

Income tax, subsidies to education and human capital accumulation in a two-sector economy

Concetta Mendolicchio
IRES, Université Catholique de Louvain
DEIR, Università di Sassari

and

Dimitri Paolini
DEIR and CRENoS, Università di Sassari

and

Tito Pietra¹
DSE, Università di Bologna

Version: February 26, 2009

The paper studies a two-sector economy with investments in human and physical capital and imperfect labor markets. Workers and firms endogenously select the sector they are active in, and choose the amount of their investments. To enter the high-skill sector, workers must pay a fixed cost that we interpret as direct cost of education. The economy is characterized by two different pecuniary externalities. Given the distribution of the agents across sectors, at equilibrium there is underinvestment in both human and physical capital, due to non-contractibility of investments. A second pecuniary externality is induced by the self-selection of the agents in the two sectors.

When the difference between total factor productivities is sufficiently large, subsidies to investments in education in the low skill sector and fixed taxes on the direct costs of education increase expected total surplus, while subsidies to investments in the high skill sector can actually reduce it.

JEL classification: J24; H2

Keywords: Human capital; Efficiency; Labour income tax

Corresponding author: D. Paolini, Tel.: +39 079 2017338; fax +39 079 2017312. *E-mail address:* dpaolini@uniss.it

¹We acknowledge the financial support of MIUR - PRIN 2006. C. Mendolicchio acknowledges the financial support of the "programma visiting professor" of the Università di Sassari (Italy).

1. INTRODUCTION

Causes and consequences of the investments in human capital have been a central field of research in the last few decades. Several distinct motivations have concurred to focus the analysis on this issue. Among them, the relevance of human capital externalities in growth theory (see, Lucas (1988), Romer (1990) and the literature growing out of these seminal papers), and the issues posed by changing wage premia and, more generally, income distribution (see, for instance, Heckman and Krueger (2003), Goldin and Katz (2007), Gordon and Dew-Becker (2008) and their extensive references). The analysis of human capital externalities is still far from settled from both the empirical and the theoretical viewpoints. Empirically, it is not obvious that (at the level of subsidies prevailing in most Western countries) there are significant, positive differences between social and private returns². From a theoretical viewpoint, the precise microeconomic mechanism generating the externality is not fully understood (see, however, Acemoglu (1996)). Even if one is willing to take for granted that, at least in many countries, there are no significant, unexploited, positive externalities, this result is typically obtained with very high subsidies to education³. A better understanding of the nature of the externality has, therefore, policy relevance.

In this paper, we extend the microeconomic analysis of the externalities related to investments in human capital. We also derive some results on the welfare effects of different policies: fixed tax/subsidies to education, which affect its direct cost, and tax/subsidies on labor income⁴.

We consider a class of economies with two key features. First, we adopt the notion of human capital put forth in Roy (1951) (see, also, Willis and Rosen (1979)): there are distinct markets for (non substitutable) skilled and unskilled labor. However, contrary to what is often assumed in models *à la* Roy, once a worker has selected the type of human capital he wants to acquire, he still has to decide how much effort to invest. The human capital so acquired translates one-to-one into efficiency units of high skill (low skill, respectively) labor⁵. When agents, through schooling, self-select into different labor markets, the effect of public policies on total (expected) surplus works via two different channels. The first is the standard one: their impact on the amount of effort of the agents acquiring a specific skill

²For the U.S.A., a negative conclusion is reached, for instance, by Heckman, Layne-Farrar, and Todd (1996) and by Acemoglu and Angrist (2001). For E.U. countries, the results in De la Fuente (2003) are also negative. See also Krueger and Lindhal (2000).

³In 2005, according to OECD (2008, Table B3.1, p.251), in the OECD average 85.5% of the direct cost of education (all levels included) is financed by public sources. The EU19 average is 90.5%. At the tertiary level, the percentages are, respectively, 73.1% and 82.5% (Table B3.2b, p. 253).

⁴In both cases, we also introduce uniform lump-sum taxes/subsidies on workers, so that the public budget is balanced.

⁵As usual, we can also interpret effort as elastic supply of labor of a given skill.

(we will refer to it as *incentive effect*). The second is their impact on the agents' distribution across markets (following Charlot and Decreuse (2005), we call it *composition effect*). The results in terms of policy implications crucially depend on the interaction of the two effects. As we will argue in more detail, in "pure" Roy's economies (with self-selection, but no choice of the investment effort) only the composition effect is at play. In "pure" efficiency-units economies (without self-selection) only the incentive effect is at work. Our (simple, but) encompassing model allows us to study the interaction between the two. The second essential feature of our economy is that investments in human and physical capital are not contractible ex-ante, so that agents must base their investment decisions on the (conditional) distribution of their payoffs. Lack of contractibility generates constrained inefficiency of the market equilibrium, i.e., the equilibrium total (expected) surplus is lower than its constrained efficient level.

There are (at least) two classes of economies exhibiting these key features. The first is based on a two-sector version of the economy considered in Acemoglu (1996). In his model, firms and workers choose the amount of their investments. After that, they are matched randomly, production takes place, and income distribution is determined by a bargaining process. In our version, there are two separate sectors. In the first one, firms use capital and skilled labor. In the second, capital and unskilled labor are used. Firms and workers first choose the sector they will be active in. Afterward, each sector is just an Acemoglu's economy (so that workers choose their optimal effort in the acquisition of human capital).

A second class of economies is characterized by a continuum of separate islands. On each island there is a continuum of workers and firms. They choose simultaneously the sector they are going to be active in, and the amount of their investments. While all the firms are identical ex-ante, workers are identical in each island, but heterogeneous across them. The source of heterogeneity is some parameter affecting the individual labor supply. Its realization on each island is private information of the workers. After investments take place, competitive labor markets open, wages are determined at their competitive market clearing values, and production takes place. In this set-up, investments are not contractible ex-ante and, when investments are made, firms and workers base their actions on their conditional expectations of future wages. In Appendix 2, we show that the closed form of the rational expectation equilibrium of this economy is, essentially, identical to the one obtained for the previously model⁶. Given that the first type of economies are simpler from the notational viewpoint, we consider them in the main body of the paper.

As already mentioned, the two key features of our economy are lack of contractibility of investments and a Roy's model of human capital. Most of

⁶There are differences with respect of the exact nature of constrained inefficiency. They change the details of the analysis, but not the central message.

the recent literature adopts a different viewpoint, considering the efficiency unit interpretation of human capital (see Griliches (1970)). This rules out by assumption all the consequences of self-selection, which appear, instead, to be relevant from the theoretical and the empirical viewpoints⁷.

Acemoglu (1996) studies the microeconomic foundations of externalities in human capital accumulation. Due to the two-sector structure of our economy, we obtain sharply different policy prescriptions, because of the composition effect. Not surprising, some of the other results are identical. For instance: in both classes of economies, the externality is related to the average level of human capital (which coincides with its conditional expectation), not to its aggregate level (as postulated in Lucas (1988)). In Acemoglu (1996), subsidies to labor income (or to investments in human capital) are unambiguously beneficial (actually, they are Pareto improving). Only the incentive effect is at play: a subsidy to the investments in human capital of *any subset* of agents increases, as a first order effect, their investments and, therefore, the expected human capital. This has a positive impact on the firms' investment decisions. In turn, an increase in the expected physical capital increases the optimal investment in human capital of all the workers. Hence, these subsidies are always Pareto improving. To reformulate the point differently: in one-sector economies, there is a unique (pecuniary) positive externality related to the level of investments in human (and physical) capital. Any policy increasing the investments of any subset of agents is Pareto improving.

In a two-sector economy, the incentive effect of a policy can be strengthened or weakened by its composition effect. Specifically, in the final section of the paper, we consider a (balanced budget) policy based on tax/subsidies to the direct costs of education and on skill-contingent subsidies to labor income. If the difference in total factor productivities across sectors is sufficiently high, subsidies to low skilled labor income always increase expected total surplus, because their positive effect on individual effort in this sector is strengthened by the composition effect, i.e., by the "improvement of the expected quality" of the pool of workers in both markets. An increase in taxes on the direct costs of education, also increases total expected surplus, because of its composition effect. On the other hand, subsidies to high skilled labor incomes have a positive incentive effect for these workers, but

⁷A survey supporting this claim is in Sattinger (1993). For more recent discussions of the different empirical implications of efficiency units vs. Roy's models see, for instance, Carneiro, Heckman, and Vytlačil (2001). They conclude (p.32) that "The data suggest that comparative advantage is an empirically important phenomenon governing schooling choices and that naive efficiency units models of the labor market are empirically inappropriate". Investments in human capital in a two-sector economy with frictions due to random matching have been studied in Sattinger (2003), Charlot and Decreuse (2005), and Mendolicchio, Paolini, and Pietra (2008). The composition effect plays an essential role in their results. However, they consider economies with perfectly inelastic supply of human and physical capital, so that the incentive effects of public policies are absent.

a negative composition effect. We provide robust examples where the net effect on total expected surplus is negative. To reformulate the point, in two-sector economies there are two distinct pecuniary externalities at play. The one related to the investments in the low-skill sector is always positive for all the agents in the economy. To the contrary, the one related to the investments in the high-skill sector is always negative for low-skilled workers (and firms active in that sector), it may be positive or negative for agents active in the high-skill sector.

We consider a simple parametric class of economies, with Cobb-Douglas production functions and quasi-linear utility functions. This allows us to compute explicitly the equilibrium, so that we can evaluate the welfare impact of the policies by direct computation. Because of quasi-linearity of preferences, our analysis abstracts from any substantive consideration of distributional issues to focus only on aspects related to the (expected) total surplus. Quasi-linearity also implies that the lump-sum taxes used to balance the government budget have no incentive effect. This clearly simplifies the argument. However, similar result must hold for any economy at least if income effects are sufficiently small.

The economy is highly stylized. For instance, acquisition of human capital is deterministic and instantaneous, so that there are no opportunity costs of education. An extension of the analysis to a richer environment is possible, but at an high cost in terms of analytical tractability. What it matter most, we believe that the basic intuition behind the welfare results is strong and that they should be robust (maybe, in a less sharp form) to many possible extensions of the basic set-up. We take as benchmark an economy with no subsidies to the direct costs of education and no income taxes. In the last section, we analyze the welfare effects of (small) taxes on direct costs and (small) subsidies on labor income. We would obtain exactly the same results taking as a benchmark a more descriptively realistic economy where there are subsidies on the direct costs of education and a flat tax on labor income. The obvious reinterpretation of our result is in terms of lower direct subsidies and some (small) degree of progressiveness in the tax schedule. Finally, subsidies to labor income are relevant in our economy for their indirect effects on the amount of effort invested in education. We focus the analysis on income subsidies for three reasons: they deliver analytically tractable, closed form, values of the equilibrium variables. With the (obvious) reinterpretation provided above, they are a feature of real world economies. The effect of labor income taxes on investment in human capital has been extensively analyzed in the literature, and, therefore, our results can be easily compared with previous work in this field. However, it is worth to point out that analytically identical results can be obtained considering properly specified direct subsidies to the effort in education⁸.

⁸Alas, to obtain exactly the same closed forms of the equilibrium variables, public incentive to effort must take on a specific (and non particularly appealing) functional

There is a very large literature on the effects of subsidies to education and of labor income taxes on accumulation of human capital. The usual arguments favoring subsidies hinge either on their positive externality effects or on the existence of liquidity constraints. While these second are in principle important, they are (deliberately) irrelevant in our set-up. The classical analysis of the effects of labor income tax on the individual choices regarding investments in human capital started with the seminal papers by Ben-Porath (1970), Boskin (1975) and Heckman (1976)⁹. A flat labor income tax has a negative impact on human capital accumulation just because of non-deductibility of the direct costs of education. On the other hand, by depressing the net interest rate, in fully specified life-cycle models of consumer behavior an *earning* tax rate may actually have a positive effect on investments in human capital. Eaton and Willis (1980) extend the analysis to (uninsurable) multiplicative wage uncertainty, pointing out that a flat earning tax affects investments in human capital through its effects on the riskiness of the investment and (via an income effect) on the attitude toward risk.

The canonical conclusion regarding progressive income taxes (compared with a revenue neutral flat rate) is that they discourage investments in human capital at the high skill level, while they may encourage it by the less skilled. More recent theoretical contributions include Anderberg and Andersson (2003), Bovenberg and Jacobs (2003), Jacobs (2003, 2007), Jacobs and Bovenberg (2008), Jacobs, Schindler and Yang (2009) (see, also, Heckman, Lochner and Taber (1998, 1999)). However, in all these papers, there is no self-selection into different skills so that the key mechanism at work in our economy is absent. Also, notice that, at the equilibrium, workers face no uncertainty, so that the mechanism pointed out in Eaton and Willis (1980) and subsequent work focussed on the effects of income taxation under wage uncertainty is absent here.

The structure of the paper is the following. Section 2 discusses the general features of the model. Section 3 and 4 discuss equilibria in the benchmark, Walrasian, economy, and in the economy with imperfect labor markets. Section 5 studies the properties in terms of welfare of the equilibria of the economy with frictions. Most of the proofs are in Appendix 1. Appendix 2 sketches the analysis of the "island" model outlined above.

form (see note 13 below). Moreover, to introduce incentives of this kind begs the problem of direct observability of individual effort.

