Reputation and Collusion in Procurement*

Giacomo Calzolari† Giancarlo Spagnolo‡

PRELIMINARY AND INCOMPLETE
October, 2005

Abstract

When gains from trade exist both along contractible and non-contractible dimensions and procurement is repeated, non-contractible gains from trade can be realized through reputation/relational contracting. A buyer may restrict participation to his recurrent auctions to a subset of potential sellers and threaten replacement of sellers that perform poorly along non-contractible dimensions. In such a dynamic procurement framework, keeping the optimal number of eligible sellers endogenous, we find that: a) *there is a general trade-off between reputation for quality and collusion*: shorter contracts - more frequent re-auctioning - facilitate non-contractible quality provision, but also collusive agreements among suppliers; and b) when non-contractible quality and variability in suppliers' efficiency are both important, *short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off.*

JEL Classification Numbers: ...
Keywords: Reputation, Collusion, Procurement, Auction, Non-contractible quality.

*We have benefited from comments of seminar participants at IUI – Stockholm, European Summer Symposium in Economic Theory 2005 CEPR/Studienzentrum Gerzensee, Econometric Society World Congress 2005 and EARIE conference 2005.

†Department of Economics, University of Bologna, Italy. E-mail: *calzolari@economia.unibo.it*

‡Stockholm School of Economics, Consip Research Unit, and CEPR. E-mail: *giancarlo.spagnolo@tesoro.it, giancaspagnolo@yahoo.com.*

1 Introduction

Non contractible dimensions are present in different measure in every economic exchange.[1] When these dimensions are important in terms of gains from trade, letting suppliers compete - say, in an auction - may lead to a very inefficient outcome, for a buyer and in general.[2] Exchanges, however, are often regularly repeated. Reputational forces may then help governing transactions on non-contractible aspects. An opportunistic supplier that overstates the non-contractible quality of an experience good or that purposely reduces non-verifiable but ex post observable qualitative aspects to cut costs and bust profits can be punished by its buyer(s), e.g. with exclusion from future trade.[3]

This paper analyzes repeated procurement processes (recurrent auctions or other forms of search) where non-contractible dimensions are an important source of gains from trade and moral hazard, and suppliers' relative efficiency changes over time. As in Manelli and Vincent (1995) the optimal amount of search done before proposing a contract - e.g. the number of suppliers admitted to participate at the recurrent auctions - is reduced by non-contractible quality, but for a somewhat different reason than their "bad selection" effect. Even absent adverse selection effects, in a dynamic procurement process one may want to restrict the number of trading partners at the cost of reduced screening and savings on contractible dimensions. This is done to leave them sufficient future rents so that reputational commitments to provide acceptable levels of non-contractible quality (refrain from moral hazard) are sustainable.[4]

Duration of supply contracts is a crucial aspect in dynamic procurement. Abstracting from (important) technological aspects such as the rate of obsolescence, a shorter duration of supply contracts implies more frequent re-selection/search. This higher frequency of interaction makes it easier for a buyer to obtain high non-contractible quality levels from sellers by threatening exclusion from future trade.[5] On the other hand, for analogous reasons, more frequent procurement auctions also facilitate collusive behavior between the selected suppliers! Hence we find a rather general trade-off between reputation for quality and collusion in dynamic procurement: longer duration of supply contracts - less frequent auctions - deter collusion among eligible suppliers but reduce non-contractible quality levels obtainable from them; shorter contracts - more frequent auctions - facilitate supplier collusion but also the enforcement of non-contractible quality standards.

Note however that by increasing the selling price, collusion increases the expected gains from participating in procurement auctions, i.e. the cost of being excluded, hence the highest non-

[1]Reasons why some dimensions of exchanges are not explicitly contractible include complexity and prohibitive legal cost of verification; see Hart (1995) for an in depth discussion and Tirole (1999) for an evaluation of the debate on contracts incompleteness.

[2]Within a static trading environment with adverse selection, Manelli and Vincent (1995) show that when gains from trade are concentrated on non-contractible hidden characteristics, auctions may lead to the worse possible outcome and are dominated by random take-it-or-leave-it offers.

[3]Classic references include Klein and Leffler (1981), Shapiro (1983) and Allen (1984).

[4]Fehr et al. (2004) show experimentally how in a dynamic exchange environment, when non contractible aspects become important, agents do not search for the best offer each period but rather stick to the same partner and cooperate with him all periods.

[5]This point was made by Shapiro (1983) and follows from repeated games logic: with more frequent interaction the threat of exclusion is closer in time and gains from "cheating" are smaller.

contractible quality level enforceable. This consideration leads to our second main result. In a world in which firms' cost efficiency changes over time, buyers may use auctions to select the most efficient supplier each period; but the strong competition induced by auctions drives down future rents making reputation ineffective and non-contractible quality hard to obtain. When both gains from trade on non-contractible quality and the variability in firm cost efficiency across time are sufficiently large, one would like at the same time to restrict competition to generate sufficient future rents that reputational mechanisms work and to select the most efficient firm from a sufficiently large pool of firms. We show that in this situation, if firms in a cartel are able to share information on each other's costs at the time of each auction, then short contract duration that induces *a collusive agreement between eligible suppliers may leave the buyer better off.* One can interpret this possibility as the buyer stimulating the formation of firms consortia.

Relation with the literature. Our work is most closely related, at least in spirit, to Manelli and Vincent's (1995) analysis of optimal trading mechanisms in presence of non contractible qualitative aspects of the good. They cast their model in terms of adverse selection (sellers are of different quality which is reflected in the produced good), focus on single transactions, and show that when gains from trade are concentrated on the non-contractible qualitative dimension, auctions deliver the worst possible outcome, as they select firms producing the good at the lowest cost but with the lowest level of non-contractible quality. Sequential take-it-or-leave-it offers to randomly selected sellers is then a better mechanism. We also consider procurement when non-contractible quality is important, but allow the buyer to repeatedly purchase, and sellers to choose the level of non-contractible quality provided after the procurement auction .In this sense our model is close to Kim (1998), who study a repeated auctions model with moral hazard, i.e. where after the auction the firm that won can choose the level of non-contractible quality to deliver, and shows that it may be good to restrict participation to auctions and threaten exclusion if the level of non-contractible quality is too low. As Manelli and Vincent (1995), we find that the strong competition induced by an auction may be bad, but that it may be more efficient to reduce such competition by letting sellers collude rather than giving way all gains from screening from a pool of sellers.[6]

Our paper also contributes to the literature on firm's reputation for quality when this cannot be contracted for, stating with the seminal work of Klein and Leffler (1981), Shapiro (1983) and Allen (1984). This early work was concerned with the compatibility of "quality-assuring" reputational equilibria - requiring rents that make the effort of maintaining reputation worthwhile - with free entry in the market, but did not analyze in detail firms' competitive interaction (firms' incentives to steal business from each other).[7] Stiglitz (1989) raised the question how could reputation be compatible with perfect competition, which driving firms to undercut each other should eliminate any future supracompetitive gains that could motivate costly investment in reputation. Kranton (2003) offers a model that captures this dilemma, where in presence of moral hazard on quality competition makes high quality equilibria unfeasible; and suggests restricting competition in indus-

[6] See also Bulow and Klemperer (1996) and Bajari and Tadelis (2001).

[7] In Klein and Leffler and Shapiro firms face a perfectly elastic demand at the quality assuring price; in Allen consumers are randomly allocated among the firms charging the lowest price weakly above the "quality-assuring" one.

tries where non-contractible quality is important. In a different model Bar-Isaac (2004) confirms Kranton and Stiglitz's view at the limit, but shows that at intermediate levels of competition a further increase in competition (number of firms and substitutability) may well increase equilibrium product quality. But it is Hoerner (2002) that first offers an elegant answer to Stiglitz's question: in his model with heterogeneous consumers, adverse selection and moral hazard high prices signal high quality and make competition compatible with (in fact necessary for) reputation to work. In our model, and with auctions in general, (price) signalling is impossible because the lower price is chosen by the mechanism, and the trade off quality-competition reappears. The degree of competition is measured by the number of firms admitted to the auctions, and indeed restricting participation increases the level of sustainable quality. Among the additional elements considered in our model are the efficiency costs of restricting competition linked to reduced screening; the interplay between reputation and collusion among sellers; the role of contract duration or frequency of interaction in this trade-off (Shapiro noted that the frequency of interaction facilitates the operation of reputational mechanisms, but did not deal with collusion); and the different effects of different ways to limit competition often presented as substitutes (collusion versus entry restrictions).

Other related strands of literature connected with the present paper are the analysis on: incentives in procurement (e.g. Laffont and Tirole (1993)); incomplete and Relational Contracts (the Hart and Moore paradigm, McLeod and Malcomson (1989), Levin (2003), Bernheim and Whinston (1998), Baker, Gibbons and Murphy (1994, 2001), Blonski and Spagnolo (2003)); Collusion in Repeated Auctions (e.g. Aoyagi (2003), Blume and Heidhues (2004)).

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 analyzes procurement and quality with competing firms. Section 4 discusses the effect of collusion on implementable quality. compares regulations and discusses bank's representation preference. Section 5 illustrates optimal procurement. Section 6 extends the base model and discusses its main assumptions. Finally, Section 7 concludes.

2 Model Setup

A buyer B needs a unit of a good (or a service) at any date (we will refer to buyer, auctioneer and procurer as synonymous) and N firms can procure the good. Time horizon is infinite and all the players have a constant common discount factor equal to δ.

The buyer cares for the non-contractible quality of the good. His valuation of the good at any date $V(q)$ depends on quality q, with $V(0) = 0$ and $V' \geq 0$.

