Consumption, Wealth, the Elasticity of Intertemporal Substitution and Long-Run Stock Market Returns.*

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Abstract

The elasticity of intertemporal substitution is a parameter of crucial importance for macroeconomic policy. The available macroeconomic evidence on the value of this parameter for the US is conflicting. Calibrated dynamic models require a value close to one of the EIS to match the data, while estimated Euler equations concentrating on high frequency fluctuations in consumption deliver much lower values not significantly different from zero. Some recent empirical evidence indicates that the well-known asset pricing puzzles might be solved by considering consumption in the long-run. We extend these results to obtain an empirical estimate of the EIS consistent with that used in calibrated models.

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1 Introduction

The elasticity of intertemporal substitution (EIS) in consumption pins down the response of consumption growth to fluctuations in the real interest rates. It is a parameter of central importance in determining the link between macroeconomics and finance. The EIS determines (i) the comovement between consumption and real interest rates over the business cycle and hence the power of monetary policy in smoothing fluctuations in aggregate demand in basic Dynamic Structural General Equilibrium models (see, for example, Woodford, 2003, chapter 4); (ii) the importance of the macroeconomic effects of capital income taxation (King and Rebelo, 1990) (iii) the importance of the burden of government debt or unfunded social security (Hall, 1988).

As recently pointed out by Guvenen(2003), calibrated models and estimated Euler equation deliver opposite views on this parameter. On the one hand, the consistency of calibrated dynamic macroeconomic models with aggregate data requires a large value of the EIS and, after the seminal paper of Kydland and Prescott(1982) where the EIS was calibrated to 0.66, most of the real business cycle literature uses a value around unity. On the other hand, direct estimates of the EIS from the first order conditions for the solution of the consumer's intertemporal optimization problem deliver much lower values: Hall (1988) argues that the EIS is very close to zero and subsequent literature has provided further support to this evidence (Campbell and Mankiw(1989), Yogo(2004)). The objective of this paper is to evaluate how this conflicting evidence could be reconciled when the EIS is estimated by concentrating on long-run consumption growth.

It is very well known that using high-frequency fluctuations in consumption to estimate structural parameters of interest is problematic; some of the best known puzzles in the macro-finance literature depend on the fact that the single period Euler equation for US consumption generates implausible and imprecise estimates of the taste parameters. A CRRA utility function requires a risk aversion coefficient of about 40 to match asset price fluctuations and short-term fluctuations in consumption. Even if that implausibly high coefficient is accepted, the unconditional first two moments of the distribution of one-period consumption growth observed in the data require a negative discount rate to generate plausible values for the risk-free rate.[1]

[1] See, for example, Campbell, Lo and McKinlay(1997) chapter 7, for an excellent discussion of the equity premium puzzle and of the risk-free rate puzzle.

The recent literature has produced some hope for matching consumption growth and asset pricing fluctuations, without changing the basic assumption of the standard models[2], by concentrating on long-run consumption growth.

This literature has analyzed multiperiod Euler equations and cointegrating relations for consumption.

In the first strand, Parker and Julliard(2005) use the multiperiod moment condition, which they consider a moment condition robust to measurement error in consumption and simple "mistakes" by consumers, to find that this model accounts for the value premium, i.e. the difference in average returns of value vs. growth stocks. Bansal-Yaron(2005) also argue that average returns of value vs. growth stocks can be understood by their different covariance with long-run consumption growth. In fact, they examine long-run covariances of earnings with consumption, rather than the covariance of returns with consumption. Hansen, Heaton and Li(2005) show that the recursive Epstein-Zin-Weil utility variety produces a model in which asset returns at date $t+1$ are priced by their exposure to the long-run consumption risk. Importantly, clear microeconomic foundations are provided to the empirical evidence in Bansal-Yaron(2005). However, the title of the Hansen et al. paper ends with a question mark, which is justified by the empirical evidence that the results on the differences between value and growth stocks depend crucially on whether one includes a time-trend in the regression of earnings on consumption.

The second strand of the literature examines long-run consumption and asset prices from the perspective of a cointegrating relation.

Lettau and Ludvigson(1991), LL from now on, observe that the linearized intertemporal consumer's budget constraint implies that excess consumption with respect to wealth, its long-run equilibrium value, should be positively related to future returns from the market portfolio and negatively to future expected consumption growth. LL observe that aggregate wealth—specifically the human capital component of it—is unobservable. They then argue that the important predictive components of the consumption–aggregate wealth ratio for future market returns may be expressed in terms of observable variables, namely in terms of consumption, asset holdings, and current labor income. Their model implies that the log of consumption, labor income,

[2] Ferson and Constantinides(1991), for example, obtain estimate of the EIS close to one, althoug rather imprecisely estimated, by concentranting on conditional moments and by introducing habit persistence in consumption.

and asset holdings share a common stochastic trend. They are cointegrated. The parameters of this shared trend are the average shares of human capital and asset wealth in aggregate wealth. Under the maintained assumption that expected consumption growth is not too volatile, stationary deviations from the shared trend among these three variables produce movements in the consumption–aggregate wealth ratio and predict future asset returns. LL estimate a relation between *cay* (the excess consumption with respect from its long-run target) and future stock market returns to find that *cay* is a good predictor of future stock market returns.

It seems rather natural at thi stage to bring together the first order conditions for the solution of the consumer problem with the linearized intertemporal budget constraint and extend the framework offered in LL to estimate structural parameters. This is the objective of this paper. In fact, under Epstein-Zin-Weil preferences, the solved-out consumption function can be written as a relation between the log consumption-wealth ratio and expected returns on wealth, that depends only on one parameter: the EIS. Therefore, long-run fluctuations in consumption and in expected returns to wealth can be used to estimate this parameter and assess if the success of the recent literature in matching long-run consumption growth and asset pricing fluctuations can be extended to the reconciliation of the conflicting macroeconomic evidence on the EIS in consumption.