⁹As mentioned above, in our set-up, one obtains substantially identical results considering direct (non-linear) subsidies to effort and subsidies to the direct costs of education. Previous, related work includes Blankenau (2004), Blankenau and Camera (2006, 2008), Caucutt and Kumar (2003), Lloyd-Ellis (2000), Sahin (2004), and Su (2004).

2. THE MODEL

The economy is composed by two separate production sectors, denoted by $s \in \{ne, e\}$. Workers (denoted by a subscript i when we refer to individuals, I when we refer to the set) and firms (denoted by j and J , respectively) can choose to enter one of the two sectors, paying a fixed cost. Workers' costs, (c_I^{ne}, c_I^e) , are exogenous, and can be interpreted as private, fixed costs of education (tuitions and the like). We denote firms' costs (d_J^{ne}, d_J^e) . They are endogenously determined, and will be discussed later on.

There are two intervals of equal length of workers, $\Omega_I = (0, 1)$, and firms, Ω_J , both endowed with the Lebesgue measure. Let $\nu(\Omega_I^s)$ ($\nu(\Omega_J^s)$) denote the measure of the set Ω_I^s (Ω_J^s , respectively). At equilibrium, each interval is partitioned into two sets, $\{\Omega_I^{ne}, \Omega_I^e\}$ and $\{\Omega_J^{ne}, \Omega_J^e\}$, determined endogenously. In sector s , production requires a firm j (with physical capital k_j^s) and a worker i (with stock of human capital h_i^s). Once the partitions $\Omega_I^P \equiv \{\Omega_I^{ne}, \Omega_I^e\}$ and $\Omega_J^P \equiv \{\Omega_J^{ne}, \Omega_J^e\}$ are given, each sector of the economy reduces to the set-up studied in Acemoglu (1996). The only difference is that his analysis is mostly devoted to the case of homogeneous workers and firms, while we (necessarily) always consider the case of heterogeneous workers. Firms are identical, and choose their investments in physical capital to maximize their expected profits. Workers choose their investments in human capital to maximize their expected utilities.

The economy lasts one period, divided into several subperiods. We consider two versions of the basic model. In the frictionless (or Walrasian) version, in subperiod zero, firms and workers enter (paying a fixed cost) one of the two sectors. In subperiod 1, each firm active in sector s is matched with a worker active in the same sector (we will be more precise on the matching issue later on) and firms and workers can sign binding contracts on the amount of human and physical capital that they will supply. In subperiod 2, investments are carried out. In the final subperiod, 3, exchanges and production take place, and agents are paid on the basis of their marginal product.

In the second version of the model, the total output of each match is split according to the Nash bargaining solution with (exogenous) weights β and $(1 - \beta)$ (for a rationalization of this allocation rule in this context, see the Appendix in Acemoglu (1996)). Moreover, and most important, agents cannot contract with their partner a given level of investment, because they are carried out before matches take place. To summarize: in subperiod 0, agents choose to enter one of the two sectors, paying a fixed entry cost. In subperiod 1, they choose their levels of investment. In subperiod 2, they are randomly matched and then, finally, production and exchanges take place.

When workers are heterogeneous, the friction in the labor market, due to the determination of labor income through bargaining, has a very lim-

ited role in determining the efficiency properties of equilibria. The crucial feature is that agents choose their investments considering the conditional expectation of the investments of a (non trivial) set of potential future partners. Indeed, as long as investments are non-contractible, the same qualitative results hold, even if the spot labor markets are perfectly competitive¹⁰. This case is briefly considered in Appendix 2.

To avoid additional complications (not really germane to the main issue analyzed here), we want to avoid to introduce the possibility of unemployment. This requires that, at equilibrium, each worker, and each firm, active in a sector is actually matched with a partner. Technologies are described by a pair of Cobb-Douglas production functions with constant returns to scale. Therefore, in the Walrasian set-up, equilibrium profits are zero, entry costs d_j are zero for each s , and the equilibrium partition Ω_J^P is essentially arbitrary. Therefore, we can set $\Omega_J^s = \{j \in \Omega_J | j = i, i \in \Omega_I^s\}$, each s . On the contrary, in the economy with frictions, expected producers' surpluses are positive in both sectors and, as we will show later on, larger in sector e . We can obtain that the equilibrium value of Ω_J^P is indeterminate (so that we can put $\Omega_J^s = \{j \in \Omega_J | j = i, i \in \Omega_I^s\}$, each s) as follows¹¹. Assume that, as in Acemoglu (1996), the matching function guarantees with probability one a match to each agent, provided that $\nu(\Omega_I^s) = \nu(\Omega_J^s)$. A commonly used matching function which delivers this property is $\pi_j^s = \frac{\min\{\nu(\Omega_I^s), \nu(\Omega_J^s)\}}{\nu(\Omega_J^s)}$

(and $\pi_i^s = \pi_j^s \frac{\nu(\Omega_I^s)}{\nu(\Omega_I^s)}$), where π_j^s is the probability of a match for a firm active in sector s . The partition Ω_J^P is determined endogenously. The easiest way to guarantee that $\nu(\Omega_I^s) = \nu(\Omega_J^s)$ is to introduce a feature of the economy such that the equilibrium expected profits are always equal in the two sectors, so that the equilibrium value of Ω_J^P is indeterminate. One way to obtain this property is to assume that the technology exploited in sector ne is free, while the one adopted in sector e is protected by a patent, owned by some outside agent (clearly, nothing would change if both technologies were subject to distinct patents)¹². Rights to use the patent are auctioned off to firms before the match firm-worker obtains. Given that, at an equilibrium, expected profits in both sectors must be identical, the equilibrium royalties (that we identify with d_J) must be equal to the (positive) difference between the expected producer's surpluses in the two sectors. It follows

¹⁰In Acemoglu (1996), the benchmark is an economy with identical workers and firms. Evidently, in this case, if spot labor markets were competitive, we would end up with the Pareto efficient, complete markets allocation.

¹¹An alternative solution is to assume that firms cannot move across sectors. A non-null measure of firms is exogenously assigned to each sector. The matching function guarantees that each firm is matched with a worker (and conversely) for each non-trivial partition of the workers. As long as there is a continuum of workers and firms in each sector, this can be done. Of course, this approach would break down if we had a finite number of agents and, anyhow, is based on a very *ad hoc* trick.

¹²Of course, any input used only in sector e and with perfectly inelastic supply would do. We consider the case of a patent to simplify as much as possible the model.

that, at each equilibrium, Ω_j^F is essentially arbitrary. The property we are looking for.

Without any loss of generality, we can take the prices of both kinds of output to be equal to 1 and, therefore, omit them.

Finally, notice that there are always three additional equilibria: the ones where all workers and firms are in one of the two sectors, and the one where none is active in any sector. This is because, evidently, if no firm is active in sector s , every worker moves to the other sector (and conversely). As usual, we will ignore these trivial equilibria.

3. THE FRICTIONLESS ECONOMY

When active in sector s , and matched with worker i with human capital h_i^s , firm j has production function

$$y_{ij}^s = A^s h_i^{s\alpha} k_j^{s(1-\alpha)},$$

with $A^e > A^{ne}$.

Let μ be the unit price of physical capital, that we assume to be equal in the two sectors. This implies some loss of generality, but allows for more straightforward computations. Under suitable restrictions on the values of $\frac{A^s}{\mu^s}$, similar results could be obtained for $\mu^e \neq \mu^{ne}$.

In sector s , given a match with worker i , firm j solves optimization problem

$$\text{choose } k_j^s \in \arg \max A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu k_j^s - w_{ij}^s h_i^s, \quad (\Pi^{W^s})$$

where we omit the fixed entry costs, because, at equilibrium, they must be zero.

For each worker active in sector s , the utility function is

$$U_i^s(C_i^s, h_i^s) = C_i^s - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma},$$

where C_i^s denotes consumption, h_i^s is the amount of human capital (or the labor supply). Let c_I^s be the (fixed) cost of the investment in sector s human capital. Then, in the absence of taxes and subsidies, if worker i is active in sector s and matched with firm j , $C_i^s = (w_{ij}^s h_i^s - c_I^s)$. Workers are heterogeneous because of the parameter δ_i . Without any essential loss of generality, we assume $\delta_i = i$, so that this parameter is uniformly distributed on $(0, 1)$. To introduce more general assumptions on the distribution of δ_i and its support would introduce additional computational complexities without changing the essential results. Given that, in the sequel, we will introduce uniform lump sum taxes, we must (alternatively) allow for negative consumption, or assume that workers have a strictly positive (and sufficiently large) initial endowment of consumption good. Given the properties of the

utility functions, the most convenient (essentially, notation-wise) solution is the first one.

By a straightforward computation, the equilibrium amount of agent i 's investment in human capital in sector s is given by

$$H^{Ws}(\delta_i) \equiv \left[\delta_i \alpha A s^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\mu} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{1}{\Gamma}},$$

where the superscript W denotes the frictionless, Walrasian economy. Given that, at the equilibrium, profits are always zero and firms are identical, there is no loss of generality in assuming that firm j is always matched with worker $i = j$. With this convention, the (equilibrium) demand for physical capital of firm $j = i$ is

$$K^{Ws}(\delta_i) \equiv \left[\delta_i A s^{\frac{1+\Gamma}{\alpha}} \alpha \left(\frac{1-\alpha}{\mu} \right)^{\frac{1+\Gamma-\alpha}{\alpha}} \right]^{\frac{1}{\Gamma}}.$$

Let's now consider the equilibrium partition Ω_I^P . For convenience (and without any loss of generality), set $c_I^e = 0$ and $c_I^s > 0$. Let

$$V_i^{Ws}(\delta_i, c_I^s) \equiv U_i^s(H^{Ws}(\delta_i), K^{Ws}(\delta_i), c_I^s).$$

Then, worker i chooses to enter sector e if and only if

$$V_i^{We}(\delta_i, c_I^e) - V_i^{Wne}(\delta_i) \geq 0,$$

i.e., by direct computation, if and only if

$$\delta_i \geq \delta^W(c_I^e) \equiv \left[\frac{(1+\Gamma)\mu^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}}{\Gamma \left[\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \right]^{\frac{1+\Gamma}{\Gamma}} \left[A e^{\frac{1+\Gamma}{\alpha\Gamma}} - A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \right]} c_I^e \right]^{\Gamma}. \quad (1)$$

Hence, for c_I^e positive and sufficiently small, there is a unique threshold value $\delta^W(c_I^e)$, strictly increasing in c_I^e .

Clearly, the physical-human capital ratio is δ_i -invariant, with $\frac{K^{Ws}(\delta_i)}{H^{Ws}(\delta_i)} = \left(\frac{(1-\alpha)A^s}{\mu} \right)^{\frac{1}{\alpha}}$ and $\frac{K^{We}(\delta_i)}{H^{We}(\delta_i)} > \frac{K^{Wne}(\delta_i)}{H^{Wne}(\delta_i)}$, each i .

4. THE ECONOMY WITH FRICTIONS

The key feature of this economy is that matches take place after the non-contractible investments are carried out. Hence, agents must base their choices on their conditional expectations of the investments of the (potential) partners.

Given any random variable x^s , with $x^s : \Omega_I^s \rightarrow \mathbb{R}$, (or y^s , with $y^s : \Omega_J^s \rightarrow \mathbb{R}$), let

$$E_{\Omega_I^s}(x_i^s) \equiv \frac{\int_{\Omega_I^s} x_i^s di}{\nu(\Omega_I^s)}$$

$(E_{\Omega_J^s}(y_j^s))$ be the conditional expectation of x_i^s over the set Ω_I^s (of y_j^s over Ω_J^s). Later on, we will show that, at the equilibrium, it is always $\Omega_I^e = [\delta^*, 1)$. Therefore, in the sequel, the partitions Ω_I^P and Ω_J^P will be defined by the threshold level δ^* . To emphasize it, we will often use the notation $\Omega_J^s(\delta^*)$ and $\Omega_I^s(\delta^*)$. We will also use δ^F to denote the equilibrium threshold value, where the superscript F denotes the economy with frictions.

For future reference, let's determine the optimal amount of investments assuming that there is a public intervention defined by a pair of vectors $\xi^s = (\tau^s, \zeta^s, \Delta c_I^s, T)$, $\xi = (\xi^e, \xi^{ne})$ describing (possibly) sector specific subsidies and taxes. We assume that there are linear subsidies on labor income (described by the subsidy rates τ^s , $s = ne, e$), and on the cost of the investments in physical capital (with rates ζ^s , $s = ne, e$) and fixed taxes on the direct costs of education, Δc_I^s . T denotes a lump-sum tax, defined so that the public budget is balanced. We write the subsidy rates as sector specific just to simplify the notation. As we will show in the sequel, at the equilibrium, the investment in physical capital is always larger in sector e , and the labor income of each worker in sector e is always strictly greater than the one of any worker active in sector ne . Therefore, the subsidy system considered is (at the equilibrium) isomorphic to a standard system of step-linear subsidies to labor income and to investments in physical capital¹³.

