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Abstract

In an English auction, a bidder.s strategy depends on the prices at which his competitors drop out, because these convey information on the value of the object on sale. A ring of colluding bidders can strategically manipulate the information transmitted through its members.bids, in order to mislead other bidders into bidding less aggressively and thus allow a designated ring bidder to bid more aggressively. Collusion increases the probability that the ring wins the auction and reduces the price it pays. The presence of a ring harms other bidders (as well as the seller) and reduces efficiency.

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Contents

1. Introduction

The possibility that bidders collude during an auction is a crucial concern for the seller: there is considerable evidence that collusion is a widespread phenomenon in auctions, and collusion typically results in a substantial loss of revenue for the seller.[1] We analyze collusion in English (or ascending) auctions in which bidders' valuations are not independent and show how a ring of bidders can exploit the characteristics of the bidding process in order to win more often and pay a lower price, when other bidders do not know they are facing a ring. Specifically, ring bidders strategically modify their behavior in order to send misleading signals that affect the strategies of their competitors: bidders use their bids as a vehicle of misinformation.

Most of the existing literature on collusion assumes that a ring designates a single bidder who participates in the auction on behalf of all colluding bidders, while other ring members have no active task and do not participate in the auction at all.[2] So the ring reduces competition in the auction by reducing the number of active bidders. This may reduce the price paid by the auction winner, but it cannot influence the probability that a ring bidder wins the auction.[3] When valuations are not independent, however, the ring can do better than simply eliminating competition among its members — the ring can induce its competitors to bid less aggressively, thus biasing the outcome of the auction to its advantage.

Consider, as an example, an auction for wildcat oil leases. Part of the value of a tract is determined by the amount of oil it contains, and this is common to all bidder. But bidders are usually very uncertain about this value and have access to different information, such as different seismic studies on the tract. Knowing the information possessed by competitors would allow bidders to make a better estimate of the tract's value.

In an English auction, a bidder can infer his competitors' information on the tract's value by observing their bids. Therefore, ring bidders may strategically manipulate the information transmitted through their bids, in order to influence the bidding strategies of their opponents.

[1] Many observers argued that the outcome of the European auctions for 3G mobile-phone licenses was affected by collusion and antitrust agencies investigated bidders' behavior in Italy, the Netherlands and Switzerland (Klemperer, 2004). According to Hendricks and Porter (1989), 81% of the 319 Sherman Act Section 1 criminal cases filed by the U.S. Department of Justice from November 1979 to May 1988 were in auction markets. The U.S. Department of Justice's antitrust chief (as quoted by McAfee and McMillan, 1992) reports that collusive behavior among bidders in auctions for highway contracts increased building costs by at least 10%. Klemperer (2004) argues that preventing collusive behavior is one of the main challenges faced by the auction designer.

[2] See, for example, Robinson (1985), McAfee and McMillan (1992), and Mailath and Zemsky (1991). However, Graham and Marshall (1987) show that, when a ring includes all bidders, bidders can place random bids in order to conceal the presence of the ring from the seller. Porter and Zona (1993) provide empirical evidence of this type of strategic behavior. Moreover, Marshall and Marx (2007) show that, in a first-price auction, a ring can require bidders to place similar and relatively high bids, in order to prevent deviation by its members.

[3] In the words of Graham and Marshall (1987): "a coalition [...], which contains K of the N bidders at an auction, gains in expected terms by removing $K-1$ bidders from the competitive bidding. If the coalition does not contain the two bidders with the two highest valuations from the N bidders, then the ring realizes no gain beyond what each member could have obtained acting non-cooperatively."

If some ring bidders drop out of the auction at a low price, pretending their estimate of the tract's value is low, non-ring bidders are misled into reducing their own estimate of the tract's value and into bidding less aggressively. Thus a remaining ring bidder can bid more aggressively since he suffers a lower "winner's curse," and the ring shares the enhanced profit.

Our analysis yields the following insights:

- In addition to reducing competition among ring bidders, collusion misleads the behavior of non-ring bidders who are not aware of the presence of the ring. Hence, collusion reduces the price paid by the ring and increases its probability of winning.
- All collusive bidders have an active role in the auction.
- Collusion may reduce efficiency (when the auction prize does not have a pure common value), because the ring may win the auction even if it competes against bidders with higher valuations.
- Collusion makes non-ring bidders strictly worse off because they are induced to bid less aggressively, win the auction with a lower probability, and pay a higher price when they do win.

