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Colluding through Suppliers

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Colluding through Suppliers

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Abstract
In a dynamic game between N retailers and a large number of suppliers, I show that inefficient contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When efficient collusion is not sustainable, this complementarity allows retailers to rely on inefficient input supply, entailing double marginalization and negative franchise fees, to squeeze the wedge between collusive and deviation profits. I also study the role of communication on the equilibrium outcomes of games where retailers have the initiative. It turns out that communication is indeed fundamental to strengthen cartels' sustainability, although generating efficiency losses.

Keywords: Bertrand competition, double marginalization, collusion, competing hierarchies.

JEL classification: D21, D43, L42.

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1 Introduction

Manufacturer-retailer relationships have been widely studied by the recent IO literature. Existing models have underscored several important aspects of these games. For instance, by studying the link between pre-commitment effects and renegotiation (e.g., Caillaud et al., 1995, and Katz, 1991), the rationale behind alternative forms of vertical restraints (e.g., Blair and Lewis, 1994, Gal-Or, 1991a, Khun, 1997, Jullien and Rey, 2007, Martimort, 1996, Rey and Stiglitz, 1995, Rey and Tirole, 1986, and Semenov and Wright, 2009), or by emphasizing the welfare effects of non-exclusive deals (e.g., Bernheim and Whinston, 1986 and 1998, Gal-Or, 1991b, Martimort and Stole, 2008, and Rey et al., 2008).

But this body of work has mainly taken a static approach, and has thus often neglected the strategic aspects stemming from the intertemporal dimension of vertical contracting. Hence, the effects of different wholesale trading rules on the outcome of the repeated interaction between upstream and downstream firms have been poorly understood (few recent exceptions being Jullien and Rey, 2007, Nocke and White, 2007, and Schinkel et al., 2008). This gap raises a number of natural, yet unanswered issues that are related both to the recent antitrust debate over the right legal attitude towards different and evolving forms of vertical arrangements, and to the literature on the determinants of firms' boundaries. What is the link between collusion and vertical contracting in markets where the bargaining power is on the retailers' side? Can the strategic design of wholesale contracts affect competition in these games? Do public contracts help downstream firms to enforce cooperative outcomes? If so, in which markets this is more likely to occur?

The economic relevance of these issues stems from the rise in many developed countries of big box retailers — i.e., Wal-Mart in the US and Ikea in Europe — as well as from the widespread diffusion of large supermarket chains.[1] Scheelings and Wright (2006) point out that the growing influence of these big retailers has led competition authorities to renew their focus on 'buyer power'. As they observe: "Antitrust authorities in the United States have been investigating 'slotting fees' and other retail practices, while UK and EU authorities have commenced a number of inquiries into the competitiveness of the supermarket grocery retail sector. Some in competition policy circles in the United States and Europe claim that there is something sufficiently special about market power on the buyer side of vertically-related industries as to warrant special antitrust scrutiny or a separate analytical framework altogether."

[1] As argued in Rey et al. (2008), large supermarket chains often account for a high share of a manufacturer's production: in the UK, even large manufacturers typically rely on their main buyer for more than 30 percent of domestic sales. In contrast, the business of a leading manufacturer usually represents a very small proportion of business for each of the major multiples.

My analysis goes precisely in this direction. The focus of the paper is twofold. First, I highlight the role that wholesale arrangements play in softening competition in a dynamic framework where retailers have the initiative — i.e., dictate the wholesale trade rules. In a nutshell, inefficient vertical contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When collusion between retailers is not sustainable with efficient wholesale deals, this complementarity makes it advantageous for retailers to rely on inefficient input supply in order to squeeze the wedge between collusive and deviation profits, whereby weakening the incentive to deviate from the implicit agreement. In addition, (inefficient) collusive outcomes must be supported by wholesale contracts featuring negative franchise fees (i.e., payments made by manufacturers to retailers, such as listing fees and slotting allowances) a practice which is widespread in many markets.

The second main purpose of the paper is to comment on the role played by communication between competing supply chains on the set of collusive outcomes achievable in games where the contractual power is in the retailers' hands. Communication is, indeed, fundamental to strengthen cartels' sustainability, although generating some efficiency losses. I study two simple communication regimes: one where retailers share information about wholesale contracts (public contracts), and the other where bilateral negotiations between downstream and upstream firms are secret (private contracts). I show that collusion possibilities considerably broaden under public contracts and argue that the value of communication increases the less patient firms are and the larger the number of competing retailers is. This point seems particularly relevant because, as documented by Briley et al. (1994), U.S. courts often hesitate to prohibit information exchanges between competing supply chains when the bargaining power seems concentrated on the retailers' side, a view that is contrasted by my findings.[2]

I consider an industry where, in each period, N retailers sell a homogenous good and compete by setting prices. The final good must be recovered from an intermediate input, which is supplied by upstream firms (suppliers) each being in an exclusive relationship with a single retailer. The interaction is repeated over an infinite horizon and, in each period, retailers dictate the wholesale trading rules (two-part tariffs) by making take-it or leave-it offers to suppliers. Therefore, in contrast to the pioneering papers on the subject (Jullien and Rey, 2007, Nocke and White, 2007, and Schinkel et al. 2008) dealing with collusion between upstream firms, I focus on collusion between retailers.

With public contracts, retailers observe the wholesale contracts offered by rivals before competing in the downstream market. Hence, in this regime, it is possible to tailor retail price

[2] Briley et al. (1994) pg. 10 — e.g., *Belliston v. Texaco.*

decisions to the game contractual history. I show that, as long as wholesale contracts purposefully specify inefficient trading rules, retailers can make positive profits by charging prices higher than marginal costs (wholesale prices in my model) even when the discount factor δ falls short of the critical value $(N-1)/N$.[3] While in the static game the unique symmetric equilibrium features no double marginalization — i.e., zero wholesale prices — and null franchise fees, retailers might prefer to pay positive wholesale prices for collusive purposes in the repeated game.

There is one main trade-off shaping the self-enforceability conditions needed to sustain collusion in the downstream market with public contracts. On the one hand, excessively high wholesale prices introduce double marginalization, which stifles the difference between deviation and collusive profits. To understand this effect remember that when retailers face zero (or very low) marginal costs, by undercutting the monopoly price a deviant retailer grabs a spot gain close to the monopoly profit. This is no longer true when retailers are committed to pay large wholesale prices, undercutting would then secure lower profits to the deviant. On the other hand, a too large wholesale price — i.e., low downstream margins — can induce a retailer not only to undercut rivals, but also to change its wholesale contract in such a way to gain a competitive advantage over them and obtain a higher profit from deviation: what I will define a '*public deviation*'.

Building on the trade-off between those two effects, I characterize the optimal collusion strategy and show that the (efficient) monopoly profit is sustainable for large values of the discount factor, in this parameter region downstream firms charge the monopoly price, set a wholesale price equal to zero and uniformly share final demand. For intermediate values of the discount factor, the monopoly profit is still sustainable, but only via inefficient contracting, wholesale prices need to be positive and franchise fees are negative to sustain full collusion. Below this region, instead, collusion is still viable with inefficient contracting, but retail prices fall short of the monopoly level. Clearly, for very impatient firms (δ close to zero) the unique equilibrium is perfect competition.

In the second part of the analysis I consider the case of private (unobservable) contracts. I show that when retail pricing decisions cannot be contingent on the rivals' contracts, preventing retailers from grabbing spot deviation gains becomes impossible below the critical discount factor $(N-1)/N$. The reason is that, with private contracts, firms that are cheated cannot instantaneously react to deviations relying on a wholesale contract different than that specified by the (implicit) agreement. This insight has a simple, but novel testable implication concerning the link between market concentration, firms' intertemporal preferences and the role of communication. Public contracts help downstream collusion in very competitive environments (N large) or in

[3]Below this threshold collusion would not arise in the standard repeated Bertrand game, that is, in the game where retailers do not need to rely on suppliers to produce the final good.

circumstances where firms' discount factor is not too large (δ small).

Summarizing, the paper offers two novel insights to the literature on dynamic competition between competing supply chains. First, it emphasizes the coordination role that suppliers play in dynamic games where retailers jointly gain by fixing downstream prices. The analysis robustly shows that there exists a mechanism which allows to sustain collusion even in the region of parameters where self-enforceability would not be met in the standard (repeated) Bertrand analysis.

Second, the paper provides a novel rationale for payments made by suppliers to retailers — e.g., slotting allowances — as well as for excessively high wholesale prices (double marginalization). While earlier models have discussed different reasons for double marginalization to be welfare detrimental, less research has been done on negative franchise fees: a contractual practice that, as a matter of fact, can be spot quite easily by antitrust authorities. This practice is common in many markets: according to the US Federal Trade Commission, since 1998 manufacturers' expenses in slotting allowances have increased sharply from a share of 28% of their total expenses in promotional activities up to 50%. Ever since this practice began, state and federal agencies conducted numerous investigations, but none have resulted in a conclusion against slotting allowances. On February 2001, the Federal Trade Commission released a staff report addressing slotting allowances and other related practices in the supermarket industry. The report notes that such arrangements have the potential to lead to the exclusion of rival suppliers or to anti-competitive horizontal collusion among groups of suppliers or retailers.[4] According to a former FCT Chairman (Robert Pitofsky) there is still little theoretical work on the topic to issue guidelines on slotting allowances; this line was reaffirmed by the FTC staff in 2002, when it was claimed that more studies need to be conducted to learn more about this practice before intervening. My paper contributes to this important debate. While the existing literature has mainly focused on suppliers' incentive to use this instrument, the evidence corroborates the view that negative franchise fees are positively correlated with the exercise of buyer power.[5] In this respect, my model is the first to emphasize that, in repeated games, negative fees can be used for collusive purposes. Moreover, while in the U.S. courts generally regard positive (and high) franchise fees as a signal of price fixing — see, e.g., Briley et al. (1994) — I show that this is not necessarily true when the contracting power is on the retailers' side.

Finally, results are robust to several extensions regarding voluntary information sharing among downstream firms, more general contracts and alternative punishment schemes. The rest of the

[4]Report on the Federal Trade Commission Workshop on Slotting Allowances and Other Marketing Practices in the Grocery Industry, available at http://www.ftc.gov/opa/2001/02/slotting.htm.

[5]See Rey et al. (2008) for a detailed discussion on this point.

paper is organized in the following way: Section 2 below links my contribution to the existing literature. I introduce the model in Section 3. Section 4 characterizes optimal collusion with public contracts. The case of private contracts is analyzed in Section 5. I study the value of public contracting in Section 6. Section 7 discusses some robustness issues and Section 8 concludes. All proves are provided in the Appendix.

2 Related literature

My analysis shares common features with the existing vertical contracting literature.

Given its dynamic perspective, it is related to papers studying the repeated interaction between upstream and downstream firms — e.g., Jullien and Rey (2007), Nocke and White (2007) and Schinkel et al. (2008). There is one key difference between these papers and mine: while I am interested in downstream collusion, they all study the opposite case of upstream collusion. Moreover, while in Nocke and White (2007) vertical mergers have detrimental welfare effects by broadening collusion possibilities, in my model the opposite obtains: delegation of input supply is what makes collusion possible via inefficient contracting. Jullien and Rey (2007) identify the negative effects of RPM, a practice often seen as efficiency enhancing in static models of complete information (the Chicago school argument), my model concludes that collusion can be sustained by way of contracting rules that are inefficient in the static game. Finally, while Schinkel et al. (2008) show that upstream collusion requires low wholesale prices when the bargaining power is in the suppliers' hands, I find the opposite prediction with buyer power.

My analysis also overlaps with the literature on 'buyer power'. Shaffer (1991) analyzes a static duopoly model with differentiated products and buyer power where slotting allowances and resale price maintenance (RPM) are substitutes. He shows that the equilibrium features slotting allowances with public contracts. This outcome cannot occur in my stage game because of Bertrand competition, therefore my focus on the repeated game. Marx and Shaffer (2008) consider a (static) model with buyer power where strong retailers can exclude competitors by offering 'three-part tariffs' that include slotting allowances. In contrast to their model, where negative fees are a way of creating negative externalities (exclusionary purposes) between downstream firms, in my framework these instruments create positive externalities and are used as a tool to sustain cooperative behavior. Finally, Rey et al. (2008), analyze the competitive effects of up-front payments in a contracting situation where rival retailers offer contracts to a single manufacturer. In contrast to Bernheim and Whinston (1986, 1998), they show that two-part tariffs do not suffice to implement the monopoly outcome in a static game, and argue that more complex arrangements, which combine slotting allowances and standard two-part tariffs, are necessary to internalize all the contractual externalities stemming from common agency.[6] My model departs from Rey et al. (2008) in two main respects. On one hand, I study a dynamic framework while they focus on a static game, so my paper is a complement to them.[7] On the other, while I purposefully abstract from common agency issues, their results mainly rely on the externalities that these games feature and do not hold in a model with downstream Bertrand competition, which is instead the building block of my analysis.[8]

3 The model

Players. Consider $N \geq 2$ independent and identical downstream firms (retailers), each denoted by R_i $(i = 1, .., N)$, selling a homogenous good and competing by setting prices. The demand for the final good is $D(p)$; so that, given a vector of retail prices $\mathbf{p} = (p_i)_{i=1}^N$, each downstream firm i faces the individual demand

$$D_i(\mathbf{p}) = \begin{cases} 0 & \text{if } p_i > p_j \text{ for some } j \neq i, \\ D(p_i) & \text{if } p_i < \min_{j \neq i} p_j, \\ \frac{D(p)}{\#\{j : p_j = \min\{p_1, ..., p_N\}\}} & \text{if } p_i = \min\{p_1, ..., p_N\}. \end{cases} \tag{1}$$

Retailers' production technologies are linear and marginal costs are normalized to zero. Nevertheless, the final output must be recovered from an intermediate input that is produced by upstream firms (suppliers). Following the literature[9], I assume that suppliers, each denoted by S_i $(i = 1, .., N)$, are in exclusive relationships with retailers.[10] The intermediate input is transformed into the final output according to a one-to-one technology and, for simplicity, upstream production technologies are linear, with zero marginal costs.