Any firm i can procure a unit of the good with a per-period cost that depends on a cost (in-)efficiency parameter θ_i and it is increasing with the level of non-contractible quality provided by the firm, i.e. the per-period cost of firm i is $\theta_i + \psi(q_i)$, where $\psi(\cdot)$ is a positive real valued function with $\psi' \geq 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

The buyer is not fully informed on firms' costs so that she runs auctions to procure the goods. The auction awards a procurement contract that requires the winning supplier to procure the good for $x \geq 1$ periods with a contract length x determined by the buyer. Contracts that last more than one period cannot be unilaterally renegotiated (i.e. reneging is ruled out by law), but bilateral renegotiation is admissible. Any firm participating an auction makes an offer b_i and the winner is the bidder that offers the lowest acceptable price.[8] Then, the auction rules map the bids vector $\mathbf{b}$ to the per-period payment p that the winner receives for any of the x periods comprised by the contract. The buyer may set a reservation price r so that acceptable bids must satisfy $r \geq b_i$ and, if $r < b_i$ for any i, the buyer does not award the contract. Furthermore, the buyer can decide to limit to $n \leq N$ the number of bidders that are eligible and admitted to the auction process.

Organizing an auction for a contract lasting x periods costs a fixed amount $K \geq 0$ to the buyer.

The firm that is awarded the contract sets the level of quality once and for all the duration x of the contract.[9] Moreover, quality is observable but not verifiable so that it is not contractible. However, if the quality provided by the auction winning firm does not satisfy a quality requirement that the buyer specifies before the auction, then the buyer can exclude this firm for some future auction rounds.[10]

We model this exclusion rule, indicated by σ, with a minimum quality requirement $q \geq \underline{q}$ so that if the firm procures a good of quality $q < \underline{q}$, the buyer can discretionally decide to exclude this seller for the next T auctions.[11] Hence, the contract that is agreed upon between the buyer and the winning firm is composed by a part that is verifiable and court-enforced (i.e. contract duration x and per-period payment p) and a part that is non-contractible and discretionary (provided quality and the minimum quality standard $\underline{q}$). A court can enforce the mechanism that identifies the procuring firm with the one offering the lowest price (or the best offer in terms of contractible features), the price paid to the winning firm and the contract length x. On the contrary, the discretionary part of the contract is the exclusion rule with its minimum quality requirement and actual quality q.[12]

The **timing of the game** is as follows.

$t = -2$: Once and for all, the buyer sets the contract length x, the number of bidders $n \leq N$ that are admitted to the auction stage and a reservation price r and announces an exclusion rule $\sigma = (\underline{q}, T)$.

$t = -1$: The buyer randomly selects the n firms amongst the N potential sellers.[13]

$t = 0$: An infinite repetition of the following stage game (or auction game) takes place:

Stage Game

[8] To simplify exposition, we let the price summarize contractible qualitative features. A more general interpretation of our model is that the buyer evaluates all contractible elements in firms' offers with a scoring rule that is here represented by the price, which can be thought of as a score.

[9] Our result would be qualitatively unaffected if firms could be free to choose the quality level for any period. In this case, if a firm decided not to provide the minimum required quality, then it would certainly do at the first period and for all the duration of the contract, exactly as in the case we study.

[10] Clearly, in the private sector a buyer is free to discretionally exclude firms. In the public sector this may be problematic. However, national rules for public procurement often contemplate some discretion in excluding disloyal firms. This is explicitly the case for US and UK. In EU, a recent European Directive (EC, 2004) allows for a two-stages procedure where the buyer first determine a pool of firms with a competitive procedure and then with discretion she is free to chose a supplier among them.

[11] In Section 6 we explore the possibility for the buyer to use scoring rules on non-contractible quality.

[12] This relates to the literature on relational contracts (e.g. Levin 2003, Blonski and Spagnolo 2003).

[13] In Section 5 we will consider the possibility that the buyer asks a participation fee to the firms.

At time t_1, an auction is run that awards the procurement contract, all firms observe who wins and at what price; the auction winner sets the level of quality q of the good it will provide;

At any period $t \in [t_1 + 1, t_1 + x]$ the winner procures the good of quality q;

At time $t_1 + x$, the buyer observes the quality provided during the contract and excludes the winning firm i for T periods in case $q_i < \underline{q}$.

If the firm that procured the good has been excluded, a new firm enters (otherwise the pool n remains unchanged) and a new stage game starts.

For the sake of simplicity we make the following working assumptions which will be further discussed in Section 6.

Assumption 1 *(i) In each period, for any firm i in the pool of n firms the cost parameter θ_i is drawn from a distribution $f(\theta_i)$ on $\Theta = [\underline{\theta}, \overline{\theta}]$;*

(ii) The Buyer does not know the realization of $\boldsymbol{\theta} = (\theta_{1,...,}\theta_n)$ but she knows f and Θ;

(iii) Firms are fully informed.

Assumption 2 *The number of potential suppliers N is infinite.*

Assumption 3 *The buyer needs to be served at any period, i.e. $V(\varnothing) = -\infty$.*[14]

Assumption 2 will be relaxed in Section 6. With assumption 3 interrupting the flow of goods / services is extremely costly and this makes the analysis simpler by guaranteeing buyer's participation.

A strategy for the buyer includes all the elements that compose the contractible rules of the procurement process: r, n, x.[15] In addition, the buyer's strategy is complemented with non-contractible ingredients of the procurement relationship, namely the exclusion rule $\sigma = (\underline{q}, T)$ composed by the minimum quality requirement $\underline{q}$ and the exclusion procedure lasting T periods for a firm that served a given contract but did not provided the required quality.

A strategy for a firm i is composed by a participation decision, a bid b_i and a decision on procured quality q_i in case the firm wins an auction at any auction stage.

In Sections 3-5 we will consider the tougher exclusion rule available that punishes firms with exclusion forever from future auctions whenever the quality requirement is not met, i.e. $T = \infty$. This exclusion rule is credible because the (infinite) firms are ex-ante identical, so that replacing a firm has no cost for the buyer, and constitutes an optimal penal code in the sense of Abreu (1988) that allows us to characterize the maximum level of non-contractible quality that can be implemented through this implicit contract.[16]

[14] Equivalently, one could assume that being served with any quality q provides the buyer with a value $v + V(q)$ where v is positive and very large.

[15] For simplicity in the exposition we will treat n and x as continuous variables.

[16] With finite N the threat of irreversible exclusion is still credible, but it is not robust w.r.t. coalitions of deviants. See the discussion in Section 6.

3 Implementable Quality with Competing Suppliers

When the buyer does not know the realization of firms' costs, she sets the elements of the procurement process, the exclusion rule and the quality requirement once and for all stage games.[17]

Let $\theta'(n)$ be the cost of the most efficient firm in the pool of n firms admitted to production, i.e. $\theta'(n) \equiv \min\{\theta_1, ..., \theta_n\}$. Note that the minimum cost statistic $\theta'(n)$ for a given set of firms' costs $\boldsymbol{\theta} = (\theta_1, ..., \theta_n)$ is a decreasing function of n.

The maximum welfare that could be determined at each auction is

$$w(q, x, n) \equiv \sum_{k=1}^{x} \delta^{k-1} \left[V(q) - \theta'(n) - \psi(q)\right] - K = \frac{1-\delta^x}{1-\delta} \left[V(q) - \theta'(n) - \psi(q)\right] - K$$

so that the *overall maximum welfare* for infinite repetitions of the auction stage is equal to

$$W(q, x, n) = w(q, x, n) \sum_{k=0}^{\infty} \delta^{kx} = \frac{1}{1-\delta} \left[V(q) - \theta'(n) - \psi(q)\right] - \frac{1}{1-\delta^x} K.$$

Similarly, the buyer's surplus at any auction stage is

$$s(q, x, n) \equiv \frac{1-\delta^x}{1-\delta} V(q) - b - K$$

and her overall surplus is $S(q, x, n) = s(q, x, n) \sum_{k=0}^{\infty} \delta^{kx}$.

When quality is not contractible, any firm i that wins an auction may choose to satisfy the quality requirement $\underline{q}$, i.e. to provide a quality $q \geq \underline{q}$ or not. Clearly, being quality non contractible, none of the firms has incentive to provide any quality larger than the minimum requirement so that $q = \underline{q}$ if the firm's decision is to satisfy the quality requirement. Furthermore, we consider exclusion rules that punish firms with exclusion for one or more periods whenever the quality requirement is not met, so that if a firm prefers to do so, it optimally set the least cost quality, i.e. if fixes $q = 0$.

At any auction stage, the most efficient firm (i.e. the one with cost $\theta'(n)$), wins the auction by bidding a price b which is equal either to the minimum between the reservation price r and the second most efficient firm's cost. Hence, in case firms satisfy the quality requirement the winning bid is

$$b_w \equiv \min\{r, \theta''(n) + A\}$$

where $\theta''(n) \equiv \min\left\{\boldsymbol{\theta}/\theta'(n)\right\}$ is the second lowest efficiency parameter in vector $\boldsymbol{\theta}$ and $A = \delta\frac{1-\delta^{x-1}}{1-\delta} E[\theta] + \frac{1-\delta^x}{1-\delta} \psi\left(\underline{q}\right)$.

The most efficient firm can decide to win the auction and shirk on quality, thus providing $q = 0$. This strategy lets the firm save on production costs but it induces exclusion from all future

[17] Buyer's information is the same at any stage game, except for the quality provided by firms in previous contracts. Hence, his optimal strategy is time invariant.

auctions, with an overall profit equal to $b_w - \left[\theta'(n) + \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]\right]$. On the contrary, if it provides the required quality, the most efficient firm can alternatively obtain an expected profit equal to

$$b_w - \theta'(n) - A + E\left[\Pi_c | q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x}$$

where the third term is the expected profits from future auctions of a firm always complying with the quality requirement $q \geq \underline{q}$.

Comparing these profits for the most efficient firm, the following Lemma illustrates the maximal implementable quality, where a quality requirement $\underline{q}$ is implementable if firms offer a quality at least as large as $\underline{q}$.

