

The Political Economy of Housing Supply*

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FIRST DRAFT
December 19, 2005

Abstract

We model an economy where people choose housing consumption, housing investment, and vote at a local level on the number of new building permits to be issued. Inefficiencies arise due to a vicious circle between households' desire to own housing as a hedge against income shocks and their support for housing supply restrictions that make housing a better hedge against income shocks. The equilibrium supply of housing depends on the rule that governs the appropriation of the proceeds from issuing new building permits. There is hysteresis in housing supply: the more restricted the initial housing supply, the smaller the city size selected by the voting process. We analyze the effects of a number of policies: (1) A reform of the housing permit allocation process; (2) Incentives for home ownership; (3) Centralization of planning decisions; and (4) Elimination of barriers to fractional ownership. The model highlights the tension between the two housing policy objectives of increasing homeownership and housing affordability.

1 Introduction

Between 1975 and 2004, real housing prices in Santa Barbara more than tripled. They more than doubled in Boston, Los Angeles, San Diego and San Francisco. The UK experienced as much housing price growth as Santa Barbara. A number of recent papers put the blame on housing-supply regulations. Glaeser, Gyourko and Saks (2005a) present evidence that man-made restrictions in housing supply are responsible for the soaring house prices in the most expensive US markets since the mid-Seventies. Quigley and Raphael (2005) concur in their analysis of the drivers of the recent housing price growth in California. Green, Malpezzi and Mayo (2005) agree that housing supply regulations are a key driver of differences in housing supply elasticities across US metropolitan areas.¹ For the UK, Barker (2003, 2005)

*We are grateful to Erzo G. Luttmer, Monika Piazzesi and seminar participants at Warwick University for useful comments.

¹For further evidence on the critical impact of housing regulations on housing supply, see Rose (1989), Malpezzi, Chun and Green (1998), Mayer and Somerville (2000), Glaeser and Gyourko (2003), Glaeser, Gyourko and Saks (forthcoming).

identifies the regulatory constraints on the release of land for development as the primary reason behind the unresponsiveness of housing supply to price increases.

Restrictive housing supply regulations cannot survive without political support. Glaeser, Gyourko and Saks (2005b) argue that the ability of local residents to block city expansions has increased since the Seventies in the US. Quigley and Raphael (2005) point to the power of local voters in regulating their own city to explain why housing prices in California have increased so much more than in the rest of the country.

Why do voters support growth restrictions? Local residents may be concerned about negative externalities caused by congestion (pollution, traffic, etc). Or they may worry about the worsening of public services and negative peer effects if lower income households move in. We focus on a direct housing price effect: homeowners fear that an increase in supply will reduce the value of their investment. There is no doubt that this is one of the key drivers of the political support behind supply restrictions (Fischel, 2001). However, this just begs the question: what drives households to buy their home rather than rent it? Households may have fiscal incentives to own. They may want to avoid the agency costs involved in renting. We focus on a demand for homeownership of a different nature.

In a world where the productivity of a household depends on its location, housing costs too depend on location, as long as the supply of housing is not perfectly elastic. A household that rents its home is vulnerable to local positive productivity shocks that affect some other households more than itself, like an influx of talented newcomers (the availability of skilled workforce in Western Europe after the EU enlargement), a new local business opportunity (the financial sector boom of the 90's in London and New York), or a new technology (the IT revolution in the Bay Area). As a result of such shocks, the renter suffers a loss of disposable income after housing expenditures that may price him out of the opportunity to take advantage of the benefits of his local labor market.

Homeownership provides insurance against such shocks. If the household sells its home in response to the rise in housing cost, he captures some of the benefits of the positive productivity shock. Capital gains on the home provide compensation for the loss of surplus incurred due to the move. In our model, the strength of this hedging motive drives households' housing investment and thus tenure choice. As the three examples above illustrate, the magnitude of these shocks can be substantial (think of an academic who has been living

in London, New York, or the Bay Area for the past decade and did not buy a house when it was still affordable).²

The goal of this paper is to provide a first attempt at modelling an economy that captures this complex set of interactions between the labor market, the housing market, and collective decision-making on urban growth. In the model, households support for urban growth restrictions depends on their own housing investment decision which in turn depends on the risks they face in both the housing and labor markets and the mechanism whereby the benefits of urban growth are allocated. The model can be used to assess the extent and the causes of the under-supply of housing and evaluate the effect of a number of policies that have been proposed.

We consider a country with one city and a vast countryside. A continuum of agents live for two periods. In the first period, every agent is assigned a productivity level. Productivity and location are complementary: the more productive agents are even more productive if they live in the city rather than in the countryside.

Agent productivity may change from the first to the second period. In particular, there may be a technological innovation with two effects: an increase in average productivity and turbulence in the productivity levels of individual agents. Turbulence involves a reordering of individual productivity levels. For instance, the IT revolution that occurred in the Bay Area in the 90's boosted overall productivity but had a more positive effect for certain workers (e.g software engineers) than for others (e.g. economics professors).

Agents who move to the city in the first period must choose whether to buy or to rent. To focus on our core question, we assume away all market imperfections and transaction costs (the only missing market relates to the direct provision of insurance against income fluctuations – hence the desire to use housing as a hedge). In particular, in our world there are no credit constraints, no re-location costs, and no agency issues that may make renting intrinsically more or less costly than buying. All houses are identical and can only accommodate one agent. We also assume that home ownership is a continuous choice variable, going from unboundedly negative (short-selling city real estate) to unboundedly positive (owning multiple homes in the city, or derivatives on the city housing price index).³

²An analogous class of localized shocks can be obtained through consumption rather than labor. This is happening with places – like Florida or Southern France – that have become attractive places to live, mostly for retirees. Housing costs rise and locals are “priced out”. Ex ante, residents can insure themselves against these shocks by buying property.

³In Section 6, we study the effect of legal restrictions which make home ownership a binary choice between renting one home or buying one home.

At the end of the first period, city residents vote over housing supply in the second period: they select the number of construction licenses to be issued. A key element of the model is the institutional mechanism that regulates the distribution of new licenses. The windfall gain deriving from new construction can accrue to homeowners (e.g. licenses allowing for extensions to existing homes), residents (licenses are sold to developers and the revenues are used for local services), or to a set of measure zero of the population (lucky or clever developers in case of an arbitrary mechanism, or well-connected developers and perhaps corrupt public officials in case of favoritism). Our analysis focuses mostly on situations in which only a small portion of the gain accrues to residents or homeowners.⁴

Equilibrium in our model is determined by two sets of interactions. On the housing market, the rent-buy choice of the agents depends on their productivity. More productive agents have a stronger hedging motive and they are more likely to buy rather than rent. On the political side, agents who are more invested in housing are more likely to favor a restrictive licensing regime. This leads to a first, unsurprising result: in equilibrium, housing is under-supplied. Voters support artificial supply restrictions in order to protect their investment. They invest because they expect the value of their housing investment to be protected by urban growth restrictions. They value the hedging benefits of home ownership because of the risks of turbulence in the labor market at times of productivity growth.

More importantly, we show hysteresis in housing supply. The degree of undersupply in the second period is an increasing function of undersupply in the first period. This is not due to construction costs (there are none) but to the interaction between hedging demand and politics. A city with a small number of houses has an initial population of high-productivity agents who pay a high housing price or rent relative to their income. The median voter is then highly invested in housing and he is keen to keep the city small and housing expensive. The opposite occurs in a city that starts from a relatively high housing supply and thus low housing costs relative to income.

Once we identify the potentially vicious circle between home ownership and housing supply, we can begin to discuss the effects of a number of institutional reforms that have been

⁴To our knowledge, there is little systematic evidence on the distribution of windfall gains. In her comprehensive review of housing supply in Britain, Barker (2003, Chapter 5) argues that: (1) Developers hold option agreements on large tracts of land currently without building permission; (2) Developers have significant local market power; (3) While local authorities have a legal avenue to demand monetary transfers in exchange for issuing building permission, the payments obtained in this way are quite low (of the order of £2000/8000 per unit built – See Table 8.2 in Barker). These three facts taken together seem to indicate that most of the windfall gains accrue to developers.

suggested. First and foremost, one needs to question the current mechanism for allocating housing permits. While it varies widely across countries (and even within countries), we are unaware of any place where housing permits are allocated through an auction mechanism. Our model formally identifies a strong link between the housing undersupply and the share of windfall gains that accrues to the median voter. The most natural way to break the vicious circle is to create simple legal instruments through which local communities can appropriate windfall gains.⁵

We also study the effect of making city planning decisions at a more or less centralized level. We have assumed that housing supply decisions are taken at a level that corresponds to the local labor market (i.e. a metropolitan area). In practice, city planning may occur at a different level. At one extreme, the UK Town and Country Act of 1948 and subsequent laws give the national government an enormous power on planning decisions. The government can in practice force local communities to accept large-scale land development. On the other extreme, a number of metropolitan areas around the world (this is true for most large US cities) are not under a unified jurisdiction: planning decisions are made by a number of autonomous local governments. Our paper shows that there exists a U-shaped relation between the degree of centralization and equilibrium housing supply. A very centralized system and a very decentralized one result in more construction than a system where local government coincides with local labor markets. A centralized government wants more housing supply because it takes into account the welfare of countryside residents (who may move to the city if more houses are built). A very decentralized system falls into a beggar-thy-neighbor equilibrium, whereby local residents do not internalize the negative price effect that construction in their community imposes on the rest of the metropolitan area.