[6] The same point has been recently made by Semenov and Wright (2009) in a model where suppliers have full bargaining power.

[7] This is consistent with the legal approach taken in Carstensen (2000 and 2004) who argues that antitrust treatment of buyer power should be sensitive to the differences in the economic incentives to collude or unilaterally exercise monopsony power between buyers and sellers.

[8] My analysis extends to product differentiation, see, e.g., Shaffer (1991).

[9] For instance Jullien and Rey (2007) and Schinkel et al. (2008).

[10] In an earlier version of the paper I show that results do not change whenever the number of suppliers is larger than N and retailers have full bargaining power. The opposite case where the number of suppliers is lower than that of retailers would not present differences with the analysis performed in the current paper if exclusivity can be enforced. Were this not possible, the analysis would be more involved because of common agency problems (see, e.g., Rey et al. 2008). To simplify the analysis I rule out this possibility.

Wholesale contracts. Retailers have full bargaining power in dictating the wholesale trading rules and make take-it or leave-it offers to suppliers, whose reservation utility is normalized to zero. A wholesale contract between R_i and S_i is a two-part tariff, $C_i \equiv (T_i, w_i)$, specifying a wholesale price w_i for each unit of intermediate input ordered by R_i and a franchise fee T_i. Franchise fees are paid up-front and are thus sunk when downstream firms set retail prices. Once final demand materializes, R_i buys inputs from S_i and pays the negotiated unit price w_i. I will discuss in a concluding section more general contracts.

Information. Two different regimes concerning contracts' observability are considered:

- *Public contracts.* The contract signed between R_i and S_i is observed by all other players before downstream price competition takes place.[11]

- *Private contracts.* Wholesale contracts are secret — i.e., a retailer-supplier pair cannot observe the contracts signed by its opponents.

With observable contracts downstream firms can condition retail prices on competitors' wholesale offers. This is not possible with unobservable contracts. For most of the paper these two regimes will be treated as exogenous features of the environment, I will discuss in an extension the case of voluntary communication.

The existing literature often assumes that wholesale contracts, or some of their dimensions, are public (Jullien and Rey, 2007, Nocke and White, 2007, and Rey and Stiglitz, 1995, among many others). According to Briley et al. (1994) there exists substantial evidence showing that information sharing agreements about wholesale contracts are widespread in several U.S. retail industries. For instance, this seems to be the established praxis in business format franchising where the mandatory disclosure of franchising contracts required by the Federal Trade Commission since 1979 allows firms to have almost free access to their rivals' past contracts.[12]

In practice, communication among competing firms is facilitated by information sharing agreements — i.e., strategic alliances and (retail) trade associations. Moreover, an important trend in product distribution is the growth of information-intensive channels. These are usually characterized by channel partners who invest in bundles of sophisticated information technology like telecommunication and satellite linkages, bar coding and electronic scanning systems, database

[11] Nothing would change if this information can be observed only by retailers and not by suppliers.

[12] *Entrepreneur's Magazine* collects yearly data about franchise fees that are published in the Franchise 500 survey. As Lofantaine and Shaw (1999) report, these fees are very stable over time — i.e., around 75% of franchisors never changed their royalty rate or franchise fee over a 13-year time period.

management systems etc., to not only disseminate information within a given organization, but also among competitors — see, e.g., Stern et al. (1996).[13]

Timing. I consider an infinitely repeated game with discrete time, $\tau = 0, .., +\infty$. The sequence of events within the stage game, thereafter $\mathcal{G}$, unfolds as follows:

(T=1) *Contracting.* Retailers simultaneously offer contracts to suppliers.

(T=2) *Acceptance.* Suppliers simultaneously accept or refuse the received offers. If a contract is finalized, the franchise fee is paid.

(T=3) *Retail competition.* Contractual rules become public information across all players depending on the observability regime — i.e., public *vs* private contracts. Downstream firms set retail prices and the market clears: final demands materialize and input orders are placed.

Each downstream firm has an infinite life-time horizon and its objective is to maximize the discounted sum of profits. The common discount factor is $\delta \in [0, 1]$. Following the approach taken Jullien and Rey (2007), I assume that suppliers live only for one period (alternatively, retailers can only commit to spot distribution contracts). All parties are risk neutral and have a zero reservation utility level.

Histories. With observable contracts, all players observe the same public history h^τ at the end of the contracting stage τ (before retailers compete in stage τ): $h^\tau \equiv (\mathbf{p}^\tau, \mathbf{C}^\tau)$ and contains the sequence of retail prices charged in previous stages $\mathbf{p}^\tau \equiv (\mathbf{p}_i^\tau)_{i=1}^N$ (with $\mathbf{p}_i^\tau \equiv (p_i^t)_{t=0}^{\tau-1}$) along with the sequence of wholesale contracts $\mathbf{C}^\tau \equiv (\mathbf{C}_i^\tau)_{i=1}^N$ (with $\mathbf{C}_i^\tau \equiv (C_i^t)_{t=0}^{\tau}$) offered by each retailer up to τ. Hence, in this regime, the game is one of perfect monitoring: all past actions become common knowledge at the end of each play. With private contracts, instead, at stage τ downstream firm i only observes past retail prices but not contractual histories — i.e., R_i's information set $h_i^\tau \equiv (\mathbf{p}^\tau, \mathbf{C}_i^\tau)$ still contains past retail price decisions ($\mathbf{p}^\tau$) but only its own contractual history ($\mathbf{C}_i^\tau$).

Collusion. I look for symmetric and stationary pure strategy equilibria such that retailers seek to collude whenever possible. The optimal implicit agreement between them (cartel) maximizes

[13] As noted by Niraj and Narasimhan (2004), major retailers such as Sainsbury and Marks & Spencers in U.K. as well as A&P grocery stores, Super Valu Stores and Von's supermarket in U.S. have made substantial investments in these technologies. Similarly, leading manufacturers such as Procter and Gamble have responded to the availability of greater information by developing tracking and information systems at the retail store level.

(downstream) industry profits subject to the relevant self-enforceability constraints and the suppliers' participation constraints. A collusive symmetric and stationary strategy, thereafter $\hat{\sigma}$, requires all downstream firms to offer the contract $C^c \equiv (T^c, w^c)$ and set the retail price p^c in the collusive phase, and to offer the contract $C^p \equiv (T^p, w^p)$ and charge a retail price p^p in the punishment phase.

I focus on punishment codes requiring infinite Nash reversion — i.e., following a deviation by one retailer, rivals will offer the competitive and efficient contract $C^* \equiv (0, 0)$ and price at marginal costs for the rest of the game. The main difference between public and private contracts concerning punishments will be discussed in Sections 4 and 5.

Equilibrium concept. The equilibrium concept is Subgame Perfect Nash Equilibrium (SPNE) for the regime of public contracts, and Perfect Bayesian Equilibrium (PBE) with the added 'passive beliefs' refinement[14] for that with private contracts. I will be more precise on the additional refinements needed to pin down equilibria of the games studied throughout the analysis in Sections 4 and 5.

Simplifying assumptions. The analysis will be developed under the following standard assumptions:

A1 The demand function $D(p)$ is strictly decreasing and twice continuously differentiable. It satisfies standard Inada conditions: (i) $D(0) > 0$, and (ii) there exists an upper-bound $(\overline{p})$ on the retail price such that $D(p) > 0$ for all $p < \overline{p}$ and $D(p) = 0$ for all $p \geq \overline{p}$.

Let $\phi(p) \equiv D(p)p$,

A2 The function $\phi(.)$ is single peaked: it features a unique internal maximum p^m identified by the first-order necessary and sufficient condition:

$$D'(p^m)\, p^m + D(p^m) \equiv 0.$$

A3 Whenever indifferent between accepting a wholesale contract and remain inactive, suppliers prefer to secure input supply.

This hypothesis simply allows to restrict attention to the class of equilibria with positive sales.

[14] Given an equilibrium candidate of the game with private contracts, if a suppliers is offered an unexpected contract, it believes that all other suppliers are offered the equilibrium contract.

4 Public contracts

In this section I provide the equilibrium characterization with observable contracts. I first analyze the static game and then move to the repeated game.

4.1 The stage game

Characterizing the equilibrium of the stage-game is the first necessary step to approach the case of repeated interaction. With public contracts, $\mathcal{G}$ is a two-stage game. In the first stage downstream firms simultaneously choose wholesale contracts; then, given these offers, they set retail prices in the second stage. Since retailers are ex ante identical, I focus on symmetric pure strategy equilibria where they all offer the wholesale contract $C^e \equiv (w^e, T^e)$, charge the final price p^e and sell the same positive amount of final good $D(p^e)/N$.

Before stating the main result of the section few preliminary but important remarks are worthwhile. First, since franchise fees are paid up-front once wholesale deals are finalized, they are sunk when final prices are chosen. Hence, given the opponents' prices $\mathbf{p}_{-i} = (p_j)_{j \neq i, j=1}^{N}$, each retailer R_i solves

$$\max_{p_i \in \Re} D_i(p_i, \mathbf{p}_{-i})(p_i - w_i).$$

Second, given a symmetric equilibrium candidate (C^e, p^e), 'off-equilibrium' histories — i.e., those situations where one or more unexpected offers are observed — might lead to multiple Nash equilibria in the corresponding downstream subgame, exactly as in the standard Bertrand model with asymmetric costs. A refinement criterion must then be chosen in order to pin down an equilibrium. To this purpose, I posit that in every subgame featuring multiple equilibria, downstream firms coordinate on that satisfying *weak Pareto-dominance*. Formally,

Definition 1 *(Weak Pareto-dominance) Consider a contractual history* $\mathbf{C}$ *such that* $C_j = C^e$ *for all* $j \neq i$ *and* $C_i \neq C^e$*, with* $w_i < w^e$*. Let* $\mathbf{p}^e(\mathbf{C}) \in \Re^N$ *and* $\hat{\mathbf{p}}^e(\mathbf{C}) \in \Re^N$ *be the price vectors associated with two Nash equilibria of the corresponding subgame and assume that*

$$D_j(\mathbf{p}^e(\mathbf{C}))(p_j^e(\mathbf{C}) - w_j) = D_j(\hat{\mathbf{p}}^e(\mathbf{C}))(\hat{p}_j^e(\mathbf{C}) - w_j) = 0 \ \forall \ \ j = 1, ..., N, \quad j \neq i. \tag{2}$$

Then, I shall say that $\mathbf{p}^e(\mathbf{C})$ *weakly dominates* $\hat{\mathbf{p}}^e(\mathbf{C})$ *in the sense of Pareto if and only if:*

$$D_i(\mathbf{p}^e(\mathbf{C}))(p_i^e(\mathbf{C}) - w_i) > D_i(\hat{\mathbf{p}}^e(\mathbf{C}))(\hat{p}_i^e(\mathbf{C}) - w_i). \tag{3}$$

Essentially, given the multiplicity of Pareto-rankable equilibria in each subgame where one retailer is more efficient than its rivals, there exists a non-trivial coordination problem between the downstream firms, and consequently a selection issue. The weak Pareto-dominance criterion selects the price vector as the weakly payoff-dominant equilibrium.[15] Accordingly, the equilibrium concept that I will use to solve the retail subgame is as follows:

Definition 2 *(Retail stage solution concept) Given the profile of contracts* $\mathbf{C} = (C_i)_{i=1}^{N}$*, the vector of prices* $\mathbf{p}^e(\mathbf{C}) = (p_i^e(\mathbf{C}))_{i=1}^{N}$ *is a Nash equilibrium satisfying weak Pareto-dominance if and only if:*

(i) Each price $p_i^e(\mathbf{C})$ *is a best reply to* $\mathbf{p}_{-i}^e(\mathbf{C})$ *— i.e.,*

$$p_i^e(\mathbf{C}) \equiv \underset{p_i \in \Re}{\arg\max} D_i\left(p_i, \mathbf{p}_{-i}^e(\mathbf{C})\right)(p_i - w_i) \;\; \forall\; i = 1, .., N. \tag{4}$$

(ii) For any contractual history $\mathbf{C}$ *such that* $C_j \equiv (w, T)$ *for all* $j \neq i$*, while* $C_i \neq C$ *with* $w_i < w$*, and all vectors of prices* $\hat{\mathbf{p}}^e(\mathbf{C}) \neq \mathbf{p}^e(\mathbf{C})$ *that satisfy (2) and (4), equation (3) must also hold.*

Essentially, given a contractual history $\mathbf{C}$, the equilibrium prices chosen in the corresponding downstream subgame must: (i) form a Nash equilibrium — i.e., satisfy the standard best reply criterion; and (ii) fulfill the additional weak Pareto-dominance refinement (whenever this criterion can be applied). I will argue that a symmetric equilibrium satisfying these requirements always exists in my model.