Lemma 1 (Maximal implementable quality) *A necessary condition for a quality requirement* $\underline{q}$ *to be implementable is* $\underline{q} \leq \overline{q}(x,n)$ *where* $\overline{q}(x,n)$ *solves*

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} = \psi(\overline{q})\frac{1-\delta^x}{1-\delta}, \tag{1}$$

with

$$\Delta\theta(n) \equiv \begin{cases} \theta''(n) - \theta'(n) & \text{if } n > 1 \\ \overline{\theta} - E[\theta] & \text{if } n = 1 \end{cases}$$

and $\overline{q}(x,n) = 0$ *if either* $n = \infty$ *or* $x = \infty$. *In all cases,* $\frac{\partial \overline{q}}{\partial x} < 0$, $\frac{\partial \overline{q}}{\partial n} \leq 0$ *and the negative effect of* x *(*n*) on implementable quality reduces with* n *(*x*).*

Proof. First we need to discuss the optimal reservation price for the buyer. The buyer sets a reservation price r in order to guarantee firms' participation for any realization of costs. This is a consequence of Assumption 3 that implies she bears a high cost in case of no procurement of the good. To this end r must satisfy the following participation condition

$$r - \overline{\theta} - A \geq 0$$

so that the optimal reservation price is $r^s = \overline{\theta} + A$, where $A = \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] + \frac{1-\delta^x}{1-\delta}\psi\left(\underline{q}\right)$.[18] Hence, at any auction stage the winning bid of the most efficient firm is

$$b_w = \min\{\overline{\theta} + A, \theta''(n) + A\} = \theta''(n) + A.$$

In fact, if $b_w > \theta''(n) + A$, then the second most efficient firm can undercut the most efficient firm and Bertrand competition leads to an equilibrium price $b_w = \theta''(n) + A$.

For what stated in the text, the winning firm prefers to provide the required quality if

$$b_w - \theta'(n) - \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] - \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} + E\left[\Pi_c | q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x} \geq b_w - \theta'(n) - \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]$$

[18] Note that we assume an interim participation decision on the part of the firms, i.e. a firm accepts a contract if, knowing its type θ for the first period it accepts a contract even if it does not know its costs for subsequent periods in the contract.

or

$$E\left[\Pi_c | q \geq \underline{q}\right] \frac{\delta^x}{1-\delta^x} \geq \psi\left(\underline{q}\right) \frac{1-\delta^x}{1-\delta}.$$

>From independence, for a given θ_i, we have $\Pr(\theta_i \leq \theta_j, \forall j \neq i | \theta_i) = [1-F(\theta_i)]^{n-1}$. Hence, ex-ante the probability of being the lowest cost firm at any stage game $p_w(n)$ is $\Pr(\theta_i \leq \theta_j, \forall j \neq i) = \int [1-F(\theta_i)]^{n-1} f(\theta_i)\, d\theta_i = 1/n$. Moreover, applying the optimal reservation price, in any future auction the winning firm which satisfies the required quality obtains a profit equal $\Delta\theta(n)$. We can then rewrite the expected profits for any future auction as $E\left[\Pi_c | q \geq \underline{q}\right] = E\left[\Delta\theta(n)\right]/n$ so that the firm prefers to satisfy the quality requirement if

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} \geq \psi\left(\underline{q}\right) \frac{1-\delta^x}{1-\delta}$$

Hence, any quality requirement $\underline{q} > \overline{q}(x,n)$ as defined in (1) will be never satisfied by the most efficient firm.

We now show that by offering a price $b_w = \theta''(n) + A$, the most efficient firm indeed wins the auction, i.e. none of less efficient firm will have any incentive to offer a lower price and shirk on quality. For this it suffices to consider the second most efficient firm. This will not undercut the most efficient one by planning to provide a quality lower than the requirement if and only if the following is verified:

$$\begin{aligned} \theta''(n) + A - \theta''(n) - \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] - \frac{1-\delta^x}{1-\delta}\psi(0) &\leq E\left[\Pi_c | q \geq \underline{q}\right] \frac{\delta^x}{1-\delta^x} \Longleftrightarrow \\ \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} &\leq E\left[\Pi_c | q \geq \underline{q}\right] \frac{\delta^x}{1-\delta^x}. \end{aligned}$$

Hence, if (1) is verified then also the previous inequality is verified and can then be disregarded.

The value of $\overline{q}(x,n)$ for $n=\infty$, $x=\infty$ is immediate from (1). For the comparative statics on $\overline{q}$ w.r.t. x and n, differentiate (1) to obtain

$$\frac{\partial \overline{q}}{\partial n} = \frac{\delta^x(1-\delta)}{(1-\delta^x)^2}\left[\frac{\partial E\left[\Delta\theta(n)\right]}{\partial n}\frac{1}{n} - \frac{E\left[\Delta\theta(n)\right]}{n^2}\right]\frac{1}{\psi_q(\overline{q})}$$

with $\frac{\partial E[\Delta\theta(n)]}{\partial n} \leq 0$ so that $\frac{\partial \overline{q}}{\partial n} \leq 0$, and also

$$\frac{\partial \overline{q}}{\partial x} = \frac{(1-\delta)(1+\delta^x)\delta^x Log[\delta]}{(1-\delta^x)^3}\frac{E\left[\Delta\theta(n)\right]}{\psi_q(\overline{q})} \leq 0.$$

If $n=1$, the unique firm with cost θ will ask a price equal to the reservation price $r = \overline{\theta} + A$ and prefers to provide required quality if

$$\left[\overline{\theta} - E[\theta]\right]\frac{\delta^x}{1-\delta^x} \geq \frac{1-\delta^x}{1-\delta}\psi\left(\underline{q}\right)$$

where $\overline{\theta} - E[\theta] = E\left[\Pi_c | q \geq \underline{q}\right]$ in case $n=1$.

Finally, for future reference we also notice that

$$\frac{\partial^2 \overline{q}(x,n)}{\partial x \partial n} = \left[\frac{\partial E[\Delta\theta(n)]}{\partial n}\frac{1}{n} - \frac{E[\Delta\theta(n)]}{n^2}\right]\left[\frac{(1-\delta)(1+\delta^x)\delta^x Log[\delta]}{(1-\delta^x)^3\psi_q(\overline{q})} - \frac{\delta^x(1-\delta)}{(1-\delta^x)^2\psi_{qq}(\overline{q})}\frac{\partial \overline{q}}{\partial x}\right] \geq 0$$

so that a larger n makes $\frac{\partial \overline{q}}{\partial x}$ less negative and a larger x makes $\frac{\partial \overline{q}}{\partial n}$ less negative: with a larger n (x) the negative effect of x (n) on quality is reduced. ■

The term $E[\Delta\theta(n)]/n$ is the informational rent that firms can expect when winning future auctions. The Proposition shows that with uncontractible quality, firms are ready to provide quality in return of future profits so that if expected profits are small then the implementable quality is also small.[19] Furthermore, if all firms are admitted, i.e. $n = \infty$, any firm faces a negligible probability of being again the most efficient firm in tomorrow auctions and the expected cost difference between the most and the second most efficient firm $E[\Delta\theta(n)]$ is small. In this case future rents are null and there is no incentive to provide quality. This has been already pointed by Stiglitz (1989) who first showed that perfect competition clashes with firms' incentives to provide quality.

Furthermore, the condition on the maximal implementable quality (1) shows that if firms' heterogeneity is small, i.e. the expected cost difference between the most and the second most efficient firm $E[\Delta\theta(n)]$ is small, or the number of admitted firms is large (i.e. there is strong competition in the bidding process), then the buyer can only implement a low level of quality. In addition, the longer is the contract length x the smaller is the maximal implementable quality for exclusion of a quality shirking firm is retarded when x is large. In the limit, when the buyer sets a once-and-for-all contract with $x = \infty$, the unique implementable quality is the null quality $\underline{q} = 0$.

Lemma 1 illustrates the negative effect of x and n on the maximal implementable quality. However, x and n have also other effects on the buyer's payoff that we now illustrate.

First, a longer contract (i.e. larger x) implies smaller auction costs K, but it also implies that along the contract the buyer is stuck with a firm that may no longer be the most efficient firm in the pool of n admitted ones.

Second, a larger n implies that the price asked by the most efficient firm $b_w = \theta''(n) + A$ decreases because $\theta''(n)$ is a decreasing function of n.

The existing trade-offs that emerge from these considerations are well illustrated in the following optimization program ($\mathcal{P}$) for the buyer

$$(\mathcal{P}) \quad \begin{cases} \max\limits_{\underline{q},x,n} \; S\left(\underline{q},x,n\right) = \frac{1}{1-\delta}\left[V(\underline{q}) - \psi\left(\underline{q}\right)\right] - \frac{E[\theta''(n)]}{1-\delta^x} - \frac{\delta\left(1-\delta^{x-1}\right)E[\theta]}{(1-\delta^x)(1-\delta)} - \frac{1}{1-\delta^x}K \\ s.t.\overline{q}(x,n) \geq \underline{q} \end{cases}$$

which is obtained by simply substituting the winning price b_w into $S\left(\underline{q},x,n\right)$ and that arises whenever the strategy of all firms contemplates the provision of required quality. Instead of providing an explicit solution to the program ($\mathcal{P}$), we now illustrate an important property of the optimal solution that follows from Lemma 1.

[19] With a single firm admitted (i.e. $n = 1$) buyer is obliged to buy at the reservation price so that auction profits are equal to $\max\theta - E[\theta] \geq \Delta\theta(n)$ for any $n > 1$.