Pick an arbitrary threshold value δ^* . If active in sector s , firm j selects the value of k_j^s solving optimization problem

$$\begin{aligned} & \max_{k_j^s} E_{\Omega_I^s(\delta^*)} \left((1 - \beta) A^s h_i^{s\alpha} k_j^{s(1-\alpha)} - \mu (1 - \zeta^s) k_j^s \right) - d_J^s \\ & = (1 - \beta) A^s E_{\Omega_I^s(\delta^*)} (h_i^{s\alpha}) k_j^{s(1-\alpha)} - \mu (1 - \zeta^s) k_j^s - d_J^s. \quad (\Pi^{Fs}) \end{aligned}$$

As mentioned before, we interpret d_J^s as royalties paid to access the technologies used in the two sectors. At equilibrium, we set $d_J^{ne} = 0$ and d_J^e is equal to the (positive, as we will show) difference between the conditional expected producer's surpluses in the two sectors. Therefore, each firm is indifferent between the two sectors (and has non-negative conditional expected profits). The first order conditions (FOCs in the sequel) of problem (Π^{Fs}) imply that

$$k_j^s(E_{\Omega_I^s(\delta^*)}(h_i^{s\alpha}), \xi^s) = \left[\frac{(1 - \beta)(1 - \alpha) A^s E_{\Omega_I^s(\delta^*)}(h_i^{s\alpha})}{\mu(1 - \zeta^s)} \right]^{\frac{1}{\alpha}} \quad (2)$$

¹³Exactly the same closed form of the equilibrium is obtained considering a direct subsidy to effort in education of the form $\tau^s h_i^{s\alpha}$, which requires direct observability of effort.

The optimization problem of worker i (if s) is

$$\begin{aligned} & \max_{h_i^s} E_{\Omega_j^s(\delta^*)} (U_i^s(\cdot)) && (U^{Fs}) \\ & = (1 + \tau^s) \beta A^s h_i^{s\alpha} E_{\Omega_j^s(\delta^*)} (k_j^{s(1-\alpha)}) - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1 + \Gamma} - (c_I^s + \Delta c_I^s + T). \end{aligned}$$

Its FOCs imply that

$$h_i^s \left(E_{\Omega_j^s(\delta^*)} (k_j^s), \xi \right) = \left[\delta_i \alpha \beta (1 + \tau^s) A^s E_{\Omega_j^s(\delta^*)} (k_j^{s(1-\alpha)}) \right]^{\frac{1}{1+\Gamma-\alpha}}. \quad (3)$$

Given that firms in sector s are, ex-ante, identical, $k_j^s(\cdot) = k^s(\cdot)$, and $E_{\Omega_j^s(\delta^*)} (k_j^s(\cdot)^{(1-\alpha)}) = k^s(\cdot)^{(1-\alpha)}$. Solving (2) and (3), we obtain

$$\begin{aligned} K^{Fs}(\delta^*, \xi^s) & = \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)} E_{\Omega_i^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right]^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \\ & \quad \times ((1 + \tau^s) \alpha \beta)^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}}, \end{aligned} \quad (4)$$

and

$$\begin{aligned} H^{Fs}(\delta_i, \delta^*, \xi^s) & = \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)} E_{\Omega_i^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right]^{\frac{1-\alpha}{\alpha\Gamma}} \\ & \quad \times \delta_i^{\frac{1}{1+\Gamma-\alpha}} ((1 + \tau^s) \alpha \beta)^{\frac{1}{\Gamma}} A^{s\frac{1}{\alpha\Gamma}}. \end{aligned} \quad (5)$$

From (4) and (5), agent i 's (conditional) expected utility is

$$\begin{aligned} V_i^s(\delta_i, \delta^*, \xi^s) & \equiv E_{\Omega_j^s(\delta^*)} (U_i^s(\delta_i, \delta^*, \xi^s)) = -(c_I^s + \Delta c_I^s + T) && (6) \\ & + \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)} E_{\Omega_i^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right]^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \\ & \quad \times \delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \beta^{\frac{1+\Gamma}{\alpha\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}} [(1 + \tau^s) \alpha]^{\frac{1}{\Gamma}} \frac{1 + \Gamma - (1 + \tau^s) \alpha}{1 + \Gamma}, \end{aligned}$$

while the (conditional) expected producer' surplus is

$$\begin{aligned} E_{\Omega_i^s(\delta^*)} (\Pi^{Fs}(\delta_i, \delta, \xi)) & = \left[\frac{(1-\alpha)(1-\beta)}{\mu(1-\zeta^s)} E_{\Omega_i^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right]^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \\ & \quad \times \frac{\mu(1-\zeta^s) a [(1 + \tau^s) \alpha \beta]^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}}}{(1-\alpha)}. \end{aligned} \quad (7)$$

Worker i enters sector e if and only if

$$V_i^{Fe}(\delta_i, \delta^*, \xi^e) - V_i^{Fne}(\delta_i, \delta^*; \xi^{ne}) \geq 0.$$

Then, by direct computation, the equilibrium threshold value δ^F is obtained solving

$$\begin{aligned}
0 &= f(\delta^*, \xi) - a(c_I^e + \Delta c_I^e) \\
&\equiv \sum_s (-1)^{\chi(s)} \delta^{* \frac{\alpha}{1+\Gamma-\alpha}} \left(A^s E_{\Omega_I^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{(1-\alpha)} \right)^{\frac{1+\Gamma}{\alpha}} \\
&\quad \times \frac{(1+\tau^s)^{\frac{1}{\Gamma}} (1+\Gamma - (1+\tau^s)\alpha)}{(1-\zeta^s)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}} - a(c_I^e + \Delta c_I^e),
\end{aligned} \tag{8}$$

where $\chi(s) = 1$ if $s = ne$, $\chi(s) = 2$ if $s = e$, and

$$a \equiv \frac{1+\Gamma}{\alpha^{\frac{1}{\Gamma}} \beta^{\frac{1+\Gamma}{\Gamma}}} \left(\frac{\mu}{(1-\alpha)(1-\beta)} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}}.$$

Remark 1. Consider an economy associated with $\xi = 0$. Given that $E_{\Omega_I^e(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) > E_{\Omega_I^{ne}(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)$, and $A^e > A^{ne}$, we always have $E_{\Omega_I^e(\delta^*)} \left(\Pi^{Fe}(\delta_i) \right) > E_{\Omega_I^{ne}(\delta^*)} \left(\Pi^{Fne}(\delta_i) \right) > 0$. Hence, as claimed above, at the equilibrium,

$$d_J^{e*} = \left[E_{\Omega_I^e(\delta^F)} \left(\Pi^{Fe}(\delta^F) \right) - E_{\Omega_I^{ne}(\delta^F)} \left(\Pi^{Fne}(\delta^F) \right) \right] > 0.$$

The following Proposition summarizes the fundamental properties of the equilibria. The proof is in Appendix 1.

PROPOSITION 1. *Fix (Γ, α, β) . Given (A, c_I, ξ) , with $\xi = 0$, there is $\tilde{C} > 0$ such that, for each $c_I^e \in (0, \tilde{C})$, there is an equilibrium with threshold value $\delta^F(A, c_I, \xi) \equiv \delta^F(\cdot) \in (0, 1)$. Moreover, given A^{ne} , there is A^{e*} such that, for each $A^e > A^{e*}$, at $\xi = 0$, the equilibrium is unique and $\frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F} > 0$, $\frac{\partial \delta^F(\cdot)}{\partial \tau^e} < 0$, $\frac{\partial \delta^F(\cdot)}{\partial \tau^{ne}} > 0$ and $\frac{\partial \delta^F(\cdot)}{\partial \Delta c_I^e} > 0$.*

In the sequel, we will mostly consider the leading case where $\frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F} > 0$ at each equilibrium threshold.

Remark 2. Given $\xi = 0$ and $(A^{ne}, \Gamma, \alpha, \beta)$, for A^e sufficiently close to A^{ne} the economy can exhibit multiple equilibria. This is established in Example A1 in Appendix. There, we consider an economy with $\frac{\partial f(\cdot)}{\partial \delta^*} > 0$ for δ^* sufficiently close to 0, and $\frac{\partial f(\cdot)}{\partial \delta^*} < 0$ for δ^* sufficiently close to 1. Given that $\frac{\partial f(\cdot)}{\partial \delta^*}$ is a continuous function on $(0, 1)$, $f(\cdot)$ has a local maximum, $\bar{\delta}$. For $c_I^e < f(\bar{\delta})$ and close enough to $f(\bar{\delta})$, this economy has at least two equilibria.

Remark 3. It is easy to check that, modulo a redistribution of output, the Walrasian allocation is the unique Pareto efficient allocation of this economy. With elastic supply of both human and physical capital, no allocation rule (i.e., no value of β) can guarantee Pareto efficiency of the equilibrium allocation, because $K^{Fs}(\delta^*, \xi^s)$ is δ_i -invariant.

Remark 4. The physical/human capital ratio is given by

$$\frac{K^{Fs}(\delta^F)}{H^{Fs}(\delta_i, \delta^F)} = \frac{K^{Ws}(\delta_i)}{H^{Ws}(\delta_i)} \left[\frac{(1-\beta)^{\frac{1}{\alpha}} E_{\Omega_i^s}(\delta^F) \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{1}{\alpha}}}{\delta_i^{\frac{1}{1+\Gamma-\alpha}}} \right].$$

In sector ne , and for sufficiently small δ_i , the term in square brackets is always greater than one, so that $\frac{K^{Fne}(\delta^F)}{H^{Fne}(\delta_i, \delta^F)} > \frac{K^{Wne}(\delta_i)}{H^{Wne}(\delta_i)}$ for δ_i small enough. This immediately implies that agents with a sufficiently low δ_i are always better off at the equilibrium of the frictional economy. Hence, the Walrasian equilibrium allocation is not Pareto superior to the one of the economy with frictions (of course, it still dominates it in terms of expected total surplus)¹⁴.

Remark 5. In the economy with frictions, the threshold value δ^F can be either lower or higher than its value in the Walrasian economy, as we establish with the following example.

EXAMPLE 1. Let $\xi = 0$. Consider the economy with $A^e = 2$, $A^{ne} = 1$, $\alpha = \beta = 1/2$, and $\Gamma = 1$. By direct computation, we obtain the two following maps

$$f^F(\delta^*) = \frac{27\sqrt[3]{\delta^*}}{8192} \left(\frac{16 \left(\delta^{*\frac{4}{3}} - 1 \right)^2}{\delta^{*2} - 2\delta^* + 1} - \delta^{*\frac{2}{3}} \right)$$

(defined in (8) above) in the economy with frictions, and

$$f^W(\delta^*) = \frac{15}{32} \delta^*$$

(similarly defined using eq. (1) above) in the Walrasian economy. They are shown in Figure 1. One can verify that, for $c_I^e < 0.019$, $\delta^F < \delta^W$, while, for $c_I^e > 0.019$, the opposite occurs.

¹⁴Given the structure of preferences, if the expected total surplus in the economy with frictions were larger than the one of the Walrasian economy, we would contradict the first fundamental theorem of welfare economics.

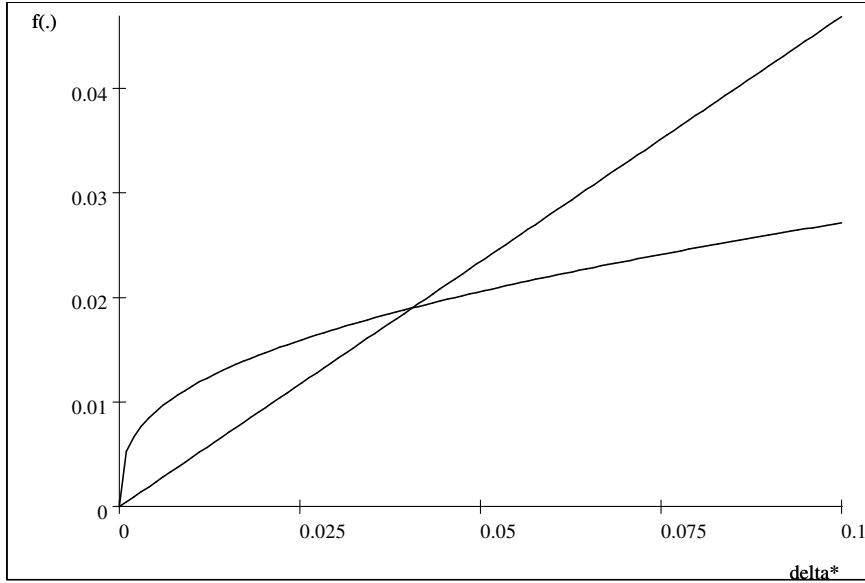


Figure 1

5. INEFFICIENCY PROPERTIES OF THE ECONOMY WITH FRICTIONS

We have already seen (in Remark 3) that the equilibria of the economy with frictions are Pareto inefficient. We will show that they do not satisfy either a weaker criterion of *constrained optimality* (CO in the sequel) which takes into account the imperfections which characterize the economy. Most interesting is the analysis of their inefficiency in terms of its effects on the amount, and the type, of investments. In the sequel, we will mainly refer to the investments in human capital. Similar considerations hold for the ones in physical capital.

In our set-up, inefficiencies can be of two different types. First, an individual can choose an *amount* of investment different from the CO one, *given* the partition Ω_I^P associated with the CO allocation. We will refer to this possible source of inefficiency as *underinvestment* (or *overinvestment*) *in educational effort*. Secondly, an agent can choose to invest in a *type* of education different from the one assigned to her at the CO allocation. We will say that there is *underinvestment in educational type* when agent i invests in education ne , while, at the CO allocation, she should invest in education level e .

