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Collusive Effects of Vertical Restraints under Asymmetric Information

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Jakub Kastl* and Salvatore Piccolo**

Abstract

This paper points out that vertical delegation, implemented through the design of quantity discount contracts, may allow upstream producers, as well as downstream retailers, to achieve profits higher than those obtained under vertical integration or contracts based on price restrictions. Our result shows that when downstream competition is sufficiently tough, the design of suitable vertical restraints implements a market outcome closer to the monopoly benchmark, which has a detrimental effect on consumer surplus. Moreover, we argue that legally banning price restricting contracts is suboptimal, the reason being that they remove a double-marginalization effect created by asymmetric information between upstream producers and downstream retailers.

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1 Introduction

THE RECENT INDUSTRIAL ORGANIZATION LITERATURE has devoted great attention to the analysis of economic environments where competition develops among vertical organizations. Much of this literature[1] shares the common theme that the choice of a particular *organizational mode*, once viewed as neutral with respect to the business strategies pursued by the firms, together with the implementation of suitable *contractual schemes* may, to some extent, soften competition. More specifically, the question of whether the delegation of some marketing and/or production decisions[2] to downstream agents may help firms to enforce *anticompetitive* behavior has been a central concern for *competition policy* and *antitrust authorities* for a long time. Along this line of research, this paper considers a model where *vertical delegation* of production decisions may be seen as a device used by upstream producers to facilitate collusive behaviors in the downstream market. Vertical integration in our setting is meant in a general sense. In particular, one may consider either the case where one upstream firm and one downstream firm fully merge or, alternatively, we may discriminate between vertical delegation and vertical integration according to the type of contracts[3] implemented between upstream producers (or manufacturers) and downstream retailers (or dealers). We shall use the basic lessons of incentives theory to show that, under the hypothesis of adverse selection, vertical delegation may be in the common interest of the upstream producers, as it may result in higher equilibrium profits. Our analysis identifies a *trade-*

[1] See Bonanno and Vickers (1988), Rey and Stiglitz (1988), Gal-Or (1991), Caillaud and Rey (1995), Martimort (1996)

[2] Such as for instance pricing and advertising decisions.

[3] We shall show that contracts imposing *price restrictions* in our model lead to the same market outcome as if the market game was played by a vertically integrated firm; whereas *quantity discount* arrangements correspond to vertical delegation as downstream dealers enjoy positive rents uncder this contractual scheme.

off between an effect of *relaxing competition*, achieved by means of a downward distortion of market quantities due to the *traditional rent extraction-efficiency trade-off,* and a *direct* negative impact of the informational rents on the profits earned by upstream producers.

Although several theoretical results suggest that one should be very cautious about promoting policies against the use of vertical restraints, over the last decades the antitrust authority in the United States has argued unambiguously against the use of such contractual arrangements. Refiners, for instance, have long been a favorite target of antitrust enforcement. Court decisions have pronounced as unlawful *exclusive dealing* contracts for gasoline as well as several contractual schemes through which the retail price is controlled by the upstream refiners. Legislation aimed of restricting the *nature* of vertical restraints have then been widely introduced either at a state or federal level. In this respect a crucial question to answer is what are the welfare effects of those legal restrictions. It is in fact still not clear whether restricting the nature of vertical contracts limits producers' ability to enforce anticompetitive behavior trough vertical contracting.

The idea that vertical delegation may have interesting *strategic aspects* has been widely discussed in the IO literature. A number of recent papers have pointed out that the design of particular vertical restraints not only affects the internal organization of each vertical hierarchy[4] (*direct effect*) but, crucially, it also influences the behavior of the rivals (*strategic effect*). The seminal work of Bonanno and Vickers (1988), for instance, shows that *delegation* of pricing decisions to downstream agents *(under the hypothesis of complete information)* leads competitors to behave in a more friendly manner and thus to set higher equilibrium

[4]¿From hereafter we will define a vertical hierarchy (or organization) as the coalition upstream-downstream producers.

prices. Many authors[5], however, have strongly criticized the assumption of complete information, arguing that this approach is too restrictive as it may neglect relevant contexts where *internal agency constraints*[6] could complicate the analysis. Specifically, three *nontraditional aspects* may play a crucial role in this environment: *(i)* how those internal agency problems affect the efficiency frontier of each vertical hierarchy, *(ii)* whether the choice of specific contractual *instruments* may soften competition in the final market and how the nature of product market competition affects the optimal choice of those instruments and, finally, *(iii)* whether those contracts are enforceable by simple means which are commonly observed in real market situations. In this paper we shall attempt to answer those questions and derive some *normative* implications for antitrust and regulatory policies.

A convenient and at the same time natural way to bring incomplete information into the framework analyzed by the previous literature is to study how the commitment to specific *informational channels* may force downstream competition towards prices closer to the monopoly benchmark. Rey and Caillaud (1995) developed this idea by considering a two stage differentiated duopoly game where first the upstream producers simultaneously select their informational channels and then downstream firms compete in prices. In this paper, following Caillaud and Rey, we explicitly model an agency problem involving asymmetric information in an industry where downstream firms *compete in quantities*[7]. In addition, we assume that the market demand for the final good is affected by a *common shock* which is private information to the downstream agents.[8] The structure of the model, in particular,

[5]See Rey and Tirole (1988).

[6]Typically in the form of adverse selection and/or moral hazard.

[7]Hence our attention focuses on the interaction between strategic substitutability of choice variables and the value of acquiring some relevant information.

[8]As our attention focuses on the anticompetitive scope for vertical delegation, the assumption of common

enables the upstream producers to learn this information (*if they commit to do so*) through the observation of the ex post realization of retail prices. Accordingly to the previous literature we allow upstream producers to choose among two alternative informational channels (or organizational modes): *(i) vertical integration* which is achieved trough the inclusion of *price restrictions* in the contract offered by upstream producers to downstream dealers[9]. *(ii) vertical delegation,* where upstream producers commit not to monitor prices; hence, in order to learn the realized state of nature, they must thus leave an informational rent to downstream dealers[10]. Moreover, in contrast to the previous literature, in our model the strategic effect of vertical separation is explicitly framed in a dynamic setting. More specifically, under the common assumption that each manufacturer is able to observe the *type* but not the *terms* of it's competitors' contracts (see Rey and Jullien (2000) for an example) we show that price restrictions may be used as a credible punishment threat in order to enforce anticompetitive behavior through vertical delegation.

Differently from Caillaud and Rey we show that quantity competition, in the static setting, may lead to a simple *prisoner's dilemma* outcome depending upon the degree of products' substitutability. Although in the static game the choice of a finer informational channel[11] turns out to be a dominant strategy, as it forces downstream firms to behave

shocks looks very appealing for particular market configurations such as for instance gasoline, electricity, steel and chemistry sectors where (i) collusive behavior has been identified empirically, and (ii) the production technology is relatively homogeneous across firms and the shocks come mainly from the demand side of the market.

[9]Crucially, if prices are observable ex post, the assumption of *common shocks* together with the traditional symmetry assumption allows upstream producers to costlessly learn the realization of the market conditions and implement the complete information allocation. This is the reason why we label this kind of contract as vertical integration.

[10]We shall argue later that this nonlinear mechanism looks like a *quantity discount arrangement*.

[11]This is the case where the observation of prices enables upstream producers to implement the complete information Cournot-Nash outcome.

more aggressively[12], we shall characterize sufficient conditions under which profits obtained by upstream producers under price restrictions turn out to be lower than those obtained under a vertically delegated market structure or alternatively on quantity discount contracts. Under these conditions, the positive strategic effect of vertical delegation (eliciting double marginalization) dominates the negative impact of informational rents.

The main implications of our analysis suggest that, depending upon the degree of competition on the market (namely the degree of substitutability between products), the choice of quantity discount contracts can be regarded as a collusive behavior. In a more general sense the model underlines a close parallel between the traditional double marginalization problem and the effects of the informational rent on the retail market price in a principal-agent relationship. An alternative way of interpreting our results is to consider vertical delegation in a traditional sense. Our assumptions can be indeed easily framed in a model where the upstream producers, instead of choosing among different types of contracts, must decide whether to hire a downstream agent (or an expert) which owns private information about the market and\or technology characteristics; or serving the downstream market on his own and thus acquire the relevant information without any cost[13].

Our results are supported by the empirical findings in gasoline retail markets presented by Shepard (1990). Referring to Slade's (1986 -1987) estimations, she argues that the refiner-run outlets are associated with lower price (higher quantity) gas stations as opposed to

[12] It implements "higher" reaction functions in the market game.

[13] Even though this interpretation might seem natural and the commitment to hire a retailer is very likely to be publicly observable, it is easy to notice that this formulation would then imply vertical integration to serve as a punishment device within the model. The extent to which such punishment is credible is not clear, since further problems may arise. For example, the integration process may take time and involve some transaction costs that are not modeled, etc.

higher prices charged at stations operated by dealers. In addition, it is worth noticing that Slade collects sufficient evidence to support the hypothesis that the reaction functions are significantly different from single period best replies. Her explanation relies on price discrimination theory. Our model offers an alternative explanation. We can view the dealer-run gasoline stations (or in other words vertical structures) as an example of collusion among the manufacturers, where the vertical structure is being used to *artificially* separate the manufacturer from the market relevant information, and she thus has to costly extract it from her dealer, which results in departures from single period best replies.

The structure of the paper is as follows. Section 2 presents the basic setup of the model. We discuss the price restriction contracts in Section 3. The case of quantity discounts is presented in Section 4. Section 5 summarizes the solution of the model in the static setting, whereas the dynamic extension is considered in Section 6. Section 7 concludes. Most of the proofs are relegated to the appendix.