We also examine reducing subsidies to homeowners, and allowing for fractional property (as suggested by Caplin et al., 1997).

Our model is unique in the fact that it combines endogenous location and tenure choice in a dynamic and stochastic housing market equilibrium where the housing supply is determined by a vote. Glaeser, Gyourko and Saks (2005b) provide a detailed model of the

⁵This point seems to have escaped governments concerned with housing affordability. For instance, a recent comprehensive report sponsored by the UK Treasury (Barker 2005) uses a wealth of information to show that housing inflation in the UK is due to undersupply of land, which in turn is due to the unwillingness of local authorities to make more land available. However, the policy recommendation is to tax windfall gains and transfer the proceeds to central government, and to deprive local government of the only existing channel to appropriate some of the developers' rent (the so called "section 106" – see Barker (2005, p.7, recommendation 29)).

decision process involved in authorizing housing development. They study the effects of changing judicial tastes, decreasing ability to bribe regulators, rising incomes and greater tastes for amenities, and improvements in the ability of homeowners to organize and influence local decisions. As mentioned above, they find the main driver for the rise in urban growth restrictions to be a significant increase in the ability of local residents to block new projects. They conclude that cities have changed from urban growth machines to homeowners' cooperatives. Our paper foregoes some of the political process complexity of Glaeser et al. in order to endogenize the composition of the local population, their tenure decision, and hence their preferences for urban growth.

We identify a new benefit of homeownership: homeownership provides a hedge against the risk of being priced out of the opportunity to enjoy returns from the local labor market by people better positioned to take advantage of a local gain in productivity. Homeowners who lose from a productivity shock that benefits others more than them are compensated by the capital gains they realize when selling their home. As mentioned earlier, although we focus on productivity, the same reasoning would apply in a world where households differ according to their preferences for local amenities. Homeownership provides a hedge against the loss of the enjoyment of a particular location when other households experience an increased willingness to pay for the same location.

This hedging benefit differs from that discussed in the recent literature that analyzes tenure choice by risk averse households in a stochastic environment (e.g., Ortalo-Magné and Rady, 2002, Sinai and Souleles, 2005, Hilber, 2005, and Davidoff, forthcoming). The literature has focused on the fact that housing prices move with expected future rents. As a consequence, owning a home provides a hedge against future housing expenditures to households who plan to stay in their home for a long time. Here, we emphasize the hedging benefit from ownership that accrues to households who find it optimal to move out because there are other households who can extract a greater surplus from the same home. The capital gains realized by the households who sell provides compensation from the loss of productivity (or direct utility) incurred when moving out.

From a modelling standpoint the key innovations here concern the heterogeneity of households surplus from a same location and the stochastic nature of this surplus. In addition, we embed the households' location and tenure choice in a housing market equilibrium where housing supply is the outcome of a voting process.⁶

⁶Ortalo-Magné and Rady (2005) analyze a dynamic and stochastic equilibrium model of the housing market where homeownership provides a hedge against rent fluctuations. In that model, the supply of

Fischel (2001) provides detailed arguments and empirical evidence that downside risk motivates homeowners to participate in the planning process. He argues that “homevoters” are motivated by the risk of loss on their home because of the difficulty to diversify this risk away. Here, we take a more general standpoint, arguing that households are motivated not only by potential loss from further development but also by the prospects of enjoying capital gains if and when they move out. More generally, we take the view that households derive benefits from shaping the nature of the financial returns provided by their home: they decide how their home value responds to shocks through the political process.

To keep our model simple, we make a number of strong assumptions. Most importantly, we abstract from the issue of local taxation for the provision of local public goods and the sorting of household across communities.⁷ The financing of local public goods varies dramatically from country to country. At one extreme, the US tends to fund key public services, such as education and police, at a local level. At another extreme, in some EU countries local authorities have no tax power and central government provides uniform funding for local public goods. Clearly, these different funding systems create different incentives for local government. However, the kind of real estate phenomenon we try to explain does not seem to be circumscribed to countries with a certain public good provision mechanism. For instance, it is hard to imagine two systems that are more different than the ones in place in the Netherlands and in California, yet they have both witnessed the same boom-and-bust pattern in the last decade. Clearly, local public good provision is extremely important and more research is needed to relate it to the political economy of housing supply. Yet, a model like ours – based on the hedging motive only – can already provide some meaningful insights into this issue.

The plan of the paper is as follows. Section 2 lays out the model. For expositional purposes, we first analyze the model holding housing supply fixed (Section 3) and we then endogenize supply and study the political equilibrium (Section 4). We discuss the hysteresis result in Section 5. Section 6 studies the effect of institutional reforms. Section 7 concludes. All proofs are in the Appendix.

housing is taken as given and households do not differ in the deterministic surplus they derive from each location.

⁷See Fernandez and Rogerson (1997) and Calabrese, Epple and Romano (2004) for studies of the political economy of zoning regulation and its interaction with the provision of local public goods.

2 Model

Consider a two period model of an open economy with two locations, the City and the Countryside. There are two commodities, housing and a numeraire consumption good. For simplicity, we abstract from housing construction costs. Housing in both communities consists in homogeneous plots of land. The economy is populated by a mass 1 of agents with identical CARA utility defined over consumption of second period numeraire only, $u(c) = -e^{-ac}$.

The endowment of numeraire good each agent receives every period depends on his location and his productivity index $i \in [0, 1]$. We normalized the productivity in the Countryside to zero.⁸ Let y^i be a random variable uniformly distributed on $[0, 1]$. In the first period, if agent i works in the City, his productivity is $y_1^i = y^i$. In period 2, the distribution of earnings evolves as follows. With probability $1 - \pi$, no shock occurs, if agent i works in the City, his productivity is $y_2^i = y_1^i = y^i$. With probability π , there is a positive shock to aggregate productivity: average productivity increases by $g > 0$. Conditional on this shock occurring, with probability $1 - \gamma$, the productivity of agent i in the City is $y_2^i = y^i + g$. With probability γ , all agents draw a new productivity parameters \tilde{y}^i from the initial productivity distribution. The productivity of agent i in the City is then $y_2^i = \tilde{y}^i + g$. The expected aggregate growth rate of the economy between periods 1 and 2 is therefore πg . Future earnings of individual are stochastic. The greater the probability γ , the more insecure are the agents about their future productivity.

Working in the City requires consuming one unit of City housing. We denote l_t^i the housing consumption by agent i in period t where $l_t^i = 1$ if the agent locates in the City, $l_t^i = 0$ otherwise.

Independently of their housing consumption choice, agents may also invest in City housing. Let $h_t^i \in (-\infty, \infty)$ be the measure of City housing that agent i owns in period t . We do not restrict this measure to be a positive integer. A non-integer h_t^i indicates fractional property. A negative h_t^i means that the agent has sold City housing short for period t . In practice, there are serious obstacles to fractional property and to shortselling properties. We abstract from this type of market imperfection for the time being. We shall return to this issue later.

In the Countryside, the supply of housing is perfectly elastic at a cost normalized to zero. There are no moving costs between City and Countryside.

⁸Our results could easily be extended to a more general setting in which the productivity of agent i at time t is y_t^i in the City and ay_t^i in the countryside, where $a \in (0, 1)$.

There is a measure N_1 of housing in the City at the start of period 1 owned initially by a large number of international real estate investment trusts (REITs), which maximize the expected value of their real estate investment. At the end of period 1, City residents choose the measure N_2 of houses available in the City in period 2, . We assume that existing houses cannot be destroyed and no depreciation of the housing stock, $N_2 \geq N_1$. City residents therefore vote on the number of building permits that will be issued, $N_2 - N_1 > 0$. Building permits are sold immediately, yielding total revenue denoted by b .

The parameters ϕ and τ characterize the distribution of the proceeds from the sale of building permits. A proportion $1 - \phi$ of the proceeds is allocated to a set of measure zero of the population. We think of this set of people as City officials and their friends (the cronies). The remainder is allocated to voters. A proportion $\phi\tau$ is allocated to current residents. A proportion $\phi(1 - \tau)$ goes to the owners of existing properties in the City.

The property markets open at the start of each period. Let r_t denotes the rent and p_t denote the unit price of housing in the City at period t . Competition among REITs ensures that the price of housing is equal to its expected rent return plus any benefit that accrues from the sale of building permits. We assume an exogenous interest rate of zero between periods 1 and 2 for ease of exposition.

To sum up, the timing of the model is as follows:

1. Period $t = 1$ begins, agents learn y^i .
2. The property market opens, agents choose h^i and l^i .
3. City residents vote on the measure of City houses N_2 to be made available in period 2. A measure $N_2 - N_1$ of building permit is issued in proportion $\phi\tau$ to City residents (in equal parts) and in proportion $\phi(1 - \tau)$ to owners. These permits can be traded immediately.
4. Each City resident receives y_1^i and pays the rent r_1 . Permit holders can build new houses in the City at zero cost.
5. Period $t = 2$ begins, both aggregate and idiosyncratic shocks are realized, agents learn y_2^i
6. The property market opens, agents choose l_2^i .
7. Each City residents receives y_2^i and pays the rent r_2 .

8. Agents consume their accumulated wealth.

Accumulated wealth of agent i at the end of the game is

$$w^i = l_1^i \left(y_1^i + \phi \tau \frac{N_2 - N_1}{N_1} b - r_1 \right) + l_2^i (y_2^i - r_2) + h^i \left(r_2 + r_1 + \phi (1 - \tau) \frac{N_2 - N_1}{N_1} b - p_1 \right). \quad (1)$$

Housing consumption and investment are separable. An agent who lives in the City always rents his home. If he owns real estate, he receives a rent on it.