The buyer may accept a good of null non-contractible quality instead of no procurement at all (by assumption 3). When she sets a null quality requirement $\underline{q} = 0$, she also maximizes her surplus $S(0, x, n)$ by optimizing the number of admitted firms n and the contract length x. In this case then we immediately have

Lemma 2 *If the buyer sets a nil quality standard $\underline{q}^s = 0$, then the optimal number of admitted firms is $n_0^s = \infty$ and the optimal contract length is $x_0^s = \infty$ if $K \geq E[\theta] - \underline{\theta}$ and $x_0^s = 1$ otherwise.*

Proof. The function $S\left(\underline{q}, x, n\right)$ with $\underline{q} = 0$ is non increasing in n and it is non increasing in x if $K \geq E[\theta] - \underline{\theta}$ and increasing in x otherwise. Furthermore, we know from Lemma 1 that $\overline{q}(x, n)$ is nil if at least one amongst x and n is infinite. Hence, the result follows. ■

Let $S(0, x_0^s, n_0^s)$ denote the maximal surplus associated with procurement of nil quality. We then have

$$S(0, x_0^s, n_0^s) = \begin{cases} S(0, \infty, \infty) = -\underline{\theta} - \frac{E[\theta]\delta}{1-\delta} - K & \text{if } K \geq E[\theta] - \underline{\theta} \\ S(0, 1, \infty) = -\frac{\underline{\theta}}{1-\delta^x} - \frac{\delta\left(1-\delta^{x-1}\right)E[\theta]}{(1-\delta^x)(1-\delta)} - \frac{1}{1-\delta^x}K & \text{otherwise} \end{cases}$$

Indeed, $S(0, x, n)$ is increasing in n so that the optimal n is always infinite when the procured quality is nil (i.e. $n_0^s = \infty$) and the optimal contract length depends on the sign of the derivative of $S(0, x, \infty)$ w.r.t. x which is proportional to $K - [E[\theta] - \underline{\theta}]$. Even if the procured quality is nil and competition is maximal with all potential suppliers competing at any auction, still the buyer trades off the cost of running auctions more frequently (i.e. K) with the cost of being stuck with an inefficient firm for a long period (i.e. $E[\theta] - \underline{\theta}$). To see the latter, consider a contract that lasts two periods, i.e. $x = 2$, and is assigned, say, to firm i. In the second period the buyer knows that with $n = \infty$ in the pool of n firms there will be one with cost $\underline{\theta}$ which is more efficient than the procuring firm i and with an efficiency gap equal to $E[\theta] - \underline{\theta}$. Note that the expected maximal surplus associated with $\underline{q} = 0$ can be also rewritten with the following intuitive expression,

$$S(0, x_0^s, n_0^s) = S(0, \infty, \infty) + \min\{0, K - [E[\theta] - \underline{\theta}]\} \frac{\delta}{1-\delta}.$$

If non-contractible quality is not a matter for the buyer, then she optimally sets a contract length x_0^s, admits all the firms n_0^s at the auction and the implemented quality is nil. However, when quality is valuable then at the optimum the buyer may be induced to reduce both the number of firms thus reducing competition and the contract length so that she can react promptly against a quality shirking firm.

To investigate the effects of these changes of x and n on her payoff, we can decompose the difference $S(q, x, n) - S(0, x_0^s, n_0^s)$ in several terms, as follows

$$\begin{array}{l} S(q, x, n) - S(0, x_0^s, n_0^s) = \frac{1}{1-\delta}\left[V(q) - \psi(q)\right] + \\ -\left[E\left[\theta''(n)\right] - \underline{\theta}\right] + \left[E\left[\theta''(n)\right] - E[\theta]\right]\frac{\delta^x}{1-\delta^x} \\ -K\frac{\delta^x}{1-\delta^x} - \min\{0, K - [E[\theta] - \underline{\theta}]\}\frac{\delta}{1-\delta} \end{array} \tag{2}$$

The first term is clearly the positive and direct effect of quality induced by relaxing the implementable quality constraint with a reduction of x and or n. The second line shows the cost of reducing n in terms of being served by a firm which is not the most efficient one (in the first period of the first auction when $x_0^s = \infty$ and in the first period of any future auction when $x_0^s = 1$). The third line illsutrates that more frequent auctions imply larger organization costs (the first term when $_0^s = \infty$ and also the last one when $x_0^s = 1$ because from tomorrow and for any period, the buyer organizes an auction at any period with a net cost $K - [E[\theta] - \underline{\theta}]$).

When the sum of these effects is positive, then the buyer may want to fix n and x so that $\overline{q}(x, n) > 0$ and she can set a minimum quality requirement $\underline{q}$ strictly larger than zero. This is certainly the case if the (net) per period value of quality $V(q) - \psi(q)$ is sufficiently large for certain values of q. In particular, we consider the following measure of how much non-contractible quality is important in terms of producing welfare. We state that *quality is "important"* if increasing quality from a zero level significantly augments the welfare, i.e. $V_q(0) - \psi_q(0)$ is large. We can now state the following result

Proposition 1 (Optimal Procurement with Competing Firms) *If non-contractible quality is sufficiently important (i.e. $V_q(0) - \psi_q(0)$ is positive and large), then (i) the buyer optimally sets quality q^s at its maximal implementable level $\overline{q}(x, n)$; (ii) she restricts $n^s < \infty$ and $x^s \leq x_0^s$; (iii) optimal n^s and x^s are complement.*

Proof. Consider $S\left(\underline{q}, x, n\right)$ evaluated at $\underline{q} = 0$ x_0^s, n_0^s so that the maximal implementable quality constraint $\overline{q}(x, n) \geq \underline{q}$ is trivially binding. We now check whether the buyer can increase her payoff $S(0, x, n)$ by reducing x and / or n.

For what stated in Lemma 1, reducing x by a small amount ϵ (in case $x_0^s = 1$, simply leave x unchanged) and/or n by a small amount ε, the buyer can relax the implementable quality constraint by an amount $\Delta > 0$. The net effect of these changes in $\overline{q}(x, n), x, n$ in the buyer's payoff is approximated by the following expression

$$\begin{aligned}
&\frac{1}{1-\delta}\left[V_q(0) - \psi_q(0)\right]\Delta + \\
&- \left[E\left[\theta''\left(n_0^s - \varepsilon\right)\right] - \underline{\theta}\right] + \\
&- \left\{K - \left[E[\theta] - E\left[\theta''\left(n_0^s - \varepsilon\right)\right]\right]\right\}\frac{\delta^{x_0^s - \epsilon}}{1 - \delta^{x_0^s - \epsilon}} + \\
&- \min\left\{0, K - \left[E[\theta] - \underline{\theta}\right]\right\}\frac{\delta}{1-\delta}.
\end{aligned}$$

The first term is positive, the second is negative, the third has ambiguous sign and the forth is positive. Hence, if $V_q(0) - \psi_q(0)$ is sufficiently large then the buyer prefers to restrict the number of n and x.

In order to check that a reduction of n and x is indeed optimal we need to check whether an equilibrium can emerge where, even if the buyer sets a strictly positive minimum quality requirement $\underline{q} > 0$, still all the firms offer nil quality. It is immediate to check that this set of strategies cannot be an equilibrium because the winning bid would be $b_w = \theta''(n) + A$ for a nil quality so that the buyer would deviate optimally setting $\underline{q} = 0$ obtaining the same level of quality but at a smaller price.

We are now interested in analyzing the relationship between optimal x and n when the implementable quality constraint binds. Differentiating with respect to n the first order condition for x, $\frac{\partial S(\overline{q}(x,n),x,n)}{\partial x} = 0$ we obtain $\frac{\partial x}{\partial n} = -\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} / \frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial x}$ where $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial x} \leq 0$ for the second order condition. The expression $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n}$ can be decomposed as follows

$$\begin{aligned} \frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} &= \frac{1}{1-\delta}\left[V_q(\overline{q}(x,n)) - \psi_q(\overline{q}(x,n))\right]\frac{\partial^2 \overline{q}(x,n)}{\partial x \partial n} + \\ &+ \frac{1}{1-\delta}\left[V_{qq}(\overline{q}(x,n)) - \psi_{qq}(\overline{q}(x,n))\right]\frac{\partial \overline{q}(x,n)}{\partial n}\frac{\partial \overline{q}(x,n)}{\partial x} \\ &- \frac{\delta^x Log(\delta)}{(1-\delta^x)^2}\frac{\partial E[\theta''(n)]}{\partial n} \end{aligned}$$

where $\frac{\partial^2 \overline{q}(x,n)}{\partial x \partial n} \geq 0$ as shown in the proof of Lemma 1. Then, if $V_q - \psi_q$ is large enough, $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} \geq 0$ and $\frac{\partial x}{\partial n} \geq 0$, i.e. n and x are complements, otherwise, if the second and third term in $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n}$ prevail, we have $\frac{\partial^2 S(\overline{q}(x,n),x,n)}{\partial x \partial n} \leq 0$ so that $\frac{\partial x}{\partial n} \leq 0$ and the two are substitutes. ■

This result shows that when non-contractible quality matters the buyer prefers excluding some firms and reduce competition.[20] By reducing firms' profits competition also reduces firms' incentive to maintain their "reputation" by complying with the buyer's quality requirement.

In addition, the buyer has another instrument to improve quality, namely contract length, so that when quality is a real concern, she can increase quality by reducing the length of the contract. As we have discussed above, this is not without costs for running auctions more frequently is costly, but it also has the advantage of avoiding to end up stuck with inefficient firms for long periods.

We have shown that, in the analysis of x and n when the buyer implements a nil quality, the optimal values of these two strategic variables are independent. In fact, the buyer always admits all the firms and sets a contract length uniquely on the basis of the sign of the difference $K - [E[\theta] - \underline{\theta}]$ which is independent of n. This independence property of the optimal x and n breaks down when quality is a concern and the buyer prefers to obtain a strictly positive quality. In this case the two instruments can be both complements or substitutes. However, if quality is sufficiently important for the buyer they turn out to be complements so that any event which causes an optimal reduction of firms admitted at any auction also fosters a reduction of the contract length and *viceversa*.

4 Quality and Collusion

In this section we explicitly account for the possibility that firms may collude. At $t = -1$ firms decide whether to collude in the auction supergame. If a collusive agreement is reached, the most

[20] As mentioned in the introduction, the negative effect of competition on quality and reputation is also emphasized by Stiglitz (1989). By comparing an auction with all N participating firms against bilateral negotiation Manelli and Vincent (1995) show that it is better to have auction (i.e. a "larger" N) if the procured good is a standardized one and bargaining (i.e. smaller N) when the value of q is large (they have a trade off between screening and quality implementation).

efficient firm is awarded the contract, procures the good for x periods and receives the payment by the buyer which the firm sets at the highest admissible price, i.e. the reservation price r. All the other firms abstain from bidding or submit not acceptable / winning bids so that collusion takes place with bid rotation. Deviation from a collusive agreement is punished in the harshest way. If a defection is observed, firms compete forever in all the following auctions (i.e. we employ grim trigger strategy). All the other features of the game are unchanged and described in Section 2.

