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Learning, Cascades and Transaction Costs

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Abstract

The paper analyzes the effect of transaction costs on social learning in an asset market with asymmetric information, sequential trading and a competitive price mechanism. Both fixed and proportional transaction costs reduce the information content of trading orders and lead to informational cascades. If transaction costs are very high, an informational cascade may occur not only when beliefs converge on a specific asset value, but also when there is absolute uncertainty about the asset's fundamental value. Finally, if the value in the bad state is sufficiently low, proportional transaction costs lead to an informational cascade only when prices are very high.

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1 Introduction

Both market practitioners and academic economists have recently shown renewed interest in herd behavior in financial markets.[1] Much of the literature on market microstructure holds that when agents are asymmetrically informed and trades occur sequentially, the market should gradually learn the fundamental information possessed by market participants, so that prices should eventually converge to fundamental values.[2] In the long run, therefore, asset markets are viewed as informationally efficient.[3] However, many interesting regularities observed in capital markets are not easily addressed under the efficient-markets hypothesis. Many of them can be explained by herd behavior.[4] For example, the empirical evidence on episodes of soaring prices and subsequent collapse, like the Dutch Tulip Mania in the seventeenth century, the stock market crashes of 1929 and 1987 and, more recently, the international financial crisis in southeast Asia in 1997-8 and the overpricing of US technology stocks in the late 1990s, would be consistent with short-run mispricing caused by rational herd behavior.[5] Also, the recurrent high volatility of financial market prices even in the absence of justifying news could be the consequence of information aggregation failure due to herding.[6]

Originally, imitative behavior was not considered rational: herds were described as people blindly following others.[7] More recently, however, the literature on social learning has reconciled herd behavior with rationality in many economic environments.[8] The models of informational cascades – that is, situations where imitative behavior leads market participants to disregard completely their own private information – belong to this strand of literature on "rational herding".[9]

This paper contributes to the analysis of rational herding in financial markets, inquiring into the existence and the empirical implications of informational cascades arising from transaction costs in stock markets with asymmetric information, sequential trading and a competitive price mechanism.

Standard models of informational cascades apply to environments where prices are exogenous. So they hardly apply to asset markets, where prices adjust continuously to reflect the changing information revealed by orders and trades.[10] One may imagine that this informational role of prices would eliminate the tendency of agents to herd, that is, to trust

[1] See Hirshleifer and Teoh [13] for a survey on the theoretical and empirical literature on herd behavior in capital markets.

[2] See Glosten and Milgrom [12], Kyle [15].

[3] Informational efficiency refers to how much information is revealed by the price process. This is important in economies where information is dispersed among many individuals. Prices are information efficient if they fully and correctly reflect the relevant information (Fama, [10]). If prices do not correctly and fully reflect public information, then there would be a profitable trading opportunity for individuals. In general, this is ruled out in models with rational utility-maximizing agents.

[4] See Kortian [14] for an extensive review of recent theoretical literature on asset price formation and capital market behavior.

[5] See Avery and Zemsky [2].

[6] See Lee [16].

[7] For references to irrational herding see Devenow and Welch [9].

[8] See Chamley [5] for a thorough analysis of the causes of rational herds.

[9] The term "informational cascade" was introduced by Bikhchandani, Hirshleifer and Welch [4] to describe a situation in which, in a sequential trade framework, every agent, based on the observation of previous agents, makes the same choice independent of his private information. Banerjee [3] also wrote another paper on rational imitative behavior due to informational externalities.

[10] Avery and Zemsky [2] prove that, in the standard Glosten-Milgrom framework, the competitive price mechanism prevents informational cascades.

the quantity signals issued by other market participants. But some recent theoretical models have shown that herding – and in some cases informational cascades – may arise even in financial markets. The situations in which this may occur include (i) multidimensional uncertainty, (ii) reputational concerns of asset managers, (iii) heterogeneous preferences of market participants, and (iv) transaction costs. The present model is predicated on the last assumption, as we shall see below.

Multidimensional uncertainty and, more generally, non-monotonic signals open the possibility of herd behavior and short-run mispricing of assets, as shown by Avery and Zemsky [2]. Intuitively, when traders observe multidimensional signals the market price, being a one-dimensional variable, is unable to convey all their information to other market participants. Since market makers and informed traders interpret the history of trades differently, this model generates short-run mispricing, though not informational cascades: herding does not stem altogether the flow of information in the market, so that it does not prevent prices from converging to fundamental values. However, Gervais [11] shows that uncertainty about the precision of investors' private signals can lead to complete information blockage.

Herding in financial markets may also arise from reputational concerns of asset managers, in the context of a principal-agent relationship with investors, as first shown by Scharfstein and Stein [18]. Dasgupta and Prat [7] examine the effect of reputational concerns on the informational efficiency of prices in asset markets *à* la Glosten and Milgrom. In a standard sequential trading model with managers who trade on behalf of other investors, they show that if institutional traders care about their reputation for ability, the market is informationally inefficient and there is an incentive for conformism.

Informational cascades may arise in asset markets also as a result of the heterogeneous preferences of market participants. Decamps and Lovo [8] show that they may develop if informed traders are risk-averse and market makers risk-neutral, assuming that there is a floor and a ceiling to the trade size per period. An informational cascade occurs because as prices become more informative, the traders' informational advantage vanishes and orders only reflect the inventory imbalance. Cipriani and Guarino [6] reach similar results considering heterogeneous traders in a multiple security setting, finding that informational cascades can also lead to contagion across markets.

Finally, trading frictions in price formation may induce informational cascades insofar as they deter informed market participants to place orders. Lee [16] shows that, in a stock market with fixed transaction costs and sequential trading, the market may fail to aggregate information, and partial or total informational cascades may occur. However, in his framework the market maker is not a fully rational intermediary. At each trading round, he is assumed to set the price equal to the expected asset value conditional on the history of past trades, rather than setting bid and ask prices so as to compete for incoming orders. Indeed, since his prices do not condition on the information conveyed by the order flow, he makes losses on average.

The first question that the present paper addresses is whether Lee's insights on informational cascades in financial markets with transaction costs are robust to the introduction of competitive and profit maximizing market makers, that is, to full modelling of the price mechanism. Second, while Lee's model predicts that informational cascades occur as public beliefs concentrate at the tails of the asset value distribution, it is worthwhile to ask if they may occur also if the market is completely uncertain about the fundamental value. Finally, I shall investigate if in such a model herding is more often associated with crashes than with frenzies. One of the main purposes of models on herd behavior is to explain the empirical evidence of financial bubbles and crashes, which are not fully addressed in standard asset pricing models. But these models have no apparent bias for crashes: herd behavior can just as easily generate frenzies, in contrast with the empirical evidence that frenzies are much less common than crashes.

The paper addresses these issues in a stock market economy *à* la Glosten and Milgrom, where sequential trades for a single risky asset are channelled through risk-neutral market makers who set competitive prices. Some traders have better information about the asset's fundamental value and trade so as to exploit this edge; the others are uninformed and trade for exogenous reasons. A key assumption is that market makers bear an exogenous cost for executing orders.

I show that when market makers bear a fixed cost per transaction, the market is not informationally efficient. That is, the competitive price mechanism cannot prevent the occurrence of informational cascades. Further, in tune with the finding of Lee [16], during the informational cascade no informed traders will trade. Equilibrium bid and ask prices can be decomposed into two components: expectations of the asset's value, given all relevant public information, and the exogenous transaction cost. The informational advantage of traders is the difference between the expected value conditional on their private signal and the expected asset value for market makers. As the price becomes a more precise estimate of fundamental value, the difference shrinks. Before the uncertainty about fundamental value is completely resolved, informed traders stop trading because their informational advantage is smaller than the fixed transaction cost. From then on, no new information reaches the market, so prices stay constant and the bid-ask spread narrows, because the adverse selection component disappears.

The model predicts a positive correlation between bid-ask spreads and informed trading volume. This is because traders getting private signals with moderate information content assign a large weight to previous price history. So before a cascade develops the traders with less informative signals refrain from trading because of transaction costs, orders become less informative, and the spread gradually narrows.

Moreover, if transaction costs are high enough, an informational cascade may occur even when the market is totally uncertain about the fundamental value. High transaction costs mean that traders observing a "bearish" signal will sell only when the bid price is high enough and those observing a "bullish" signal will buy only when the ask price is low enough. Equilibrium prices are low if the market attaches a high probability to the low value, high in the opposite case. When the market is completely uncertain, prices are in the middle of their distribution and in equilibrium no informed trader chooses to trade. This result runs counter to the previous literature, which holds that cascades develop when there is a convergence of beliefs.

Finally, I extend the analysis to include the case of proportional – rather than fixed – transaction costs. In this setting, if the fundamental value in the bad state of nature is sufficiently low, then the probability of an informational cascade when prices are low approaches zero. The impact of proportional transaction costs on the expected profits of informed traders decreases as prices fall and tends to zero if the asset value in the bad state of nature is close to zero. This result has the interesting implication that cascades will be asymmetrical. They will emerge seldom in depressed markets, more often in bull markets. As a consequence, they are more likely to result in crashes than in manias.

The paper proceeds as follows. The basic model with fixed transaction costs is presented in section 2. Section 3 defines and derives the market equilibrium. Section 4 demonstrates the occurrence of informational cascades, and section 5 analyzes the market equilibrium before and during a cascade. The case of proportional transaction costs is examined in section 6. Finally, section 7 explores some extensions and concludes. Proofs of propositions, corollaries and lemmas are given in the appendix.