2 The Model

Consider a symmetric differentiated duopoly model in which two downstream firms (retailers) (D_i, D_j) producing gross substitutes goods compete in quantities. Let $R^i(q_i, q_j, \theta)$ be the revenue/profit function of the downstream firm entering market i and assume that the *market value* is uncertain in the sense that the profits of both firms are affected by a common random variable θ. Let θ be distributed according to the cumulative distribution function $F(\theta)$ on the compact support $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$, and denote by $f(\theta) = F'(\theta)$ its density function. Assume θ affects positively profits, meaning that higher realizations for θ

denote better market conditions[14] or positive aggregate technological shocks. To carry out production both downstream firms must buy an input from their *exclusive* suppliers denoted by (S_i, S_j). Crucially, we assume that before the market game takes place each upstream producer decides *secretly the terms* of a contract to offer to her own retailer by means of a take-it or leave-it offer.

Invoking the revelation principle we assume that each vertical hierarchy - *producer-retailer* - plays a direct communication game in which the retailer reports a message $\hat{\theta}$ to his own producer and according to this message an incentive compatible allocation is selected. Our analysis restricts attention to two alternative contractual regimes: *(i)* S_i may *commit not to* observe the ex post level of price in market i and propose a nonlinear mechanism[15] $\mathcal{Q}_i = (q_i(\hat{\theta}_i), t_i(\hat{\theta}_i))$ specifying a quantity schedule $q_i(\hat{\theta}_i)$ and a transfer function $t_i(\hat{\theta}_i)$ both contingent on D_i's message $\hat{\theta}_i$. *(ii)* Conversely, S_i may *commit to* observe the ex post price in market i, $p^i(\theta, \hat{\theta}_i, \hat{\theta}_j)$, and offer a nonlinear mechanism $\mathcal{R}_i = (q_i(\hat{\theta}_i), t_i(\hat{\theta}_i, p^i(\theta, \hat{\theta}_i, \hat{\theta}_j)))$ specifying a quantity schedule which depends upon $\hat{\theta}_i$ and a transfer function which depends not only on D_i's message but also on the ex post realization of price in his own market.

Let $\mathcal{C} = \{\mathcal{Q}, \mathcal{R}\}$ be the contract space. The game proceeds in the following way:

- at time t=0: a realization of θ occurs and the retailers observe it, but the producers do not.

[14]In fact we assume that θ is common to both markets, hence we can view it as "global" market conditions - like level of household or government spending etc, or local market conditions which affect all single good markets in a similar way.

[15]According to standard principal-agent model we assume that the uninformed agent makes a "take-it-or-leave-it" offer.

- at time t=1: the upstream producers decide simultaneously which mechanism $\mathcal{M}_i \in \mathcal{C}$ to implement.

- at time t=2: the choices made at t=1 become common knowledge, and according to them the market game takes place and payments are made.

The simultaneous move game played at time t=1 is summarized in Table 1.

Table 1: Game played at t=1

i\j	Quantity Discounts ($\mathcal{Q}$)	Price Restrictions ($\mathcal{R}$)
$\mathcal{Q}$	$\pi^i(\mathcal{Q},\mathcal{Q})$, $\pi^j(\mathcal{Q},\mathcal{Q})$	$\pi^i(\mathcal{Q},\mathcal{R})$, $\pi^j(\mathcal{Q},\mathcal{R})$
$\mathcal{R}$	$\pi^i(\mathcal{R},\mathcal{Q})$, $\pi^j(\mathcal{R},\mathcal{Q})$	$\pi^i(\mathcal{R},\mathcal{R})$,$\pi^j(\mathcal{R},\mathcal{R})$

Each upstream producer playing the game has then two possible actions defined as $\mathcal{M}_i \in \{\mathcal{Q},\mathcal{R}\}$, where $\mathcal{Q}$ stays for *quantity discounts (or vertical delegation)* and $\mathcal{R}$ stays for *price restrictions (or vertical integration)*. Given the information structure implied by each of these actions, we may define a mapping $\Phi_i : \mathcal{C} \to \Omega_i(q_j)$, according to which for every upstream producer-i's action there is one reaction function played in the market game[16]. In fact one can notice that in this game the producers are actually choosing reaction functions. Let *"PR"* and *"QD"* denote respectively *price restrictions* and *quantity discounts* and define by $\Omega_i(q_j) = \left\{q_i^{PR}(\theta, q_j), q_i^{QD}(\theta, q_j)\right\}$ the set of the admissible reaction functions which will be characterized below. Let $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$ denote the inverse hazard rate. We impose the following technical assumptions:

[16]This is true for the case of strategic substitutes goods see Martimort (1996) for a discussion.

Assumption 1 $R^i \in C^3$; $R^i(0, q_j, \theta) = 0$ *for any* (q_j, θ), $R^i_{ii} = \frac{\partial^2 R^i}{(\partial q_i)^2} < 0$, $R^i_{ji} = R^j_{ij} \leq 0$; $R^i_\theta > 0$; $R^i_{\theta i} \geq 0$; $R^i_{\theta j} \leq 0$; $R^i_j < 0$, $R^i_i(q_i, q_j, \theta) = 0$ *has a unique solution in* q_i *for all* (q_j, θ).

Most of these assumptions are fairly standard. We want the revenue function to be three times continuously differentiable; further we need the second order conditions and Slutsky symmetry to hold, and to guarantee existence of a Nash equilibrium we assume that the game is *supermodular* (see Vives (2001) for a formal discussion). We also require that higher values of θ result in higher profits for each owner, and that this effect becomes stronger the higher is own quantity, while it becomes weaker the higher is the quantity produced by the rival. Constant sign assumption of the first derivative just makes sure that goods are always substitutes whereas uniqueness of the best response is just a convenient mathematical simplification.

Assumption 2 $R^i_{ij\theta} \geq 0$, $R^i_{\theta ii} \geq 0$ *and* $R^i_{\theta i\theta} \leq 0$ *for all* (q_i, q_j, θ).

Assumption 2 is required in order to guarantee respectively: concavity of the Hamiltonian which solves the producer's problem under quantity discounts (vertical delegation) and monotonicity (with respect to θ) of the equilibrium market quantity.

Assumption 3 *The inverse hazard rate is non-increasing, e.g.* $\dot{h}(\theta) \leq 0$

Assumption 3 is standard in the literature on adverse selection. Also, for the sake of clarity of our analysis, we restrict our attention to the case where marginal costs are zero, and we rule out storable quantities, as the latter would complicate analytical derivation of our results without any qualitative changes. [17]

Assumption 4 *Production does not involve any cost, and storing is not allowed.*

Finally, following Martimort (1996) the next assumption is equivalent to a sort of Spence-Mirlees condition in the context of competing vertical hierarchies:

Assumption 5 (Aggregate payoff condition) $\frac{\partial}{\partial q_i}\left(\frac{R^i_j}{R^i_\theta}\right) = 0$

Assumption 5 can equivalently be stated as $\frac{\partial}{q_i}\left(\Lambda\left(q_j,\theta\right)\right) = \frac{\partial}{\partial q_i}\left(\frac{R^i_j}{R^i_\theta}\right) = 0$ or $R^i_{ij}R^i_\theta = R^i_{\theta i}R^i_j$. Moreover, notice that under the restrictions imposed by assumption 1, assumption 5 implies (as commonly assumed in the literature) that whenever goods are gross substitutes, the choice variables are also strategic substitutes, i.e. $sign(R^i_{ij}) = sign(R^i_j)$ (see Dixit (1986) for an intuition).

In the remainder we shall solve the game shown in Table 1. We will consider first the situations where owners play symmetrically, $\mathcal{M} = (\mathcal{R},\mathcal{R})$ and $\mathcal{M} = (\mathcal{Q},\mathcal{Q})$, then we shall prove that the unique Nash equilibrium of the (static) game involves vertical integration, namely both of them playing price restrictions. Finally, we will provide conditions, under which coordinating on quantity discount contracts would be more profitable for the producers, even though harmful to the final consumer.

[17]For more discussion see Blair and Lewis (1994).

3 Price Restrictions

In this section we consider the case where both upstream producers impose price restrictions and achieve an outcome on the downstream market equivalent to one achieved under vertical integration meant in the traditional sense (merger of one upstream and one downstream firm). Notice that here S_i's problem is *fully equivalent* to one we would have to solve in the simple complete information setting. The lemma below establishes formally that indeed price restrictions implement the complete information Cournot-Nash market allocation.

Lemma 1 *In the communication game, when both upstream producers implement price restrictions, the following property holds: a Nash equilibrium entails both downstream retailers revealing truthfully the information and no rents are left to them.*

Making use of lemma 1 we can write S_i's optimization program as follows

$$\max_{q_i \in \Re_+} \quad R^i\left(q_i, q_j, \theta\right)$$

Focusing on a symmetric Nash equilibrium, a Cournot-Nash market allocation is defined by the following first order condition (FOC) (from hereafter we shall suppress the superscript i since the game is symmetric)

$$R_i(\mathbf{q}^{PR}, \theta) = 0 \tag{1}$$

where the vector $\mathbf{q}^{PR} = (q^{PR}(\theta), q^{PR}(\theta))$ defines the state contingent pair of Cournot-Nash equilibrium quantities if both producers implement price restrictions. Notice that as the game is submodular, that is $R_{ij} \leq 0$, the set of Nash equilibria is *not empty*, *bounded above and below* and a *symmetric equilibrium* always exists.

