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Sorry Winners

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Abstract

Bidders who receive both "common-value" and "private-value" signals about the value of an auction prize cannot fully infer their opponents' information from the bidding, so may overestimate the value of the prize and, subsequently, regret winning. With multiple objects, prices in later auctions provide information relevant to earlier ones, and sequential auctions appear more vulnerable to overpayment and inefficiency than simultaneous auctions. However, aggregating across all auctions in a simple model, winners still earn positive profit ex-post. With information inequality among bidders, the seller's revenue is influenced by two competing effects. On the one hand, simultaneous auctions reduce the winner's curse of less informed bidders and allow them to bid more aggressively. On the other hand, sequential auctions induce less informed bidders to bid more aggressively in early auctions to acquire information.

Keywords: asymmetric bidders, auctions, overpayment.

JEL Classification: D44.

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1. Introduction

During the years 2000-2001, many European countries sold 3G mobile-phone licenses by sequential (ascending) auctions. Prices were generally higher in earlier auctions, and much lower in later ones. For example, in terms of euros per capita, prices were 650 and 615 in the UK (March 2000) and Germany (July 2000) respectively, but were less than 100 in all the year-2001 auctions.[1] Based in part on the evidence provided by the lower prices in later auctions, winners of the earlier ones complained they overpaid and successfully lobbied governments for improved licenses' conditions. Following the results of later auctions, in Germany and the UK winning firms were allowed to share infrastructure building costs (with the approval of the European Commission),[2] in Italy licences were lengthened from 15 to 20 years, and in France (where licenses were sold by a beauty contest at prices higher than in auctions run later) firms were allowed to share costs, licenses were lengthened and prices reduced.[3]

But standard auction theory does not make clear how rational players can overbid during an auction. We argue that, before an auction, bidders generally receive both *common-value* and *private-value* signals about the value of the object on sale.[4] A common-value signal affects the valuation of all bidders alike (e.g., the future demand estimate for mobile-phones) while a private-value signal only affects the valuation of the bidder who receives it (e.g., a firm's production efficiency and cost). When different types of information influence the value of the prize, bidding behaviour depends on all of them but it does not fully reveal any of them. Then even a perfectly rational bidder may overestimate the prize value, if he faces a rival with a large private signal, and hence overbid and regret winning the auction.

[1] The complete sequence of prices in the European 3G auction is: 650 (UK–March 2000), 170 (Netherlands–July 2000), 615 (Germany–July 2000), 240 (Italy–October 2000), 100 (Austria–November 2000), 20 (Switzerland–November 2000), 45 (Belgium–March 2001), 45 (Greece–July 2001) and 95 (Denmark–September 2001). For an analysis of the European 3G auctions and an evaluation of how differences in the auctions' design affected prices, see Klemperer (2002).

[2] It is commonly argued that infrastructure sharing not only reduces costs but also makes collusion among firms easier.

[3] In Germany, winners are also lobbying the government to be allowed to merge keeping all the spectrum blocks acquired in the auction, even if for competition policy this was explicitly forbidden by the auction rules.

[4] Milgrom and Weber (1982) first analyzed auctions where players' payoffs depend on their personal preferences as well as the intrinsic qualities of the prize. Maskin (1992), Jehiel and Moldovanu (2000) and Goeree and Offerman (2003) prove that, with multidimensional signals, auctions can be ex-post inefficient because, for example, a bidder with a high common signal can win against an opponent who has a higher private valuation and, hence, is more efficient. Goeree and Offerman (2002) provide experimental evidence. Similarly, Compte and Jehiel (2002) argue that in ascending auctions more competition does not necessarily promote efficiency. But Pesendorfer and Swinkels (2000) prove that, in multi-object auctions in which bidders receive two-dimensional signals, efficiency is restored as the number of bidders and the number of objects go to infinity. De Frutos and Pechlivanos (1999) and Pagnozzi (2004) analyze the effects of the presence of advantaged bidders in auctions with private and common signals.

In real auctions, we sometimes observe winners unwilling to pay a price they were willing to bid during the auction. For example, soon after winning the 1996 C-Block spectrum auction in the US, NextWave Telecom and General Wireless filed for bankruptcy to avoid paying their bids (Board, 1999). Clearly, winners may obtain additional information from exogenous sources that make them reduce the value estimate of the prize. For instance, a new technical analysis with pessimistic views about the possible use of a mobile-phone license may become available after the end of the auction. But winners may also obtain information from their opponents' bid in other auctions, as the type of information that is revealed by bidders' strategies varies from market to market.

When multiple objects with dependent values are auctioned, the order of sale becomes crucial for information revelation: prices in later auctions are a source of information about the prize value in earlier ones and, hence, may provide bidders with proof of overpayment. In particular, winning a second auction "soon after" winning a first one conveys bad news since it reveals that opponents' estimate of the prize value is lower than a player expected when winning the first auction. In sequential auctions, it is low prices in auctions which are run later that conveys bad news; in simultaneous auctions, it is similar (even if high) prices in auctions which terminate later that conveys bad news. However, in simultaneous auctions bidders obtain more information while they can still modify bidding strategies, and the type of information necessary to induce winners' regret is less likely.[5] So simultaneous auctions reduce the risk of winners' regret. For the same reasons, simultaneous auctions also increase expected efficiency since they allow bidding to depend on more information about the object's value.

But in a simple model, even if with sequential sales winners may learn they are losing money in one auction, aggregating across all auctions they may still earn positive expected profit ex-post. A lower price in a later auction conveys bad news about the profitability of an earlier auction but, at the same time, allows winners to obtain the prize relatively cheaply. So sellers should be cautious when evaluating bidders' complaints of overpayment, if these are based on lower prices than expected in some auctions.

The order of sale is also relevant because it affects the seller's revenue.[6] When information is unequally distributed among bidders, there are two contrasting effects. On the one hand, simultaneous auctions, by revealing more information during the bidding, reduce information asymmetries between bidders and hence

[5] But the kind of bad news a winner obtains in simultaneous auctions can be "worse" on aggregate than the kind of bad news a winner obtains in sequential auctions. The reason is that, in simultaneous auctions, winning a second auction at a similar price after winning a first auction at a high price, can convey negative information about both auctions' profit. On the other hand, in sequential auctions, winning a second auction at a low price after winning a first auction at a high price conveys negative information about the first auction's profit, but it also implies high profit in the second auction.

[6] With multiple objects and private and common signals, the revenue equivalence theorem does not hold since the allocation of prizes among bidders depends on the auction format.

their potential winner's curse, so that less informed bidders can bid more aggressively. On the other hand, sequential auctions give less informed bidders incentive to bid more aggressively in early auctions in order to obtain information which could be valuable in later ones. When the second effect is absent, simultaneous auctions yield a higher expected revenue for the seller than sequential auctions.

Summing up, when selling multiple objects, the auctioneer may prefer a simultaneous auction to a sequential one since a simultaneous auction reduces the risk of winners' regret and increases efficiency. However, which auction type should be chosen to maximize seller's revenue is ambiguous and depends on the specific context.

So our analysis suggests that the European Union could have done better by organizing a simultaneous sale of mobile-phone licenses in all European countries, instead of letting each of them sell sequentially: it would have then been more difficult for winning firms to argue they overpaid and lobby to alter licenses' conditions and reduce the competitiveness of the industry, and efficiency would have been higher.

The effect on governments' revenue of running simultaneous instead of sequential auctions is ambiguous in our analysis.[7] But given the choice of independent sales, to maximize revenue each government had an incentive to run its auction first since uninformed bidders tend to bid more aggressively in earlier auctions. However, with sequential sales, the risk of winners' regret is maximal in the first auction, when less information is available and, hence, auctioning first implies a higher risk of bidders' complaint and litigations.

The paper is organized in two parts. Section 2 starts by considering a single-object auction with both private- and common-value signals and shows how a rational bidder may overpay for an auction prize. We then present two different multi-object auction models that allow us to obtain different insights on the effects of the sale's order on bidders' and sellers' profit. Part I analyzes the first and more symmetric model. Section 3 analyzes sequential auctions and argues bidders may be sorry winners. Section 4 analyzes simultaneous auctions and compares them to sequential ones in terms of possible winners' regret. Part II considers the second simplified model. This is the simplest tractable model that allows us to explicitly solve for equilibrium bidding strategies in both simultaneous and sequential auctions, hence allowing us to compare them in terms of revenue and efficiency, which we do in Section 7. Before that, in Sections 5 and 6 we discuss how winners' regret may (or may not) arise in this different setting. The last

[7] There are other reasons, which are not considered here, why a simultaneous auction may have raised less revenue. For example, signalling among firms is easier during a simultaneous auction and firms can use it to sustain collusion by agreeing on a division of the pie (Brusco and Lopomo, 2002). And punishing deviations from a collusive agreement can also be easier. On the other hand, a simultaneous auction may have eliminated the sharp reduction in the number of bidders after the first European auction. Moreover, it would have been more difficult for firms to "learn to play the game" and develop collusive strategies or mergers on the base of preceding auctions (Klemperer, 2002).

section concludes. Omitted proofs are collected in Appendix A, while the other appendixes discuss two variants of our models and compare static to dynamic auctions.

2. Single-Object Auction

We start by analyzing a single-object auction in order to illustrate that a winner may overestimate the value of the object on sale when observing his competitors bidding aggressively, and end up paying more than the object is actually worth.

An object, for example a mobile-phone license, is sold by an ascending auction with two potential buyers, called E and I_1, whose valuations are:[8]

$$\begin{cases} V_E = \theta + t_E, \\ V_1 = \theta + t_1. \end{cases}$$

We call θ the common-value signal (or simply common signal) and t_i the private-value signal of bidder i (or simply private signal). Bidders' valuations are partly private and partly common: the signal θ affects all valuations alike while the signal t_i only affects the valuation of bidder i. It is assumed that each bidder is privately informed about his private signal t_i but only bidder I_1 knows the common signal θ.[9] Signals are independently and identically distributed.

We can interpret t_i as expressing bidder i's production cost (or efficiency level); or the financial cost that bidder i has to pay in order to raise money to bid, which depends on the particular credit condition he can obtain from financial institution. And θ can be interpreted as an intrinsic characteristic of the object; for example the future level of demand for mobile-phone services, which affects the profitability of the licenses for all bidders.

The next lemma describes equilibrium bidding strategies.

Lemma 1. *In the unique equilibrium of the single-object auction, bidder I_1 bids up to $\theta + t_1$ and bidder E bids up to $2t_E$.*

Proof. First note that it is a dominant strategy for I_1 to bid up to his valuation. If I_1 drops out of the auction at price p, then E expects the common signal to be equal to

$$\mathbb{E}\left[\theta \middle| \theta + t_1 = p\right] = \frac{p}{2},$$

because signals are *i.i.d.*[10] Therefore E expects his valuation to be equal to

$$\mathbb{E}\left[V_E \middle| E \text{ wins at price } p\right] = t_E + \frac{p}{2}.$$

[8] We use an additive value-function as in the "wallet game" of Klemperer (1998) and Bulow and Klemperer (2002), and as in Compte and Jehiel (2002).

[9] Results analogous to those presented in this section would be obtained by assuming that both bidders receive a signal about θ, but no bidder knows θ.

[10] Here and throughout the paper, to save notation we denote by $\mathbb{E}\left[\theta \middle| \mathcal{I}\right]$ the expectation of the random variable whose realization is θ computed by bidder E given the information $\mathcal{I}$.