The rest of the paper is organized as follows. The next section derives an explicit long-run consumption function by using the first order conditions for the consumer optimization problem in the linearized budget constraint. The following section deals with measurement issues and specification strategy to derive an empirical specification to identify the relevant parameter of interests. We then consider the empirical performance of the model and the robustness of the results to some modification of the baseline model related to the measurement of consumption and human capital. The last section concludes.

2 Theory: the derivation of a long-run consumption function.

Consider a representative agent economy in which all wealth, including human capital, is tradable. Let W_t be aggregate wealth, i.e. human capital plus asset holdings in period t, C_t is consumption and $R_{m,t+1}$ is the net return on aggregate wealth, i.e. the market portfolio. The accumulation equation for aggregate wealth may be written as:

$$W_{t+1} = (1 + R_{m,t+1})(W_t - C_t) \tag{1}$$

Define $r_{m,t+1} = \log(1 + R_{m,t+1})$, and use lowercase letters to denote log variables throughout. As LL we follow Campbell and Mankiw (1989) and assume that the consumption–aggregate wealth ratio is stationary. In this case the budget constraint may be approximated by taking a first-order Taylor expansion of equation (1), to obtain

$$\begin{aligned} \Delta w_{t+1} &= r_{m,t+1} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \qquad (2) \\ \rho &= 1 - \exp\left(\overline{c - w}\right) \end{aligned}$$

where k, is a constant of normalization, not relevant for the problem at hand.

By solving (2)forward, we have :

$$c_t - w_t = E_t \left[\sum_{j=1}^{\infty} \rho^j (r_{m,t+j} - \Delta c_{t+j}) \right] + \frac{\rho k}{1 - \rho} \tag{3}$$

LL point out that (3) shows that the consumption–wealth ratio is a function of expected future returns to the market portfolio in a broad range of optimal consumption models, so they concentrate in finding a proxy for $c_t - w_t$ and in assessing its performance for forecasting market returns. However (3) is almost an identity, so the predictive evidence on LL is only partially informative on consumer's behavior: it tells us that expected consumption growth does not fluctuate too much and that the proxy derived by LL, using cointegration analysis, for $c_t - w_t$ is not a bad one. Moreover, given that the predictive regressions in LL relate ex-post realized stock market returns at long-horizons[3] and excess consumption, their results tell us also that ex-post realized long-run returns are somewhat correlated with ex-ante expected long-run returns.

[3]Note that long-horizons returns are computed in LL just by cumulating period returns, in other words by assuming that $\rho = 1$. Such assumption, as we will show in the next section, is counter-factual.

In fact, it is possible to derive a relation between the consumption-wealth ratio and expected future returns that allows the identification and estimation of the EIS in consumption.

To do so, we follow the recent literature concentrating on multiperiod Euler equation for consumption and consider the Epstein-Zin-Weil objective function[4], defined recursively by:

$$\begin{aligned} U_t &= \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\delta}} + \delta \left(E_t \left(U_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ \theta &= \frac{1-\gamma}{1-\frac{1}{\psi}} \end{aligned}$$

ψ is the elasticity of intertemporal substitution, when $\theta = 1$ we have the familiar recursion and the model reduces to the time separable power utility one. Note that only in this special case ψ is restricted to be equal to the reciprocal of the coefficient of relative risk aversion γ.

The utility function and the budget constraint imply an Euler equation of the form:

$$1 = E_t \left[\left\{ \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^{\theta} \left\{ \frac{1}{(1+R_{m,t+1})} \right\}^{1-\theta} ((1+R_{i,t+1})) \right] \tag{4}$$

Where $R_{i,t+1}$ is the return of the generic asset i. If asset returns and consumption are homoscedastic and jointly lognormal, then we can derive expression for the riskless real rate $r_{f,t+1}$ and for the return of any generic asset $r_{i,t+1}$:

$$\begin{aligned} r_{f,t+1} &= -\log \delta + \frac{\theta - 1}{2} \sigma_m^2 - \frac{\theta}{2\psi^2} \sigma_c^2 + \frac{1}{\psi} E_t \left(\Delta c_{+1} \right) && (5) \\ E_t \left(r_{i,t+1} \right) - r_{f,t+1} + \frac{\sigma_i^2}{2} &= \theta \frac{\sigma_{ic}}{\psi} + (1-\theta) \sigma_{im} && (6) \end{aligned}$$

where σ_i^2 is the variance of the return on the generic asset i,σ_m^2 is variance of the return on the market portfolio, σ_c^2 is the variance of consumption

[4]The following derivation is standard in the literature, see Campbell et al. Ch.8, pages 319-320.

growth, σ_{ic} is the covariance between the return on asset i and consumption growth and σ_{im} is the covariance between the return on asset i and the return on the market portfolio. By using (6)[5] in equation (5) to solve out for future expected consumption growth in the intertemporal budget constraint we obtain:

$$c_t - w_t = (1-\psi)\, E_t \left[\sum_{j=1}^{\infty} \rho^j r_{m,t+j} \right] + \frac{\rho\,(k-\mu_m)}{1-\rho} \qquad (7)$$

The solved-out consumption function (7) shows that the log consumption-wealth ratio is a constant plus $(1-\psi)$ times the discounted value of expected future returns on invested wealth. The EIS parameter can be identified and estimated from (7) given the availability of some proxy for future expected returns. Values of the EIS ψ lower than one imply that the income effect of higher returns dominates the substitution effect, while if ψ is greater than one, then the substitution effect dominates and the consumption-wealth ratio falls when expected returns rise. The combination of the intertemporal budget constraints with the first order condition of the consumer optimization problem under Eptein-Zin-Weil preferences makes the relation between excess consumption and expected long-term returns tighter than in the intertemporal budget constraints. Moreover, it is now explicit that the correlation between consumption and long-horizon returns depends on the combined effect of income and substitution effects. A positive relation implies that the income effect dominates, this is what Lettau and Ludvigson meant when stating "...**If expected consumption growth is not too volatile**, stationary deviations from the shared trend among these three variables produce movements in the consumption–aggregate wealth ratio and predict future asset returns...". Clearly the evidence in LL is suggestive of a value for the EIS smaller than one, but the estimation of the linearized intertemporal budget constraint cannot be helpful in reconciling the available conflicting evidence on the empirical value of such parameter.