Hence, we provide an explanation of why players are hurt by collusive agreements among their competitors and typically try to prevent or denounce such agreements, if they become aware of them. This contrasts with standard economic analysis (and previous models of collusion in auctions), that instead suggests that all players in a market, even non-colluding ones, (weakly) benefit from collusion, since competition and prices are lower and all players' profit are higher in a market where some players collude.[4] Moreover, in the existing literature collusion affects neither the behavior of non-ring bidders nor the probability of the ring winning the auction.

According to the US Department of Justice, a common form of collusion in procurement auction involves a "bid suppression scheme" in which "one or more competitors who otherwise would be expected to bid, or who have previously bid, agree to refrain from bidding or withdraw a previously submitted bid."[5] We argue that this strategy may also be aimed at signalling that the auction prize is relatively unattractive to non-colluding bidders who are not aware of the ring's presence. Feinstein, Block and Nold (1985) provide evidence that, in

[4] McAfee and McMillan (1992) even show that, with private valuations, non-colluding players may earn higher expected profit than colluding ones. A notable exception is Asker (2010) who examines data on bidding by a ring of stamp dealers that operated in North America in the 1990s and shows that non-ring bidders paid higher prices because of collusion. The reason was that the side payment that a ring bidder received in the collusive mechanism was increasing in the valuation he declared, and this determined his bid in the auction. Hence, collusion induced ring bidders to overbid (see also Graham, Marshall and Richard, 1990). By contrast to our analysis, however, this behaviour did not increase the ring profit.

[5] See "Price Fixing & Bid Rigging — They Happen: What They Are and What to Look For," available at http://www.usdoj.gov/atr/public/guidelines/211578.htm.

a series of repeated procurement auctions for highway construction contracts held in North Carolina between 1975 and 1981, colluding bidders submitted phoney bids to manipulate the expectation and the choices of the auctioneer, who was unaware of the ring's presence. They conclude that rings "appear to be actively engaged in misinforming purchasers." We show how a ring may also misinform other non-colluding bidders.

After the 3G mobile-phone licenses auctions in the UK and Germany (which raised considerable revenue for the governments), various potential buyers failed to enter other European auctions.[6] This was possibly a consequence of a genuine concern about licenses' profitability or deteriorating bidders' credit rating. However, by failing to bid firms caused a drastic reduction in markets' estimate of the licenses' value and in the auction prices in many European countries (Klemperer, 2004). Our analysis suggests that failure to bid may have been an explicit strategic choice, aimed to signalling that the licenses were not valuable.[7]

Following the literature, we assume that non-ring bidders are unaware of the presence of a ring in the auction, even after they observe a number of bidders drop out at low prices. For example, this happens if non-ring bidders remain unaware of the presence of a ring as long as the probability of the bidding behaviour they observe in the auction being generated by independent non-colluding bidders is higher than a certain threshold. This assumption is consistent with the fact that rings usually manage to conceal their presence, in order to avoid being prosecuted. In Section 6.2, however, we show that, in an almost common-value model, bidders' strategies and the auction's outcome are the same both when non-ring bidders know they are facing a ring, and when non-ring bidders are unaware of the presence of a ring.[8] This (somewhat counter-intuitive) result suggests that, in an almost common-value model, even if non-ring bidders only place some positive probability on the existence of a ring in the auction, the ring can always credibly signal its presence and obtain the same outcome as it does under our assumption.

Our insights extend to sequential auctions, even when bidders have private and independent values, since bidders infer the level of competition in later auctions by observing their competitors' strategies in earlier ones. Therefore, a ring can induce non-colluding bidders to bid less aggressively in earlier auctions, by having some of its member drop out at low prices, thus pretending that they have a low valuations for the objects on sale, and hence that they will not bid aggressively in later auctions. When they do so, non-colluding bidders expect to be able to win a later auction at a low price.

[6] There were 13 bidders (for 5 licenses) in the UK auction but, for example, only 6 (for 5 licenses) in Italy and the Netherlands and 4 (for 4 licenses) in Switzerland (Klemperer, 2004).