2 Trading mechanism and information structure

We take a sequential trade model similar to Glosten and Milgrom's [12], with a single risky asset whose value $\widetilde{V}$ depends on the state of nature. If the state is good, the value is $\overline{V}$; if bad, $\underline{V}$, with $\overline{V} > \underline{V} \geq 0$. We assume that the initial prior probability $\pi_0 = P(\overline{V})$ of the high value is non-degenerate, that is, $\pi_0 \in (0, 1)$. There are traders and market makers. Trades are sequential, and at any given point in the time, only one trader is allowed to transact. Before a trader arrives, market makers simultaneously announce their bid and ask prices. We assume that market makers are risk-neutral and act competitively. We further suppose that they bear an exogenous transaction cost c.[11] The trader arriving in the market observes the prices and has the option of selling or buying one unit of the asset at the best prices or refraining from trading. Once he has had the opportunity to trade, the trader leaves the market. He may trade further, but only after returning to the pool of traders and being selected again.[12]

A fraction μ of traders are uninformed liquidity traders while $1 - \mu$ are informed.[13] Liquidity traders trade for reasons exogenous to the model. To simplify the analysis, we assume that they choose to sell, buy, or refrain from trading with equal probability.[14] Informed

[11] The cost c represents all transaction costs unrelated to the underlying value of the stock, and includes clearing and settlement fees.

[12] As usual in models à la Glosten and Milgrom [12], we make the simplifying assumption of fixed trade size equal to one unit of asset. The analysis would be totally unaffected by the normalization if transaction costs were proportional to the quantity traded (not its value). Moreover, it can be shown that the results on informational cascades are robust with respect to the hypothesis of different order sizes.

[13] Liquidity traders are needed to guarantee that trading occurs. In the absence of traders who trade for reasons other than speculation, the no-trade theorem of Milgrom and Stokey [17] applies and the market breaks down.

[14] In Glosten and Milgrom [12], liquidity traders trade as long as the reservation value dictated by their liquidity needs exceeds the ask (bid) price for the buy (sell) order. As a consequence, if there are too many informed traders, then the market maker may have to set a spread so large as to preclude any trading at all. In order to avoid market collapse, we assume that the

traders are risk-neutral, price-taking agents[15] who privately observe a signal θ correlated with the value of the asset. They trade to maximize their expected profit. We denote as $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$ the set of private signals, and assume that these are conditionally independent and satisfy the monotone likelihood property:

$$0 < \lambda_1 < \lambda_2 < \cdots < 1 < \cdots < \lambda_{N-1} < \lambda_N < \infty$$

where $\lambda_n = P(\theta_n|\widetilde{V} = \underline{V})/P(\theta_n|\widetilde{V} = \overline{V})$.

The signal θ_n is "good" if the probability of observing θ_n when the true asset value is $\overline{V}$ exceeds that probability conditional on $\underline{V}$, that is $\lambda_n < 1$. In the opposite case it is "bad".

It is easy to see that a good signal is more informative when λ is lower, a bad signal when λ is higher. The information content of a good and a bad signal, θ_i and θ_j, are equal if $\lambda_j = 1/\lambda_i$. For simplicity, we assume that good and bad signals are symmetrical, that is $\lambda_1 = 1/\lambda_N$, $\lambda_2 = 1/\lambda_{N-1}$, and so on.

Both market makers and traders are Bayesian agents who know the structure of the market. We denote by π_t the probability of $\overline{V}$ conditional on the publicly observable history of trades up to time t. Thus, the public belief about the asset value at t is:

$$E_t[\widetilde{V}] = \pi_t \cdot \overline{V} + (1 - \pi_t) \cdot \underline{V} = \underline{V} + \pi_t \cdot (\overline{V} - \underline{V})$$

3 Equilibrium

Before each trading round market makers set their prices. Bertrand competition and risk neutrality lead market makers to earn zero expected profit for every possible trade.

After prices are set, a trader is randomly selected to trade. He can place a sell order (s) at the highest bid price, B_t, place a buy order (b) at the lowest ask price, A_t, or refrain from trading (r). We denote by $\mathcal{A} \equiv \{s, b, r\}$ the traders' action space.

With probability μ the trader selected will be uninformed; with probability $1-\mu$, informed. Agents do not observe the type of trader.

Uninformed traders trade for exogenous reasons and submit orders in the probabilistic way specified above. Informed traders observe the prices quoted and choose the strategy that maximizes expected profit. We denote by $\sigma \equiv \{\sigma_\theta\}_{\theta \in \Theta}$ the informed traders' strategies, where σ_θ is the mixed strategy of the informed trader if he observes θ. Clearly:

$$\sigma_\theta \equiv (\sigma_{\theta,s}, \sigma_{\theta,b}, \sigma_{\theta,r})$$

probability that liquidity traders buy and sell in each period is sufficiently high and is stationary.

[15]This assumption rules out their strategic behavior. It is reasonable given that there is an infinite number of informed traders in the pool and for each of them the probability of trading again is zero.

where $\sigma_{\theta,i}$ is the probability of i, with $i \in \mathcal{A}$, if the trader observes θ, $\Sigma_{i\in\mathcal{A}}\ \sigma_{\theta,i} = 1$, and $\sigma_{\theta,i} \geq 0\ \ \forall\, i \in \mathcal{A}$.

The expected profit of a market maker is equal to $E_t[\widetilde{V}|satB] - B - c$, if he buys at B, and it is equal to $A - E_t[\widetilde{V}|batA] - c$, if he sells at A. The expected profit of a trader getting signal θ, when the price schedule is $P = \{B,\ A\}$ and he plays the strategy $\sigma \in \Delta(\mathcal{A})$, is $E_t[\Pi_\theta(\sigma|P)] = \sigma_{\theta,s}(B - E_t[\widetilde{V}|\theta]) + \sigma_{\theta,b}(E_t[\widetilde{V}|\theta] - A)$.

Assuming that market makers act competitively and that for each informed trader the probability of trading again is zero rules out all intertemporal strategic behavior. Therefore, each trading round can be viewed as a single two-stage game. In the first stage market makers set their prices, and in the second the trader observes the quotes and plays his strategy. What we are concerned with here is the Nash equilibrium.

Definition 1 *At each trading round* t*, the equilibrium in the asset market consists of a trading strategy correspondence* $\sigma^*_\theta(P|t) : R^2_+ \longrightarrow \Delta(\mathcal{A})$ *for each informed trader* $\theta \in \Theta$*, and a price schedule* $P^*_t = \{B^*_t,\ A^*_t\}$ *such that:*

1. $\sigma^*_\theta(P|t) = \arg\max_{\sigma\in\Delta(\mathcal{A})} E_t[\Pi_\theta(\sigma|P)], \quad \forall P \in R^2_+, \ \ \forall\theta \in \Theta$
2. $B^*_t \in E_t[\widetilde{V}|satB^*_t,\ \sigma^*(P^*_t|t)] - c$
3. $A^*_t \in E_t[\widetilde{V}|batA^*_t,\ \sigma^*(P^*_t|t)] + c$
4. $B^*_t \leq A^*_t$

where $\sigma^*(P^*_t|t) = \{\sigma^*_\theta(P|t)\}_{\theta\in\Theta}$.

The first equilibrium condition states that informed traders maximize their expected profit, given the price schedule, trade by trade. Conditions (2) and (3) make market makers determine the price schedule so that they expect zero profits from each trade. Evidently, in the absence of exogenous transaction costs equilibrium bid and ask prices are equal to the expectation of $\widetilde{V}$ conditional, respectively, on a sell and on a buy order. The last condition says that at the equilibrium the market makers' buy price must be lower than (or equal to) the sell price.

We now offer a useful characterization of the equilibrium in terms of the informed traders' optimal strategy, and then prove the existence and the uniqueness of zero-profit equilibrium prices.

Informed traders are profit-maximizing agents. Hence, for any price schedule $P = \{B,\ A\}$ such that $B \leq A$, an informed trader places a sell order when his expected asset value is lower than the bid price, and places a buy order when it is higher. When the bid and ask prices straddle his valuation, he will not trade. Finally, if his expected value is equal to the bid or to the ask price, he is indifferent between all mixed strategies defined on the simplex $\Delta(s,\ r)$ in the first case or on the simplex $\Delta(b,\ r)$ in the second.

The valuation of the asset for informed traders depends on their information set, which includes both the whole history of trades and the private signal. From the maximum likelihood ratio property of signals, the expected value for traders observing signal θ_i is higher than that for those observing θ_j for any $j < i$. So, if it is profitable for traders with signal θ_n to sell when the price schedule is P, then also all traders observing signals θ_{n+i}, with $i \in \{1, 2, ..., N-n\}$, optimally prefer to sell. Similarly, if the traders getting signal θ_n prefer to buy, then so do all traders getting signals θ_{n-j}, with $j \in \{1, 2, ..., n-1\}$. Proposition 1 resumes this result.