Solving out for expected consumption growth allows the estimation of the intertemporal elasticity of substitution and provides an immediate interpretation of the correlation between excess-consumption and long-horizon

[5] Note that (6)determines the risk premium, adjusted for a Jensen inequality term, in terms of a weighted average of the Capital Asset Pricing Model and the Consumption Capital Asset Pricing Model.

returns on the market portfolio. Empirical estimation of (7) is the natural step to take at this stage. We shall devote the next section to this issue.

3 Measurement Issues and Identification Strategy

The empirical estimation of the relation of our interest requires the solution of a number of measurement issues and of an identification problem. In fact, w_t and $r_{m,t}$ are not directly observed, moreover the identification of ψ is possible only after having found a proxy for future expected returns on the total wealth. Measurement issues are also relevant for c_t as the theoretical relation is derived for total consumption but most empirical investigations use consumption of nondurable goods and services as the empirical counterpart of c_t.

To illustrate how we take equation (7) to the data note that, following Campbell(1996), we approximate the log of total wealth as:

$$w_t = va_t + (1 - v)\, h_t$$

where v is a constant of linearization, equal to the average share of asset holdings in total wealth, a_t is the log of asset holdings and h_t is the log of human capital. While we have available data for financial wealth, the measurement of h_t is not immediate. To find an empirical counterpart of this variable consider that labour income can be interpreted as a dividend on human capital (see Julliard(2004)):

$$1 + R_{h,t+1} = \frac{H_{t+1} + Y_{t+1}}{H_t}$$

Log-linearizing this relation around the steady state human capital-labor income ratio ($\frac{Y}{H} = \frac{1}{\rho_h} - 1$) we have:

$$r_{h,t+1} = (1 - \rho_h)\, k_h + \rho_h\, (h_{t+1} - y_{t+1}) - (h_t - y_t) + \Delta y_{t+1}$$

By solving this relation forward and by imposing the transversality condition we have:

$$h_t = y_t + \sum_{i=1}^{\infty} \rho_h^{i-1}\, (\Delta y_{t+i} - r_{h,t+i}) + k_h$$

so the log of human capital to income ratio is determined by discounted sum of future labour income growth and human capital returns.

Consistently with our linearization for wealth, the total return on wealth can be approximated by:

$$r_{m,t} = v r_{a,t} + (1-v)\, r_{h,t} + k_r$$

we decompose the unobservable $r_{h,t}$ into a part correlated with $r_{a,t}$ and a part orthogonal to it:

$$r_{h,t} = \beta r_{a,t} + \epsilon_t$$

By substituting all these relationships in the optimality condition we have:

$$\begin{aligned} c_t - v a_t - (1-v)\, y_t &= (1-\psi)\, E_t \left[\sum_{j=1}^{\infty} \rho^j \left(v + (1-v)\,\beta\right) r_{a,t+j} \right] + k + \\ & \sum_{j=1}^{\infty} E_t \rho_h^{j-1} \left(\Delta y_{t+j} - \beta r_{a,t+j}\right) + \eta_t \qquad (8) \\ \eta_t &= \sum_{j=1}^{\infty} \rho_h^{j-1} \epsilon_{t+j} \end{aligned}$$

where η_t is an unobservable stationary component.

Our strategy for identifying and estimating ψ comes in two steps. We first estimate a cointegrating relation between c_t, a_t,and y_t.Such a cointegrating relation is implied by the intertemporal budget constraint, that defines the consumption-wealth ratio as a stationary variable. We then proceed to estimate the following stationary VAR[6]:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{A}\mathbf{X}_{t-1} + \mathbf{u}_t \qquad (9) \\ \mathbf{X}_t &= \begin{bmatrix} r_{m,t} \\ (c_t - c_t^*) \\ \Delta y_t \\ \Delta a_t \end{bmatrix}. \end{aligned}$$

[6]We adopt a first order representation of our VAR, if the estimated VAR is of higher order all following results are applicable to the stacked representation of the VAR

(9) is constructed by considering the stationary VAR representation of a cointegrated system proposed by Campbell and Shiller(1987) and formally derived in Mellander et al.(1993). In practice, we adopt the same VAR estimated by LL and augment it by another stationary variable, the quarterly returns on financial wealth.