In the one-sector model, equilibria are unambiguously characterized by underinvestment in educational effort and in physical capital. In our set up, the same effect is at work: in each sector, given δ^F , an increase in

the investments of firms and workers leads to a Pareto improvement. The argument is identical to the one exploited by Acemoglu (1996): fix δ^F and consider a small change in h_i^s and k_j^s , each i and j . The changes in utilities and producers' surplus evaluated at the equilibrium pair (h_i^{s*}, k_j^{s*}) (and taking into account that $k_j^{s*} = k^{s*}$, each j and s), are given by

$$0 < \left(\alpha \beta A^s \left[\frac{k^{s*}}{h_i^{s*}} \right]^{1-\alpha} - \frac{1}{\delta_i} h_i^{s*\Gamma} \right) dh + \left((1-\alpha) \beta A^s \left(\frac{h_i^{s*}}{k^{s*}} \right)^\alpha \right) dk, \quad (9)$$

and

$$0 < \left((1-\alpha)(1-\beta) A^s \frac{E_{\Omega_i^s(\delta^F)}(h_i^{s\alpha})}{k^{s*\alpha}} - \mu \right) dk + \left(\alpha(1-\beta) A^s \frac{k^{s*(1-\alpha)}}{E_{\Omega_i^s(\delta^F)}(h_i^{s(1-\alpha)})} \right) dh, \quad (10)$$

respectively. The inequalities hold because the first terms in parenthesis in both (9) and (10) are zero (at the optimal solutions of (Π^{Fs}) and (U^{Fs})), while the second terms are clearly positive. Hence, in each sector, there is underinvestment in both educational effort and physical capital. This establishes in a more direct way Pareto inefficiency of equilibria in the economy with frictions.

In the two-sector case, there is a second potential source of inefficiency, because changes in the value of δ^* may also entail Pareto improvements. An increase in the threshold value δ^* increases the conditional expected amount of human capital in both sectors at the same time and, consequently, induces an increase in the amount of physical investments of firms in both sectors. Indeed, given that $\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}$ is strictly monotonically increasing,

$$\frac{\partial E_{\Omega_i^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)}{\partial \delta^*} > 0, \text{ for each } s \text{ and } \delta^*, \quad (11)$$

and, consequently, using (4) and (5),

$$\frac{\partial H^{Fs}(\delta_i, \delta^*)}{\partial \delta^*} > 0 \text{ and } \frac{\partial K^{Fs}(\delta^*)}{\partial \delta^*} > 0, \text{ for each } s \text{ and } \delta^*. \quad (12)$$

More relevant, from (6), (7) and (11), one obtains immediately that

$$\frac{\partial V_i^{Fs}(\delta_i, \delta^*)}{\partial \delta^*} > 0 \text{ and } \frac{\partial E_{\Omega_i^s(\delta^*)}(\Pi^{Fs}(\delta_i, \delta^*))}{\partial \delta^*} > 0, \text{ for each } i \text{ and } \delta^*. \quad (13)$$

These properties do not suffice to establish our claim, because a change in the threshold induces a jump in the producer's surplus for the firms shifting from one sector to the other. However, as we will formally establish

below (in Proposition 3), under suitable restrictions on equilibria, sufficiently small increases of the threshold value increase expected aggregate surplus, because of this composition effect.

To complete the analysis of the welfare properties of equilibria, it is convenient to introduce an explicit notion of (constrained) efficiency. As usual in economies with frictions, we consider the metaphor of a benevolent planner choosing an allocation while facing constraints aiming to capture the ones the agents face in the decentralized economy. We provide two results. First, we show that there are constrained optimal allocations (Proposition 2) and that they can be attained with an appropriate system of taxes and subsidies. The amount of subsidies and taxes is entirely dictated by the features of the CO allocation and they can (in fact, are) quite large. That's why, in Proposition 3, we study the effects of *small* taxes and subsidies on the expected total surplus evaluated at the market equilibrium, taking as given the demand and supply functions of the agents.

Bear in mind that, in the sequel, we always consider changes in the conditional expectation of the total surplus. We are not concerned with actual Pareto improvements. However, given that the utility functions are quasi-linear, an increase of the expected total surplus immediately translates (modulo an appropriate - and i -contingent - system of lump-sum taxes and transfers) into a Pareto improvement. Also, we will show that, given the structure of the economy, the systems of taxes and transfers can be designed so to guarantee a balanced budget.

5.1. Constrained optimal allocations

The objective function of the planner is given by the sum of the conditional expected utilities and producers' surpluses, i.e.,

$$P(h_i^s, k_j^s, \Omega_I^s, \Omega_J^s) \equiv \sum_s \int_{\Omega_I^s} \left[\beta E_{\Omega_J^s} \left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)} \right) - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma} - c_I^s \right] di \\ + \sum_s \int_{\Omega_J^s} \left[(1-\beta) E_{\Omega_I^s} \left(A^s h_i^{s\alpha} k_j^{s(1-\alpha)} \right) - \mu k_j^s \right] dj.$$

The policy instruments are the partitions Ω_I^P and Ω_J^P and a pair of maps $(H^{COs}(\delta_i, \delta^*), K^{COs}(\delta^*))$. We restrict the partitions to have the structure $\Omega_I^e(\delta^*) = \{i \in \Omega_I | \delta_i \geq \delta^*\}$, and $\Omega_J^e(\delta^*) = \{j \in \Omega_J | j = i, i \in \Omega_I^e(\delta^*)\}$. Given that firms are (ex-ante) identical, the informational constraints embedded into the definition of $P(\cdot)$, and the properties of the (implicit) matching function, to impose this structure on Ω_J^P does not entail any loss of generality. The optimal choice k_j^s is clearly j -invariant and, by assumption, $\nu(\Omega_I^s(\delta^*)) = \nu(\Omega_J^s(\delta^*))$. Hence, the planner's objective function can

be rewritten as

$$\begin{aligned}
P(h_i^s, k^s, \delta^*) &\equiv \sum_s \int_{\Omega_I^s(\delta^*)} \left(\beta A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma} \right) di - c_I^s \nu(\Omega_I^s(\delta^*)) \\
&\quad + \sum_s \left((1-\beta) A^s \frac{\int_{\Omega_I^s(\delta^*)} h_i^{s\alpha} di}{\nu(\Omega_I^s(\delta^*))} k^{s(1-\alpha)} - \mu k^s \right) \nu(\Omega_J^s(\delta^*)) \\
&= \sum_s \int_{\Omega_I^s(\delta^*)} \left(A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma} \right) di \\
&\quad - (c_I^s + \mu k^s) \nu(\Omega_I^s(\delta^*)).
\end{aligned}$$

Its optimization problem is, then,

$$\max_{(h_i^s, k^s, \delta^*)} P(h_i^s, k^s, \delta^*). \quad (P)$$

It is convenient to decompose (P) into three problems. First, given an arbitrary value δ^* , we determine the maps $(H^{COs}(\delta_i, \delta^*), K^{COs}(\delta^*))$ solving, for each s , the optimization problem

$$\begin{aligned}
\max_{(h_i^s, k^s)} P_{\delta^*}^s(h_i^s, k^s) &\equiv \int_{\Omega_I^s(\delta^*)} \left[A^s h_i^{s\alpha} k^{s(1-\alpha)} - \frac{1}{\delta_i} \frac{h_i^{s(1+\Gamma)}}{1+\Gamma} \right] di (P_{\delta^*}^s) \\
&\quad - (c_I^s + \mu k^s) \nu(\Omega_I^s(\delta^*)).
\end{aligned}$$

Next, given the value functions $P^s(\delta^*)$ of the two problems $(P_{\delta^*}^s)$, $s = e, ne$, we recast problem (P) as

$$\max_{\delta^*} \bar{P}(\delta^*) \equiv P^e(\delta^*) + P^{ne}(\delta^*), \quad (\bar{P})$$

finding the optimal value of δ^* , δ^{CO} .

PROPOSITION 2. *Under the maintained assumptions, each economy with frictions has a CO allocation. Given the CO threshold value δ^{CO} ,*

$$\begin{aligned}
a. \quad K^{COs}(\delta^{CO}) &= A^{s \frac{1+\Gamma}{1-\alpha}} \alpha^{\frac{1}{\Gamma}} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s(\delta^{CO})} \left(\delta_i^{\frac{1}{1+\Gamma-\alpha}} \right) \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}, \\
b. \quad H^{COs}(\delta_i, \delta^{CO}) &= \delta_i^{\frac{1}{1+\Gamma-\alpha}} \alpha^{\frac{1}{\Gamma}} A^{s \frac{1}{\alpha\Gamma}} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s(\delta^{CO})} \left(\delta_i^{\frac{1}{1+\Gamma-\alpha}} \right) \right)^{\frac{1-\alpha}{\alpha\Gamma}}.
\end{aligned}$$

Moreover, equilibrium allocations are never CO and they are characterized by underinvestment in the amount of physical capital and in educational effort. Both under and overinvestment in educational level are possible.

Proof. In Appendix 1. ■

The source of inefficiency considered by Acemoglu (1996) reappears in our set-up, because, given any threshold level δ^* , $H^{COs}(\delta_i, \delta^*) > H^{Fs}(\delta_i, \delta^*)$,

for each δ_i , and $K^{COs}(\delta^*) > K^{Fs}(\delta^*)$. On the other hand, the relation between the CO value of the threshold, δ^{CO} , and its equilibrium level, δ^F , is not univocal. Example A2 in Appendix 1 shows two economies: the first has $\delta^{CO} < \delta^F$, the other $\delta^{CO} > \delta^F$. This does not contradict our previous claim (that an increase in δ^F can improve expected total surplus). There, we were evaluating the reactions of firms and workers according to their actual demand and supply functions. Here, the optimal value δ^{CO} is determined using the demand/supply functions determined by the planner.

Remark 6. In our set-up (as well as in Acemoglu (1996)), equilibria of the economy with frictions are constrained inefficient for each value of β , because, at $\xi = 0$, even if $\delta^{CO} = \delta^F$,

$$\frac{H^{Fs}(\delta_i, \delta^{CO})}{H^{COs}(\delta_i, \delta^{CO})} = (1 - \beta)^{\frac{1-\alpha}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}}, \text{ for each } s \text{ and } i,$$

while

$$\frac{K^{Fs}(\delta^{CO})}{K^{COs}(\delta^{CO})} = (1 - \beta)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}}, \text{ for each } s.$$

Evidently, $(1 - \beta)^{\frac{1-\alpha}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}} \neq 1$ and $(1 - \beta)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \beta^{\frac{1}{\Gamma}} \neq 1$ for each $\beta \in (0, 1)$. To the contrary, in random matching models, efficiency obtains when the Hosios' condition is satisfied, i.e., when β is equal to the value of the elasticity of the matching function. As observed by Acemoglu (1996, p.789), here, given any threshold δ^* , the externalities are related to "the value of the future matches and are always positive."

It is easy to see that the CO distribution of investments in human and physical capital can be attained with an appropriate system of subsidies to investments in physical capital and labor income, and of fixed taxes or subsidies on the educational choice. Moreover, given that preferences are quasi-linear, the system of tax and subsidies can be balanced using uniform lump-sum taxes (T) on workers (notice that, in the absence of positive endowments of consumption goods, this could entail negative consumption for some subset of agents).

COROLLARY 1. *There is a (balanced budget) system of taxes and subsidies ξ such that the associated equilibrium allocation is CO.*

Proof. See Appendix 1. ■

5.2. The effect of human capital policies on expected total surplus

We conclude considering the welfare effects of alternative, active, balanced budget, human capital policies. In particular, we study the (local,

around 0) effect on expected total surplus of changes in the vector ξ , with $(\zeta^e, \zeta^e) = 0$. We show that, depending upon the sign of $\frac{\partial \delta^F}{\partial \delta^*} |_{\delta^* = \delta^F}$ (and provided that the threshold level δ^F is not "too high"), distinct subsets of policy instruments have unambiguous effects on expected total surplus.

Our main result is that an increase in the cost of education (redistributing the revenues as lump-sum transfer), or an increase of the subsidies to labor income in the "low skill" sector ne (financed with lump-sum taxes) always increase expected total surplus. On the contrary, an increase in the subsidy to labor income in the high skill sector (again, financed with lump-sum taxes) may decrease it. The logic of these results is the following. As established in Proposition 1, if $\frac{\partial f(\delta^*, \xi)}{\partial \delta^*} |_{\delta^* = \delta^F(0)} > 0$ (a maintained assumption in the sequel), $\frac{\partial \delta^F(\xi)}{\partial \tau^e} |_{\xi=0} < 0$, $\frac{\partial \delta^F(\xi)}{\partial \tau^{ne}} |_{\xi=0} > 0$ and $\frac{\partial \delta^F(\xi)}{\partial \Delta c_I^e} |_{\xi=0} > 0$. A subsidy τ^{ne} to labor income in sector ne has a direct, positive effect on effort in this sector, and a positive, indirect, effect on the effort in both sectors, because it induces an increase in the equilibrium value of $\delta^F(\xi)$. For the same reason, a tax on higher education $\Delta c_I^e > 0$ has an indirect, positive effect on effort in both sectors. Therefore, they always lead to an increase in expected total surplus. The third policy, making sector e more attractive to workers, induces some workers with $\delta_i < \delta^F(0)$ to move to sector e . This has an unambiguous, negative effect on the welfare of the workers remaining in sector ne (and on the expected profits in this sector). Moreover, the negative effect on the welfare of the workers in sector e , due to the composition effect, may overcome the positive effect of the incentives on effort in this sector too, so that we may end up with a lower expected total surplus. This is established in Proposition 3 and by a final example.