Proposition 1 *If $\sigma^*_{\theta_n,s}(P|t) \neq 0$, then $\sigma^*_{\theta_{n+i},s}(P|t) = 1$ for any $i \in \{1, 2, ..., N-n\}$. If $\sigma^*_{\theta_n,b}(P|t) \neq 0$, then $\sigma^*_{\theta_{n-j},b}(P|t) = 1$ for any $j \in \{1, 2, ..., n-1\}$.*

Proposition 1 implies that a sell order generally indicates $\underline{V}$ because the probability of its occurrence is greater in the bad state of nature, while a buy order generally indicates $\overline{V}$. To see this, note that the proposition implies that the likelihood ratio of a sell order at t, given the price schedule $P = \{B, A\}$, is:

$$\lambda_t^s(B) \equiv \frac{P_t(s\ at\ B|\underline{V})}{P_t(s\ at\ B|\overline{V})} = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^s(B)}^{N} P(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^s(B)}^{N} P(\theta_i|\overline{V})}$$

and the likelihood ratio of a buy order is:

$$\lambda_t^b(A) \equiv \frac{P_t(b\ at\ A|\underline{V})}{P_t(b\ at\ A|\overline{V})} = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^b(A)} P(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^b(A)} P(\theta_i|\overline{V})}$$

where $\frac{\mu}{3}$ is the probability that the order comes from a liquidity trader, and $\theta_{n_t^s(B)}$ and $\theta_{n_t^b(A)}$ are respectively the signals of marginal selling and buying traders.[16] Since $\sum_{i=n}^{N} P_t(\theta_i|\underline{V}) \geq \sum_{i=n}^{N} P_t(\theta_i|\overline{V})$ for all $n = 1, 2, \ldots, N$, the probability of a sell order is higher in the bad state of nature and, since $\sum_{i=1}^{n} P_t(\theta_i|\underline{V}) \leq \sum_{i=1}^{n} P_t(\theta_i|\overline{V})$ for all $n = 1, 2, \ldots, N$, the probability of a buy order is higher in the good state.

Another important implication of Proposition 1 is that if equilibrium bid and ask prices exist, then they straddle the unconditional expected asset value, which is the price that would prevail in the absence of adverse selection and fixed transaction costs. Indeed, since $\lambda_t^s(B) \geq 1$ and $\lambda_t^b(A) \leq 1$ for all t and P, the expected value conditional on a sell order never

[16]Given the price schedule $P = \{B, A\}$, the marginal selling and buying traders at t, are those getting, respectively, signals $\theta_{n_t^s(B)}$ and $\theta_{n_t^b(A)}$ such that:

$$E_t[\widetilde{V}|\theta_{n_t^s(B)}] \leq B \quad \text{and} \quad E_t[\widetilde{V}|\theta_{(n_t^s(B)-1)}] > B$$

and

$$E_t[\widetilde{V}|\theta_{n_t^b(A)}] \geq A \quad \text{and} \quad E_t[\widetilde{V}|\theta_{n_t^b(A)+1)}] < A.$$

For simplicity, here we suppose that the marginal traders choose to place an order also if they are indifferent between trade and do not.

exceeds the unconditional expectation, while the expected value conditional on a buy order is never less than that expectation.

The next proposition states the existence and the uniqueness of equilibrium bid and ask prices.

Proposition 2 *In each period t there exists a unique equilibrium price schedule $P_t^* = \{B_t^*,\ A_t^*\}$.*

4 Informational cascades with costly market-making

In this section we study the occurrence of informational cascades in the market equilibrium just described.

If all informed traders make the same choice regardless of their private signal, no new information reaches the market. Hence, during an informational cascade, the market equilibrium is such that: $\sigma_\theta^*(P_t^*|t) = \sigma^*(P_t^*|t)$ for all $\theta \in \Theta$, and $B_t^* = E_t[\widetilde{V}] - c$ and $A_t^* = E_t[\widetilde{V}] + c$ since orders are uninformative about the asset value, that is: $\lambda_t^s(B_t^*) = \lambda_t^b(A_t^*) = 1$.

It is straightforward to see that in equilibrium traders observing a bad signal never buy and those getting a good signal never sell. Indeed, the valuation of traders with bad news is always lower than the unconditional expected value, while that of traders with good news is always higher. As we have seen, by Proposition 1 the unconditional expected value is the upper bound of the equilibrium bid price and the lower bound of the equilibrium ask price. Therefore, traders with a bad signal will never find it worthwhile to buy, while those with a good signal will never place a sell order. This implies that in the market equilibrium an informational cascade in which all informed traders place the same type of order will never occur.

Nevertheless, an informational cascade can still occur if all informed traders abstain from trading, since also in this case they behave identically. This may occur if market-making has a cost. The intuition is the following. Transaction costs induce wider bid-ask spreads, but the expectation of market makers and that of informed traders converge as the number of transactions increases.[17] Hence, there would be a moment at which the informational advantage is so small relative to the exogenous transaction cost that it is no longer profitable for any informed trader to place an order.

A no-trade informational cascade starts if $E_t[\widetilde{V}|\theta] \in (B_t^*,\ A_t^*)$ for any $\theta \in \Theta$. Clearly, if an informational cascade occurs, the orders have no information content and then $B_t^* = E_t[\widetilde{V}] - c$ and $A_t^* = E_t[\widetilde{V}] + c$.

The next proposition specifies a necessary and sufficient condition for the occurrence of informational cascades in market equilibrium.

[17] See Glosten and Milgrom [12].

Proposition 3 $B_t^* = E_t[\widetilde{V}] - c$ *if, and only if:*

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$$

and $A_t^* = E_t[\widetilde{V}] + c$ *if, and only if:*

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c.$$

Proposition 3 states that the equilibrium bid price is equal to the public belief about the fundamental value less the transaction cost c if and only if the informational advantage of the traders with the most informative bad signal θ_N is smaller than the fixed transaction cost. Symmetrically, the equilibrium ask price is equal to the public belief plus c if and only if the informational advantage of traders endowed with the most informative good signal θ_1 is smaller than the fixed transaction cost.

The belief about the asset value of traders getting bad signals less informative than θ_N, exceeds that of traders observing θ_N. This means that:

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c \Longrightarrow E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta] \leq c$$

for all bad signals $\theta \in \Theta$. Likewise, the belief of traders with good signals less informative than θ_1 is lower than that of traders who observe it. This means that:

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c \Longrightarrow E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}] \leq c$$

for all good signals $\theta \in \Theta$. If the informational advantage of all traders is lower than the transaction cost, no informed trader will place a order. Since all traders observing a private signal act alike, no new information reaches the market and an informational cascade is triggered.

It is straightforward that until the uncertainty is resolved, the informational advantage of informed traders is strictly positive. As a consequence, in the absence of exogenous transaction costs, an informational cascade would never occur. On the other hand, if for any possible history of trades the transaction cost is greater than the advantage of informed traders, then orders will never be information-based. This last property is stated in the following lemma.

Lemma 1 *There exists a threshold cost* $\bar{c} = \frac{|1-\sqrt{\lambda_1}|}{1+\sqrt{\lambda_1}} \cdot (\overline{V} - \underline{V})$ *such that, if* $c > \bar{c}$*, in equilibrium all informed traders refrain from trading regardless of trading history.*

Since the threshold cost $\bar{c}$ is increasing in the information content of the best private signals, one can conclude that in markets where private information is less sharp, informed

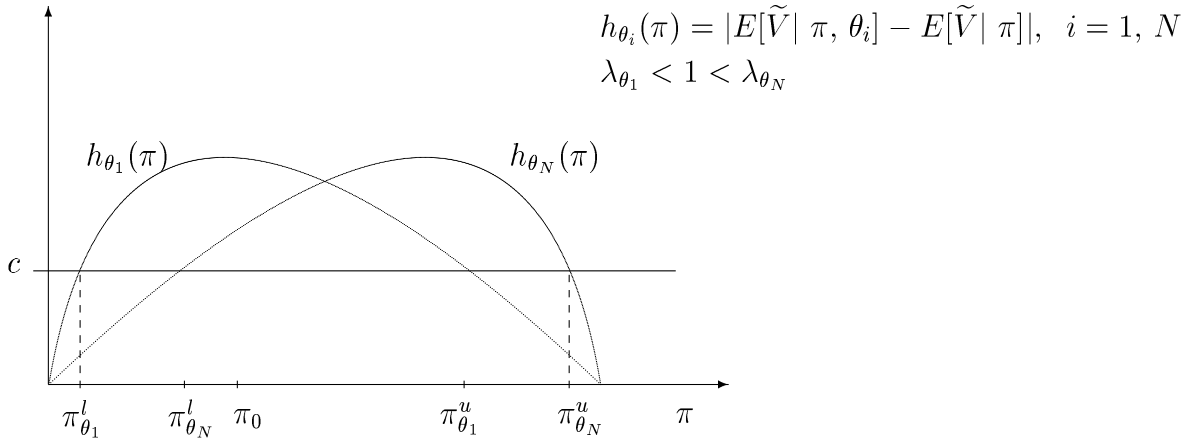

Figure 1: *Informational cascades in a market with fixed transaction costs.*

traders will be more likely to refrain from trading, so price changes will not reflect information about the fundamental.

The next proposition establishes that when transaction costs are lower than the threshold cost, an informational cascade occurs as the public belief approaches the extremes of the distribution of the true asset value.

Proposition 4 *If $c \in (0,\ \bar{c}\ )$ there exist unique $\pi^l_{\theta_1}$ and $\pi^u_{\theta_N}$, with $\pi^l_{\theta_1} < \pi^u_{\theta_N}$, such that when $\pi \in [0,\ \pi^l_{\theta_1}) \cup (\pi^u_{\theta_N},\ 1]$ all informed traders refrain from trading in equilibrium.*

If the transaction cost does not exceed $\bar{c}$, there exist a lower bound $\pi^l_{\theta_1}$ and an upper bound $\pi^u_{\theta_N}$ of public beliefs such that if the unconditional expected value is lower than $E[\widetilde{V}|\pi^l_{\theta_1}]$ or higher than $E[\widetilde{V}|\ \pi^u_{\theta_N}]$, then no informed trader prefers to trade.