Direct implication of FOC is that the slope of the reaction functions is negative and defined as follows

$$\frac{\partial q_i^{PR}(q_j,\theta)}{\partial q_j}=\frac{R_{ij}(\mathbf{q}^{PR},\theta)}{|R_{ii}(\mathbf{q}^{PR},\theta)|}$$

Whereas the slope of the equilibrium quantity $q^{PR}(\theta)$ with respect to θ is positive and defined by the following expression

$$\dot{q}^{PR}(\theta)=\frac{R_{i\theta}(\mathbf{q}^{PR},\theta)}{|R_{ii}(\mathbf{q}^{PR},\theta)|+|R_{ij}(\mathbf{q}^{PR},\theta)|}\geq 0 \tag{2}$$

Finally, one can easily derive the expression showing how profits earned under price restrictions $\pi^{PR}(\theta)$ vary with respect to θ.

Lemma 2 *The profit earned by each producer under price restrictions is an increasing function of θ.*

Proof. Using together the first order condition (1) and the aggregate-payoff condition (Assumption 5) one gets

$$\dot{\pi}^{PR}(\theta)=R_{i\theta}(\mathbf{q}^{PR},\theta)\left(\frac{\left|R_j(\mathbf{q}^{PR},\theta)\right|}{|R_{ij}(\mathbf{q}^{PR},\theta)|}-\frac{\left|R_j(\mathbf{q}^{PR},\theta)\right|}{|R_{ii}(\mathbf{q}^{PR},\theta)|+|R_{ij}(\mathbf{q}^{PR},\theta)|}\right) \tag{3}$$

which is positive under the assumptions we have made. ■

4 Quantity Discounts

In this section we turn to study the case where both upstream producers *commit not to* observe prices. The market structure is thus characterized by *vertical separation* since the upstream producers decide to strategically separate themselves from the market information and induce revelation of this information through a nonlinear mechanism as in Mussa and

Rosen (1978) or Maskin and Riley (1984). Notice that each retailer learns the realization of θ before the contracting stage takes place.

Throughout the analysis we will extensively refer to the revelation principle (Myerson 1982); more specifically, any equilibrium of the game in which producers compete through indirect mechanisms implemented by some nonlinear schedules generates payoffs for the producers and the retailers that can also be achieved when each producer offers a direct truth-telling mechanism[18]. We also restrict our attention to the class of differentiable and deterministic mechanisms.

Finally, notice that the assumption of the actual contractual terms being *secret* is crucial in this framework and that we *implicitly* assume that the market is sufficiently valuable in the sense that shut down of some types is never optimal.

Assuming that the informational rent is increasing in θ (which will be checked ex-post), the mechanism $\mathcal{Q}_i = (q_i(\theta), t_i(\theta))$[19] solves the following problem

$$\begin{gathered}
\max_{\langle q_i(\theta), U_i(\theta)\rangle} \int_\Theta \left[R(q_i, q_j, \theta) - U_i(\theta)\right] f(\theta)\, d\theta \\
s.t. \\
(IR) \quad U_i(\theta) \geq 0 \;\forall\, \theta \in \Theta \\
(IC_1)\; \dot{U}_i(\theta) = R_\theta(q_i, q_j, \theta) + R_j(q_i, q_j, \theta)\, \dot{q}_j(\theta) \\
(IC_2) \quad \dot{q}_i(\theta)\left[R_{i\theta}(q_i, q_j, \theta) + \dot{q}_j(\theta)\, R_{ji}(q_i, q_j, \theta)\right] \geq 0
\end{gathered} \tag{4}$$

According to the standard techniques (see for instance Laffont and Tirole (1993)), we

[18]See Martimort (1996) pg. 6 for a sketch of the proof.

[19]The direct mechanism $\mathcal{Q}_i = (q_i(\theta), t_i(\theta))$ is implemented by a nonlinear schedule $T_i(q_i) = t_i(\theta(q_i))$ such that each upstream producer gives up any direct comunication with the retailer and lets him choose the quantity, within the schedule $T_i(q_i)$, which looks like a quantity discount contract. This procedure is known as the *taxation principle*, see Laffont and Martimort (2001) (Ch.9) for a more detailed discussion.

define a relaxed problem which does not include (IC_2), and we will check it later. As the informational rent increases in θ, the individual rationality constraint binds only for the most inefficient type, namely at $\underline{\theta}$

$$U_i\left(\underline{\theta}\right) = 0$$

The solution to the relaxed problem must optimize the following Hamiltonian

$$H\left(q_i, q_j, \theta, \lambda\left(\theta\right)\right) = f\left(\theta\right)\left(R(q_i, q_j, \theta) - U_i\left(\theta\right)\right) + \lambda\left(\theta\right)\left(R_\theta(q_i, q_j, \theta) + \dot{q}_j R_j(q_i, q_j, \theta)\right)$$

where $\lambda\left(\theta\right)$ defines the multiplier associated with the local incentive compatibility constraint (IC_1). Assuming that $H\left(q_i, q_j, \theta, \lambda\left(\theta\right)\right)$ is concave[20] in q_i the Pontryagin's principle applies

$$\dot{\lambda}\left(\theta\right) = -\frac{\partial H}{\partial U_i} = f\left(\theta\right)$$

As $\lambda\left(\overline{\theta}\right) = 0$ since there is no transversality condition on $U_i(\theta)$, one gets

$$\lambda\left(\theta\right) = -\left(1 - F\left(\theta\right)\right)$$

Optimizing with respect to q_i we have

$$R_i(q_i, q_j, \theta) = h(\theta)(R_{\theta i}(q_i, q_j, \theta) + \dot{q}_j(\theta)R_{ji}(q_i, q_j, \theta)) \tag{5}$$

Let $\mathbf{q}^{QD}$ be the two dimensional state-dependent vector of the (symmetric) equilibrium

[20] We will show later that under Assumption 5 this is true.

solutions to the market game. In a symmetric Nash equilibrium the optimal allocation $q^{QD}(\theta)$ is defined as the solution to the following differential equation

$$\dot{q}^{QD}(\theta) = \frac{R_i(\mathbf{q}^{QD}, \theta) - h(\theta) R_{\theta i}(\mathbf{q}^{QD}, \theta)}{h(\theta) R_{ij}(\mathbf{q}^{QD}, \theta)} \tag{6}$$

together with the boundary condition

$$q^{QD}\left(\overline{\theta}\right) = q^{PR}\left(\overline{\theta}\right) \tag{7}$$

Next proposition, which is just a slight alteration of Proposition 2 in Martimort (1996), then follows

Proposition 1 *The market outcome defined by (6) and (7) is unique. Moreover it satisfies*

$$q^{M}(\theta) \leq q^{QD}(\theta) \leq q^{PR}(\theta) \ \forall\, \theta \in \Theta$$

with both equalities at $\theta = \overline{\theta}$, and where $q^{M}(\theta)$ is the symmetric solution to

$$R_i(q_i, q_j, \theta) = h(\theta) R_{\theta i}(q_i, q_j, \theta) \tag{8}$$

Notice that competition here plays a crucial role; indeed, as choice variables in the market game are strategic substitutes, the *rent minimization - efficiency* trade-off turns out to be less severe. In particular, in the contracting game played at time $t = 1$ there emerges a positive externality between the upstream producers. When θ increases, as $q_j(\theta)$ increases too and $R_j(q_i, q_j, \theta)$ is negative for all (q_i, q_j, θ), producer i receives a *discount* in terms of informational rent that she must give up to his retailer in order to induce truthful revelation. Hence, since the costs associated with asymmetric information decrease via this externality, the (optimal) quantity distortion needed to induce truthful revelation is going to be lower than the one arising were this externality absent, e.g. $q^{M}(\theta)$.

In the appendix we show that $\dot{q}^{QD}(\theta)$ is positive, that at the symmetric Nash equilibrium the monotonicity constraint holds, that the informational rent increases with θ, that the global incentive compatibility constraint holds for all θ, and finally that the Hamiltonian is strictly concave, which are the properties that we indeed needed to hold ex post in the equilibrium which we want to study.

5 Solution to the Static Game

In this section we shall give a characterization for the Nash equilibrium of the static game defined in Table 1. Crucially, our result relies on the fact that when an upstream producer chooses to implement price restrictions, it is as if the producer knew perfectly the realization of θ, and the contracting stage took place under complete information. We shall prove that the Nash equilibrium of the static game involves both producers imposing price restrictions. The proof is based on a simple revealed preference argument.

Proposition 2 *At the unique subgame perfect Nash equilibrium of the game both producers choose to implement price restrictions, that is* $\mathcal{M}^{*}=(\mathcal{R},\mathcal{R})$.

Proposition 2 confirms an important, yet very intuitive point. In a quantity setting game the upstream producers would like to force their downstream retailers to behave as aggressively as possible. Vickers (1985) in this respect showed that in the case of strategic substitutes (quantity setting game) and complete information, it would be in the *individual,* *but not collective interest* of each producer to vertically separate, make the retailer a residual claimant of the profits and sell to him below marginal cost, in order to commit to being more aggressive (shift reaction function out) and thus achieve higher profits, which would be extracted through a fixed fee. Observing that implementing quantity discount contract is equivalent to vertical separation and price restriction to vertical integration, our main result suggests that Vicker's point might no longer hold if the vertical separation involves informational asymmetries. In particular as we will see in the next section, it might be in *joint interest* of the producers to vertically separate.