The consumption function (**??**) puts a set of restrictions on the VAR that can be exploited to estimate the parameter to our interest. In fact, we have:

$$\begin{aligned} \mathbf{e}'_{cay}X_t &= (1-\psi)\,E_t\left[\sum_{j=1}^{\infty}\rho^j\,(v+(1-v)\,\beta)\,\mathbf{e}'_rA^jX_t\right]+ \\ &+\sum_{j=1}^{\infty}E_t\rho_h^{j-1}\left(\mathbf{e}'_{\Delta y}-\beta\mathbf{e}'_r\right)A^jX_t \end{aligned} \tag{10}$$

where $\mathbf{e}'_{cay}$, $\mathbf{e}'_r$,and $\mathbf{e}_{\Delta y}$ are selector vectors for cay, $r_{m,t}$,and Δy_t correspondingly (i.e. row vectors of the length of the vector $\mathbf{X}$, all of which elements are zero except for the 2nd element of $\mathbf{e}'_{cay}$ and the first element of $\mathbf{e}_{\Delta y}$ and the third element of $\mathbf{e}'_r$,which are unity). Since the above expression has to hold for general z_t, and, given stationarity of the VAR, the sum converges, it must be the case that:

$$\mathbf{e}'_{cay}=(1-\psi)\,(v+(1-v)\,\beta)\,\mathbf{e}'_r\rho A(I-\rho A)^{-1}+\left(\mathbf{e}'_{\Delta y}-\beta\mathbf{e}'_r\right)A(I-\rho_hA)^{-1} \tag{11}$$

which implies:

$$\mathbf{e}'_{cay}(I-\rho A)=(1-\psi)\,(v+(1-v)\,\beta)\,\mathbf{e}'_r\rho A+\left(\mathbf{e}'_{\Delta y}-\beta\mathbf{e}'_r\right)A(I-\rho_hA)^{-1}(I-\rho A) \tag{12}$$

by imposing the restrictions on the cointegrated VAR, conditionally upon ρ,ρ_h and v, ψ is identified and it can be estimated in the restricted VAR. The estimation procedure considers jointly the forward looking consumption function and a VAR used to generate projections of the relevant variables and it avoids the problem of generated regressors that would be encountered by a two-step procedure in which future expected variable are projected first and then they are substituted in the forward-looking consumption function to estimate the parameters of interest.

We shall apply our framework to three specifications:

1) A baseline model in which $r_{h,t}=r_{a,t}$, $h_t=y_t$, and $c_t=\lambda c_{n,t}$.In this case current labour income is the proxy for human capital, the only relevant

returns are those on financial wealth and the log of total consumption is assumed proportional to log of consumers expenditure on durable goods and services. This is the case considered by LL. The consumption function takes the following specification:

$$c_{n,t} - \frac{v}{\lambda}a_t + \frac{(1-v)}{\lambda}y_t = \frac{(1-\psi)}{\lambda}E_t\left[\sum_{j=1}^{\infty}\rho^j r_{m,t+j}\right] + \frac{\rho\,(k-\mu_m)}{1-\rho} + u_t \quad (13)$$

The model predicts cointegration between consumption of non-durable and services, financial wealth and labour income. The stationary VAR relevant for the estimation of the EIS is:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{A}\mathbf{X}_{t-1} + \mathbf{u}_t \quad (14) \\ \mathbf{X}_t &= \begin{bmatrix} r_{a,t} \\ c_{n,t} - \frac{v}{\lambda}a_t + \frac{(1-v)}{\lambda}y_t \\ \Delta y_t \\ \Delta a_t \end{bmatrix}. \end{aligned}$$

and the parameter of interest is estimated by imposing the following restrictions on (14) :

$$\mathbf{e}'_{cay}(I-\rho A) = \frac{(1-\psi)}{\lambda}\mathbf{e}'_r\rho A \quad (15)$$

Note that for this specification when $\psi < 1$ the income effect dominates the substitution effect and excess consumption with respect to its long-run equilibrium value implies positive expected returns on financial wealth. This is the prediction of the model exploited by LL to show that excess consumption is a good predictor of future stock market returns.

2) We augment baseline model to explicitly consider non durable expenditure, so we maintain the first two assumptions of the baseline case but we relax the third one, $c_t = \lambda c_{n,t}$. This is a relevant extension in that it has been shown (Ogaki and Reinhart, 1998) that the estimates of the EIS are downward biased when the intratemporal substitution between non durable consumption goods and durable consumption goods is ignored.

In our derivation of the equilibrium relation for consumption in the previous two cases we have followed the assumption that the log of total real consumption has been constantly proportional to the log of real consumption of nondurable and services. The empirical evidence does not favour the hypothesis of constancy for the parameter λ in the relation $c_t = \lambda c_{n,t},$ .Time variation in λ does have some implications for our structural estimate of the elasticity of intertemporal substitution ψ, as this parameter is identified conditional to an estimate for λ.

Following the idea put forward by Fernandez-Corugedo et al.(2003) we consider the possibility of capturing the time-variation in λ by the fluctuations in the relative price of durable goods to nondurable.

So we re-do our empirical exercise assuming a long-run linear relationship between durable consumption, non-durable consumption and the relative price:

$$c_t = \phi_1 c_{n,t} - \phi_2 p_t^d$$

Our revised structural model is :

$$c_{n,t} - \frac{\omega}{\phi_1} a_t - \frac{(1-\omega)}{\phi_1} y_t - \frac{\phi_2}{\phi_1} p_t^d = \frac{(1-\psi)}{\phi_1} E_t \left[\sum_{j=1}^{\infty} \rho^j r_{a,t+j} \right] + k + u_t \quad (16)$$

The cointegrating relation is now different in that it includes the relative price of durable to non–durable goods. The stationary VAR becomes now:

$$\begin{aligned} \mathbf{X}_t &= \mathbf{A}\mathbf{X}_{t-1} + \mathbf{u}_t \quad (17) \\ \mathbf{X}_t &= \begin{bmatrix} r_{m,t} \\ caynd_t \\ \Delta y_t \\ \Delta a_t \\ p_t^d \end{bmatrix} \\ caynd_t &= c_{n,t} - \frac{\omega}{\phi_1} a_t - \frac{(1-\omega)}{\phi_1} y_t - \frac{\phi_2}{\phi_1} p_t^d \end{aligned}$$