Consider the optimal procurement strategies described in the previous section, including the maximum implementable quality $\overline{q}(x,n)$ and exclusion rule σ discussed therein (where, to ease notation, in the following we will suppress the apex s). We now study firms' incentives to collude under these strategies.

For the collusive agreement to be sustainable at any auction stage, the second most efficient firm with cost θ'' is the one with the highest incentive to deviate and should prefer not to undercut the most efficient one. If it does not deviate from the collusive agreement, this firm as well as any firm other than the most efficient one, can expect a collusive payoff Π^C.

Firms' costs are drawn anew at any auction stage from the interval $[\underline{\theta},\overline{\theta}]$ so that, contrary to standard models of collusion, incentives to deviate are not fixed once and for all but are stochastic. Hence, we need to consider a more sophisticated collusive agreement that also contemplates temporary phases of competition when firms' costs make too strong incentives to deviation. More precisely, colluding firms observing a cost vector realization $\boldsymbol{\theta}$ know that with a collusive pricing r the second most efficient firm is too prone to deviation and to avoid a break down of the cartel prefer to temporarily revert to competition, until in a subsequent auction stage costs allow to sustain high collusive pricing. Clearly, all this does not affect firms' ability to detect deviations when costs status would concede collusive pricing and to punish such deviation. Hence, the expected future collusive profit Π^C is

$$\begin{aligned}\Pi^C = {}& \tfrac{\delta^x}{n(1-\delta^x)} E_\theta\left[\overline{\theta} + A - \theta'(n) - \delta\tfrac{1-\delta^{x-1}}{1-\delta}\theta - \psi\left(\underline{q}\right)\tfrac{1-\delta^x}{1-\delta}\right]\Pr(b_w = r) + \\ &+ \tfrac{\delta^x}{n(1-\delta^x)} E_\theta\left[\theta''(n) + A - \theta'(n) - \delta\tfrac{1-\delta^{x-1}}{1-\delta}\theta - \psi\left(\underline{q}\right)\tfrac{1-\delta^x}{1-\delta}\right][1-\Pr(b_w = r)] = \\ & \tfrac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(b_w = r) + E\left[\Delta\theta(n)\right][1-\Pr(b_w = r)]\right\}\end{aligned}$$

where $\Pr(b_w = r)$ represents the probability that costs allow collusive pricing and $[1-\Pr(b_w = r)]$ is the probability associated to competitive pricing. Note that we are assuming that colluding firms agree to comply with the quality requirement and in the following we will study whether this is indeed the case or not.[21]

On the contrary, by deviating the second most efficient firm can ask a price slightly smaller than r thus winning the auction. This firm may decide to deviate from the collusive agreement but to comply with quality so that it obtains the following payoff,

$$\Pi_q^D = \overline{\theta} - \theta''(n) + \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$$

[21] It is well known (Rotemberg and Saloner 1986) that fixed price collusive agreements need not be optimal in a stochastic environment. As it is customary in this literature, we will not consider more sophisticated collusive schemes that smoothly vary with the realizations of costs. They are extremely difficult to derive analytically and are expected to imply similar results.

where again $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ is the expected payoff for future auctions upon deviation. Alternatively, the deviating firm can also decide to shirk on quality so that its profit turns out to be[22]

$$\Pi^D_{no-q} = \overline{\theta} - \theta''(n) + \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$$

Hence, the second most efficient firm at any auction does not deviate if the following incentive compatibility constraint is satisfied[23]

$$\Pi^C \geq \max\left\{\Pi^D_q, \Pi^D_{no-q}\right\} = \overline{\theta} - \theta''(n) + \max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi(\underline{q})\frac{1-\delta^x}{1-\delta}\right\} \tag{3}$$

The right hand side shows that the smaller is θ'' the more profitable is the deviation for this firm and less probable is that collusion can take place. For a given procurement rules (r, x, n), quality requirement $\underline{q}$ and exclusion rule σ, (3) may or may not be satisfied depending on the value of θ''. This shows that an optimal collusive agreement has to specify what firms should do when the value of θ'' is small and does not satisfy (3). To this respect, consider the boundary $\hat{\theta}$ which is implicitly defined by the following indifference condition for collusion

$$\begin{array}{l} \frac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(\theta'' \geq \hat{\theta}) + E[\Delta\theta(n)]\Pr(\theta'' < \hat{\theta})\right\} = \\ \overline{\theta} - \hat{\theta} + \max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi(\underline{q})\frac{1-\delta^x}{1-\delta}\right\} \end{array} \tag{4}$$

where the left hand side is a rewriting of Π^C with $\Pr(b_w = r) = \Pr(\theta'' \geq \hat{\theta})$ and $1 - \Pr(b_w = r) = \Pr(\theta'' < \hat{\theta})$. When firms collude, they agree to do refrain from competition at the bidding phase only when $\theta'' \geq \hat{\theta}$ and, on the contrary, when firms observe that $\theta'' < \hat{\theta}$ (recall that firms are fully informed), they know that the collusive agreement would induce the second most efficient firm to deviate and then prefer to temporarily revert to competition (till the next auction stage when $\theta'' \geq \hat{\theta}$). In these cases, the agreement requires that firms compete so that the most efficient firm wins at a price $b_w = \theta''(n) + A$, as in the previous section. However, the buyer may make this event less or more probable by varying her strategy and then the boundary $\hat{\theta}$.

Consistently with the literature on collusion in stochastic environments, in the following we will indicate that the buyer is able to deter collusion only if her strategy is such that there are no realizations of costs which satisfy incentive compatibility, i.e. uniquely when $\hat{\theta} \geq \overline{\theta}$. With this respect, from (4) we have that, if the contract lasts for infinitely many periods (e.g. if $x \to \infty$), or similarly, if all firms are admitted at the auction (i.e. if $n \to \infty$), then $\hat{\theta} > \overline{\theta}$ so that $\theta'' \geq \hat{\theta}$ is never met and collusion is deterred.

We can now state our first result concerning collusion.

[22] Alternatively to collusion with bid rotation, firms may be able to use undetectable side transfers so that the most efficient firm at any stage game always wins and share the collusive surplus with all the other firms. In case the winning firm shirks on quality it will be excluded from future auctions but can be still compensated also in the future by the firms that are allowed by the buyer to participate. The number of firms that belongs to the collusive ring increases with time but collusion with quality shirking can be still an equilibrium if firms use transfers that decline with time.

[23] It can be shown that if a deviating firm prefers also to shirk on quality, then it does so immediately.

Proposition 2 (Collusion inducing n and x.) *Assume non-contractible quality is sufficiently important. While increasing implementable quality, small x and n facilitate collusion among suppliers.*

Proof. As illustrated in the text, collusion holds if the incentive compatibility constraint (3) is verified. Furthermore, Π^C is larger the smaller is the threshold $\hat{\theta}$ because $\left[\overline{\theta} - E[\theta'(n)]\right] \geq E\left[\Delta\theta(n)\right]$. Hence, collusion is facilitated by a smaller $\hat{\theta}$ as defined in (4). We then need to show that smaller x and n imply a larger implementable quality $\overline{q}(x,n)$ but also a smaller $\hat{\theta}$.

Clearly, if the buyer wants to reduce n and x, she does so in order to increase the implementable quality by relaxing the quality constraint $\underline{q} \leq \overline{q}(x,n)$. Hence, we will consider $\hat{\theta}$ defined by (4) when $\underline{q} = \overline{q}(x,n)$.

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(\theta'' \geq \hat{\theta}) + E\left[\Delta\theta(n)\right]\Pr(\theta'' < \hat{\theta})\right\} = \overline{\theta} - \hat{\theta} + \max\left\{\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right.$$

If $\max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\} = \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ then (4) can be written as

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\overline{\theta} - E[\theta''(n)]\right\}\Pr(\theta'' \geq \hat{\theta}) = \overline{\theta} - \hat{\theta} \tag{5}$$

A small x increases the L.H.S. so that to preserve the equality, $\hat{\theta}$ must reduce. The effect of n is similar but more complex. On one side, a small n increases the probability that a firm wins the auction, thus increasing the L.H.S. in (5). On the other side, a small n means that the winning firm, on average is less efficient than it could have expected to be when n is large. This is captured by the fact that the term $E[\theta''(n)]$ in (5) is increasing in n thus implying that a small n implies a small L.H.S. However, the net effect is necessarily positive. In fact, for a given path of cost realizations for any firm i, reducing the number of competitors makes this firm the most efficient one in the same cases as with the large n plus some additional cases (in which it would not have been the most efficient firm with n large).

Note also that with a smaller $\hat{\theta}$, firms can secure the larger collusive rents with higher probability thus increasing expected profits.

If $\max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\} = \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}$ the effects described above are enhanced because we know that $\overline{q}(x,n)$ increases with smaller x and n so that, to restore the equality in (4) $\hat{\theta}$ has to further reduce. ■

Proposition 1 illustrates that prominence of quality may induce the buyer to both restrict the pool n of bidders at any auction and reduce the length of contracts x, thus having more frequent auctions. However, we know that a small number of firms that frequently compete may be induced to collude. Indeed, Proposition 2 shows that the smaller are n and x, the larger is the scope for collusion for the firms admitted in the pool and the easier is to sustain collusive agreements.

This Proposition illustrates a seeming trade-off between quality implementable through reputation and the risk of inducing collusion among suppliers. However, it is first instructive to check when the collusive ring prescribes to offer the required quality or not. For what we have stated above, we need to check colluding firms' incentive to provide quality both when the cost structure $\boldsymbol{\theta}$ allows for collusive pricing and when it does not so, i.e. respectively when $\theta'' \geq \hat{\theta}^c$ and $\theta'' < \hat{\theta}^c$.

When $\theta'' \geq \hat{\theta}^c$ so that collusive pricing takes place, one needs to compare the profit for the most efficient firm with the profit this firm can obtain when it shirks on quality. The former is clearly $\overline{\theta} - \theta'(n) + \Pi^C$, on the contrary, the latter is $\overline{\theta} + \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} - \theta'(n)$. With $\theta'' < \hat{\theta}$ competitive pricing takes place and, by providing quality, the most efficient firm obtains a profit $\theta''(n) - \theta'(n) + \Pi^C$ and $\theta''(n) - \theta'(n) + \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$ if it shirks on quality. Hence, in any event, the firm prefers to provide the required quality if

$$\Pi^C \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}. \tag{6}$$

We are now in a position to compare the maximal implementable quality when firms collude with that analyzed in Section 3 and associated with competition.