6 Price Restrictions vs Quantity Discounts: a Dynamic Perspective

So far we have shown that the unique *SPNE* of the static game described in Table 1 excludes situations where upstream producers choose to be strategically ignorant and implement quantity discounts contracts, whereby committing not to observe the ex post price realization. This result, however, is not in line with the empirical evidence showing that (i) quantity discounts contracts (or equivalently vertical delegation) is used in a large numbers of industries, and (ii) estimated reaction functions in many markets are sensibly different than the predicted static ones. To this extent the present section is aimed to address the issue in a dynamic perspective. In particular, we aim to characterize sufficient conditions under which quantity discounts contracts allow the producers to achieve a more collusive market outcome than price restrictions. The basic idea is the following: vertical delegation, in a context characterized by *adverse selection*, is analytically equivalent to a standard Cournot game with differentiated products where firms compete at *higher marginal costs*. This rise in marginal costs is due to the informational rents that upstream producers must pay to their retailers to induce truthful information revelation. The issue then becomes to characterize how the informational rent affects the producers' profits. The answer naturally involves the identification of a *trade-off*. Specifically - just as in the traditional double marginalization problem - on the one hand, as the reaction functions shift downward, and market quantities are getting closer to the monopoly benchmark, the producers are better-off *(strategic effect)*. On the other hand, there is a negative counterbalancing effect due to the loss that producers incur as the additional cost of informal rent distributed to the retailers has a direct negative impact on profits *(direct effect)*. Obviously, when the former effect dominates the latter one, although the unique SPNE equilibrium of the static game involves the integrated market structure (or price restrictions), it would be in the collective interest of both producers to coordinate toward a vertically separated market structure (quantity discounts).

A full characterization of conditions under which a vertically disintegrated market structure yields payoffs higher than those obtained under vertical integration would require to compare the respective upstream producers' payoffs in expectation over θ. Nevertheless, as this task in its general form lacks tractability, we will approach the problem by studying this difference on a pointwise basis. In this respect the conditions we shall characterize below are only *sufficient*.

Let π^{QD} the expected profit earned by each producer when quantity discounts are played. Integration by parts of $\dot{U}(\theta)$ together with aggregate payoff condition (see the appendix for a proof) yield the following expression for the owner's state contingent virtual surplus

$$\pi^{QD}(\theta) = R(\mathbf{q}^{QD}, \theta) - h(\theta)R_\theta(\mathbf{q}^{QD}, \theta)(1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta)) \tag{9}$$

In the remainder of the section we shall prove two results. *First* we shall characterize a sufficient condition under which the *strategic effect* discussed above is present. *Second* we prove that under some quite intuitive assumptions, a *sufficient* condition for quantity discounts to be strictly preferred to price restrictions is that competition on the downstream market must be sufficiently tough. Before going trough the analytical results, for a notational convenience it is worth to explain what we mean by sufficiently tough competition. We believe that an appropriate notion of competition is a measure of the steepness of the reaction functions, that means that the higher is the degree of substitutability between products, the steeper are the reaction functions, and the tougher is the competition. Therefore we will say that competition is sufficiently tough if the steepness of the reaction function exceeds some lower bound characterized in the result we present.

Definition 1 *For any given positive function $k(q_i, q_j, \theta)$ we shall say that competition is sufficiently*[21] *tough if $|R_{ij}(q_i, q_j, \theta)| \geq k(q_i, q_j, \theta)$ for all (q_i, q_j, θ).*

The following proposition characterizes a sufficient condition for the *strategic effect* to exist, namely situations in which $R(\mathbf{q}^{QD}, \theta) > R(\mathbf{q}^{PR}, \theta)$.

Proposition 3 *Vertical delegation (quantity discount contract) involves a strategic effect if competition is sufficiently tough.*

[21] *"Sufficiently" is meant relative with respect to the function $k(\cdot)$.*

The last proposition confirms the intuition that if the competition in the downstream market is strong, then if both upstream producers were to reduce the quantity supplied to the downstream retailers, the resulting market outcome would involve higher profits to both producers.[22] Of course any contract between the two upstream firms directly attempting to secure a reduction in quantities supplied to downstream retailers would be considered anti-competitive, and would thus be severely punished by the anti-trust authority if detected. Offering a quantity discount contract to the downstream retailer rather than a price restricting contract is, however, hardly considered harmful to competition. On the contrary, price restricting contracts are usually regarded as more dangerous to competition than quantity discounts. However, our following analysis provides conditions under which precisely the unexpected can happen, that is quantity discount contracts lead to more collusive outcome than price restrictions.

Let us look for a sufficient condition, under which not only the strategic effect exists, but it also *overcomes* the direct negative effect of the informational rent left by the upstream producer to the downstream retailer. We will see that once again such a condition requires competition being sufficiently tough. Apart from this, we also require the revenue functions being convex or slightly concave in θ - which is summarized in the following assumption:

Assumption 6 $\frac{R_{\theta\theta}(q_i,q_j,\theta)}{R_\theta(q_i,q_j,\theta)} \geq \frac{R_{\theta i\theta}(q_i,q_j,\theta)}{R_{\theta i}(q_i,q_j,\theta)}$ *for all admissible* (q_i, q_j, θ)

The economic rationale for this assumption will be discussed later. Let $\Delta(\theta) = \pi^{QD}(\theta) - \pi^{PR}(\theta)$ denote the pointwise difference between profits earned by the upstream producers

[22]On the other hand if the competition on the downstream market is weak, the retailers are enjoying almost a local monopoly position. Therefore a reduction in their quantity sold leads to a reduction in their revenue.

under vertical delegation and under vertical integration. As $\Delta\left(\bar{\theta}\right) = 0$, to show that $\Delta\left(\theta\right) \geq 0$ for all θ we just need to characterize conditions such that $\dot{\Delta}\left(\theta\right) \leq 0$ for all θ.

Proposition 4 *Suppose assumption 6 holds and that competition is sufficiently tough, then quantity discount contracts achieve higher profits than price restrictions.*

The intuition for the additional requirement on the curvature of the revenue function with respect to market uncertainty can be provided quite easily. The basic lessons from incentive theory tell us that the highest informational rent is paid out to the "highest types". If the revenue function were concave in the market uncertainty, under quantity discounts each additional "high type" would cost the upstream producer a lot in terms of a high informational rent without a substantial increase in revenue - the direct negative effect of separation from the market information (i.e. the informational rent) would overcome the positive strategic effect associated with softened competition achieved through quantity reduction. The upper bound on the concavity of the revenue function in the market uncertainty is precisely such that the additional dollar of rent given up to one additional "high type" is still outweighed by the gain through quantity distortion.

A simple example

Finally, since much of the IO literature considers as a standard benchmark case an oligopoly model with linear demand functions[23], we restrict our attention to the case of quadratic revenue functions. For this case we prove a strong version of our previous results. Specifically,

[23] An example would be the following demand system: $q_1\left(p_1, p_2\right) = \alpha + \theta - \beta p_1 + \gamma p_2$ and $q_2\left(p_1, p_2\right) = \alpha + \theta - \beta p_2 + \gamma p_1$.

in this case it turns out that upstream producers obtain payoffs under quantity discounts that are *always higher* than those obtained under price restrictions, no matter how strong the competition is.

Linearity of downstream demand functions requires in our model the following assumption, which is in fact weaker:

Assumption 7 *All third derivatives of the revenue function $R(q_i, q_j, \theta)$ are zero.*

We begin by a technical lemma, which will be useful later:

Lemma 3 *Assumptions 5 and 7 together imply $\frac{\partial}{\partial q_j}\left(\frac{R_j(q_i,q_j,\theta)}{R_\theta(q_i,q_j,\theta)}\right) = \frac{\partial}{\partial \theta}\left(\frac{R_j(q_i,q_j,\theta)}{R_\theta(q_i,q_j,\theta)}\right) = 0$ which in turn entail $R_{\theta\theta} \geq 0$, $R_{jj} \geq 0$ and $\frac{R_{jj}}{R_{\theta j}} = \frac{R_{j\theta}}{R_{\theta\theta}} = \Lambda$.*

The last lemma confirms that indeed the extra assumption of "not too concave" revenue function in the proposition 4 will be satisfied for a linear demand model satisfying our initial assumptions. It would be an immediate implication of proposition 4 that with tough competition, coordination on quantity discount contracts would achieve higher profits than price restricting contracts. The following proposition, however, shows that in the case of linear demands the degree of competition is not even important for this result:

Proposition 5 *Under assumption 7, profits achieved under quantity discounts are pointwise weakly higher than under price restrictions.*

Proof. First notice that $(1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta)) = \frac{R_i(\mathbf{q}^{QD},\theta)}{h(\theta)R_{\theta i}}$ is a direct implication of (6) and $\Lambda = \frac{R_{ij}}{R_{\theta i}}$. Plugging this expression into (9) one gets $\pi^{QD}(\theta) = R(\mathbf{q}^{QD}, \theta) -$

$\frac{R_\theta(\mathbf{q}^{QD},\theta)R_i(\mathbf{q}^{QD},\theta)}{R_{\theta i}}$, using an envelope argument, total differentiation of $\pi^{QD}(\theta)$ with respect to θ yields

$$\begin{aligned}\dot\pi^{QD}(\theta) &= R_\theta(\mathbf{q}^{QD},\theta) + \dot q^{QD}(\theta)R_j(\mathbf{q}^{QD},\theta) - \frac{R_\theta(\mathbf{q}^{QD},\theta)}{R_{\theta i}}(R_{\theta i} + \dot q^{QD}(\theta)R_{ij}) \\ &\quad - \frac{R_i(\mathbf{q}^{QD},\theta)}{R_{\theta i}}(R_{\theta\theta} + \dot q^{QD}(\theta)R_{\theta j})\end{aligned}$$