The main difficulty in showing these results is that a change in the threshold induces a discontinuous jump in the expected producer's surplus for the firms changing sectors. We provide *one* sufficient condition which guarantees that, at the equilibrium, the total, conditional expected total surplus is increasing in the value of the threshold. This condition is far from necessary to obtain our result. The implicit restriction on the equilibrium values is not unreasonable. For instance, for $\alpha = \frac{2}{3}$, we certainly have $\frac{\partial P(\delta^*, \xi)}{\partial \delta^*} |_{\delta^* = \delta^F} > 0$ if $\delta^F < 0.5$ and $\Gamma = 0.2$, if $\delta^F < 0.35$ and $\Gamma = 0.4$ and so on. Notice that, given (α, β, Γ) , this is essentially a restriction on the ratio $\frac{A^e}{A^{ne}}$. An alternative sufficient condition is that β is "large enough".

Up to now we have consider a sector-contingent vector of subsidy rates (τ^e, τ^{ne}) . This is certainly an unusual feature for a system of income subsidies. The same results can be obtained with a commonly used system of step-linear subsidies, of the type

$$T(w) = \begin{cases} \tau^{ne} w & \text{if } w \leq w^{ne}(\delta^F, \delta^F) \\ \tau^{ne} w^{ne}(\delta^F, \delta^F) + \tau^e (w - w^{ne}(\delta^F, \delta^F)) & \text{if } w > w^{ne}(\delta^F, \delta^F) \end{cases},$$

where w denotes a generic labor income, while $w^{ne}(\delta^F, \delta^F)$ is the labor income of the worker with $\delta_i = \delta^F$ at the equilibrium associated with the threshold level δ^F . All that is required to establish this result is to observe, that, at the equilibrium associated with any δ^F , at $\xi = 0$, the actual labor income of each worker in sector e is larger than the labor income of any worker in sector ne . Indeed, by direct computation, the labor income of worker i active in sector s is given by

$$w^s(\delta_i, \delta^F) = \left[\beta \left(\frac{(1-\alpha)(1-\beta)}{\mu} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} (\alpha\beta)^{\frac{1}{\Gamma}} \right] \times A^s \frac{1+\Gamma}{\alpha\Gamma} E_{\Omega_I^s}(\delta^F) \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \delta_i^{\frac{\alpha}{1+\Gamma-\alpha}}, \quad (14)$$

and, evidently, $w^e(\delta'_i, \delta^F) > w^{ne}(\delta''_i, \delta^F)$ for each $\delta'_i \geq \delta^F \geq \delta''_i$.

Remark 7. We are taking as a reference point an economy where $\xi = 0$. Evidently, if we introduce a flat tax rate t on labor incomes, we would obtain exactly the same results considering changing the marginal tax rates.

To conclude, let's make formal the heuristic argument above. Define the expected social surplus as a function of the vector of policy instruments,

$$\xi \equiv (\tau^e, \tau^{ne}, \Delta c_I^e, T).$$

Given ξ , workers and firms choose their individually optimal behavior. Consider $\delta^F(\cdot)$ as a function of ξ . Omitting the superscript "F" whenever convenient, the expected total surplus is given by

$$P(\xi) \equiv \sum_s \int_{\Omega_j^s(\delta^F(\xi))} E_{\Omega_I^s(\delta^F(\xi))} \left(\Pi_j^s(\delta_i, \delta^F(\xi), \xi) \right) dj + \sum_s \int_{\Omega_I^s(\delta^F(\xi))} V_i^s(\delta_i, \delta^F(\xi), \xi) di - T, \quad (15)$$

with total lump-sum taxes given by

$$T = \left[\sum_s \tau^s \int_{\Omega_I^s(\delta^F(\xi))} w^s(\delta_i, \delta^F(\xi), \xi) di - \Delta c_I^e \nu \left(\Omega_I^e(\delta^F(\xi)) \right) \right],$$

so that the budget is balanced.

PROPOSITION 3. *Consider an equilibrium associated with $\xi = 0$ and such that $\frac{\partial f(\cdot)}{\partial \delta^*} \Big|_{\delta^* = \delta^F(0)} > 0$ and*

$$\frac{1 - \delta^F(0)^{\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\left(1 - \delta^F(0)\right) \delta^F(0)^{\frac{\alpha}{1+\Gamma-\alpha}}} \geq \frac{(1+\alpha)(1+\Gamma)}{1+\Gamma-\alpha}.$$

Then,

- i. $\Delta c_I^e > 0$, and sufficiently small, increases the expected total surplus,
- ii. $\tau^{ne} > 0$, and sufficiently small, increases the expected total surplus,
- iii. $\tau^e > 0$, and sufficiently small, may decrease the expected total surplus.

The proofs of (i, ii) are in Appendix 1, where we also establish that the welfare effect of a subsidy τ^e is, in general, indeterminate. We now provide a strategy to construct economies where an increase in τ^e decreases the total expected surplus. The third statement, therefore, is established by the following example.

EXAMPLE 3. *Welfare-reducing subsidies to investments in human capital in sector e.*

From eq. (8), we can easily see that the sign of $\frac{\partial f}{\partial \delta^*} |_{\delta^* = \delta^F(\cdot)}$ depends upon the parameters $(\frac{A^e}{A^{ne}}, \alpha, \Gamma)$, and the equilibrium level $\delta^F(\cdot)$, while it doesn't depend directly on β . Moreover, given $(\frac{A^e}{A^{ne}}, \alpha, \Gamma)$, the effect of changes of β on the value of $\delta^F(\cdot)$ can always be neutralized by appropriate changes in the parameter c_I^e .

Using the notation introduced in the proof of Proposition 3,

$$\frac{\partial P(\cdot)}{\partial \tau^e} = \left(\frac{\partial P(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F(0)} \right) \frac{\partial \delta^F(\xi)}{\partial \tau^e} + D^e(\delta_i, \delta^F(\xi), \xi).$$

We want to construct an economy such that, at the equilibrium, $\frac{\partial P(\cdot)}{\partial \tau^e} < 0$. Given that $D^e(\delta_i, \delta^F(\xi), \xi)$ is easily seen (looking at its parametric structure reported in that proof) to be bounded above, the required result is established if we can construct an equilibrium with $\left(\frac{\partial P(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F(0)} \right) \frac{\partial \delta^F(\xi)}{\partial \tau^e} < 0$ and arbitrarily large in absolute value. As established in Example A1 in Appendix 1, for $\frac{A^e}{A^{ne}}$ sufficiently small there are economies such that $\frac{\partial f(\cdot)}{\partial \delta^*} > 0$, for δ^* sufficiently small and $\frac{\partial f(\cdot)}{\partial \delta^*} < 0$ for δ^* large enough. Given that the function $\frac{\partial f(\cdot)}{\partial \delta^*}$ is continuous, this implies that, for economies in this set, there is $\bar{\delta}$ such that $\frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \bar{\delta}} = 0$ and $\frac{\partial f(\cdot)}{\partial \delta^*} > 0$ at each $\delta^* < \bar{\delta}$. Given the values of all the parameters, but the actual direct cost of education, c_I^e , pick a sequence $\{c_I^{ev}\}_{v=1}^\infty$ such that $c_I^{ev} < \bar{c}_I^e = f(\bar{\delta})$, for each v , and $c_I^{ev} \rightarrow \bar{c}_I^e$. Along this sequence, by construction, $\delta^F(c_I^{ev}) < \bar{\delta}$, and $\delta^F(c_I^{ev}) \rightarrow \bar{\delta}$. Therefore, $\frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F(c_I^{ev})} > 0$, for each v , and $\lim_{v \rightarrow \infty} \frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F(c_I^{ev})} = 0$. By the implicit function theorem, $\frac{\partial \delta^F(\cdot)}{\partial \tau^e} = -\frac{\frac{\partial f(\cdot)}{\partial \tau^e}}{\frac{\partial f(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F}}$. Given that $\frac{\partial f(\cdot)}{\partial \tau^e}$ is positive and bounded away from zero (see Proposition 1), the sequence $\frac{\partial \delta^F(\cdot)}{\partial \tau^e}$ associated with $\left\{ \delta^F(c_I^{ev}) \right\}_{v=1}^\infty$ is negative and divergent.

To prove our result we still need to show that there are economies such that, in a neighborhood of $\bar{\delta}$, $\frac{\partial P(\cdot)}{\partial \delta^*} |_{\delta^* = \delta^F} \geq \varepsilon$, for some $\varepsilon > 0$. In

the proof of Proposition 3 (in Appendix), we have defined the expression $\Delta P^e(\delta^F)$, governing the sign of $\frac{\partial P(\cdot)}{\partial \delta^*}|_{\delta^*=\delta^F}$. It is easy to check that, given δ^F , $\Delta P^e(\delta^F)$ is strictly positive for β close enough to 1. Pick a value of $\beta, \bar{\beta}$, such that, at $\bar{\delta}$, $\frac{\partial P(\cdot)}{\partial \delta^*} > 2\varepsilon$, for some $\varepsilon > 0$. By continuity, $\frac{\partial P(\cdot)}{\partial \delta^*} > \varepsilon$ at each δ close enough to $\bar{\delta}$. Change the sequence $\{c_I^{ev}\}_{v=1}^\infty$ to neutralize the change in $\delta^F(c_I^{ev})$ due to the new value of β . For the economy so constructed, for some c_I^e , it must be $\frac{\partial P(\cdot)}{\partial \tau^e} < 0$ and arbitrarily large in absolute value, as claimed

6. APPENDIX 1: PROOFS OF THE PROPOSITIONS

Proof of Proposition 1. Bear in mind that here we are assuming $\xi = 0$ and, therefore, we can omit it from the notation whenever convenient.

Pick the partition $\Omega_I^P(\delta^*)$ induced by any arbitrary δ^* . Assume that there is an agent i' such that $\delta_{i'} = \delta^*$ at δ^* solving $f(\delta^*) - ac_I^e = 0$. It is easy to check that $V_i^{Fe}(\delta_i, \delta^*) - V_i^{Fne}(\delta_i, \delta^*) > 0$ for each $\delta_i > \delta^*$ and $V_i^{Fe}(\delta_i, \delta^*) - V_i^{Fne}(\delta_i, \delta^*) < 0$ for each $\delta_i < \delta^*$. Hence, each equilibrium partition $\Omega_I^P(\delta^F)$ such that $\Omega_I^s(\delta^F) \neq \emptyset$, each s , satisfies $\Omega_I^e(\delta^F) = \{i \in \Omega_I | \delta_i \geq \delta^F\}$, where δ^F is the (unique) threshold value.

By direct computation, for each threshold δ^* ,

$$E_{\Omega_I^e(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) = \left(\frac{1+\Gamma-\alpha}{1+\Gamma} \frac{1-\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1-\delta^*} \right)$$

and

$$E_{\Omega_I^{ne}(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) = \left(\frac{1+\Gamma-\alpha}{1+\Gamma} \delta^{*\frac{\alpha}{1+\Gamma-\alpha}} \right).$$

Evidently, both functions are continuous in the (arbitrarily selected) threshold value δ^* and increasing in δ^* . Let

$$\bar{c}_I^e = ac_I^e (1+\Gamma-\alpha) \left(\frac{1+\Gamma}{1+\Gamma-\alpha} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}$$

and, with a some abuse of notation, let

$$\begin{aligned} & \delta^{*\frac{\alpha}{1+\Gamma-\alpha}} A^e \frac{1+\Gamma}{\alpha\Gamma} \left(\frac{1-\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1-\delta^*} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \\ & - \delta^{*\frac{\alpha}{1+\Gamma-\alpha}} A^{ne} \frac{1+\Gamma}{\alpha\Gamma} \left(\frac{\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{\delta^*} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} - \bar{c}_I^e \\ \equiv & f(\delta^*) - \bar{c}_I^e. \end{aligned}$$

$f(\delta^*)$ is clearly continuous for $\delta^* \in (0, 1)$. Given that $E_{\Omega_I^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)$, each s , is bounded, $\lim_{\delta^* \rightarrow 0} f(\delta^*) = 0$.

$$\text{Given that } \lim_{\delta^* \rightarrow 1} \frac{1 - \delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}}}{1 - \delta^*} = \frac{\partial \left(\delta^{*\frac{1+\Gamma}{1+\Gamma-\alpha}} \right)}{\partial \delta^*} \Big|_{\delta^*=1} = \frac{1+\Gamma}{1+\Gamma-\alpha},$$

$$\lim_{\delta^* \rightarrow 1} f(\delta^*) = \left(A^{e\frac{1+\Gamma}{\alpha\Gamma}} \frac{1+\Gamma}{1+\Gamma-\alpha} - A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \right) \equiv \bar{C} > 0$$

Hence, by the intermediate value theorem, for each c_I^e such that $\bar{c}_I^e \in (0, \bar{C})$, there is an interior equilibrium, whose threshold value δ^F is given by the solution to $f(\delta^*) - \bar{c}_I^e = 0$.