Equipped with this characterization, I can now introduce the solution concept for game $\mathcal{G}$.

Definition 3 *(Two-stage game solution concept) A symmetric SPNE of* $\mathcal{G}$*, with the added refinement of weak Pareto-dominance in the downstream game, features a wholesale contract* C^e *and a retail price* p^e *such that:*

(i) For all $i = 1, ..., N$*, the following is true:*

$$C^e \equiv \underset{C_i \in \Re^2}{\arg\max} \left\{ D_i\left(\tilde{\mathbf{p}}^e(C^e, C_i)\right)\left(p_i^e(C_i, C^e) - w_i\right) - T_i : D_i\left(\tilde{\mathbf{p}}^e(C^e, C_i)\right) w_i + T_i \geq 0 \right\}, \tag{1}$$

[15] Alternatively, instead of weak Pareto-dominance one could use a refinement based on a modified notion of weakly dominated strategies. Essentially, one could assume that among the possible (asymmetric) Nash equilibria of the downstream subgame, the retailers that are not supposed to sell, because they are less efficient than rivals, never charge a price lower than their marginal costs. Arguably, this behavior allows to avoid losses in case the most efficient retailers, that are expected to sell in equilibrium, mistakenly charge a too high price so that those who were expected not to sell actually face a positive demand and make negative profits because selling at a price below the marginal cost.

$$\frac{D(p^e)\, w^e}{N} + T^e = 0, \tag{2}$$

where $\tilde{\mathbf{p}}^e(C^e, C_i) \equiv (p^e(C^e, C_i), p_i^e(C_i, C^e))$, *with* $p_j = p^e(C^e, C_i)$ *for all* $j \neq i$, *is the price vector chosen in the selected Nash equilibrium of the subgame corresponding to the history where* $C_j = C^e$ *for all* $j \neq i$ *and* $C_i \neq C^e$. *While* $p^e = p_i^e(\mathbf{C}^e)$ *is the retail price obtained in the symmetric equilibrium.*

(*ii*) *If the subgame corresponding to the contractual history such that* $C_j = C^e$ *for all* $j \neq i$ *and* $C_i \neq C^e$, *with* $w_i < w$, *features multiple Nash equilibria, retailers coordinate on the equilibrium* $\tilde{\mathbf{p}}^e(C^e, C_i)$ *that satisfies Definition 2 part* (*ii*) *and where the most efficient retailer serves the whole market.*

Condition (1) simply states that the equilibrium wholesale contract must satisfy the best response criterion given the equilibrium of the subgame triggered by any unilateral 'contract' deviation. Condition (2), instead, is the suppliers' participation constraints, which must bind in equilibrium since retailers have full bargaining power. As already noted before, to pin down reasonable equilibria of the two stage game $\mathcal{G}$, I restrict attention to equilibria satisfying Definition 2. The main proposition of the section can be then stated.

Proposition 1 *Assume A1-A3. Then game* $\mathcal{G}$ *features a unique SPNE satisfying the added weak Pareto-dominance refinement. In this equilibrium all players make zero profits, retailers offer the competitive and efficient contract* ($C^e = C^*$) *and set retail prices equal to marginal costs* ($p^e = 0$).

The intuition for this result is straightforward. First, because retailers are in Bertrand competition, they must make zero profits in a symmetric equilibrium of the one-shot game. Second, since contracts with positive wholesale prices limit retailers' ability to undercut one another, the unique equilibrium of the game must feature efficient wholesale contracts — i.e., $w^* = T^* = 0$.

As a final remark, note that the analysis is developed under the hypothesis of Bertrand competition for a specific purpose. In contrast to static models analyzing Cournot or differentiated Bertrand competition[16], in my framework the static game has a unique perfectly competitive equilibrium where, even with public contracts, retailers price at marginal costs and wholesale prices are set to zero. This shows that the strategic value of public contracts cannot always be rationalized in static models, whereby motivating the dynamic analysis which is at the core of the next section.

[16] See, e.g., Shaffer (1991).

4.2 Repeated interaction

Consider now the infinitely repeated game. In the following I will identify the conditions under which downstream collusion is sustainable, and then characterize the properties of the implicit agreement that supports such cooperative outcome. As before, also in this case I will focus on symmetric equilibria where in the collusive phase all retailers offer the contract C^c, charge the retail price p^c and evenly share final demand.

To gain insights about the key forces shaping the equilibrium of the repeated game note that, with public contracts, there are two types of deviations that a retailer may envision. First, it may stick to the collusive contractual rule C^c and cheat its rivals only by undercutting the collusive price p^c: a *secret* (or *unobservable*) deviation. Second, a retailer may 'announce' its forthcoming deviation to rivals by (publicly) offering a wholesale contract different than C^c, and then charge the retail price in accordance with the Nash equilibrium that is expected to be played in the following subgame: a *public* (or *observable*) deviation.

Of course, these two different types of deviations can be punished in different manners. While secret deviations are observed after demand has materialized, and so they can be punished only from the next period onwards, a public deviation is instantaneously spot, and it therefore triggers a reaction already in the very same period where it occurred. For the sake of crispiness, and consistently with the equilibrium refinement stated in Definition 2, for the moment I assume that retailers never coordinate on equilibria of the downstream (pricing) game that do not satisfy weak Pareto-dominance:

A4 In any contractual history where the downstream game features multiple Nash equilibria, retailers coordinate on those satisfying weak Pareto-dominance whenever this criterion can be applied.

Arguably, this refinement also rules out asymmetric equilibria of the retail game where least efficient downstream firms charge a price so low that they would make losses were the most efficient competitors mistakenly charging a too high price. Even if reasonable under some circumstances, this hypothesis clearly rules out *minmaxing* behavior within stages where a public deviation occurs. In Section 7, I argue that results are even stronger if retailers can play equilibrium strategies that do not satisfy weak Pareto-dominance when punishing a public deviation, and show that harsher punishments lead to results with the same qualitative features of those characterized below.[17] It is important to note, however, that a minmaxing behavior occurs following a secret deviation. This is because when retailers spot a secret price cut, they will revert for the rest of

[17] I thank two insightful referees for bringing this point to my attention.

the game to the unique equilibrium of the stage game that leads each player to get the lowest possible payoff.

Before turning to the cartel's optimization program it is important to note that, if at each stage of the game downstream firms symmetrically offer the competitive and efficient contract C^*, the unique equilibrium features collusion supported by Nash reversion trigger strategies if and only if the discount factor δ is larger than the standard threshold $(N-1)/N$.

Lemma 1 *Suppose that the equilibrium outcome path has the competitive and efficient contract C^* offered in each period. Then every retail price level between monopoly (p^m) and perfect competition (0) can be supported by Nash reversion trigger strategies if and only if $\delta \geq (N-1)/N$. Otherwise, for $\delta < (N-1)/N$, there exists a unique SPNE featuring perfect (efficient) competition — i.e., retail and wholesale prices are both equal to zero.*

This lemma restates in a slightly more complex environment the textbook version of Folk Theorem for the Bertrand game with perfect monitoring. It will be helpful to understand the insights provided for the more interesting case where wholesale contracts can be distorted for collusive purposes. I shall argue that equilibrium outcomes featuring positive retail prices can emerge also in the region of parameters where the standard version of the Folk Theorem does not hold — i.e., for $\delta < (N-1)/N$.

To understand more clearly the trade-off that makes this new collusion opportunities viable, it is worthwhile discussing in detail the incentive constraints that an implicit agreement between downstream firms must satisfy. To this purpose, let me first introduce some useful notation. Given a public history h^τ, denote by $s(h^\tau) \in \{0,1\}$ the dichotomous state variable taking value 1 if at the end of stage $\tau-1$ the game is in a cooperative phase — i.e., all retailers have obeyed to the implicit agreement up to $\tau-1$ — and 0 if a deviation occurred. Moreover, within the subset of histories such that no deviation occurred up to the end of period $\tau-1$ — i.e., those for which the state variable $s(h^\tau)$ takes value 1 — denote by $z(h^\tau) \in \{0,1\}$ the dichotomous state variable taking value 0 if within period τ a public deviation occurs, and 1 otherwise. A stationary and symmetric collusion strategy $\hat{\sigma} \equiv (\hat{\sigma}^\tau(h^\tau))_{\tau=0}^{+\infty}$ enforcing the non-competitive retail price p^c and the wholesale contract C^c on the cooperative phase can be thus formally described as follows,

Definition 4 *Under A4, the optimal collusive strategy $\hat{\sigma}$ requires each retailer i:*

(i) To offer C^c and charge p^c if $s(h^\tau) = 1$ and $z(h^\tau) = 1$;

(ii) To offer C^* and charge a retail price equal to 0 from τ onwards if $s(h^\tau) = 0$;

(*iii*) *To charge the retail price* $p^e(C^c, C_j) = w^c$ *if* $s(h^\tau) = 1$, $z(h^\tau) = 0$ *and* $C_j \neq C^c$ *with* $\min_{j \in \{j: C_j \neq C^c\}} w_j < w^c$, *or if* $w_j > w^c$ *for all rivals but one (say* i'*) who offers* $C_{i'} = C^c$.

(*iv*) *To charge the retail price*

$$p^e(C^c, C_j) = \min_{j \in \{j: C_j \neq C^c\}} w_j - \varepsilon$$

with $\varepsilon > 0$ *and small enough if* $s(h^\tau) = 1$, $z(h^\tau) = 0$ *and* $C_j \neq C^c$, *with* $w_j > w^c$ *for all* $j \neq i$.

Where, as noted in Section 4.1, $p^e(C^c, C_j)$ is the Nash equilibrium retail price satisfying weak Pareto-dominance charged by each retailer $i \neq j$ when one or more unexpected offers occur. Essentially, the collusive strategy $\hat{\sigma}$ prescribes: (*i*) to keep cooperating as long as no one has deviated; (*ii*) to offer the competitive and efficient contract C^* for the rest of the game as long as a deviation has occurred before stage τ; and (*iii*) to charge the Nash equilibrium price satisfying weak Pareto-dominance (i.e., $p^e(C^c, C_j)$) if up to stage $\tau - 1$ the game was in a cooperative phase, but at least one retailer j has offered an unexpected contract in stage τ.

For $\hat{\sigma}$ to be self-enforceable, all players must find it convenient to follow its rules — i.e., downstream and upstream firms must not have profitable deviations. First, suppliers must make non-negative profits when accepting C^c, as otherwise they would refuse to supply the intermediate input — i.e.,

$$\frac{1}{N} D(p^c) w^c + T^c \geq 0. \tag{3}$$

This constraint simply implies that, in the collusive phase, the sum of the franchise fees and each supplier's sales revenue is non-negative.

In contrast, two types of deviations must be considered for downstream firms. Each retailer must not find it worthwhile to offer the collusive wholesale contract C^c, and then undercut p^c (secret deviation). Given the strategy $\hat{\sigma}$, this type of behavior cannot be profitable as long as the intertemporal gain from collusion exceeds the spot gain that a retailer can grab by secretly cutting its retail price. Formally:

$$V^c \equiv \frac{\frac{1}{N} D(p^c)(p^c - w^c) - T^c}{1 - \delta} \geq D(p^c)(p^c - w^c) - T^c. \tag{4}$$

Moreover, retailers must also not find it convenient to deviate 'publicly' — i.e., they must not gain by offering both a contract and a retail price different than those specified by $\hat{\sigma}$. Given the punishment code described in Definition 4, this type of deviation is not profitable as long as the following self-enforceability constraint is met:

$$V^c \geq \max_{C_i \in \Re^2} \left\{ D_i \left(\tilde{\mathbf{p}}^e \left(C^e, C_i \right) \right) \left(p_i^e \left(C_i, C^e \right) - w_i \right) - T_i \ : \ D_i \left(\tilde{\mathbf{p}}^e \left(C^e, C_i \right) \right) w_i + T_i \geq 0 \right\}, \tag{5}$$

where $\tilde{\mathbf{p}}^e (C^e, C_i) = (p^e (C^e, C_i), p_i^e (C_i, C^e))$ is the Nash equilibrium vector of prices satisfying A4 — i.e., weak Pareto-dominance if this criterion can be applied — played in the subgame that is triggered by the deviation $C_i \neq C^e$. This leads to the following result:

Lemma 2 *Under A4, the maximal profit that a retailer can make via a public deviation is* $\phi (w^c) \equiv D (w^c) w^c$. *The incentive constraint (5) thus rewrites as:*

$$V^c \geq \phi (w^c) . \tag{6}$$

The intertemporal profit that each retailer earns on the cooperative path must exceed the profit that it would make by cutting the price down to $w^c - \varepsilon$, with ε small enough — i.e., slightly below the punishment price charged by its competitors when spotting a public deviation. This is because in every 'off-equilibrium' subgame that is triggered by a public deviation, the minimal price that cheated firms can charge and that is consistent with Nash equilibria and weak Pareto-dominance is their marginal cost.

Next definition formally introduces the notion of an optimal symmetric collusive strategy with public contracts.