Using Assumption 5 together with lemma 3 and rearranging it is easy to show that $\dot\pi^{QD}(\theta)$ becomes

$$\dot\pi^{QD}(\theta) = -R_i(\mathbf{q}^{QD},\theta)(1+\dot q^{QD}(\theta)\Lambda)\frac{R_{\theta\theta}}{R_{\theta i}}$$

which is non-positive as $R_i(\mathbf{q}^{QD},\theta) \geq 0$, with equality at $\bar\theta$ (see also proof of lemma 5), $R_{\theta\theta} \geq 0$, $R_{\theta i} > 0$ and $(1+\dot q^{QD}(\theta)\Lambda) \geq 0$. The result is then a direct implication of $\dot\pi^{QD}(\theta) \leq 0$ and proof of lemma 2. ■

Discussion

The results presented in the previous paragraphs are quite surprising. As noted earlier much of the antitrust work dealing with vertical contracting focuses on price restrictions fearing that regulation of prices by upstream producers may lead to collusive outcomes. The effect identified in our model can be, however, quite opposite. In particular, as we can see, price restricting contracts can *eliminate an adverse selection problem*, which would otherwise exist in an upstream producer - downstream retailer relationship. This adverse selection problem can indeed be *intentionally* used by upstream producers to soften the downstream competition and achieve a more collusive downstream market outcome. Therefore quantity discount contracts, which are quite common in the real upstream - downstream relationships and have not been so far viewed as potentially harmful to competition, can be viewed as a tool for implementing collusion, because they lead to artificial separation of the producer from the downstream market, and to competition softening. Once again we can see the parallel between the current argument and the solution to the traditional double marginalization problem as proposed by the Chicago school economists.

Alternatively, we can view this result as a vertical hierarchy (upstream producer - downstream retailer coalition) being preferred to a vertically integrated firm. Again the driving force being the existence of the adverse selection problem within a vertical hierarchy as opposed to an integrated firm, which softens competition on the downstream market.

Dynamic Perspective

So far we have been analyzing a static problem. In real life situations, however, producers and their retailers interact repeatedly, which significantly broadens their sets of possible strategies. Using the results from our static analysis we will now discuss how framing the model in the dynamic setting can affect the market outcome.

Consider an infinite repetition of the stage game described in the previous sections. In this case, when other instruments implementing perfect collusion are not available to the producers, the option of choosing a particular vertical restraint can provide an opportunity for partial collusion, which brings the quantities closer to the monopoly benchmark, but does not achieve it. Moreover, in the absence of information about all prices[24] it may as well provide a device to communicate and implement collusive behavior, because it is reasonable to imagine that the decision of the producer whether to implement price restrictions or quantity discounts would be relatively easily observable by the rivals, whereas the exact

[24] For example if public price lists as in Allbaek et al (1997) were banned by the antitrust authority, or were unreliable.

contractual terms that might have been agreed upon within a vertical hierarchy would not.

The repeated game in the easiest formulation can be thought of as follows: at each period $\tau = (1, .., \infty)$ a particular realization of θ_τ occurs (we assume for simplicity that these realizations are independent and identically distributed over time, i.e. $\theta_\tau \perp \theta_{\tau'}$) and that given this realization the game described in the previous sections takes place.

In this context, the folk theorem with delegation applies:

Proposition 6 *(Fershtman, Judd and Kalai (1991)) If* $\pi^l \left(q^{QD}, q^{QD}\right) \geq \pi^l \left(q^{PR}, q^{PR}\right)$ *for* $l \in \{i, j\}$ *then for sufficiently patient players a Subgame Perfect Nash Equilibrium (SPNE)*[25] *of a repeated game with delegation is characterized by strategies* q_l^{QD} *if* q_{-l}^{QD} *in all previous stages,* q_l^{PR} *otherwise.*

Hence as proposition 4 shows, if competition is sufficiently tough and the revenue function is not too concave in the market uncertainty θ, then the (expected) payoff associated with both producers choosing quantity discount contracts (i.e. choosing to artificially separate themselves from the market information) is strictly higher then the payoff associated with both manufacturers choosing price restrictions (i.e. vertically integrated market structure). Moreover, by proposition 6 there exists a critical value for the discount factor δ^* such that for all $\delta \geq \delta^*$, a pair of quantity discount contracts is sustainable as an SPNE of the repeated game.

This result may then suggest an alternative justification for the presence of vertical hierarchies. It especially applies to markets, in which it is not entirely clear what the vertical organization is useful for. A producers could choose to voluntarily separate themselves from

[25] *Folk theorem with delegation*

the source of information (the downstream market) through creating a vertical hierarchy, i.e. choose to be *strategically ignorant*, because such behavior is virtually impossible to be challenged by the anti-trust authority, and yet it can lead to a collusive outcome.

7 Conclusions

The analysis developed in this paper points out that vertical delegation, implemented through the design of quantity discount contracts, may allow upstream producers, as well as downstream retailers, to achieve profits higher than those obtained under vertical integration or contracts based on price restrictions. Our dynamic result shows that when *downstream competition is sufficiently tough* the design of suitable vertical restraints, while leaving to the downstream dealers a positive informational rent, implements a market outcome closer to the monopoly benchmark, which has a detrimental effect on consumer surplus. This finding is not in line with much of the antitrust work that has recently pronounced against price regulation by upstream producers. In contrast to this view, we argue that the choice of *quantity discount contracts*, which are quite common in producer-retailer relationships and viewed generally as unharmful by competition policy authorities, may *artificially* produce a double marginalization effect contributing in turn to softening of competition on the downstream market.

On the one hand our model seems to confirm from an alternative viewpoint the classical Chicago School's argument that in the static model price restrictions may be socially desirable as they avoid a double marginalization effect produced in our framework by adverse selection. On the other hand, however, we show that price restrictions can also serve as a credible punishment threat, which helps to enforce collusion through vertical delegation in the dynamic setting, when downstream competition is tough.

The insight we draw from our results is that making price restrictions illegal does not always fulfill the objective of avoiding anticompetitive behaviors. Legally banning vertical contracting based on retail price restrictions would indeed in some sense produce two possible sources of inefficiency: *(i)* when competition is sufficiently weak this policy is clearly *suboptimal* in the pareto sense as it forces upstream producers and downstream dealers to sign contracts which are not only unprofitable from the vertical hierarchy's viewpoint but that also impose a negative externality on consumers[26], *(ii)* when competition is sufficiently tough, banning price restrictions is harmful to consumers as it *forces* upstream producers to propose the contract that they prefer the most. In this case, in fact, for each admissible discount factor the market game will lead to anticompetitive solutions.

To summarize, our model underlines how on normative grounds, rather than legally banning some types of vertical restraints, the antitrust authorities should leave upstream producers and downstream dealers free to sign any contract they wish and then, given the nature of those contracts and the underlying market conditions, infer whether some form of collusion has been attempted.

[26]When competition on the downstream market is sufficiently weak, price restrictions not only are welfare improving, but they also emerge as an equilibrium outcome of the market game without the need of any institutional intervention.

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A APPENDIX

A.1 Proof of lemma 1

The proof is developed in two steps:

Step 1- *Each deviation $\hat{\theta}_i \neq \theta$ is detected at the equilibrium.*

Proof. We will use a backward argument. First any $\hat{\theta}_i \notin \Theta$ is detected trivially. Moreover, notice that (without loss of generality) for any given $\hat{\theta}_i \in \Theta$, as the game is symmetric and the shock θ is common, producer i will anticipate that if retailer i has revealed truthfully the information, it must be the case that $q_i(\hat{\theta}_i) = q_j$. Hence observing ex post q_j would be a sufficient statistic to infer whether retailer i lied. Suppose then that retailer j reveals truthfully his information to producer j, i.e. $\hat{\theta}_j = \theta$, and assume that retailer i deviates, i.e.. $\hat{\theta}_i \neq \theta$ with $\hat{\theta}_i \in \Theta$. Given any quantity allocation $q_i(\hat{\theta}_i)$, producer i will observe ex post a retail price realization $p^i(\theta, \hat{\theta}_i) = P^i(q_i(\hat{\theta}_i), q_j(\theta), \theta)$, as she knows the quantity level $q_i(\hat{\theta}_i)$ sold to retailer i and the functional form of the inverse market demand $P^i(q_i, q_j, \theta)$ is common knowledge, it is easy to back out $q_j(\theta)$ from $p^i(\theta, \hat{\theta}_i)$. Given $q_j(\theta)$ any deviation $\hat{\theta}_i$ will be detected. *QED*

For any given allocation $\langle q_i, t_i \rangle$ let $U^i = R^i(q_i, q_j, \theta) - t_i$ be retailer-i's utility function and p^i the ex post realization of market-i's retail price. Consider the following transfer schedule $t_i(\hat{\theta}_i, p^i)$ such that *(i)* if $p^i \neq P^i(q(\hat{\theta}_i), q(\hat{\theta}_i), \hat{\theta}_i)$ then $t_i(\hat{\theta}, p^i) = p^i q_i(\hat{\theta}_i)$, and *(ii)* if $p^i = P^i(q(\hat{\theta}_i), q(\hat{\theta}_i), \hat{\theta}_i)$ then $t_i(\hat{\theta}_i, p^i) = R(q_i(\hat{\theta}_i), q_j(\hat{\theta}_i), \hat{\theta}_i) - \varepsilon$ for $\varepsilon > 0$ arbitrarily small. The following step completes the proof

Step 2- *For any quantity schedule $q_i(\hat{\theta}_i)$ the transfer schedule $t_i(\hat{\theta}_i, p^i)$ leaves retailers with zero rents and implements truthful revelation.*

Proof. The proof is simple, for a given allocation $q_i(\hat{\theta}_i)$, any feasible deviation $\hat{\theta}_i \neq \theta$ retailer i may play in the communication game will be detected and he will end up with zero rent. Hence, for any arbitrarily small $\varepsilon > 0$ truthful revelation is a Nash equilibrium of the communication game. *QED*