[7] In private conversation, the CEO of a major European telecom company admitted that he tried to "talk down" the value of the 3G licenses before the auctions.

[8] When non-ring bidders know they are facing a ring, a common-value English auction has a continuum of equilibria (Bikhchandani and Riley, 1991). However, by analyzing a pure common-value auction as the limit of an almost common-value auction, we prove that it is natural to select a unique equilibrium in which bidding strategies are the same as when non-ring bidders are unaware of the presence of a ring.

The rest of the paper is organized as follows. After a review of the theoretical literature on collusion in auctions, Section 2 discusses a simple example, based on a pure common-value model, to introduce the main idea of the paper. In Section 3, following Milgrom and Weber (1982), we consider an English auction with affiliated valuations. Section 4 presents a collusive mechanism that results in all ring members truthfully reporting their signal. Section 5 analyzes the effects of collusion on bidding strategies and the profit obtained by colluding bidders. Section 6 extends the analysis to almost common-value auctions and to sequential private-value auctions. The last section concludes. All proofs are contained in the Appendix.

Related Literature There is an extensive theoretical literature on collusion in auctions.[9] Robinson (1985) shows that, when all bidders join a ring and select a single bidder to participate in the auction, collusion is easier to sustain in a second-price auction than in a first-price auction, because in a second-price auction the designated winner can bid infinitely high and other bidders have no incentive to cheat. But while Robinson (1985) assumes that ring members know their valuations, one of the main problems faced by a ring is how to induce its members to report their information truthfully. This problem arises because the division of the ring profit depends on bidders' valuations; hence, ring members have an incentive to misreport them. So the ring has to design a mechanism that efficiently and incentive-compatibly designates the winner and divides the collusive profit.

McAfee and McMillan (1992) analyze rings that include all bidders in an auction with independent and private valuations and introduce an efficient and ex-post budget balanced mechanism. After winning the auction, the ring allocates the object by a first-price "knockout," with the winner paying each ring bidder (including himself) an equal share of his bid.[10] The mechanism is incentive-compatible since a losing bidder's payoff does not depend on his bid; hence, in the knockout each bidder bids exactly as in a standard first-price auction without collusion.[11] Notice that the revenue equivalence theorem holds in the main auction, but each bidder's surplus is higher by a fixed amount than in an auction without collusion.

Graham and Marshall (1987) show that, with independent and private valuations, ring bidders can efficiently allocate the object among themselves in dominant strategies by running a second-price knockout before the main auction, the winner of which pays (the second-highest bid to) a risk-neutral "ring center" who previously paid all ring bidders an equal share of the

[9] For a review of the empirical literature on collusion see Porter (2005).

[10] See also Graham, Marshall and Richard (1990) and Deltas (2002) for descriptions and analysis of knockouts.

[11] This is a special case of the mechanism proposed by Cramton, Gibbons and Klemperer (1987) to assign an object jointly owned by a group of agents. When bidders cannot make side-payments, however, McAfee and McMillan (1992) prove that the ring cannot extract any information from its members and can do no better than randomize the right to bid in the main auction. For an analysis of collusion in repeated auctions when bidders cannot make side payments see Aoyagi (2003), Athey, Bagwell and Sanchirico (2004), Skrzypacz and Hopenhayn (2004), and the references therein.

expected payment by the winner (so that payment received by ring bidders do not depend on their bids). This mechanism, however, is only budget balanced in expectation. The authors also extend the result to rings that do not include all bidders in second-price and English auctions. Mailath and Zemsky (1991) analyzes the case of heterogenous bidders, and show that efficient collusion can be achieved without the need for a ring center.

Hendricks, Porter and Tan (2008) extend the ex-post budget balanced mechanism of McAfee and McMillan (1992) to auctions with affiliated values in which all bidders collude. By contrast, we consider rings that do not include all bidders in auctions with affiliated values and extends the ex-ante budget balanced mechanism of Graham and Marshall (1987) that allows colluding bidders to equally share the expected collusive profits.

In general, however, ring bidders may want to cheat at the main auction. Marshall and Marx (2007) show that, when the ring cannot directly control its members' bids and the collusive mechanism cannot rely on the auction outcome, collusion is more difficult at first-price than at second-price auctions. This confirms the results of Robinson (1985).[12] Moreover, Marshall and Marx (2009) show that various details of the design of second-price and ascending auctions are crucial for reducing the profitability of collusion.