This result is a consequence of the convergence of beliefs. To see this, consider the following example. Suppose that at $t = 0$ the unconditional expected asset value $E[\widetilde{V}|\pi_0]$ is such that at least the traders getting signals θ_1 and θ_N choose to trade (see figure 1). Suppose also that at $t = 1, 2, ..., T$ a sequence of buy orders arrives. Because of the new information that reaches the market, the public belief about the asset value moves toward $\overline{V}$. Hence the bid and ask prices rise, and so does the valuation of informed traders. If public belief rises enough (π exceeds $\pi^u_{\theta_1}$), the informational advantage of traders with signal θ_1 becomes so small that it is no longer worthwhile for them to buy. Moreover, at the beginning of the arrival process the expected profit of traders getting signal θ_N increases because the bid price rises. Nevertheless, if the buy orders are very numerous, the expected profit from selling begins to fall off and eventually turns negative, because the traders' evaluation depends both on the history of trades and on their private signal. When the public belief converges to 1 or to 0, the weight of the information inferred from the history of trades increases and that of private information decreases. Then, if enough buy orders are placed and the unconditional expected value goes above $E[\widetilde{V}|\pi^u_{\theta_N}]$, no informed trader will sell even though the bid price is very high. At that point, an informational cascade occurs. An analogous argument demonstrates the occurrence of an informational cascade when the public belief tends to $\underline{V}$.

The previous example emphasizes that an informational cascade develops because the informed traders'expectation depends not only on their private information but also on the market history. But the weight of market history in the updating of informed traders' expectations also depends on the sharpness of their private signal. If there are perfectly informed traders, an informational cascade never occurs. Indeed, if signals θ_N and θ_1 are perfectly informative (that is, $P(\theta_N|\underline{V}) = P(\theta_1|\overline{V}) = 0$) the expected asset value conditional on those signals does not depend on the history of trades. As a consequence, the profit upon sale of a trader observing θ_N increases when the public belief tends to $\overline{V}$ because of the price effect. Similarly, the expected profit from buying of traders getting θ_1 increases when the public belief tends to $\underline{V}$.

In the following we show that if transaction costs are high enough, an informational cascade may start even when the market is totally uncertain over the true value, that is, when π is close to $\frac{1}{2}$.

Notice that, when the market assigns higher probability to the high value, the informational advantage of traders getting a good signal is greater than that of traders getting an equally informative bad signal; in the opposite case, it is smaller. Moreover, if signals are symmetrical, when the market is totally uncertain about the true value, the informational advantage of the traders getting signal θ_1 is equal to that of the traders observing θ_N.

Let be $\underline{c} \equiv E[\widetilde{V}|\ \pi = \frac{1}{2},\ \theta_1] - E[\widetilde{V}|\ \pi = \frac{1}{2}]$. Proposition 5 establishes that if the fixed transaction cost exceeds $\underline{c}$, there exists a neighborhood of $E[\widetilde{V}|\ \pi = \frac{1}{2}]$ such that an informational cascade occurs also if the unconditional expected asset value is within this neighborhood.

Proposition 5 *If $\underline{c} < c < \overline{c}$ then there exist $\pi^u_{\theta_1} \in (\pi^l_{\theta_1},\ \frac{1}{2})$ and $\pi^l_{\theta_N} \in (\frac{1}{2},\ \pi^u_{\theta_N})$ such that an informational cascade occurs when $\pi \in (\pi^u_{\theta_1},\ \pi^l_{\theta_N})$.*

To understand intuitively why cascades can occur under the conditions described in Proposition 5, consider the following argument.

High transaction costs require large informational advantages to induce informed traders to enter the market. If $\pi < \frac{1}{2}$, both the signal θ_N and the public belief indicate that the true value is $\underline{V}$. Hence, when the market assigns greater probability to the low value, the bad signal θ_N is "worth" less than the good signal θ_1, which contradicts the public belief. By the same argument, when $\pi > \frac{1}{2}$, θ_1 is "worth" less then θ_N. This implies that if the transaction cost is greater then $\underline{c}$, no trader with a bad signal chooses to sell when $E_t[\widetilde{V}] < \frac{\overline{V}-\underline{V}}{2}$, and no trader observing a good signal chooses to buy when $E_t[\widetilde{V}] > \frac{\overline{V}-\underline{V}}{2}$ (see figure 2). As a consequence, sell orders are uninformative about the true value when prices are low and buy orders are

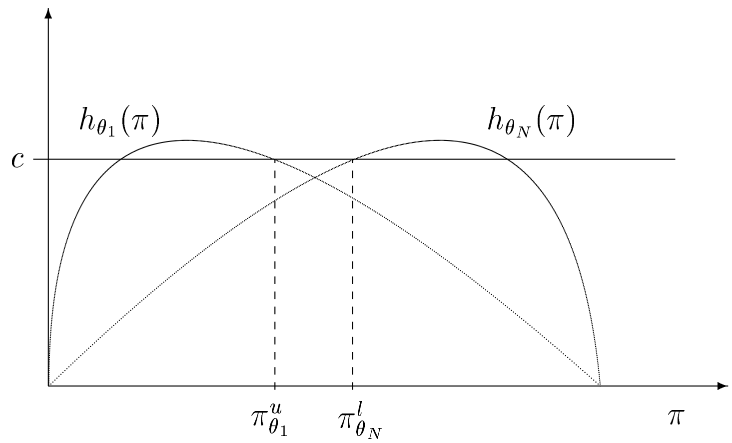

Figure 2: *Informational cascades in a market with high fixed transaction costs.*

uninformative when prices are high. And, if $E_t[\widetilde{V}]$ is in the interval $(E[\widetilde{V}|\pi^u_{\theta_1}],\ E[\widetilde{V}|\pi^l_{\theta_N}])$, no informed trader places an order and there is an informational cascade. Therefore, if transaction costs are high enough, an informational cascade may develop even when the public belief is not concentrated on $\overline{V}$ or $\underline{V}$. This result contrasts with the typical finding of the literature, namely that cascades occur when there is convergence of beliefs.

5 Information content of orders and fixed transaction costs

In this section we analyze the effect of transaction costs on the information content of orders in equilibrium. First, we prove that in the absence of exogenous transaction costs the competitive price mechanism produces equilibrium bid and ask prices that maximize the information content of orders. Then we show that transaction costs reduce the amount of information that the market can infer from both buy and sell orders.

An order's information content is related to its likelihood ratio. An order is totally uninformative about the true value of the asset if and only if the likelihood ratio is equal to 1; the more the ratio differs from 1, the more informative the order. More specifically, a sell order is more informative when the likelihood ratio is higher, a buy order when it is lower. So we can define the information content of a sell order as that order's likelihood ratio and that of a buy order as the reciprocal of its likelihood ratio.

The likelihood ratio of an order depends on the set of informed traders who prefer to place the order. From Proposition 1 we know that, given the price schedule $P = \{B, A\}$, the set of informed traders who sell the asset is $\Theta^s_t(B) = \{\theta_n \in \Theta : n \geq n^s_t(B)\}$ and the set of those who buy is $\Theta^b_t(A) = \{\theta_n \in \Theta : n \leq n^b_t(A)\}$, with $n^s_t(B)$ and $n^b_t(A)$ being the signals of the marginal selling and buying traders, respectively.

If the set of buying (selling) traders is very numerous but also includes many traders with very noisy signals, that is, $\lambda_{n_t^b(A)}$ ($\lambda_{n_t^s(B)}$) near to 1, the information content of a buy (sell) order is low. However, the information content of an order may be low even when the set of active informed traders includes only a few traders observing very accurate signals.

Lemma 2 establishes that there exists a set of selling traders, $\Theta_{n^s} = \{\theta_n \in \Theta : n \geq n^s\}$, and a set of buying traders, $\Theta_{n^b} = \{\theta_n \in \Theta : n \leq n^b\}$, that maximize the information content of sell and buy orders, and that these sets do not depend on the public belief.

Lemma 2 *There exist a bad signal θ_{n^s} and a good signal θ_{n^b} such that, for all trading histories:*

- *the information content of a sell order is maximum if $n_t^s(B) = n^s$,*
- *the information content of a buy order is maximum if $n_t^b(A) = n^b$.*

A price schedule $P = \{B, A\}$ maximizes the information content of orders at t if it prompts these two sets of traders to place orders at t, that is, $\Theta_t^s(B) = \Theta_{n^s}$ and $\Theta_t^b(A) = \Theta_{n^b}$. Proposition 6 states that in the absence of fixed transaction costs, perfect competition among market makers leads to equilibrium bid and ask prices such that $\Theta_t^s(B_t^*) = \Theta_{n^s}$ and $\Theta_t^b(A_t^*) = \Theta_{n^b}$ for all trading histories.

Proposition 6 *If $c = 0$, the equilibrium bid and ask prices, B_t^* and A_t^*, are such that $\Theta_t^s(B_t^*) = \Theta_{n^s}$ and $\Theta_t^b(A_t^*) = \Theta_{n^b}$ $\forall$ t.*

To gain an intuitive understanding of this result, denote by $E_t[\widetilde{V}|\Theta_{n^b}]$ the expected asset value conditional on a buy order, when the set of informed buying traders is Θ_{n^b}.

In the absence of exogenous transaction costs, the market makers' expected profit from selling is given by the difference between the ask price and their conditional expectation. Bertrand competition between market makers implies that, in equilibrium, this difference is equal to zero.

