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Separation of Functions, Collusion and Supervisors Financial Participation

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Abstract

We derive the optimal principal-supervisor-agent relationship in an incomplete contract setting. We show that while having a three-layer structure with full separation of tasks has no efficiency consequence under a complete contract, it becomes crucial under an incomplete contract and can even lead to a result of irrelevance of the contractual incompleteness. When it is not possible to achieve this outcome, managers non-compliance arises as equilibrium behaviour despite the presence of complying supervisors.

However, although always more efficient relative to a bilateral structure, the three-layer structure is prone to collusion and renegotiation. We show that the response to such drawback is to involve financially the supervisor into the venture, with the extent of the participation depending on how costly it is to collude and renegotiate the contract terms.

Last, we derive the welfare properties of the various contract forms showing that they depend on the relative scarcity of each form of financing (monitoring/non monitoring), on transaction costs of collusion and on the distribution of cash flows relative to the project size. This allows us to derive the conditions for the emergence of a bilateral structure in which the financing and supervisory task are centralised on to the same subject and to argue that riskier firms, with high risk of collusive behaviour, are more likely to be financed by a single monitoring lender and thus face a higher cost of credit.

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Introduction

Why do pure monitors exist? Why, despite their presence, do we observe management misbehaviour? Why do they participate financially into the venture? This paper is an attempt to answer some of these questions.

It is well known that the design of the optimal financial contract when the manager has private information which is costly to verify has in-built problems of commitment to the verification policy (Hart, 1995). While committing to a verification policy ex ante can improve incentives for truth-telling, carrying it out is ex post inefficient, and therefore not credible: parties will renegotiate the contract to eliminate the inefficiency.

The contract must therefore be set so as to eliminate the ex post incentive to renegotiate it. The literature has proposed two ways to achieve this: 1. give the incentives to the principal/financier to monitor so as to induce the agent/manager to truthfully reveal his cash flows (Jost (1996), Persons (1997)); 2. have the agent/manager "misrepresenting" the true state with positive probability, thus using the possibility of collecting a penalty for detected false reporting as an incentive to monitor (Khalil (1997), Persons (1997), Choe (1998), Khalil and Parigi (1998), Menichini and Simmons (2003)). In either case the principal is given the incentives to monitor but the resulting contract is third best.

To mitigate, or even overcome the inefficiencies introduced by contractual incompleteness, we propose to separate tasks and assign the supervising and financing tasks on to two different subjects. We find in particular that, whenever a truth-telling contract is implementable, the no commitment contract performs as well as the second best contract. When only a misrepresentation contract is implementable, the no commitment contract with separation is dominated by the second best one, but dominates always the contract in which both tasks are carried out by the same subject.[1]

However, this solution suffers from other drawbacks. First of all, it works only if there are sufficient cash-flows in the low state to cover observation cost. If not, the only way to induce the supervisor to monitor is to have the entrepreneur cheating with positive probability. The possibility of collecting a penalty for detected false reporting can be used as an incentive to monitor, but a truth-telling contract will no longer be implementable, thus leading to a third best outcome.

Moreover, it opens the possibility of collusion between the entrepreneur and the supervisor at the expenses of the investor both before and after the report has been made. We show that in either case any collusive agreement can be made ineffective by involving financially the supervisor into the venture, thereby providing a rationale for the widespread observation of "monitoring investors".

Last, it leaves room to renegotiation: because the structure of the contract is such that the investor gets paid only in unmonitored states, after a low cash flow report she has always an incentive to persuade the supervisor not to monitor and share the savings in observation cost. Anticipating this, the entrepreneur

[1] This result is based on the assumption that the two subjects have the same cost of capital. A higher cost of monitoring capital (as in Holmstrom-Tirole (1997)) clearly reinforces it.

will always claim low cash flows and the report will no longer be credible. Also in this case, we show that the only way to stop this is to give the supervisor a financial stake, thus confirming the need for monitoring investors.

We then derive the welfare properties of the contract showing that they depend on the relative scarcity of each form of financing (monitoring/non monitoring), on transaction costs of collusion and on the distribution of cash flows relative to the size of the project.

While the role of the first two factors is quite obvious, the role of the distribution of cash flows is less clear. A very spread out distribution increases the collusion stake and, for sufficiently low transaction costs of collusion, reduces the investor's willingness to participate into the venture, requiring the involvement of monitoring capital. When this spread is the highest possible, i.e. all revenues accrue in the good state, and transaction costs are low, then the investor will not participate at all and all capital will have to be provided by the monitor. A bilateral structure will arise and the benefits of separation of tasks will be lost.

This result has important implications as it seems to suggest that riskier firms, i.e. those with distribution of cash flows highly spread out (low collateral ones), with high risk of collusive behaviour, are more likely to be financed by a single monitoring lender and face higher cost of credit.

A last remark concerning some related literature. Multiple relationships as a tool for mitigating incentive problems has also been studied by Persons (1997) Strausz (1997) and Menichini and Simmons (2003). Whereas Persons focuses only on cases in which low state revenues exceed observation cost, thus making truth-telling contracts implementable, Menichini and Simmons look at the complementary parameter space and thus at misrepresentation contracts only. In the present paper, instead, we consider the entire parameter space and thus both truth-telling and misrepresentation contract.

Moreover, Persons studies the properties of the three party contract in a context in which there cannot be separation of tasks -one or two lenders can finance the project- and contracting with each investor entails a fixed cost. He does not derive the optimal ownership structure and does not study the effects of the risk of collusion, as the present paper.

The main difference of the present paper with Menichini and Simmons is that, as well as non contractible monitoring, they study the effects of non observability of the act of monitoring. We assume instead non observability to be restricted to the result, rather than to the act of monitoring. The difference is an important one as it affects the repayments in their paper in such a way to make the resulting optimal contract both collusion-proof and renegotiation-proof. In our paper, instead, this effect on repayments is not present and collusion and renegotiation become an issue again.

Last, Strausz also studies a principal-supervisor-agent contract with moral hazard and non contractible and non observable monitoring, showing that delegation is always preferable to centralisation. However, to induce the agent to choose the high effort level, he must be given an incentive contract. This creates room for collusion between the supervisor and the agent which limits the scope for delegation. In the extreme case of zero cost of collusion the contract without delegation performs as well as the contract with delegation. In our setting, instead, the beneficial effect of delegation remains, even with zero cost of collusion. This is because we allow the supervisor to act as a financier, as well as a monitor.

The paper is organised as follows. The next section sets up the model assumptions; section 2 analyses the bilateral commitment contract; section 3 studies the implications of contractual incompleteness presenting the contract with separation of tasks; section 4 introduces the risk of collusion while section 5 deals with renegotiation; section 6 derives the optimal collusion proof contract; section 7 concludes.

1 Setup and model assumptions

A risk neutral entrepreneur seeks funding to finance an investment project which costs him I and gives him access to a technology generating a state contingent financial return $f(\sigma) = f_\sigma$ with $\sigma \in \{H, L\}$ and $f_H > f_L$. State $\sigma = H$ occurs with probability p.

The necessary funding can be raised from a risk neutral investor demanding an expected rate of return r_P normalised to 1. Let P_s be the state contingent repayments from the entrepreneur to the investor, $s \in \{HL, H, L, LL\}$, with P_H as the repayment if the entrepreneur reports the high state; P_{LL} the repayment if he reports the low state truthfully and is monitored; P_L the repayment if he makes a low report and is not monitored; and P_{HL} be the payment to investors when a low state report is monitored and found to be false.

Realised cash flows are private information to the entrepreneur and unobservable to investors. However, a supervisor can be hired to monitor, at a cost ϕ, the entrepreneur and reveal the realised state. The act of monitoring is observable and it reveals without error, but to the supervisor only, the realised state. Hard evidence about the state must be produced by the monitor to communicate it credibly to other parties and claim the repayment due.

We assume that evidence can be destroyed, but it cannot be forged. This means that, when the low state is realised, the supervisor cannot produce hard evidence showing the occurrence of the high state. However, evidence can be destroyed: after a false low state report, the supervisor can either make the result of the audit public, so as to convict the entrepreneur, or hide it, confirming the false report made by the entrepreneur. To prevent the supervisor from destroying the evidence, she must be paid a premium whenever she detects cheating: defining by S_{LL} the repayment she gets if the audit detects compliance, and by S_{HL} the repayment she gets if the audit detects cheating, we must have $S_{HL} > S_{LL}$.

The capital necessary to finance the project can also be provided by the supervisor at a cost $r_S \geq 1$. When she provides funding, she has the right to a repayment also in unmonitored states: S_H is the repayment she gets if a high report is received, and S_L the repayment if a low report is received and this is not monitored.

We assume that all parties are protected by limited liability, so that repayments to each party are non-negative.

To make the problem interesting, we make moreover the following assumptions: the fixed financing requirement is such that

$$f_L < I; \tag{H.1}$$

otherwise the firm could just pay a constant repayment to the investor in each state s and meet her participation constraint, have no reporting incentive problem and face no monitoring. Under this assumption, a feasible contract will require S_s, P_s to vary by state, which in turn induces incentive constraints on the entrepreneur and on the supervisor. Moreover, in line with the existing literature, we assume that the investment is socially profitable:

$$pf_H + (1-p)f_L - I - \phi > 0, \tag{H.2}$$

i.e. the expected return from the project is sufficiently high to cover the observation cost ϕ.

2 Contractible monitoring

In this section we take as a benchmark the case in which parties can credibly contract on the level of monitoring. Since the separation of tasks does not have any efficiency consequence, we can assume that the supervisory and financing role are centralised on to the same subject, say S. We can therefore set $P_s = 0$, $s \in \{HL, H, L, LL\}$ and work with S_s.