Bidders do not necessarily want to join a ring. When bidding is costly, Tan and Yilankaya (2007) show that high-value bidders may signal their valuation by refusing to participate in a ring, thus inducing their competitors not to bid in the main auction. In common-value auctions, Hendricks, Porter and Tan (2008) show that bidders' who have good information on the value of the prize may prefer to bid non-cooperatively, even when there is no bidding cost.

The use of bids as a signalling device has already been underlined by Bikhchandani (1988) and Brusco and Lopomo (2002). Bikhchandani (1988) shows that, in sequential common-value auctions without collusion, a bidder can establish a reputation for bidding aggressively, thus inducing his competitors to bid less aggressively in future auctions. Brusco and Lopomo (2002) analyze (tacit) collusion in multi-unit ascending auctions and prove that a bidder can use his bid to truthfully signal his valuations to his competitors, in order to agree on a division of the objects and end the auction at low prices.[13] By contrast, we prove how bidders can use their bids to communicate misleading information regarding their valuations.

[12] Moreover, Lopomo, Marshall and Marx (2005) show that, in an English auction, collusion generates inefficiency if ring members cannot communicate information regarding their values before the auction and the collusive mechanism has to be ex-post budget balanced.

[13] Cramton and Schwartz (2000) argue that this type of signalling strategy was adopted during the FCC spectrum auctions in the 1990s. Weber (1997), Ausubel and Cramton (1998), Englebrecht-Wiggans and Kahn (1998), Grimm et al. (2003), and Pagnozzi (2010) also analyze collusion in multi-unit ascending auctions.

2. An Example: Common Value

Consider an English auction for a prize whose value is exactly the same for all bidders.[14] There are three bidders — called 1, 2 and 3 — and each bidder i receives a non-negative private signal x_i about the value of the prize. Signals are independently and uniformly distributed. Similarly to the "wallet game" of Klemperer (1998) and Bulow and Klemperer (2002), the common value of the auction prize is:

$$V(x_1, x_2, x_3) = x_1 + x_2 + x_3 - c,$$

where c is a strictly positive small number that represents a fixed cost that the winner has to pay in order to use the prize. A strategy for a bidder specifies the price at which he drops out if no other bidder has dropped out yet, and the price at which he drops out after one other bidder dropped out.

In the unique symmetric equilibrium of the auction, if no bidder has dropped out of the auction yet, bidder i bids up to:[15]

$$\max\{V(x_i, x_i, x_i);\ 0\} = \max\{3x_i - c;\ 0\}. \tag{2.1}$$

That is, a bidder bids up to the price at which he makes no money if he wins the auction when all other bidders have his same signal (and, therefore, he is indifferent between winning or losing), provided this value is not negative.[16] A bidder with a signal lower than $\underline{x} \equiv \frac{c}{3}$ drops out of the auction at price zero, because he can never win and obtain a positive profit. Dropping out at price zero can be interpreted as failing to bid more than the reserve price, or not participating in the auction at all, or exiting immediately, as soon as the auction starts.[17]

When a bidder quits the auction, he reveals information about his signal to the remaining bidder(s), who update their bidding strategies accordingly. If two or more bidders drop out at price zero, the auction ends immediately. If one bidder drops out at price zero, he reveals that his signal is at most $\underline{x}$; hence on average it is equal to $\frac{c}{6}$. Then in the unique symmetric

[14] In an English auction the price starts at zero and is raised continuously by the auctioneer until only one active bidder is left. A bidder who wishes to be active at the current price depresses a button and, when he releases it, he is withdrawn from the auction. The price level and the number of active bidders are continuously displayed.

[15] Notice that this strategy, as well as the one in (2.3), is independent of the signals' distributions and does not require the distributions to be symmetric.

[16] To see that this is an equilibrium, suppose bidder i deviates when other bidders bid according to (2.1), stays longer in the auction and wins at price $3x_i - c + \varepsilon$, when both the other bidders drop out. Then, however, each of the other 2 bidders has signal $\frac{1}{3}(3x_i + \varepsilon)$ and the value of the prize is $x_i + 2\left(x_i + \frac{1}{3}\varepsilon\right) - c < 3x_i - c + \varepsilon$. Hence, bidder i pays more than the prize is worth. By contrast, at price $3x_i - c - \varepsilon$ bidder i knows that, if both the other bidders drop out, he wins and pays less than the value of the prize. Hence, he has no incentive to drop out. It is straightforward to show that this is the unique symmetric equilibrium.