Traders observing θ_{n^b} refrain from trading when the ask price is greater than $E_t[\widetilde{V}|\theta_{n^b}]$. But if $A > E_t[\widetilde{V}|\theta_{n^b}]$, the market makers' expected profit from selling is positive because $E_t[\widetilde{V}|batA] \leq E_t[\widetilde{V}|\Theta_{n^b}] \leq E_t[\widetilde{V}|\theta_{n^b}]$ $\forall$ A, by the definition of Θ_{n^b}. Hence, the equilibrium ask price cannot be greater than $E_t[\widetilde{V}|\theta_{n^b}]$ and the set of informed buying traders includes Θ_{n^b}.

On the other hand, if traders observing good signals less informative than θ_{n^b} buy the asset, i.e. $A < E_t[\widetilde{V}|\theta_{(n^b+i)}]$, the information that market makers can infer from a buy order is more accurate than the signal $\theta_{(n^b+i)}$. This implies that $E_t[\widetilde{V}|batA]$ is higher than the valuation of marginal traders, hence market makers' expected profit from selling is negative. As a consequence, the equilibrium ask price belongs to the interval $(E_t[\widetilde{V}|\theta_{(n^b+1)}], E_t[\widetilde{V}|\theta_{n^b}]]$, and $\Theta_t^b(A_t^*) = \Theta_{n^b}$.

The results of Proposition 6 suggest that in an asset market with a competitive price mechanism, if adverse selection is the only source of the bid-ask spread, then in equilibrium the information content of both sell and buy orders is always at its maximum.

With costly market-making, the equilibrium bid price is lower than the conditional expectation of the value given a sell order, and the equilibrium ask price is greater than the conditional expectation of the asset value, given a buy order. As a consequence, transaction costs reduce the information content of both types order, for any given history of trades.

An implication of Proposition 6 is that in the absence of transaction costs, the set of informed traders who are active in the market does not depend on the trading history and is constant over time. As a result, the probability of an order, conditional on the true asset value, is the same at all t. If we consider the expected trading volume as the occurrence probability of an order, Proposition 6 suggests that, in equilibrium, the expected trading volume is constant over time and does not depend on the public belief about the true value of the asset.

This result does not extend to a market with exogenous transaction costs. During an informational cascade, no order comes from informed traders and the expected volume is equal to the probability of a liquidity trader's being selected to trade and placing an order. Moreover, before an informational cascade starts, the expected volume gradually decreases because the traders getting less informative signals refrain from trading. To show this, suppose that at $t = 0$ traders getting signal θ_n choose to buy, that is:

$$E_0[\widetilde{V}|\theta_n] - A_0^* > 0.$$

Suppose, then, that there is a sequence of buy orders. The informational advantage of the traders getting signal θ_n decreases as the public belief about the asset value approaches $\overline{V}$. And, it turns negative before π gets larger than $\pi^u_{\theta_1}$ because traders observing θ_n have a lower valuation than those observing θ_1. Because the ask price exceeds the unconditional expectation of the asset value, the traders getting signal θ_n refrain from trading before an informational cascade starts. Therefore, the probability of an order is not constant over time. We conclude that with exogenous transaction costs volume decreases as the public belief approaches $\overline{V}$ or $\underline{V}$, while it is positively correlated with the bid-ask spread. Moreover, since the difference between the equilibrium prices with and without transaction costs is increasing in c, when transaction costs are higher, orders convey less information. This suggests that price volatility should decrease as c increases.

To conclude this section, note that the effect of transaction costs on the speed of convergence to a cascade is ambiguous. On the one hand, an increase in c implies higher $\pi^l_{\theta_1}$ and lower $\pi^u_{\theta_N}$, which should obviously speed up convergence. But, an increase in c also involves lower information content of orders, slowing convergence down. Thus, the sign of the final effect of an increase of transaction costs on the speed of convergence to a cascade depends on which of these two effects prevails.

6 Proportional transaction costs

Let us now examine informational cascades when the market makers sustain a transaction cost proportional to the price of the asset. The only difference from the framework of the previous sections is the expected profit of the market maker: this is now equal to $E_t[\widetilde{V}|satB]-(1+c)B$ if he buys at B, and to $(1-c)A - E_t[\widetilde{V}|batA]$ if he sells at A.

Perfect competition gives market makers zero expected profit on either side of the market. Hence, the equilibrium price schedule, $P_t^* = \{B_t^*, A_t^*\}$, is such that:

$$B_t^* \in \frac{E_t[\widetilde{V}|satB_t^*, \sigma^*(P_t^*|t)]}{1+c} \tag{1}$$

$$A_t^* \in \frac{E_t[\widetilde{V}|batA_t^*, \sigma^*(P_t^*|t)]}{1-c}. \tag{2}$$

Since equilibrium bid and ask prices straddle the unconditional value expectation, in equilibrium traders with a bad signal will never buy, those with a good signal will never sell. Then, as before, in equilibrium there will never be informational cascades in which all informed traders place the same type of order. However, the price schedule may be such that the informed traders refrain from trading. In this case, the orders convey no information; hence $B_t^* = \frac{E_t[\widetilde{V}]}{1+c}$ and $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$.

The optimal correspondence strategies of informed traders do not change if transaction costs are proportional rather than fixed. Thus, no informed trader wishes to sell when the bid price is lower than the valuation of traders observing signal θ_N, i.e. $E_t[\widetilde{V}|\theta_N] > B_t$. Symmetrically, none wishes to buy when the ask price is higher than the valuation of traders observing signal θ_1, i.e. $E_t[\widetilde{V}|\theta_1] < A_t$. Proposition 7 now states that an informational cascade will occur in equilibrium if and only if the informational advantage of the traders getting the most informative good and bad signals is lower than the transaction cost multiplied by their expected asset value.

Proposition 7 $B_t^* = \frac{E_t[\widetilde{V}]}{1+c}$ *if, and only if:*

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c \cdot E_t[\widetilde{V}|\theta_N]$$

and $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$ *if, and only if:*

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c \cdot E_t[\widetilde{V}|\theta_1].$$

To gain an insight into this proposition, consider the ask side of the market and denote by $C(A) \equiv cA$ the cost to the market maker of selling the asset at the price A. In equilibrium, when the set of informed buyers is non-empty, $A^* \leq E[\widetilde{V}|\theta_1]$ and then $C(A^*) \leq c \cdot E[\widetilde{V}|\theta_1]$.

This signals that $c \cdot E[\widetilde{V}|\theta_1]$ represents the upper bound of the cost to the market maker of selling the asset in the market equilibrium. So the traders getting signal θ_1 will buy the asset as long as their informational advantage exceeds $c \cdot E[\widetilde{V}|\theta_1]$. On the other hand, if the public belief is such that $E[\widetilde{V}|\theta_1] - E[\widetilde{V}] \leq c \cdot E[\widetilde{V}|\theta_1]$, then in equilibrium informed traders do not trade, because their expectations are below $\frac{E[\widetilde{V}]}{1-c}$, which is the lower bound of equilibrium ask prices.

The total cost to the market maker of selling or buying now depends on price. Higher bid and ask prices produce higher transaction costs, so their effect on social learning should be greater when the probability that the market assigns to $\overline{V}$ is high and should depend on the magnitude of $\underline{V}$ and $\overline{V}$. In particular, if the low asset value is zero, transaction costs vanish as π converges to 0. Hence, the two cases, $\underline{V} > 0$ and $\underline{V} = 0$, should be considered separately. First consider $\underline{V} > 0$. As in the previous framework, excessively high transaction costs can inhibit informed traders from trading regardless of public beliefs. In particular if c is always greater than the relative informational advantages of both traders getting θ_N and traders getting θ_1, that is, if:

$$c \geq \sup\{\frac{E[\widetilde{V}|\pi] - E[\widetilde{V}|\pi,\, \theta_N]}{E[\widetilde{V}|\pi,\, \theta_N]};\ \frac{E[\widetilde{V}|\pi,\, \theta_1] - E[\widetilde{V}|\pi]}{E[\widetilde{V}|\pi,\, \theta_1]}\}$$

for all $\pi \in [0,\, 1]$, then in equilibrium orders will never be information-based.

Lemma 3 *There exists a threshold cost $\overline{c}_p = \frac{\overline{V}-\underline{V}}{(\sqrt{\overline{V}}+\sqrt{\lambda_N \underline{V}})^2} \cdot (\lambda_N - 1)$ such that, if $c > \overline{c}_p$, in equilibrium all informed traders refrain from trading, regardless of the history of trades.*

Proposition 3 states that if the low asset value is strictly positive and the proportional transaction cost does not exceed $\overline{c}_p$, an informational cascade develops both when the public belief about the asset value tends to $\overline{V}$ and when it approaches $\underline{V}$. Also, if the transaction costs are high enough and if the bid-ask spreads are not too large, an informational cascade may also occur when the probabilities that the market assigns to $\underline{V}$ and to $\overline{V}$ are not far apart.