A.2 Characterization of IC constraints and monotonicity condition

Assume producer-i decides to implement quantity discounts (i.e. to vertical separate). Given the mechanism $\mathcal{Q}_i = (q_i(\hat{\theta}_i), t_i(\hat{\theta}_i))$, retailer i's objective function is given by:

$$U_i(\theta, \hat{\theta}_i) = R(q_i(\hat{\theta}_i), q_j(\theta), \theta) - t_i(\hat{\theta}_i)$$

As the allocation $(q_i(\hat{\theta}_i), t_i(\hat{\theta}_i))$ must induce truthful revelation of the retailer's type (which in this case corresponds to the market conditions θ), the following global IC must hold for all θ

$$R(q_i(\theta), q_j(\theta), \theta) - t_i(\theta) \geq R(q_i(\hat{\theta}_i), q_j(\theta), \theta) - t_i(\hat{\theta}_i) \;\; \forall\, (\theta, \hat{\theta}_i) \in \Theta$$

As it is usually done in this literature (see for instance Martimort (1996)), this global incentive compatibility constraint can be replaced by a local one and by a monotonicity condition:

$$\left.\frac{\partial U_i(\theta,\hat{\theta}_i)}{\partial\hat{\theta}_i}\right|_{\theta=\hat{\theta}_i}=0$$

Using an envelope argument we then have:

$$(IC_1)\quad \dot{U}_i\left(\theta,\theta\right)=R^i_\theta\left(q_i,q_j,\theta\right)+R^i_j\left(q_i,q_j,\theta\right)\dot{q}_j\left(\theta\right)$$

Moreover, a second order incentive compatibility condition follows:

$$\left.\frac{\partial^2 U_i\left(\theta,\hat{\theta}_i\right)}{\partial\hat{\theta}_i^2}\right|_{\theta=\hat{\theta}_i}\leq 0$$

This condition implies $\left.\frac{\partial^2 U_i\left(\theta,\hat{\theta}_i\right)}{\partial\hat{\theta}_i\partial\theta}\right|_{\theta=\hat{\theta}_i}\geq 0$ which directly entails

$$(IC_2)\qquad \left.\frac{\partial^2 U_i\left(\theta,\hat{\theta}_i\right)}{\partial\hat{\theta}_i\partial\theta}\right|_{\theta=\hat{\theta}_i}=\dot{q}_i\left(\theta\right)\left(R_{i\theta}\left(q_i,q_j,\theta\right)+\dot{q}_j(\theta)R_{ij}\left(q_i,q_j,\theta\right)\right)\geq 0$$

A.3 Proof of proposition 1

The proof follows closely Martimort (1996). Consider the differential equation (6) and the boundary condition $q^{QD}(\bar{\theta})=q^{PR}(\bar{\theta})$, let's study the local behavior of the solution around the point $\bar{\theta}$. We first transform them into a system of homogenous differential equations for which $q(.)$ and $\theta(.)$ are functions of some parameter $t\in\Re$:

$$\dot{q}^{QD}(t)=R^i_i(\mathbf{q}^{QD}(t),\theta(t))-h\left(\theta(t)\right)R^i_{\theta i}(\mathbf{q}^{QD}(t),\theta(t))$$

$$\dot{\theta}(t)=h(\theta(t))R^i_{ji}(\mathbf{q}^{QD}(t),\theta(t))$$

Let $X(t)$ and $Y(t)$ be defined as follows:

$$X(t)\ =\ q^{PR}(\overline{\theta})-q^{QD}(t)$$

$$Y(t)\ =\ \overline{\theta}-\theta\left(t\right)$$

As $\left.\frac{dh(\theta)}{d\theta}\right|_{\theta=\bar{\theta}}=-1$, linearizing the system[27] around $\bar{\theta}$ we have:

[27] The values of second order derivatives of $R^i(.)$ are evaluated at $\theta=\overline{\theta}$ and $q^{ci}(\overline{\theta})$.

$$\dot{X}(t) = X(t)(R^i_{ii} + R^i_{ij}) + 2Y(t) R^i_{i\theta} \tag{A-1}$$

$$\dot{Y}(t) = -Y(t)R^i_{ij} \tag{A-2}$$

Direct integration of equation (A-2) gives $Y(t) = K\exp(-tR^i_{ij})$. Equation (A-1) then becomes:

$$\dot{X}(t) = X(t)(R^i_{ii} + R^i_{ij}) + 2K\exp(-tR^i_{ij})R^i_{i\theta}$$

solution of which yields:

$$X(t) = K_1 \exp((R^i_{ii} + R^i_{ij})t) + t_0 K \exp(-tR^i_{ij})$$

where K is some constant of integration and $t_0 = \frac{2R^i_{i\theta}}{(R^i_{ii}+2R^i_{ij})} < 0$. The following expression for $q^{QD}(\cdot)$ then obtains:

$$q^{QD}(\theta) = q^{PR}(\bar{\theta}) + t_0(\bar{\theta} - \theta) + \beta(\bar{\theta} - \theta)^K \tag{A-3}$$

where $K = -\frac{(R^i_{ii}+2R^i_{ij})}{R^i_{ij}} < 0$ and β is some constant of integration. As $R^i_{ij} < 0$ and $K < 0$, it follows that equation (A-3) has only one admissible solution for β, which is $\beta = 0$. Therefore q^{QD} has a derivative equal to $(-t_0)$ at $\theta = \bar{\theta}$.

We must now study the global behavior of $q^{QD}(\theta)$. We need to prove that $q^{QD} < q^{PR}$ for all $\theta < \bar{\theta}$. This property is certainly locally true, since $q^{QD}\left(\bar{\theta}\right) = q^{PR}\left(\bar{\theta}\right)$ and also:

$$\dot{q}^{QD}\left(\bar{\theta}\right) = -t_0 = \frac{2R^i_{i\theta}}{|R^i_{ii}| + 2|R^i_{ij}|} > \dot{q}^{PR}\left(\bar{\theta}\right) = \frac{R^i_{i\theta}}{|R^i_{ii}| + |R^i_{ij}|} \tag{A-4}$$

Assume for a moment that there is some highest $\theta^* < \bar{\theta}$ such that $q^{QD}(\theta) = q^{PR}(\theta)$ $\forall\, \theta > \theta^*$. But then we have from (6) and (1) that

$$\dot{q}^{QD}(\theta^*) = \frac{R_{i\theta}(\mathbf{q}^{QD}(\theta^*), \theta^*)}{|R_{ij}(\mathbf{q}^{QD}(\theta^*), \theta^*)|}$$

It is easy to check that $\dot{q}^{QD}(\theta^*) > \dot{q}^{PR}(\theta^*)$. Therefore for ε small enough and positive, we must have $q^{QD}(\theta^* + \varepsilon) > q^{PR}(\theta^* + \varepsilon)$, which is a contradiction.

A similar argument can be applied to show that $q^M(\theta) \leq q^{QD}(\theta)$, and the equality holds again only at $\bar{\theta}$. *QED*

The required ex post features of the equilibrium (in particular $\dot{q}(\theta) \geq 0$ and local second-order conditions) are checked in the following paragraphs.

A.4 Checking ex-post features of the equilibrium

Lemma 4 *$\dot{q}^{QD}(\theta)$ is positive.*

Proof. The denominator of equation (6) is negative since goods are strategic substitutes, $R^i_{ij}(.) \leq 0$ for all (q_i, q_j, θ). The numerator, however, is negative too since it is zero when the upstream producer selects the quantity schedules defined by equation (8) and it is concave in q_i and decreasing in q_j, e.g. $R_{ii}(.) \leq 0$ and $R^i_{ij\theta}(.) \geq 0$ for all (q_i, q_j, θ). *QED* ∎

Lemma 5 *At the symmetric Nash equilibrium when A5 holds then the following properties are satisfied:* (i) *the monotonicity condition IC_2 is always satisfied and* (ii) *the retailer's rent is increasing in θ.*

Proof. *(i)* As $\dot{q}^{QD}(\theta)$ is strictly positive (see lemma 4), to satisfy IC_2 we need the following inequality to hold at the Nash symmetric equilibrium:

$$R_{i\theta}(\mathbf{q}^{QD}, \theta) + \dot{q}_j\left(\theta\right) R_{ij}(\mathbf{q}^{QD}, \theta) \geq 0 \tag{A-5}$$

this can be rewritten as:

$$\dot{q}^{QD}\left(\theta\right) \leq \frac{R_{i\theta}(\mathbf{q}^{QD}, \theta)}{|R_{ji}(\mathbf{q}^{QD}, \theta)|} \tag{A-6}$$

However, equation (6) implies:

$$\dot{q}^{QD}\left(\theta\right) = -\frac{R_i(\mathbf{q}^{QD}, \theta)}{h(\theta)|R_{ji}(\mathbf{q}^{QD}, \theta)|} + \frac{R_{i\theta}(\mathbf{q}^{QD}, \theta)}{|R_{ji}(\mathbf{q}^{QD}, \theta)|}$$

Using proposition 1 we know that $q^{QD}\left(\theta\right) \leq q^{PR}\left(\theta\right)$, moreover combining this fact, Assumption 1 (strict concavity of $R^i\left(.\right)$ in q_i and q_j) and equation (1) we conclude that $R_i(\mathbf{q}^{QD}, \theta) \geq 0$ with equality at $\bar{\theta}$. Therefore, inequality (A-6) directly follows.