In an ascending auction, a player bids up to the expected value of the prize, conditional on winning. Hence, E bids up to p^* such that

$$p^* = \mathbb{E}\left[V_E \middle| E \text{ wins at price } p^*\right],$$

which implies $p^* = 2t_E$. ■

We then have the following result:

Proposition 1. *Bidder E wins the auction but obtains negative profit if and only if $t_1 > t_E > \frac{1}{2}(\theta + t_1)$.*

Proof. Bidder E wins the auction if and only if $2t_E > \theta + t_1$. His profit is negative if and only if $t_1 > t_E$. Rearranging yields the statement. ■

Bidder E's profit can be negative if bidder I_1's private signal is sufficiently higher than the common signal, because then I_1 bids aggressively and E rationally expects a high common signal. Knowing his opponent's bid does not tell bidder E all relevant information since I_1's bid is not a sufficient statistic for his private information (as it is the case when private information has only one dimension). Whenever bidder E overpays, the outcome of auction A is also inefficient since E has a lower valuation than I_1 but obtains the prize.[11]

Definition 1. *A* ***sorry winner*** *is a bidder who realizes after winning an auction that the value of the prize is lower than the price he paid.*

So a sorry winner regrets winning at a price he was willing to bid during the auction. In the single-object auction, however, bidder E is not aware he overpaid (until the profit is actually realized). But after an auction ends, the winner may learn additional negative information about the common signal or about his rival's private signal (even before the profit is actually realized). We are going to argue that, when multiple objects are on sale, other auctions become a source of information and the sequence of sale is crucial for the revelation of information. Moreover, when multiple objects are on sale, a bidder may find it convenient to bid more than the expected value of the prize in some auctions, and so willingly overpay (even before any additional information is revealed).

Part I: Model I

Consider an ascending auction for two objects (A and B) – e.g. two mobile phone licenses – with three bidders (E, I_1 and I_2). Bidders' valuations for the

[11] The possibility of inefficiency in auctions with multiple signals is discussed by Maskin (1992), Jehiel and Moldovanu (2001) and Goeree and Offerman (2003).

two objects are:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E = \theta + t_E$		$V_2^B = \theta + t_2$

As in the single-object case, θ is the common signal affecting all valuations alike while t_i is the private signal that only affects the valuation of bidder i.

All bidders' signals (θ, t_E, t_1 and t_2) are independently drawn from a uniform distribution on $[0, 1]$. We assume each bidder is privately informed about his private signal and, in addition, bidders I_1 and I_2 also know θ. We will think of bidders I_1 and I_2 as incumbent firms in market A and B respectively who are better informed about their own market's characteristics (e.g., they are able to better estimate future demand) and are only interested in bidding in their home market. Bidder E is a potential new entrant who competes in both auctions.

We assume E faces different incumbents in the two auctions, so that we do not have to take into account a possible signalling strategy by a single informed incumbent. For example, a single incumbent may want to strategically manipulate his bid in one auction in order to signal misleading information about θ to the entrant and affect his bidding strategy in the other auction. We deliberately neglect these issues in order to focus on the effects of interest.

Bidders I_1 and I_2 know their valuation for the objects on sale. Therefore, in ascending auctions, it is a dominant strategy for them to bid up to their valuation for the objects.

Lemma 2. *It is a dominant strategy for bidder I_1 to bid up to $\theta + t_1$ in auction A and for bidder I_2 to bid up to $\theta + t_2$ in auction B.*

So the bidding strategy of I_1 and I_2 does not depend on the order of sale. On the contrary, the strategy of bidder E may be affected by the order of sale. In each auction, E's bidding behaviour depends on two factors: (i) his expected valuation for the auction's prize, conditional on winning and on all the information available about the common signal; (ii) the information that the bidding process reveals, that could be valuable in the other auction. Both of these factors may be affected by the order of sale. In the following sections, we describe the different bidding strategies of bidder E, depending on the order of sale, and discuss how bidders' profits are affected.

3. Sequential Auction I

Assume the two objects are sold by sequential ascending auctions; without loss of generality, the auction for object A is run first. Then bidder E's strategy in auction B depends on the outcome of auction A.

If E wins auction A at price p_A, then he learns that $\theta + t_1 = p_A$. Therefore, conditional on also winning B at price p_B, he expects the common signal to be equal to:

$$\mathbb{E}\left[\theta \left| \theta + t_1 = p_A,\ \theta + t_2 = p_B \right.\right]$$
$$= \begin{cases} \frac{1}{2}\min\{p_A; p_B\} & \text{if } \max\{p_A; p_B\} \le 1 \\ \frac{1}{2}(p_A + p_B - 1) & \text{if } \min\{p_A; p_B\} < 1 < \max\{p_A; p_B\} \\ \frac{1}{2}\max\{p_A; p_B\} & \text{if } 1 \le \min\{p_A; p_B\}. \end{cases}$$

Bidder E bids up to the expected value of the prize in auction B, conditional on all the information acquired. Hence, we have the following result.

Lemma 3. *In sequential auctions:* (i) *if bidder E wins auction A at price $p_A < 2t_E$, then in auction B he bids up to*

$$\beta_1(t_E, p_A) = \begin{cases} t_E + \frac{1}{2}p_A & \textit{if } \ p_A \le 2(1 - t_E) \\ 2t_E + p_A - 1 & \textit{if } \ 2(1 - t_E) < p_A \le 1 \\ 2t_E & \textit{if } \ 1 < p_A; \end{cases}$$

(ii) *if bidder E wins auction A at price $p_A \ge 2t_E$, then in auction B he bids up to*

$$\beta_1(t_E, p_A) = \begin{cases} 2t_E & \textit{if } \ p_A \le 1 \\ 2t_E + p_A - 1 & \textit{if } \ 1 < p_A \le 2(1 - t_E) \\ t_E + \frac{1}{2}p_A & \textit{if } \ 2(1 - t_E) < p_A. \end{cases}$$

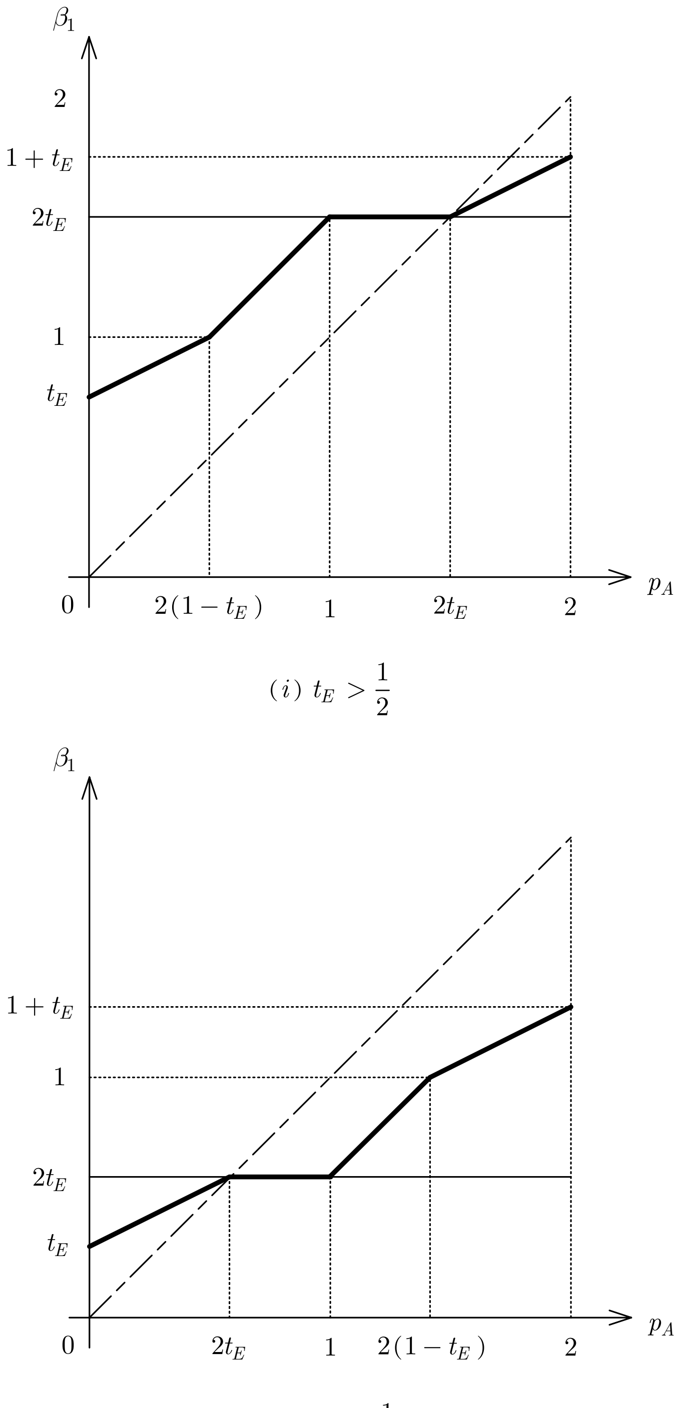
Figure 3.1 represents E's bidding strategy in auction B, conditional on winning auction A at price p_A; in the two cases: $t_E < \frac{1}{2}$ and $t_E > \frac{1}{2}$. Note that, for $p_A \le 2t_E$, $\beta_1(t_E, p_A) \le 2t_E$: winning auction A at a low price is bad news about θ and leads E to bid less aggressively in auction B (compared to a single–object auction).

Consider now the first auction. Raising the price in auction A provides bidder E with valuable information about the common signal. Moreover, after winning at a price lower than $2t_E$ (when the expected value of the object conditional on winning is equal to the auction price), bidder E earns positive expected profit in auction A. Hence, we have the following result.

Lemma 4. *In sequential auctions, bidder E does not drop out of auction A at a price lower than $2t_E$.*

However, we cannot conclude that bidder E drops out at price $2t_E$ in auction A, since he may have an incentive to bid even higher in order to obtain more information about θ and have a better bid in auction B.

Lemma 5. *In sequential auctions, bidder E does not drop out of auction A when the price reaches his expected valuation conditional on winning.*



$(i)\ t_E > \frac{1}{2}$

$(ii)\ t_E < \frac{1}{2}$

Figure 3.1: E's bid in auction B after winning auction A.

The proof of the lemma, which is contained in Appendix A, proceeds as follows. We argue that bidder E is always better off bidding up to $2t_E+\varepsilon$, for ε small enough, rather than dropping out at $2t_E$, regardless of whether, as a consequence, he wins auction A or not. Firstly, if bidder E loses auction A at price $2t_E+\varepsilon$ anyway, he only learns valuable additional information about the common signal, and can bid more accurately in auction B. Secondly, if bidder E wins auction A at price $2t_E+\varepsilon$, then he loses, in expectation, an amount of order ε in auction A. But we prove that, given the additional information about the common signal, bidder E also reduces his bid in auction B by an amount of order higher than ε. And this can prevent bidder E form winning auction B at a price higher than his valuation, and losing an amount of order higher than ε.

So bidder E prefers to bid slightly more than his expected valuation conditional on winning in auction A because, even when this leads him to win and overpay in auction A, he avoids the risk of overpaying much larger amounts in auction B.[12] However, bidder E's equilibrium bidding functions are intractable. (See Appendix A for the extremely complex bidding function in auction B, when bidder E follows even a very simple strategy of dropping out at price $2t_E$ in auction A.)

Summing up, in sequential auctions: (*i*) bidder E bids more aggressively (compared to a single-object auction) in the first auction in order to learn information about the common signal which he can use in the second auction; (*ii*) if he wins the first auction at a price lower than $2t_E$, then bidder E bids less aggressively in the second auction (compared to a single-object auction); (*iii*) if he wins the first auction at a price higher than $2t_E$ or if he loses the first auction, then bidder E bids more aggressively in the second auction (compared to a single-object auction).