And the restrictions relevant to estimate the EIS are:

$$\mathbf{e}'_{cay}(I - \rho A) = \frac{(1-\psi)}{\phi_1} \mathbf{e}'_r \rho A \quad (18)$$

3) Finally we further augment the model to consider the role of future labour income in determining the value of human capital (See Julliard(2005)). In

this case we relax also the first two assumptions of case one.The consumption function becomes now:

$$
\begin{aligned}
c_{n,t} - \frac{\omega}{\phi_1} a_t - \frac{(1-\omega)}{\phi_1} y_t - \frac{\phi_2}{\phi_1} p_t^d &= \frac{(1-\psi)}{\phi_1} E_t \left[\sum_{j=1}^{\infty} \rho^j \left(v + (1-v)\beta \right) r_{a,t+j} \right] + \\
&+ \frac{1}{\phi_1} \sum_{j=1}^{\infty} E_t \rho_h^{j-1} \left(\Delta y_{t+j} - \beta r_{a,t+j} \right) + k + \eta_t \qquad (19) \\
\eta_t &= \sum_{j=1}^{\infty} \rho_h^{j-1} \epsilon_{t+j}
\end{aligned}
$$

The cointegrating relation is not different from case 2), and the specification of the stationary VAR is the same, however the relevant restrictions to estimate the parameters of interest become now:

$$
\mathbf{e}'_{cay}(I - \rho A) = \frac{(1-\psi)}{\phi_1} \left(v + (1-v)\beta \right) \mathbf{e}'_r \rho A + \frac{1}{\phi_1} \left(\mathbf{e}'_{\Delta y} A - \beta \mathbf{e}'_r A \right) (I - \rho_h A)^{-1} (I - \rho A) \qquad (20)
$$

that shows clearly that excess consumption is now related to expected returns on financial wealth, and expected labour income growth. As consequence, the relevant variable to predict future returns on financial wealth is not excess consumption but excess consumption minus discounted future expected changes in labour income.

4 Empirical Results

Our empirical exercise concentrates on US data. We consider an extended version of the Lettau and Ludvigson original data-set to analyze over the period 1952:4-2003:2, quarterly observations for the following series: $c_{n,t}$ (log of) real consumption of non-durable and services, a_t,(log of) real financial wealth, y_t (log of) real labour income,$r_{m,t}$ quarterly returns on the S&P composite index, p_t^d the relative price of consumer durable goods to non durable and services.[7]

[7] The first three series are taken directly from the authors' websites: http://www.ny.frb.org/rmaghome/economist/lettau/lettau.html and

4.1 The Baseline Model

We start by adopting the LL specification and use a cointegrating relation between the log of real consumption of non durable goods and services the log of real financial wealth and the log of real labour income as a proxy for the log of the consumption to wealth ratio:

$$c_t - w_t \sim c_{n,t} - \hat{\beta}_a a_t - \hat{\beta}_y y_t = cay_t$$

Note that the parameters of the cointegrating relation will be $[1, -(1/\lambda)v, -(1/\lambda)(1-v)]$, so we can identify $\lambda = \frac{1}{\hat{\beta}_a + \hat{\beta}_y}$.

The results of the empirical analysis of cointegration are reported in Table 1. The null of at most zero cointegrating relation is strongly rejected, while the null of at most one cointegrating relation cannot be rejected by the Maximum Eigenvalue test proposed by Johansen(1995). The estimated values for $\hat{\beta}_a$ and $\hat{\beta}_y$ take values of respectively 0.26 and 0.62, which are slightly different form those reported by LL on a restricted sample[8].

On the basis of our long-run analysis we specify an unrestricted stationary VAR, note that the representation is totally equivalent to a VECM but it allows to impose directly the restrictions of interest on the estimated parameters determining the short-run dynamics of the system. In order to do so we calibrate $\rho = 0.955$, this number is obtained as the complement to one of the average consumption to financial wealth ratio (0.17) multiplied by one-third. We multiply by one third the average consumption-financial wealth ratio as the cointegration results suggest that the share of financial wealth in total wealth is about one-third.

The results from estimation of the unrestricted and restricted VAR are reported in Tables 1.2-1.3.

The results in Table 1.2 confirms the interesting properties of the data on consumption, wealth, labor income and stock market returns explored by LL. The asset growth equation shows that cay_t predicts asset growth,

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http://www.ny.frb.org/rmaghome/economist/ludvigson/ludvigson.html. A detailed description on the construction of these series is provided in the appendix to Lettau and Ludvigson(2001). The S&P composite index has been taken from Robert Shiller's webpage. The relative price of durable to non durables is taken by the FRED Database at the Federal Reserve of St.Louis (http://research.stlouisfed.org/fred2/).

[8] In fact, if we run the Johansen procedure on the same sampl with LL we obtain estimates numerically equivalent to those reported by LL.

implying that deviations in asset wealth from its shared trend with labor income and consumption uncover an important transitory variation in asset holdings. The equation for stock returns confirms that cay_t predicts asset growth because the estimated trend deviation forecasts asset returns. There is very little predictability of labour income growth, that in fact is less predictable than stock market returns. cay_t reverts towards its mean but it does so rather slowly. Table 1.3 shows that imposing the restrictions necessary to estimate the EIS penalizes the log-likelihood very little, and that point estimates of the unrestricted coefficients are not significantly different from those reported in Table 1.2. ψ is estimated rather precisely at 0.85. This a value that is much higher than all those obtained by estimating Euler equation on short-term fluctuations in consumption and it is instead very close to that usually adopted in calibrated dynamic business cycle models. On the basis of this results we can clearly explain the positive relation between excess-consumption and long-run expected returns originally found by LL.