We need parameter assumptions $g < \frac{1}{a}$ and

$$\phi (1 - \tau) \frac{1}{N_1} (1 - N_1 + \pi g) < 1.$$

The rationale for these assumption will become clearer in the next sections.

3 Housing Market Equilibrium – Exogenous Housing Supply

As a first step, suppose that N_1 and N_2 are exogenously given and known to the agents with $N_2 \geq N_1$. Under this condition, we characterize the set of City housing prices and rents such that the REITs are indifferent between renting and selling properties and the housing markets clear in each period when the agents' housing consumption and investments policies maximize their utility taking the prices and rents as given.

Once the period-two shock is realized, agent i faces a simple one period deterministic location choice problem. He lives in the City if and only if City earnings more than compensate the rent: $y_2^i \geq r_2$. In equilibrium the N_2 most productive agent live in the City. The second-period market rent is then given by the earnings of the N_2 th most productive agent:

$$r_2 = \begin{cases} 1 - N_2 & \text{if there is no aggregate shock} \\ 1 - N_2 + g & \text{with an aggregate shock} \end{cases}$$

The price of a building permit issued at time 1 is the expected value of a unit of housing in period 2 evaluated prior to the realization of the shock

$$b = E[r_2] = 1 - N_2 + \pi g.$$

The location choice in period 1 is simple as well. The N_1 most productive agents live in the City. The market rent is determined by the willingness to pay of the N_1 th most productive agent:

$$r_1 = 1 - N_1 + \phi \tau \frac{N_2 - N_1}{N_1} b,$$

where the third term account for the fact that city residents receive a proportion $\phi\tau$ of the building permits.

The price of a house in period 1 is the expected present values of period 1 and period 2 rents plus the value of the building permits that ownership of a house gives rights to:

$$p_1 = E \left[r_2 + r_1 + \phi(1 - \tau) \frac{N_2 - N_1}{N_1} b \right] = 1 - N_1 + \left(1 + \phi \frac{N_2 - N_1}{N_1} \right) b + E[r_2].$$

The period 1 price of a house captures the benefits of the building permits allocated to both renters and owners. The payoff of buying a unit of real estate in period 1 depends on the realization of the shock in period 2: investors loose πg if there is no aggregate shock and gain $(1 - \pi)g$ if there is a shock. Given that period 2 is the last period of the model, $p_2 = r_2$.

Substituting equilibrium rents and prices into equation (1) yields the following expression for final wealth:

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i D,$$

where $D = -\pi g$ if there is no aggregate productivity shock and $D = (1 - \pi)g$ otherwise.

Now that we know how agents choose where to locate and the benefits of ownership, we can write the final utility of agent i conditional on his choice of h^i as follows:

- If there is no shock:

$$U_{NN} = u \left(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2) - h^i \pi g \right)$$

- If only the aggregate shock occurs:

$$U_{SN} = u \left(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2) + h^i (1 - \pi) g \right)$$

- If both the aggregate and the idiosyncratic shock occur:

$$U_{SS} = E_{\tilde{y}} \left[u \left(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2) + h^i (1 - \pi) g \right) \right]$$

When no idiosyncratic shock occurs, the income of city residents increases by the same amount as the city rent. This explains why city earnings in period 2 are identical in the first two expressions above. When an idiosyncratic shock occurs, agents face the possibility of reduced earnings in a state when second period rents are high because of the aggregate shock. Such a realization makes housing a useful asset to own because it delivers gains

$(1 - \pi)g$ at the time when the agents faces the risk of a decrease in earnings, at the cost of losses in states when the agents face constant or increased earnings.

To decide how much housing to buy, agent i solves

$$\max_{h^i} (1 - \pi)U_{NN} + \pi(1 - \gamma)U_{SN} + \pi\gamma U_{SS}$$

Proposition 1 *Given $0 < N_1 \leq N_2 < 1$, there is a unique market equilibrium with the following properties:*

(i) *In period t , agent i lives in the City if and only if $y_t^i \geq 1 - N_t$;*

(ii) *An agent with first-period income y^i buys \hat{h}^i units of housing, where \hat{h}^i is the unique solution of*

$$-U'_{NN}(\hat{h}^i, y^i) + (1 - \gamma)U'_{SN}(\hat{h}^i, y^i) + \gamma U'_{SS}(\hat{h}^i, y^i) = 0,$$

with

$$\begin{aligned} U'_{NN}(h^i, y^i) &= u'(\max(y^i - 1 + N_1, 0) + \max(y^i - 1 + N_2, 0) - h^i \pi g) \\ U'_{SN}(h^i, y^i) &= u'(\max(y^i - 1 + N_1, 0) + \max(y^i - 1 + N_2, 0) + h^i(1 - \pi)g) \\ U'_{SS}(h^i, y^i) &= E_{\tilde{y}^i} [u'(\max(y^i - 1 + N_1, 0) + \max(\tilde{y}^i - 1 + N_2, 0) + h^i(1 - \pi)g)] \end{aligned}$$

We can now characterize the comparative statics of the market equilibrium:

Proposition 2 *In equilibrium:*

(1) *Housing investment is nondecreasing in y^i (strictly increasing for $y^i > 1 - N_2$);*

(2) *There exists $y^* > 1 - N_2$ such that agents with $y^i < y^*$ choose $h^i < 0$ and agents with $y^i > y^*$ choose $h^i > 0$;*

(3) *If γ is sufficiently high with respect to g , $h^i > 1$ for all agents with $y^i > y^* + \varepsilon$, $\varepsilon > 0$;*

(4) *A marginal increase in N_2 induces agents with $y^i < 1 - N_2$ to buy less housing and agents with $y^i > 1 - N_2$ to buy more housing.*

(5) *A marginal increase in N_1 keeping $N_2 > N_1$ has no effect on housing investment decisions.*

To understand property (1), note first that if an agent chooses to live in the city in period 1, he remains in the city in period 2 unless he suffers an idiosyncratic income shock that decreases his period 2 productivity below the new rent. We saw above that investing in city housing generates a loss if no aggregate productivity shock occurs, and a gain otherwise. Investing in city housing therefore allows residents to transfer wealth from the state when no

aggregate shock occurs to the states when an aggregate shock occurs possibly concurrently with an idiosyncratic shock. The higher an agent's productivity in the first period, the greater the probability that an idiosyncratic shock will result in a loss of earnings, therefore the greater his demand for insurance against idiosyncratic shocks and the greater his housing investment.

Property (2) builds from the fact that city residents with the lowest productivity pay a rent equal to their earnings. They get not benefit from living and working in the city in period 1. They also get not benefit from the city in period 2 if no idiosyncratic shock occurs. If the idiosyncratic shock, any marginal agent who draws a lower productivity moves to the countryside and gets a surplus of zero. Any marginal agent who draws a higher productivity stays in the city and enjoys a positive surplus. Due to risk aversion, marginal city residents wants to shift resources away from the state in which an idiosyncratic shock occurs. He therefore sells city housing short in the first period. At the other extreme, the agents who start with the highest productivity can only loose from an idiosyncratic shock. To insure against this loss, they take a long position in city housing in period 1. By monotonicity of the optimal housing investment policy, there must be city residents who do not own any city housing. Any resident with lower productivity goes short on housing, and vice versa.

Property (3) exploits the fact that housing investment is increasing in the agent's productivity and in the probability they suffer an idiosyncratic shock. The greater an agent's productivity and the greater the probability of an idiosyncratic shock, the more the agent stands to loose if the idiosyncratic shock occurs. Therefore, the greater the hedging demand of this agent for housing, hence the greater his purchase of housing in period 1.

A higher N_2 yields a second period rent lower by the same amount in all states and a lower city housing price in the first period. A same quantity of housing investment in the first period therefore yields less resource transfers in the second period across states. Since a change in N_2 does not bring about a change in the income risk faced by agents, they need to take larger positions in the housing market. Agents who short the housing market take a bigger negative position. Agents who take a long position in the market buy more housing. This explains property (4).

Finally, property (5) follows from the fact that the housing supply in period 1, N_1 affects the rent in period 1 and the price in period 1. It does not change the extent to which housing investment allows agents to shift resources across states in period 2. This explains why per se a change in N_1 does not affect housing investment.

Example

We will use $N_1 = N_2 = \frac{1}{2}$, $\pi = \frac{1}{2}$, $g = \frac{1}{12}$, $\gamma = \frac{1}{2}$. The CARA coefficient is 2.