[17] Only in English auctions, and not in sealed-bid auctions, do players observe their opponents bidding, and hence know whether or not they are participating in the auction.

equilibrium a remaining bidder bids up to:

$$\begin{aligned} \mathbb{E}\left[V\left(x_i, x_i, x_j\right) | x_j \leq \underline{x}\right] &= 2x_i + \mathbb{E}\left[x_j \,| x_j \leq \underline{x}\right] - c \\ &= 2x_i - \tfrac{5}{6}c \end{aligned} \tag{2.2}$$

If no bidder drops out at zero and, say, bidder j drops out at a positive price, he reveals his signal x_j. Then in the unique symmetric equilibrium a remaining bidder bids up to:

$$V\left(x_i, x_i, x_j\right) = 2x_i + x_j - c. \tag{2.3}$$

Basically, the auction proceeds in two phases. In the first one, the bidder with the lowest signal drops out and reveals (some or all of) his private information. If no more than one bidder drops out at price zero, in the second phase the two remaining bidders engage in a second-price auction using the information acquired in the first phase. In each phase, a bidder bids up to his estimate of the prize value, conditional on all the information he has and on winning against opponent(s) with his same signal, provided this estimate is positive. To update his estimate of the prize value, a bidder infers his competitors' private information from their bidding behavior.[18]

Suppose now that two bidders, say 1 and 2, join a ring and that the third one does not know they do, nor does she suspect it.[19] (We are going to relax this assumption in Section 6.2.) To make the analysis interesting, suppose that both bidders' signals are higher that $\underline{x}$. (If at least one ring bidder has a signal lower that $\underline{x}$, then collusion has no effect on the auction outcome.) Since the bidding strategy of bidder 3 may depend on the price at which a ring bidder drops out, the ring can induce bidder 3 to bid less aggressively.

Assume, without loss of generality, that $x_1 > x_2$ and assume that ring members know each other's signals.[20] The bidder with the highest signal (i.e., bidder 1) is the *designated* bidder while the bidder with the lowest signal (i.e., bidder 2) drops out of the auction at price zero. This misleads bidder 3 into thinking that bidder 2 has a signal weakly lower than $\underline{x}$. If bidder 3's signal is lower than $\underline{x}$, collusion does not affect her strategy anyway. But if bidder 3's signal is higher than $\underline{x}$, she reduces her own estimate of the prize value and, by equation (2.2), she only bids up to $2x_3 - \frac{5}{6}c$.

As a result, bidder 1 suffers a lower winner's curse and can bid more aggressively. Indeed, if bidder 1 wins the auction at price p, he knows the value of the prize is:

$$x_1 + x_2 + \tfrac{1}{2}\left(p + \tfrac{5}{6}c\right).$$

[18] For example, suppose that only one bidder drops out at price zero and bidder j uses the bidding strategy described by (2.2). Then if bidder i wins the auction at price p, his expected valuation is $x_i + \frac{1}{6}c + \frac{1}{2}\left(p + \frac{5}{6}c\right) - c$. This is lower than p if and only if p is lower than (2.2).

[19] We adopt the convention of using feminine pronouns for the non-ring bidders.

[20] In Section 4 we are going to prove that the ring can design a mechanism such that it is incentive compatible for each colluding bidder to truthfully reveal his signal.

Bidder 1 stays in the auction as long as the price is lower than the prize value — i.e., he bids up to p^* such that:

$$p^* = x_1 + x_2 + \frac{1}{2}\left(p^* + \frac{5}{6}c\right) \quad \Leftrightarrow \quad p^* = 2(x_1 + x_2) + \frac{5}{6}c.$$

So the ring wins the auction if and only if:

$$x_1 + x_2 + \frac{5}{6}c > x_3.$$

By contrast, without collusion bidder 3 bids up to $2x_3 + x_2 - c$ (assuming that $x_3 > \underline{x}$) and bidder 1 wins the auction if and only if $x_1 > x_3$.