4.2 The model with the relative price of durable goods to non-durable goods

Ogaki and Reinhart, (1998) have shown that the estimates of the EIS are downward biased when the intratemporal substitution between non durable consumption goods and durable consumption goods is ignored. In our derivation of the equilibrium relation for consumption we have followed the assumption that the log of total real consumption has been constantly proportional to the log of real consumption of nondurable and services. We plot in Figure 1 the ratio of these two variables. The empirical evidence does not favour the hypothesis of constancy for the parameter λ in the relation $c_t = \lambda c_{n,t}$, .and the estimate of λ derived directly form the data differs slightly from that implied by the coefficients in the cointegrating vector, although the restriction that $\hat{\beta}_a + \hat{\beta}_y = \frac{\bar{c}_t^n}{\bar{c}_t} = \frac{1}{1.04}$ is not rejected in our full sample estimation of the cointegrating vector. Time variation in λ does have some implications for our structural estimate of the elasticity of intertemporal substitution ψ. In fact, when we estimate our deep parameter of interest using GMM on the solved out forward looking consumption function we keep λ constant at the value 1/0.88 estimated from the cointegrating relation, therefore time variation in λ would cause time variation in our estimate of the EIS.

Following the idea put forward by Fernandez-Corugedo et al.(2003) we consider the possibility of capturing the time-variation in λ by the fluctuations in the relative price of durable goods to nondurable. The graphical evidence of Figure 2 shows that indeed this variable has the capability of explaining the increase in the expenditure on nondurable that caused the increase in λ over the last part of our sample.

We implement cointegration analysis by augmenting by one variable, p_t^d, the VAR originally considered by LL. The results of the cointegrating analysis are reported in Table 2.1. The evidence is mixed in that it shows that the results on the number of cointegrating relations is robust to this extension of the model, but the null that the coefficient on p_t^d can restricted to zero in the cointegrating relation is rejected, although not strongly.

Table 2.2 and 2.3 reports the results from the estimation of the unrestricted and restricted stationary VAR, that now include one more variable than in the baseline case. All the coefficient statistically significant in Table 1.2 and 1.3 are so in Table 2.2 and 3.3, moreover excess consumption has also some predictive power for fluctuations in p_t^d.The restrictions necessary to identify the EIS can be validly imposed on the VAR and the point estimate of ψ takes now a value of 0.77, which is not statistically different from the 0.84 estimated in the baseline version of the model.

4.3 The model with future labour income

Finally, we extend the model to explicitly consider the role of future labour income and future returns to determine the value of human capital. These extensions do not alter the cointegrating analysis and the unrestricted VAR estimated for the preceeding case but it does have some implications for the specification of the restricted VAR and the estimation of the EIS. In fact, when the role of future discounted labour income fluctuations is taken on account to determine current human capital, two main modifications are introduced in the baseline model. First excess consumption does not only reflect higher future returns on wealth but also higher discounted labour income growth, second the restricted model features two more parameters: β, that determines the relation between returns on financial assets and returns on human capital and ρ^h that determines how future incime growth and returns on human capital are discounted. ρ^h is usually calibreted at 0.955 in the available literature (see. for example, Baxter and Jermann(1997) and Julliard(2004)) so we set this parameter equal to ρ. We are now left with a different set of restrictions and just one additional parameter in the restricted model than in the baseline case. Given that we include labour income growth in our VAR, the further set of restrictions is easily imposed on the VAR coefficients. Then we conduct a grid search on values of β in the range between 0 and 1. The empirical results show that the β is rather difficult to identify, as the likelihood functions is rather flat for the range of values we have considered for this parameter. the likelihood peaks at 0.9, although the height of the peak is such that all values between 0 and 1 are not statistically different from 0.9. So some caution as to be exercised when interpreting the results reported in Table 3, which refer to the case of $\beta = 0.9$. The estimated EIS takes now the value of 0.77, which again show the robustness of our original estimate to this extension of the model.

5 Conclusions

In this paper we have argued that it is possible to reconcile the conflicting macroeconomic evidence on the magnitude of the elasticity of intertemporal substitution in consumption. We have shown that estimated values of EIS based on long-run US consumption fluctuations are consistent with those needed to reconcile macroeconomic series generated by calibrated dynamic models with the observed data. Our results are in contrast with the evidence of a low EIS derived by estimating Euler Equation on high-frequency fluctuations in consumption. They suggest that the recent evidence on the solution of some well-known asset pricing puzzles based on empirical investigation concentrating on the low-frequency fluctuations in consumption can be extended to derive estimated values of the EIS in line with those calibrated in dynamic models. In particular, we have used a recursive Epstein-Zin utility function and the linearized intertemporal budget constraint to derive an explicit long-run consumption function. The forward looking consumption function constitutes a tight relation between long-run stock market returns and a cointegrating relation linking consumption to wealth. Such a relation is determined by the elasticity of intertemporal substitution. The empirical estimation of the forward-looking consumption function delivers a precise estimate of the elasticity of intertemporal substitution not far from one and it shows that deviation of consumption from its long-run trend has indeed some predictive power for long-run expected stock market returns. The empirical results of an high value of the EIS are robust to explicit inclusion of non-durable consumption in the empirical model and to the consideration of the role of future discounted labour income growth in determining the current value of human capital.

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Table 1.1: The Baseline Model. Cointegration Analysis

This panel contains the results of the application of the Johansen (1995) procedure allowing an intercept in the cointegrating vector and in the VAR. The panel below reports the estimate of the cointegrating parameters and the result of the cointegration test over different samples.