Evidently, $\frac{\partial f(\cdot)}{\partial \Delta c_I^e} < 0$, and, by direct computation ,

$$\frac{\partial f(\cdot)}{\partial \tau^s} \Big|_{\xi=0} = \left[\delta^F \left(A^s E_{\Omega_I^s(\delta^F)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{(1-\alpha)} \right)^{\frac{1+\Gamma}{\alpha\Gamma}} \frac{(1+\Gamma)(1-\alpha)}{\Gamma} \right] (-1)^{\chi(s)} > 0,$$

with $\chi(e) = 2$, $\chi(e) = 1$, while the term in square brackets is obviously positive, so that $\frac{\partial f(\cdot)}{\partial \tau^e} \Big|_{\xi=0} > 0$ and $\frac{\partial f(\cdot)}{\partial \tau^{ne}} \Big|_{\xi=0} < 0$. Unfortunately, the sign of $\frac{\partial f(\cdot)}{\partial \delta^*} \Big|_{\delta^*=\delta^F}$ depends upon the specific parameters of the economy. By direct computation, at $\xi = 0$, setting $\gamma \equiv \frac{1+\Gamma}{1+\Gamma-\alpha}$,

$$\begin{aligned} \frac{\partial f(\cdot)}{\partial \delta^*} \Big|_{\delta^*=\delta^F} &= (\gamma - 1) \frac{1}{\delta^F} f(\delta^*) + \frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma} \frac{\delta^{F\gamma-1}}{\delta^F} A^{ne\frac{1+\Gamma}{\alpha\Gamma}} \\ &\times \left(\left(\frac{A^e}{A^{ne}} \right)^{\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega_I^e(\delta^F)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \eta^e(\delta^*) - E_{\Omega_I^{ne}(\delta^F)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \eta^{ne}(\delta^*) \right), \end{aligned}$$

where $\eta^s(\delta^*)$ is the elasticity of $E_{\Omega_I^s(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)$. By direct computation, $\eta^{ne}(\delta^*) = (\gamma - 1)$, while $\eta^e(\delta^*) = \frac{-\gamma\delta^{*\gamma}(1-\delta^*) + \delta^*(1-\delta^{*\gamma})}{(1-\delta^*)(1-\delta^{*\gamma})}$. With a straightforward manipulation, we obtain that

$$\begin{aligned} &\left(\frac{1+\Gamma-\alpha}{1+\Gamma} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \frac{\Gamma}{A^{ne\frac{1+\Gamma}{\alpha\Gamma}}} \frac{\partial f(\cdot)}{\partial \delta^*} \Big|_{\delta^*=\delta^F} \\ &= \left(\left(\frac{A^e}{A^{ne}} \right)^{\frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{1-\delta^{F\gamma}}{\delta^F} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \times \left(\frac{\alpha\Gamma}{1+\Gamma-\alpha} + \frac{(1-\alpha)(1+\Gamma)}{\alpha} \eta^e(\delta^F) \right) - 1 \right) \\ &\times \delta^F \end{aligned}$$

If $\eta^e(\delta^*) \geq 0$ at each $\delta^* \in (0, 1)$, and $\frac{1-\delta^\gamma}{(1-\delta)\delta^{\gamma-1}}$ is bounded away from zero, the right hand side of the eq. above is always positive, for $\frac{A^e}{A^{ne}}$ sufficiently large. Therefore, $\frac{\partial f(\cdot)}{\partial \delta^*} \Big|_{\delta^*=\delta^F} > 0$ at each equilibrium, for A^e large

enough. Evidently, if $\frac{\partial f(\cdot)}{\partial \delta^*}|_{\delta^*=\delta^F} > 0$ at each solution to $f(\delta^*) - \bar{c}_I^e = 0$, the solution must be unique. Moreover, by the implicit function theorem, $\frac{\partial F(\cdot)}{\partial \delta^*}|_{\delta^*=\delta^F} > 0$ at each equilibrium also implies that $\delta^F(\cdot)$ satisfies $\frac{\partial \delta^F(\cdot)}{\partial \tau^e}|_{\xi=0} < 0$, $\frac{\partial \delta^F(\cdot)}{\partial \tau^{ne}}|_{\xi=0} > 0$ and $\frac{\partial \delta^F(\cdot)}{\partial \Delta c_I^e} > 0$. Hence, to conclude the proof we need two additional results:

Fact 1. $\eta^e(\delta) \geq 0$, at each $\delta \in (0, 1)$.

By direct computation, $\eta^e(0) = 0$ and $\eta^e(1) = \frac{\gamma-1}{2} > 0$. Hence, either there is $\bar{\delta} \in (0, 1)$ such that $\eta^e(\bar{\delta}) = 0$ or $\eta^e(\delta) > 0$ for each $\delta \in (0, 1)$, as claimed. Consider the numerator of $\eta^e(\delta)$, call it $g(\delta)$,

$$g(\delta) = -\gamma\delta^\gamma(1-\delta) + \delta(1-\delta^\gamma).$$

Given that the denominator is strictly positive for each $\delta \in (0, 1)$, $\eta^e(\delta) \leq 0$ if and only if $g(\delta) \leq 0$. Clearly, $g(0) = g(1) = 0$. Given that

$$\frac{\partial g}{\partial \delta} = (1 - \gamma^2\delta^{\gamma-1} + (\gamma^2 - 1)\delta^\gamma),$$

$\frac{\partial g}{\partial \delta}|_{\delta=0} > 0$ and $\frac{\partial g}{\partial \delta}|_{\delta=1} = 0$. Moreover,

$$\frac{\partial^2 g}{\partial \delta^2}|_{\delta=1} = \gamma(\gamma^2 - 1)\delta^{\gamma-1} - \gamma^2(\gamma - 1)\delta^{\gamma-2} = \gamma(\gamma - 1) > 0,$$

so that $\delta = 1$ is a local minimum of $g(\delta)$. This implies that, if there is a $\bar{\delta} \in (0, 1)$ such that $g(\bar{\delta}) = 0$, there must also be a $\hat{\delta} \in (0, 1)$ such that $g(\hat{\delta}) = 0$ and $\frac{\partial g}{\partial \delta}|_{\delta=\hat{\delta}} > 0$. Given that, by assumption, $\hat{\delta} \in (0, 1)$, $\hat{\delta} \neq 0$, and, therefore, $\frac{g(\hat{\delta})}{\hat{\delta}} = 0$, and $\left(\frac{\partial g}{\partial \delta}|_{\delta=\hat{\delta}} - \frac{g(\hat{\delta})}{\hat{\delta}}\right) > 0$. However,

$$\begin{aligned} \frac{\partial g}{\partial \delta}|_{\delta=\hat{\delta}} - \frac{g(\hat{\delta})}{\hat{\delta}} &= -\gamma^2\hat{\delta}^{\gamma-1} + (\gamma^2 - 1)\hat{\delta}^\gamma + \gamma\hat{\delta}^{\gamma-1}(1-\hat{\delta}) + \hat{\delta}^\gamma \\ &= (\gamma - \gamma^2)(1-\hat{\delta})\hat{\delta}^{\gamma-1} < 0, \end{aligned}$$

because $\gamma > 1$. Hence, $g(\delta) > 0$ and, therefore, $\eta(\delta) > 0$, at each $\delta \in (0, 1)$.

Fact 2. Let $G(\delta) \equiv \left(\frac{1-\delta^\gamma}{1-\delta} \frac{\delta}{\delta^\gamma}\right)$. Then, $G(\delta) > 1$, for each $\delta \in (0, 1)$.

Given that $\gamma > 1$, the result is quite obvious from the geometrical viewpoint. Also, using Taylor's expansion, for each $\bar{\delta} \in (0, 1)$, we can write $\bar{\delta}^\gamma = 1 + \gamma\hat{\delta}^{\gamma-1}(\bar{\delta} - 1)$, for some $\hat{\delta} \in (\bar{\delta}, 1)$. Similarly, $\bar{\delta}^\gamma = 0 + \gamma\tilde{\delta}^{\gamma-1}\bar{\delta}$ for some $\tilde{\delta} \in (0, \bar{\delta})$. Hence, $G(\bar{\delta}) = \left(\frac{1-(1+\gamma\hat{\delta}^{\gamma-1}(\bar{\delta}-1))}{1-\bar{\delta}} \frac{\bar{\delta}}{\gamma\tilde{\delta}^{\gamma-1}\bar{\delta}}\right) = \left(\frac{\hat{\delta}}{\tilde{\delta}}\right)^{\gamma-1} > 1$, because $\frac{\hat{\delta}}{\tilde{\delta}} > 1$.

EXAMPLE A1. We show that it can actually be the case that $\frac{\partial f(\cdot)}{\partial \delta^*}|_{\delta^*=\delta^*} < 0$. Fix $\xi = 0$. Assume that $\alpha = \frac{1}{2}, \Gamma = 10, A^{ne} = 1, A^e = 11/10$. By direct computation,

$$f(\delta^*) = \frac{105}{10} \left(\frac{105}{110}\right)^{\frac{11}{10}} \delta^{*\frac{1}{21}} \left(\left(\frac{11}{10} \left(\frac{1 - \delta^{*\frac{110}{105}}}{1 - \delta^*} \right)^{\frac{1}{2}} \right)^{\frac{11}{5}} - \delta^{*\frac{1}{20}} \right).$$

$\frac{\partial f(\delta^*)}{\partial \delta^*}$ is strictly positive for δ^* sufficiently small, and negative for all δ^* larger than some critical value δ^+ . For instance, one can check that $\lim_{\delta^* \rightarrow 1} \frac{\partial f(\delta^*)}{\partial \delta^*} < -0.001$, while $\left\{ \frac{\partial f(\delta^*)}{\partial \delta^*} \Big|_{\delta^{*v}} \right\}_{v=0}^{\infty}$ with $\delta^{*v} \rightarrow 0$ is unbounded above. Therefore, by choosing appropriately c_I^e , every δ^* can be made an equilibrium $\delta^F(c_I^e)$.

Proof of Proposition 2. Given that optimization problem $(P_{\delta^*}^s)$ is concave, each s , its solution is completely characterized by the FOCs:

- i. $\frac{\partial P_{\delta^*}^s(h_i^s, k^s)}{\partial h_i} = \alpha A^s k^{s(1-\alpha)} h_i^{s(\alpha-1)} - \frac{1}{\delta_i} h_i^{s\Gamma} = 0,$
- ii. $\frac{\partial P_{\delta^*}^s(h_i^s, k^s)}{\partial k} = (1-\alpha) A^s k^{s(-\alpha)} \int_{\Omega_I^s(\delta^*)} h^{s\alpha} - \mu \int_{\Omega_I^s(\delta^*)} di = 0,$

which imply

- a. $K^{COs}(\delta^*) = A^s \frac{1+\Gamma}{\Gamma\alpha} \alpha^{\frac{1}{\Gamma}} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}},$
- b. $H^{COs}(\delta_i, \delta^*) = \delta_i^{\frac{1}{1+\Gamma-\alpha}} \alpha^{\frac{1}{\Gamma}} A^s \frac{1}{\alpha\Gamma} \left(\frac{1-\alpha}{\mu} E_{\Omega_I^s} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) \right)^{\frac{1-\alpha}{\alpha\Gamma}}.$

It follows that $K^{COs}(\delta^*) > K^{Fs}(\delta^*)$ and $H^{COs}(\delta_i = \delta^*, \delta^*) > H^{Fs}(\delta_i = \delta^*, \delta^*)$, for each value δ^* . Therefore, equilibria are always characterized by under-investment in physical capital and in the effort in education.

Demand and supply functions are clearly well-defined and continuous at each $\delta^* \in (0, 1)$ and $P^s(\delta^*)$ has the same properties. Hence, problem (\bar{P}) has a solution, either internal or at one of the boundary points, and, therefore, CO allocations exist.

Compare a market allocation and any CO allocation. If they have the same threshold value δ^* , $K^{COs}(\delta^*) \neq K^{Fs}(\delta^*)$ and the market allocation is not CO. Otherwise, the threshold values are different, and constrained inefficiency follows immediately.