If $y^i \geq \frac{1}{2}$, the agent lives in the City and his marginal utilities are:

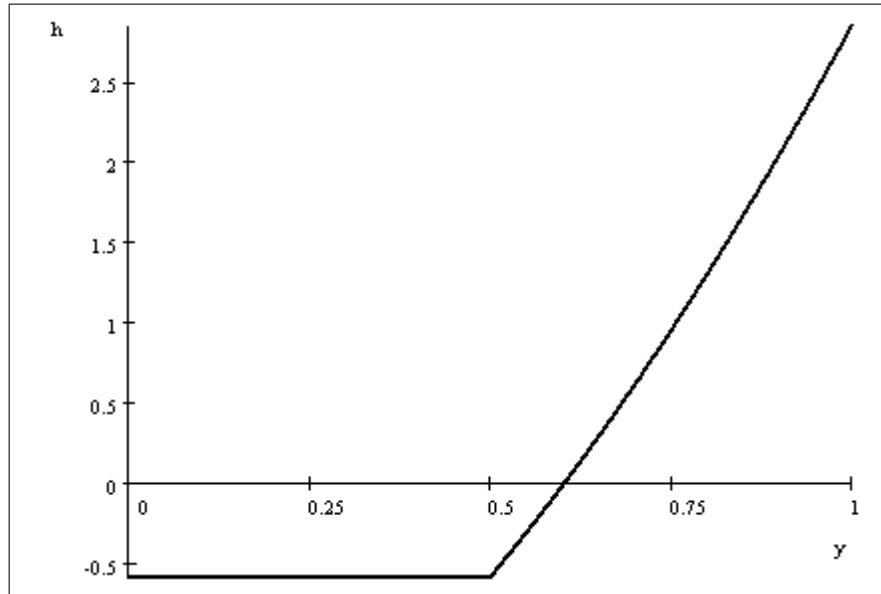
$$\begin{aligned} U'_{NN}(h^i, y^i) &= \exp \left[-2 \left(2 \left(y^i - \frac{1}{2} \right) - \frac{1}{24} h^i \right) \right] \\ U'_{SN}(h^i, y^i) &= \exp \left[-2 \left(2 \left(y^i - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] \\ U'_{SS}(h^i, y^i) &= \int_{1-N_2}^1 \exp \left[-2 \left(y^i - \frac{1}{2} + \max \left(0, \tilde{y}^i - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] d\tilde{y}^i \end{aligned}$$

The first-order condition for i is

$$\begin{aligned} 0 &= -\exp \left[-2 \left(2 \left(y - \frac{1}{2} \right) - \frac{1}{24} h^i \right) \right] + \frac{1}{2} \exp \left[-2 \left(2 \left(y - \frac{1}{2} \right) + \frac{1}{24} h^i \right) \right] \\ &\quad + \frac{1}{2} \exp \left[-2 \left(y - \frac{1}{2} + \frac{1}{24} h^i \right) \right] \left(1 - \frac{1}{2} \exp[-1] \right) \end{aligned}$$

If $y^i < \frac{1}{2}$, the agent lives in the Countryside and buys an amount of housing that is independent of y^i and equal to the amount of housing bought by the marginal city resident (If $y^i = \frac{1}{2}$, the optimal amount of housing is $h^i = -0.578$).

The optimal amount of housing is plotted below:



4 Endogenous Housing Supply

We now revert to the full-fledge game introduced in Section 2. Suppose the game has a pure-strategy equilibrium where voters select a level N_2 (we will see that the game does indeed have a pure-strategy equilibrium). We let \hat{N}_2 denote a possible deviation from N_2 . The agents take as given the equilibrium p_1 and r_1 that correspond to the supply N_2 . They internalize the fact that changing housing supply affects the rent in period 2 and the value of the housing permits.

The final wealth of agent i is

$$w^i = l_1^i \left(y_1^i + \phi\tau \frac{\hat{N}_2 - N_1}{N_1} \hat{b} - r_1 \right) + l_2^i (y_2^i - \hat{r}'_2) + h^i \left(\hat{r}'_2 + r_1 + \phi(1 - \tau) \frac{\hat{N}_2 - N_1}{N_1} \hat{b} - p_1 \right)$$

where $\hat{b} = 1 - \hat{N}_2 + \pi g$ and $\hat{r}_2 = 1 - \hat{N}_2$ if there is no aggregate shock, and $\hat{r}_2 = 1 - \hat{N}_2 + g$ otherwise.

Proposition 3 *Given deviation \hat{N}_2 , agent i 's final wealth is*

$$w^i = l_1^i (y_1^i - 1 + N_1 + \phi\tau\Omega) + l_2^i (y_2^i - 1 + \hat{N}_2) + h^i (D + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega),$$

where $D = -\pi g$ if there is no shock and $D = (1 - \pi)g$ if there is a shock, and Ω represents the effect of deviation to \hat{N}_2 on the total market value of new housing permits per unit of existing housing:

$$\Omega = \frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g).$$

To understand Proposition 3, note that on the equilibrium path $\hat{N}_2 = N_2$, $\Omega = 0$, and the expression for i 's wealth boils down to:

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i D.$$

In equilibrium, the number of new permits $N_2 - N_1$ is fully internalized in house prices and rents. The number of houses in the second period only affects agent i through its real effect: more people will be able to move to the city if N_2 goes up.

Off the equilibrium path, a deviation from the conjectured number of houses creates a potential windfall gain for city residents (in proportion $\phi\tau$) and home owners (in proportion $\phi(1 - \tau)$), but it also generates a price drop for home owners, which is captured by the term $N_2 - \hat{N}_2$.

We now need to resort to the technical assumption mentioned earlier. We assume that:

$$\phi(1-\tau)\frac{1}{N_1}(1-N_1+\pi g) < 1.$$

This restriction is satisfied if N_1 or τ are not too small or ϕ is not too large. For example, this condition is satisfied when none of the benefits from the building permits accrue to the residents or homeowners. The assumption guarantees that the benefit of a marginal deviation from N_2 to $N_2 + \epsilon$ is non-increasing in the amount of housing owned by agent i .⁹

With this assumption, the preferences of voters over \hat{N}_2 are single-peaked and we can apply Downs Theorem. The equilibrium amount of housing supply in period 2 corresponds to the \hat{N}_2 preferred by the median city resident, which we call m . The median city resident has the median income among city residents: i.e. $y^m = 1 - \frac{N_1}{2}$.

The final wealth of m is given by

$$w^m = \frac{N_1}{2} + \phi\tau\Omega + l_2^m \left(y_2^m - 1 + \hat{N}_2 \right) + h^m \left(D + N_2 - \hat{N}_2 + \phi(1-\tau)\Omega \right).$$

Proposition 4 *The necessary and sufficient conditions for the existence of an equilibrium in which $N_1 < N_2 < 1$ are:*

- (i) *The conditions for a market equilibrium*
- (ii) *the no-political-deviation condition:*

$$\left(h^m - \phi(\tau + h^m(1-\tau)) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2=N_2} \right) U' = U'_{city} \quad (2)$$

where

$$U' = (1-\pi)U'_{NN} + \pi(1-\gamma)U'_{SN} + \pi\gamma U'_{SS}$$

and

$$U'_{city} = (1-\pi)U'_{NN} + \pi(1-\gamma)U'_{SN} + \pi\gamma \int_{1-N_2}^1 u' \left(\frac{N_1}{2} + \phi\tau\Omega + \max(0, \tilde{y}^m - 1 + N_2) + h^m((1-\pi)g + \phi(1-\tau)\Omega) \right) d\tilde{y}^m$$

⁹To see this,

$$\frac{\partial^2 w^i}{\partial \hat{N}_2 \partial h^i} = -1 + \phi(1-\tau) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2=N_2},$$

and that

$$\frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2=N_2} = \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g) < \frac{1}{N_1} (1 - N_1 + \pi g).$$

The necessary and sufficient conditions for an equilibrium with $N_1 = N_2$ are as above but (2) is replaced with

$$\left(h^m - \phi(\tau + h^m(1 - \tau)) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2 = N_2} \right) U' \geq U'_{city}.$$

The equilibrium with endogenous housing supply must satisfy the conditions set out in Proposition 1. Every agent i must choose the optimal housing consumption and investment given the number of houses in the two periods, N_1 and N_2 . However, there is now an additional condition. The number of houses in the second period is endogenously determined by the preference of the median city resident. In turn, the median voter's preferences depend on the amount of housing h^m he owns. Thus, the conditions in (i) and (ii) constitute a system of equations which are necessary and sufficient.

The left-hand side of equation (2) captures the marginal cost in terms of utility from the capital gain/loss for the median voter as a result of an increase in the housing stock. There are two components: the term h^m is due the linear and negative effect of an increase in N_2 on $r_2 = 1 - N_2$; the other, more complex term consists of the revenue that accrues to home-owners and residents from the sale of housing permits. Part of the benefits accrue to the agents because they are city residents, $\phi\tau$. The remainder accrue to the agents as a proportion of the properties they own, $\phi h^m(1 - \tau)$. The right-hand side of the equation represents the marginal benefit of a change in city size computed at the equilibrium size. This benefit comes from two sources. First, a bigger city means a smaller second period rent. Second, a bigger city means a greater probability of earning a surplus from living and working in the city if the idiosyncratic shock occurs.

In an equilibrium where city size does not expand to its maximum value, the marginal benefit of increasing the size of the city cannot be greater than the marginal cost of increasing the size of the city. However, because citizens are not allowed to reduce the size of their city, it can happen that the marginal cost of city expansion exceeds the marginal benefit.

For future reference, we use the fact that agents have CARA utility to re-write the two first order conditions in a parametric form:

Proposition 5 *Let $a > 1$ be the risk-aversion coefficient. The first-order condition for a housing market equilibrium is:*

$$(1 - \gamma) \exp[-ah^m g] + \gamma \exp \left[-a \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2] \right) = 1,$$

and the first-order condition for a political equilibrium is

$$\begin{aligned} 1 - h^m + \phi(\tau + h^m(1 - \tau)) \left(\frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g) \right) \\ = \pi\gamma \exp \left[-a \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] (1 - N_2). \end{aligned}$$

5 Hysteresis in Housing Supply

We now ask how the second period equilibrium city size depends on the initial city size.