Because of commitment, the contract can include a truth-telling constraint on the firm which is enforced by a possibly random verification policy of investors/supervisor. The direct revelation mechanism involves the firm making a report on the state; hence the contract repayments can be conditioned on the report that is made and, in cases where it is monitored, the truthfulness of the report.

We know from the revelation principle that, if the parties con credibly commit to the terms of the contract, truth-telling can be imposed without loss of efficiency.

If m is the probability with which a low state report is monitored, then since the emtrepreneur is risk neutral, the truth-telling constraint has the form $S_H \leq mS_{HL} + (1-m)S_L$ and the commitment contract problem has the form:

$$\max\ p\left(f_H - S_H\right) + (1-p)\left(f_L - (1-m)S_L - mS_{LL}\right)\tag{2.1}$$

$$\text{st } pS_H + (1-p)[m\left(S_{LL} - \phi\right) + (1-m)S_L] \geq I \tag{2.2}$$

$$mS_{HL} + (1-m)S_L \geq S_H \tag{2.3}$$

$$f_H - S_{HL} \geq 0 \tag{2.4}$$

$$f_H - S_H \geq 0 \tag{2.5}$$

$$f_L - S_L \geq 0 \tag{2.6}$$

$$f_L - S_{LL} \geq 0 \tag{2.7}$$

where (2.2) is the individual rationality constraints for the investor, (2.3) is the truth-telling constraint, (2.4), (2.5), (2.6) and (2.7) are the feasibility and limited liability conditions.

If the observation cost ϕ is too high then monitoring may be undesirable but under assumption H.2, the solution to this problem can be summarised by the following proposition:

Proposition 1 *(Khalil and Parigi 1998) With commitment an optimal contract involving a positive probability of verifying low reports has:*

- *truth-telling and random monitoring;*
- *maximum punishment in false reported states, positive rent to the entrepreneur in truthfully reported high states and zero rent in the low state, whether monitored or not:* $f_H = S_{HL} > S_H = \dfrac{(f_H - f_L)(I - (1-p)f_L) - f_L(1-p)\phi}{p(f_H - f_L) - (1-p)\phi} > f_L = S_L = S_{LL}$;
- *probability of monitoring defined by* $m^c = \dfrac{I - f_L}{p(f_H - f_L) - (1-p)\phi}$;
- *and entrepreneur's maximal utility*

$$EU^c = p\,(f_H - S_H) = Ef - I - m^c(1-p)\phi = \frac{p\,(f_H - f_L)\,(Ef - I - (1-p)\phi)}{p\,(f_H - f_L) - (1-p)\phi}. \tag{2.8}$$

The proof is in KP, but we sketch here its main points.

The truth-telling constraint (2.3) binds in the contract; if it were slack then the firm could reduce m^c saving on observation costs.

It is optimal to set maximum punishment: $S_{HL} = f_H$; since the contract involves truth-telling, the punishment is never paid, but by setting it at the highest level possible, the firm has the incentive to tell the truth with a lower value of m^c and hence lower observation costs.

It is optimal to give the firm a rent in high truthfully reported states, $f_H > S_H$, but no rent in the low state, $f_L = S_L = S_{LL}$. Since the entrepreneur is risk neutral, in itself this relatively high variance of firms net profits across states imposes no loss on it, but having all rent delivered in the high state with the truthful report minimises the incentive to cheat on the state.

These results, derived in Khalil and Parigi (1998), constitute the benchmark with which to confront our model with separation of tasks.

3 Separation of tasks

The drawback with the above formulation is that it leaves room for renegotiation: while committing to a verification policy ex ante can improve incentives for truth-telling, carrying it out is ex post inefficient, and therefore not credible: parties will revise the contract to eliminate the inefficiency.

To make monitoring credible, the lender has to be given the right incentives. Conditional on receiving a low state report, only full reimbursement of the monitoring cost incurred will make the lender indifferent between monitoring and not monitoring and hence induce truth-telling (Jost [3], Persons [8]). However, under limited liability, the reimbursement can be made only out of low state cash flows. If these do not suffice to cover the observation cost ($f_L < \phi$), the lender will strictly prefer not to monitor, thus inducing the entrepreneur to cheat. This, in turn, will restore the incentive to monitor, as the lender can collect a penalty for detected false reporting, and an equilibrium in mixed strategies will arise (Khalil (1997), Persons (1997), Khalil and Parigi (1998)).

Whatever the contract that arises, whether a truth-telling one or a mixed strategies one (misrepresentation), the cited literature finds that it is always suboptimal relative to the commitment contract. To mitigate, or even overcome the inefficiencies introduced by contractual incompleteness, we propose to separate tasks and assign monitoring to a third party, the supervisor. We find in particular that, whenever a truth-telling contract is implementable, the no commitment contract performs as well as the second best contract. When only a misrepresentation contract is implementable, the no commitment contract with separation is dominated by the second best one, but dominates the contract with specialisation.

Before we get into the determination of the properties of each of the above mentioned contract forms, we study the time-line of the game and the subgame starting after the contract has been signed:

- At time zero a financial contract is offered by the entrepreneur to the supervisor and to the investors specifying the repayments due in monitored or non monitored states.
- At time one the state of nature is realised and observed by the entrepreneur only.
- At time two the entrepreneur sends a public report to the investor and to the supervisor, who decides whether to verify the report. These strategies are chosen as mutual best responses.
- At time three, conditional on the reported state and on the result of monitoring, if any, the relevant transfers are made.

At t=2 the decision variable for the entrepreneur is the probability l with which it falsely declares the low state when the high state H has occurred. For the supervisor, the decision variable is the probability m with which she will audit the entrepreneur's low state report $\hat{L}$. As standard in models of this type (e.g. in Khalil [4]), we assume that the entrepreneur never cheats when the low state occurs; and the supervisor never monitors when a high state report is received.

Moreover, because we are analysing the case in which there is separation of tasks, the supervisor has to be paid only when she monitors and we can set $S_H = S_L = 0$.[2]

Given that the entrepreneur's lies with probability l, the supervisor chooses the probability m with which to audit any low income report to maximise her expected profit:

$$E\pi_S = m\left(plS_{HL} + (1-p)\,S_{LL} - (1-p+pl)\,\phi\right). \tag{3.1}$$

[2] Remunerating the supervisor in unmonitored states makes it harder to entice monitoring.

According to whether

$$\frac{\partial E\pi_S}{\partial m} = pl\left(S_{HL} - \phi\right) + (1-p)\left(S_{LL} - \phi\right) \lesseqgtr 0, \tag{3.2}$$

the best response is $0 \leq m \leq 1$.

Given that the supervisor monitors with probability m, the entrepreneur's best reporting strategy l maximises his expected utility:

$$\begin{aligned} E\pi_E \quad &= \quad p\left((1-l)(f_H - P_H) + l\left(m(f_H - S_{HL} - P_{HL}) + (1-m)(f_H - P_L)\right)\right) \\ &\quad + (1-p)\left(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - P_L)\right). \end{aligned} \tag{3.3}$$

and according to whether

$$\frac{\partial E\pi_E}{\partial l} = P_H - P_L - m(S_{HL} + P_{HL} - P_L) \lesseqgtr 0, \tag{3.4}$$

the best response is $0 \leq l \leq 1$.

Last, given that the supervisor monitors with probability m and the entrepreneur lies with probability l, the investor's expected profits are:

$$E\pi_P = p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L + (1-p)mP_{LL} - I. \tag{3.5}$$

According to the size of the observation cost relative to low state revenues, the Nash equilibrium may involve truth-telling or mixed strategies. If low state revenues are sufficiently high to cover the observation costs, both types of equilibria are feasible, but the truth-telling one is best. If instead $f_L < \phi$, no truth-telling equilibrium can arise and some diversion of cash flows will occur in equilibrium.[3]

3.1 Truth-telling: $f_L \geq \phi$

If $f_L \geq \phi$, the payoff from monitoring a truthful low state report is sufficient to cover the supervisor's observation cost and the Nash equilibrium will involve $l = 0$ and $0 < m < 1$.

When $l = 0$, using (3.2), we must have $S_{LL} = \phi$ for $0 < m < 1$ to be a best response. Analogously, when $0 < m < 1$, using (3.4), we must have $P_H \leq m\left(S_{HL} + P_{HL}\right) + (1-m)\,P_L$ for $l = 0$ to be a best response. Taken together these yield the incentive constraints $S_{LL} = \phi$ and $P_H \leq m\left(S_{HL} + P_{HL}\right) + (1-m)\,P_L$.

The firm will therefore choose P_H, P_L, P_{HL}, P_{LL}, S_{HL}, S_{LL} to

$$\max\ p(f_H - P_H) + (1-p)\left(f_L - (1-m)P_L - m(S_{LL} + P_{LL})\right) \tag{3.6}$$

$$\text{st } pP_H + (1-p)(1-m)P_L + (1-p)mP_{LL} \geq I \tag{3.7}$$

$$m\left(S_{HL} + P_{HL}\right) + (1-m)\,P_L \geq P_H \tag{3.8}$$

$$S_{LL} = \phi \tag{3.9}$$

[3] Other equilibria are either inefficient or infeasible (Simmons and Garino 2003). For example, a pooling equilibrium in which the entrepreneur always cheats is not feasible, as this requires a single repayment contract, which, given fixed loan size, has been ruled out by assumption H.1.

and to the limited liability conditions (3.17), (3.18) and (3.19), where (3.7) is the participation constraint on the investor, (3.8) and (3.9) are the incentive constraints.