Proposition 3 (Implementable Quality with Collusion) *For given contract length x, number n of firms admitted to the auction pool and reservation price r, the maximal implementable quality when firms collude is larger than with competing firms.*

Proof. We need to compare inequality $\Pi^C \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$ with the equivalent one in the case of (always) competing firm, i.e.

$$\frac{E\left[\Delta\theta\left(n\right)\right]\delta^x}{n(1-\delta^x)} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$$

Note that Π^C is composed by two terms weighted with different probabilities. The first component is simply $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ and the second is $\left[\overline{\theta} - E[\theta'(n)]\right]\frac{\delta^x}{n(1-\delta^x)} \geq \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$. Hence, for any $\Pr(\theta'' \geq \hat{\theta})$ we immediately have $\Pi^C \geq \frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}(\geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta})$. ■

The result is an immediate consequence of the fact that when firms collude they can expect a larger profit as compared with competition so that they are more reluctant to give up those larger (future) profits for an immediate but once and for all gain by shirking on quality. Note also that in principle the implementable quality is also constrained by incentive compatibility for collusion. In fact, a larger quality requirement $\underline{q}$ increases $\hat{\theta}$ so that with a larger probability the second most efficient firm is induced to deviate from collusion. However, for any probability associated to the event $\theta'' \geq \hat{\theta}$, the expected profit from collusion Π^C is always (weakly) larger than the expected profit with competing firms $\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}$ and this gives the result.

Proposition 3 shows that the seeming trade-off between implementable quality and collusion may well be misleading. Indeed inducing collusion may further increase the implementable quality so that a trade-off may now arise between competition and collusion, as we discuss in the next Section.

5 Optimal Procurement

Proposition 1 shows that, if the buyer is worried about quality, she may want to restrict participation n and contract length x. However, by Proposition 2, both these two decisions tend to induce collusion. In addition, as shown in Proposition 3, the buyer may not be necessarily impaired by collusion because this may allow implementing higher quality. At first grasp this second may sound difficult because, in general, collusion makes the buyer paying a larger price for the good. However, this is not the unique effect of collusion when quality is not contractible. Indeed, the buyer may optimally want to further restrict n and/or x, exactly because this induces collusion and increases the implementable quality. Hence, we now explore how the emerging trade-off between collusion and competition is solved by the buyer.

If firms have the ability to collude, the buyer has to keep this possibility into account and adjust her strategies accordingly. Namely, she may chose a procurement contract (r^s, x^s, n^s), a quality requirement $\underline{q}^s$ and an exclusion rule σ^s, so as to sistematically prevent collusion. In this case, in addition to what we have studied in the previous Section, the buyer must be sure that her strategies do not induce collusion, i.e. that with this procurement contract, $\underline{q}^s$ and σ^s, constraint (3) is not verified.

Alternatively, the buyer may want to set procurement rules which do induce collusion. When the buyer anticipates firms' collusion, she sets a contract (r^c, x^c, n^c), quality requirement $\underline{q}^c$ and exclusion rule σ^c so that $\overline{\theta} < \hat{\theta}^c$ where the boundary $\hat{\theta}^c$ is defined by

$$\begin{array}{l} \frac{\delta^{x^c}}{n^c(1-\delta^{x^c})} \left\{ \left[\overline{\theta} - E[\theta'(n^c)] \right] \Pr(\theta'' \geq \hat{\theta}^c) + E\left[\Delta\theta\,(n^c) \right] \Pr(\theta'' < \hat{\theta}^c) \right\} = \\ \overline{\theta} - \hat{\theta}^c + \max\left\{ \frac{E[\Delta\theta(n^c)]\delta^{x^c}}{n^c(1-\delta^{x^c})}, \psi\left(\underline{q}^c\right) \frac{1-\delta^{x^c}}{1-\delta} \right\} \end{array} \tag{7}$$

The left hand side of (7) shows that when a firm deviates, this overtakes both the other firms and the buyer.[24] However, from the auction just after the deviation, the firms recognize that a deviation occurred and adapt their strategies accordingly reverting to competition.

Even if collusion may allow the buyer to increase the implementable quality, the analysis of the optimal procurement contract in this case is even subtler than this. In fact, if the buyer's unique concern were the highest non-contractible quality, then Lemma 1 tells us that she should set $n = 1$ (possibly also with a short contract, i.e. x small). Clearly, with a single firm, collusion would not be an issue and the implementable quality would be larger than with collusion associated with any number of firms $n^c > 1$. However, this strategy of restricting n to a single firm implies a large cost in terms of *inefficiency in production.* Indeed, as we have discussed in the analysis of surplus (2), by restricting the number of admitted firms n, the expected efficiency parameter of the winning firm reduces. With this respect, the advantage of collusion then is in allowing a large implementable quality associated with a (moderately) large n. In other terms, for any level of $n > 1$, colluding

[24] The buyer sticks to her collusion-inducing strategy even if she observes a defection. In Section 6 we analyze this behavior and show that, whenever the buyer has the commitment power to leave the contractual terms and the quality requirement unchanged upon cartel defection, then she has interests to do so and then she prefers to restrict to stationary strategies.

firms are ready to provide larger quality than when they compete and, at the same time, they produce more efficiently than a single firm would do.

There are several reasons that may induce the buyer to be concerned also with efficiency in production. For the sake of simplicity, here we assume that the buyer can ask for a participation fee to the sellers who are admitted in the restricted pool of n firms (as in the case of "selective tendering"). We denote with τ^s the price the buyer asks for participation when the procurement contract induces competition and with τ^c when it induces collusion. Firms have a zero outside option if they are not allowed to participate so that we define the set of rational prices as $\tau^i = \gamma\Pi^i$, $i = s, c$, where Π^i represents the profit in the two regimes and the parameter γ captures the fraction of firms' surplus that can be appropriated by the buyer.[25]

Proposition 4 (Optimal procurement) *(i) Assume the buyer does not care for efficiency in production (i.e. $\gamma = 0$). Then, the buyer prefers negotiating with a single firm when quality is a concern. Otherwise, she prefers running auctions with many competing firms.*

(ii) If efficiency is important for the buyer (i.e. γ is large), then the buyer prefers negotiating with a single firm when quality is a major concern; she prefers running collusion-inducing auctions for intermediate relevance of quality and auctions with many competing firms when quality is not particularly important.

Proof. The first statement of the proposition is immediate. Consider the optimal collusion-inducing contract with n^c, x^c and associated implementable quality $\overline{q}^c$. Then, by setting $n = 1$ and keeping the contract length at x^c, the maximal implementable quality becomes larger than $\overline{q}^c$ because the single firm can expect larger rents than the n^c colluding firms. Not caring for efficiency, the buyer is better off by restricting n to 1 for she obtains a larger quality at the same price which, in both cases, is equal to the reservation price $r = \overline{\theta} + A\left(\underline{q}\right)$ with $A\left(\underline{q}\right) = \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] + \frac{1-\delta^x}{1-\delta}\psi\left(\underline{q}\right)$. Recall that this is indeed the unique reservation price that guarantees provision of the good with any cost realization.

Assume now that $\gamma > 0$. We compare the buyer's payoff with and without collusion and when the buyer admits a single firm (i.e. $n = 1$). To simplify the analysis assume that

$$V(q) - \psi(q) = \lambda \geq 0$$

for any q, so that in any case the buyer wants to implement the maximal quality. The proof also holds for a strictly concave $V(q) - \psi(q)$, as long as this function reaches the maximum for a sufficiently large q.

The buyer avoids collusion if $\hat{\theta} < \overline{\theta}$ with $\hat{\theta}$ defined by (4) (which here does not depend on $\overline{q}$) and the maximal implementable quality $\overline{q}$ is defined by

$$\frac{E\left[\Delta\theta\left(n\right)\right]\delta^x}{n(1-\delta^x)} = \psi\left(\overline{q}\right)\frac{1-\delta^x}{1-\delta}$$

[25] An alternative is to relax Assumption 3. By so doing, the buyer is no more constrained to set the reservation price r at the highest value in order to always guarantee procurement. In this case, setting a smaller reservation price may serve the buyer to appropriate part of the sellers' surplus which increases with their efficiency.

In this case, welfare is

$$\begin{aligned} S\left(\overline{q}, x, n\right) &= \frac{1}{1-\delta}\left[\lambda \overline{q} - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}} E[\theta] - \frac{1-\delta}{1-\delta^{x}} K\right] + \\ & -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right) E[\theta''\left(n\right)] + \gamma E[\theta'\left(n\right)]\right\} \end{aligned}$$

With a single firm $n = 1$ the maximal implementable quality $\overline{q}(n = 1)$ is defined by

$$\frac{\delta^{x}}{1-\delta^{x}}\left\{\overline{\theta} - E[\theta]\right\} = \psi\left(\overline{q}\right)\frac{1-\delta^{x}}{1-\delta}$$

(recall that $E[\theta'\left(n = 1\right)] = E[\theta]$) and welfare is

$$\begin{aligned} S\left(\overline{q}(n=1), x, 1\right) &= \frac{1}{1-\delta}\left[\lambda \overline{q}(n=1) - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}} E[\theta] - \frac{1-\delta}{1-\delta^{x}} K\right] + \\ & -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right)\overline{\theta} + \gamma E[\theta]\right\} \end{aligned}$$

Finally, collusion holds if $\hat{\theta}^{c} \geq \overline{\theta}$ (recall that for what stated above, here $\hat{\theta}^{c}$ does not depend on the implementable quality without collusion) and the associated maximal implementable quality $\overline{q}^{c}$ is defined by setting the R.H.S. equal to the L.H.S. in (6), i.e.