Proposition 6 *The equilibrium number of houses in the second period N_2 is a strictly increasing function of the number of houses in the first period N_1 .*

The proposition is proven in two steps. Start with N_1 : there are two cases. If $N_2 = N_1$ in equilibrium, then it is immediate that an increase in N_1 causes an increase in N_2 . If instead the initial N_1 yielded an interior solution, we can use the first-order conditions to determine the effect of an increase in N_1 . We know from point (5) of Proposition 2 that an increase in N_1 does not make agent i change his housing investment. However, an increase in N_1 changes the identity of the median city resident, who is now poorer and buys less housing. Hence, the voting outcome changes and allows for a strictly higher N_2 .

This proposition captures a phenomenon of great practical importance. Consider a community that, for exogenous reasons (natural barriers) or past events (a period of very fast local growth), is today characterized by a low housing supply (recall that low housing supply in the model is equivalent to high housing price to income ratio). Our analysis predict that this unbalance will persist over time. Its residents choose to be highly invested in local housing and will vote against growth.

This observation should guide empirical work: if we observe a strong correlation between the existence of natural barriers and house prices, we should not infer that high prices are due exclusively to the barriers. The vicious cycle between ownership and supply plays a role. This is important for policy purposes: as the next section argues, we can find a number of instruments to mitigate this effect.

Example of Hysteresis

Before moving to the analysis of institutional reform, we briefly discuss a numerical example of hysteresis. Assume that

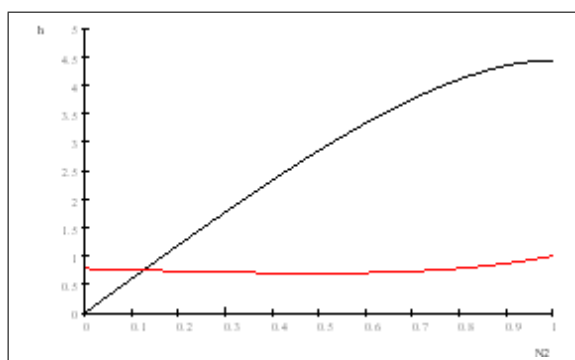
$$a = 2, \gamma = \frac{1}{2}, g = \frac{1}{12}, \phi = 0, \pi = \frac{1}{2}.$$

Then, the two first-order conditions become

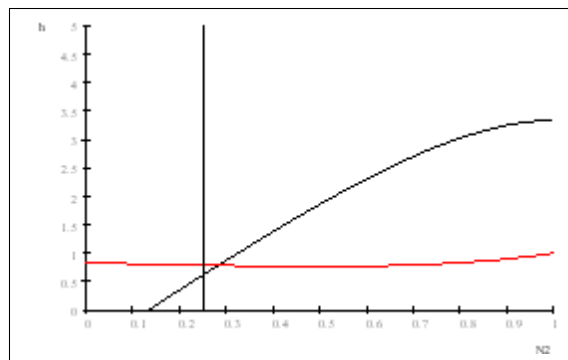
$$-1 + \frac{1}{2} \exp \left[-2 \left(\frac{1}{12} h \right) \right] + \frac{1}{2} \exp \left[-2 \left(\frac{n}{2} - N + \frac{1}{12} h \right) \right] \left(\frac{3}{2} - N - \frac{1}{2} \exp [-2N] \right) = 0,$$

$$1 - h = \frac{1}{4} \exp \left[-2 \left(\frac{n}{2} - N + \frac{1}{12} h \right) \right] (1 - N).$$

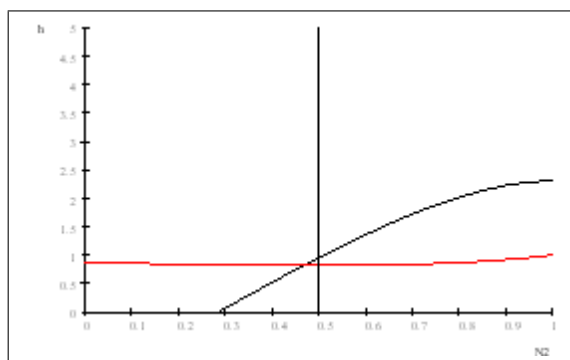
We now plot the locus of the market equilibrium (black line) and the locus of the political equilibrium (red line) for various values of N_1 . The intersection of the two loci is the solution to the unconstrained problem (disregarding the requirement that $N_2 \geq N_1$). If the intersection is to the right of N_1 , then it is the solution of the problem. If it is to the left, then the solution is given by the intersection of N_1 and the market equilibrium locus.



$N_1 = 0$



$N_1 = \frac{1}{4}$



$N_1 = \frac{1}{2}$

6 Institutional Reform

This section uses the model to study the effects of a number of policies that have been proposed.

6.1 Housing Permit Allocation

The first institutional feature is, in our opinion, the most important. We shall now ask how the distribution of windfall gains affect equilibrium housing supply.

Proposition 7 *Start from an equilibrium where the sale of housing permit does not benefit citizens directly ($\phi = 0$) and new housing supply is restricted ($N_2 < \frac{1+N_1}{2}$). Then, a marginal increase in ϕ causes an increase in housing supply.*

Now, compare the effect of an increase in ϕ when owners get all the benefit ($\tau = 0$) or residents get all the benefit ($\tau = 1$). Start from an equilibrium where $\phi = 0$, $N_2 < \frac{1+N_1}{2}$ and the median voter owns \hat{h}^m units of housing. A marginal increase in ϕ causes an increase in housing supply that is greater when $\tau = 0$ rather than $\tau = 1$ if and only if $\hat{h}^m > 1$.

This result will come as no surprise to political economists. Giving the median voter a larger share of housing windfalls (either as a resident or as a homeowner) will make him want to vote for more urban growth. As we argued in the Introduction, what is more surprising is that this simple point still escapes most policy makers. The most extreme case is probably Britain. The history of urban planning in the UK starting from the Town and Country Planning Act of 1949 is characterized by an attempt to strip local authorities of any benefit connected with new construction. The latest proposal (Barker, 2005) continues in this tradition.

Example

We now present a simple example to illustrate the effect of institutional changes in the distribution of windfall gains. As before, we will use $N_1 = \frac{1}{2}$, $\pi = \frac{1}{2}$, $g = \frac{1}{12}$, $\gamma = \frac{1}{2}$. The CARA coefficient is 2.

The median city resident has income $y^m = \frac{3}{4}$.

We shall look for the set of parameters (ϕ, τ) such that no new houses are built in equilibrium: $N_2 = N_1 = \frac{1}{2}$. In such an equilibrium, the housing decision of the median voter is given by (see Example in previous section):

$$-\exp\left[-2\left(\frac{1}{2} - \frac{h}{24}\right)\right] + \frac{1}{2}\exp\left[-2\left(\frac{1}{2} + \frac{h}{24}\right)\right] + \frac{1}{2}\exp\left[-2\left(\frac{1}{4} + \frac{h}{24}\right)\right] \left(1 - \frac{1}{2}\exp[-1]\right) = 0$$

with solution $\hat{h}^m = 0.956$.

The condition for $\hat{N}_2 = \frac{1}{2}$ is that the marginal utility of issuing an extra permit is negative for the median city resident. The political equilibrium condition is:

$$\begin{aligned} & \left(\exp \left[-2 \left(\frac{1}{2} - \frac{\hat{h}^m}{24} \right) \right] + \frac{1}{2} \exp \left[-2 \left(\frac{1}{2} + \frac{\hat{h}^m}{24} \right) \right] \right. \\ & \quad \left. + \frac{1}{2} \exp \left[-2 \left(\frac{1}{4} + \frac{\hat{h}^m}{24} \right) \right] \left(1 - \frac{1}{2} \exp[-1] \right) \right) K \\ & \leq \frac{1}{4} \exp \left[-2 \left(\frac{1}{4} + \frac{\hat{h}^m}{24} \right) \right]. \end{aligned}$$

Substitute $\hat{h}^m = 0.956$ and solve

$$\begin{aligned} & \left(\exp \left[-2 \left(\frac{1}{2} - \frac{0.956}{24} \right) \right] + \frac{1}{2} \exp \left[-2 \left(\frac{1}{2} + \frac{0.956}{24} \right) \right] \right. \\ & \quad \left. + \frac{1}{2} \exp \left[-2 \left(\frac{1}{4} + \frac{0.956}{24} \right) \right] \left(1 - \frac{1}{2} \exp[-1] \right) \right) K \\ & = \frac{1}{4} \exp \left[-2 \left(\frac{1}{4} + \frac{0.956}{24} \right) \right]. \end{aligned}$$

where $K = \phi \left(\tau + \hat{h}^m (1 - \tau) \right) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2 = N_2} + 1 - \hat{h}^m$. This yields $\bar{K} = 0.175$. We have that $\hat{N}_2 = \frac{1}{2}$ if and only if $K \leq \bar{K}$.