EXAMPLE A2. By tedious computation, one can verify that δ^{CO}

is the solution to

$$\begin{aligned}
& \frac{1+\Gamma}{1+\Gamma-\alpha} \left(\frac{\mu}{1-\alpha} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \frac{1}{\alpha^{\frac{1}{\Gamma}}} c_I^e \\
= & A^{e\frac{1+\Gamma}{\Gamma\alpha}} \delta^{CO\frac{\alpha}{1+\Gamma-\alpha}} E_{\Omega_I^e(\delta^{CO})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \left(1 - \frac{(1-\alpha) \left(1 - \delta^{CO\frac{1+\Gamma}{1+\Gamma-\alpha}} \right)}{\delta^{CO\frac{\alpha}{1+\Gamma-\alpha}} (1 - \delta^{CO})} \right) \\
& - \alpha A^{ne\frac{1+\Gamma}{\Gamma\alpha}} \alpha^{\frac{1}{\Gamma}} \delta^{CO\frac{\alpha}{1+\Gamma-\alpha}} E_{\Omega_I^{ne}(\delta^{CO})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}},
\end{aligned}$$

while δ^F is the solution to

$$\begin{aligned}
& \delta^F A^{e\frac{1+\Gamma}{\Gamma\alpha}} E_{\Omega_I^e(\delta^F)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \\
& - \delta^F A^{ne\frac{1+\Gamma}{\Gamma\alpha}} E_{\Omega_I^{ne}(\delta^F)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \\
= & \frac{1+\Gamma}{1+\Gamma-\alpha} \left(\frac{\mu}{1-\alpha} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \frac{1}{\alpha^{\frac{1}{\Gamma}}} \left(\frac{1}{\beta(1-\beta)^{\frac{1-\alpha}{\alpha}}} \right)^{\frac{1+\Gamma}{\Gamma}} c_I^e.
\end{aligned}$$

It is easy to construct economies with $\delta^{CO} > \delta^F$ and with $\delta^F < \delta^{CO}$. Assume that $\Gamma = \mu = A^{ne} = 1$, while $\alpha = \beta = \frac{1}{2}$, $c_I^e = \frac{1}{8}$. Then, the two conditions above reduce to

$$\frac{1}{3} = A^{e4} \delta^{CO\frac{1}{3}} \left(\frac{4}{3} \frac{1 - \delta^{CO\frac{4}{3}}}{1 - \delta^{CO}} \right)^2 \left(1 - \frac{1}{2} \frac{1 - \delta^{CO\frac{4}{3}}}{\delta^{CO\frac{1}{3}}(1 - \delta^{CO})} \right) - \delta^{CO\frac{1}{3}} \left(\frac{4}{3} \delta^{CO\frac{1}{3}} \right)^2$$

and

$$\frac{16}{3} = \delta^F A^{e4} \left(\frac{4}{3} \frac{1 - \delta^F}{1 - \delta^F} \right)^2 - \delta^F \left(\frac{4}{3} \delta^F \right)^2.$$

For $A^e = 2$, $\delta^{CO} = 0.18290$ while $\delta^F = 0.00643$. At $A^e = \frac{13}{10}$, $\delta^{CO} = 0.39928$, while $\delta^F = 0.57283$.

Proof of Corollary 1. Obviously, there are many different tax-subsidy schemes implementing the CO allocation. We will focus the analysis on linear subsidies on labor income and investments in physical capital and on fixed fees and lump-sum taxes. Fix $\bar{\zeta}^e = \bar{\zeta}^{ne} = \beta$ and $\bar{\tau}^e = \bar{\tau}^{ne} = \frac{1-\beta}{\beta}$. It is easy to check that, given any threshold value $\bar{\delta}$, the FOCs of the individual optimization problem in the actual economy imply that the FOCs of (constrained) planner's optimization problem are satisfied. Let $\delta(\xi)$ be the market threshold value associated with ξ .

By direct computation, at the CO allocation, expected profits are zero in both sectors. Hence, firms are indifferent among sectors. Therefore, at

each optimal solution $\bar{\delta} \in (0, 1)$, the FOCs of optimization problem (\bar{P}) are simply given by

$$- [U_i^e(H^{COe}(\delta_i = \bar{\delta}, \bar{\delta}), K^{COe}(\bar{\delta})) - U_i^{ne}(H^{COne}(\delta_i = \bar{\delta}, \bar{\delta}), K^{COne}(\bar{\delta})) - c_I^e] + \sum_s \int_{\Omega_I^s(\bar{\delta})} \frac{\partial U_i^s(H^{COs}(\delta_i = \bar{\delta}, \bar{\delta}), K^{COs}(\bar{\delta}))}{\partial \bar{\delta}} = 0.$$

Set

$$\bar{\Delta c}_I^e = \sum_s \int_{\Omega_I^s(\bar{\delta})} \frac{\partial U_i^s(H^{COs}(\delta_i = \bar{\delta}, \bar{\delta}), K^{COs}(\bar{\delta}))}{\partial \bar{\delta}}.$$

Then, given education fees equal to $(c_I^e + \bar{\Delta c}_I^e)$, the threshold value in the market economy (with subsidies) coincides with $\bar{\delta}$. Therefore, given $\bar{\xi}$, the equilibrium level of social surplus, $\delta^F(\bar{\xi})$ coincides with its CO level. Finally, redistribute the total net revenues (or costs) of the fee-subsidy scheme across workers using i -invariant lump-sum taxes so to balance the budget.

Proof of Proposition 3. Let $Y^s(\delta_i, \delta^F, \xi)$ be the equilibrium level of the output of a match with worker i in sector s . Using the properties of the two sets $\Omega_I^s(\delta^F)$ and $\Omega_J^s(\delta^F)$, we can rewrite $P(\xi)$ as

$$\begin{aligned} & P(\delta^F(\xi), \xi) \\ \equiv & \sum_s \int_{\Omega_I^s(\delta^F(\xi))} \left(\beta Y^s(\delta_i, \delta^F, \xi) - \frac{1}{\delta_i} \frac{H^s(\delta_i, \delta^F, \xi)^{1+\Gamma}}{1+\Gamma} - c_I^s \right) di \\ & + \sum_s \int_{\Omega_J^s(\delta^F(\xi))} \left((1-\beta) Y^s(\delta_i, \delta^F(\xi), \xi) - \mu K^s(\delta^F(\xi), \xi) \right) dj \\ = & \sum_s \int_{\Omega_I^s(\delta^F(\xi))} \left(Y^s(\delta_i, \delta^F(\xi), \xi) - \frac{1}{\delta_i} \frac{H^s(\delta_i, \delta^F(\xi), \xi)^{1+\Gamma}}{1+\Gamma} - c_I^s \right) di \\ & - \sum_s \int_{\Omega_J^s(\delta^F(\xi))} \mu K^s(\delta^F(\xi), \xi) dj. \end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial P(\cdot)}{\partial \tau^s} &= \left(\frac{\partial P(\cdot)}{\partial \delta^*} \Big|_{\delta^* = \delta^F} \right) \frac{\partial \delta^F(\xi)}{\partial \tau^s} \\
&+ \left[\sum_s \int_{\Omega_I^s(\delta^F(\cdot))} \left(\frac{\partial Y^s(\cdot)}{\partial \tau^s} - \frac{1}{\delta_i} H^{Fs}(\cdot) \frac{\partial H^s(\cdot)}{\partial \tau^s} \right) di - \int_{\Omega_I^s(\delta^F(\cdot))} \mu \frac{\partial K^s(\cdot)}{\partial \tau^s} dj \right] \\
&\equiv \left(\frac{\partial P(\cdot)}{\partial \delta^*} \Big|_{\delta^* = \delta^F} \right) \frac{\partial \delta^F(\xi)}{\partial \tau^s} + D^s(\delta_i, \delta^F(\xi), \xi), \\
\frac{\partial P(\cdot)}{\partial \Delta c_I^e} &= \left(\frac{\partial P(\cdot)}{\partial \delta^*} \Big|_{\delta^* = \delta^F} \right) \frac{\partial \delta^F(\cdot)}{\partial \Delta c_I^e}.
\end{aligned}$$

Clearly, $D^s(\delta_i, \delta^F(\xi), \xi)$ is defined by the term in square brackets. Replacing (4) and (5) into $D^s(\delta_i, \delta^F(\xi), \xi)$, we obtain

$$\begin{aligned}
D^s(\delta_i, \delta^F(\xi), \xi) &= \sum_s ((1 + \tau^s) \alpha \beta)^{\frac{1}{\Gamma}} \left[\alpha + \beta - \alpha \beta - \frac{(1 + \tau^s) \alpha \beta}{1 + \Gamma} \right] \\
&\left[v \left(\Omega_I^s(\delta^F(\xi)) \right) E_{\Omega_I^s(\delta^F(\xi))} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} A^{s \frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{(1-\alpha)(1-\beta)}{\mu} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \right]
\end{aligned}$$

and, therefore,

$$\begin{aligned}
\frac{\partial D^s(\cdot)}{\partial \tau^s} &= \left[v \left(\Omega_I^s(\delta^F(\xi)) \right) E_{\Omega_I^s(\delta^F(\xi))} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} A^{s \frac{1+\Gamma}{\alpha\Gamma}} \left(\frac{(1-\alpha)(1-\beta)}{\mu} \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \right] \\
&\times \frac{(\alpha\beta)^{\frac{1}{\Gamma}}}{\Gamma} (\alpha + \beta - 2\alpha\beta).
\end{aligned}$$

The term in square brackets is clearly positive. Given that $\alpha < 1$ and $\beta < 1$, $(\alpha + \beta - 2\alpha\beta) > (\alpha^2 + \beta^2 - 2\alpha\beta) = (\alpha - \beta)^2 > 0$. Hence, $D^s(\cdot) > 0$, for each s .

By direct computation,

$$\begin{aligned}
&\frac{\partial P(\delta^F(\xi), \xi)}{\partial \delta^F} \\
&= - \left[V_i^e(\delta_i = \delta^F, \delta^F, \xi) - V_i^{ne}(\delta_i = \delta^F, \delta^F, \xi) \right] \\
&- \left[\Pi_j^e(\delta_i = \delta^F, \delta^F, \xi) - \Pi_j^{ne}(\delta_i = \delta^F, \delta^F, \xi) \right] \\
&+ \sum_s \int_{\Omega_I^s(\delta^F)} \frac{\partial V_i^s(\cdot)}{\partial \delta^F} di + \sum_s \int_{\Omega_J^s(\delta^F)} \frac{\partial E_{\Omega_I^s(\delta^F)}(\Pi_j^s(\cdot))}{\partial \delta^F} dj,
\end{aligned}$$

By definition of δ^F , the first term in square brackets is zero. We have already established (see eqs. (13)) that the last four terms are positive and $\Pi_j^{ne}(\delta_i = \delta^F, \delta^F, \xi)$ is also clearly positive.

Hence, a *sufficient* condition for $\frac{\partial P(\delta^F(\xi), \xi)}{\partial \delta^F} > 0$ is

$$0 \leq \Delta P^e \equiv \int_{\Omega_i^e(\delta^F)} \frac{\partial V_i^e(\cdot)}{\partial \delta^F} di + \int_{\Omega_j^e(\delta^F)} \frac{\partial E_{\Omega_i^e(\delta^F)}(\Pi_j^s(\cdot))}{\partial \delta^F} dj - \Pi_j^e(\delta_i = \delta^F, \delta^F, \xi).$$

Setting $\gamma \equiv \frac{1+\Gamma}{1+\Gamma-\alpha}$, by direct computation,

$$\begin{aligned} & \Pi_j^e(\delta_i = \delta^F, \delta^F, \xi) \\ &= \left[\frac{1}{\gamma} A e^{\frac{1+\Gamma}{\alpha\Gamma}} (\alpha\beta)^{\frac{1}{\Gamma}} \left(\frac{(1-\alpha)(1-\beta)}{\mu} \frac{1}{\gamma} \left(\frac{1-\delta^{F\gamma}}{1-\delta^F} \right) \right)^{\frac{(1+\Gamma)(1-\alpha)}{\alpha\Gamma}} \right] \\ & \quad \times (1-\beta) \left(\gamma \delta^{F\gamma-1} - (1-\alpha) \left(\frac{1-\delta^{F\gamma}}{1-\delta^F} \right) \right) \\ & \equiv B^e (1-\beta) \left(\gamma \delta^{F\gamma-1} - (1-\alpha) \left(\frac{1-\delta^{F\gamma}}{1-\delta^F} \right) \right). \end{aligned}$$

Moreover, using (8) and (9), and the definition of B^e (i.e., the expression in square brackets),

$$\begin{aligned} & \int_{\Omega_i^e(\delta^F)} \frac{\partial V_i^e(\cdot)}{\partial \delta^F} di \\ &= B^e \frac{(1+\Gamma)(1-\alpha)}{\gamma\alpha\Gamma} \beta^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \left(\frac{-\gamma \delta^{F\gamma-1} (1-\delta^F) + 1 - \delta^{F\gamma}}{1-\delta^F} \right) \end{aligned}$$

and

$$\begin{aligned} & \int_{\Omega_j^e(\delta^F)} \frac{\partial E_{\Omega_j^e(\delta^F)}(\Pi_j^e(\cdot))}{\partial \delta^F} dj \\ &= B^e (1-\beta) \alpha \left(\frac{1 - \gamma \delta^{F\gamma-1} + (\gamma-1) \delta^{F\gamma}}{1-\delta^F} \right). \end{aligned}$$

Hence, by a straightforward computation,

$$\begin{aligned} \frac{\Delta P^e}{\delta^{F\gamma-1} B^e} &= \left[\beta^{\frac{1+\Gamma-\alpha}{\alpha F}} \frac{(1+\Gamma)(1-\alpha)}{\gamma\alpha\Gamma} \left(\frac{1-\delta^{F\gamma}}{(1-\delta^F)\delta^{F\gamma-1}} - \gamma \right) \right] \\ &\quad + (1-\beta) \left[\frac{1-\delta^{F\gamma}}{(1-\delta^F)\delta^{F\gamma-1}} - (1+\alpha)\gamma \right]. \end{aligned}$$

The first term in square brackets is strictly positive (because, as we have shown above, $\frac{1-\delta^{F\gamma}}{(1-\delta^F)\delta^{F\gamma-1}}$ is bounded below by γ). The assumption in the statement of this Proposition means that the second term is also positive. Notice that the inequality is also satisfied for β large enough.