3.1. Sorry Winners

As in a single-object auction, bidder E may win auction A but pay more than the object is worth; and, when this happens, the auction is also inefficient ex-post.[13]

Proposition 2. *In sequential auctions, if* $t_1 > t_E > \frac{1}{2}(\theta + t_1)$, *then bidder* E *overpays for object* A.

In addition, since E has an incentive to bid more than his expected valuation conditional on winning, when he does so and wins he overpays and is aware of it as soon as the bidding in auction A terminates.

However, with sequential auctions, E also learns additional information about the common signal from bidder I_2's bidding in auction B, and hence he may also become a sorry winner only after auction B terminates. Indeed, suppose E wins

[12] In Appendix B we provide other examples of this strategic behaviour.

[13] Auction A is also inefficient ex-post whenever I_1 wins but has a lower valuation than E, i.e. if $t_1 < t_E < \frac{1}{2}(\theta + t_1)$.

auction A at price p_A, and then wins auction B at price $p_B < p_A$. This is generally bad news about the common signal and, hence, about his valuation for object A.

Proposition 3 (Sorry Winner in Expectation). *In sequential auctions, bidder E regrets winning A at price p_A after winning B at price p_B if and only if*

$$t_E + \mathbb{E}\left[\theta \left|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right.\right] < p_A.$$

Example 1. *Assume $t_1 = \frac{3}{4}$, $\theta = t_2 = \frac{1}{4}$ and $t_E = \frac{1}{2} + \varepsilon$, where ε is small. Then E wins A at price $p_A = 1$ and B at price $p_B = \frac{1}{2}$. However, after auction B, E expects his value for object A to be $t_E + \mathbb{E}\left[\theta \left|p_A = 1,\ p_B = \frac{1}{2}\right.\right] = \frac{1}{2} + \varepsilon + \frac{1}{4} < p_A$.*[14]

Hence, bidder E may *expect* to have paid more than the prize value in the first auction if, conditional on the information obtained in both auctions, his expected valuation for A is less than the price he paid.[15] Moreover, after winning auction B, bidder E also learns that θ is at most equal to $\min\{p_B, 1\}$. Therefore, the price in auction B can be so low that bidder E learns to have overpaid for sure in auction A.[16]

Proposition 4 (Sorry Winner with Certainty). *In sequential auctions, after winning both objects bidder E is certain he lost money from object A if and only if $t_E + \min\{p_B, 1\} < p_A$.*

Example 2. *Assume $t_1 = 1$, $\theta = t_2 = 0$ and $t_E = \frac{1}{2} + \varepsilon$, where ε is small. Then E wins A at price $p_A = 1$ but, after auction B, he learns that $\theta = 0$ and hence that he lost $\frac{1}{2} - \varepsilon$ in auction A.*

The possibility that the auction's winner overpays, and can prove it with certainty given the result of a later auction, is relevant from a political point of view since it can create serious embarrassment, for instance, to a government selling an asset.

The auction sequence provides additional information about the common signal and therefore about bidder E's value for A. E may discover that his valuation is lower than he expected based on the bidding behaviour of bidder I_2 and, therefore, he may be a sorry winner. Of course, bidder E may also discover that he lost money in auction A if some information about θ is exogenously revealed after the auction. But our model suggests that, with sequential auctions, the order of sale endogenously reveals relevant information and can induce bidders' regret.

Moreover, the auction sequence affects E's bidding behaviour and may lead him to willingly pay more than his expected valuation in the first auction, in

[14] With signals uniformly distributed on $[0,1]$, for $p_A, p_B \leq 1$, $\mathbb{E}\left[\theta \left|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right.\right] = \mathbb{E}\left[\theta \left|\theta + t_1 = \min\{p_A, p_B\}\right.\right] = \frac{1}{2}\min\{p_A, p_B\}$ since the highest price is uninformative about θ.

[15] Note that bidder E can be a sorry winner in expectation even if he does not actually overpay for object A.

[16] This result is independent of the distribution of signals.

order to improve his bid in the second one. In this case, bidder E may also earn negative expected profit on aggregate (i.e. considering both auctions) if, for instance, he overpays in auction A and then just breaks even in auction B.

4. Simultaneous Auction I

Consider now a sale by a simultaneous auction, in which the prices for the two objects rise simultaneously and continuously. Each auction terminates when one of the bidders drops out. In this context, player E's bidding in each auction depends on whether the other auction is still running or not.

If E wins one auction, say auction A, at price p while the other auction is still running, he knows that $\theta + t_2$ is higher than p and hence he expects the common signal to be equal to:[17]

$$\mathbb{E}\left[\theta \left| \theta + t_1 = p,\ \theta + t_2 > p\right.\right] = \begin{cases} \frac{3p-p^2}{6-3p} & \text{if} \quad p < 1 \\ \frac{4-3p^2+p^3}{3(2-p)^2} & \text{if} \quad p \geq 1. \end{cases} \tag{4.1}$$

Bidder E bids up to the expected value of the object in auction A, conditional on the information conveyed by winning.[18] Hence, we have the following result.

Lemma 6. *In simultaneous auctions, if bidders I_1 and I_2 are still active, bidder E drops out of both auction simultaneously at price*

$$\beta_2(t_E) = \begin{cases} \frac{3}{4}\left(1 + t_E - \sqrt{1 - \frac{10}{3}t_E + t_E^2}\right) & \text{if} \quad t_E < \frac{1}{3}, \\ \frac{1}{2} + \frac{3}{2}t_E & \text{if} \quad t_E \geq \frac{1}{3}. \end{cases}$$

Figure 4.1 represents player E's bidding strategy in simultaneous auctions as a function of his signal, conditional on both his opponents being still active. Note that $\beta_2(t_E) > 2t_E$: compared to a single-object auction, simultaneous auctions

[17] Let $X, Y, W \backsim U[0,1]$. Equation (4.1) follows from:

$$\begin{aligned} & \mathbb{E}\left[X \left| X + Y = p,\ X + W > p\right.\right] \\ = & \int x \Pr\left(X \in [x, x+dx] \left| X + Y = p,\ X + W > p\right.\right) \\ = & \int x \frac{\Pr\left(X + Y = p,\ X + W > p \left| X \in [x, x+dx]\right.\right) \Pr\left(X \in [x, x+dx]\right)}{\Pr\left(X + Y = p,\ X + W > p\right)} \\ = & \frac{\int x f_Y(p-x) F_W(p-x) f(x)\, dx}{\int f_Y(p-x) F_W(p-x) f(x)\, dx}. \end{aligned}$$

For example, for $p < 1$, the above expression is equal to $\frac{\int_0^p x(1-p+x)dx}{\int_0^p (1-p+x)dx} = \frac{3p-p^2}{6-3p}$. All other conditional expectations are computed in a similar way.

[18] If prices do not rise simultaneously in all auctions, then in simultaneous auctions, like in sequential auctions, bidder E may have an incentive to raise the price in one of the auctions first, and bid more than his expected valuation conditional on winning, in order to obtain more precise information about θ. See also note 24.

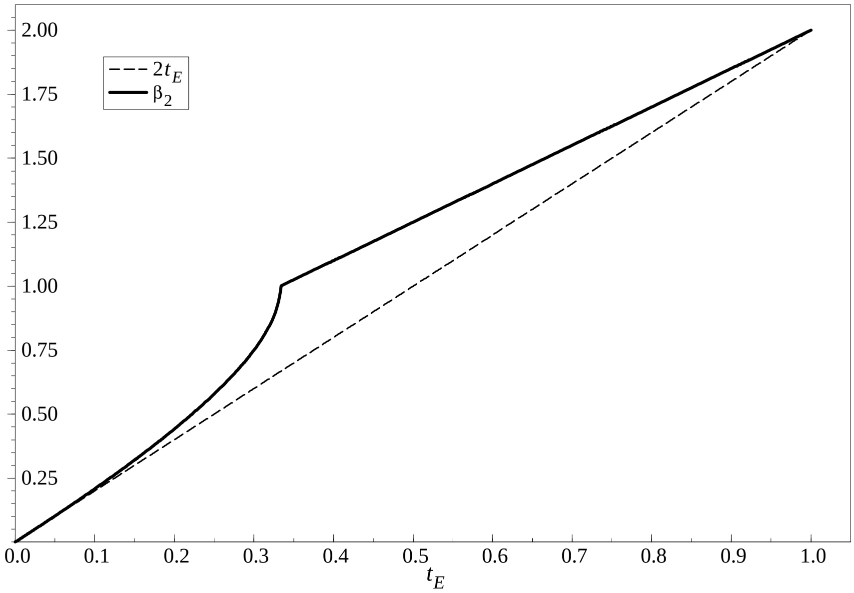


Figure 4.1: E's bid in simultaneous auctions if he wins no object.

allow bidder E to bid more aggressively because, by observing both his opponents' bidding simultaneously, E learns additional information about θ and this reduces his potential winner's curse.

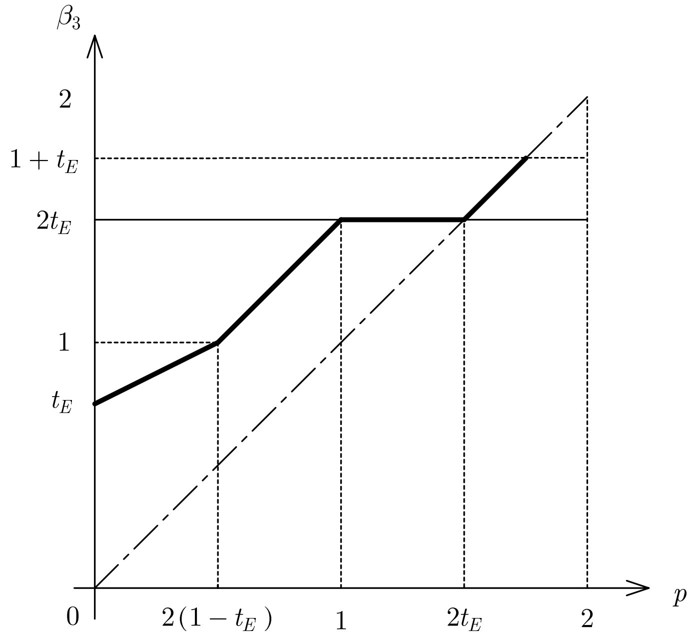
Now suppose E wins one auction, say auction A, at price p_A, and hence learns that $\theta + t_1 = p_A$. Conditional on also winning B at price $p_B \geq p_A$, he expects the common signal to be equal to $\mathbb{E}\left[\theta \left|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right.\right]$. Bidder E bids up to the expected value of the prize in auction B, conditional on all the information acquired. However, if E's expected value is less than the current price, then he drops out immediately of auction B. Hence, we have the following result.