The properties of this contract form are summarised in the following proposition:

Proposition 2 *When the low state cash flows suffice to cover the observation cost, the principal-supervisor-agent contract has the following properties:*

- *it features truthtelling*[4] *and random monitoring, with*

$$m^t = \frac{I - f_L}{p\,(f_H - f_L) - \phi(1-p)} = m; \tag{3.10}$$

- *the entrepreneur gets no rent in audited states, whether truthful or not, i.e.* $S^t_{LL} + P^t_{LL} = f_L$ *and* $S^t_{HL} + P^t_{HL} = f_H$. *However because the audit reveals compliance, the penalty for misreporting is never paid and the distribution of repayments to the supervisor and the investor is indeterminate. Thus, whenever she audits, the supervisor gets the reimbursement of the audit costs incurred* $S^t_{LL} = \phi$, *whilst the investor gets the residual* $P^t_{LL} = f_L - \phi$;

- *in non audited states only the investor gets paid: whenever a low state report is made and this is not monitored she gets* $P^t_L = f_L$, *whereas, when a high state report is made, she gets*

$$P^t_H = f_L + \frac{(f_H - f_L)\,(I - f_L)}{p\,(f_H - f_L) - \phi(1-p)} < f_H; \tag{3.11}$$

- *the entrepreneur's profits are given by*

$$EU^t = p\left(f_H - P^t_H\right) = Ef - I - m^t(1-p)\phi = \frac{p\,(f_H - f_L)\,(Ef - I - (1-p)\phi)}{p\,(f_H - f_L) - \phi(1-p)} \tag{3.12}$$

The solution to this problem is comparable with the one obtained under contractible monitoring and summarised in Proposition 1. In particular the probability of monitoring is the same across the two contracts and they share the same welfare properties. The only difference with respect to the commitment contract is the presence of the supervisor who gets paid only in monitored states, with the reimbursement of the monitoring cost; the repayment due to the investor in such states is correspondingly reduced by the same amount, as in the commitment contract. Hence, there is no substantial difference with the commitment contract. The inability of the supervisor to commit to monitor is therefore irrelevant, provided each party is assigned a different task and low state cash flows are sufficiently high to cover observation cost. Moreover, because this contract replicates the full second best contract, we deduce that it is always superior to the corresponding bilateral contract with non contractible monitoring.

These results parallel Persons (1997), who has studied the properties of the three party contract in a context in which there is no specialisation of tasks -one or two lenders can finance a project- and contracting with each investor entails a fixed cost. Unlike the present paper, Persons studies only the case

[4] A misrepresentation contract might also arise when $f_L \geq \phi$, but it would be dominated by a truthtelling contract.

in which low state cash flows are sufficiently high to cover observation cost and ranks different contract forms -truthtelling with a single lender, truthtelling with two lenders and misrepresentation with a single lender- according to the project's required investment. He finds, as we do, that it is efficient in monitored states to fully reimburse the monitoring investor for the costs incurred and give the non monitoring one the residual cash flows, thus showing the existence of a conflict between security-holders. However, because of the assumption of fixed contracting cost, the various contract forms cannot be ranked univocally and the result of irrelevance of noncontractibility does not follow.

3.2 Misrepresentation: $f_L < \phi$

When $f_L < \phi$, the payoff from monitoring a truthful low state report is not sufficient to cover the supervisor's observation cost. To be induced to monitor the supervisor must get a payoff also from catching a false low state report, which can be obtained by inducing the entrepreneur to lie with positive probability. The truth-telling equilibrium is therefore lost and some diversion of cash flows will occur in equilibrium.

Using (3.4), the probability of monitoring m is:

$$m = \frac{P_H - P_L}{S_{HL} + P_{HL} - P_L}. \tag{3.13}$$

Using (3.2), the probability of lying is:

$$l = \frac{(1-p)(\phi - S_{LL})}{p(S_{HL} - \phi)}. \tag{3.14}$$

To ensure $0 < m, l < 1$ requires $S_{HL} + P_{HL} > P_H \geq 0$ to give $0 < m < 1$ and $\phi - S_{LL} > 0$; $S_{HL} - \phi > 0$; $pS_{HL} + (1-p)S_{LL} - \phi > 0$ (we cannot have $\phi < S_{LL}$ as $f_L < \phi$ implies that $f_L < S_{LL}$, thus violating the entrepreneur's limited liability).

The firm will choose P_H, P_L, P_{HL}, P_{LL}, S_{HL}, S_{LL} to

$$\max \; p(f_H - P_H) + (1-p)(f_L - (1-m)P_L - m(S_{LL} + P_{LL})) \tag{3.15}$$

$$\text{st } p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L + (1-p)mP_{LL} \geq I \tag{3.16}$$

$$f_H - S_{HL} - P_{HL} \geq 0 \tag{3.17}$$

$$f_L - S_{LL} - P_{LL} \geq 0 \tag{3.18}$$

$$f_L - P_L \geq 0 \tag{3.19}$$

to the incentive constraints defined by the ex post probabilities of monitoring and lying (3.13) and (3.14). Here (3.16) is the lender's participation constraint and (3.17), (3.18) and (3.19) are the limited liability conditions.

The results of this type of contract are summarised in the following proposition:

Proposition 3 *When the low state cash flows are too low to cover the observation cost, the principal-supervisor-agent contract has the following properties:*

- *it features misrepresentation and random monitoring;*

- *whenever there is an audit, the supervisor gets all the available cash flows, i.e.* $S_{LL}^m = f_L$ *and* $S_{HL}^m = f_H$;

- *the investor gets a repayment only in non audited states, i.e. whenever a low state report is made and this is not audited* $P_L^m = f_L$, *or when a high state report is made*

$$P_H^m = f_L + \frac{(f_H - \phi)(I - f_L)}{pf_H - \phi} < f_H; \tag{3.20}$$

otherwise she gets $P_{HL} = P_{LL} = 0$;

- *the probabilities of lying and monitoring are respectively given by:*

$$l^m = \frac{(1-p)(\phi - f_L)}{p(f_H - \phi)}; \tag{3.21}$$

$$m^m = \frac{(f_H - \phi)(I - f_L)}{(f_H - f_L)(pf_H - \phi)}; \tag{3.22}$$

- *the entrepreneur maximal profits equal*

$$EU^m = Ef - I - m^m(1 - p + pl^m)\phi = Ef - I - \frac{(I - f_L)(1-p)\phi}{pf_H - \phi}. \tag{3.23}$$

Remark 1 *This contract unambiguously dominates a contract in which the financing and the supervisory functions are centralised*[5] *(eg a contract à la Khalil and Parigi (1998), in which the investor and the supervisor are a single entity, or a contract à la Khalil and Lawarrée (2002) in which the investor pays out a fixed repayment to the supervisor and incurs the losses or keeps the proceeds of the audit).*

Adding a superscript 1 to the variables to denote the presence of a single investor/monitor, Khalil and Parigi main results are as follows:

$$f_H = S_{HL}^1 > S_H^1 = f_L + \frac{(I - f_L)(f_H - f_L - \phi)}{p(f_H - f_L) - \phi} > f_L = S_{LL}^1 = S_L^1$$

$$l^1 = \frac{(1-p)\phi}{p(f_H - f_L - \phi)} \tag{3.24}$$

$$m^1 = \frac{(I - f_L)(f_H - f_L - \phi)}{[p(f_H - f_L) - \phi](f_H - f_L)}. \tag{3.25}$$

Comparing the probabilities of lying and monitoring in the case with separation and in the case with centralisation, we find that they are higher under centralisation:

$$l^1 = \frac{(1-p)\phi}{p(f_H - f_L - \phi)} > \frac{(1-p)(\phi - f_L)}{p(f_H - \phi)} = l^m \tag{3.26}$$

$$m^1 = \frac{(f_H - f_L - \phi)(I - f_L)}{(f_H - f_L)(p(f_H - f_L) - \phi)} > \frac{(f_H - \phi)(I - f_L)}{(f_H - f_L)(pf_H - \phi)} = m^m, \tag{3.27}$$

which implies that also the expected monitoring costs are higher.

[5] Under the assumption that the cost of monitoring capital is no higher than the cost of non monitoring capital.

This result can be motivated as follows: the centralisation of the financing and the supervisory functions implies that the supervisor is remunerated not only in monitored but also in unmonitored states, with expected profits respectively for the supervisor and for the entrepreneur:

$$\begin{aligned} E\pi_S &= (1-m)(1-p+pl)\,S_L + m\,(plS_{HL} + (1-p)\,S_{LL} - (1-p+pl)\,\phi)\,, \\ E\pi_E &= p\,((1-l)(f_H - S_H) + l\,(m(f_H - S_{HL}) + (1-m)(f_H - S_L))) \\ &\quad + (1-p)\,(m(f_L - S_{LL}) + (1-m)(f_L - S_L))\,. \end{aligned}$$

Each player will maximise its payoff function given the other players action, thus giving two reaction functions and a Nash equilibrium in the probability of auditing (m) and the probability of lying (l). Assuming that a mixed strategy equilibrium arises, this is defined by:

$$\frac{\partial E\pi_S}{\partial m} = pl\,(S_{HL} - \phi) + (1-p)\,(S_{LL} - \phi) - (1-p+pl)\,S_L = 0, \tag{3.28}$$

$$\frac{\partial E\pi_E}{\partial l} = S_H - S_L - m(S_{HL} - S_L) = 0. \tag{3.29}$$

With two parties, the optimal sharing rule gives all of the low state revenues in unmonitored states to the monitor ($S_L^1 = f_L$), which makes the her moral hazard problem more serious, thus increasing the firms probability of cheating (remunerating the monitor also when she does not monitor reduces the marginal benefit of monitoring (3.28) and raises l).

Hence, S_L ought to be reduced to raise the marginal benefit of monitoring. However, S_L cannot be set equal to zero as this has adverse effects on the marginal benefit of lying (3.29) via two channels: 1) it increases the return from cheating $f_H - S_L$, and 2) requires a higher S_H to remunerate the monitoring investor for the capital provided, thus lowering the reward for compliance. Both effects increase the marginal benefit of lying thus increasing m and the expected observation cost.