The Cointegrating Relation				
Sample	Cointegrating Parameters			
1952:3-2003:2	$c_{n,t}$	a_t	y_t	CONST
$\lambda = \frac{1}{0.26+0.62}$				
$\nu = \lambda * 0.26 = 0.3$	1.000	-0.26	-0.62	-0.76
		(0.029)	(0.03)	

Cointegration Rank Test (Maximum Eigenvalue)				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.16	34.86	22.30	0.0005
At most 1	0.07	14.56	15.89	0.0799

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level.

* denotes rejection of the hypothesis at the 0.05 level.

**MacKinnon-Haug-Michelis (1999) p-values

Table 1.2: The Baseline Model. The unrestricted stationary VAR

1952:3-2003:2	Equation			
Dependent Variable	$r_{m,t}$	Δy_t	Δa_t	cay_t
$r_{m,t-1}$	0.156	0.023	0.032	-0.007
(s.e.)	(0.17)	(0.019)	(0.047)	(0.014)
Δy_{t-1}	-0.39	-0.002	-0.018	0.123
(s.e.)	(0.69)	(0.061)	(0.189)	(0.06)
Δa_{t-1}	-0.046	0.038	-0.011	-0.015
(s.e.)	(0.60)	(0.077)	(0.145)	(0.045)
cay_{t-1}	1.449	-0.019	0.347	0.872
(s.e.)	(0.53)	(0.075)	(0.133)	(0.05)
$CONST$	-0.109	0.0067	-0.023	0.01
(s.e.)	(0.046)	(0.0065)	(0.011)	(0.004)
$\bar{R}^2$	0.09	0.07	0.06	0.73
Log Likelihood	2424.3			

Table 1.3: The Baseline Model. The restricted stationary VAR.

1952:3-2003:2	Equation			
Dependent Variable	$r_{m,t}$	Δy_t	Δa_t	cay_t
$r_{m,t-1}$	0.074	0.029	0.019	$-0.074 * \frac{1-\psi}{\lambda}$
(s.e.)	(0.0852)	(0.015)	(0.038)	
Δy_{t-1}	-0.835	0.031	-0.08	$0.835 * \frac{1-\psi}{\lambda}$
(s.e.)	(0.49)	(0.057)	(0.18)	
Δa_{t-1}	0.089	0.028	0.009	$-0.089 * \frac{1-\psi}{\lambda}$
(s.e.)	(0.34)	(0.067)	(0.113)	
cay_{t-1}	1.339	-0.01	0.331	$\frac{1}{\rho}-1.339 * \frac{1-\psi}{\lambda}$
(s.e.)	(0..44)	(0.07)	(0.126)	
$CONST$	0.029	0.0059	-0.021	0.010
(s.e.)	(0.015)	(0.006)	(0.01)	(0.004)
$\bar{R}^2$	0.08	0.07	0.06	0.72
Log-Likelihood	2423.1			
ψ	0.849			
(s.e.)	(0.038)			

This table reports estimated coefficients (with standard errors within brackets) from cointegrated sytem estimated by FIML. cay_t is $c_{n,t} - 0.26a_t - 0.62y_t - 0.76$.

Table 2.1: The Model with the relative price of durables to non durables. Cointegration Analysis

This panel contains the results of the application of the Johansen (1995) procedure allowing an intercept in the cointegrating vector and in the VAR. The panel below reports the estimate of the cointegrating parameters and the result of the cointegration test over different samples.

The Cointegrating Relation					
Sample	Cointegrating Parameters				
1952:3-2003:2	$c_{n,t}$	a_t	y_t	pd_t	CONST
$\varphi_1 = \frac{1}{0.35+0.63}$					
$\nu = \varphi_1 * 0.35 = 0.36$	1.000	-0.35	-0.63	-0.14	0.43
$\varphi_2 = \varphi_1 * 0.63 = 0.65$		(0.038)	(0.034)	(0.06)	

Cointegration Rank Test (Maximum Eigenvalue)				
Hypotesized No. of CE(s)	Eigenvalue	Max - Eigen Statistic	0.05 Critical Value	Prob.**
None*	0.20	46.88	28.58	0.0001
At most 1	0.09	18.48	22.29	0.1568
Test of the null that the coeff on pd_t=0				$\chi^2(1) = 4.72(0.03)$

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level.

* denotes rejection of the hypothesis at the 0.05 level.

**MacKinnon-Haug-Michelis (1999) p-values

Table 2.2: The Model with the relative price of durables to non durables. The unrestricted stationary VAR

1952:3-2003:2	Equation				
Dependent Variable	$r_{m,t}$	Δy_t	Δa_t	$caypd_t$	Δpd_t
$r_{m,t-1}$	0.16	0.023	0.033	-0.008	0.036
(s.e.)	(0.18)	(0.019)	(0.05)	(0.015)	(0.017)
Δy_{t-1}	-0.44	-0.012	-0.043	0.13	-0.045
(s.e.)	(0.80)	(0.06)	(0.22)	(0.07)	(0.07)
Δa_{t-1}	-0.08	0.032	-0.027	- 0.01	-0.11
(s.e.)	(0.62)	(0.08)	(0.15)	(0.047)	(0.07)
$caypd_{t-1}$	1.37	-0.03	0.316	0.88	-0.006
(s.e.)	(0.58)	(0.08)	(0.151)	(0.05)	(0.012)
Δpd_{t-1}	0.31	0.061	0.145	-0.05	0.342
	(0.73)	(0.10)	(0.21)	(0.07)	(0.073)
$CONST$	-0.10	0.008	-0.019	0.009	-0.002
(s.e.)	(0.05)	(0.007)	(0.013)	(0.004)	(0.001)
$\bar{R}^2$	0.09	0.07	0.06	0.72	0.15
Log Likelihood	3122.7				

Table 2.3: The Model with the relative price of durables to non durables. The restricted stationary VAR.