$$\frac{\delta^{x}}{n(1-\delta^{x})}\left\{\overline{\theta}\Pr(\theta'' \geq \hat{\theta}^{c}) + E[\theta''\left(n\right)]\Pr(\theta'' < \hat{\theta}^{c}) - E[\theta'\left(n\right)]\right\} = \psi\left(\overline{q}^{c}\right)\frac{1-\delta^{x}}{1-\delta}$$

In this case welfare can be written as

$$\begin{aligned} S\left(\overline{q}^{c}, x, n\right) &= \frac{1}{1-\delta}\left[\lambda \overline{q}^{c} - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}} E[\theta] - \frac{1-\delta}{1-\delta^{x}} K\right] + \\ & -\frac{1}{1-\delta^{x}}\left\{\left(1-\gamma\right)\left[\Pr(\theta'' \geq \hat{\theta}^{c})\overline{\theta} + \Pr(\theta'' < \hat{\theta}^{c})E[\theta''\left(n\right)]\right] + \gamma E[\theta'\left(n\right)]\right\} \end{aligned}$$

We can now compare the buyer's expected surplus with and without the collusive agreement. To induce collusion we know that the buyer must set a number of bidders and a contract length such that $n^{c} \leq n^{s}$, $x^{c} \leq x^{s}$. The difference in the expected surplus can then be written as

$$\begin{aligned} & S\left(\overline{q}^{c}, x^{c}, n^{c}\right) - S\left(\overline{q}^{s}, x^{s}, n^{s}\right) = \tfrac{1}{1-\delta}\lambda\left(\overline{q}^{c} - \overline{q}^{s}\right) + \\ & -\tfrac{\delta E[\theta]}{1-\delta}\left[\tfrac{1-\delta^{x^{c}-1}}{1-\delta^{x^{c}}} - \tfrac{1-\delta^{x^{s}-1}}{1-\delta^{x^{s}}}\right] + \\ & -K\left[\tfrac{1}{1-\delta^{x^{c}}} - \tfrac{1}{1-\delta^{x^{s}}}\right] + \\ & -\tfrac{1}{1-\delta^{x^{c}}}\left\{\left(1-\gamma\right)\left[\Pr(\theta'' \geq \hat{\theta}^{c})\overline{\theta} + \Pr(\theta'' < \hat{\theta}^{c})E[\theta''\left(n^{c}\right)]\right] + \gamma E[\theta'\left(n^{c}\right)]\right\} + \\ & +\tfrac{1}{1-\delta^{x^{s}}}\left\{\left(1-\gamma\right)E[\theta''\left(n^{s}\right)] + \gamma E[\theta'\left(n^{s}\right)]\right\} \end{aligned}$$

The positive effects of collusion are indicated by the first and the second term. The first clearly relates to the positive effect that collusion has on quality because $\overline{q}^c \geq \overline{q}^s$. As for the second term, recall that for any period after the first in any contract, firms do not earn any rent because they do not have private information with respect to the buyer for those periods who has to reimburse the expected cost $E[\theta]$ for any period. Hence, the longer is the contract, the larger is this reimbursement, as we have in the comparison between collusion and competition, $x^c \leq x^s$. On the contrary, the negative effects of collusion are represented by all the other terms. In particular, collusion requires more frequent auctions and this is costly (the third term). In addition, collusion is costly because it implies a larger price (this is effect is captured by setting $\gamma = 0$ in the last two terms) and / or it implies that producing firms are less efficient (this is described by setting $\gamma = 1$). In fact, for any $n^c \leq n$ we have $E[\theta''(n)] \leq E[\theta''(n^c)] \leq \overline{\theta}$ and $E[\theta'(n)] \leq E[\theta'(n^c)]$. Note also that the these costs of collusion are strengthened by the fact that they arise (more) frequently because $x^c \leq x^s$ implies $\frac{1}{1-\delta^{x^c}} \geq \frac{1}{1-\delta^{x^s}}$. In any case, if λ is sufficiently large, i.e. quality is important, then the first term dominates.

Finally, we compare collusion against negotiation with a single firm. In this latter case, the buyer sets $n = 1$ and let the implementable quality be $\overline{q}^1$ and the contract length x^1, Then, considering for simplicity $x^c = x^1$ we then have

$$S\left(\overline{q}^c, x^c, n^c\right) - S\left(\overline{q}^1, x^c, 1\right) = \frac{1}{1-\delta}\lambda\left(\overline{q}^c - \overline{q}^1\right) +$$
$$-\frac{1}{1-\delta^{x^c}}\left\{(1-\gamma)\left[\Pr(\theta'' \geq \hat{\theta}^c)\overline{\theta} + \Pr(\theta'' < \hat{\theta}^c)E[\theta''(n^c)]\right] + \gamma E[\theta'(n^c)] - (1-\gamma)\overline{\theta} - \gamma E[\theta]\right\}$$

The first term is negative because $\overline{q}(n=1) \geq \overline{q}^c$. On the contrary, the second is positive. In fact, for any n^c we have $E[\theta''(n^c)] \leq \overline{\theta}$ and $E[\theta'(n^c)] \leq E[\theta]$. Indeed, reducing to $n = 1$ the number of firms implies both that the buyer will have to pay a larger price (on average) and that the producing firm will be less efficient, as compared with collusion and $n^c > 1$. It follows that if λ is sufficiently large, then it is better to set $n = 1$ than having more and colluding firms and vice versa. ■

As we have discussed above, collusion leaves large rents to the firms and rents are necessary to induce them to provide high level of quality. However, if the buyer does not care for efficiency in production, then all what she can obtain in terms of quality with collusion can be replicated by admitting a single firm to the auction. Indeed, this guarantees the maximal rent and then the highest implementable quality. However, a single firm comes at the cost of inefficient production because the expected cost of the firm is higher than the expected cost of the most efficient firm when n firms are admitted at the auction stage, i.e. $E[\theta] \geq E[\theta'(n)]$. If the buyer does care for efficiency, then a trade off arise. In this case, restricting n and x so that collusion emerges may become optimal because it allows for balance between higher implementable quality as compared with competition and higher efficiency in production as compared with $n = 1$.

Finally we also note that if firms are homogenous as for costs so that $E\left[\Delta\theta(n)\right]$ is small, then the implementable quality with competition is also small (see condition (1)) because informational rents (profits) are necessary to provide firms' incentives in quality procurement. Hence, we have the following:

Corollary 1 *If firms' cost heterogeneity is small, then auctioning with competing firms is dominated either by bilateral negotiation or collusion-inducing procurement.*

If firms are very homogeneous in terms of costs so that $E\left[\theta''(n)\right] \simeq E\left[\theta'(n)\right]$, competing firms earn very low rents and the implementable quality is then very low. This is also the case where restricting competition to $n = 1$ may turn to be optimal because the loss in terms of efficiency in production is small. On the other hand, if firms are very heterogeneous, then the buyer can implement a high quality also with many competing firms. Hence, collusion inducing procurement shows its maximal strength for intermediate values of heterogeneity in costs, exactly because it mediates quality with efficiency.

We think the analysis in this section contributes to the literature on the mode of transaction for procurement. On the two extremes, one could seek several suppliers thus relying on the benefits of competition, or, otherwise, one could bargain with a single seller to avoid the drawbacks of competition when quality is not contractible. If auction mechanisms are the natural implementation of competitive tendering, there are several protocols the buyer may rely on to bargain with a single seller. Manelli and Vincent (1995) have analyzed sequential bargaining with take-it-or-leave-it offers designed by the buyer. In a different context, Bulow and Klemperer (1996) reach the conclusion that a seller should prefer using auctions to the best possible negotiation with one less buyer. Bajari and Tadelis (2001) bundle the choice between auctions and negotiation with the choice of the contractual form (with the two extremes of fixed-price and cost-plus contracts). They argue that fixed-price contracts typically lend themselves to competitive bidding in auctions whilst cost-plus contracts are normally negotiated.

It is important to notice that the few papers that have dealt with the choice of the mode of transaction have limited the analysis mainly to a framework with no repetition. However, as we have previously emphasized, a main ingredient of procurement is the need to repeat the procuring process over time and, with non-contractible quality, the level of competition (i.e. the number n of firms admitted at the auction) is only one relevant dimension in the procurement process. Indeed, the duration of the relationship is important. A long term relationship creates an implicit incentive so that procuring firms have incentives to establish reputation and the buyers may prefer long lasting contracts when quality is not contractible. This creates the bridge between our analysis and the important strand of literature dealing with trust and reputation formation in long-term relationships (Fehr, Brown and Falk 2004). Hence, our analysis introduces the novelty of combining these two elements in the choice of a trading procedure, the degree of competition and the length of the awarded contract.

6 Extensions

In this section we consider a number of extension of our base procurement model and check the robustness of our results.

Finite number N of firms. The analysis in the previous section has relied on Assumption 2 stating that the number of potentially active firms N is infinite. Clearly, when the buyer prefers to negotiate with a single firm, then it is immaterial whether N is finite or not.

When the buyer induces competition between $n > 1$ firms, assume first that the firms admitted at the auction are $n < N$ and consider a candidate strategy for firms such that at any auction stage the most efficient firm provides the minimum required quality and the buyer's exclusion rule $\widetilde{\sigma}$ contemplates the replacement of cheating firms with one among the $N - n$ firms that were previously not allowed to participate, otherwise all firms are kept into the pool of qualified n firms. We now check whether a single firm may have any incentive to deviate and cheat on quality. If he does so, according to $\widetilde{\sigma}$ the buyer will replace the cheating firm with one among those $N - n$ firms and, given that all other firms will provide the required quality, the pool of active firms n will remain the same for all the subsequent auction stages. Hence, the deviating firm that cheats on quality will be excluded forever. This implies that as long as $n < N$ and with the exclusion rule $\widetilde{\sigma}$, the maximal implementable quality for the buyer is again $\overline{q}(x, n)$ as defined in Lemma 1. Importantly, note that $n < N$ makes the threat of exclusion of cheating firms by the buyer a credible one so that the buyer has no incentive to deviate from the exclusion rule $\widetilde{\sigma}$. In fact, at any subsequent auction all firms are identical from the buyer' view point so that replacing one firm with another in the pool of $N - n$ firms is costless. Hence, the main difference with the case $N = \infty$ is that, here, a necessary condition for $\overline{q}(x, n) > 0$ is $n < N$. In fact if $n = N$ with N finite, there is no exclusion rule that can guarantee a strictly positive maximal implementable quality.[26,27] It follows that exclusion here last for a minimum of $N - n$ periods.