By introducing separation of tasks, S_L can be set as low as zero, thereby powering the incentive to monitor and lowering l (to restore indifference). Such reduction in S_L has no adverse effect on the incentive to cheat as it is compensated by a rise in P_L, as it can be seen in (3.4). However, the fall in l slackens the investor participation constraint (3.16), thus reducing the repayment due to compensate her for financial participation P_H. This, in turn, determines a reduction in m, via the reduction in the marginal benefit of lying (3.4), and thus an overall reduction in expected observation cost.

This is confirmed by comparing the repayment to the investor in truthful high state report with centralisation S_H^1 with the one obtained under separation P_H^m. S_H^1 is unambiguously higher than under P_H^m :

$$\left(f_L + \frac{(I-f_L)\,(f_H - f_L - \phi)}{p(f_H - f_L) - \phi}\right) - \left(f_L + \frac{(f_H - \phi)(I - f_L)}{pf_H - \phi}\right) = (1-p)\,\phi > 0.$$

4 Collusion

In this section we turn to study whether these contracts are robust to collusive agreements taking place between two of the three contracting parties.

As we have seen in section 2, under contractible monitoring, having the principal monitoring the agent directly leads to no loss in terms of efficiency and leaves obviously no room for collusion. When introducing a three-layer structure, the risk of collusion arises. The higher efficiency gained by the three-tier structure might be partly or entirely lost.

To investigate this issue, we first check the parties which are more likely to be involved in a coalition. Given that the agent has private information and the supervisor has monitoring power, the most natural coalition to form is between **entrepreneur and supervisor:** both parties might have an interest in striking a deal. However, a feasible coalition is also that between **supervisor and investor**: in this case it would be mostly in the investor's interest to make a deal. An **entrepreneur-investor coalition,** instead, is less likely to form: given that both these parties would like to alter the supervisor monitoring decision, such a coalition will not form because neither of them can affect it.

A **supervisor-investor coalition** might emerge after a report $\hat{L}$ has been received. In such circumstances, because the structure of the repayments is such that after a report $\hat{L}$ the investor gets f_L if the supervisor does not monitor, and at most $f_L - \phi$ if she does monitor,[6] the investor might try to persuade the supervisor not to monitor and share the saving in observation cost. However, because such agreement would not penalise the entrepreneur, it would be a renegotiation rather than a collusive agreement and we will deal with it in the next section.

As regards the collusive agreement between the entrepreneur and the supervisor, notice first of all that, because monitoring is public, the supervisor cannot claim to have monitored while she has not. However, because monitoring involves randomisation by the supervisor, there ought to be public display of this process (i.e. even though the firm and the investor can compute the equilibrium probability of monitoring, to ensure that the supervisor actually uses this needs a public lottery). If there is no such a public device, then the supervisor can behave opportunistically and decide not to monitor if she finds convenient to do so, even if the randomisation process prescribes to do it. Hence, although the act of monitoring is verifiable, the randomisation process is not.

We can therefore distinguish between two cases:

- offer made by the supervisor

  1. **after monitoring**, there's no point in making any offer: whatever the outcome of the audit, there is no offer the supervisor wants to make

[6] This is paid only under a truth-telling contract if a monitored low report is found to be true. The investor gets always zero instead if monitoring detects false reporting ($P_{HL} = 0$).

2. **after the report, but before monitoring:** the supervisor has an interest in making an offer only when the $\hat{L}$ report is received, not when the $\hat{H}$ report is received. If the $\hat{L}$ report is received, she gets $S_{LL} = \phi$ for sure under a truth-telling contract and expects to get $(1-p)(S_{LL}-\phi)+pl(S_{HL}-\phi)=0$ under a misrepresentation contract. Since she expects to get at most zero, she is ready to give up monitoring for any bribe $b>0$. The acceptance by the entrepreneur reveals his type: a true low type cannot afford any deal as all the low state resources under no monitoring are exhausted in repaying the investor, $f_L = P_L$. Thus, only the H type can afford to make a deal and acceptance reveals that he had cheated. However, the high type entrepreneur might strategically reject the offer, so as to signal to be truly low type and not be monitored. Anticipating this, there will be no offer that the supervisor is willing to make and thus no collusive agreement can occur after the report has been made.

3. **after the realisation of the state, but before the report**: also in this case the supervisor could try to convince the entrepreneur to make her a transfer $b>0$ in exchange for no monitoring. While the true low type, for the same reasons as in case 2, has no interest in accepting such an offer, the high type could accept, thus revealing his type, and then declare $\hat{L}$. He could, however, strategically reject the offer, so as to signal to be truly low type and not be monitored. Foreseeing this, the supervisor will not make any collusive offer.

- offer made by the entrepreneur:

1. **after monitoring**: following a low state report, the supervisor detects noncompliance and does not report it in exchange for a bribe from the entrepreneur. Such an agreement is clearly conceivable only under a misrepresentation contract, but never under a truth-telling contract, in which the supervisor never detects noncompliance. However, because there cannot be any money injection by the entrepreneur, even under a misrepresentation contract such collusive agreement will never occur since the maximum bribe the entrepreneur can pay is equal to the payoff the auditor gets for unravelling noncompliance (maximum deterrence). She will therefore have no incentive to hide the results of the audit and will cash the prize for monitoring.

2. **after the report, but before monitoring:**

   $\hat{H}$ type will not make any offer to the supervisor, since a high report is never monitored and hence there is nothing upon which to make a deal.

   False $\hat{L}$ type might have an interest in making an offer under a misrepresentation contract, but since only the high type could afford it, it would be type revealing.[7] This implies that supervisor would be willing to accept so long as the offer is higher than what she otherwise

[7] In true low state all the entrepreneur's resources are used to repay either the supervisor or the investor and there are no resources left to make any offer.

gets by monitoring and detecting non-compliance, i.e. $B \geq f_H - \phi + \varepsilon$.[8] The entrepreneur would thus be left with the residual $\phi - f_L - \varepsilon$, given that the pure investor is still to receive f_L after an unmonitored low state report. Under such conditions there is no point in making any offer for the entrepreneur, as what he would get by truth-telling (or by lying and being monitored with probability m)[9] would be strictly higher than what he gets with the collusive agreement. We can check this: for a collusive agreement to occur after a low report has been made we need $f_H - P_H^m \leq \phi - f_L - \varepsilon$, i.e.

$$f_H - f_L - \frac{(f_H - \phi)(I - f_L)}{p f_H - \phi} < \phi - f_L$$

which boils down to:

$$p f_H + (1 - p)\, f_L - I - \phi + p f_L < 0,$$

a contradiction.

A true $\hat{L}$ type, both under a misrepresentation and a truth-telling contract, has no gain in not being monitored, given that she gets zero anyway, except that she could try to signal to be truly low type so as to avoid the inefficient waste of resources.[10] This opens the possibility for the false $\hat{L}$ type to pursue the same strategy and fake the monitor, making her believe to be a truly low type. Being unable to screen between the true and the false low type, the supervisor will turn down the offer and the signaling will have no effect on her monitoring strategy.

Only false $\hat{L}$ type will therefore make an offer, which the supervisor will never reject as it pays her more than she gets by monitoring and detecting cheating.

3. **after the realisation of the state, but before the report**:

   state H realises and the entrepreneur persuades the auditor not to carry out the audit, with the promise to report low and share the excess resources on low state cash flows. Such an agreement, conceivable both under a truth-telling and a misrepresentation contract when there is no public display of the randomisation process, can be advantageous for both the entrepreneur and the auditor at the expenses of the investor, who gets a repayment lower than the amount invested $P_L = f_L < I$. The collusion stake is therefore the saving on the repayment due to the investor $P_H - P_L$. Any $0 < B \leq P_H - P_L$ would be mutually advantageous for the entrepreneur and the auditor at the expenses of the investor, who no longer recoups the amount invested given that $P_L = f_L < I$.[11]

[8] The supervisor knows that after a low report only the high type can afford to make a collusive offer. If this is not good enough, she will reject it and monitor, thus getting all the high state cash flows net of the observation cost, $f_H - \phi$.

[9] By the entrepreneur's incentive constraint, defined by (3.4) $m(S_{HL} + P_{HL}) + (1 - m)\, P_L = P_H$, we can look both at what the entrepreneur expects to pay if she runs the risk of being monitored and what she pays by telling the truth.

[10] This would be a renegotiation agreement, as the investor would not be damaged by it and the auditor would save the costs incurred by monitoring a true low state.

[11] We are assuming here that only the high type can make such an offer and not considering the possibility of an offer

If state L realises, the entrepreneur has no other choice than report $\hat{L}$. She might nevertheless try to signal to be truly low type so as to avoid the inefficient waste of resources. As in the previous case, this opens the possibility for the H type to pursue the same strategy in order to fake the monitor and make her believe to be a low type. Being unable to screen between the types, the supervisor will turn down the offer and the signaling will have no effect on her monitoring strategy.

It turns out that only type H entrepreneur can credibly make collusive offers before any report is made, and any such offer will be accepted by the supervisor. Suppose not, i.e. suppose that the supervisor rejects the offer in order to potentially exploit the information about the type. Anticipating this, the entrepreneur will revise his reporting strategy to $l' = 0$ reporting truthfully, and the supervisor will never be called to monitor, thus getting zero payoff. She will does strictly prefer to accept the offer.