1952:3-2003:2	Equation				
Dependent Variable	$r_{m,t}$	Δy_t	Δa_t	$caypd_t$	Δpd_t
$r_{m,t-1}$	0.02	0.026	0.003	$-0.02 * \frac{1-\psi}{\varphi_1}$	0.034
(s.e.)	(0.05)	(0.018)	(0.024)		(0.017)
Δy_{t-1}	0.099	-0.035	0.06	$-0.099 * \frac{1-\psi}{\varphi_1}$	-0.039
(s.e.)	(0.26)	(0.065)	(0.12)		(0.07)
Δa_{t-1}	0.04	0.021	-0.009	$-0.04 * \frac{1-\psi}{\varphi_1}$	-0.11
(s.e.)	(0.20)	(0.07)	(0.07)		(0.06)
cay_{t-1}	1.26	-0.072	0.23	$\frac{1}{\rho}-1.26 * \frac{1-\psi}{\varphi_1}$	-0.006
(s.e.)	(0.59)	(0.078)	(0.14)		(0.012)
Δpd_{t-1}	0.05	0.071	0.09	$-0.05 * \frac{1-\psi}{\varphi_1}$	0.34
	(0.24)	(0.10)	(0.11)		(0.07)
$CONST$	-0.09	0.011	-0.013	0.0006	-0.001
(s.e.)	(0.05)	(0.006)	(0.012)	(0.002)	(0.001)
$\bar{R}^2$	0.07	0.07	0.05	0.95	0.14
Log Likelihood	2943.7				
ψ	0.77				
(s.e.)	(0.091)				

$caypd_t$ is $c_{n,t} - 0.35a_t - 0.63y_t - 0.14pd_t + 0.43$.

Table 3.3: Model 3. The restricted stationary VAR:1952:3-2003:2.

	Equation				
Dep. Var.	$r_{m,t}$	Δy_t	Δa_t	$caypd_t$	Δpd_t
$r_{m,t-1}$	-0.05	0.012	-0.007	$0.05 * \frac{1-\psi}{\varphi_1} * \beta - \frac{1}{\varphi_1 \rho}(-0.05 - \beta * 0.012)$	0.026
(s.e.)	(0.07)	(0.018)	(0.03)		(0.018)
Δy_{t-1}	-0.52	-0.018	-0.032	$0.52 * \frac{1-\psi}{\varphi_1} * \beta - \frac{1}{\varphi_1 \rho}(-0.52 - \beta * -0.018)$	0.04
(s.e.)	(0.35)	(0.069)	(0.18)		(0.061)
Δa_{t-1}	-0.12	0.04	-0.037	$0.12 * \frac{1-\psi}{\varphi_1} * \beta - \frac{1}{\varphi_1 \rho}(-0.12 - \beta * -0.037)$	-0.057
(s.e.)	(0.26)	(0.077)	(0.097)		(0.07)
$caypd_{t-1}$	0.73	-0.045	0.21	$\frac{1}{\rho}$-0.73 $* \frac{1-\psi}{\varphi_1} * \beta - \frac{1}{\varphi_1 \rho}(0.73 - \beta * 0.21)$	0.137
(s.e.)	(0.38)	(0.085)	(0.11)		(0.048)
Δpd_{t-1}	-0.11	0.052	0.06	$0.11 * \frac{1-\psi}{\varphi_1} * \beta - \frac{1}{\varphi_1 \rho}(-0.11 - \beta * 0.06)$	0.284
	(0.35)	(0.096)	(0.16)		(0.068)
$CONST$	0.02	0.005	0.006	-0.0009	-0.003
(s.e.)	(0.007)	(0.001)	(0.002)	(0.0008)	(0.00089)
$\bar{R}^2$	0.07	0.04	0.04	0.95	0.14
Log L	2943.9				
ψ	0.77	β	0.9		
(s.e.)	(0.09)				

This table reports the sum of estimated coefficients from cointegrated sytem estimated by FIML of the column variable on the row-variable, standard errors for the sum are reported in parentheses. $caypd_t$ is $c_{n,t} - 0.35a_t - 0.63y_t - 0.14pd_t + 0.43$.

Figure 1: Ratio between total consumption and consumption of non-durable and services

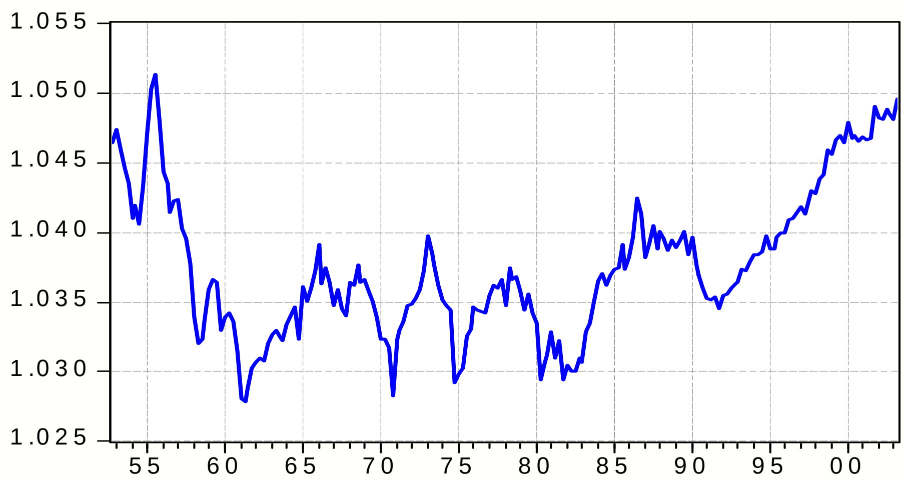

Figure 2: The relative price of durables to non durables goods