Proposition 8 *If $c \in (0, \overline{c}_p)$ and if $\underline{V}$ is strictly positive, there exist unique $\underline{\pi}$ and $\overline{\pi}$, with $\underline{\pi} < \overline{\pi}$, such that when $\pi \in [0,\, \underline{\pi}) \cup (\overline{\pi},\, 1]$ all informed traders refrain from trading in equilibrium. Further, if $c \in (\underline{c}_p,\, \overline{c}_p)$, with $\underline{c}_p = \frac{\overline{V}-\underline{V}}{\overline{V}+\underline{V}} \cdot \frac{1-\lambda_1}{1+\lambda_1}$, and $\frac{\underline{V}}{\overline{V}}$ is greater than λ_1, there exist $\underline{\underline{\pi}} \in (\underline{\pi},\, \frac{\underline{V}}{\overline{V}+\underline{V}})$ and $\overline{\overline{\pi}} \in (\frac{\underline{V}}{\overline{V}+\underline{V}},\, \overline{\pi})$ such that an informational cascade occurs also when $\pi \in (\underline{\underline{\pi}},\, \overline{\overline{\pi}})$.*

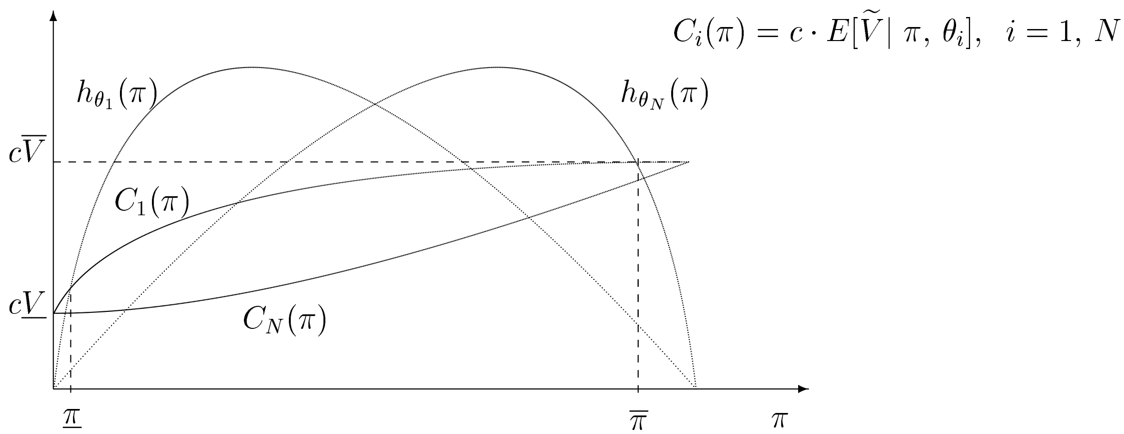

Figure 3: *Informational cascades in a market with proportional transaction costs and* $\frac{\underline{V}}{\overline{V}} < \lambda_1$.

Informed traders find it profitable to trade as long as their informational advantage is greater than the transaction costs. The informational advantage decreases both when π approaches 0 and when it tends to 1, and when π is exactly equal to 0 or 1, it is nil. The transaction costs are an increasing function of the public belief and are always strictly positive if $\underline{V}$ is greater than zero. As a result, the transaction cost exceeds the informational advantage of informed traders so a cascade occurs both when the public belief about the asset value is close to $\underline{V}$ and when it is near to $\overline{V}$ (see figure 3).

What is more, when the proportional transaction cost is high, an informational cascade can take place even without the convergence of beliefs. However, differently from the case of fixed costs, with proportional costs a cascade can occur when π is near $\frac{1}{2}$ only if the cost to the market maker when buying is not too much smaller than the cost when selling; that is, if $\frac{\underline{V}}{\overline{V}} > \lambda_1$ (see figure 4).

If $\underline{V}$ is close to zero, the transaction costs are very small when the market assigns a high probability to the low asset value. Then, the probability of an informational cascade with low prices declines as $\underline{V}$ approaches 0, and in the extreme case of $\underline{V} = 0$, it falls to zero. This result is stated in Proposition 9.

Proposition 9 *If $c \in (0,\, \overline{c}_p)$ and if $\overline{V} = 0$, there exists unique $\overline{\pi}$ such that if and only if $\pi > \overline{\pi}$, all informed traders refrain from trading in equilibrium.*

Intuitively, the transaction cost of the market maker decreases as π tends to zero and reaches its minimum, $c\underline{V}$, when $\pi = 0$. Clearly, if $\underline{V}$ is zero, proportional transaction costs

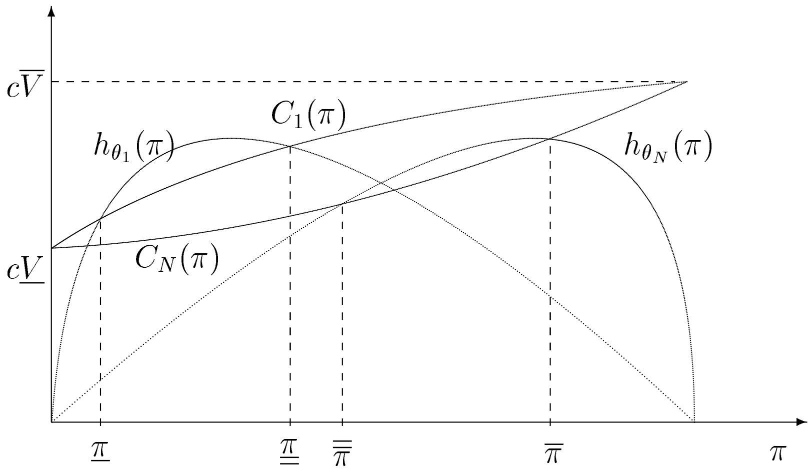

Figure 4: *Informational cascades in a market with proportional transaction costs and* $\frac{\overline{V}}{\underline{V}} > \lambda_1$.

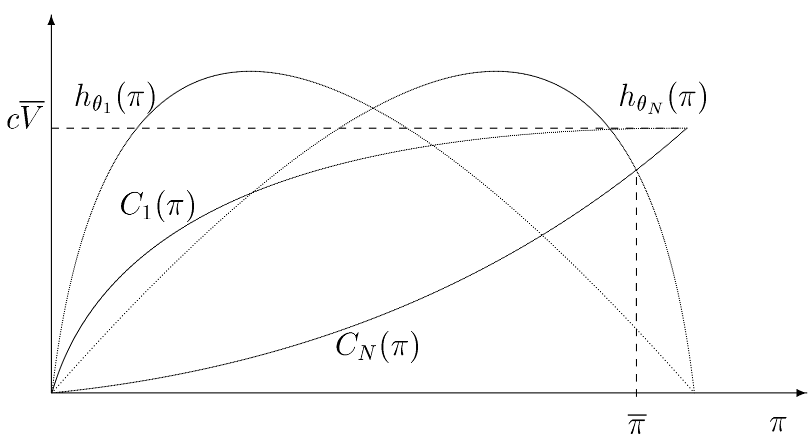

Figure 5: *Informational cascades in a market with proportional transaction costs and* $\underline{V} = 0$.

vanish when $\pi = 0$ (see figure 5). As a consequence, an informational cascade will never occur when equilibrium prices are low.

This finding carries the interesting implication that cascades tend to be asymmetrical. In depressed markets, they will almost never occur while they are more likely to be present in bull markets. As a consequence, cascades are more likely to trigger a market crash than a buying surge.

7 Concluding remarks and extensions

We have presented a model of the effect of costly market-making on the informational efficiency of prices in an asset market characterized by asymmetric information and sequential trading, examining the impact of both fixed and proportional transaction costs on price discovery.

Standard microstructure models show that in the short run trading frictions may decouple market prices from fundamentals, but assume that prices ultimately converge to fundamental values (Glosten and Milgrom [12], Kyle [15]). Models of informational cascades question this conclusion, by showing that the deviations can be persistent, because there are circumstances where markets may stop impounding fundamental information in prices. One of these circumstances is precisely the presence of trading frictions, as shown by Lee [16], who considers fixed transaction costs in a framework of asymmetric information and sequential trading. The limitation of Lee's model however is that it does not allow for optimizing behavior by market makers as normally done in microstructure models, so that the question arises if informational cascades can still arise when this assumption is made.

This paper answers this question. When market makers are assumed to behave optimally and competitively in setting prices, informational cascades can still occur in equilibrium. Moreover, transaction costs may allow a cascade not only when the market assigns a high probability to the high or to the low value of the asset, so that prices are either very high or very low, as in Lee [16], but even when there is uncertainty about the fundamental value.

The paper also studies what happens if transaction costs are proportional to asset prices, rather than fixed. The main novel finding is that in this case the probability of an informational cascade is greater when prices are high. Specifically, if the fundamental value in the bad state approximates zero, an informational cascade will never occur when the price is low. This implies that when transaction costs depend on prices, cascades are asymmetrical: they are rare in depressed markets, and more likely to develop in bull markets.

Recent work on informational cascades in financial markets seeks to explain bubbles and crashes (Avery and Zemsky [2], Dasgupta and Prat [7], Lee [16]). These empirical phenomena generally come with increasing trading volume. However, like Lee [16], we find that as the market tends to an informational cascade, the expected volume of trading decreases because informed traders stay on the sidelines. The model assumes that liquidity traders submit orders in a probabilistic way according to a stationary distribution. This is a very strong assumption. In reality, if liquidity traders have any discretion as to the timing of their orders, one should expect their trading to be negatively correlated with the bid-ask spread.[18] Since the bid-ask spread tends to decrease before and during informational cascades, due to the reduction of informed trading, we should find an increase in liquidity trading before and during cascades, when the adverse selection component of the spread disappears. In a such a setting, informational cascades should involve a change in the composition of total volume in favor of liquidity trading. And, if liquidity traders are more numerous, total expected volume could actually increase during a cascade. A natural extension of this paper would be to investigate the empirical implications of informational cascades in financial markets with discretionary liquidity traders.