Definition 5 *With public contracts, an optimal symmetric and stationary collusive strategy* $\hat{\sigma}$ *maximizes retailers' joint profits subject to the self-enforceability and participation constraints described above — i.e.,*

$$\mathcal{P} : \begin{cases} \max\limits_{(p,C) \in \Re^3} D (p) (p - w) - NT, \\ s.t. \ (3), \ (4) \ and \ (6). \end{cases}$$

The best symmetric and stationary collusive strategy has to maximize the (downstream) industry profits provided that no player can profitably deviate. The cartel's payoff is defined as total industry profits $D (p) (p - w)$ net of the aggregate franchise fees NT.

Obviously, in the region of parameters where positive profits cannot be sustained, the optimal collusive strategy must require retail prices equal to marginal costs (perfect competition) and efficient wholesale contracting. Next lemma shows that whenever retailers offer the efficient contract on the equilibrium path and collude, then $\hat{\sigma}$ supports the monopoly profit — i.e., each downstream firm gets $\pi_i^C = \phi (p^m) / N$:

Lemma 3 *As long as $\hat{\sigma}$ requires retailers to offer the efficient and competitive contract C^* and to charge positive retail prices in the cooperative phase, the monopoly price p^m can be sustained in equilibrium (full collusion).*

The intuition for this result is straightforward. Recall that when all retailers offer the competitive and efficient contract C^* in the cooperative phase, the self-enforceability constraints (4) and (6) rewrite as:

$$\frac{\phi(p^c)}{N(1-\delta)} \geq \max\{0, \phi(p^c)\}.$$

Hence, with efficient contracts, downstream firms can share the monopoly profits only if collusion is viable in the simple repeated Bertrand analysis where retailers own the upstream technology and can produce the intermediate input at no cost — e.g., they are vertically integrated. As already discussed above, this result can hold only if δ exceeds the threshold $(N-1)/N$.

When retailers are not sufficiently patient to sustain monopoly profits via the efficient contract, the analysis becomes much more complex. A natural question is then whether there exists a symmetric collusive strategy, still securing positive profits to downstream firms, which is supported by positive wholesale prices and negative franchise fees. Clearly, the answer depends on the interplay between the incentive constraints (4) and (6). Since the suppliers' participation constraint must bind in the optimum, substituting $T^c = -(1/N) D(p^c) w^c$ into both equations leads to the following more intuitive and compact formula for incentive compatibility:

$$\frac{\phi(p^c)}{N(1-\delta)} \geq \max\left\{D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), \phi(w^c)\right\}. \tag{7}$$

There is one fundamental trade-off shaping this self-enforceability condition. Clearly, increasing the wholesale price makes public deviations more profitable — i.e., $\phi(w^c)$ is increasing in w^c for all $w^c \leq p^m$. This is because a high wholesale price — i.e., low downstream margin — makes public deviations more attractive: the within-period punishment is less severe the higher the collusive downstream margin is. However, a low wholesale price — i.e., high downstream margin — makes secret deviations more attractive. This is because undercutting the collusive price yields a larger profit when retailers face zero (or very low) marginal costs than when they are committed to pay large wholesale prices for the intermediate input.

Hence, given a collusive retail price $p^c > 0$, the relevant question is whether changes in w^c make one or the other type of deviations profitable. To see this point, denote the right hand side of (7) by

$$\psi(w^c; p^c) \equiv \max\left\{D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), \phi(w^c)\right\}.$$

Next lemma summarizes the trade-off just discussed by characterizing the shape of $\psi(.)$; it shows that a necessary condition for $\hat{\sigma}$ to enforce collusion in the range $\delta < (N-1)/N$ requires a positive wholesale price $w^c > 0$.

Lemma 4 *Assume A1-A4. Then the following properties hold:*

(*i*) *For any $p^c \in [0, p^m)$, there exits a threshold $\underline{w}^c(p^c) \in (0, p^m)$ such that:*

$$\psi(w^c; p^c) = \begin{cases} D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right) & \text{for } w^c \leq \underline{w}^c(p^c), \\ \\ \phi(w^c) & \text{for } w^c \in (\underline{w}^c(p^c), p^m]. \end{cases}$$

(*ii*) *If the equilibrium features collusion for values of δ below the threshold $(N-1)/N$, the contract C^c must require positive wholesale prices ($w^c > 0$) and negative franchise fees ($T^c < 0$).*

As explained above, for small values of the collusive wholesale price (w^c) retailers' best deviation involves only a secret price cut, while for large values of w^c public deviations are more profitable. For a given p^c, there exists a unique value of w^c at which cheating retailers are indifferent between one or the other type of deviation.

Building on this simple and intuitive result, the next theorem identifies the conditions that the optimal implicit collusive agreement between downstream firms must satisfy. Its objective is to identify a range of discount factors lying below the critical value $(N-1)/N$ where a collusive outcome can still emerge in equilibrium.

Theorem 2 *Assume A1-A4. Then, the optimal collusive strategy satisfies the following properties:*

(*i*) *For $\delta \geq (N-1)/N$ full collusion is compatible with the efficient contract. Retailers share uniformly the monopoly profit $\pi^m = \phi(p^m)$ and offer the efficient contract C^*.*

(*ii*) *There exists a lower bound $\underline{\delta}^1(N) < (N-1)/N$, such that for $\delta \in [\underline{\delta}^1(N), (N-1)/N)$ full collusion is still viable. In this region of parameters retailers share uniformly the monopoly profit but offer the inefficient contract $C^m = (w^m, T^m)$, with $w^m > 0$ solving:*

$$\frac{p^m - w^m}{p^m} = \frac{\phi(w^m)}{\phi(p^m)} - \frac{w^m}{p^m N}, \tag{8}$$

with corresponding franchise fee $T^m = -(1/N)\, D(p^m)\, w^m$.

(*iii*) *There exists a threshold* $\underline{\delta}^0(N) < \underline{\delta}^1(N)$ *such that for all* $\delta \in [\underline{\delta}^0(N), \underline{\delta}^1(N))$ *collusion is still viable but inefficient. In this range the collusive retail price* ($\underline{p}$) *and the wholesale price* ($\underline{w}$)*, with* $\underline{p} \in [0, p^m)$ *and* $\underline{w} \in [0, \underline{p})$*, solve the following system of equations:*

$$\frac{\underline{p}-\underline{w}}{\underline{p}} = \frac{1}{N(1-\delta)} - \frac{\underline{w}}{\underline{p}N}, \tag{9}$$

$$\frac{\underline{p}-\underline{w}}{\underline{p}} = \frac{\phi(\underline{w})}{\phi(\underline{p})} - \frac{\underline{w}}{\underline{p}N}, \tag{10}$$

with corresponding franchise fee $\underline{T} = -(1/N)\, D(\underline{p})\underline{w}$.

(*iv*) *The bound* $\underline{\delta}^1(N)$ *is*

$$\underline{\delta}^1(N) = 1 - \frac{\phi(p^m)}{N\phi(w^m)}. \tag{11}$$

Moreover, let $\underline{p}(\delta, N)$ *be the solution of (9) and (10), the bound* $\underline{\delta}^0(N)$ *is identified by the zero retail price condition* $\underline{p}(\delta, N) = 0$.

The intuition for this result is simple and it builds on the results stated before. If retailers are patient enough collusion is efficient exactly as in the infinitely repeated Bertrand game where each retailer is integrated with its own supplier. However, while in a model without suppliers, retailers' temptation to undercut each other would be so high for $\delta < (N-1)/N$ to frustrate any attempt of cooperation, with vertical relationships this is not always true. A careful design of wholesale contracts can still make cooperation viable in this region of parameters. The implicit cost of collusion, though, is double marginalization: even if contracts are such that suppliers break-even, positive wholesale prices and negative franchise fees are needed to squeeze the wedge between collusive and deviation profits. Clearly, for discount factors close to zero collusion becomes not sustainable at all — i.e., for $\delta < \underline{\delta}^0(N)$.

Summing up, the optimal collusive strategy specifies a retail price p^c and a wholesale price w^c having the following features:

$$p^c = \begin{cases} 0 & \text{for } \delta < \underline{\delta}^0(N) \\ \underline{p} & \text{for } \delta \in [\underline{\delta}^0(N), \underline{\delta}^1(N)) \\ p^m & \text{for } \delta \geq \underline{\delta}^1(N) \end{cases}, \quad w^c = \begin{cases} 0 & \text{for } \delta < \underline{\delta}^0(N) \\ \underline{w} & \text{for } \delta \in [\underline{\delta}^0(N), \underline{\delta}^1(N)) \\ w^m & \text{for } \delta \in (\underline{\delta}^1(N), (N-1)/N) \\ 0 & \text{for } \delta \geq (N-1)/N \end{cases}.$$

Clearly, besides the aforementioned static work on buyer power, there might be other stories that could square inefficient wholesale deals and public contracting — see, e.g., the large body of work on double marginalization, Motta (2000, Ch. 6). But, in this literature, where the initiative is on the suppliers' hands, franchise fees are typically positive and have beneficial welfare effects insofar as they prevent double marginalization. My analysis, instead, unambiguously suggests that antitrust and competition policy authorities should regard as anticompetitive *per se* forms of communication between retailers, which make it possible to share information about wholesale contracts and are bundled with negative franchise fees.

Comparative statics. To understand how firms' intertemporal preferences and the degree of concentration in the downstream market affect retailers' market power, it is interesting to study the equilibrium mark-up (downstream margin), $m^c = (p^c - w^c)/p^c$ and show how it changes with respect to the underlying parameters of the model (δ and N).

With full and efficient collusion the mark-up is mainly shaped by the characteristics of demand through the first-order condition in **A2**. By contrast, when collusion is supported by inefficient contracts the mark-up also depends on the characteristics of supply through the self-enforceability constraints. This leads to the following result:

Proposition 3 *Assume A1-A4. The mark-up m^c satisfies the following properties:*

(i) It is weakly increasing in δ;

(ii) For $\delta < \underline{\delta}^1(N)$ it is decreasing in N, while the opposite obtains for $\delta \geq \underline{\delta}^1(N)$.

Industries with more patient firms feature higher price-cost margins. As δ increases, retailers become more patient and value less spot deviation gains. Hence, there is less need to distort upward wholesale prices to sustain positive downstream margins. By the same token, when the stakes of collusion are shared by fewer downstream firms, a higher margin can be sustained as long as collusion is inefficient ($p^c < p^m$). Differently, in the region of parameters where full collusion is sustainable through inefficient contracting, the opposite relationship between the mark-up and N obtains. In this region of parameters a larger number of downstream competitors reduces the gain from a secret deviation, and hence it induces a lower distortion in the wholesale price required to sustain monopoly profits. In other words, when the number of competitors increases, each retailer will find it less convenient to deviate secretly because its production costs — i.e., $w^m D(p^m)$ — will be too large relative to what he would pay in equilibrium — i.e., $w^m D(p^m)/N$.

The linear example. I now construct a simple example putting Theorem 2 at work. Consider the linear demand function $D(p) = \max\{0, 1-p\}$, such that $p^m = 1/2$ and $\overline{p} = 1$. First, solving

(8) one gets:

$$w^m(N) = \frac{3}{4} - \frac{1}{4N} - \frac{1}{4N}\sqrt{5N^2 - 6N + 1},$$

one can check that w^m is decreasing in N and that $w^m \in (0, 1/2)$. This expression together with (11) yields

$$\underline{\delta}^1(N) = 1 - \frac{1/4}{N(1 - w^m(N))\, w^m(N)},$$

where simple algebraic manipulations allow to verify that $\underline{\delta}^1(N) \in (0, (N-1)/N)$ and that $\underline{\delta}^1(N)$ is increasing in N.

Consider now the region of parameters where collusion is inefficient, $\delta < \underline{\delta}^1(N)$. Solving the system of equations (9)-(10) we have:

$$\underline{p}(\delta, N) = \frac{(N-1)(1-\delta)\left(N^2(1-\delta) - 2N + 1\right)}{3N + \delta - 2N\delta - 3N^2 + N^3 + 3N^2\delta - 2N^3\delta + N^3\delta^2 - 1},$$

and

$$\underline{w}(\delta, N) = \frac{(N(1-\delta) - 1)\left(N^2(1-\delta) - 2N + 1\right)}{3N + \delta - 2N\delta - 3N^2 + N^3 + 3N^2\delta - 2N^3\delta + N^3\delta^2 - 1},$$

which, as one can check with simple algebra, imply $0 \leq \underline{w}(.) \leq \underline{p}(.)$ for $\delta < (N-1)/N$ and $\underline{p}(\delta, N) = \underline{w}(\delta, N) = 0$ for

$$\underline{\delta}^0(N) = \left[\frac{N-1}{N}\right]^2.$$

Moreover, it is easy to check that $\underline{\delta}^0(N)$ is increasing in N, that $\Delta\underline{\delta}(N) = \underline{\delta}^1(N) - \underline{\delta}^0(N) \geq 0$ and $\Delta\underline{\delta}(N)$ is inverted-U shaped with respect to N with $\lim_{N\to+\infty} \Delta\underline{\delta}(N) = 0$.