The sequence of events is as follows:

- the firm offers the supervisor to report $\hat{L}$ in exchange for no monitoring. Because only the true H type has an interest in making such an offer, it is type revealing;[12]
- the offer is accepted and the collusive agreement is enforced: the firm reports $\hat{L}$ and pays a bribe $B \leq P_H - P_L$ which is worth kB to the supervisor.[13]

To rule out the occurrence of such collusive agreement, the auditor must be compensated for the bribe she gives up by turning down the offer, i.e. she must receive a payoff that ensures that she will certainly turn down the collusive offer. Since the highest possible collusive offer is one that pays the supervisor the informational rent the firm keeps from hiding information, i.e. $P_H - P_L = \Delta P$, for a coalition not to form the supervisor must receive a payoff S_B in all unmonitored states (H, L) no lower than the highest possible collusive offer B discounted by the transaction cost of collusion k. This makes the supervisor indifferent between not monitoring upon receiving a bribe and not monitoring upon sticking to the unverifiable verdict of the randomisation process. Thus, not only a collusive offer will be rejected, but it will also be type-revealing. Anticipating this, no offer will ever be made.

coming from the low type. This is because, similar to case 2, a true low type cannot afford any positive payoff to the auditor in exchange for no monitoring, as low state cash flows are entirely used as a payment to the auditor and the investor in monitored and unmonitored low states respectively. Thus it is indifferent between being monitored or not monitored when the state is truly low. It could however try and signal the auditor to be low type in order to avoid her the cost due to monitoring a true low state (a net loss under a misrepresentation contract, given that $S_{LL} = f_L < \phi$). However if the signaling is not credible, the auditor is unable to screen between the two types of firms and receiving a low state report does not alter her monitoring strategy.

[12] Notice that while an offer reveals with certainty the type of the firm (H), receiving no offer does not reveal anything -it could be both the high and the low type that make no offer- and the monitoring probability is not altered.

[13] If the offer was rejected, the firm would report $\hat{H}$ and the supervisor would bet zero payoff - because the offer is type revealing, reporting $\hat{L}$ would trigger monitoring for sure and lead to firm punishment. Thus an offer by the entrepreneur will never be rejected.

The coalition incentive constraint reads as follows:

$$S_B \geq kB = k\Delta P. \tag{4.1}$$

Misrepresentation As argued in the previous section, to prevent firm-supervisor collusion, a reward S_B has to be offered to the supervisor in any unmonitored state (H, L) in order to induce her to turn down potential bribes coming from the firm. Suppose the reward was paid only in high unmonitored state H. Then, after the state is realised and before the report is made, the entrepreneur might still have an interest in making a collusive offer to the supervisor. If he decides to tell the truth, he will have to pay P_H to the investor and S_B to the supervisor, provided he can afford with the high cash flows to make such repayments. However, because the repayment P_H is due to the investor, there is still room to bribe a bit more the supervisor and offer $S_B + \varepsilon$, $\varepsilon \leq P_H - P_L$, in exchange for not monitoring. The supervisor will be again better off accepting.[14] Hence, to prevent any side agreement, the reward necessary to induce the monitor to turn down the offer has to be paid in **any unmonitored state,** i.e. with probability $p(1-l) + (1-p+pl)(1-m)$ and not only in high unmonitored state.

The auditor's expected profits are:

$$\begin{aligned} E\pi_S = {} & m\left(plS_{HL} + (1-p)S_{LL} - (1-p+pl)\phi\right) \\ & + \left(p(1-l) + (1-m)(1-p+pl)\right)S_b \geq 0 \end{aligned} \tag{4.2}$$

The extra repayment ΔP that has to be made to the auditor in unmonitored states modifies the entrepreneur's expected profits in the way that follows:

$$\begin{aligned} E\pi_E \ = \ & p\left((1-l)(f_H - \Delta P - P_H) + l\left(m(f_H - S_{HL} - P_{HL}) + (1-m)(f_H - \Delta P - P_L)\right)\right) \\ & + (1-p)\left(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - \Delta P - P_L)\right) \geq 0. \end{aligned} \tag{4.3}$$

Because $P_H > P_L$, the profits in unmonitored low states turn negative, $f_L - P_H < 0$, thus violating the limited liability condition. To preserve limited liability while preventing firm-supervisor coaliton formation it is necessary to set

$$P_H \leq P_L. \tag{4.4}$$

Given that the highest possible repayment in low state is f_L, it turns out that $P_H \leq P_L = f_L$. Using $P_{HL} = P_{LL} = 0$ in the investor participation (3.16), it follows that $p(1-l)P_H + (1-p+pl)(1-m)f_L \geq I$, thus implying that $f_L > I$, which is impossible given that by assumption (H.1) low state revenues are not sufficient to finance the venture.[15]

[14] She will not reject, since she knows that at this stage (i.e. after a collusive offer has been made) S_B is the most she can get. This is because the offer is type revealing: upon rejection the entrepreneur has no other choice than making a truthful report, given that a low state report triggers monitoring and leads to maximum punishment.

[15] If P_H were set at f_L, the highest possible value compatible with the coalition incentive constraint, the investor participation would read $(1 - m(1-p+pl))f_L \geq I$, a contradiction, given that the LHS is unambiguously less than I.

Truth-telling Also under a truthtelling equilibrium it is possible that the firm and the supervisor collude at the expense of the investor. To prevent this, a reward has to be paid in any unmonitored state to induce the supervisor to turn down potential bribes coming from the firm. Given that $l = 0$, the reward has to be offered with probability $p + (1-p)(1-m) = 1 - m(1-p)$. Thus the auditor's expected profits are:

$$E\pi_S = m((1-p)S_{LL} - (1-p)\phi) + (p + (1-m)(1-p))S_b \geq 0$$

The extra repayment that has to be made to the auditor in unmonitored states, i.e. the collusion stake ΔP, modifies the entrepreneur's expected profits as follows:

$$\begin{aligned} E\pi_E &= p(f_H - \Delta P - P_H) + (1-p)(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - \Delta P - P_L)) \geq 0 \\ &= p(f_H - 2P_H + P_L) + (1-p)(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - P_H)) \geq 0. \end{aligned}$$

However, because $P_H > P_L = f_L$, again the profits in unmonitored low states turn negative, violating the limited liability condition. Thus to preserve limited liability while preventing firm-supervisor coaliton formation it is necessary to set $P_H \leq P_L$. Given that the highest possible repayment in low state is f_L, it turns out that $P_H \leq P_L = f_L$. Using $P_{HL} = P_{LL} = 0$ in the investor participation (3.7), it follows that $pP_H + (1-p)(1-m)f_L \geq I$, thus implying that $f_L > I$, which is impossible given that by assumption (H.1) low state revenues are not sufficient to finance the venture.[16]

Anticipating that she will never recoup the amount invested, the investor will never finance the venture and the project will not be carried out. We have therefore derived the following result:

Proposition 4 *Both in the truthtelling and misrepresentation contract, when auditors can collude with the firm, the project is not financed because the investment has negative net present value for investors.*

How to overcome the problems induced by the risk of collusion while not losing the efficiency of the 3-tier structure? One way to overcome this problem is to have the investor providing only a share α of finance, just sufficient to prevent collusion, and ask the supervisor to provide the balance. In this way it is possible to prevent collusion, while still financing the project.

5 Renegotiation

In the previous section we have considered which are the coalitions that can form between two of the three contracting parties at the expense of the party left out of the coalition and seen that collusion can have dramatic implications on the properties of the contract. In this section we investigate what happens when the parties can renegotiate the terms of the contract and which are the incentives to do that. We consider in particular what happens after a report $\hat{L}$ has arrived. In such circumstances, because the

[16] If P_H were set at f_L, the highest possible value compatible with the coalition incentive constraint, the investor participation would read $(1 - m(1-p))f_L \geq I$, a contradiction, given that the LHS is unambiguously less than I.

structure of the repayments is such that the investor gets f_L if the supervisor does not monitor, and at most $f_L - \phi$ if she does monitor,[17] she might try to persuade the supervisor not to monitor and share the saving in observation cost.[18]

In this way the supervisor would get $0 < e < \phi$ and the investor would get $f_L - e > 0$. So long as $e > 0$, the supervisor has always an incentive to accept the offer as she gets a payoff higher than her reservation utility. The only way to reduce the chances that renegotiation may take place is to increase the stake, i.e. the minimum payoff the supervisor is to be paid to give up monitoring, which implies paying her a rent. However, because rents are costly, the entrepreneur can ask the supervisor to pay for them by investing into the venture.

6 Supervisor as a venture capitalist?

Having the auditor contributing to the financing of the venture implies that she has not only to be compensated for the monitoring task, but also for the financing task. Given that she has to get all available cash flows in monitored states to provide the monitoring incentives, it follows that the compensation for the financing task has to be provided in unmonitored states.

Misrepresentation Denoting by $1 - \alpha$ the share of finance provided by the supervisor, by S_H her payoff in truthful high cash flow states and by S_L her payoff in unmonitored low cash flow states (and by $r_S = 1 + \rho_s$, with $\rho_s \geq 0$, the minimum rate of return that has to be paid to the supervisor to provide financing), the supervisor's expected profits are:

$$\begin{aligned} E\pi_S = p\,(1-l)\,S_H + m\,(plS_{HL} + (1-p)\,S_{LL} - (1-p+pl)\,\phi) \\ + (1-m)\,(1-p+pl)\,S_L \geq (1-\alpha)\,r_S I. \end{aligned} \tag{6.1}$$

Given that the entrepreneur has to make a transfer to the supervisor also in unmonitored states, his expected profits are modified in the way that follows:

$$\begin{aligned} E\pi_E \;=\;& p\,((1-l)(f_H - S_H - P_H) + l\,(m(f_H - S_{HL} - P_{HL}) + (1-m)(f_H - S_L - P_L))) \\ & + (1-p)\,(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - S_L - P_L))\,. \end{aligned} \tag{6.2}$$

Using (6.1) and (6.2), the incentive constraints on l, m are given by:

$$l \;\in\; \arg\max_{l'}\; E\pi_E \tag{6.3}$$

$$m \;\in\; \arg\max_{m'}\; E\pi_S. \tag{6.4}$$

[17] This is paid only under a truth-telling contract if a monitored low report is found to be true. The investor gets always zero instead if monitoring detects false reporting ($P_{HL} = 0$).

