), *Parallel optimization*, Oxford University Press.
- Chari V.V. and Jagannathan R. (1988), "Banking Panics, Information, and Rational Expectation Equilibrium", *Journal of Finance*, 3, 749-760.
- Cocco J.F, Gomes F.J and Martins N.C. (2003), "Lending Relationships in the Interbank Market", mimeo.
- De Bandt O. and Hartmann P. (2000), "Systemic Risk: A Survey", European Central Bank, Working Paper Series 35 (November 2000).
- Degryse H. and Nguyen G. (2006), "Interbank Exposures: An Empirical Examination of Systemic Risk in the Belgian Banking System", *Tilec Discussion Paper*, 2006-016, Tilburg University.
- Diamond D.W. and Dybvig P. H. (1983), "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy*, 3, 401-419.

- Elsinger H., Lehar A. and Summer M. (2002), "Risk Assessment for Banking Systems", Oesterreichische Nationalbank, Working Paper 79.
- Freixas X. and Parigi B. (1998), Contagion and Efficiency in Gross and Net Interbank Payment Systems, *Journal of Financial Intermediation*, 7, 3-31.
- Freixas X., Parigi B. and Rochet J.C. (2000), "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *Journal of Money, Credit and Banking*, 32, 611-638.
- Furfine C. H. (2003), "Interbank Exposures: Quantifying the Risk of Contagion", *Journal of Money, Credit and Banking*, 35, 111-28.
- Iori G., Jafarey S. and Padilla F.G. (2006), "Systemic risk on the interbank market", *Journal of Economic Behavior & Organization*, vol.61, pp.525-542.
- Jacklin C. J. and Bhattacharya S. (1988), "Distinguishing Panics and Information-Based Bank Runs: Welfare and Policy Implications", *Journal of Political Economy*, 3,568-592.
- James C. (1991), "The Loss Realized in Bank Failures", *Journal of Finance*, 46, 1223-42.
- Kaufman G.G. (1994), "Bank Contagion: A Review of the Theory and Evidence", *Journal of Financial Services Research*, 8, 123-50.
- van Lelyveld I. and Liedorp F. (2004), "Interbank Contagion in the Dutch Banking Sector", De Nederlandsche Bank, mimeo.
- Rochet J.C. and Tirole J. (1996), "Interbank Lending and Systemic Risk", *Journal of Money Credit and Banking*, 28, 733-62.
- Sheldon G. and Maurer M. (1998), "Interbank Lending and Systemic Risk: An Empirical Analysis for Switzerland", *Swiss Journal of Economics and Statistics*, Vol.134, pp.685-704.
- Upper C. (2006), "Contagion Due to Interbank Credit Exposures: What Do We Know, Why Do We Know It, and What Should We Know?", mimeo, Bank for International Settlements.

Upper C. and Worms A. (2004), “Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion?”, *European Economic Review*, Vol.8, pp.827-49 09/02.

van Lelyveld, I. and Liedorp F. (2006) “Interbank Contagion in the Dutch Banking Sector: A Sensitivity Analysis”, *International Journal of Central Banking*, Vol.2, pp.99-133

Wells S. (2004), “Financial Interlinkages in the United Kingdom’s Interbank Market and the Risk of Contagion”, *Bank of England*, Working Paper n.230.

Appendix A
The empirical investigation on financial contagion

Reference	Country	Institutions	Type of Data	Period
Angelini, Maresca and Russo (1996)	Italy	288 Banks	Bilateral end-of-day net balances	January 1992
Bech, Madsen and Natorp (2002)	Danmark	Danish Banks	Interbank Multilateral Netting System, Daily Data	Dec. 21 st 2001 - Jan. 25 th 2002
Blåvarg and Nimander (2002)	Sweden	Four Largest Swedish Banks	15 Largest Bilateral Interbank Exposures	September 1999 - September 2001
Degryse and Nguyen (2006)	Belgium	Belgian Banks	Estimated Bilateral Interbank Exposures (RAS Algorithm), Large Bilateral Exposures (December 2003), Quarterly data	1993-2002
Elsinger, Lehar and Summer (2003)	Austria	Austrian Banks	Estimated Bilateral Interbank Exposures (RAS Algorithm)	September 2001
Furfine (2003)	United States	Fedwire Participants (719 Commercial Banks)	Federal Funds Bilateral Exposures (daily data)	February 1998 - March 1998
Sheldon and Maurer (1998)	Switzerland	Swiss Banks	Estimated Bilateral Interbank Exposures (RAS Algorithm), Overnight Interbank Loans	1987-95
Upper and Worms (2002)	Germany	German banks and foreign banks branches operating in Germany	Estimated Bilateral Interbank Exposures (RAS Algorithm)	December 1998
Wells (2004)	United Kingdom	UK banks	24 Largest Individual Exposures and Estimated Bilateral Interbank Exposures (RAS Algorithm)	December 2000

Appendix B

The maximum entropy methodology for obtaining interbank bilateral exposures

The interbank linkages may be represented by the following $N \times N$ matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix}$$

where $a_i = \sum_{j=1}^N x_{ij}$, $l_j = \sum_{i=1}^N x_{ij}$ are, respectively, the total amount of money bank i lends to other banks and bank j raises from other banks. In absence of any assumption about the distribution of bilateral exposures the matrix X cannot be identified, since $N^2 - 2N$ unknowns have to be estimated.

The common approach is to assume that banks maximize the dispersion of their interbank exposures. Following an appropriate normalization this implies that bilateral exposures are given by a simple solution: $x_{ij}^* = a_i l_j$. However, this solution would imply that for banks that are both lender and borrower in the market they lend to themselves. In order to rule out this outcome is necessary to assume that $x_{ij}^* = 0, \forall i = j$.

Then the problem is to estimate bilateral exposures such that the matrix \hat{X} obtained by the maximization gets as close as possible to matrix X^* . This is generally obtained by minimizing the cross-entropy between the two matrices:

$$\begin{aligned} & \min_{\hat{x}_{ij}} \sum_{i=1}^N \sum_{j=1}^N \ln \left(\frac{\hat{x}_{ij}}{x_{ij}^*} \right) \\ & \text{s.t.} \\ & \sum_{i=1}^N \hat{x}_{ij} = a_i \quad \sum_{j=1}^N \hat{x}_{ij} = l_j \quad \hat{x}_{ij} \geq 0 \quad \forall i \neq j \quad \hat{x}_{ij} = 0 \quad \forall i = j \end{aligned}$$

This problem has been solved numerically by using the RAS algorithm.¹⁸

¹⁸ See Censor and Zenos (1997) for details.