In the standard duopoly model ($N = 2$), for discount factors above the critical value 1/2 full collusion is sustainable with the efficient contract. It is easy to check that $w^m = 1/4$ and $\underline{\delta}^1(2) = 1/3$. So, in the region of parameters where $\delta \in [1/3, 1/2)$ the murk-up is 1/2 and full collusion is still viable but it must be sustained by an inefficient contract. Differently, below the threshold $\underline{\delta}^1(2) = 1/3$, one has:

$$\underline{p}(.) = \frac{(1-\delta)(1-4\delta)}{1 + 8\delta^2 - 7\delta},$$

and

$$\underline{w}(.) = \frac{(1-2\delta)(1-4\delta)}{1 + 8\delta^2 - 7\delta},$$

where $\underline{\delta}^0(2) = 1/4$. Both the retail and the wholesale prices are increasing in δ in the relevant range of parameters: more patient players can enforce implicit agreement sustaining larger retail prices. Clearly, the larger is the collusive price the higher the wholesale price needs to be in order to refrain retailers from deviating. Finally, it is easy to show that in the range $\delta \in (1/4, 1/3)$ both $\underline{p}(.)$ and $\underline{w}(.)$ are positive, with $\underline{p}(.) \geq \underline{w}(.)$, and the mark-up $m(\delta) = \delta/(1-\delta)$ is lower than 1/2 and increasing with respect to δ.

5 Private contracts

Suppose now that wholesale contracts are unobservable. The key difference with the case of public contracts is that, in this framework, public deviations can no longer be distinguished by secret ones. When bilateral negotiations are private, wholesale contracts loose their strategic value: a deviation in contracts can no longer be detected instantaneously. Hence, the punishment phase must start with one period delay under all circumstances.

The objective of the section is to show that this limit on retailers' communication dramatically stifles the collusion possibility frontier — i.e., it (discretely) increases the lowest discount factor above which positive profits can be sustained in equilibrium. Before introducing the formal analysis, it is worthwhile noting that also with private contracts the static game features a unique competitive and efficient equilibrium.

Lemma 5 *Assume A1-A3. Then, with private contracts the stage game features a unique competitive PBE. All players make zero profits, retailers symmetrically offer C^*, set a retail price equal to zero and uniformly share demand.*

The intuition is straightforward. With private contracts the stage game $\mathcal{G}$ is one of imperfect information. Retailers actually play a simultaneously move game where their strategy is to choose independently a wholesale contract and a retail price. Hence, they cannot condition retail prices on the observed contractual history of the game. Hence, since undercutting positive retail prices was profitable in the regime with public contracts, by revealed preferences it must also be with private contracts.

Once again, I shall focus on symmetric and stationary implicit agreements: retailers play symmetrically both in the cooperative and punishment phases. As before, I assume that the cartel's penal code follows Nash reversion trigger strategies, which in this case also imply minmaxing. With private contracts the public history only contains information about past retail prices — i.e., $h^\tau \equiv (\mathbf{p}_i^\tau)_{i=1}^N$. Denote by $\hat{\sigma} \equiv (\hat{\sigma}^\tau(h^\tau))_{\tau=0}^{+\infty}$ the collusive strategy which specifies for each public history h^τ at stage τ a retail price $\hat{p}^\tau$ and wholesale contract $\hat{C}^\tau$. Moreover, denote by $y(h^\tau) \in \{0, 1\}$ the dichotomous state variable taking value 1 if after stage $\tau - 1$ the game is in a cooperative phase — i.e., all retailers obeyed to the implicit agreement up to $\tau - 1$ — and 0 if a deviation has occurred before τ. A stationary and symmetric collusion strategy enforcing the non-competitive retail price p^c and the wholesale contract C^c on the cooperative phase can be thus formally described as follows:

$$\forall\ \tau \text{ and } h^\tau \in H^\tau : \ \hat{\sigma}^\tau (h^\tau) \equiv \begin{cases} (p^c, C^c) & \text{if } y (h^\tau) = 1, \\ \\ (0, C^*) & \text{if } y (h^\tau) = 0. \end{cases}$$

Of course, the strategy $\hat{\sigma}$ must satisfy suppliers' participation constraint:

$$\frac{1}{N} D (p^c) w^c + T^c \geq 0.$$

The main difference with the previous analysis rests on the self-enforceability constraint. Given (p^c, C^c), self-enforceability here requires that no downstream firm must find it profitable to deviate by undercutting its rivals with a retail price slightly below p^c and by issuing the best wholesale contract given that rivals stick to C^c. Formally:

$$V^c \equiv \frac{\frac{1}{N} D (p^c) (p^c - w^c) - T^c}{1 - \delta} \geq \max_{C_i \in \Re^2} \{D (p^c) (p^c - w_i) - T_i : \ D (p^c) w_i + T_i \geq 0\} . \tag{12}$$

There is one key difference between this self enforceability constraint and the one described in equation (5) for the case of public contracts: when a downstream firm deviates from the collusive agreement and wholesale contracts are private, the best retail price it can charge is $p^c - \varepsilon$ (with $\varepsilon > 0$ and small enough). Since firms cannot cannot react to public deviations, undercutting now secures the largest possible profit.

Using the participation constraint $T_i = -D (p^c) w_i$, the above incentive compatibility constraint rewrites as:

$$\frac{\frac{1}{N} D (p^c) (p^c - w^c) - T^c}{1 - \delta} \geq \phi (p^c) . \tag{13}$$

This leads to the following definition:

Definition 6 *With private contracts, an optimal symmetric and stationary collusive strategy $\hat{\sigma}$ maximizes retailers' joint profits subject to the relevant self-enforceability and participation constraints — i.e.,*

$$\mathcal{P}' : \begin{cases} \max\limits_{(p,C) \in \Re^3} D (p) (p - w) - NT, \\ s.t. \ (3) \text{ and } (13). \end{cases}$$

As before, the implicit collusive agreement has to maximize the downstream industry joint profits subject to the relevant participation and self-enforceability constraints. Using the participation constraint $T^c = -(1/N)\, D(p^c)\, w^c$ in equation (13) it is immediate to see that wholesale contracts play no role on collusion when these contracts are private. The next proposition formalizes this intuition:

Proposition 4 *Assume A1-A3. Then, with private contracts collusion can be sustained only in the region of parameters where $\delta \geq (N-1)/N$. In this range downstream firms are able to sustain monopoly profits (π^m) and offer the competitive and efficient contract. For $\delta < (N-1)/N$, the unique PBE of the repeated game features perfect competition and efficient wholesale contracts.*

Limits on retailers' communication ability dramatically reduce collusion possibilities. If there is no way of making wholesale contracts observable to rivals, deviation spot gains become so large to prevent any scope for cooperation below the critical discount factor $(N-1)/N$. While with public contracts retailers can avoid this moral hazard problem by changing instantaneously their retail price in response to a public deviation, with private contracts this mechanism is no longer viable.

6 The collusive value of communication

What is the link between the anticompetitive role of public contracts, firms' discount factor and competition in retail markets where the bargaining power is on the retailers' side?

My analysis offers a simple answer to this question. Taken together, the results stated in Theorem 2 and Proposition 4 imply that public contracts broaden the collusive possibility frontier as long as the downstream industry features a discount factor δ that falls short of the critical value $(N-1)/N$. In this region of parameters, the possibility of making wholesale contracts public leads to new collusive outcomes relying on inefficient input supply. Hence:

Proposition 5 *Since the threshold $(N-1)/N$ is increasing in N, public contracts have a stronger anticompetitive role for lower levels of δ and larger levels of N.*

This result offers simple testable predictions on the link between the anticompetitive use of wholesale contracts on the one hand, and the downstream market structure and retailers' time preferences on the other. Information sharing agreements between retailers should be more likely to harm consumers in environments featuring a larger number of competitors in the downstream market and/or more shortsighted firms.

7 Extensions and robustness

So far the analysis was developed under few simplifying assumptions. First, I assumed that the observability regime is exogenous — i.e., retailers cannot choose whether to have or not an information sharing agreement. Second, I considered only contracts involving two-part tariffs. Third, in the regime with public contracts, I assumed that retailers never coordinate on equilibria that do not satisfy weak Pareto-dominance in the punishment phase following a public deviation. In this section I show that the qualitative insights of the previous analysis remain unaltered if these hypotheses are weakened each in turn. Finally, I also discuss the reasons why, at least under the assumptions made by the earlier literature on repeated interactions between upstream and downstream firms, my analysis seems more compelling for the 'buyer power' case rather than for the polar situation where the bargaining power is on the suppliers' side.

7.1 Endogenous communication

Instead of assuming that contracts are (exogenously) either public or private, suppose now that it is a decision of a retailer whether to make its contract public. Two different scenarios can be analyzed depending on the retailers' commitment power. First, one can imagine that retailers can commit to share information about wholesale contracts at the very outset of the game before engaging in price competition, and if so, they cannot renege on this choice: *full commitment*. Second, when retailers lack such long term commitment ability, they must decide whether to disclose their contracts at each stage of the game: *lack of commitment*. Below I will analyze each regime in turn. For obvious reasons, in both cases I assume that, once retailers decide to communicate, it must be feasible for them to credibly disclose their wholesale contracts to rivals — i.e., if disclosed, contracts are hard information.[18]

Full commitment. Assume that information is shared if and only if all downstream firms agree to do so, and that the cost of setting up such reporting system is negligible.[19] Clearly, for $\delta \geq (N-1)/N$ there is no need to share information, as shown in Theorem 2 and Proposition 4. Nevertheless, for $\delta < (N-1)/N$ retailers will find it optimal to share information if $\delta \in [\underline{\delta}^0(N), \underline{\delta}^1(N)]$. Hence, with full commitment retailers spontaneously share information whenever this allows them to achieve a collusive outcome.

[18] Hard information is quantitative, easy to store and transmit in impersonal ways, and its content is independent of the collection process. In this sense, a legally binding contract specifying the wholesale price w_i and the franchise fee T_i can be interpreted as hard information.

[19] Of course, if the individual (per retailer) cost of setting-up the information sharing system is too high, there will be no communication whatsoever.

Lack of commitment. Consider now the case where retailers lack commitment power and must take their communication decisions on a period-by-period basis. Suppose that if at stage τ a retailer discloses its contract, rivals observe such information before choosing their final prices. This form of voluntary information sharing expands the strategy space of game $\mathcal{G}$: in addition to choose a wholesale contract and a retail price, each downstream firm must also decide whether to disclose its contract at every stage of the game. The issue that I analyze below is whether, in this extended game, there exists a self-enforceable collusive strategy with communication among retailers.

To make this point in the simplest possible way, consider a strategy profile with the same features of that in Definition 4 and, in addition, suppose that each downstream firm charges a final price equal to w^c if not all contracts have been disclosed in stage τ and it offers the competitive and efficient contract C^* and charges a retail price equal to zero from the next period onwards. As one can easily show — see, e.g., the proof of Theorem 2 — this strategy enforces collusion on the equilibrium path and contracts are made voluntarily public. This is because the self enforceability constraints are the same as those characterized in Section 4. Indeed, not disclosing the contract at all or disclosing a contract different from C^c is pay-off equivalent for a retailer's viewpoint in the deviation phase. Hence, voluntary information sharing emerges endogenously as an equilibrium of the extended game where downstream firms choose non-cooperatively their disclosure policy at every stage.

Of course, this result hinges on the somewhat natural hypothesis that wholesale contracts can be credibly disclosed to rivals. The idea that firms can credibly share information has been made in the earlier IO and banking literature dealing with information sharing in oligopolies.[20] In the case at hand, the idea that retailers can credibly share information about their wholesale contracts seems reasonable in all circumstances where, to be legally binding, contracts need to be recorded in a 'Public Registry' or require verifiable legal certifications.

7.2 More general contract spaces

I have already discussed why in the region of parameters where $\delta \in [\underline{\delta}^1(N), (N-1)/N]$ simple two-part tariffs allow to implement full collusion as long as wholesale prices are positive and franchise fees negative. In this section I will explain why also in the region of parameters where $\delta < \underline{\delta}^1(N)$ more complex wholesale contracts, such as resale price maintenance and quantity forcing contracts, cannot improve upon the allocation characterized in Theorem 2 under some

[20] See, e.g., either the IO literature on information sharing in oligopoly (Gal-Or, 1985, Raith, 1996), or the banking literature stemming from Jappelli and Pagano (1993).

natural conditions on the legal environment.

Resale price maintenance. One may reasonably wonder whether contracts imposing retail price restrictions (RPM) broaden collusion possibilities in my game. Clearly, as long as the law does not ban this form of contracts and there exists a credible mechanism which makes this commitment reliable, the equilibrium will trivially feature full collusion. These contracts, though, are often seen as anticompetitive: antitrust and competition policy authorities have largely debated the opportunity of forbidding vertical price restrictions, and the recent attitude is to ban them in almost all circumstances. Taking this legal view, my focus on the anticompetitive nature of simpler two-part tariffs (typically seen as pro-competitive) seems more compelling. Moreover, even if RPM contracts were not forbidden, but their enforceability would be limited by moral hazard or (secret) renegotiation, the equilibrium characterization would boil down to that presented in Section 4. Essentially, whenever there is no legal way to prevent a retailer from announcing a strictly positive price p^c and then charge $p^c - \varepsilon$ at the expense of its competitors[21], the self enforceability conditions (4) and (5) remain the necessary and sufficient conditions for the existence of a collusive agreement.