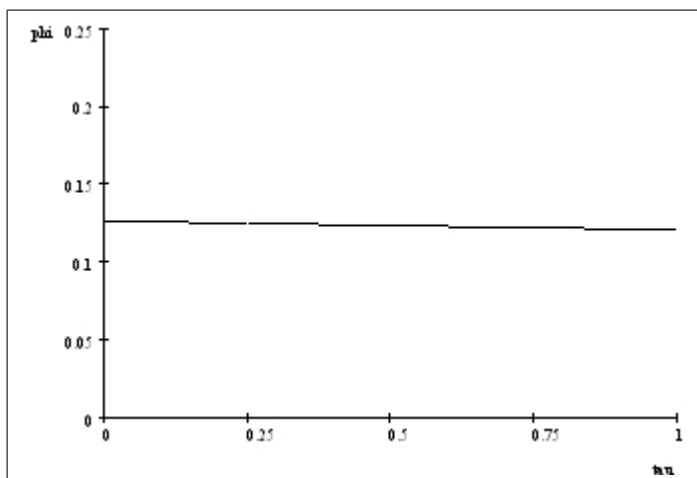
Now note that

$$\begin{aligned} \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2 = N_2} &= \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g) \\ &= 2 \left(1 + \frac{1}{2} - 1 + \frac{1}{24} \right) = \frac{13}{12} \end{aligned}$$

Hence,

$$\begin{aligned} K &= \phi \left(\tau + \hat{h}^m (1 - \tau) \right) \frac{\partial \Omega}{\partial \hat{N}_2} \Big|_{\hat{N}_2 = N_2} + 1 - \hat{h}^m \\ &= \frac{13}{12} \phi (\tau + 0.956 (1 - \tau)) + 1 - 0.956 \\ &= \begin{cases} 0.044 & \text{if } \phi = 0 \\ 0.044 + \frac{13}{12} \phi & \text{if } \tau = 1, \text{ implying that } K < \bar{K} \text{ if } \phi < 0.168 \\ 0.044 + \frac{13}{12} \phi (0.9561) & \text{if } \tau = 0, \text{ implying that } K < \bar{K} \text{ if } \phi < 0.134 \end{cases} \end{aligned}$$

The set of (ϕ, τ) pairs for which no new houses are built is the one under this curve. The effect of ϕ is clear: if it is low, the median resident has more costs than benefits from new housing. The effect of τ is very small: that's because the median voter's housing is almost 1 (if it were exactly 1, τ has no effect).



6.2 Incentives for Home Ownership

Suppose that the tax system creates distortions between renting and buying. What happens in the second period is irrelevant because it gets discounted. Suppose that there is a tax or a subsidy on house purchases on the part of citizens (but not REITs). For every dollar of housing purchased the state offers a subsidy of s cents (or a tax if s is negative).

Assume that $\phi = 0$. The final wealth is now

$$w^i = l_1^i (y_1^i - r_1) + l_2^i (y_2^i - r_2) + h^i (r_2 + r_1 - (1 - s)p_1).$$

That is,

$$w^i = l_1^i (y_1^i - 1 + N_1) + l_2^i (y_2^i - 1 + N_2) + h^i (D + s(2 - N_1 - N_2)).$$

It is easy to see that, ceteris paribus, a higher s increases h^i . Hence, in equilibrium the median voters buys more housing and votes for less housing expansion.

Proposition 8 *A subsidy (tax) on house ownership reduces housing supply.*

Most Western countries have a tax regimes that favor owning over renting. Apart from other documented distortions, we find that these regime generate less housing supply.

6.3 Level at which Planning Decisions are Made

We have assumed that housing supply is decided by city residents. Suppose that all citizens vote on housing supply; i.e., the political process takes into consideration not only the interest of city residents but also those of potential future residents of the city. By proposition

2, the amount of housing is an increasing function of productivity. The median citizen owns less housing than the median city residents. Hence

Proposition 9 *If city planning decisions are made at a national level, housing supply in the city increases*

However, the relation between planning centralization and housing supply is U-shaped. To see this, suppose that the current city is divided up into m independent cities. The cities are initially identical: each with a stock of housing $\frac{N_1}{m}$. The model is as before but now at the end of periods 1 each of the m cities hold a separate election to decide the local housing supply. If $\phi = 0$, there is no difference: the benefit from a deviation is unchanged and the equilibrium is exactly as in Proposition 4. If however $\phi > 0$, things change. If the city is smaller, the median voter in m gets a larger share the revenues accruing from selling construction licenses in his city. Formally, the marginal benefit of an extra license becomes

$$\left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} = \frac{m}{N_1} \left(1 + \frac{N_1}{m} - 2N_2 + \pi g \right),$$

which is increasing in m (and tends to 1 when the number of cities tends to infinity). It is then immediate from the first-order political economy condition of Proposition 4 that the second period supply must increase (in an interior equilibrium):

Proposition 10 *If city planning decisions are made at a more decentralized level, housing supply in the city increases.*

6.4 Barriers to Fractional Ownership

In practice, there are serious barriers to fractional ownership. Caplin et al. (1997) have argued that this imposes a cost on households. Our model assumed that fractional ownership is allowed. In this section, we shall assume that fractional ownership is impossible.

Suppose $h^i \in \{0, 1\}$. Our first-order condition on the amount of housing owned by the median voter is replaced by a comparison between the level of wealth that the median voter gets if he buys a house rather than no house.

Proposition 11 *If allowing for fractional ownership reduces (increases) the amount of housing that the median voter owns, housing supply increases (decreases).*

7 Conclusions

We present a theoretical framework focused on the interactions between households' labor income risk, their housing consumption and investment decision and their support for urban growth restrictions. The model highlights the role of homeownership as a hedge against the risk of being left behind.

The model provides insights into the links between housing supply constraints and labor income volatility. When aggregate productivity growth brings about more individual income uncertainty, households have a stronger incentive to own housing and to vote for tighter restrictions on the growth of their city. Gottschalk and Moffitt (1994) report evidence of increasing volatility in US worker's earnings starting in the Seventies. It is interesting to note the coincidence in timing with the increase in growth controls in some US cities.

To make sense of housing price growth differences across US metropolitan areas since the Seventies, we note that the model predicts differences in voting outcome according to the initial housing price to income ratio. The more expensive markets end up with the tighter growth restrictions in the model. California was already an expensive housing market in the Seventies. It ranked in the top three states in the US in terms of median housing price to household income ratio in 1970.

The model provides insights into the interaction between housing supply constraints and policies that promote homeownership. The more housing households own, the greater their incentive to protect their investment through city growth restrictions. It might be worth noting that Australia, the UK and the US have much higher ownership rates than Germany and Switzerland and have experienced much stronger housing price growth since the Seventies (Evans and Hartwich, 2005).

The political consensus, at least in the US, is that homeownership should be encouraged because it generates better socio-economic outcomes for local residents. Our results shows that this objective is in potential conflict with the other often-cited goal of housing policy: affordability.

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Appendix

Proof of Proposition 1

The first-order condition on h^i is

$$-(1-\pi)\pi gU'_{NN} + \pi(1-\gamma)(1-\pi)gU'_{SN} + \pi\gamma(1-\pi)gU'_{SS} = 0.$$

We write

$$\Psi(h^i, y^i) \equiv -U'_{NN} + (1-\gamma)U'_{SN} + \gamma U'_{SS}.$$

The first-order condition is satisfied if and only if $\Psi(h^i, y^i) = 0$. Note also that

$$\Psi_h(h^i, y^i) = \pi gU''_{NN} + (1-\pi)g(1-\gamma)U''_{SN} + (1-\pi)g\gamma U''_{SS} < 0.$$

Hence, for every y^i there exists a unique h_1 such that $\Psi(h^i, y^i) = 0$. ■

Proof of Proposition 2

To prove (1), note that

$$\Psi_y(h^i, y^i) = \begin{cases} 0 & \text{if } y^i < 1 - N_2 \\ -U''_{NN} + (1-\gamma)U''_{SN} & \text{if } 1 - N_2 < y^i < 1 - N_1 \\ -2U''_{NN} + 2(1-\gamma)U''_{SN} + \gamma U''_{SS} & \text{if } y^i > 1 - N_1 \end{cases}$$

Re-write the last expression as

$$\Psi_y(h^i, y^i) = 2\left(-\frac{U''_{NN}}{U'_{NN}}\right)U'_{NN} - 2(1-\gamma)\left(-\frac{U''_{SN}}{U'_{SN}}\right)U'_{SN} - \gamma\left(-\frac{U''_{SS}}{U'_{SS}}\right)U'_{SS}.$$

As the utility is CARA, we can write

$$\begin{aligned} \Psi_y(h^i, y^i) &\propto 2U'_{NN} - 2(1-\gamma)U'_{SN} - \gamma U'_{SS} \\ &= U'_{NN} - (1-\gamma)U'_{SN} \\ &= \gamma U'_{SS} > 0 \end{aligned}$$

where the two equalities are due to the foc. Similarly, the second expression can be rewritten as

$$\Psi_y(h^i, y^i) = U'_{NN} - (1-\gamma)U'_{SN} = \gamma U'_{SS} > 0.$$

Recall that Ψ_h was derived above and was found to be always negative. By the implicit function theorem,

$$\frac{\partial h^i}{\partial y^i} = -\frac{\Psi_y(h^i, y^i)}{\Psi_h(h^i, y^i)} \begin{cases} = 0 & \text{if } y^i < 1 - N_2 \\ > 0 & \text{if } y^i > 1 - N_2 \end{cases}$$

To prove (2), examine

$$\Psi(0, y^i) \equiv -U'_{NN}(0, y^i) + (1-\gamma)U'_{SN}(0, y^i) + \gamma U'_{SS}(0, y^i).$$

Note that

$$\begin{aligned} U'_{NN}(0, y^i) &= u'(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2)) \\ U'_{SN}(0, y^i) &= u'(\max(0, y^i - 1 + N_1) + \max(0, y_1^i - 1 + N_2)) \\ U'_{SS}(0, y^i) &= E_{\tilde{y}}[u'(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2))] \end{aligned}$$

Recall that $\Psi_y(h^i, y^i)$ is non-decreasing in y^i . First, if $y^i < 1 - N_2$,

$$\begin{aligned} U'_{NN}(0, y^i) &= u'(0) \\ U'_{SN}(0, y^i) &= u'(0) \\ U'_{SS}(0, y^i) &= E_{\tilde{y}^i} [u'(\max(0, \tilde{y}^i - \bar{y}_{N_2}))] < u'(0) \end{aligned}$$

Hence, if $y^i < 1 - N_2$,

$$\Psi(0, y^i) < -u'(0) + (1 - \gamma)u'(0) + \gamma u'(0) = 0.$$

Second, it is easy to see that there exists a threshold such that, for a higher y^i ,

$$\begin{aligned} &u'(\max(0, y^i - 1 + N_1) + \max(0, y_1^i - 1 + N_2)) \\ &< E_{\tilde{y}} [u'(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2))]. \end{aligned}$$

In that case, $\Psi(0, y^i) > 0$. As $\Psi(0, y^i)$ is non-decreasing in y^i , there exists a threshold $y^* \geq 1 - N_2$ such that $\Psi(0, y^i) \geq 0$ if and only if $y^i \geq y^*$. This means that agents below (above) y^* choose a negative (positive) amount of housing.