Lemma 7. *In simultaneous auctions:* (i) *if bidder E wins one auction at price $p < 2t_E$, then in the other auction he bids up to*

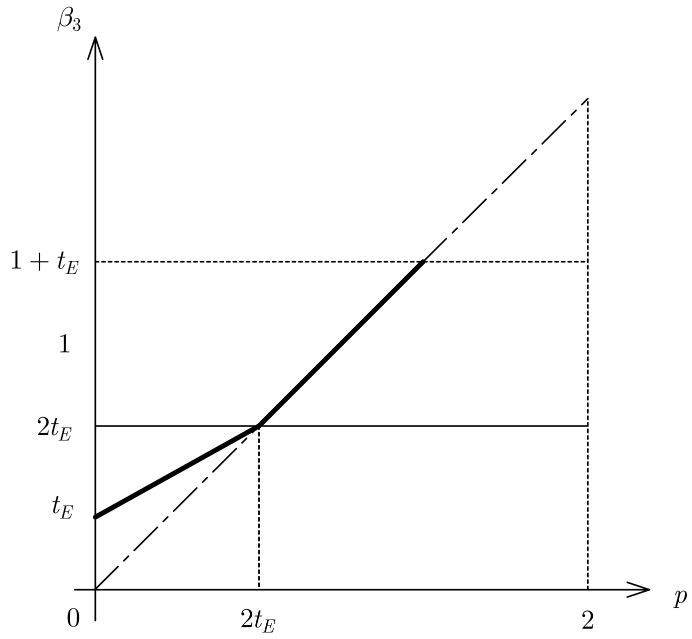
$$\beta_3\left(t_E, p\right) = \begin{cases} t_E + \frac{1}{2}p & \text{if} \quad p \leq 2\left(1 - t_E\right), \\ 2t_E + p - 1 & \text{if} \quad 2\left(1 - t_E\right) < p < 1, \\ 2t_E & \text{if} \quad 1 \leq p; \end{cases}$$

(ii) *if bidder E wins one auction at price $p \geq 2t_E$, then he drops out of the other auction immediately.*

Figure 4.2 represents player E's bidding strategy in one auction, conditional on E winning the other auction at price p. Note that, for $p < 2t_E$, $\beta_3\left(t_E, p\right) \leq 2t_E$: as in sequential auctions, winning one auction at a price lower than $2t_E$ is bad



$(i)\ t_E > \frac{1}{2}$



$(ii)\ t_E < \frac{1}{2}$

Figure 4.2: E's bid in the remaining simultaneous auction, after winning one auction at price p.

news about θ and results in E bidding less aggressively in the other auction (compared to a single-object auction).

Summing up, in a simultaneous auction: (*i*) bidder E bids more aggressively (compared to a single-object auction) in both auction as long as both his competitors remain active; (*ii*) after winning one auction at a relatively low price, bidder E bids less aggressively in the remaining one; (*iii*) after winning one auction at a relatively high price, bidder E drops out immediately in the remaining one.

With sequential auctions the information obtained in later auctions may make winners realize they overpaid in earlier ones. In a simultaneous auction, on the other hand, E's bidding strategy depends on all information revealed by his competitors' bidding, and E learns whether any of them has low signals while the auctions are still running and he can still modify his bidding strategy.

Proposition 5. *In simultaneous auctions, bidder E is never a sorry winner.*

Proof. In a simultaneous auction, bidder E keeps bidding in each auction if and only if his expected valuation, conditional on all the information revealed in both auctions, is higher than the current price. Hence, E is never a sorry winner at the time he wins an auction. However, additional information may be revealed in the other auction, which make him revise his valuation. But if E wins one auction at a price lower than $2t_E$, then any outcome of the other auction leads him to expect a valuation strictly higher than $2t_E$. If, on the other hand, he wins one auction at a price higher than $2t_E$, then he drops out of the other auction immediately, so he has no chance to acquire additional information. Therefore, with simultaneous auctions the outcome of a second auction can never reveal bidder E's valuation to be lower than he price he paid in a first auction. ■

Note that bidders may overpay in a simultaneous auction too, since they can still only estimate the value of the common signal; but a simultaneous auction design does not allow them to realize it during the auction process, and does not endogenously provide evidence of overpayment.

But which of the two multi-object auction format (simultaneous or sequential) raises more revenue for the seller? To answer this question we need to compute E's bidding strategy in sequential auctions. We therefore turn to a different model which eliminates the incentive for E to bid more than his expected valuation in the first of two sequential auctions.

Part II: Model II

In this second part of the paper we analyze a simplified version of Model I, that allows us to compute closed-form equilibrium bidding strategy in sequential auctions and hence compare simultaneous and sequential auctions in terms of seller's revenue and potential inefficiency. Consider an ascending auction for two objects (A and B) with three bidders (E, I_1 and I_2). Bidders' valuations are:

		Bidder E	Bidder I_1	Bidder I_2
Auction	A	$V_E^A = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E^B = 1 + t_E$		$V_2^B = \theta + t_2$

As in our first model, θ represents the common signal and t_i the private signal of bidder i; bidder I_1's and bidder I_2's valuations and bidder E's valuation in auction A are partly private and partly common. Now, however, bidder E's valuation in auction B only depends on his private signal.

We still assume that all bidders' signals (θ, t_E, t_1 and t_2) are independently drawn from a uniform distribution on $[0, 1]$, that each bidder is privately informed about his private signal and that bidders I_1 and I_2 also know θ.

We analyze this "asymmetric" model, in which bidder E's valuation is different in the two auctions, in order to be able to solve for equilibrium bidding strategies and focus on the effects of interest, without having to take into account more complex strategic bidding. When, as in Model I, bidder E's valuation in auction B depends on θ too, additional strategic effects arise. For example, as proved in Lemma 5, in sequential auction E may want to bid more than his expected valuation in auction A, and lose money in that auction if he happens to win, in order to obtain more precise information about θ and be able to make a better bid in auction B. With the value functions of Model II there is no such incentive because, during auction A, bidder E can learn no information relevant for auction B.

Our results, however, do not depend on E's valuation in auction B being higher than his valuation in auction A. In Appendix C we show that the result of Proposition 6 still holds in a different simplified model in which E's valuation is the same in both auctions.

Bidders I_1 and I_2 and bidder E in auction B all know their valuations for the objects on sale. Moreover, E's valuation for B is always higher than his valuation for A and hence, even with simultaneous sales (as long as prices rise simultaneously), E always drops out of auction A at a price lower than $1 + t_E$. So E has no incentive to bid over his valuation in auction B.[19] Therefore neither player I_1's and player I_2's bidding strategies nor player E's bidding strategy in auction B are affected by the order of sale.

[19] If E can affect the pace of the auctions in simultaneous sales, or if the objects are sold in the sequence B-A, this is not true anymore since E may want to bid over $1 + t_E$ in auction B in order to learn additional information about $\theta + t_2$ and hence have a more precise estimate of θ and a better bid in auction A.

Lemma 8. *It is a dominant strategy for bidder I_1 to bid up to $\theta + t_1$ in auction A, for bidder I_2 to bid up to $\theta + t_2$ in auction B, and for bidder E to bid up to $1 + t_E$ in auction B.*

In the following sections, we describe the different bidding strategies of bidder E in auction A, depending on the order of sale, and discuss how bidders' and seller's profits are affected.

5. Sequential Auction II: A–B

Consider a sale by sequential ascending auctions, and assume the auction for object A is run first. In this model, as we discuss later, the auction sequence matters.

Lemma 9. *In sequential auctions, bidder E bids up to price $2t_E$ for object A.*

Basically, as far as the bidding is concerned, the two sequential auctions are like two separate single-object auctions, because in the first auction bidder E can obtain no useful information for the second auction. Exactly as in a single-object auction, and as in Model I, with sequential auctions bidder E wins auction A but pay more than the object is worth if and only if $t_1 > t_E > \frac{1}{2}(\theta + t_1)$. (See Propositions 1 and 2.)

With sequential auctions, E learns additional information about the common signal from bidder I_2's bidding in auction B, and hence he may become a sorry winner. Indeed, winning auction B at a low price after winning auction A at a higher price is bad news about the common signal and, hence, about E's valuation for object A. Moreover, after winning auction B, bidder E also learns that θ is at most equal to $\min\{p_B, 1\}$. Proposition 3 and Proposition 4 still hold: bidder E *expects* to have overpaid in the first auction if, conditional on the information obtained in both auctions, the expected value of A is less than the price he paid;[20] and the price in auction B can even be so low that bidder E is *certain* to have overpaid in the first auction.

5.1. Sorry Winners?

We argued that, with sequential auctions, bidder E may obtain negative profit in auction A. But can bidder E obtain negative aggregate profit (i.e., considering both objects)?

Proposition 6 (No Sorry Winner on Aggregate). *Aggregating across both auctions, bidder E always earns non-negative expected profit ex-post.*

[20] But, in this model, because bidder E has no incentive to overbid in auction A, he can never expect to have overpaid before auction B starts.

Proof. If E wins only one auction, or if he wins both auctions at increasing prices (i.e., $p_A < p_B$), then he expects positive profit in each auction. Assume therefore that E wins both auctions and $p_A \geq p_B$. His total expected profit is:

$$\begin{aligned}\mathbb{E}[\pi_E] &= \mathbb{E}\left[V_E^A \,|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right] + V_E^B - p_A - p_B \\ &= \mathbb{E}[\theta\,|\theta + t_1 = p_A,\ \theta + t_2 = p_B] + 1 + 2t_E - p_A - p_B.\end{aligned}$$

For $p_A \geq p_B$, $\mathbb{E}[\theta\,|\theta + t_1 = p_A,\ \theta + t_2 = p_B] \geq \mathbb{E}[\theta\,|\theta + t_1 = p_B,\ \theta + t_2 = p_B] = \frac{p_B}{2}$.[21] Then:

$$\mathbb{E}[\pi_E] \geq 1 + 2t_E - p_A - \frac{p_B}{2} \geq 0,$$

since $2t_E \geq p_A$ and $2 \geq p_B$. ■

Therefore, our model suggests that, although in a sequential auction a bidder may overpay for the first object on sale and discover this when winning the second one at a low price, his expected total profits after the auctions are still positive. The bad news about the common signal conveyed by a low price in the second auction is also itself a good news about profit in the second auction. For example, in the European 3G spectrum auctions, telecom firms may have regret winning in the UK and Germany at relatively high prices given the outcome of later auction;[22] but they were mainly the same firms that won all auctions and the price they paid in later auctions was so low that, on aggregate, they were arguably still earning positive profit. Governments should be suspicious of winning firms complaining they overpaid, and should not accept lobbying based on lower prices in later auctions with the same winners.

In our model bidder E's value for object B is higher than his value for object A. It may seem that the "no sorry winner" result rests on this assumption. In Appendix C we obtain an analogous result for a more symmetric model, in which E's valuation is the same in both auctions. Of course, it is still possible to construct a model in which E obtains negative expected profit on aggregate, for example if he does not participate to the second auction at all, but only observes its outcome.

6. Simultaneous Auction II

Consider now a sale by a simultaneous auction, in which the prices for the two objects rise simultaneously and continuously. In this case, player E's bidding in auction A depends on whether he wins auction B or not. In particular, auction A is analogous to a single-object auction in which E learns when the current price is equal to $\theta + t_2$.

[21] Here we are using the fact that, with uniform distribution, $\mathbb{E}[\theta\,|\theta + t_1 = \theta + t_2 = p] = \frac{p}{2}$ and $\mathbb{E}[\theta\,|\theta + t_1 = p_1,\ \theta + t_2 = p_2]$ is increasing in p_1 and p_2.

[22] In terms of euros per capita, auction prices in Italy, for instance, where less than half (about 2/5) auction prices in Germany.

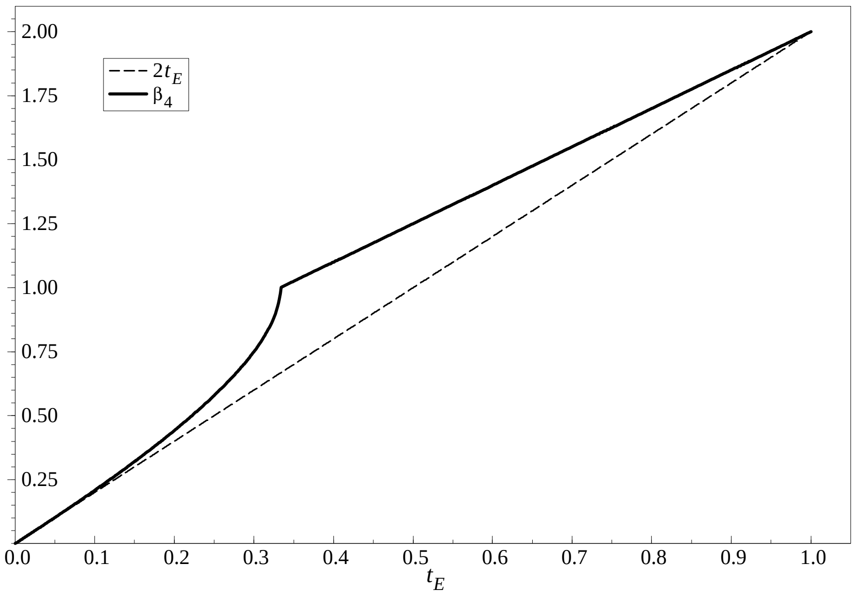


Figure 6.1: E's bid in simultaneous auction A if auction B is still running.