Quantity forcing contracts. The same considerations made for RPM apply to quantity forcing contracts. Were each retailer able to commit in a credible way to sell only $1/N$-th of the monopoly quantity, the equilibrium should lead once again to full collusion. It is hard, though, to imagine legal rules allowing for this type of behavior. Contracts that enforce collusion in such an explicit way should be forbidden by the antitrust authority. Moreover, even if public but non-binding announcements about the amount of inputs that each retailer is going to purchase were allowed, my equilibrium characterization would still remain the relevant one. Whenever there is no legal way to prevent a retailer from announcing a capacity $D(p^m)/N$ and then secretly change its contract so as to freely charge $p^m - \varepsilon$ and poach the whole market at the expense of its competitors, the self enforceability conditions (4) and (5) studied before remain the necessary and sufficient conditions for the existence of a collusive equilibrium.

7.3 Harsher punishments following a public deviation

So far, it was assumed that in the regime with observable contracts, following a public deviation retailers never select equilibria that do not satisfy weak Pareto-dominance. This implies that if a retailer deviates publicly by offering a wholesale price lower than w^c, its rivals will charge a final

[21] This would be for instance the case when only public but non-binding recommendations about retail prices are allowed.

price equal to w^c in the corresponding subgame. The deviant retailer will then charge a price equal to $w^c - \varepsilon$ (with $\varepsilon > 0$ and small) so that rivals will not sell, even though they gain the sunk fee T^c at the expense of their suppliers. Suppose, instead, that following such a deviation the cheated firms charge a price such that they make (overall) zero profits — i.e., they charge the price $\widehat{p}$ solving the following zero profit condition:

$$\frac{1}{N} D\left(\widehat{p}\right)\left(\widehat{p} - w^c\right) - T^c = 0,$$
$$\text{s.t. } T^c = -\frac{1}{N} D\left(p^c\right) w^c.$$

Arguably, retailers should never price below $\widehat{p}$ in the punishment phase following a public deviation: if so, given the sharing rule specified in (1) they would make negative profits as long as the deviant rival mistakenly charges a price equal or larger than $\widehat{p}$.[22] In this sense such a price can be thought of as a minmaxing punishment. Hence, given (p^c, w^c), the price $\widehat{p}\left(w^c, p^c\right)$ solves

$$D\left(\widehat{p}\left(w^c, p^c\right)\right)\left(\widehat{p}\left(w^c, p^c\right) - w^c\right) + D\left(p^c\right) w^c \equiv 0. \tag{14}$$

This punishment scheme simply implies that cheated retailers charge the lowest possible price that would yield non-negative profits in a situation where each of them (including the deviant) prices at $\widehat{p}(.)$ and get a share $1/N$-th of demand. Of course, for (14) to hold, one must have $\widehat{p}(.) < w^c$. Hence, public deviations become less profitable because the maximal profit that the deviant firm can appropriate is approximately $\phi\left(\widehat{p}\right)$ when charging a retail price equal to $\widehat{p} - \varepsilon$ (with $\varepsilon > 0$ and small). In this case, the self enforceability constraints for collusion to be viable rewrite as:

$$\frac{\phi\left(p^c\right)}{N(1-\delta)} \geq \max\left\{D\left(p^c\right)\left(p^c - \frac{w^c(N-1)}{N}\right), \phi\left(\widehat{p}\left(w^c, p^c\right)\right)\right\}, \tag{15}$$

where $\phi\left(\widehat{p}(.)\right) < \phi\left(w^c\right)$ since $\widehat{p}(.) < w^c < p^m$. This shows that with such kind of behavior following a public deviation, collusion may actually be sustained for a larger range of discount factors than that characterized in Theorem 2. Hence,

Proposition 6 *When public deviations are punished with the price $\widehat{p}(.)$, collusion can emerge in a range of parameters larger than that characterized in Theorem 2.*

Consider, for instance, downstream firms wishing to enforce the monopoly price p^m. Condition

[22] In this event, those retailers that were not expected to sell in equilibrium should rationally opt-out and refuse to sell.

(15) would then require:

$$\frac{\phi\left(p^{m}\right)}{N\left(1-\delta\right)}\geq\max\left\{D\left(p^{m}\right)\left(p^{m}-\frac{w^{c}\left(N-1\right)}{N}\right),\phi\left(\widehat{p}\left(w^{c},p^{m}\right)\right)\right\}.$$

Hence, the wholesale price ($w^{m}_{\widehat{p}}$) that maximizes the range of discount factors where full collusion is feasible solves:

$$w^{m}_{\widehat{p}}\equiv\underset{w^{c}\in\Re}{\arg\min}\left\{\max\left\{D\left(p^{m}\right)\left(p^{m}-\frac{w^{c}\left(N-1\right)}{N}\right),\phi\left(\widehat{p}\left(w^{c},p^{m}\right)\right)\right\}\right\}. \tag{16}$$

To show an example of how this works, consider the linear example studied in Section 4.2, and assume for simplicity that $N=2$. Recall that by (14) the price $\widehat{p}\left(.\right)$ solves:

$$\left(1-\widehat{p}\right)\left(\widehat{p}-w^{c}\right)+\frac{w^{c}}{2}=0,$$

yielding:

$$\widehat{p}\left(w^{c},p^{m}\right)=\frac{1}{2}w^{c}-\frac{1}{2}\sqrt{\left(w^{c}\right)^{2}+1}+\frac{1}{2}.$$

Coming back to (16), one can easily check that $w^{m}_{\widehat{p}}$ is the solution of

$$\left(1-p^{m}\right)\left(p^{m}-\frac{w^{c}}{2}\right)=\left(1-\widehat{p}\left(.\right)\right)\widehat{p}\left(.\right),$$

yielding $w^{m}_{\widehat{p}}\approx0.43>w^{m}$ and so $\widehat{p}(.)\approx0.17$. Finally, the critical value of δ above which the monopoly price can be still sustained ($\underline{\delta}^{1}_{\widehat{p}}\left(2\right)$) solves

$$\frac{\left(1-p^{m}\right)p^{m}}{2\left(1-\delta\right)}=\left(1-\widehat{p}\left(.\right)\right)\widehat{p}\left(.\right),$$

yielding $\underline{\delta}^{1}_{\widehat{p}}\left(2\right)\approx0.12<\underline{\delta}^{1}\left(2\right)\approx0.3$. This example shows quite clearly that even if the 'minmaxing' rule derived from (14) makes collusion easier to sustain, it cannot guarantee the monopoly outcome at every discount factor $\delta\in[0,1]$. Hence, for low discount factors, competing retailers will again need to rely on inefficient collusion to sustain positive profits.

7.4 Bargaining power on the suppliers' side

Finally, in this section I briefly discuss how my results change when the bargaining power is concentrated on the suppliers' side. To this purpose, consider a game where, differently than before, each supplier makes a take-it or leave-it offer to its exclusive retailer. Suppose that contracts again feature two-part tariffs and that they are public. The timing is the same as that in game $\mathcal{G}$, with the difference that here suppliers propose contracts and retailers decide whether to accept the offers received.

Following the literature dealing with upstream collusion — see, e.g., Jullien and Rey (2007) — I consider the case where suppliers are infinitely lived and discount future at the rate $\delta \in [0, 1]$, whereas retailers are short-lived and just maximize their spot profit — i.e., they are too short-sighted to collude at their level. In this game, because of Bertrand competition, every symmetric profile of wholesale contracts leads each downstream firm to set its retail price equal to the wholesale price — i.e., $p_i = w^c$ for all i as long as all suppliers offer $C^c \equiv (w^c, T^c)$. Hence, upstream firms can sustain collusion only by offering a contract $C^c \equiv (w^c, T^c)$, specifying a positive wholesale price ($w^c > 0$). As before, I assume that following any deviation at stage τ, suppliers offer the competitive and efficient contract C^* from the next stage onwards.

In this game the concepts of public and secret deviations are equivalent: anticipating its retailer's behavior a supplier can deviate from an implicit collusive agreement only by charging a wholesale price slightly lower than w^c. So, in order for collusion to be viable, the following conditions must be satisfied:

(i) Since in the collusive phase $p^c = w^c$, the retailer's participation constraint requires:

$$-T^c \geq 0. \tag{17}$$

(ii) Suppliers must not gain by undercutting C^c — i.e., S_i cannot profit from offering a contract specifying a wholesale price equal to $w^c - \varepsilon$, with $\varepsilon > 0$ and small enough. For this behavior not to be convenient, the following incentive constraint must hold:

$$\frac{\frac{\phi(p^c)}{N} + T^c}{1-\delta} \geq \phi(p^c) + T^c \Longrightarrow \phi(p^c)\left[\frac{1 - N(1-\delta)}{N}\right] \geq -\delta T^c. \tag{18}$$

Combining equations (17) and (18), it is easy to show that any price between competition and monopoly can be sustained in equilibrium for $\delta \geq (N-1)/N$. However, since retailers will never accept to pay a positive franchise fee, as implied by the constraint in (17), collusion cannot occur below the threshold $(N-1)/N$. This leads to the following result:

Proposition 7 *If the bargaining power is on the suppliers' side and retailers are shortsighted, even with public contracts collusion can only be sustained for $\delta \geq (N-1)/N$. Otherwise, only the competitive and efficient equilibrium exists.*

This shows that public contracts are more likely to enforce collusive agreements when the bargaining power is on the retailers' hand and that, precisely in these instances, negative franchise fees might actually reflect some form of implicit collusion going on. What is interesting, though, is that while with buyer power collusion can be sustained by the efficient contract C^* in the region where $\delta \geq (N-1)/N$, a positive wholesale price is always needed when the bargaining power is on the other side of the market.

Clearly, the result shown in Proposition 7 may dramatically change when retailers are long-lived and behave strategically vis-à-vis suppliers. In this case the equilibrium outcome can exhibit features similar to those described in Theorem 2 in the range of parameters where $\delta < (N-1)/N$. But, the way things work in this more complex game heavily rely on how the collusive surplus $\phi(p^c)/N$ is shared within each supply chain, and it also depends on whether the sharing rule that would sustain collusion is itself self-enforceable. More precisely, to achieve collusion in this environment, suppliers might want to reward retailers — i.e., by committing not to extract a fraction larger than α of the total surplus $\phi(p^c)/N$ — as long as they 'behave well' by keeping retail prices above marginal costs in the collusive phase, and punish them otherwise. Were this possible, the same logic developed above would enforce collusion even below the critical value $(N-1)/N$, as it can be seen from the constraint (18). However, in this scenario, one needs to be careful: retailers might hold up suppliers by grabbing the ex ante side payment $(1-\alpha)\phi(p^c)/N$, and then undercut rivals ex post by increasing their individual demand up to $D(p^c)$, so as to enjoy larger sales profits at the expense both of suppliers and rivals. This extra moral hazard problem adds a source of complexity to my model that is certainly worth studying, but that goes behind the scope of the current paper, whose main focus is on buyer power. I plan to address this and related issues in future research.

8 Concluding remarks

Two main objectives have been pursued throughout the analysis. First, the model throws new light on the hidden determinants of vertical contracting, and on the role that a careful design of wholesale arrangements might play in softening competition in a dynamic framework with buyer power. Inefficient vertical contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When collusion between retailers turns out not to be sustainable with efficient wholesale deals, this complementarity makes it advantageous for them to rely on inefficient input supply provision in order to squeeze the wedge between collusive and deviation profits, whereby weakening deviation incentives. In addition, profitable collusive market outcomes must be supported by wholesale contracts featuring negative franchise fees, a practice intensely debated by antitrust and competition policy authorities.

The second main insight of the paper is about the effect of communication between competing retailers on the set of collusive outcomes. Communication turns out to be fundamental to strengthen cartels' sustainability, although generating some efficiency losses. Moreover, its collusive value increases in environments where firms are less patient the number of competitors in the downstream market is larger.

Appendix

Proof of Proposition 1. Consider the class of symmetric equilibria of game $\mathcal{G}$ where downstream firms share evenly final demand, offer the same wholesale contract $C^e \equiv (w^e, T^e)$ and charge the same retail price p^e. Under the hypothesis that retailers select equilibria of the downstream game according to weak Pareto-dominance, in the following steps I show that there exists a unique equilibrium with such features where $C^e = C^*$ and $p^e = 0$.

Step 1. *There cannot exist a SPNE where $p^e > w^e > 0$.*

The proof is by contradiction. Suppose that there exists a SPNE where $p^e > w^e > 0$. In this equilibrium candidate each retailer earns a profit of

$$\pi^i \left(p^e | C^e\right) = \frac{D\left(p^e\right)}{N} \left(p^e - w^e\right) - T^e, \quad \forall\ i = 1, ..., N.$$

Consider the following deviation: R_i offers C^e, but charges a final price $p_i = p^e - \varepsilon$, with ε positive and close to zero. Given the rivals' equilibrium strategies, this deviation yields to R_i a profit of

$$\pi^i \left(p^e - \varepsilon | C^e\right) = D\left(p^e - \varepsilon\right) \left(p^e - \varepsilon - w^e\right) - T^e.$$

For $p^e > w^e$ and ε close to zero, this implies

$$\pi^i \left(p^e - \varepsilon | C^e\right) \approx D\left(p^e\right) \left(p^e - w^e\right) - T^e > \pi^i \left(p^e, w^e\right) = \frac{D\left(p^e\right)}{N} \left(p^e - w^e\right) - T^e.$$

Hence, R_i can profitably undercut p^e and steal the entire market from its rivals. This provides the desired contradiction. A symmetric SPNE must then necessarily entail $p^e \leq w^e$.