Now, turn to (3). For any positive ε , consider an agent with $y^i = y^* + \varepsilon$. Consider $h^i = 1$ and note that

$$\begin{aligned} U_{NN}(1, y^i) &= u(\max(0, y^i - 1 + N_1) + \max(0, y^i - 1 + N_2) - \pi g) \\ U_{SN}(1, y^i) &= u(\max(0, y^i - 1 + N_1) + \max(0, y_1^i - 1 + N_2) + (1 + \pi)g) \\ U_{SS}(1, y^i) &= E_{\tilde{y}^i} [u(\max(0, y^i - 1 + N_1) + \max(0, \tilde{y}^i - 1 + N_2) + (1 + \pi)g)] \end{aligned}$$

Note that $U_{NN}(1, y^i)$, $U_{SN}(1, y^i)$, and $U_{SS}(1, y^i)$ do not depend on γ , and that

$$U'_{NN}(1, y^* + \varepsilon) < U'_{SN}(1, y^* + \varepsilon).$$

Hence, for γ sufficiently high,

$$\Psi(1, y^* + \varepsilon) > 0,$$

and the optimal h^i must be greater than 1.

To prove (4), consider

$$\begin{aligned} \Psi_{N_2}(h^i, y^i) &= -U''_{NN} + (1 - \gamma)U''_{SN} \\ &\quad + \gamma \int_{1-N_2}^1 u''(\max(0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi)g) d\tilde{y}^i \quad \text{if } y_1^i > 1 - N_2 \end{aligned}$$

and

$$\Psi_{N_2}(h^i, y^i) = \gamma \int_{1-N_2}^1 u''(\max(0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi)g) d\tilde{y}^i \quad \text{if } y_1^i < 1 - N_2.$$

If $y_1^i > 1 - N_2$, CARA implies

$$\begin{aligned} \Psi_{N_2}(h^i, y^i) &\propto U'_{NN} - (1 - \gamma)U'_{SN} - \gamma \int_{1-N_2}^1 u'(\max(0, y^i - 1 + N_1) + \tilde{y}^i - 1 + N_2 + (1 + \pi)g) d\tilde{y}^i \\ &> U'_{NN} - (1 - \gamma)U'_{SN} - \gamma U'_{SS} = 0. \end{aligned}$$

Instead, if $y_1^i < 1 - N_2$, $\Psi_{N_2}(h^i, y^i) < 0$. We then have

$$\frac{\partial h^i}{\partial N_2} \begin{cases} < 0 & \text{if } y^i > 1 - N_2 \\ > 0 & \text{if } y^i < 1 - N_2 \end{cases}$$

To prove (5), note that, if $y^i < 1 - N_1$, $\Psi_{N_1}(h^i, y^i) = 0$. If $y^i > 1 - N_1$,

$$\Psi_{N_1}(\hat{h}^i, y^i) = -U''_{NN}(\hat{h}^i, y^i) + (1 - \gamma)U''_{SN}(\hat{h}^i, y^i) + \gamma U''_{SS}(\hat{h}^i, y^i),$$

which, by CARA, can be re-written as $\Psi(\hat{h}^i, y^i)$ and, by the first-order condition, is equal to zero. ■

Proof of Proposition 3

Given the conjectured level of N_2 , the agents face first period rent and price as follows:

$$r_1 = 1 - N_1 + \phi\tau \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g)$$

$$p_1 = 1 - N_1 + \left(1 + \phi \frac{N_2 - N_1}{N_1}\right) (1 - N_2 + \pi g)$$

The payoff of living and working in the city for agents y^i assuming a deviation to \hat{N}_2 is

$$\begin{aligned} & y_1^i + \phi\tau \frac{\hat{N}_2 - N_1}{N_1} \tilde{b} - r_1 \\ = & y_1^i - 1 + N_1 + \phi\tau \left(\frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g) \right) \end{aligned}$$

We denote Ω the change in the total value of the housing permits per unit of existing housing due to a political deviation from N_2 to \hat{N}_2 :

$$\Omega \equiv \frac{\hat{N}_2 - N_1}{N_1} (1 - \hat{N}_2 + \pi g) - \frac{N_2 - N_1}{N_1} (1 - N_2 + \pi g).$$

The expected gain from investing in a unit of city housing is affected by a deviation from N_2 to \hat{N}_2 as follows

$$\tilde{r}_2 + r_1 + \phi(1 - \tau) \frac{\hat{N}_2 - N_1}{N_1} \tilde{b} - p_1 = D + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega$$
■

Proof of Proposition 4

The median city resident decides the election. His wealth in the three possible outcomes is

- If there is no shock:

$$\begin{aligned} w_{NN} &= y^m - 1 + N_1 + \phi\tau\Omega + y^m - 1 + \hat{N}_2 + h^m \left(-\pi g + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega \right) \\ &= \hat{N}_2 + \phi\tau\Omega + h^m \left(-\pi g + N_2 - \hat{N}_2 + \phi(1 - \tau) \Omega \right) \end{aligned}$$

- If the aggregate shock alone occurs

$$w_{SN} = \hat{N}_2 + \phi\tau\Omega + h^m \left((1 - \pi)g + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega \right)$$

- If both the aggregate and the idiosyncratic shock occur

$$w_{SS} = \frac{N_1}{2} + \phi\tau\Omega + \max\left(0, \tilde{y}^m - 1 + \hat{N}_2\right) + h^m \left((1 - \pi)g + N_2 - \hat{N}_2 + \phi(1 - \tau)\Omega \right)$$

Let

$$U^m = (1 - \pi)u(w_{NN}) + \pi(1 - \gamma)u(w_{SN}) + \pi\gamma u(w_{SS})$$

The condition for N_2 to be the equilibrium choice is that

$$\left. \frac{dU^m}{d\hat{N}_2} \right|_{N_2=\hat{N}_2} = 0$$

That is

$$\frac{dU^m}{d\hat{N}_2} = (1 - \pi)u'(w_{NN})\frac{dw_{NN}}{d\hat{N}_2} + \pi(1 - \gamma)u'(w_{SN})\frac{dw_{SN}}{d\hat{N}_2} + \pi\gamma E \left[u'(w_{SS})\frac{dw_{SS}}{d\hat{N}_2} \right].$$

Note that

$$\begin{aligned} \frac{dw_{NN}}{d\hat{N}_2} &= \frac{dw_{SN}}{d\hat{N}_2} = 1 + \phi\tau\frac{d\Omega}{d\hat{N}_2} + h^m \left(-1 + \phi(1 - \tau)\frac{d\Omega}{d\hat{N}_2} \right) \\ &= 1 - K \end{aligned}$$

where

$$K = h^m - \phi(\tau + h^m(1 - \tau))\frac{d\Omega}{d\hat{N}_2}$$

and

$$\begin{aligned} \frac{dw_{SS}}{d\hat{N}_2} &= \phi\tau\frac{d\Omega}{d\hat{N}_2} - \frac{d}{d\hat{N}_2} \max(0, \tilde{y}^m - 1 - N_2) + h^m \left(-1 + \phi(1 - \tau)\frac{d\Omega}{d\hat{N}_2} \right) \\ &= \phi\tau\frac{d\Omega}{d\hat{N}_2} + I_{\tilde{y}^m > 1 - \hat{N}_2} + h^m \left(-1 + \phi(1 - \tau)\frac{d\Omega}{d\hat{N}_2} \right) \\ &= \begin{cases} 1 - K & \text{if } \tilde{y}^m > 1 - \hat{N}_2 \\ -K & \text{if } \tilde{y}^m < 1 - \hat{N}_2 \end{cases} \end{aligned}$$

Note also that

$$\left. \frac{\partial\Omega}{\partial\hat{N}_2} \right|_{\hat{N}_2=N_2} = \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g).$$