If E wins auction A at price p_A while auction B is still running, he knows that $\theta + t_2$ is higher than p_A and hence he expects the common signal to be equal to:

$$\mathbb{E}\left[\theta \left|\theta + t_1 = p_A,\ \theta + t_2 > p_A\right.\right] = \begin{cases} \frac{3p_A - p_A^2}{6-3p_A} & \text{if} \quad p_A < 1 \\ \frac{4-3p_A^2+p_A^3}{3(2-p_A)^2} & \text{if} \quad p_A \geq 1. \end{cases}$$

Bidder E bids up to the expected value of the object in auction A, conditional on the information conveyed by winning. Hence, we have the following result.

Lemma 10. *In simultaneous auctions, if bidder I_2 is still active in auction B, then bidder E drops out of auction A at price*

$$\beta_4\left(t_E\right) = \begin{cases} \frac{3}{4}\left(1 + t_E - \sqrt{1 - \frac{10}{3}t_E + t_E^2}\right) & \textit{if} \quad t_E < \frac{1}{3} \\ \frac{1}{2} + \frac{3}{2}t_E & \textit{if} \quad t_E \geq \frac{1}{3}. \end{cases}$$

Figure 6.1 represents player E's bidding strategy in auction A, conditional on I_2 being still active in auction B.[23] Note that $\beta_4\left(t_E\right) \geq 2t_E$: not winning auction B is good news about θ, and allows E to bid more aggressively in auction A (compared to a sequential auction). Simultaneous auctions reduce E's potential winner's curse by revealing additional information about θ, so that he may be happy of winning auction A at price higher than $2t_E$.

[23] This figure is identical to Figure 4.1.

If, on the other hand, E wins B at price p_B, then he learns that $\theta + t_2 = p_B$ and, conditional on also winning A at price p_A, he expects the common signal to be equal to $\mathbb{E}\left[\theta \left| \theta + t_1 = p_A,\ \theta + t_2 = p_B \right.\right]$. Then in auction A, bidder E bids up to the expected value of the prize, conditional on the information acquired. If the expected value is lower than the current price, E drops out immediately of auction A. Hence, we have the following result.

Lemma 11. *In simultaneous auctions:* (i) *if bidder* E *wins object* B *at price* $p_B < 2t_E$*, then for object* A *he bids up to*

$$\beta_5\left(t_E, p_B\right) = \begin{cases} t_E + \frac{1}{2}p_B & \text{if} \quad p_B \leq 2\left(1 - t_E\right), \\ 2t_E + p_B - 1 & \text{if} \quad 2\left(1 - t_E\right) < p_B < 1, \\ 2t_E & \text{if} \quad 1 \leq p_B; \end{cases}$$

(ii) *if bidder* E *wins object* B *at price* $p_B \geq 2t_E$*, then he drops out of auction* A *immediately.*

Summing up, with simultaneous auctions, in auction A bidder E: (i) bids more aggressively (compared to sequential auctions) if auction B is still running, (ii) bids less aggressively after winning auction B at a relatively low price, (iii) drops out immediately after winning B at a relatively high price.[24]

In this model, even if bidder E learns more information during the bidding process (and precisely because of this), he can still be a sorry winner in simultaneous auctions. Bidder E may be happy of winning auction A because auction B is still running, but then discover that his opponent's signals in auction B were not so high after all. Winning auction A at a relatively high price, and immediately after winning auction B, conveys bad news about the common signal and may lead E to regret winning auction A.

Proposition 7 (Sorry Winner in Simultaneous Auctions). *In simultaneous auctions, bidder* E *regrets winning auction* A *after winning auction* B *if and only if*

$$\mathbb{E}\left[V_E^A \left| \theta + t_1 = p_A,\ \theta + t_2 = p_B \right.\right] < p_A.$$

[24] If bidder E could control the pace of the auctions (for instance, in the simultaneous ascending auction often used to sell spectrum licenses, by bidding only on one object — but this is often forbidden by the "activity rule" which is designed precisely to prevent strategic delaying of the end of some auctions and to speed up the pace of the auctions), he would complete auction B first, in order to learn as much information as possible about θ (and, in particular, whether $(\theta + t_2) \in [\beta_5, 1 + t_E]$) and hence have a better bid in auction A. The same would happen if auctions are run in the sequence B-A. However, in both these cases E would also have an incentive to bid over $1 + t_E$ in auction B in order to learn more information about θ (exactly as in Model I, with sequential sales, E has incentive to bid more than his expected valuation in the first auction), and this prevents us from explicitly solving for equilibrium bidding strategies in these cases.

Example 3. *Assume that* $t_E = \frac{1}{3}$, $\theta = t_2 = \frac{1}{2}$ *and* $t_1 = \frac{1}{2} - \varepsilon$, *where* ε *is small. Then* E *is willing to bid up to* $\beta_4\left(\frac{1}{3}\right) = 1$ *in auction* A, *and so he wins* A *at price* $p_A = 1 - \varepsilon$ *and* B *at price* $p_B = 1$. *However, after winning* B, E *expects his value for object* A *to be* $t_E + \mathbb{E}\left[\theta \,| p_A = 1 - \varepsilon,\ p_B = 1\right] = \frac{1}{3} + \frac{1-\varepsilon}{2} = \frac{5}{6} - \frac{\varepsilon}{2} < p_A$.

To be a sorry winner in simultaneous auctions, it is necessary that bidder E wins auction B after winning A at a price higher than $2t_E$, which is his expected valuation conditional only on the fact of winning auction A. This is because, if E loses auction B after winning A, then his expected valuation for object A is no lower than right after winning the first auction, and hence he cannot regret winning A. And, for $p_B > p_A$, the condition of Proposition 7 yields:

$$p_A > \mathbb{E}\left[V_E^A \,| \theta + t_1 = p_A,\ \theta + t_2 = p_B\right]$$

$$\begin{aligned}
&\Leftrightarrow \quad \begin{cases} p_A > t_E + \frac{1}{2}p_A & \text{if } p_B \leq 1 \\ p_A > t_E + \frac{1}{2}\left(p_A + p_B - 1\right) & \text{if } p_A < 1 < p_B \\ p_A > t_E + \frac{1}{2}p_B & \text{if } 1 \leq p_A \end{cases} \\
&\Leftrightarrow \quad \begin{cases} p_A > 2t_E & \text{if } p_B \leq 1 \\ \left(p_A - p_B + 1\right) > 2t_E & \text{if } p_A < 1 < p_B \\ 2p_A - p_B > 2t_E & \text{if } 1 \leq p_A \end{cases} \\
&\Rightarrow \quad p_A > 2t_E.
\end{aligned}$$

So, in both simultaneous and sequential auctions, bidder E can be a sorry winner if he wins a second auction soon after winning a first one. In sequential auctions, a bad news about the common signal consists in winning auction B at a low price, after winning A at a high price; in simultaneous auctions, a bad news consists in winning auction B soon after winning A at a high price. Intuitively, the kind of bad news necessary to make E a sorry winner in simultaneous auction is less likely than that in sequential auctions, and this is confirmed by numerical simulations.[25]

Moreover, in simultaneous auction bidder E cannot be a sorry winner with certainty. In fact, E regrets winning auction A in expectation only if $p_B > p_A$ but, in order for E to be sure he overpaid, p_A must be higher than bidder E's highest possible valuation, given the information he obtains from the auctions' outcome, i.e. it is necessary that $p_A > t_1 + \max\{p_A, p_B\} \Rightarrow p_A > p_B$. Also, bidder E cannot be sorry on aggregate. However, in Appendix D we analyze a different version of Model II where bidder E can expect to lose money on aggregate in simultaneous auctions.

[25] The probability of E being a sorry winner in sequential auctions is approximately equal to 0.042, while the probability of E being sorry in simultaneous auctions is approximately equal to 0.016.

7. Revenue and Efficiency Comparison

How does the order of sale in Model II affect the expected revenue of the seller and the efficiency of the final allocation? Players' bidding behaviour in auction B is not affected by whether the seller uses simultaneous auctions or sells the objects sequentially, in the order A-B. But compared to a sequential auction, with simultaneous auctions bidder E bids less aggressively in auction A after winning auction B at a low price, while he bids more aggressively if his opponent does not drop out of auction B. What is the net effect on the seller's revenue?

The expected revenue of the seller from auction B is approximately 0.958; while the expected revenue from auction A is 0.7 in a sequential auction (like in a single object auction), and approximately 0.73 in a simultaneous auction.[26] Therefore:

Proposition 8. *The expected revenue of the seller is higher in simultaneous auctions than in sequential auctions.*

In both types of auctions each bidder wins with probability $\frac{1}{2}$; player I_1's bidding behaviour is the same and bidder E's expected payment conditional on winning the auction is the same. However, in a simultaneous auction bidder I_1's expected payment is higher since E on average bids more aggressively whenever he loses the auction.[27] Hence, the seller should prefer a simultaneous auction, while the better informed bidders should prefer a sequential one. And, for example, in a sale of mobile-phone licenses we expect the incumbent and better informed firms to lobby governments to sell through sequential, rather than simultaneous, auctions.

The intuition for this result is that in auction A bidders are unequally informed about their valuations and hence the less informed bidder has to bid more cautiously in auction A to avoid the winner's curse (since E does not θ but he bids against an opponent who does). So inequality of information reduces the seller's revenue. But then any additional information revealed about θ reduces the inequality of information among bidders and raises revenue. And a simultaneous auction reveals more information than a sequential one.

When bidders receive both private and common value signals about the value of the object on sale, it is possible that the final allocation is inefficient, in the sense that the winner is not the bidder with the highest valuation for the prize. This cannot happen in auction B, since each bidder knows his valuation and bids up to it. On the contrary, the outcome of auction A is inefficient whenever the

[26] With sequential auctions, the expected revenue in auction A is:

$$\mathbb{E}\left[2t_E \left| \theta + t_1 > 2t_E\right.\right] \Pr\left(\theta + t_1 > 2t_E\right) + \mathbb{E}\left[\theta + t_1 \left| 2t_E > \theta + t_1\right.\right] \Pr\left(2t_E > \theta + t_1\right) = 0.7.$$

The expected revenue in the other two cases is computed by numerical simulations.