It may be also possible that the coalition of all firms shirks on the quality requirement (both with and without collusion) providing nil quality. In this case, the minimal exclusion of $N - n$ auction rounds becomes also the maximal exclusion length.[28] Interestingly, this shows that a finite N introduces another motive for reducing the pool of admitted firms. In fact, a smaller n now makes the punishment for quality shirking tougher so that the buyer may be induced to further restrict n.

Non-stationary strategies. In the previous sections we have assumed stationary strategies.

[26] Consider the exclusion rule σ' such that a firm is excluded forever in case he cheats on quality and retained otherwise (similarly if exclusion takes place for a shorter and finite number of periods). By excluding a cheating firm the buyer limits the number of firms admitted to the next auction stage which becomes $N - 1$. However, this exclusion at the following auctions is not credible because the buyer gains by increasing competition with a larger number of bidders and would like to admit the cheating firm at next auctions. Any firm anticipates that there will be no punishment for quality cheating and the quality provided in equilibrium will be nil.

[27] With N finite, an equilibrium always exists where all firms provide null quality, the buyer admits all the firms (i.e. $n = N$) and the exclusion rule σ keeps all firms independently of the procured quality or, equivalently, the minimum quality requirement is kept at $\underline{q} = 0$. Indeed, admitting all firms at the auction is optimal for the buyer in this case, for expected cost and price are obviously decreasing in the number of potential suppliers. This type of equilibrium is clearly of limited interest.

[28] A coalition-proof exclusion rule is easily obtained by complementing the initial rule with one that says that if more firms deviate, firms excluded before are reintegrated in the auction process only after all never-excluded firms have been chosen, and in order of exclusion. The punishment for multiple deviations would then be exclusion for at least $N - n$ periods. See the discussion in Section 6.

In particular, when collusion is desirable, the buyer does not change her strategy when she realizes that collusion has broken down due to a deviation (see the right hand side of equation (7)).

Assume now that, on the contrary, the buyer may react when she observes a deviation. In particular, by observing a low winning bid, the buyer learns a deviation occurred and then reverts to the optimal contract for competition. This is the case when b_w is such that $b_w \geq \hat{\theta}^c + \psi\left(\underline{q}^c\right)\frac{1-\delta^{x^c}}{1-\delta}$ where the right hand side is the maximum price that would emerge under the collusion agreement when firms temporarily abstain from colluding.

Now, recall that the buyer may want to induce collusion only if this allows to increase the implementable quality. It the follows that the required quality with collusion $\underline{q}^c$ cannot be afforded by firms when they are induced to competition by a defection. Hence, when collusion breaks down, if the buyer sticks to her collusion-inducing strategies, firms will be induced to shirk on quality so that they will be excluded from future auctions. This, in turn, implies that firms' payoff following a deviation are very low and collusion is strengthened. In other words, when the buyer can commit and collusion is desirable, it is in the buyer's interest not to revert to competition in future auctions if a deviation from collusion occurs. This provides support to our choice of considering stationary strategies in the previous Sections.

If, on the other hand, the buyer cannot commit to her strategies, then her sequentially optimal strategy upon (detectable) cartel defection is the optimal strategy with competition. In this case, the payoff of a firm deviating from the cartel would be larger and collusion more difficult to sustain. It is however clear that collusion can still allow a larger implementable quality than competition, albeit smaller than when the buyer can commit to her strategies. Hence, our main results qualitatively hold.

Discontinuing procurement. Assumption 3 requires that the buyer procures the good at any point in time. This clearly puts her in a very weak position with colluding firms. In fact, to guarantee procurement, she has to set a reservation price r which is sufficient to pay back the cost of the most inefficient firm for any level of required quality. Clearly, colluding firms can extract all the surplus by asking a price equal to r. It is then immediate that abandoning this assumption, r can be optimally set at a lower value even if this may discontinue procurement for certain realization of costs. This is decision has a strong effect in limiting colluding firms' rents so that our previous results on the desirability of collusions are strengthened.

Scoring rules. In principle, firms' offers could be formed by a price bid and a quality level that the firm is ready to supply. The buyer could then rank offers according to a scoring rule function of price and (contractible together with) non-contractible quality offered. The buyer may then exclude the winning firm in case the latter does not provide the promised non-contractible quality. However, this form of competition with bid-quality offers and scoring rule may be preferable to the one we study only if there is heterogeneity in firms' cost for quality, i.e. in case the per-period cost of firm i is $\psi(q,\theta)$ with $\frac{\partial^2\psi}{\partial q \partial \theta} \neq 0$. [@To be completed]

Asymmetrically informed firms. [@ To be done: the intricacies of collusion in auction with asymmetrically informed bidders. See Compte (1994, 1998), and Kandori and Matsushima (1998): public communication in repeated games with private monitoring.]

7 Concluding Remarks

In this paper we have analyzed the relationships between reputation, non-contratible quality and collusion in a repeated procurement context. Repetition in the procurement relationship allow the emergence of reputation as an incentive device inducing firms to supply non-negligeable quality. Restricting participation and contractual length, the buyer increases firms' incentives to provide quality and hence, maximal implementable quality. On the other hand, running more frequent auctions among few bidders facilitates collusive agreements among suppliers. We have analyzed this trade-off showing that when non-contractible quality and variability in suppliers' efficiency are both important, short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off. If quality is a major concern, the buyer can do even better by negotiating with a single firm, even if this may clash with efficiency in production. Hence, we show that the optimal procurement strategy involves a subtle balance between firms' rents, incentives for quality, collusion and efficiency.

As discussed in the Introduction, a possible interpretation of the buyer inducing collusion can be that she allows the formation of consortia. With respect to collusive agreements, a consortium to be viable may not need to satisfy incentive compatibility and it is interesting to notice that, in any case, the buyer cannot rely on consortia involving all the potential firms because, otherwise, there would be no room for punishment. This analysis is left for future research.

[@to be completed]

References

[1] Abreu, D. 1988, "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica*, 56, 383-396

[2] Allen, F., 1984, "Reputation and Product Quality," *The RAND Journal of Economics*, 15, 311-27.

[3] Aoyagi, M., 2003, "Bid Rotation and Collusion in Repeated Auctions," *Journal of Economic Theory*, 112, 79-105.

[4] Baker, G., R. Gibbons and K. J. Murphy, 1994, "Subjective Performance Measures in Optimal Incentive Contracts," *Quarterly Journal of Economics*, 109, pp. 1125-56.

[5] Baker, G., R. Gibbons and K. J. Murphy, 2001, "Relational Contracts and the Theory of the Firm," *Quarterly Journal of Economics* , forthcoming 2001.

[6] Bar-Isaac, H., 2004, "A review of an idiosyncratic selection of literature loosely related to interactions between individual and collective reputations", mimeo new York University.

[7] Bernheim, D. and M. Whinston, 1998, "Incomplete Contracts and Strategic Ambiguity." *American Economic Review* 88: 902-32.

[8] Blonski, M. and Spagnolo, G., 2003, "Relational Contracts and Property Rights", CEPR Discussion Paper No. 3460.

[9] Blume, A. and P. Heidhues, 2004, "Modeling Tacit Collusion in Auctions," mimeo U. of Pittsburgh.

[10] Compte, O., 1998, "Communication in Repeated Games with Imperfect Private Monitoring," *Econometrica*, 66, 597-626.

[11] Bulow, J. and Klemperer P. 1996. "Auctions versus Negotiations," *American Economic Review*, vol. 86(1), pages 180-94.

[12] EC, 2004, "On the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts", Directive 2004/18/EC of the European Parliament and of the Council, March 31st, 2004.

[13] Fehr, E, M. Brown and A. Falk, 2004, "Relational Contracts and the Nature of Market Interactions," *Econometrica* Vol. 72, No. 3: 747-780.

[14] Hart, O. "Firms, Contracts and Financial Structure", Clarendon Press, Oxford, 1995

[15] Hörner, J., 2002,. "Reputation and Competition." *American Economic Review*, June, 92 (3), pp. 644- 663.

[16] Kandori, M. and H. Matsushima,1998, "Private observation, communication and collusion," *Econometrica*, 66, 627-652.

[17] Klein, B. and Leffler, K., 1981, "The Role of Market Forces in Assuring Contractual Performance", *Journal of Political Economy*, 89, 615-41.

[18] Kranton, K., 2003, "Competition and the Incentive to Produce High Quality", *Economica*, 70: 385–404.

[19] Laffont, J. and J. Tirole, 1993, *A Theory of Incentives in Regulation and Procurement*, Cambridge, MA: M.I.T. Press.

[20] Levin, J. 2003, "Relational Incentive Contracts," *American Economic Review*, 93(3): 835-847.

[21] McLeod, W. B., et J. Malcomson (1989). "Implicit Contracts, Incentive Compatibility, and the Involuntary Unemployment", *Econometrica*, vol. 57, no 2, p. 447-480.

[22] Manelli A. and D. R. Vincent, 1995, "Optimal Procurement Mechanisms." *Econometrica*, 63, 3: 591-620.

[23] Rotemberg, J. and G. Saloner (1986), "A Super-game Theoretic Model of Business Cycles and Price Wars during Booms," American Economic Review, 76(3), 390-407

[24] Shapiro, C., 1983, "Premiums for High Quality Products as Returns on Reputation", *Quarterly Journal of Economics*, 98, 659-80.

[25] Stiglitz, J. E., 1989, "Imperfect Information in the Product Market," in Richard Schmalensee and Robert Willig, eds., *Handbook of industrial organization*, volume 1. Amsterdam: Elsevier Science Publishers, : 771–847.

[26] Tirole, J., 1999. "Incomplete Contracts: Where Do We Stand?," *Econometrica*, vol. 67(4), pages 741-782, July.