[18] Because this agreement does not penalise the entrepreneur, we envisage it to be a renegotiation rather than a collusive agreement.

Under the assumption that $f_L < \phi$, no truth-telling equilibrium can arise and the incentive constraints (6.3) and (6.4) become indifference conditions associated with a mixed strategy equilibrium:

$$S_H + P_H - S_L - P_L = m(S_{HL} + P_{HL} - S_L - P_L); \tag{6.5}$$

$$\frac{pl}{1-p+pl} S_{HL} + \frac{1-p}{1-p+pl} S_{LL} - \phi = S_L, \tag{6.6}$$

which give the ex post probabilities of lying and monitoring as:

$$l = \frac{(1-p)(\phi - S_{LL} + S_L)}{p(S_{HL} - S_L - \phi)} \tag{6.7}$$

$$m = \frac{P_H + S_H - P_L - S_L}{S_{HL} + P_{HL} - S_L - P_L}. \tag{6.8}$$

Using (6.5) into (6.2) and (6.6) into (6.1),the firm's and the supervisor expected profits become respectively:

$$E\pi_E = p(f_H - S_H - P_H) + (1-p)(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - S_L - P_L)) \geq 0, \tag{6.9}$$

$$E\pi_S = p(1-l) S_H + m(plS_{HL} + (1-p) S_{LL} - (1-p+pl)\phi) \tag{6.10}$$
$$+ (1-m)(1-p+pl) S_L \geq (1-\alpha) r_S I,$$

and the entrepreneur's problem becomes:

$$\max \; p(f_H - S_H - P_H) + (1-p)(m(f_L - S_{LL} - P_{LL}) + (1-m)(f_L - S_L - P_L)) \tag{6.11}$$

$$p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L + (1-p)mP_{LL} \geq \alpha I \tag{6.12}$$

$$p(1-l) S_H + (1-p+pl) S_L \geq (1-\alpha) r_S I \tag{6.13}$$

$$f_H - S_{HL} - P_{HL} \geq 0 \tag{6.14}$$

$$f_H - S_H - P_H \geq 0 \tag{6.15}$$

$$f_L - S_{LL} - P_{LL} \geq 0 \tag{6.16}$$

$$f_L - S_L - P_L \geq 0 \tag{6.17}$$

to the coalition incentive constraint (4.4) and to the ex post probabilities of lying and monitoring (6.7) and (6.8). Here (6.12) and (6.13) are the investor and the supervisor participation constraints, (6.14) to (6.17) the feasibility and limited liability conditions.

Relative to the problem with specialised tasks, we have added the participation constraint (6.13) for the supervisor (PC_S), two extra feasibility conditions, (6.15) and (6.17) and the coalition incentive constraint (4.4).

To preserve the monitoring incentives, it is still optimal to set $S_{HL}^{c.m} = f_H$, $P_{HL}^{c.m} = 0$, and $S_{LL}^{c.m} = f_L$, $P_{LL}^{c.m} = 0$. Moreover, assuming binding coalition incentive constraint (4.4),[19] and maximal financial

[19] The lower is P_H wrt P_L, the lower the share α provided by the investor, and the higher the balance that has to be provided by the monitor to ensure project realisation.

participation compatible with deterring collusion, $P_H^{c.m} = P_L^{c.m} = f_L$. This implies that $S_L^{c.m} = 0$ and that the supervisor can be compensated for financial participation only in high truthfully reported states.

The problem then is to choose $S_H^{c.m}$ and $\alpha^{c.m}$ to

$$\max\ p\left(f_H - f_L - S_H^{c.m}\right) \tag{6.18}$$

$$\left(p(1-l) + (1-p+pl)(1-m)\right) f_L \geq \alpha^{c.m} I \tag{6.19}$$

$$p\,(1-l)\, S_H^{c.m} \geq \left(1-\alpha^{c.m}\right) r_S I \tag{6.20}$$

whence it is straightforward to derive $\alpha^{c.m} = \dfrac{fL(pfh + (1-p)fL - rI(1-p) - \phi)}{I(pfh - (1-p)(r-1)fL - \phi)}$, $S_H^{c.m} = \dfrac{r_S(I-f_L)(f_H-\phi)}{pf_H - (1-p)(r_S-1)f_L - \phi}$. Notice that while the probability of lying is independent of the cost of monitoring capital, the probability of monitoring is increased with respect to the specialised tasks:

$$m^{c.m} = \frac{r_s(f_H-\phi)\,(I-f_L)}{(f_H-f_L)\,(pf_H-\phi-(1-p)(r_s-1))} > \frac{(f_H-\phi)\,(I-f_L)}{(f_H-f_L)\,(pf_H-\phi)} = m^m$$

The reason why the probability of monitoring is higher in a collusion proof contract has to do with the higher cost of monitoring capital relative to non-monitoring capital: the higher the monitoring capital, the lower the rent to the entrepreneur, the higher the marginal benefit of lying, the higher the value of $m^{c.m}$ necessary to restore indifference on the monitor. This implies also higher expected observation cost $m^{c.m}\,(1-p+pl)\,\phi$ and lower entrepreneur's expected utility:

$$p\left(f_H - S_H^{c.m} - P_H^{c.m}\right) = p\left(f_H - f_L - \frac{r_s(f_H-\phi)\,(I-f_L)}{pf_H-\phi-(1-p)(r_S-1)f_L}\right)$$

which can also be written as:

$$\begin{aligned}
EU^{c.m} &= Ef - \alpha^{c.m} I - \left(1-\alpha^{c.m}\right) r_s I - m^{c.m}\left(1-p+pl^{c.m}\right)\phi \\
&= Ef - I - \frac{(r_s-1)\,(I-f_L)\,(Ef-\phi)}{pf_H-\phi-(1-p)\,(r_s-1)\,f_L} - m^{c.m}\left(1-p+pl^{c.m}\right)\phi \\
&= Ef - I - \frac{r_s\,(Ef-p\phi)-(Ef-\phi)}{pf_H-\phi-(1-p)(r_S-1)f_L}\,(I-f_L) \geq EU^m
\end{aligned}$$

These results allow us to state the following proposition:

Proposition 5 *When $f_L < \phi$, the collusion-proof principal-supervisor-agent contract has the following properties:*

- *it features misrepresentation and random monitoring, with* $l^{c.m} = \dfrac{(1-p)\,(\phi-f_L)}{p\,(f_H-\phi)} = l^m$; $m^{c.m} = \dfrac{r_s(f_H-\phi)\,(I-f_L)}{(f_H-f_L)\,(pf_H-\phi-(1-p)(r_s-1))} \geq m^m$;
- *the investor gets a flat repayment only in non audited states, i.e.* $P_H^{c.m} = P_L^{c.m} = f_L$ *and her financial stake is strictly less than one and increasing in low state cash flows:* $\alpha^{c.m} = \dfrac{f_L(pf_H + (1-p)f_L - r_sI(1-p) - \phi)}{I(pf_H - (1-p)(r_s-1)f_L - \phi)}$;

- *the supervisor participates financially into the venture and gets a repayment in high truthfully reported state:* $S_H^{c,m} = \dfrac{r_S(I - f_L)(f_H - \phi)}{pf_H - (1-p)(r_S - 1)f_L - \phi} < f_H$. *Moreover, whenever she audits she gets all the available cash flows, i.e.* $S_{LL}^{c,m} = f_L$ *and* $S_{HL}^{c,m} = f_H$*;*

- *a rent* $p\,(f_H - S_H^{c,m} - P_H^{c,m})$ *accrues to the entrepreneur when the high state occurs and it is truthfully reported. This is less than or equal to the rent obtained by the entrepreneur under fully specialised tasks, depending on the relative scarcity of monitoring capital and hence on whether* $r_S \geq 1$*. Hence ensuring collusion proofness can alter the welfare properties of the contract.*

Remark 2 *Unlike Tirole (1986), there is a role for a risk neutral supervisor, that does not depend on risk aversion or transaction costs of collusion.*

Truthtelling Using the supervisor's expected profits (6.1) and entrepreneur's expected profits (6.2) as modified by the transfer due to the supervisor also in unmonitored states to prevent collusion, the incentive constraints on m, l are given by:

$$\frac{\partial E\pi_S}{\partial m} = plS_{HL} + (1-p)\,S_{LL} - (1-p+pl)\,(S_L + \phi) \lesseqgtr 0 \tag{6.21}$$

$$\frac{\partial E\pi_E}{\partial l} = -(f_H - S_H - P_H) + m(f_H - S_{HL} - P_{HL}) + (1-m)(f_H - S_L - P_L) \lesseqgtr 0 \tag{6.22}$$

Under the assumption that $f_L \geq \phi$, the payoff from monitoring a truthful low state report is sufficient to cover the supervisor's observation cost and the Nash equilibrium will involve $l = 0$ and $0 < m < 1$.

When $l = 0$, using (6.21), we must have $S_{LL} - \phi = S_L$ for $0 < m < 1$ to be a best response. Analogously, when $0 < m < 1$, using (6.22), we must have $S_H + P_H \leq m\,(S_{HL} + P_{HL}) + (1-m)\,(S_L + P_L)$ for $l = 0$ to be a best response. Taken together these yield the incentive constraints $S_{LL} - \phi = S_L$ and $S_H + P_H \leq m\,(S_{HL} + P_{HL}) + (1-m)\,(S_L + P_L)$.