Step 2. *There cannot exist a SPNE where $p^e = w^e > 0$.*

The proof is again by contradiction. Suppose that there exists a symmetric equilibrium such that $p^e = w^e > 0$. Consider the following deviation: R_i offers $C^* \equiv (0, 0)$. Given such unexpected offer and the fact that franchise fees are paid up-front, in the corresponding competitive subgame each retailer R_j (with $j \neq i$) makes a total profit of

$$\pi^j \left(\mathbf{p} | C^e\right) = \begin{cases} D_j\left(\mathbf{p}\right)\left(p_j - p^e\right) & \text{if} \quad D_j\left(\mathbf{p}\right) > 0, \\ \\ 0 & \text{otherwise,} \end{cases} \tag{A.1}$$

given the vector of prices $\mathbf{p} = (p_i)_{i=1}^N$. Hence, it is straightforward to see that such a subgame features a continuum of Nash equilibria. Each of these equilibria is identified by a pair of prices (p_{-i}, p_i) such that: $p_j = p^e(C^e, C_i) \in (0, p^e]$ for all $j \neq i$, and $p_i = p_i^e(C_i, C^e) - \varepsilon$, with $\varepsilon > 0$ and small enough. However, using Definition 1, the unique Nash equilibrium that survives to weak

Pareto-dominance is the one where $p_j = p^e$ for all $j \neq i$ and $p_i = p^e - \varepsilon$. Hence, focusing on such equilibrium, R_i's deviation profit is:

$$\pi^i \left(p^e - \varepsilon | C^*\right) = D\left(p^e - \varepsilon\right)\left(p^e - \varepsilon\right) \approx \phi\left(p^e\right) > \pi^i \left(p^e | C^e\right) = \frac{\phi\left(p^e\right)}{N},$$

which yields the desired contradiction.

Step 3. *There cannot exist a symmetric SPNE where $p^e < w^e$ and retailers share evenly final demand.*

The proof of this result is straightforward. Suppose that there exists a SPNE where $p^e < w^e$ and all retailers sell the same positive amount of final good. Given its rivals' strategies, R_i would then be better-off by not selling at all — i.e., by setting $p_i = p^e + \varepsilon$, with $\varepsilon > 0$, instead of $p_i = p^e$. This is immediate because

$$-T^e > -T^e + \frac{D\left(p^e\right)}{N}\left(p^e - w^e\right) \Longrightarrow 0 > \frac{D\left(p^e\right)}{N}\left(p^e - w^e\right)$$

for $p^e < w^e$. A contradiction.

Step 4. *There cannot exist a symmetric SPNE where all downstream firms offer $C^e \equiv (w^e, T^e)$, with $T^e > 0$, and charge a positive retail price p^e.*

The argument is by contradiction. Suppose that such an equilibrium exists. Since suppliers' participation constraint must bind at equilibrium, $T^e > 0$ requires $w^e < 0$ — i.e.,

$$T^e = -\frac{D\left(p^e\right) w^e}{N} > 0 \quad \Longrightarrow \quad w^e < 0.$$

For any positive price p^e this implies

$$\pi^i \left(p^e | C^e\right) = \frac{D\left(p^e\right)}{N}\left(p^e - w^e\right) < D\left(p^e\right)\left(p^e - w^e\right) \approx D\left(p^e - \varepsilon\right)\left(p^e - \varepsilon - w^e\right),$$

for ε positive and close enough to zero. Therefore, a profitable deviation for R_i would be to announce C^e according to the equilibrium strategy, but then undercut p^e. A contradiction.

Step 5. *There exists a SPNE where all downstream firms offer $C^* \equiv (0, 0)$ and set a retail price equal to 0.*

Consider the following profile of strategies:

(i) Each downstream firm i offers $C_i = C^*$;

(ii) Each downstream firm i charges $p_i = 0$ as long as there is at least another retailer who offered C^* or if $\min_{j \neq i} w_j < 0$; while it charges $p_i = \min_{j \neq i} w_j$ if $w_j > 0$ for all $j \neq i$;

(iii) Supplier i accepts C_i if and only if $T_i \geq 0$ and

$$T_i + D_i \left(\tilde{\mathbf{p}}^e \left(C^e, C_i \right) \right) w_i \geq 0,$$

where, from Section 4.1, recall that $\tilde{\mathbf{p}}^e \left(C^e, C_i \right) = \left(p^e \left(C^e, C_i \right), p_i^e \left(C_i, C^e \right) \right)$ is the retail price vector in the Nash equilibrium satisfying weak Pareto-dominance of the subgame triggered by the deviation C_i.

Showing that $(i) - (iii)$ identifies a SPNE is immediate. First, note that, given (ii), if all retailers offer C^*, the unique Nash equilibrium of the retail game is such that $p_i = 0$ for all $i = 1, .., N$. Hence, no downstream firm can profitably deviate by changing the retail price only.

I now show that no retailer can also profitably deviate by offering a contract $C_i \neq C^*$. Given (ii), no retailer can gain by offering C_i with $T_i > 0$. This is because S_i's participation constraint would require $w_i < 0$ and in the corresponding subgame $p_j = 0$ for all $j \neq i$ according to (ii). Indeed, the maximal profit that i can expect to make from the retail price game is $-D(0) w_i > 0$, but then to accept C_i supplier S_i would require a fee

$$T_i \geq -D(0) w_i,$$

so that R_i's (ex ante) profit would be

$$-D(0) w_i - T_i \leq -D(0) w_i + D(0) w_i = 0,$$

implying that offering C_i is not profitable. Note that charging $p_j = 0$ for all $j \neq i$ satisfies perfection because this outcome is a Nash equilibrium of the game where at least two downstream firms have zero wholesale prices.

Next, suppose that R_i offers $C_i = (w_i, T_i)$, with $T_i < 0$. This is also not profitable because S_i would refuse such an offer according to (iii). This action is sequentially rational for S_i. Given (ii), S_i's participation constraint implies $T_i \geq -w_i D(0)$ and thus $w_i \geq 0$. Hence, if S_i would accept a negative fee, given (ii), all retailers $j \neq i$ will set $p_j = 0$ and R_i will then gain by not selling. Indeed, if R_i wishes to sell, it must charge a price $p_i = 0$, in this case its total profit would be $-\left(w_i D(0) \right) / N - T_i$ that is clearly lower than $-T_i$. As a consequence, in the subgame following such a deviation R_i would prefer not to sell, so that S_i would make losses by accepting C_i. So it is rational for him to refuse the offer. Showing that for all retailers $j \neq i$ setting $p_j = 0$ satisfies perfection is again immediate since this outcome is a Nash equilibrium of the game where at least two downstream firms have zero wholesale prices.

Perfection must be checked also in all other possible contractual histories — i.e., for all possible off-equilibrium wholesale offers. Assume that $C_i = C^*$. First, if $\min_{j \neq i} \{w_j\} < 0$, retailer's i strategy satisfies perfection from the argument above, and the same is true in all histories such that $\min_{j \neq i} \{w_j\} \geq 0$ and there exists at least another downstream firm offering C^*. Finally, consider the case where all retailers but i offer a wholesale contract such that $w_j > 0$ for all $j \neq i$. In this case $p_i = \min_{j \neq i} w_j$ also satisfies perfection because weak Pareto-dominance implies that no retailer with a wholesale price $w_j > 0$ will charge a price lower that w_j.

Gathering steps **1-5** yields the result. ■

Proof of Lemma 1. Suppose that all retailers offer the efficient contract C^* along the equilibrium path. A collusive equilibrium can then be sustained as long as the self-enforceability conditions (4) and (6) hold. Since $w^c = 0$, it can be easily verified that these two inequalities are met if and only if $\delta \geq (N-1)/N$. Finally, since the self-enforceability is not met when all retailers offer the efficient contract C^* and $\delta < (N-1)/N$, it is immediate to show that, in this region of parameters, the unique SPNE of the game where retailers offer the efficient and competitive contract C^* at every stage entails zero retail prices. ■

Proof of Lemma 2. Given Definition 4, increasing the wholesale price above w^c cannot be profitable for a deviant retailer. Consider then a public deviation where R_i offers $C_i = (w_i, T_i)$ with $w_i < w^c$, given that $C_j = C^c$ for all $j \neq i$. By Definition 2, there exists a unique Nash equilibrium satisfying weak Pareto-dominance of the corresponding subgame that requires all downstream firms $j \neq i$ to price at $p_j = w^c$ while $p_i = w^c - \varepsilon$. Hence, since S_i's participation constraint is binding — i.e., $T_i = -D(w^c)\, w_i$, R_i's profit from such a deviation is

$$\pi_i(w^c|C_i) \approx D(w^c)(w^c - w_i) + D(w^c)\, w_i = \phi(w^c),$$

which concludes the proof. ■

Proof of Lemma 3. The proof of this result is straightforward as explained in the text and is thus omitted. ■

Proof of Lemma 4. I will first show part (i) and then (ii).

Part (i): The argument is straightforward. First, observe that for $w^c = 0$ one has

$$\psi(0; p^c) = \max\left\{ D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), \phi(w^c)\right\} = \phi(p^c) > 0,$$

moreover, for $w^c = p^c$ one has:

$$\psi(p^c; p^c) = \max\left\{ D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), \phi(w^c)\right\} = \phi(p^c) > 0.$$

Next, observe that the function $D(p^c)(p^c - w^c(N-1)/N)$ is strictly decreasing in w^c and positive at $w^c = 0$ for any $p^c > 0$. Moreover, the function $\phi(w^c)$ is strictly concave, it is equal to zero at $w^c = 0$ and features a unique maximum at p^m. Since $\phi(x)$ is single peaked, it must then be the case that, for any $p^c > 0$, there exists a unique threshold $w^c = \underline{w}^c(p^c) < p^c$ which equalizes $D(p^c)(p^c - w^c(N-1)/N)$ and $\phi(w^c)$, and thus minimizes

$$\max\{D(p^c)(p^c - w^c(N-1)/N), \phi(w^c)\},$$

whereby showing the result. Notice also that strict concavity of $\phi(x)$ implies $D'(w^c)\, w^c + D(w^c) >$

0 whenever $p^c \leq p^m$. Hence, for all $w^c < p^c$ one must have

$$-\frac{N-1}{N} - \frac{D'(w^c)\,w^c + D(w^c)}{D(p^c)} \neq 0,$$

which by the Implicit Function Theorem immediately implies that the function $\underline{w}^c(p^c)$ is continuous and differentiable around any point $p^c < p^m$. ■

Part (ii): In order to show this result one first needs to argue that as long as there exists a collusive equilibrium featuring a (strictly) positive retail price in the region of parameters where $\delta < (N-1)/N$, the collusive wholesale contract cannot be the efficient one. The argument is by contradiction. Suppose that there exists a collusive symmetric equilibrium where retailers charge strictly positive retail prices and issue the efficient contract C^*. Then, by the self enforceability constraint (4) it is immediate to verify that secret deviations are always profitable, so that there exists no implicit agreement which sustains a positive price. This, together with suppliers' participation constraint (which in equilibrium must bind) imply that if a collusive equilibrium exists in this region of parameters, it must be either the case that wholesale prices are strictly positive (with negative franchise fees) or strictly negative (with positive franchise fees). Suppose that $w^c < 0$, and take $T^c = -(1/N)\,D(p^c)\,w^c$, then the self-enforceability condition (4) implies:

$$p^c \frac{1 - N(1-\delta)}{N(1-\delta)} + \frac{w^c(N-1)}{N} \geq 0,$$

but this cannot be true in the relevant range of parameters. In fact, for $p^c \geq 0$ and $w^c < 0$ the above condition requires $\delta > (N-1)/1$: a contradiction. Therefore, if a collusive equilibrium exists it must be the case that $w^c > 0$ and $T^c < 0$. ■

Proof of Theorem 2. To begin with, it is useful to rewrite the cartel's program as:

$$\mathcal{P}: \begin{cases} \max_{(p,C)\in\Re^3} D(p)(p-w) - NT \\ \text{s.t.} \\ \frac{1}{N}D(p)\,w + T \geq 0, \\ D(p)(p-w) \geq N\max\left\{(1-\delta)\,D(p)(p-w) + \delta T, (1-\delta)\,\phi(w) + T\right\}, \end{cases}$$

Clearly, the suppliers' participation constraint must be binding in the optimum, as otherwise retailers could profitably decrease the franchise fee or the wholesale price to gain higher profits. Then, $T = -(1/N)\,D(p)\,w$. Substituting this term into the incentive constraint and the objective of program $\mathcal{P}$ one has:

$$\mathcal{P}: \begin{cases} \max_{(p,w)\in\Re^2_+} \phi(p) \\ \text{s.t.} \\ \frac{\phi(p)}{N(1-\delta)} \geq \psi(w;p) \equiv \max\left\{D(p)\left(p - \frac{w(N-1)}{N}\right), \phi(w)\right\}. \end{cases}$$

The cartel program then amounts to find a pair (w^c, p^c) of retail and wholesale prices that maximizes joint revenues $\phi(p)$ subject to the self enforceability constraints.