Hence

$$\begin{aligned}
& \left. \frac{dU^m}{d\hat{N}_2} \right|_{\hat{N}_2=N_2} \\
&= (1-\pi) u'(w_{NN})(1-K) + \pi(1-\gamma) u'(w_{SN})(1-K) + \pi\gamma \int_{1-N_2}^1 u'(w_{SS}(h_m, \tilde{y}^m))(1-K) d\tilde{y}^m \\
&\quad + \pi\gamma \int_0^{1-N_2} u'(w_{SS}(h_m, 0))(-K) d\tilde{y}^m \\
&= -((1-\pi) u'(w_{NN}) + \pi(1-\gamma) u'(w_{SN}) + \pi\gamma u'(w_{SS})) K \\
&\quad + (1-\pi) u'(w_{NN}) + \pi(1-\gamma) u'(w_{SN}) + \pi\gamma \int_{1-N_2}^1 u'(w_{SS}(h_m, \tilde{y}^m)) d\tilde{y}^m
\end{aligned}$$

■

Proof of Proposition 5

Start with the condition for a political equilibrium:

$$\left(h^m - \phi(\tau + h^m(1-\tau)) \left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} \right) U' = U'_{\text{city}} \quad (3)$$

First, note that the first-order condition on the housing market implies

$$\begin{aligned}
U' &= (1-\pi) U'_{NN} + \pi(1-\gamma) U'_{SN} + \pi\gamma U'_{SS} \\
&= U'_{NN} \\
&= \exp[-a(N_2 - h^m \pi g)].
\end{aligned}$$

Next,

$$\begin{aligned}
U'_{\text{city}} &= (1-\pi) U'_{NN} + \pi(1-\gamma) U'_{SN} + \pi\gamma \int_{1-N_2}^1 u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m \\
&= U'_{NN} - \pi\gamma \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m.
\end{aligned}$$

Note that

$$\begin{aligned}
& \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp[-a(y^m - 1 + N_1 + h^m(1-\pi)g)] d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] (1-N_2)
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{U'_{\text{city}}}{U'} &= \frac{U'_{NN} - \pi\gamma \int_0^{1-N_2} u'_{SS}(w_{SS}(h^m, \tilde{y}^m)) d\tilde{y}^m}{U'_{NN}} \\
&= \frac{\exp[-a(N_2 - h^m\pi g)] - \pi\gamma \exp[-a(\frac{N_1}{2} + h^m(1-\pi)g)](1-N_2)}{\exp[-a(N_2 - h^m\pi g)]} \\
&= 1 - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right](1-N_2)
\end{aligned}$$

Thus, the political equilibrium condition is:

$$h^m - \phi(\tau + h^m(1-\tau)) \left. \frac{\partial \Omega}{\partial \tilde{N}_2} \right|_{\tilde{N}_2=N_2} = 1 - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right](1-N_2),$$

which can immediately be re-written as in the statement of the proposition.

The other first-order condition is:

$$-U'_{NN} + (1-\gamma)U'_{SN} + \gamma U'_{SS} = 0$$

where

$$\begin{aligned}
U'_{NN} &= \exp[-a(N_2 - h^m\pi g)] \\
U'_{SN} &= \exp[-a(N_2 + h^m(1-\pi)g)] \\
U'_{SS} &= \int_0^1 \exp[-a((y^m - 1 + N_1) + \max(\tilde{y}^m - 1 + N_2, 0) + h^m(1-\pi)g)] d\tilde{y}^m \\
&= \exp[-a((y^m - 1 + N_1) + h^m(1-\pi)g)] \int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-2N_2]\right)
\end{aligned}$$

because

$$\begin{aligned}
&\int_0^1 \exp[-a \max(\tilde{y}^m - 1 + N_2, 0)] d\tilde{y}^m \\
&= \int_0^{1-N_2} \exp[0] d\tilde{y}^m + \int_{1-N_2}^1 \exp[-a(\tilde{y}^m - 1 + N_2)] d\tilde{y}^m \\
&= 1 - N_2 - \frac{1}{2} \exp[-a(1-1+N_2)] + \frac{1}{2} \exp[-a(1-N_2-1+N_2)] \\
&= 1 - N_2 - \frac{1}{2} \exp[-aN_2] + \frac{1}{2} \\
&= \frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]
\end{aligned}$$

which re-writes as

$$\begin{aligned}
&-\exp[-a(N_2 - h^m\pi g)] + (1-\gamma) \exp[-a(N_2 + h^m(1-\pi)g)] \\
&+ \gamma \exp\left[-a\left(\frac{N_1}{2} + h^m(1-\pi)g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\
&= 0
\end{aligned}$$

and can be simplified to

$$-1 + (1 - \gamma) \exp[-ah^m g] + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) = 0.$$

■

Proof of Proposition 6

Starting from the first-order conditions in Proposition 5, let

$$\begin{aligned} f &= -1 + (1 - \gamma) \exp[-ah^m g] + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ g &= (1 - h^m) - \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) \end{aligned}$$

Compute partial derivatives and simplify using the first-order conditions:

$$\begin{aligned} f_h &= -ag(1 - \gamma) \exp[-ah^m g] - ag\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ &= -ag < 0 \end{aligned}$$

$$\begin{aligned} g_h &= -1 + ag\pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) \\ &= -1 + ag(1 - h^m) = -1 + ag - agh^m < 0 \text{ since } g < 1/a \end{aligned}$$

$$\begin{aligned} f_{N_2} &= a\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) \\ &\quad + \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (-1 + \exp[-aN_2]) \\ &= \gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(a\left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) + (-1 + \exp[-aN_2])\right) \\ &= a\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) > 0 \end{aligned}$$

$$\begin{aligned} g_{N_2} &= -a\pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) + \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \\ &= \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (-a(1 - N_2) + 1) \\ &= \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (aN_2 - a + 1) \leq 0 \\ &\quad \exp\left[-2\left(\frac{n}{2} - N + \frac{1}{12}h\right)\right] (2N - 1) \end{aligned}$$

$$f_{N_1} = -\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-aN_2]\right) < 0$$

$$g_{N_1} = \pi\gamma \exp\left[-a\left(\frac{N_1}{2} - N_2 + h^m g\right)\right] (1 - N_2) > 0$$

The implicit function theorem says that

$$\begin{bmatrix} \frac{\partial h^m}{\partial N_1} \\ \frac{\partial N_2}{\partial N_1} \end{bmatrix} = \begin{bmatrix} f_h & f_{N_2} \\ g_h & g_{N_2} \end{bmatrix}^{-1} \begin{bmatrix} -f_{N_1} \\ -g_{N_1} \end{bmatrix}$$

That is

$$\begin{bmatrix} \frac{\partial h^m}{\partial N_1} \\ \frac{\partial N_2}{\partial N_1} \end{bmatrix} = \begin{bmatrix} -\frac{g_{N_2}}{\Delta} & \frac{f_{N_2}}{\Delta} \\ \frac{g_h}{\Delta} & -\frac{f_h}{\Delta} \end{bmatrix} \begin{bmatrix} -f_{N_1} \\ -g_{N_1} \end{bmatrix}$$

where $\Delta = -f_h g_{N_2} + g_h f_{N_2}$ is the Jacobian, which must be negative if the second-order condition is satisfied. Then

$$\begin{aligned} \frac{\partial N_2}{\partial N_1} &= -\frac{g_h}{\Delta} f_{N_1} + \frac{f_h}{\Delta} g_{N_1} \\ &= \frac{1}{(-)} (-(-)(+) + (-)(-)) > 0 \end{aligned}$$

This shows that the unconstrained N_2 is increasing in N_1 . If we denote the unconstrained value with \bar{N}_2 , it is immediate to see that the constrained value $\max(\bar{N}_2, N_1)$ is increasing in N_1 a fortiori. ■

Proof of Proposition 7

Start with

$$\left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} = \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g)$$

$$\begin{aligned} f &= -1 + (1 - \gamma) \exp[-2h^m g] + \gamma \exp \left[-2 \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] \left(\frac{3}{2} - N_2 - \frac{1}{2} \exp[-2N_2] \right) \\ g &= \left(1 - h^m + \phi \left(\tau + \hat{h}^m (1 - \tau) \right) \left. \frac{\partial \Omega}{\partial \hat{N}_2} \right|_{\hat{N}_2=N_2} \right) - \pi \gamma \exp \left[-2 \left(\frac{N_1}{2} - N_2 + h^m g \right) \right] (1 - N_2) \end{aligned}$$

Assume that $\phi = 0$ and look at the effect of an increase in ϕ .

By the Implicit Function Theorem

$$\begin{bmatrix} \frac{\partial h^m}{\partial \phi} \\ \frac{\partial N_2}{\partial \phi} \end{bmatrix} = \begin{bmatrix} -\frac{g_{N_2}}{\Delta} & \frac{f_{N_2}}{\Delta} \\ \frac{g_h}{\Delta} & -\frac{f_h}{\Delta} \end{bmatrix} \begin{bmatrix} -f_\phi \\ -g_\phi \end{bmatrix}$$

Note that

$$g_\phi = \left(\tau + \hat{h}^m (1 - \tau) \right) \frac{1}{N_1} (1 + N_1 - 2N_2 + \pi g) > 0$$

if $N_2 < \frac{1+N_1}{2}$. As $f_\phi = 0$, we have

$$\begin{aligned} \frac{\partial h^m}{\partial \phi} &= -\frac{f_{N_2}}{\Delta} g_\phi = -\frac{(+)}{(-)} (+) > 0 \\ \frac{\partial N_2}{\partial \phi} &= \frac{f_h}{\Delta} g_\phi = \frac{(-)}{(-)} (+) > 0. \end{aligned}$$

■

The proofs of Propositions 8–11 follow directly from the arguments in the main text.