[27] Numerical simulations show that the expected bid by player E conditional on him losing the auction is higher in a simultaneous auction than in a sequential auction.

winner has a lower private signal then the loser and, since bidder E does not know θ, he only bids based on his expectation of it, conditional on all the information acquired. Therefore, both with simultaneous and sequential sales, E can win even if his private signal is lower than his opponent's one, and he can lose even if his private signal is higher than his opponent's one.[28]

In simultaneous auctions, however, more information about the common signal is revealed and bidder E bids based on a more precise estimate of θ, reducing therefore inefficiency. Numerical simulations show that in sequential auctions the probability of inefficiency is 0.167 and the expected difference between the loser's and the winner's valuation in an inefficient allocation is 0.125; while in simultaneous auctions the probability of inefficiency is 0.135 and the expected difference between the loser's and the winner's valuation in an inefficient allocation is 0.106. Therefore:

Proposition 9. *In simultaneous auctions, both the probability and the expected size of an inefficient allocation of the prize are lower than in sequential auctions.*

With sequential auctions, player E bids for license A like in a single-object auction in which he does not know θ; while in a simultaneous auction, player E bids for license A like in a single-object auction in which he learns when the price equals $\theta + t_2$. In order to further explore the effect of information inequality among bidders on revenue, we compare four single-object auctions, with different information structures and different degrees of information inequality. In each auction, bidders' valuations are the same as in auction A of our multiple-object model. We assume bidder I_1 always knows θ; while bidder E has the following different levels of information:

- Case (i): E does not know θ;
- Case (ii): E learns when the current price equals $\theta + t_2$;
- Case (iii): E learns when the current price equals 2θ;
- Case (iv): E knows θ.

We think of these as four natural cases: in each one bidder E learns more during the auction and the inequality of information with respect to I_1 is reduced. Case (i) corresponds to auction A in a the sequential sale, and E is completely

[28] Both in sequential and in simultaneous auctions, there is inefficiency if and only if one of the following conditions are satisfied:

1. $t_E + \mathbb{E}[\theta \,|E \text{ wins auction } A] > t_1 + \theta$ and $t_1 > t_E$,
2. $t_1 + \theta > t_E + \mathbb{E}[\theta \,|E \text{ wins auction } A]$ and $t_E > t_1$.

And whenever the first condition is satisfied, bidder E also pays object A more than it is worth to him.

uninformed; case (*ii*) corresponds to auction A in a simultaneous sale; and in case (*iv*) both bidders are perfectly (and symmetrically) informed. We think of case (*iii*) as a natural intermediate case: bidder E learn more than in case (*ii*) but is not perfectly informed. In appendix C we discuss case (*iii*) in more details and show how it can arise in a multi-object auction too.

Seller's expected revenue and bidders' expected profits in the four cases are:[29]

	(*i*)	(*ii*)	(*iii*)	(*iv*)
Seller's revenue	0.7	0.73	0.78	0.83
E's profit	0.146	0.152	0.164	$1.\overline{6}$
I_1's profit	0.292	0.27	0.214	$1.\overline{6}$

Hence, information inequality harms the seller and the less informed bidder: as inequality raises, the potential winner's curse of the less informed bidder increases and the seller's revenue is reduced. A reduction in the inequality improves the seller's revenue because it allows the less informed bidder to bid more aggressively and reduces the informed bidder's profit (since it reduces his information rent).

8. Conclusions

When players receive both private- and common-value signals about the value of the object on sale, rational bidders can regret winning an auction and be sorry winners. For multiple-object sales, with sequential auctions lower prices in later auctions can provide proof of overpayment in earlier auctions (as they supposedly did in the European 3G auctions), but winners can still expect positive profit on aggregate as lower prices also imply higher profit in later auctions. So governments should be cautious when evaluating winners' complaints of overpayment in sequential auctions. Nonetheless, sellers may prefer simultaneous auctions since they reduce the risk of sorry winners and lobbying, and increase efficiency.

The revenue comparison between simultaneous and sequential auctions with information inequality among bidders is not univocal. As we have argued in our second model, simultaneous auctions reduce the information inequality by revealing more information during the bidding process and so reduce the potential winner's curse of poorly informed bidders and allow them to bid more aggressively. On the other hand, in sequential auctions the less informed bidder may have an incentive to bid more aggressively in the first auction, in order to learn useful information for later auctions. This effect is present in our first model. It is therefore controversial which auction sequence should be chosen by the seller to maximize revenue. To the extent that the first effect predominates, as it

[29] In case (*iv*), $\mathbb{E}\left[\text{Revenue}\right] = \mathbb{E}\left[\theta\right] + \mathbb{E}\left[t_i \left| t_i < t_j \right.\right] = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, while expected firms' profit is $\mathbb{E}\left[t_i - t_j \left| t_i > t_j \right.\right] \Pr\left(t_i > t_j\right) = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$. The other numbers are obtained by numerical simulations.

does in our simple second model, with inequality of information, incumbent and better informed firms may lobby governments to prevent the use of simultaneous auctions.

Given the choice of sequential sales, when different sellers are involved, our analysis suggests that each seller prefers to run its auction first to maximize revenue, since less informed bidders have an incentive to bid more aggressively in earlier auctions to acquire information. In the European 3G auctions, it appears that the UK government tried hard to avoid postponing its sale, expecting to obtain higher revenue by auctioning before other countries did (see Binmore and Klemperer, 2002).[30]

[30] Running the first auction also maximizes entry since weak bidders have an incentive to not participate to later ones after learning that they have no chance of winning.

Appendix A: Additional Proofs

Proof of Lemma 3. Assume that E wins auction A at price p_A. Then in auction B he bids up to

$$p^* = t_E + \mathbb{E}\left[\theta \left| \theta + t_1 = p_A,\ \theta + t_2 = p^* \right.\right].$$

For $p_A \leq 2t_E$,[31]

$$p^* = t_E + \begin{cases} \frac{1}{2}p_A & \text{if} \quad p_A < p^* \leq 1 \\ \frac{1}{2}(p^* + p_A - 1) & \text{if} \quad p_A < 1 < p^* \\ \frac{1}{2}p^* & \text{if} \quad 1 \leq p_A < p^*. \end{cases} \tag{A.1}$$

If $2(1 - t_E) < 1 < 2t_E$, i.e. if $t_E > \frac{1}{2}$, (A.1) implies

$$p^* = \begin{cases} t_E + \frac{1}{2}p_A & \text{if} \quad p_A \leq 2(1 - t_E) \\ 2t_E + p_A - 1 & \text{if} \quad 2(1 - t_E) < p_A < 1 \\ 2t_E & \text{if} \quad 1 \leq p_A \leq 2t_E. \end{cases}$$

If, on the other hand, $2t_E < 1 < 2(1 - t_E)$, (A.1) implies $p^* = t_E + \frac{1}{2}p_A$.

For $p_A > 2t_E$,

$$p^* = t_E + \begin{cases} \frac{1}{2}p^* & \text{if} \quad p^* < p_A \leq 1 \\ \frac{1}{2}(p^* + p_A - 1) & \text{if} \quad p^* < 1 < p_A \\ \frac{1}{2}p_A & \text{if} \quad 1 \leq p^* < p_A \end{cases} \tag{A.2}$$

If $2t_E < 1 < 2(1 - t_E)$, (A.2) implies

$$p^* = \begin{cases} 2t_E & \text{if} \quad p_A \leq 1 \\ 2t_E + p_A - 1 & \text{if} \quad 1 < p_A < 2(1 - t_E) \\ t_E + \frac{1}{2}p_A & \text{if} \quad 2(1 - t_E) \leq p_A. \end{cases}$$

If, on the other and $2(1 - t_E) < 1 < 2t_E$, (A.2) implies $p^* = t_E + \frac{1}{2}p_A$. ■

Proof of Lemma 4. Notice that:

$$p \leq \mathbb{E}\left[V_E \middle| E \text{ wins } A \text{ at price } p\right] = t_E + \mathbb{E}\left[\theta \middle| \theta + t_1 = p\right] \quad \Leftrightarrow \quad p \leq 2t_E.$$

Therefore, by winning at any price up to $2t_E$, bidder E expects to earn positive profit in auction A since he pays less than his expected valuation, conditional on winning. Moreover, dropping out earlier would only damage him in auction B,

[31] Here we use the fact that, for $X, Y, W \backsim U[0,1]$ and $0 \leq a < b \leq 2$,

$$\mathbb{E}\left[X \left| X + Y = a,\ X + W = b \right.\right] = \begin{cases} \frac{1}{2}a & \text{if} \quad a < b \leq 1 \\ \frac{1}{2}(a + b - 1) & \text{if} \quad a < 1 < b \\ \frac{1}{2}b & \text{if} \quad 1 \leq a < b. \end{cases}$$

since it would prevent him from learning valuable information about θ. So E has no incentive to drop out of auction A before the price reaches $2t_E$. ■

Proof of Lemma 5. Bidder E's expected valuation conditional on winning auction A is equal to $2t_E$. Assume the price has reached $2t_E$ and E bids up to $p = 2t_E + \varepsilon$, for ε small. If E loses the auction anyway, then he learns valuable additional information about θ (i.e., that $\theta > 2t_E + \varepsilon$) and he can bid more accurately in auction B. Therefore, given that he loses auction A anyway, bidder E prefers not to drop out at price $2t_E$.

However, by bidding up to $p = 2t_E + \varepsilon$, bidder E runs the risk of winning auction A and lose, in expectation,

$$p - \mathbb{E}\left[V_E \middle| \theta + t_1 = 2t_E + \varepsilon\right] = 2t_E + \varepsilon - \left(t_E + \frac{2t_E + \varepsilon}{2}\right) = \frac{\varepsilon}{2}.$$

But, also in this case, given the additional information about θ, E is able to bid more accurately in auction B.

If he drops out at price $2t_E$, then conditional on winning auction B at price p, he expects the common signal to be equal to:[32]

$$\mathbb{E}\left[\theta \,|\theta + t_1 > 2t_E,\ \theta + t_2 = p\right] = \begin{cases} \frac{3p^2 - 8t_E^3}{6p - 12t_E^2} & \text{if } 2t_E < p \leq 1 \\ \frac{3 - 8t_E^3 - 3(1-2t_E)(p-1)^2 - 2(p-1)^3}{9 - 3(p-2t_E)^2 - 12t_E} & \text{if } 2t_E < 1 < p \\ \frac{4 + 3p^2 + 6p^2 t_E - 12pt_E - 2p^3}{3\left[(2-2t_E)^2 - (a-2t_E)^2\right]} & \text{if } 1 \leq 2t_E < p. \end{cases}$$

Hence, in auction B he bids up to:

$$\begin{aligned} p' &= \mathbb{E}\left[V_E \,|\theta + t_1 > 2t_E,\ \theta + t_2 = p'\right] \\ &= t_E + \mathbb{E}\left[\theta \,|\theta + t_1 > 2t_E,\ \theta + t_2 = p'\right] \end{aligned}$$

$$\Rightarrow \begin{cases} 3p'^2 - \left(12t_E^2 + 6t_E\right)p' + 20t_E^3 = 0 & \text{if } 2t_E < p' \leq 1 \\ p'^3 + (3 - 9t_E)\,p'^2 + \left(24t_E^2 - 9\right)p' - 20t_E^3 - 12t_E^2 + 15t_E + 2 = 0 & \text{if } 2t_E < 1 < p' \\ p'^3 - 3\,(3t_E - 1)\,p'^2 - 12\left(1 - t_E - t_E^2\right)p' - 24t_E^2 + 12t_E + 4 = 0 & \text{if } 1 \leq 2t_E < p'. \end{cases}$$

$$\Rightarrow p'(t_E) = \begin{cases} 2t_E^2 + t_E\left(1 + \frac{1}{3}\sqrt{36t_E^2 - 24t_E + 9}\right) & \text{if } \quad t_E < 0.3934 \\ 3t_E - 1 + \sqrt{t_E^2 - 6t_E + 4}\begin{bmatrix} \sqrt{3}\sin\left(\frac{\pi}{3} + \phi\right) \\ -\cos\left(\frac{\pi}{3} + \phi\right) \end{bmatrix} & \text{if } \quad 0.3934 \leq t_E \leq 0.5 \\ \frac{1}{2}\left[\left(\sqrt{33} - 5\right) + \left(9 - \sqrt{33}\right)t_E\right] & \text{if } \quad t_E > 0.5 \end{cases}$$