8 Appendix

Lemma 4 *The marginal selling trader increases the information content of a sell order if, and only if:*

- $\lambda_{\theta_{n_t^s(B)}} > \lambda_t^s(B)$.

The marginal buying trader increases the information content of a buy order if, and only if:

- $\lambda_{\theta_{n_t^b(A)}} < \lambda_t^b(A)$.

Proof

We prove the result for the ask side. The result for the bid side follows by symmetry.
Let $f^b(n) = \frac{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} P(\theta_i|\underline{V})}{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} P(\theta_i|\overline{V})}$. To prove the lemma for the ask side, we have to show that:

$$f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n).$$

By using algebraic calculus, it is easy to show that:

1. $f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n-1)$
2. $\lambda_n \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n)$.

By combining these results, we obtain:

$$f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n).$$

Since $f^b(n_t^b(A)) = \lambda_t^b(A)$, the lemma for the ask side is proved. □

Proof of Proposition 2

We prove the proposition for the ask price. The proof for the bid price is by symmetrical argument.

[18] Admati and Pfleiderer [1] analyze the volume and price variability in a model where discretionary liquidity traders time their trades to minimize transaction costs.

By the monotone likelihood property of signals, the expected asset value conditional on a buy order is always greater than (or equal to) the unconditional expected value. This implies that if the ask price is lower than $E[y|H]+c$, the market maker's expected profit is negative. Moreover, $E[y|H,\ b\ at\ A]$ is upper-bounded by $\overline{V}$. Hence, by the zero-expected-profit condition, the equilibrium ask price cannot be greater than $\overline{V}+c$. Let us define the correspondence $F_t^b : [E_t[\widetilde{V}]+c,\ \overline{V}+c] \rightsquigarrow [E_t[\widetilde{V}]+c,\ \overline{V}+c]$ as:

$$F_t^b(A) \equiv E_t[\widetilde{V}|b\ at\ A]+c$$

$F_t^b(A)$ is an upper semicontinuous convex-valued correspondence that maps the set $[E_t[\widetilde{V}]+c,\ \overline{V}+c]$ onto itself. By Kakutani's fixed point theorem, $F_t^b(A)$ has a fixed point A_t^*; that is, an equilibrium ask price always exists.

Uniqueness is proved using the results of Lemma 4, which states that the marginal buying (selling) trader increases the information content of a buy (sell) order. That is, the likelihood ratio of the buy (sell) order increases (decreases) when the marginal buyer (seller) refrains from trading, if the signal that he observes is more accurate than the information that the market maker infers from a buy (sell) order.

Suppose, by way of obtaining a contradiction, that there exist two equilibrium ask prices: A_1 and A_2, with $A_1 < A_2$. Denote $n_t^b(A_1)$ and $n_t^b(A_2)$ respectively the marginal buying trader given A_1 and the marginal buying trader given A_2. Clearly, since we suppose $A_1 < A_2$, it must be that $\lambda_{n_t^b(A_1)} > \lambda_{n_t^b(A_2)}$. First, note that at the equilibrium, the value assessment of the marginal buying trader is greater than (or at most equal to) the market maker's conditional expected value. Hence $\lambda_{n_t^b(A_1)} \leq \lambda_t^b(A_1)$. This implies that $\lambda_t^b(A_1) < \lambda_t^b(A_2)$ because, when the ask price rises to A_2, the traders getting the signal $\theta_{n_t^b(A_1)}$ refrain from trading, and then the likelihood ratio of a buy order increases by lemma 4. As a consequence, if the ask price is equal to A_2, the market maker's expected profit is strictly positive. Hence, by the zero-expected-profit condition, A_2 cannot be an equilibrium ask price. □

Proof of Proposition 3

We prove the proposition for the ask price; the proof for the bid price is symmetrical. First we assume that $A_t^* = E_t[\widetilde{V}]+c$ and we prove that this implies $E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$. Suppose by way of obtaining a contradiction that $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] > c$. Since at least the traders observing θ_1 prefer to buy when the ask price is $E_t[\widetilde{V}]+c$, the expected asset value conditional on a buy order at $E_t[\widetilde{V}]+c$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradicts the fact that $E_t[\widetilde{V}]+c$ is the equilibrium ask price.

On the other hand, if $E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}] \leq c$ then no informed trader will buy. Hence $\inf\{E_t[\widetilde{V}|b\ at\ E_t[\widetilde{V}]+c,\ \sigma^*(P_t^*|t)]+c\} = E_t[\widetilde{V}]+c = A_t^*$. □

Proof of Lemma 1

Define the functions $h_\theta(\pi)$ as:

$$h_\theta(\pi) \equiv |E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]| = \frac{\pi - \pi^2}{\pi + (1-\pi)\lambda_\theta} \cdot |1-\lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

$h_\theta(\pi)$ gives the informational advantage of traders observing θ for any $\pi \in [0,\ 1]$. Putting Proposition 3 and the assumption of symmetrical signals together it follows that:

$$\bar{c} = \max h_{\theta_N}(\pi) = \max h_{\theta_1}(\pi) = \frac{|1-\sqrt{\lambda_1}|}{1+\sqrt{\lambda_1}} \cdot (\overline{V} - \underline{V}).\square$$

Proof of Proposition 4

Consider the function h_θ defined in the proof of lemma 1. Note that:

- $h_\theta(0) = h_\theta(1) = 0$
- $\max h_\theta(\pi) = \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$
- $h_\theta^{''}(\pi) = -\frac{2\lambda}{(\pi+(1-\pi)\lambda)^3} \cdot |1-\lambda_i| \cdot (\overline{V} - \underline{V}) < 0\ \forall \pi \in [0,\ 1]$.

Since by assumption $0 < c \le \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$, from the strict concavity of $h_\theta(\pi)$ it follows that there exist unique π_θ^l and π_θ^u, with $\pi_\theta^l < \pi_\theta^u$, such that $h_\theta(\pi_\theta^l) = h_\theta(\pi_\theta^u) = c$ and $h_\theta(\pi) > c$ if, and only if, $\pi \in (\pi_\theta^l,\ \pi_\theta^u)$. Thus, when $\pi \in (0,\ \pi_\theta^l) \cup (\pi_\theta^u,\ 1)$, traders getting signal θ refrain from trading. Moreover,

$$\arg\max h_{\theta_1}(\pi) = \frac{\sqrt{\lambda_1}}{1+\sqrt{\lambda_1}} < \frac{1}{2} < \arg\max h_{\theta_N}(\pi) = \frac{\sqrt{\lambda_N}}{1+\sqrt{\lambda_N}}$$

and

$$h_{\theta_1}(\pi) \ge h_{\theta_N}(\pi) \quad \forall \pi \le \frac{1}{2}$$

As $\pi_{\theta_1}^l < \arg\max h_{\theta 1}(\pi)$ and $\pi_{\theta_N}^u > \arg\max h_{\theta_N}(\pi)$, then $\pi_{\theta_1}^l < \pi_{\theta_N}^l$ and $\pi_{\theta_N}^u > \pi_{\theta_1}^u$. Hence, when either $\pi < \pi_{\theta_1}^l$ or $\pi > \pi_{\theta_N}^u$, all informed traders will refrain from trading. $\square$

Proof of Proposition 5

From the proof of Proposition 4, we know that there exist $\pi_{\theta_1}^u > \pi_{\theta_1}^l$ and $\pi_{\theta_N}^l < \pi_{\theta_N}^u$ such that $h_{\theta_1}(\pi_{\theta_1}^u) = h_{\theta_N}(\pi_{\theta_N}^l) = c$. Since $h_{\theta_1}^{'}(\frac{1}{2}) < 0$ and $h_{\theta_N}^{'}(\frac{1}{2}) > 0$, and since $\frac{|1-\lambda_1|}{2(1+\lambda_1)} \cdot (\overline{V} - \underline{V}) < c < \bar{c}$, then $\pi_{\theta_1}^u < \pi_{\theta_N}^l$ and both $c > h_{\theta_1}(\pi)$ and $c > h_{\theta_N}(\pi)$ for any $\pi \in [\pi_{\theta_1}^u,\ \pi_{\theta_N}^l]$. $\square$

Proof of Lemma 2

We prove the lemma for the ask side. The proof for the bid side is analogous.

A buy order indicates the good state of nature. Hence, the information content of a buy order is maximum when $f^b(n)$ (defined in the proof of lemma 4) reaches its minimum value. If all informed traders prefer to buy the asset, then $f^b(N) = 1 < \lambda_N$. By lemma 4, it follows that the marginal buying trader reduces the information content of the buy order; that is: $f^b(N-1) < f^b(N)$. If no informed trader buys, then $f^b(0) = 1 > \lambda_1$, and the marginal buying trader increases the information content of the buy order; that is: $f^b(1) > f^b(0)$. By the maximum likelihood property of signals, it follows that there exists a unique signal θ_{n^b} such that:

$$f^b(n^b+1) > f^b(n^b) \ge \lambda_{n^b} \ \textit{and}\ f^b(n^b) \le f^b(n^b-1).$$

If the equilibrium ask price A_t^* is such that $n_t^b(A_t^*) = n^b$, then the equilibrium information content of a buy order is maximum. $\square$

Proof of Proposition 6

We prove the proposition for the ask side. The proof for the bid side can be obtained symmetrically.