The firm will therefore choose P_H, P_L, P_{HL}, P_{LL}, S_H, S_L, S_{HL}, S_{LL}, m to

$$\max\ p\,(f_H - S_H - P_H) + (1-p)\,(f_L - m(S_{LL} + P_{LL}) + (1-m)(S_L + P_L)) \tag{6.23}$$

$$pP_H + (1-p)(1-m)P_L + (1-p)mP_{LL} \geq \alpha I \tag{6.24}$$

$$pS_H + (1-p)\,S_L \geq (1-\alpha)\,r_S I \tag{6.25}$$

$$S_H + P_H \leq m\,(S_{HL} + P_{HL}) + (1-m)\,(S_L + P_L) \tag{6.26}$$

$$S_{LL} - \phi = S_L \tag{6.27}$$

to the coalition incentive constraint (4.4), to the feasibility and limited liability conditions (6.14), (6.15), (6.16), (6.17).

Also in this case, relative to the problem with specialised tasks, we have added the participation constraint (6.25) for the supervisor (PC_S), the two extra feasibility conditions, (6.15) and (6.17), and the coalition incentive constraint (4.4).

To preserve the monitoring incentives, it is still optimal to set $S_{HL}^{c.t} = f_H$, $P_{HL}^{c.t} = 0$. Moreover, assuming binding coalition incentive constraint (4.4),[20] and maximal financial participation compatible with deterring collusion, $P_H^{c.t} = P_L^{c.t} = f_L$. This implies that $S_L^{c.t} = 0$ and that the supervisor can be compensated for financial participation only in high truthfully reported states.

Using $S_L^{c.t} = 0$ and binding supervisor incentive constraint (6.27), $S_{LL}^{c.t} = \phi$ and $P_{LL}^{c.t} = f_L - S_{LL}^{c.t} = f_L - \phi \geq 0$. Hence, relative to the misrepresentation contract, there could be some resources left to repay the investor in monitored states.

The problem then is to choose $S_H^{c.t}$ and $\alpha^{c.t}$ to

$$\max\ p\left(f_H - f_L - S_H^{c.t}\right) \tag{6.28}$$

$$f_L - m^{c.t}(1-p)\phi \geq \alpha^{c.t} I \tag{6.29}$$

$$pS_H^{c.t} \geq \left(1 - \alpha^{c.t}\right) r_S I \tag{6.30}$$

$$S_H^{c.t} \leq m^{c.t}\left(f_H - f_L\right) \tag{6.31}$$

whence, solving for $S_H^{c.t}$ and $\alpha^{c.t}$, we get:

$$\begin{aligned} S_H^{c.t} &= \frac{\left(I - f_L\right) r_S \left(f_H - f_L\right)}{p\left(f_H - f_L\right) - (1-p)\phi r_S} \\ \alpha^{c.t} &= \frac{p\left(f_H - f_L\right) f_L - (1-p)\phi r_S I}{\left(p\left(f_H - f_L\right) - (1-p)\phi r_S\right) I} < 1. \end{aligned}$$

We can thus derive the probability of monitoring as

$$m^{c.t} = \frac{\left(I - f_L\right) r_S}{p\left(f_H - f_L\right) - (1-p)\phi r_S} \geq m^t.$$

Also for this case, the reason why the probability of monitoring is higher in the collusion proof contract has to do with the higher cost of monitoring capital: an increase in monitoring capital lowers the rent to the entrepreneur and exhacerbates the incentive problem (the incentive constraint (6.26) is violated). A higher $m^{c.t}$ is necessary to restore indifference on the monitor, which in turn increases expected observation cost $m^{c.t}(1-p)\phi$.

Last, we derive the entrepreneur's expected utility as:

$$\begin{aligned} EU^{c.t} &= p\left(f_H - S_H^{c.t} - P_H^{c.t}\right) = \frac{p\left(f_H - f_L\right)\left(Ef - I - (1-p)\phi - \left(I - f_L + (1-p)\phi\right)\rho_S\right)}{p\left(f_H - f_L\right) - (1-p)\phi\left(1 + \rho_S\right)} \\ &= Ef - I - \frac{\left(I - f_L\right) p\left(f_H - f_L\right)\rho_s}{p\left(f_H - f_L\right) - (1-p)\phi r_S} - (1-p)\,\phi m^{c.t} \\ &= Ef - I - \frac{\left(p\left(f_H - f_L\right) - (1-p)\,\phi\right)\rho_s - (1-p)\,\phi}{p\left(f_H - f_L\right) - (1-p)\phi r_S}\left(I - f_L\right). \end{aligned}$$

These results allow us to state the following proposition:

Proposition 6 *When $f_L \geq \phi$, the collusion-proof principal-supervisor-agent contract has the following properties:*

[20] The lower is P_H wrt P_L, the lower the share α provided by the investor, and the higher the balance that has to be provided by the monitor to ensure project realisation.

- *it features truth-telling and random monitoring, with monitoring probability increasing in the cost of monitoring capital:* $m^{c.t} = \frac{(I - f_L) r_S}{p(f_H - f_L) - (1-p)\phi r_S}$;
- *the investor gets a flat repayment in non audited states, i.e.* $P_H^{c.t} = P_L^{c.t} = f_L$ *and what is left in monitored true low states after the observation cost has been covered:* $P_{LL}^{c.t} = f_L - \phi$. *Moreover, her financial stake is strictly less than one and increasing in low state cash flows:* $\alpha^{c.t} = \frac{p(f_H - f_L) f_L - (1-p)\phi r_S I}{(p(f_H - f_L) - (1-p)\phi r_S) I}$;
- *the supervisor participates financially into the venture and gets a repayment in high truthfully reported state:* $S_H^{c.t} = \frac{(I - f_L) r_S (f_H - f_L)}{p(f_H - f_L) - (1-p)\phi r_S} < f_H$, *but no repayment in low unmonitored states. Moreover, when she audits, she recoups the observation cost in truthful low states, while she gets all the available cash flows in false low states, i.e.* $S_{LL}^{c.t} = \phi$ *and* $S_{HL}^{c.t} = f_H$. *However, because of truthtelling, the penalty for misreporting is never paid;*
- *a rent*

$$p\left(f_H - S_H^{c.t} - P_H^{c.t}\right) = Ef - I - \frac{(p(f_H - f_L) - (1-p)\phi)\rho_s - (1-p)\phi}{p(f_H - f_L) - (1-p)\phi r_S}(I - f_L)$$

accrues to the entrepreneur when the high state occurs and it is truthfully reported. This is no higher than the rent obtained by the entrepreneur under fully specialised tasks (3.12), depending on the relative scarcity of monitoring capital and hence on whether $r_S \geq 1$. *Hence ensuring collusion proofness can alter the welfare properties of the contract.*

Remark 3 *Although with a three-layer structure with separation of tasks it is possible to replicate the full second best contract even with non-contractible monitoring, this is no longer so when there is a risk of collusion.*

This proposition shows that when there is a risk of collusion, the higher efficiency of the three-layer structure relative to a bilateral one is lost. The extent of such loss depends on three factors: 1. the relative scarcity of each form of financing, i.e. the cost of monitoring (ex-post informed) relative to non monitoring (uninformed) capital; 2. the transaction costs of collusion: the higher they are, the lower the collusion stake, the lower the compensation that makes the supervisor willing to turn down a collusive offer, and thus her share of financial participation; 3. the spread between project size and low state cash flows: this is because the share α of the uninformed investor is increasing in low state cash flows. The lower the low state cash flows relative to the project size, the higher the collusion stake, the higher the need for costly monitoring capital, the lower the rent from the project. In the limit, if realised low state cash flows are zero, transaction costs of collusion are negligible (or even negative!) and there is limited liability, the share of uninformed capital tends to zero and the project is entirely financed by supervisors. A bilateral structure à la Khalil and Parigi emerges as an optimal response to the high risk of collusive behaviour. As shown in Remark 1, this structure is always suboptimal relative to the three layer structure, even if the two forms of financing available costed the same.

This result has important implications as it suggests that riskier firms, i.e. those with distribution of cash flows highly spread out (low collateral ones), and with high risk of collusive behaviour, are more likely to be financed by a single monitoring lender and face higher cost of credit.

7 Conclusions

In this paper we have derived the optimal principal-supervisor-agent contract when monitoring is not contractible. We have shown that, while the separation of tasks implied by the three-layer structure can improve efficiency and lead to a result of irrelevance of the contractual incompleteness, it is prone to collusion and renegotiation. The response to such drawbacks is to involve financially the supervisor into the venture. The extent of the participation depends on how costly it is to collude and renegotiate the contract terms. We are therefore able to derive the collusion- and renegotiation-proof ownership structure.

We have also derived a welfare analysis showing that the efficiency of each contract form is decreasing in the cost of informed capital, relative to uninformed capital, increasing in the transaction costs of collusion, and decreasing in the variance of the distribution of cash flows. Under limited liability and sufficiently low transaction costs of collusion, if all revenues accrue in the high state, the share of uninformed capital tends to zero and a suboptimal bilateral structure emerges.

Various open questions remain. Although in the present model manager's misbehaviour can arise in equilibrium, this is never so for supervisors, who never collude in equilibrium and always report their information truthfully. This is because we have assumed that the monitoring technology is perfect and the monitoring activity perfectly observable. We know however that in most cases this is not so and that it is the existence of *grey areas* of discretionality that makes often possible to circumvent the rules. This is left for future research.

A Appendix

Proof of Proposition 2. Solving the participation constraint (3.7) for P_{LL}

$$P_{LL} = \frac{I - pP_H - (1-p)(1-m)P_L}{m(1-p)} \tag{A.1}$$

and substituting out in the objective function, the problem becomes one of

$$\max\ pf_H + (1-p)f_L - I - m(1-p)\phi$$

subject to the truthtelling constraint. Because the objective is decreasing in m, the incentive constraint must bind. If not, utility could be increased by reducing m. Thus $m = \dfrac{P_H - P_L}{S_{HL} + P_{HL} - P_L}$.

Since the objective is increasing in S_{HL} and P_{HL}, optimally these must be set at their highest possible value. Thus $S_{HL} + P_{HL} = f_H$.