[32] It can be proven that, for $X, Y, W \backsim U[0,1]$ and $0 \leq a < b \leq 2$,

$$\mathbb{E}\left[X \,|X + Y = a,\ X + W > b\right] = \begin{cases} \frac{3a^2 - b^3}{6a - 3b^2} & \text{if } b < a \leq 1 \\ \frac{3 - b^3 - 3(1-b)(a-1)^2 - 2(a-1)^3}{9 - 3(a-b)^2 - 6b} & \text{if } b < 1 < a \\ \frac{4 + 3a^2 + 3a^2 b - 6ab - 2a^3}{3\left[(2-b)^2 - (a-b)^2\right]} & \text{if } 1 \leq b < a. \end{cases}$$

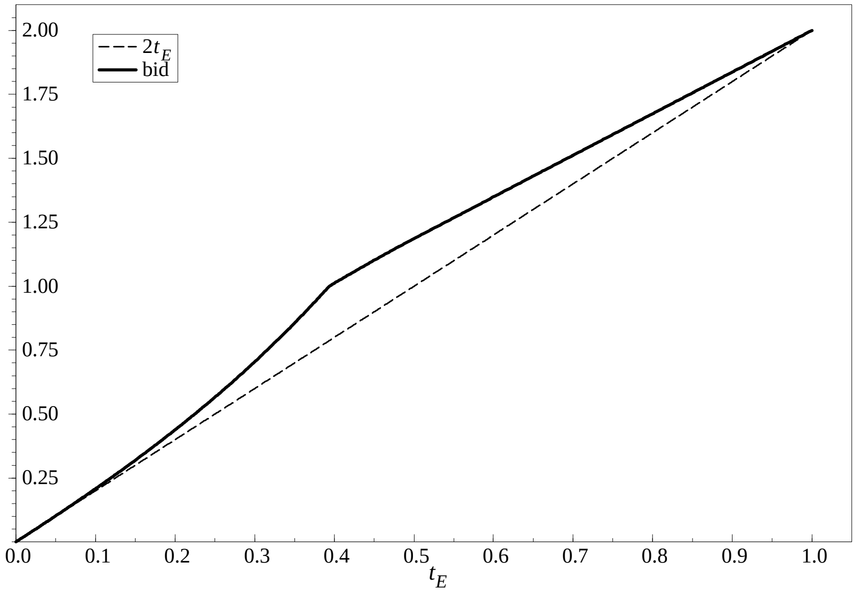


Figure 8.1: E's bid in auction B after losing A at price $2t_E$.

where $\phi = \frac{1}{3}\arctan\left(\frac{\sqrt{36t_E^4-308t_E^3+552t_E^2-372t_E+87}}{2t_E^3-18t_E^2+30t_E-13}\right)$. Figure 8.1 represents E's bid in auction B, conditional him on losing auction A at price $2t_E$. The figure shows that, apart for $t_E = 0$ and $t_E = 1$, $p'(t_E) > 2t_E$.

On the other hand, after winning auction A at price $2t_E + \varepsilon$, in auction B bidder E only bids up to:

$$\begin{aligned} p'' &= \mathbb{E}\left[V_E \left|\theta + t_1 = 2t_E + \varepsilon,\ \theta + t_2 = p''\right.\right] \\ &= t_E + \mathbb{E}\left[\theta \left|\theta + t_1 = 2t_E + \varepsilon,\ \theta + t_2 = p''\right.\right] \\ &\cong t_E + \mathbb{E}\left[\theta \left|\theta + t_1 = 2t_E,\ \theta + t_2 = p''\right.\right] \end{aligned}$$

$$\Rightarrow p''(t_E) = 2t_E.$$

So given the additional information about θ, E is able to reduce his bid in auction B by $(p' - p'')$. This reduction is almost always (i.e. apart for $t_E = 0$ and $t_E = 1$) of finite order (i.e. of order higher than ε); for example, for $t_E = \frac{1}{2}$, $p'\left(\frac{1}{2}\right) = 1.186$, $p''\left(\frac{1}{2}\right) = 1$ and hence $\left[p'\left(\frac{1}{2}\right) - p''\left(\frac{1}{2}\right)\right] = 0.186$.

So, whenever I_2's bid in auction B, i.e. $\theta + t_2$, lies in the interval $[2t_E, p']$ (which happens with a probability of order higher than ε), bidder E avoids overpaying by the difference between I_2's bid and his expected valuation, conditional on I_1's signals being equal to $2t_E + \varepsilon$ and on I_2's bid, that is by:

$$\mathbb{E}\left[\text{Overpayment}\right] = (\theta + t_2) - \mathbb{E}\left[V_E \left|\theta + t_1 = 2t_E + \varepsilon,\ (\theta + t_2)\right.\right]$$

$$
\begin{aligned}
&\cong (\theta+t_2)-t_E-\mathbb{E}\left[\theta\left|\theta+t_1=2t_E,\ (\theta+t_2)\right.\right] \\
&= (\theta+t_2)-t_E-\begin{cases} t_E & \text{if } 2t_E<(\theta+t_2)\leq 1 \\ \frac{2t_E+\theta+t_2-1}{2} & \text{if } 2t_E<1<(\theta+t_2) \\ \frac{\theta+t_2}{2} & \text{if } 1\leq 2t_E<(\theta+t_2) \end{cases} \\
&= \begin{cases} (\theta+t_2)-2t_E & \text{if } 2t_E<(\theta+t_2)\leq 1 \\ 1+\frac{1}{2}(\theta+t_2)-2t_E & \text{if } 2t_E<1<(\theta+t_2) \\ \frac{1}{2}(\theta+t_2)-t_E & \text{if } 1\leq 2t_E<(\theta+t_2) \end{cases}
\end{aligned}
$$

and this is also of order higher than ε. Therefore, bidder E also prefers to win auction A at price $2t_E+\varepsilon$ instead of dropping out at price $2t_E$ since he only suffers a loss of order ε but his expected gain, in terms of expected reduction in overpayment, is of order higher than ε.

Concluding, in auction A bidder E prefers both to win and to lose at price $2t_E+\varepsilon$, for ε small enough, instead of dropping out at price $2t_E$ and hence he bids more than $2t_E$. ■

Proof of Lemma 6. Given that one auction is still running, in the other auction bidder E bids up to p^* such that

$$
\begin{aligned}
p^* &= \mathbb{E}\left[V_E\left|\theta+t_i=p^*,\ \theta+t_j>p^*\right.\right] \\
&= t_E+\mathbb{E}\left[\theta\left|\theta+t_i=p^*,\ \theta+t_j>p^*\right.\right], \quad i,j=1,2, \quad i\neq j;
\end{aligned}
$$

$$
\Rightarrow p^*=t_E+\begin{cases} \frac{3p^*-(p^*)^2}{6-3p^*} & \text{if} \quad p^*<1 \\ \frac{4-3(p^*)^2+(p^*)^3}{3(2-p^*)^2} & \text{if} \quad p^*\geq 1. \end{cases} \tag{A.3}
$$

For $p^*<1$, (A.3) yields $p^*=\frac{3}{4}\left(1+t_E-\sqrt{1-\frac{10}{3}t_E+t_E^2}\right)$ which is the only root lower than 1, for $t_E<\frac{1}{3}$. For $p^*\geq 1$, (A.3) yields $p^*=\frac{1}{2}+\frac{3}{2}t_E$ which is the only root such that $1\leq p^*\leq 1+t_E$,[33] for $t_E\geq\frac{1}{3}$.

Moreover, once bidder E quits one auction at price p^*, he prefers to quit the other auction too. In fact, assume by contradiction that he does not. Then it must be that, for $p'>p^*$,

$$
\mathbb{E}\left[V_E\left|\theta+t_i=p',\ \theta+t_j>p^*\right.\right]>p', \quad i,j=1,2, \quad i\neq j.
$$

But since $\mathbb{E}\left[V_E\left|\theta+t_i=p',\ \theta+t_j>p\right.\right]$ is increasing in p, this implies that

$$
\mathbb{E}\left[V_E\left|\theta+t_i=p',\ \theta+t_j>p'\right.\right]>p',
$$

which is a contradiction to the fact that E chooses to quit one auction at a price lower than p'. ■

[33] Bidder E never wants to bid more than the highest possible value of the prize.

Proof of Lemma 7. The proof is analogous to the proof of Lemma 3 for sequential auctions. Note only that if bidder E wins one auction at a price $\overline{p} \geq 2t_E$, then his expected value of the price, conditional on winning the other auction too at any prize $p > \overline{p}$, is lower than p. Hence, E prefers to lose the other auction and drops out immediately.[34] ■

Proof of Lemma 8. Since E has a pure private value in auction B, he has no incentive to bid over his expected valuation, conditional on winning, for object A. Hence, in auction A he bids up to p_A^* s.t. $p_A^* = t_E + \mathbb{E}\left[\theta \middle| \, \theta + t_1 = p_A^*\right] \Leftrightarrow p_A^* = 2t_E$. ■

Proof of Lemma 10. The proof is analogous to the first part of the proof of Lemma 6. ■

Proof of Lemma 11. The proof is analogous to the first part of the proof of Lemma 7. ■

[34] From Lemma 3, in sequential auctions, E's bid in the second auction, after winning the first one at a price $\overline{p} \geq 2t_E$, is strictly lower than $\overline{p}$. But since prices rise simultaneously in simultaneous auctions, when E wins one auction at a price $\overline{p} \geq 2t_E$, the price in the other auction is also equal to $\overline{p}$. Hence, E prefers to drop out at a price strictly lower than the current one, and so he abandons the auction.

Appendix B: "Overbidding" to Discover Signals

Example B1. With sequential auctions, bidder E may want to bid more than the expected value of the prize in an early auction in order to discover additional information about the common signal and hence bid more accurately in later auctions.

We analyze a simplified version of Model I, that allows us to further investigate the "learning effect" of bidder E's bid. Consider a sequential auction with valuations:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E = \theta + t_E$		$V_2^B = 2t_2$

All signals are uniformly distributed on $[0, 1]$, and only bidder I_1 knows θ. As in Model I, players' bidding behaviour in auction A reveals information about θ which is used by bidder E to determine his bid in auction B. Here however, to simplify we assume that bidding in auction B does not reveal any information about θ.[35]

Assume that $t_E = \frac{1}{2}$ and consider whether E has an incentive in auction A to bid more than $2t_E = 1$, i.e. his expected valuation, conditional on winning. If E drops out of auction A at price 1, then in auction B he bids up to[36]

$$\begin{aligned}\mathbb{E}\left[V_E \,|\theta + t_1 > 1\right] &= t_E + \mathbb{E}\left[\theta \,|\theta + t_1 > 1\right] \\ &= \frac{1}{2} + \frac{2}{3} = \frac{7}{6}.\end{aligned}$$

If instead in auction A bidder E bids up to $1 + \varepsilon$, for ε small, he runs the risk of overpaying for the prize but he also learn valuable information. If he loses the auction, then he has a better bid in auction B (since he learns that $\theta + t_1 > 1 + \varepsilon$) at no cost. If, on the other hand, he wins auction A, then he pays more than his expected valuation and so he expects to lose

$$\begin{aligned}1 + \varepsilon - \mathbb{E}\left[V_E \,|\theta + t_1 = 1 + \varepsilon\right] &= 1 + \varepsilon - t_E - \mathbb{E}\left[\theta \,|\theta + t_1 = 1 + \varepsilon\right] \\ &= 1 + \varepsilon - \left(\frac{1}{2} + \frac{1+\varepsilon}{2}\right) = \frac{\varepsilon}{2}.\end{aligned}$$

[35] In this model, unlike in Model I, bidder E cannot learn to have overpaid in auction A during auction B, since no information about θ is revealed after auction A. However, as we are going to argue, like in Model I he may bid more than the expected prize's value in auction A and hence he may regret winning even before auction B starts.