In order to show that IC_2 holds we can also notice that, using Assumption 5, (A-5) may be rewritten as:

$$R_{\theta i}(\mathbf{q}^{QD}, \theta)(1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta)) \geq 0 \Rightarrow (1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta)) \geq 0$$

which proves the result. *QED*

(ii) Using again Assumption 5 we can rewrite IC_1 as:

$$R_{\theta}(\mathbf{q}^{QD}, \theta)(1 + \dot{q}^{QD}\left(\theta\right)\Lambda(q^{QD}(\theta), \theta)) \geq 0$$

which directly proves the result as $R_\theta(.) \geq 0$ for all (q_i, q_j, θ). *QED* ∎

Lemma 6 *The global incentive compatibility constraint holds at the symmetric Nash equilibrium:*

Proof. Following Martimort (1996) we define the global IC as the difference $\bar{\Delta}$. Then, using the local incentive compatibility constraint:

$$\dot{t}_i\left(\theta\right) = R_i(q_i\left(\theta\right), q_j\left(\theta\right), \theta)\dot{q}_i\left(\theta\right)$$

we have:

$$\begin{aligned}
\bar{\Delta}\left(\theta,\hat{\theta}_i\right) &= R(q_i(\theta),q_j(\theta),\theta)-t_i(\theta)-R(q_i(\hat{\theta}_i),q_j(\theta),\theta)+t_i(\hat{\theta}_i)= \\
&= \int_{\hat{\theta}_i}^{\theta}\left\{R_i(q_i(u),q_j(\theta),\theta)\dot{q}_i\left(u\right)-\dot{t}_i(u)\right\}du= \\
&= \int_{\hat{\theta}_i}^{\theta}\left\{R_i(q_i(u),q_j(\theta),\theta)\dot{q}_i\left(u\right)-R_i(q_i(u),q_j(u),u)\dot{q}_i\left(u\right)\right\}du= \\
&= \int_{\hat{\theta}_i}^{\theta}\dot{q}_i\left(u\right)\left(\int_u^{\theta}\left\{\dot{q}_j(t)R_{ij}(q_i(u),q_j(t),t)+R_{i\theta}(q_i(u),q_j(t),t)\right\}dt\right)du= \\
&= \int_{\hat{\theta}_i}^{\theta}\dot{q}_i\left(u\right)\left(\int_u^{\theta}\left\{R_{\theta i}(q_i(u),q_j(t),t)\left(1+\Lambda(q^{QD}(t),t)\dot{q}^{QD}\left(t\right)\right)\right\}dt\right)du
\end{aligned}$$

which is positive at the symmetric Nash equilibrium since the common factor $\left(1+\Lambda(q^{QD}(t),t)\dot{q}^{QD}(t)\right)$ is positive for all t. *QED* ■

Corollary 1 *The Hamiltonian is strictly concave in q_i.*

Proof. The proof simply comes from Lemma 4 and Assumption 5. *QED* ■

A.5 Proof of proposition 2

We use a simple revealed preference argument. Let $q_j(\theta)$ be any given continuously differentiable quantity schedule chosen by producer j. Producer i must then decide whether or not to implement quantity discounts.

(i) If producer i chooses to implement price restrictions then she solves the following problem

$$\begin{gathered}
\max_{\langle q_i,t_i\rangle}\int_{\Theta}t_i(\theta)f\left(\theta\right)d\theta \\
s.t. \\
(IR)\quad U_i(\theta)=R\left(q_i,q_j\left(\theta\right),\theta\right)-t_i(\theta)\geq 0\quad\text{for all }\theta
\end{gathered}\tag{A-7}$$

(ii) Whereas, if she commits not to observe prices, the problem becomes.

$$\begin{gathered}
\max_{\langle q_i,t_i\rangle}\int_{\Theta}t_i(\theta)f\left(\theta\right)d\theta \\
s.t. \\
(IC_1)-(IC_2) \\
(IR)\quad U_i(\theta)=R\left(q_i,q_j\left(\theta\right),\theta\right)-t_i(\theta)\geq 0\quad\text{for all }\theta
\end{gathered}\tag{A-8}$$

Let $(\widehat{q}_i,\widehat{t}_i)$ be a solution to problem (A-8). As the (IR) constraint implies $U_i(\theta)=R\left(\widehat{q}_i\left(\theta\right),q_j,\theta\right)-\widehat{t}_i(\theta)\geq 0$ for all θ, the pair $(\widehat{q}_i,\widehat{t}_i)$ is also feasible for problem (A-7). Hence producer i must receive a weakly higher expected transfer if she does not vertically separate (the argument is the same for producer j). Notice, however, that this result does not imply anything about the difference between profits earned under the two contractual regimes. *QED*

A.6 Proof of lemma 3

Aggregate payoff condition implies $\Lambda = \frac{R_j}{R_\theta} = \frac{R_{ij}}{R_{\theta i}}$; moreover, as Assumption 7 sets third derivatives to zero, it follows that $\Lambda = \frac{R_j}{R_\theta}$ is constant also with respect to q_j and θ. Therefore, simple algebra shows that $\frac{\partial}{\partial q_j}\left(\frac{R_j}{R_\theta}\right) = \frac{\partial}{\partial \theta}\left(\frac{R_j}{R_\theta}\right) = 0$ which in turn implies $\frac{R_{jj}}{R_{\theta j}} = \frac{R_{j\theta}}{R_{\theta\theta}} = \Lambda$. *QED*

A.7 Characterization of the virtual surplus

Each producer's objective function under quantity discounts (vertical delegation) is given by

$$\pi^{QD} = \int_\Theta \left[R(\mathbf{q}^{QD},\theta) - U^{QD}(\theta)\right] dF(\theta) \tag{A-9}$$

where the informational rent given up to the downstream retailer is defined by the following differential equation

$$\dot{U}^{QD}(\theta) = R_\theta(\mathbf{q}^{QD},\theta)\left(1 + (\Lambda q^{QD}(\theta),\theta)\dot{q}^{QD}(\theta)\right) \tag{A-10}$$

As $U^{QD}(\underline{\theta}) = 0$, integration of (A-10) gives an expression for $U^{QD}(\theta)$

$$U^{QD}(\theta) = \int_{\underline{\theta}}^{\theta} R_\theta\left(\mathbf{q}^{QD}(u),u\right)\left(1 + \Lambda\left(q^{QD}(u),u\right)\dot{q}^{QD}(u)\right) du$$

Plugging this expression into (A-9) and integrating by parts over Θ one gets

$$\pi^{QD} = \int_\Theta \left(R(\mathbf{q}^{QD},\theta) - h(\theta)R_\theta(\mathbf{q}^{QD},\theta)(1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta),\theta))\right) dF(\theta)$$

where the state contingent function $\pi^{QD}(\theta) = R(\mathbf{q}^{QD},\theta) - h(\theta)R_\theta\left(\mathbf{q}^{QD},\theta\right)\left(1 + \dot{q}^{QD}\Lambda\left(q^{QD},\theta\right)\right)$ defines the producer's virtual surplus. *QED*

A.8 Proof of proposition 3

A sufficient condition for the *strategic effect*[28] to exist is certainly satisfied if the function $\Delta(\theta) = R(\mathbf{q}^{QD},\theta) - R(\mathbf{q}^{PR},\theta)$ is positive for all θ. As $q^{QD}(\theta) \le q^{PR}(\theta)$ for any θ, we can write $q^{QD}(\theta) = q^{PR}(\theta) - \delta(\theta)$, where the real valued function $\delta : \Theta \to \Re^+$ defines the distortion arising because of the rent-minimization efficiency trade-off and it satisfies $\dot{\delta}(\theta) \le 0$ and $\delta(\overline{\theta}) = 0$. Consider now any given $\theta \ne \overline{\theta}$, the function $\Delta(\theta)$ may be rewritten as

$$\tilde{\Delta}(\delta,\theta) = R(q^{PR}(\theta) - \delta(\theta), q^{PR}(\theta) - \delta(\theta),\theta) - R(q^{PR}(\theta), q^{PR}(\theta),\theta)$$

With a little abuse of notation, assume that at some given θ the distortion δ is a param-

[28] *The existence of the strategic effect at least for some subset of Θ of nonzero measure is necessary to achieve an increase in expected revenue through reduction of quantity for some realizations of θ.*

eter. Consider the following limits

$$\begin{aligned}\lim_{\delta\to 0}\tilde{\Delta}(\delta,\theta) &= 0\\ \lim_{\delta\to +\infty}\tilde{\Delta}(\delta,\theta) &= -R(q^{PR},q^{PR},\theta) > 0\end{aligned}$$

Let us now take the derivative of $\tilde{\Delta}(\delta,\theta)$ with respect to δ, then we have

$$\frac{\partial\tilde{\Delta}(\delta,\theta)}{\partial\delta} = -R_i(q^{PR}-\delta,q^{PR}-\delta,\theta) - R_j(q^{PR}-\delta,q^{PR}-\delta,\theta)$$

and notice that

(i) $R_i(q^{PR},q^{PR},\theta)=0$ together with $R_j(q_i,q_j,\theta)\leq 0$ for all (q_i,q_j,θ) imply

$$\lim_{\delta\to 0}\frac{\partial\tilde{\Delta}(\delta,\theta)}{\partial\delta} = -R_j(q^{PR},q^{PR},\theta) > 0$$

(ii) The curvature of $\tilde{\Delta}(.)$ is given by:

$$\frac{\partial^2\tilde{\Delta}(\delta,\theta)}{\partial\delta^2} = R_{ii}(.) + 2R_{ij}(.) + R_{jj}(.)$$

Therefore $\tilde{\Delta}(.)$ is concave in δ if $|R_{ii}|+2|R_{ij}|\geq R_{jj}$ for all (q_i,q_j,θ), which is natural to assume, since the revenue function of producer i is usually at least weakly concave in rival's quantity. The heuristic argument for this is as follows: Given that we deal with differentiated substitutable products, if rival's quantity is small, he captures mostly the consumers that prefer his product anyway. As his quantity becomes larger, he starts capturing consumers who feel relatively neutral about the two products. Finally as he starts selling even larger quantity, he is attracting consumers who, if both products had the same price, might prefer product i, which hurts the producer i the most.