When $\frac{\partial F(\cdot)}{\partial \delta^*} |_{\delta^*=\delta^F} > 0$, $\frac{\partial \delta^F(\cdot)}{\partial \Delta c_I^e} > 0$ and $\frac{\partial \delta^F(\cdot)}{\partial \tau^{ne}} > 0$, so that $\frac{\partial P(\tau, \Delta c)}{\partial \tau^{ne}} > 0$ and $\frac{\partial P(\tau, \Delta c)}{\partial \Delta c_I^e} > 0$. It follows that a subsidy to education effort in sector ne , and/or an increase in the fixed cost of education c_I^e , increase the expected total surplus.

On the other hand, $\frac{\partial \delta^F(\cdot)}{\partial \tau^e} < 0$ and, therefore, under the maintained assumptions, the sign of $\frac{\partial P(\cdot)}{\partial \tau^e}$ is undefined.

7. APPENDIX 2: COMPETITIVE SPOT LABOR MARKETS

There is a continuum of separated islands, denoted by $j \in (0, 1)$. On each island there is an interval $(0, 1)$ of identical workers and firms. Firms (denoted by a pair $(j, \ell) \in (0, 1) \times (0, 1)$ where j denotes the island) are identical across islands. Workers (denoted by a pair $(i, \ell) \in (0, 1) \times (0, 1)$) are identical within an island (i.e., with respect to the index ℓ), but heterogeneous across islands, because of the parameter δ_i , whose realization in a given island is private information of the workers¹⁵. First, firms and workers choose the type and amount of their investments. Next, investments are mutually observable, (island specific) labor markets open and clear at the competitive wage. Given that, ex-ante, the realization δ_i is not observable, firms choose their investments taking into account the conditional expected amount of human capital of workers. Preferences and production functions are as above. Given that firms are identical, they all have the same optimal level of investments in each sector.

Hence, each worker chooses his behavior solving: given the equilibrium

¹⁵In the sequel we implicitly assume that the realization of $E_{\Omega^s(\hat{\delta})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)$ coincides with its theoretical value. A construction which guarantees that this is true is in Mendolicchio, Paolini, Pietra (2008).

maps $(w_i(\delta_i), s_\ell(j))$ and the equilibrium threshold value $\bar{\delta}$,

$$\text{choose } \left\{ \bar{s}_{i\ell}, \bar{h}_{i\ell}^s \right\} \in \arg \max_s \left\{ \max_{\bar{h}_{i\ell}^s} E_{\Omega^s(\bar{\delta})} (U_{i\ell}^s(C_{i\ell}^s, h_{i\ell}^s)) \text{ subject to } C_{i\ell} = w_i(\delta_i) \bar{h}_{i\ell}^s \right\}. \quad (\text{U2})$$

Given the equilibrium maps $(w_i(\delta_i), s_\ell(i))$ and the equilibrium threshold value $\bar{\delta}$, each firm

$$\text{choose } \left\{ \bar{s}_{j\ell}, \left(\bar{k}_{j\ell}^s, \bar{h}_{j\ell}^s \right) \right\} \in \arg \max_s \left\{ \max_{\left(\bar{k}_{j\ell}^s, \bar{h}_{j\ell}^s \right)} E_{\Omega^s(\bar{\delta})} \left(A^s h_{j\ell}^{s\alpha} k_{j\ell}^{s(1-\alpha)} - w_i(\delta_i) h_{j\ell}^{s\alpha} - \mu k_{j\ell}^s \right) \right\}. \quad (\text{II2})$$

DEFINITION 1. A rational expectations equilibrium is a map $w_i(\delta_i)$, a threshold value $\bar{\delta}$, and maps $\left\{ \bar{s}_\ell(i), \bar{h}_\ell^s(i) \right\}$ and $\left\{ \bar{s}_\ell(j), k_\ell^s(j), h_\ell^s(j) \right\}$ such that

- i.* $E_{\Omega^e(\bar{\delta})} \left(U_{i\ell}^e \left(\bar{C}_{i\ell}^e, \bar{h}_{i\ell}^e \right) \right) - E_{\Omega^e(\bar{\delta})} \left(U_{i\ell}^{ne} \left(\bar{C}_{i\ell}^{ne}, \bar{h}_{i\ell}^{ne} \right) \right) \geq 0$ if and only if $\delta_i \geq \bar{\delta}$,
- ii.* for each (i, ℓ) , $\left(\bar{s}_{i\ell}, \bar{h}_{i\ell}^s \right)$ solves (U2),
- iii.* for each (j, ℓ) , $\left\{ \bar{s}_{j\ell}, \left(\bar{k}_{j\ell}^s, \bar{h}_{j\ell}^s \right) \right\}$ solves (II2),
- iv.* for each j , $\int_{(0,1)} h_\ell^s(i) d\ell = \int_{(0,1)} h_\ell^s(j) d\ell$, for each j .

We start solving for the ex-post competitive equilibrium, contingent on the aggregate investments in physical capital. A straightforward computation shows that the equilibrium wage map is defined by

$$w^s \left(\delta_i, \bar{K}_j^s \right) = \frac{\left(\alpha A^s \bar{K}_j^{s(1-\alpha)} \right)^{\frac{\Gamma}{1+\Gamma-\alpha}}}{\delta_i^{\frac{1-\alpha}{1+\Gamma-\alpha}}}.$$

Moreover, given an arbitrary threshold value $\hat{\delta}$, ex-ante, expected profits of a firm active in sector s are given by

$$E_{\Omega^s(\hat{\delta})} (\Pi_{j\ell}) = (\alpha A^s)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) E_{\Omega^s(\hat{\delta})} \left(\frac{1}{w^s \left(\delta_i, \bar{K}_j^s \right)^{\frac{1}{1-\alpha}}} \right) k_{j\ell} - \mu k_{j\ell}.$$

Hence, the zero expected profit condition imposes

$$(\alpha A^s)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\mu\alpha} \right) E_{\Omega^s(\hat{\delta})} \left(\frac{1}{w^s \left(\delta_i, \bar{K}_j^s \right)^{\frac{1}{1-\alpha}}} \right) = 1.$$

Replacing into this condition $w^s(\delta_i, \bar{K}_j^s)$, and taking into account that \bar{K}_j^s is j -invariant, we get

$$(\alpha A^s)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\mu\alpha} \right) \frac{1}{\left(\alpha A^s \bar{K}_j^s (1-\alpha) \right)^{\frac{\Gamma}{1+\Gamma-\alpha} \frac{1-\alpha}{1-\alpha}}} E_{\Omega^s(\hat{\delta})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right) = 1,$$

or

$$\bar{K}_j^s = \left(\frac{1-\alpha}{\mu\alpha} \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} (\alpha A^s)^{\frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega^s(\hat{\delta})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}.$$

Replacing into $w^s(\delta_i, \bar{K}_j^s)$, we obtain the map

$$w^s(\delta_i, \hat{\delta}) = \frac{(\alpha A^s)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\mu\alpha} \right)^{\frac{1-\alpha}{\alpha}} E_{\Omega^s(\hat{\delta})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{1-\alpha}{\alpha}}}{\delta_i^{\frac{1-\alpha}{1+\Gamma-\alpha}}}.$$

Finally, given $\hat{\delta}$, the value of the indirect utility function (if s) is

$$V_i^s(\delta_i, \hat{\delta}) = \left(\frac{\Gamma}{1+\Gamma} \left(\frac{1-\alpha}{\mu\alpha} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \alpha^{\frac{1+\Gamma}{\alpha\Gamma}} \right) \delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} A^{s \frac{1+\Gamma}{\alpha\Gamma}} E_{\Omega^s(\hat{\delta})} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}}.$$

The map defining the equilibrium threshold is then

$$\begin{aligned} & \delta^{* \frac{\alpha}{1+\Gamma-\alpha}} \left[\left(A^e E_{\Omega_I^e(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{(1-\alpha)} \right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} - \left(A^{ne} E_{\Omega_I^{ne}(\delta^*)} \left(\delta_i^{\frac{\alpha}{1+\Gamma-\alpha}} \right)^{(1-\alpha)} \right)^{\frac{(1+\Gamma)}{\alpha\Gamma}} \right] \\ & = \left(\frac{1+\Gamma}{\Gamma \left(\frac{1-\alpha}{\mu\alpha} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \alpha^{\frac{1+\Gamma}{\alpha\Gamma}}} \right) c_I^e \end{aligned} \quad (16)$$

Modulo a multiplicative term, this expression is identical to eq. (8) in the text. Evidently, the qualitative properties of the equilibria are the same in the two economies. There is, however, an important difference with respect to efficiency. By using the same definition of CO used above, the supply of human and physical capital is CO, contingent on the threshold value, i.e., if $\delta^F = \delta^{CO}$ the market equilibrium would be CO. One can prove that (except, possibly, for an exceptional set of parameters) if δ^F solves (16), it cannot satisfy the FOC of the planner's optimization problem, so that, typically, $\delta^F \neq \delta^{CO}$. The policy conclusions reported in Proposition 3, hold when $\delta^F < \delta^{CO}$: too many agents invest in education and there is too little investment in effort.

8. REFERENCES

Acemoglu, D.T., 1996, A microfoundation for social increasing returns in human capital accumulation, *Quarterly Journal of Economics*, 111, 779-804.

Acemoglu, D., and J. Angrist, 2001, How large are the social returns to education? Evidence from compulsory schooling laws, in B. Bernanke and K. Rogoff (eds.), *NBER Macroeconomic Annual 2000*, Cambridge MA, MIT Press, 9-58.

Acemoglu, D.T., and R. Shimer, 1999, Holdups and efficiency with search frictions, *International Economic Review*, 40, 827-849.

Ben-Porath, Y., 1970, The production of human capital over time, in W. Hansen (ed.), *Education, Income and Human Capital*, NBER, 129-154.

Blankenau, W., 2005, Public schooling, college subsidies and growth, *Journal of Economic Dynamics and Control*, 29, 487-507.

Blankenau, W., and G. Camera, 2006, A simple economic theory of skill accumulation and schooling decisions, *Review of Economic Dynamics* 9, 93-115.

Blankenau, W., and G. Camera, 2009, Public spending on education and the incentives to student achievement, *Economica*, forthcoming.

Boskin, M.J., 1975, Notes on the tax treatment of human capital, NBER WP 116.

Carneiro, P., J.J. Heckman, and E. Vytlačil, 2001, Estimating the return to education when it varies among individuals, mimeo.

Caucutt, E.M., and K.B. Kumar, 2003, Higher education subsidies and heterogeneity. A dynamic analysis, *Journal of Economic Dynamics and Control*, 27, 1459-1502.

Charlot, O. and B. Decreuse, 2005, Self-selection in education with matching frictions, *Labour Economics* 12, 251-267.

De la Fuente, A., 2003, *Human capital in a global and knowledge-based economy. Part II: Assessment at the EU country level. Final Report*, European Commission, Directorate General for Employment and Social Affairs.

Eaton, J., and H.S. Rosen, 1980, Taxation, Human Capital, and Uncertainty, *American Economic Review*, 70, 705-715.

Goldin, C., and L.F. Katz, 2007, The race between education and technology: The evolution of U.S. educational wage differentials, 1890 to 2005, NBER WP 12984.

Gordon, R.J., and I. Dew-Becker, 2008, Controversies about the rise of american inequality: A survey, NBER WP 13982.

Griliches, Z., 1970, Notes on the role of education in production functions and growth accounting, in W. Hansen (ed.), *Education, Income and Human Capital*, NBER, Cambridge MA, 71-128.

Heckman, J.J., 1976, A Life-Cycle Model of Earnings, Learning, and Consumption, *Journal of Political Economy*, 84, S9-S44.

- Heckman, J.J., and A.B. Krueger (eds.), 2003, *Inequality in America: What Role for Human Capital Policy?*, MIT Press, Cambridge MA.
- Heckman, J.J., L. Lochner, and C. Taber, 1998, Tax policy and human capital formation, *American Economic Review*, 88, 293-297.
- Heckman, J.J., L. Lochner, and C. Taber, 1999, General equilibrium cost benefits analysis of education and tax policies, NBER WP 6881.
- Krueger, A.B., and M. Lindhal, 2000, Education for growth: Why and for whom?, *Journal of Economics Literature*, 39, 1111-1136.
- Lloyd-Ellis, H., 2000, Public education, occupational choice, and the growth inequality relationship, *International Economic Review*, 41, 171-202.
- Lucas, R.E., 1988, On the mechanics of economic development, *Journal of Monetary Economics*, 22, 3-42.
- Mendolicchio, C., D. Paolini and T. Pietra, 2008, Investments in education in a random matching model, CRENoS WP 2008/09.
- OECD, 2008, *Education at a Glance 2008. OECD Indicators*, OECD, Paris.
- Romer, P.A., 1990, Endogenous technological change, *Journal of Political Economy*, 98, S71-S102.
- Roy, A., 1951, Some thoughts on the distribution of earnings, *Oxford Economic Papers*, 3, 135-146.
- Sahin, A., 2004, The incentive effects of higher education subsidies on student effort, Federal Reserve Bank of New York, Staff Report 192.
- Sattinger, M., 1993, Assignment Models of the Distribution of Earnings, *Journal of Economic Literature*, 31, 831-880.
- Su, X., 2004, The allocation of public funds in a hierarchical educational system, *Journal of Economic Dynamics and Control*, 28, 2485-2510.
- Willis, R.J., and S. Rosen, 1979, Education and self-selection, *Journal of Political Economy*, 87, S7-S36.