[36] For $X, Y \sim U\left[0, 1\right]$,

$$\mathbb{E}\left[X \,|X + Y > p\right] = \frac{1}{2}\mathbb{E}\left[X + Y \,|X + Y > p\right] = \frac{1}{2}\begin{cases} \frac{2\left(3-p^3\right)}{3\left(2-p^2\right)} & \text{if } p < 1 \\ \frac{2(1-p)}{3} & \text{if } p \geq 1. \end{cases}$$

However, now bidder E reduces his bid in auction B to

$$\mathbb{E}\left[V_E \left|\theta + t_1 = 1 + \varepsilon\right.\right] = 1 + \frac{\varepsilon}{2},$$

and so he avoids overpaying and losing money in auction B when $2t_E \in \left[1 + \frac{\varepsilon}{2}, \frac{7}{6}\right]$. Therefore, he avoids an expected loss of:

$$\int_{\frac{1}{2}+\frac{\varepsilon}{4}}^{\frac{7}{12}} \left[2x - \left(1 + \frac{\varepsilon}{2}\right)\right] dx = \frac{1}{144} - \frac{1}{24}\varepsilon + \frac{1}{16}\varepsilon^2 > \frac{\varepsilon}{2}.$$

So bidder E is happy of winning A at price $1 + \varepsilon$, since the expected loss he avoids in auction B is higher than his expected loss in auction A. He prefers to bid up to $1 + \varepsilon$, for ε small, instead of quitting when the price is equal to his expected valuation conditional on winning.

Example B2. In the previous example we argued that winning at a price higher than his expected valuation in the first auction can help the less informed bidder to bid more cautiously in the second auction. But also losing at a price higher than his expected valuation reveals information about the common signal and allows the less informed bidder to bid more aggressively in the second auction. Overbidding to obtain such a result is not generally profitable with continuously distributed signals; but may be profitable for the uninformed bidder if signals are distributed over a discrete support.

Consider a sequential auction with valuations:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E = \theta + t_E$		$V_2^B = \theta + t_2$

and assume $t_E = 0.5$ while all the other signals (θ, t_1 and t_2) are equal to 0 with probability $\frac{1}{2}$, and 1 with probability $\frac{1}{2}$. Then:

$$(\theta + t_1) \text{ and } (\theta + t_2) = \begin{cases} 0 & \text{with probability } \frac{1}{4}, \\ 1 & \text{with probability } \frac{1}{2}, \\ 2 & \text{with probability } \frac{1}{4}. \end{cases}$$

As in the main model, we assume each bidder knows his private signal t but only bidders I_1 and I_2 know θ.

We ask whether bidder E should drop out of auction A when the price equals to the expected value of the prize, conditional on winning, i.e. at price $2t_E = 1$. (Note that $\mathbb{E}\left[V_E \middle| E \text{ wins at } p^A = 1\right] = 1$.)

If bidder E bids up to $1+\varepsilon$ and I_1 does not drop out, then he learns $\theta = 1$. In this case, E is better off since he can increase his bid for B from 1 to 1.5. Indeed if $t_2 = 0$ (i.e. with probability $\frac{1}{2}$), the I_2 drops out of auction B at price 1 and E wins the auction earning a profit of 0.5. Moreover, bidding up to $1+\varepsilon$ is not risky since bidder I_1 never drops out of auction A at any price in the interval $(1, 1+\varepsilon]$. Therefore, bidder E prefers bidding higher than $2t_E$ in auction A, in order to discover value of θ.

As argued in these examples, with sequential auctions, overbidding in the first auction to discover information about the common signal can lead the less informed bidder in the second auction to either bid less aggressively and avoid potential losses or to bid more aggressively and obtain additional potential profits.

Appendix C: No Sorry Winners

In this appendix we show that our "No Sorry Winner" result of Model II does not depend on E having a higher valuation in the second auction (but it depends on E not having an incentive to overbid in the first auction to discover information about the common signal). We analyze a modified version of Model I which allow us to solve for E's bidding strategy in sequential auctions.

Consider a sequential auction with two objects and three bidders, whose valuations are:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E = \theta + t_E$		$V_2^B = 2\theta$

As usual, we assume that only bidders I_1 and I_2 know θ. Bidder E has the same valuation for both objects on sale and we prove that, even in this case, he may be a sorry winners in auction A, but always obtains positive expected profit on aggregate.

Note that whenever bidder E wins the second auction, he learns the value of θ. Therefore, he has no incentive to overbid in the first auction to acquire information. The next lemmas describes equilibrium bidding strategies. The proofs are analogous to those of our main models.

Lemma 12. *It is a dominant strategy for bidder I_1 to bid up to $\theta + t_1$ in auction A, and for bidder I_2 to bid up to 2θ in auction B.*

Lemma 13. *Bidder E bids up to $2t_E$ in both auctions.*

Since E always learns the value of θ when he wins the second auction, we have the following result.

Proposition 10. *Bidder E is a sorry winner in auction A if and only if*

$$t_1 > t_E > \frac{1}{2}(\theta + t_1). \tag{C.1}$$

Proof. Whenever condition (C.1) is satisfied, bidder E wins auction A but pays more than his valuation. But (C.1) also implies that $2t_E > 2\theta$, so E wins auction B too, learns θ and discovers he has overpaid. ■

However, in order for E to be sorry in auction A, θ must be relatively small and so E pays a low price in auction B.

Proposition 11. *Aggregating across both auctions, bidder E cannot be a sorry winner.*

Proof. E can only be sorry if he wins both auctions and condition (C.1) is satisfied. In this case E's total expected profit is

$$\begin{aligned}\mathbb{E}[\pi_E] &= 2(\theta + t_E) - p_A - p_B \\ &= 2(\theta + t_E) - (\theta + t_1) - 2\theta \\ &= 2t_E - (\theta + t_1),\end{aligned}$$

which is non-negative because of (C.1) itself. ■

Appendix D: Sorry Winner on Aggregate in Simultaneous Auctions

In Model II we have argued that bidder E cannot expect to lose money on aggregate. However, in this appendix, we show by an example that this can happen in a similar model in which E's valuation in auction B is lower than his valuation in auction A.

Consider a simultaneous auction with two objects and three bidders, whose valuations are:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E^A = \theta + t_E$	$V_1^A = \theta + t_1$	
	B	$V_E^B = 1 + \varepsilon$		$V_2^B = \theta + t_2$

Basically, these are the same valuations as in Model II, apart from V_E^B which, instead of $1 + t_E$, is equal to $1 + \varepsilon$, where ε is small. We assume all variables take the same values as in Example 3, i.e. that $t_E = \frac{1}{3}$, $\theta = t_2 = \frac{1}{2}$ and $t_1 = \frac{1}{2} - \varepsilon$.

Bidder E bids up to his known value $1 + \varepsilon$ in auction B, while he bids up to $\beta_4\left(\frac{1}{3}\right) = \frac{1}{2} + \frac{3}{2}\left(\frac{1}{3}\right) = 1$ in auction A.[37] Therefore bidder E wins A at price $p_A = 1 - \varepsilon$ and, immediately after, he also wins B at price $p_B = 1$. This represents bad news about the common signal for E: after winning auction B, bidder E expects his aggregate profit to be equal to

$$\begin{aligned}
\mathbb{E}\left[\pi_E\right] &= \mathbb{E}\left[V_E^A \,|p_A = 1 - \varepsilon,\ p_B = 1\right] + V_E^B - p_A - p_B \\
&= t_E + \mathbb{E}\left[\theta \,|\theta + t_1 = 1 - \varepsilon,\ \theta + t_2 = 1\right] + 1 + \varepsilon - (1 - \varepsilon) - 1 \\
&= \frac{1}{3} + \frac{1 - \varepsilon}{2} + 2\varepsilon - 1 \\
&= \frac{3}{2}\varepsilon - \frac{1}{6} < 0.
\end{aligned}$$

In this example, bidder E is a sorry winner in auction A because he wins auction B immediately after winning A, at a price close to his reserve price. Moreover, E also loses money on aggregate since his valuations for the two object are similar and, therefore, his profit in auction B are not sufficient to compensate his losses in auction A.

[37] The function $\beta_4\left(t_E\right)$, which is derived in Lemma 10, represents E's bid in auction A when, with simultaneous sales, auction B is still running and hence he knows that the sum of his opponent's signal in auction B is higher than the current price in auction A.

Appendix E: Dynamic vs Static Auctions

In this appendix we reconsider Model I and analyze the use of second-price auctions for simultaneous sales. We compare, in terms of expected seller's revenue, simultaneous second-price auctions to simultaneous ascending auctions; that is a "static format" (a second-price auction) to a "dynamic format" (an ascending auction).

Consider an auction for two objects with three bidders and assume bidders' valuations are as in Model I. Bidder I_1's and bidder I_2's strategy is not affected by the auction format: both bidders bid up to their known valuation. On the contrary, bidder E's strategy depends on the auction format. With second-price auctions, no information about θ is revealed to E before the auctions terminate and, conditional on winning one auction, E only learns that the sum of his opponent's signals in that auction is equal to the price he pays, exactly as he does in a single-object auction.[38] Therefore, as with a single-object, in each auction bidder E bids up to his expected valuation, conditional on winning that auction, i.e. $2t_E$.

On the other hand, with ascending auctions, in each auction bidder E also learns about his opponent's signal in the other auction while the price is rising and, as we have seen in Section 4, his bidding strategy depends on whether he wins or not the other auction. In particular, if the sum of his opponents' signals in one auction is lower than $2t_E$, then, compared to a single-object auction, E bids less aggressively in the other auction; if, on the other hand, the sum of his opponents' signals in one auction is higher than $2t_E$, then, compared to a single-object auction, E bids more aggressively in the other auction.

Numerical simulations show that the expected seller's revenue in simultaneous ascending auctions is approximately equal to 0.146. While the expected seller's revenue in simultaneous second-price auctions is equal to twice the expect revenue in a single-object second-price auction, that is:

$$\begin{aligned}\mathbb{E}\left[\text{Revenue in } 2^{\text{nd}}\text{-price}\right] &= 2\begin{pmatrix}\mathbb{E}\left[2t_E\left|\theta+t_i>2t_E\right.\right]\Pr\left(\theta+t_i>2t_E\right)+\\ \mathbb{E}\left[\theta+t_i\left|2t_E>\theta+t_i\right.\right]\Pr\left(2t_E>\theta+t_1\right)\end{pmatrix}\\ &= 2\left(0.7\right)=1.4.\end{aligned}$$

So, with information inequality, the dynamic format performs better in terms of revenue since more information is revealed during the bidding process and this reduces the potential winner's curse of the less informed bidder, allowing him to bid more aggressively. With no inequality of information, i.e. if both bidders have full information about θ, revenue is even higher (and does not depend on the auction format since in both second-price and ascending auctions it is a dominant

[38] This is not necessarily true if combinatorial bids are allowed since then, conditional on winning both auctions, bidder E learns information about both of his opponents' signals. Hence, static auctions with combinatorial bids are more similar to dynamic auctions.

strategy for each bidder to bid up to his known valuation):

$$\mathbb{E}\left[\text{Revenue with } \theta \text{ known}\right] = 2\left(\mathbb{E}\left[\theta\right] + \mathbb{E}\left[t_i \left| t_i < t_j \right.\right]\right) = \frac{5}{3} = 1.\overline{6}.$$

These results reinforce the conclusion emerged from our analysis that information inequality among bidders hurts the seller, and a reduction of the inequality by the use of auction mechanisms that reveals more information during the bidding process yields higher revenue.

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