Since $c = 0$, the competitive ask price has to be equal to the expected asset value conditional on a buy order, that is:

$$A_t^* \in E_t[\widetilde{V}|b \ at \ A_t^*, \ \sigma^*(P_t^*|t)].$$

Lemma 2 states that there exists a good signal θ_{n^b} such that $f^b(n^b) \leq f^b(n)$ for every $n \in \{1, 2, ..., N\}$, and $\lambda_{n^b} \leq f^b(n^b)$. Moreover, by combining lemmas 4 and 2 it easy to see that $\lambda_{(n^b+1)} > f^b(n^b+1)$. As a consequence, for any history of trades:

$$E_t[\widetilde{V}|\theta_{n^b+1}] < \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)f^b(n^b+1)} \cdot (\overline{V} - \underline{V}) \tag{3}$$

and

$$E_t[\widetilde{V}|\theta_{n^b}] \geq \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)f^b(n^b)} \cdot (\overline{V} - \underline{V}). \tag{4}$$

3 and 4 imply that there exists an ask price A_t^* that belongs to $(E_t[\widetilde{V}|\theta_{n^b+1}], \ E_t[\widetilde{V}|\theta_{n^b}]]$ and that satisfies the market maker's zero-profit condition. By Proposition 2, we know that there exists a unique ask price that satisfies that condition. Thus, in equilibrium, $n_t^b(A_t^*) = n^b$. $\square$

Proof of Proposition 7

We prove the proposition for the ask price; the proof for the bid price is symmetrical. First assume that $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$ and prove that this implies $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$. Suppose by way of obtaining a contradiction that $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} > c$. This means that $E_t[\widetilde{V}|\theta_1] > \frac{E_t[\widetilde{V}]}{1-c}$. Hence, at the least the traders observing θ_1 prefer to buy when the ask price is $\frac{E_t[\widetilde{V}]}{1-c}$. As a consequence, the expected asset value conditional on a buy order at $\frac{E_t[\widetilde{V}]}{1-c}$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradicts the fact that $\frac{E_t[\widetilde{V}]}{1-c}$ is the equilibrium ask price.

On the other hand, if $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$ then no informed trader buys.[19] Hence:

$$\inf\{\frac{E_t[\widetilde{V}|b \ at \ \frac{E_t[\widetilde{V}]}{1-c}, \ \sigma^*(P_t^*|t)]}{1-c}\} = \frac{E_t[\widetilde{V}]}{1-c} = A_t^*. \square$$

Proof of Lemma 3

Define the functions $g_\theta(\pi)$ as:

$$g_\theta(\pi) \equiv \frac{|E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]|}{E_t[\widetilde{V}|\theta]} = \frac{\pi - \pi^2}{\pi\overline{V} + (1-\pi)\lambda_\theta\underline{V}} \cdot |1 - \lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

[19] The relative informational advantage of informed traders increases with the signal's precision.

$g_\theta(\pi)$ gives the relative informational advantage of traders observing θ for any $\pi \in [0,\ 1]$. The relative informational advantage of traders getting the signals θ_N and θ_1 is always greater than that of traders observing a different signal, hence the threshold cost $c > \overline{c}_p$ is given by:

$$\overline{c}_p = \sup\{\max_\pi g_{\theta_1}(\pi),\ \max_\pi g_{\theta_N}(\pi)\} = \frac{\overline{V} - \underline{V}}{(\sqrt{\overline{V}} + \sqrt{\lambda_N \underline{V}})^2} \cdot (\lambda_N - 1). \square$$

Proof of Proposition 8

First we prove that there exist unique $\underline{\pi}$ and $\overline{\pi}$ such that when $\pi \in [0,\ \underline{\pi}) \bigcup (\overline{\pi},\ 1]$, all informed traders will refrain from trading.

Consider the function $g_\theta(\pi)$ defined at page 23. Notice that:

- $g_\theta(0) = g_\theta(1) = 0$
- $\max g_\theta(\pi) = \frac{\overline{V} - \underline{V}}{(\sqrt{\overline{V}} + \sqrt{\lambda_\theta \underline{V}})^2} \cdot |\lambda_\theta - 1|$
- $g_\theta^{''}(\pi) = -\frac{2\lambda_\theta \underline{V}\,\overline{V}}{(\pi \overline{V} + (1-\pi)\lambda_\theta \underline{V})^3} \cdot |1 - \lambda_\theta| \cdot (\overline{V} - \underline{V}) < 0\ \forall \pi \in [0,\ 1]$.

Since by assumption $0 < c \leq \overline{c}^p$, from the strict concavity of $g_\theta(\pi)$ it follows that there exist unique $\underline{\pi}_\theta$ and $\overline{\pi}_\theta$, with $\underline{\pi}_\theta < \overline{\pi}_\theta$, such that $g_\theta(\underline{\pi}_\theta) = g_\theta(\overline{\pi}_\theta) = c$ and $g_\theta(\pi) > c$ if, and only if, $\pi \in (\underline{\pi}_\theta,\ \overline{\pi}_\theta)$. Thus, $\underline{\pi} = \inf\{\underline{\pi}_\theta\}_{\theta \in \Theta}$ and $\overline{\pi} = \sup\{\overline{\pi}_\theta\}_{\theta \in \Theta}$.

In order to prove the second part of the theorem, note first that $g_{\theta_1}(\frac{\underline{V}}{\overline{V} + \underline{V}}) = g_{\theta_N}(\frac{\underline{V}}{\overline{V} + \underline{V}}) = \underline{c}_p$, and $g_{\theta_1}(\pi) < g_{\theta_N}(\pi)$ for all $\pi > \frac{\underline{V}}{\overline{V} + \underline{V}}$. Moreover, if $\overline{V} > \underline{V}\lambda_1$ then $\frac{\underline{V}}{\overline{V} + \underline{V}} < \arg\max g_{\theta_1}(\pi) < \overline{\pi}_{\theta_1}$. As a consequence, if $\overline{V} > \underline{V}\lambda_1$ then the relative informational advantage of traders getting θ_N is strictly positive for all $\pi \in [\overline{\pi}_{\theta_1},\ \overline{\pi}_{\theta_N})$ whatever $c \in (0,\ \overline{c}_p)$. In contrast, if $\overline{V} < \underline{V}\lambda_1$ and c is greater than $\underline{c}_p$, then $\overline{\pi}_{\theta_1}$ is lower than $\underline{\pi}_{\theta_N}$ and both $g_{\theta_1}(\pi)$ and $g_{\theta_N}(\pi)$ are strictly negative for all $\pi \in (\overline{\pi}_{\theta_1},\ \underline{\pi}_{\theta_N})$. Hence an informational cascade occurs also when $\pi \in (\overline{\pi}_{\theta_1},\ \underline{\pi}_{\theta_N})$. $\square$

Proof of Proposition 9

If $\underline{V} = 0$, the relative informational advantage of traders getting the signal θ as a function of π is given by $g_\theta(\pi) = (1 - \pi) \cdot |\lambda_\theta - 1|$. Since by assumption $0 < c \leq \overline{c}^p$, it follows that for any $\theta \in \Theta$ there exists a unique $\overline{\pi}_\theta$ such that $g_\theta(\overline{\pi}_\theta) = c$, and $g_\theta(\pi) > c$ if, and only if, $\pi < \overline{\pi}_\theta$. Thus, $\overline{\pi} = \sup\{\overline{\pi}_\theta\}_{\theta \in \Theta}$. $\square$

References

[1] A. R. Admati and P. Pfleiderer. A theory of intraday patterns: Volume and price variability. *Review of Financial Studies*, 1(1):3–40, 1988.

[2] C. Avery and P. Zemsky. Multidimensional uncertainty and herd behavior in financial markets. *The American Economic Review*, 88(4):724–748, 1998.

[3] A. Banerjee. A simple model of herd behavior. *Quarterly Journal of Economics*, 107(3):797–817, 1992.

[4] S. Bikhchandani, D. Hirshleifer, and I. Welch. A theory of fads, fashion, custom and cultural change as informational cascades. *Journal of Political Economy*, 100(5):992–1026, 1992.

[5] C. Chamley. *Rational Herds. Economic models of social learning.* Cambridge University Press, 2004.

[6] M. Cipriani and A. Guarino. Herd behavior and contagion in financial markets. *Working Paper*, 2001.

[7] A. Dasgupta and A. Prat. Reputation and price dynamics: Cascades and bubbles in financial markets. *Working Paper*, 2004.

[8] J. P. Decamps and S. Lovo. Aversion and herd behavior in financial markets. *SSRN Working Paper*, 2002.

[9] A. Devenow and I. Welch. Rational herding in financial economics. *European Economic Review*, 40:603–615, 1996.

[10] E. F. Fama. Efficient capital markets: A review of theory and empirical works. *Journal of Finance*, 19:69–90, 1970.

[11] S. Gervais. Market microstructure with uncertain information: A multiperiod analysis. *Working Paper*, 1996.

[12] L. Glosten and P. Milgrom. Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14:71–100, 1985.

[13] D. Hirshleifer and S. H. Teoh. Herd behavior and cascading in capital markets: a review and synthesis. *European Financial Management*, 9(1):25–66, 2003.

[14] T. Kortian. Modern approaches to asset price formation: a survey of recent theoretical literature. *Research Discussion Paper 9501, Reserve Bank of Australia*, 1995.

[15] A. S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53:1315–1335, 1985.

[16] I. H. Lee. Market crashes and informational avalanches. *Review of Economic Studies*, 65:741–759, 1998.

[17] P. Milgrom and N. Stokey. Information, trade, and common knowledge. *Journal of Economic Theory*, 26:17–27, 1982.

[18] D. Scharfstein and J. Stein. Herd behavior and investment. *American Economic Review*, 80(3):465–479, 1990.