Moreover, the objective function is also increasing in P_L

$$\frac{\partial obj}{\partial P_L} = \frac{(1-p)\phi(f_H - P_H)}{(f_H - P_L)^2}$$

so that $P_L = f_L$.

Last, the objective is decreasing in P_H. Thus we want to minimise P_H while preserving the lenders' participation constraint. This requires increasing P_{LL} up to $f_L - S_{LL} = f_L - \phi$, thus giving, using (A.1):

$$f_L - \phi = \frac{I - pP_H - (1-p)(1-m)f_L}{m(1-p)} \tag{A.2}$$

with $m = \dfrac{P_H - f_L}{f_H - f_L}$. Solving (A.2) for P_H :

$$P_H = \frac{(f_H - f_L)(I - f_L(1-p)) - f_L(1-p)\phi}{p(f_H - f_L) - \phi(1-p)}$$

and using P_H in m

$$m = \frac{I - f_L}{p(f_H - f_L) - \phi(1-p)}$$

and m in the objective, we get the entrepreneur's profits as:

$$pf_H + (1-p)f_L - I - \frac{(1-p)\phi(I - f_L)}{p(f_H - f_L) - \phi(1-p)}$$

■

Proof of Proposition 3. Define a new variable $x_{LL} = S_{LL} + P_{LL}$ and write the objective function and the participation constraint as:

$$\max\ p(f_H - P_H) + (1-p)(f_L - (1-m)P_L - mx_{LL}) \tag{A.3}$$

$$\text{st } I \leq p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L + (1-p)m(x_{LL} - S_{LL}) \tag{A.4}$$

Given that the objective function does not depend on S_{LL}, while the participation constraint is increasing in S_{LL}, $\dfrac{\partial PC}{\partial S_{LL}} = \dfrac{(1-p)\phi(P_H - P_L)}{(S_{HL} - \phi)(P_{HL} + S_{HL} - P_L)} = \dfrac{m(1-p)\phi}{S_{HL} - \phi} > 0$, then S_{LL} can be set as high as possible. Thus, because of the feasibility condition (3.18), we can set $S_{LL} = f_L - P_{LL}$, which in turn implies that $x_{LL} = f_L$.

This gives us the objective function and participation constraint as:

$$\max\ p(f_H - P_H) + (1-p)(f_L - (1-m)P_L - mf_L) \tag{A.5}$$

$$\text{st } I \leq p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L + (1-p)mP_{LL} \tag{A.6}$$

where $l = \frac{(1-p)(\phi - f_L + P_{LL})}{p(S_{HL} - \phi)}$. Differentiating the participation constraint with respect to P_{LL}, we find that this is decreasing in P_{LL}, whilst the objective is independent of P_{LL}:

$$\frac{\partial PC}{\partial P_{LL}} = -\frac{(1-p)\phi(P_H - P_L)}{(S_{HL} - \phi)(P_{HL} + S_{HL} - P_L)} < 0.$$

Hence P_{LL} can be set as low as possible, namely $P_{LL} = 0$ and $S_{LL} = f_L$.

By (3.19), the objective function is non-decreasing in P_{HL},

$$\frac{\partial obj}{\partial P_{HL}} = \frac{(1-p)(P_H - P_L)(f_L - P_L)}{(P_{HL} + S_{HL} - P_L)^2} \geq 0,$$

and so is the participation constraint,

$$\frac{\partial PC}{\partial P_{HL}} = \frac{(P_H - P_L)\left((\phi - f_L)S_{HL} + (S_{HL} - \phi)P_L\right)(1-p)}{(S_{HL} - \phi)(-P_{HL} - S_{HL} + P_L)^2} \geq 0,$$

which implies that (3.17) is binding. Setting $S_{HL} = f_H - P_{HL}$, the objective function and the participation constraint can be thus respectively written in terms of P_H, P_L and P_H, P_L and P_{HL} :

$$\max\ p(f_H - P_H) + (1-p)(f_L - (1-m)P_L - mf_L)$$
$$\text{st } p(1-l)P_H + plmP_{HL} + (1-p+pl)(1-m)P_L \geq I$$

where $m = \dfrac{P_H - P_L}{f_H - P_{HL} - P_L}$, $l = \dfrac{(1-p)(\phi - f_L)}{p(f_H - P_{HL} - \phi)}$.

While the objective is independent of P_{HL}, differentiating the participation constraint with respect to the same variable, we find, under the assumption that $\phi > f_L$, that:

$$\frac{\partial PC}{\partial P_{HL}} = -\frac{(1-p)(\phi - f_L)(P_H - P_L)\phi}{(-f_H + P_{HL} + \phi)^2(f_H - P_L)} < 0,$$

which implies that $P_{HL} = 0$ and $S_{HL} = f_H$.

The problem can thus be written as:

$$\max_{P_H, P_L}\ p(f_H - P_H) + (1-p)(f_L - (1-m)P_L - mfL) \tag{A.7}$$
$$\text{st } p(1-l)P_H + (1-p+pl)(1-m)P_L \geq I \tag{A.8}$$

with $m = \dfrac{P_H - P_L}{f_H - P_L}$, $l = \dfrac{(1-p)(\phi - f_L)}{p(f_H - \phi)}$.

From (A.8) we see that the solution must involve $P_H > P_L \geq 0$. A contract with $P_H \leq P_L$ would require the low state repayment to at least equal f_L to meet the investor reservation utility, but it would violate assumption (H.1).

To solve this problem, we rewrite the contract problem defining $\Delta P = P_H - P_L$:

$$\max_{\Delta P, P_L}\ p(f_H - \Delta P - P_L) + (1-p)\left(f_L - P_L\right)(1-m) \tag{A.9}$$
$$\text{st } p(1-l)\left(\Delta P + P_L\right) + (1-p+pl)(1-m)P_L \geq I \tag{A.10}$$

where $m = \dfrac{\Delta P}{f_H - P_L}$. Solving the participation constraint for ΔP, we find:

$$\begin{aligned}\Delta P &= \frac{(I - P_L)(f_H - \phi)(f_H - P_L)}{pf_H(f_H - f_L) + f_H\left(f_L - P_L\right) - \phi\left(f_H - P_L\right)} \\ &= \frac{(I - P_L)(f_H - \phi)(f_H - P_L)}{f_H(Ef - P_L) - \phi\left(f_H - P_L\right)}\end{aligned}$$

which must be positive.

Using ΔP into the objective function,

$$\begin{aligned} E\pi &= p(f_H - \frac{(I-P_L)(f_H-\phi)(f_H-P_L)}{f_H(Ef-P_L)-\phi(f_H-P_L)} - P_L) + (1-p)(f_L-P_L)\frac{f_H(Ef-\phi)-I(f_H-\phi)}{f_H(Ef-P_L)-\phi(f_H-P_L)} \\ &= p(f_H - \frac{(I-P_L)(f_H-\phi)(f_H-P_L)}{f_H(Ef-P_L)-\phi(f_H-P_L)} - P_L) + (1-p)(1-m)(f_L-P_L) \end{aligned}$$

and differentiating it with respect to P_L, we find:

$$\begin{aligned} \frac{\partial E\pi}{\partial P_L} &= \frac{(1-p)\phi(f_H-f_L)\left[f_H(Ef-\phi)-I(f_H-\phi)\right]}{\left[f_H(Ef-P_L)-\phi(f_H-P_L)\right]^2} \\ &= (1-p)\phi(f_H-f_L)(1-m) \end{aligned}$$

which is unambiguously positive. P_L can thus be set at the highest possible level, which, given limited liability, is f_L.

We can use $P_L = f_L$ to get the value of P_H as:

$$\begin{aligned} P_H &= \Delta P + P_L = \frac{(I-P_L)(f_H-\phi)(f_H-P_L)}{f_H(Ef-P_L)-\phi(f_H-P_L)} + f_L \\ &= \frac{f_L(pf_H-\phi)+(f_H-\phi)(I-f_L)}{pf_H-\phi}. \end{aligned}$$

Last, we can derive the entrepreneur's expected utility as:

$$\begin{aligned} p(f_H-P_H) &= \frac{f_H(Ef-\phi)-I(f_H-\phi)}{pf_H-\phi} \\ &= Ef - I - \frac{(I-f_L)(1-p)\phi}{pf_H-\phi}. \end{aligned}$$

■

References

[1] Hart, O. 1995, "*Firms, Contracts and Financial Structure*", Clarendon Press, Oxford.

[2] Holmstrom, B. and J. Tirole 1997, "Financial intermediation, loanable funds, and the real sector", *Quarterly Journal of Economics*, **CXII**, 663-691.

[3] Jost, P.J. 1996, "On the Role of Commitment in a Principal-Agent Relationship with an Informed Principal", *Journal of Economic Theory*, **68** (3), 510-530.

[4] Khalil, F. 1997, "Auditing without Commitment", *RAND Journal of Economics*, **28**, 629-640.

[5] Khalil, F. and J. Lawarrée 2002, "Corruptible Auditors: Incentives in the Absence of Commitment", mimeo.

[6] Khalil, F. and B. Parigi 1998, "Loan Size as a Commitment Device", *International Economic Review*, **39**, 135-150.

[7] Menichini, A. and P. Simmons 2002, "Can Liars Ever Prosper?", University of York Discussion Paper, 2002/10.

[8] Persons, J.D. 1997, "Liars Never Prosper? How Management Misrepresentation Reduces Monitoring Costs", *Journal of Financial Intermediation*, **6**, 269-306.

[9] Strausz, R. 1997, "Delegation of Monitoring in a Principal-Agent Relationship", *Review of Economic Studies*, **64**, 337-357.

[10] Tirole, J. 1986, "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations", *Journal of Law and Economics*, **2** (2), 181-214.