Hence, under the assumption that $|R_{ii}|+2|R_{ij}|\geq R_{jj}$, $\tilde{\Delta}(\delta,\theta)$ is a concave function of δ with a positive maximum $\delta^*(\theta)$ defined by $\frac{\partial\tilde{\Delta}(\delta,\theta)}{\partial\delta}=0$ and a value $\hat{\delta}$ such that $\tilde{\Delta}(\hat{\delta},\theta)=0$. Now notice that it must be the case that the limit $g(\theta) := \lim_{\delta\to\hat{\delta}}\frac{\partial\tilde{\Delta}(\delta,\theta)}{\partial\delta}$ is strictly negative. Therefore, an analysis of figure 1 suggests that the function $\tilde{\Delta}(\delta,\theta)$ is positive in the following cases:

(i) if $\frac{\partial\,\tilde{\Delta}(\delta,\theta)}{\partial\delta}\geq 0$ which is true if $|R_j(q_i,q_j,\theta)|\geq R_i(q_i,q_j,\theta)$ for all (q_i,q_j,θ). Then using aggregate payoff condition we have: $|R_{ij}(q_i,q_j,\theta)|\geq\frac{R_{\theta i}(q_i,q_j,\theta)}{R_\theta(q_i,q_j,\theta)}R_i(q_i,q_j,\theta)$ for all (q_i,q_j,θ).

(ii) if $\frac{\partial\tilde{\Delta}(\delta,\theta)}{\partial\delta}<0$ we can see from the figure 1 the slope of $\tilde{\Delta}$ has to be less in absolute value than $|g(\theta)|$. Hence we need the following condition to hold: $-R_i(q_i,q_j,\theta)+|R_j(q_i,q_j,\theta)|\geq g(\theta)$ which in turn, using aggregate payoff condition, implies: $|\Lambda(q_j,\theta)|\geq\frac{R_i(q_i,q_j,\theta)}{R_\theta(q_i,q_j,\theta)}+\frac{g(\theta)}{R_\theta(q_i,q_j,\theta)}$ for all (q_i,q_j,θ). Finally, as $g(\theta)$ is negative and $\Lambda(q_j,\theta)=\frac{R_{ij}(\cdot)}{R_{\theta i}(\cdot)}$, it is straightforward to notice that a sufficient condition for $\tilde{\Delta}(\delta,\theta)$ being positive is:

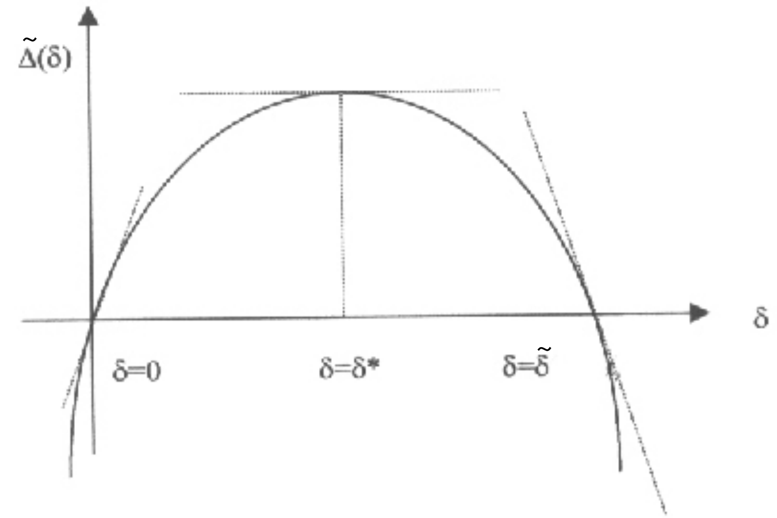

Figure 1:

$$|R_{ij}(q_i, q_j, \theta)| \geq \frac{R_{\theta i}(q_i, q_j, \theta)}{R_\theta(q_i, q_j, \theta)} \left(R_i(q_i, q_j, \theta) + g(\theta)\right) \tag{A-11}$$

Letting the RHS of (A-11) be $k(q_i, q_j, \theta)$ completes the proof.

A.9 Proof of proposition 4

We shall show that a sufficient condition for $\dot{\Delta}(\theta) \leq 0$ is $|R_{ij}(q_i, q_j, \theta)|$ being sufficiently high for all (q_i, q_j, θ). First notice that $(1 + \dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta)) = \frac{R_i(\mathbf{q}^{QD}, \theta)}{h(\theta)R_{\theta i}(\mathbf{q}^{QD}, \theta)}$ is a direct implication of (6) and $\Lambda = \frac{R_{ij}}{R_{\theta i}}$. Consider each producer's virtual surplus earned under vertical delegation $\pi^{QD}(\theta)$, substituting for $(1+\dot{q}^{QD}(\theta)\Lambda(q^{QD}(\theta), \theta))$, one gets $\pi^{QD}(\theta) = R(\mathbf{q}^{QD}, \theta) - \frac{R_\theta(\mathbf{q}^{QD}, \theta)R_i(\mathbf{q}^{QD}, \theta)}{R_{\theta i}(\mathbf{q}^{QD}, \theta)}$. Total differentiation with respect to θ together with an envelope argument yields (abusing notation a little)

$$\dot{\pi}^{QD}(\theta) = \dot{q}^{QD}(\theta)R_j + R_\theta - \frac{(R_{\theta j}R_i + R_{ij}R_\theta)\,\dot{q}^{QD}(\theta) + R_{\theta\theta}R_i + R_\theta R_{i\theta}}{R_{\theta i}} + \frac{(\dot{q}^{QD}(\theta)R_{\theta ij} + R_{\theta i\theta})R_\theta R_i}{(R_{\theta i})^2}$$

Now applying Assumption 5 and simplifying

$$\dot{\pi}^{QD} = -\frac{\dot{q}^{QD}(\theta)R_{\theta j}R_i + R_{\theta\theta}R_i}{R_{\theta i}} + \frac{(\dot{q}^{QD}(\theta)R_{\theta ij} + R_{\theta i\theta})R_\theta R_i}{(R_{\theta i})^2}$$

which can be expressed as

$$\dot{\pi}^{QD}(\theta) = -\frac{R_i(\mathbf{q}^{QD}, \theta)\Upsilon(\theta)}{(R_{\theta i}(\mathbf{q}^{QD}, \theta))^2}$$

where, for any given $\theta \in \Theta$, the function $\Upsilon(\theta)$ is defined by

$$\Upsilon(\theta) = (R_{\theta\theta}(\mathbf{q}^{QD},\theta)+\dot{q}^{QD}(\theta)R_{\theta j}(\mathbf{q}^{QD},\theta))R_{\theta i}(\mathbf{q}^{QD},\theta)-(R_{\theta i\theta}(\mathbf{q}^{QD},\theta)+\dot{q}^{QD}(\theta)R_{\theta ij}(\mathbf{q}^{QD},\theta))R_{\theta}(\mathbf{q}^{QD},\theta)$$

As $R_i(\mathbf{q}^{QD},\theta) \geq 0$ with equality at $\bar{\theta}$ (see also proof of lemma 5), and $R_{\theta i}(\mathbf{q}^{QD},\theta) \geq 0$, a sufficient condition to have $\dot{\pi}^{QD}(\theta) < 0$ is then $\Upsilon(\theta) \geq 0$. In this respect, as $R_{\theta ij}(\mathbf{q}^{QD},\theta) \geq 0$ by assumption 2, using $\dot{q}^{QD}(\theta) \leq \frac{R_{i\theta}(\mathbf{q}^{QD},\theta)}{|R_{ji}(\mathbf{q}^{QD},\theta)|}$ (see equation (A-6)) and rearranging, one may notice that the following inequality holds

$$\Upsilon(\theta) \geq R_{\theta i}R_{\theta\theta} + |R_{\theta i\theta}|R_{\theta} - [|R_{\theta j}|R_{\theta i} + R_{\theta}R_{\theta ij}]\frac{R_{\theta i}}{|R_{ij}|}$$

where again for notational convenience we have suppressed the arguments in every derivative. Finally, as $\frac{|R_{\theta i\theta}|}{R_{\theta i}} > -\frac{R_{\theta\theta}}{R_{\theta}}$ for all (q_i, q_j, θ), the previous equation in turn entails $\Upsilon(\theta) \geq 0$ if $\frac{|R_{ij}|}{R_{\theta i}} \geq \frac{\frac{|R_{\theta j}|}{R_{\theta}}+\frac{R_{\theta ij}}{R_{\theta i}}}{\frac{R_{\theta\theta}}{R_{\theta}}+\frac{|R_{\theta i\theta}|}{R_{\theta i}}}$ for all (q_i, q_j, θ). Setting $k(q_i, q_j, \theta) = R_{\theta i}\frac{\frac{|R_{\theta j}(.)|}{R_{\theta}(.)}+\frac{R_{\theta ij}(.)}{R_{\theta i}(.)}}{\frac{R_{\theta\theta}(.)}{R_{\theta}(.)}+\frac{|R_{\theta i\theta}(.)|}{R_{\theta i}(.)}}$completes the proof. *QED*