Showing that for $\delta \geq (N-1)/N$ the monopoly price is self-enforceable and is supported by the efficient contract C^* is immediate and will thus be omitted. I will therefore focus on the case where $\delta < (N-1)/N$. The proof is developed in the steps below.

Step 1. *For $\delta < (N-1)/N$, $p^c > 0$ implies $w^c > 0$.*

This fact can be easily shown by contradiction. Suppose that $w^c = 0$ and $p^c > 0$, then from the self-enforceability constraints one has:

$$\phi(p^c) \geq N(1-\delta)\max\{\phi(p^c), 0\} = N(1-\delta)\phi(p^c),$$

which cannot hold for $\delta < (N-1)/N$.

Step 2. *For $\delta < (N-1)/N$, $w^c > 0$ implies $w^c < p^m$.*

The argument is again by contradiction. Suppose that $w^c \geq p^m$ and that $p^c \geq w^c$ (a condition that I will check later), then strict concavity of $\phi(x)$, together with the fact that $\delta < (N-1)/N$ is equivalent to $1 < N(1-\delta)$, imply:

$$\phi(p^c) \leq \phi(w^c) < N(1-\delta)\phi(w^c),$$

which is incompatible with (6).

Step 3. *For $\delta < (N-1)/N$* and *$p^c \in (0, p^m]$, there exists a unique wholesale price $\underline{w}(p^c) < p^c$ such that:*

$$\underline{w}(p^c) \equiv \underset{w^c \geq 0}{\arg\min}\, \psi(w^c; p^c).$$

See the proof of Lemma 4 part (i).

Step 4. *For $\delta < (N-1)/N$, the monopoly price p^m can be sustained in a collusive equilibrium for $\delta \geq \underline{\delta}^1(N)$, with*

$$\underline{\delta}^1(N) = 1 - \frac{\phi(p^m)}{N\phi(w^m)} < \frac{N-1}{N},$$

as long as the equilibrium contract specifies a wholesale price $w^m > 0$ solving

$$\frac{p^m - w^m}{p^m} = \frac{\phi(w^m)}{\phi(p^m)} - \frac{w^m}{p^m N}.$$

The proof of this result requires to use step 3. Consider an equilibrium such that the monopoly price is self enforceable even below the critical value $(N-1)/N$. For this to be true one must have

$$\phi(p^m) \geq N(1-\delta)\max\left\{D(p^m)\left(p^m - \frac{w(N-1)}{N}\right), \phi(w^c)\right\}, \tag{A.2}$$

implying that:

$$\delta \geq 1 - \frac{\phi(p^m)}{N \max\left\{D(p^m)\left(p^m - \frac{w(N-1)}{N}\right), \phi(w^c)\right\}}.$$

By step 3 we know that there exists a unique value $w^m = \underline{w}(p^m)$ minimizing the right hand side of (A.3) and this value solves

$$D(p^m)\left(p^m - \frac{w(N-1)}{N}\right) = \phi(w^m),$$

so that the wholesale price that maximizes the region of parameters where (A.2) is met is precisely w^m. It then follows that the monopoly price can be sustained by way of the wholesale price w^m in the region of parameters where

$$\delta \geq \underline{\delta}^1(N) \equiv 1 - \frac{\phi(p^m)}{N\phi(w^m)}.$$

Clearly, by construction, below this threshold the monopoly price cannot be sustained. Moreover, it is easy to show that $\underline{\delta}^1(N) < (N-1)/N$. Indeed, for $w^c = p^m$ it follows that

$$\underline{\delta}^1(N) < 1 - \left.\frac{\phi(p^m)}{N \max\left\{D(p^m)\left(p^m - \frac{w(N-1)}{N}\right), \phi(w)\right\}}\right|_{w=p^m} = \frac{N-1}{N}.$$

Step 5. *For $\delta < \underline{\delta}^1(N)$ there exists a unique solution $(\underline{p}, \underline{w}) \in \Re^2_+$ of $\mathcal{P}$ which satisfies the following system of equations:*

$$D(p)\left(p - \frac{w(N-1)}{N}\right) = \phi(w), \tag{A.3}$$

$$\frac{p}{N(1-\delta)} = p - \frac{w(N-1)}{N}. \tag{A.4}$$

The argument follows directly from the fact that for any retail price $p^c > 0$, the cartel's optimal strategy implies $w^c = \underline{w}(p^c)$ in order to minimize the upper bound imposed by incentive compatibility on the strictly concave objective function $\phi(p)$. It then follows that the unique positive solution of $\mathcal{P}$ must solve the system (A.3)-(A.4), which is equivalent to (9) and (10) in the statement of the theorem.

It remains to verify that any collusive strategy must specify a retail price larger than the wholesale price — i.e., $p^c \geq w^c$. The proof of this fact follows immediately from (A.4). Indeed, it is easy to check that for $\delta < (N-1)/N$ it must be:

$$\frac{(N-1)(1-\delta)}{(1-\delta)N-1} \geq 1,$$

so that $p^c \geq w^c$.

Step 6. *For δ close to zero, the unique solution of program $\mathcal{P}$ that is compatible with positive sales must entail $p^c = w^c = 0$.*

Consider first $\delta = 0$. In this case, it is immediate to verify that the solution of the system (A.3)-(A.4) yields $p^c = w^c$ and $p^c \in \Pi \equiv \{p \geq 0 : \phi(p) = 0\}$. Given A1 and A2, it is easy to verify that $\Pi \equiv \{0, \overline{p}\}$. Hence, the only price level compatible with positive sales is $p^c = w^c = 0$.

Next, I show that in a neighborhood of $\delta = 0$ there cannot exist a collusive equilibrium with positive price level, so that for sales to be positive one must have $p^c = 0$. To prove this result one needs to show how the implicit function $p^c(\delta, N)$, which solves the system (A.3)-(A.4), varies around the point $\delta = 0$. Differentiating (A.3)-(A.4), one obtains:

$$\frac{\partial p^c(\delta,N)}{\partial \delta} = \frac{-p^c(\delta,N)\left(D'(w^c(\delta,N))\,w^c(\delta,N) + D(w^c(\delta,N))\right)}{(1-\delta)\,\Delta\left(p^c(\delta,N), w^c(\delta,N), \delta, N\right)}, \tag{A.5}$$

where $\Delta(.)$ is the determinant of the Jacobian corresponding to system (A.3)-(A.4),

$$\Delta(.) = \left(\frac{D'(p^c(\delta,N))\,p^c(\delta,N)}{(1-\delta)N} + D(p^c(\delta,N))\right)(1-\delta)(N-1) + \\ + \left(D'(w^c(\delta,N))\,w^c(\delta,N) + D(w^c(\delta,N))\right)(1-N(1-\delta)).$$

Taking the limit for $\delta \to 0$ and selecting the solution $\overline{p}$ such that $D(\overline{p}) = 0$ one gets:

$$\lim_{\delta\to 0} \left.\frac{\partial p^c(\delta,N)}{\partial \delta}\right|_{w^c(0,N)=p^c(0,N)=\overline{p}} = \frac{ND'(\overline{p})\,\overline{p}}{D'(\overline{p})(N-1)^2} > 0,$$

hence, around $\delta = 0$ there will be no sales as long as one selects the price $p^c(0,N) = \overline{p}$. As a consequence, positive sales around $\delta = 0$ are compatible only with $p^c(0,N) = 0$, where $D(0) > 0$ by definition. This concludes the step.

Step 7. *There exists a lower-bound $\underline{\delta}^0(N) \leq \underline{\delta}^1(N)$ such that for all $\delta \leq \underline{\delta}^0(N)$, the unique solution of $\mathcal{P}$ compatible with positive sales entails $w^c(\delta,N) = p^c(\delta,N) = 0$.*

This result can be easily shown by noticing that at $\delta = 0$ the solution of the system (A.3)-(A.4) entails $p^c(\delta,N) > \overline{p}$. Hence, for all δ close to 0 one must have $p^c(\delta,N) = 0$. Now, observe that for $\delta \to (N-1)/N$ the system (A.3)-(A.4) yields $w^c = 0$ and $\phi(p^c) = 0$, implying once again $p^c \in \Pi$. Taking the solution with the highest price $p^c = \overline{p}$, substituting $\delta = (N-1)/N$ and

$w^c = 0$ into (A.3) one has:

$$\Delta(.) = D'(\overline{p})\,\overline{p}\frac{N-1}{N},$$

and thus:

$$\left.\frac{\partial p^c(\delta, N)}{\partial \delta}\right|_{\delta \to \left(\frac{N-1}{N}\right)^-,\ p^c(.)=\overline{p}} = \frac{-D(0)\,N^2}{D'(\overline{p})(N-1)} > 0,$$

implying that $p^c(\delta, N) < \overline{p}$ for all δ close to $(N-1)/N$.

Observe that for any $\delta \in (0, (N-1)/N)$ the solution of the system (A.3)-(A.4) is continuos since the demand function $D(p)$ is continuos. From step 6 it then follows that there must exist a lower bound $\underline{\delta}^0(N) < \underline{\delta}^1(N) < (N-1)/N$ such that $p^c\left(\underline{\delta}^0(N), N\right) \equiv 0$ for all $\delta < \underline{\delta}^0(N)$. Moreover, from step 6 it must also be $0 < p^c(\delta, N) < \overline{p}$ for all $\delta > \underline{\delta}^0(N)$ and $\delta < (N-1)/N$. Finally, showing that when $p^c(\underline{\delta}(N), N) \equiv 0$ one also has $w^c(\underline{\delta}(N), N) \equiv 0$ is immediate from the system (A.3)-(A.4).

Step 8: *The price $\underline{p}$ is lower than p^m for all $\delta < \underline{\delta}^1(N)$.*

The proof of this claim is immediate and it follows from step 4: there cannot exist a zero-profit contract (w', T') — i.e., with $w'D(p^m) = -T'$ — which allows to sustain the monopoly price p^m in the region of parameters where $\delta < \underline{\delta}^1(N)$.

Finally, the statement of the theorem follows from gathering the results demonstrated in steps 1-8. ■

Proof of Proposition 3. Consider first the case where $\delta \in (\underline{\delta}^0(N), \underline{\delta}^1(N))$, the result follows immediately from differentiation of (9) with respect to δ and N. Indeed, rearranging this equation one gets:

$$\frac{\underline{w}}{\underline{p}} = \left(1 - \frac{1}{N(1-\delta)}\right)\frac{N}{N-1}.$$

Differentiating with respect to N and δ it then follows:

$$\frac{\partial(\underline{w}/\underline{p})}{\partial N} = \frac{\delta}{(1-\delta)(N-1)^2} > 0, \qquad \frac{\partial(\underline{w}/\underline{p})}{\partial \delta} = -\frac{1}{(N-1)(1-\delta)^2} < 0.$$

Since the mark-up m^c is inversely related to the ratio $\underline{w}/\underline{p}$, that is, $m^c = 1 - \underline{w}/\underline{p}$ one immediately gets that m^c is decreasing in N and increasing in δ for $\delta \in (\underline{\delta}^0(N), \underline{\delta}^1(N))$.

Next, consider the region of parameters where $\delta \in (\underline{\delta}^1(N), (N-1)/N)$, in this range the monopoly price is self-enforceable as long as the wholesale price is w^m, where

$$p^m - w^m - \frac{\phi(w^m)}{D(p^m)} + \frac{w^m}{N} = 0.$$

Clearly, since p^m does not depend on δ by **A2**, the wholesale price w^m is not a function of δ so that in the range of parameters under consideration the mark-up is invariant with respect to

δ. Differentiating the above equation with respect to N, instead, one gets:

$$\frac{\partial w^m}{\partial N} = \frac{1/N^2}{-\frac{N-1}{N} - \frac{D'(w^m)w^m + D(w^m)}{D(p^m)}},$$

where, from the strict concavity of $\phi(x)$, it must be $D'(w^m) w^m + D(w^m) > 0$ so that $\partial w^m / \partial N < 0$. This immediately implies that m^c is increasing in N for all $\delta \in (\underline{\delta}^1(N), (N-1)/N)$. ■

Proof of Proposition 4. The proof of this result relies upon the characterization of the solution of program $\mathcal{P}'$. Indeed, suppliers' break-even condition yields $T = -(1/N) D(p) w$, which leads to

$$\mathcal{P}' : \begin{cases} \max_{p \in \Re} \phi(p), \\ \text{s.t. } \frac{\phi(p)}{N(1-\delta)} \geq \phi(p). \end{cases}$$

This program immediately implies that the monopoly profit can be sustained as long as $\delta \geq (N-1)/N$ and that for $\delta < (N-1)/N$ the unique PBE of the game with private contracting implies perfect competition. ■

Proof of Proposition 5. The proof of this result simply follows from the fact that with public contracting inefficient collusion emerges in the region of parameters where $\delta < (N-1)/N$. It is then immediate to show that the critical value $(N-1)/N$ is increasing in N. ■

Proof of Proposition 6. The proof of this statement is immediate since for every $w^c \leq p^c \leq p^m$ one has

$$\phi(w^c) > \phi(\widehat{p}(w^c, p^c)),$$

Hence, (7) implies (15) but the opposite is not true. This provides the result. ■

Proof of Proposition 7. The proof of this result immediately follows from equation (18).

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