The ring achieves two objectives: (*i*) it reduces competition in the auction by eliminating one "serious" bidder; and (*ii*) it reduces the aggressiveness of the non-ring bidder. Therefore, collusion increases the probability that the designated bidder wins the auction, because the designated bidder may win even if bidder 3 has the highest signal. Moreover, the designated bidder pays a lower price when he actually wins.[21]

The extra profit obtained by the ring is given by the difference between the price the designated bidder would have paid without collusion and the price he actually pays, when he has the highest signal — i.e., $x_2 - \frac{1}{6}c$ — and by the difference between the prize value and the price the designed bidder pays, when he does not have the highest signal and wins the auction — i.e., $x_1 + x_2 - x_3 - \frac{1}{6}c$.

3. The Model

Consider an English auction with n risk-neutral bidders. Each bidder i receives a (private) signal $x_i \geq 0$ of the value of the object on sale, which is the realization of a random variable X_i. The random elements of the vector $X \equiv (X_1, \ldots, X_n)$ have joint probability density function $f(x)$. We assume that $f(.)$ is symmetric in all its arguments and, therefore, that bidders' signals are identically distributed. Following Milgrom and Weber (1982), we assume that the variables $X_1, \ldots, X_n$ are *affiliated*. Roughly, random variables are said to be affiliated when higher values for some of the variables make the other variables more likely to be high than low.

Bidder i's valuation of the object on sale is:

$$V_i = u\left(X_i; \{X_j\}_{j \neq i}\right),$$

where $u : \mathbb{R}^n \to \mathbb{R}^0_+$ and $\{X_j\}_{j \neq i}$ represents the unordered set of signals different from X_i. Hence, each bidder's valuation is a symmetric function of the other bidders' signals.

[21] For example, if signals are uniformly distributed on $[0, 1]$ and $k = 0$, the ring wins the auction with probability $\frac{5}{6}$ while, without collusion, each bidder wins with probability $\frac{1}{3}$. Before the auction, the expected price the designated bidder pays conditional on winning is equal to $\frac{9}{10}$, while without collusion the expected price he pays conditional on winning is equal to $\frac{5}{4}$.

We assume that u is continuous and (weakly) increasing in each of its arguments, which implies that bidders' valuations are affiliated too (Milgrom and Weber, 1982, Theorem 3).[22] Moreover, we assume that $\frac{\partial V_i(X)}{\partial X_i} \geq \frac{\partial V_i(X)}{\partial X_j}$ for every X and $j \neq i$. This *single crossing condition* ensures that, if bidder i's signal is higher than bidder j's one, then bidder i values the prize more than bidder j.[23]

Let $Y_1, \ldots, Y_{n-1}$ denote respectively the smallest,..., largest signal from among $\{X_j\}_{j \neq i}$. Bidder i's valuation can be written as $V_i = u(X_i; Y_{n-1}, \ldots, Y_1)$. The variables $X_i, Y_1, \ldots, Y_{n-1}$ are also affiliated (Milgrom and Weber, 1982, Theorem 2). We assume that

$$\mathbb{E}\left[V_i \mid X_i = Y_{n-1} = \ldots = Y_1 = 0\right] = \underline{V} < 0$$

— i.e., a bidder's expected valuation is negative if all bidders' signals are equal to zero — and let $\underline{x}$ be such that $\mathbb{E}\left[V_i \mid X_i = Y_{n-1} = \ldots = Y_1 = \underline{x}\right] = 0$.

In an English auction, a strategy for a bidder specifies whether, at any price level, he remains active or drops out. So if k bidders dropped out at prices $p_1 \leq \ldots \leq p_k$, bidder i's strategy can be described by a function $\alpha_k^i(x_i; p_1, \ldots, p_k)$ which specify the price at which he drops out. If the current price is higher than the price at which a bidder would like to drop out, then he drops out immediately.

Proposition 1 (Milgrom and Weber, 1982). *Without collusion, the (symmetric) strategies $\alpha^i = \left(\alpha_0^i, \ldots, \alpha_{n-2}^i\right)$, $i = 1, \ldots, n$, defined iteratively by:*

$$\alpha_0^i(x_i) = \max\left\{\mathbb{E}\left[V_i \,|X_i = Y_{n-1} = \ldots = Y_1 = x_i\right];\ 0\right\}, \tag{3.1}$$

$$\alpha_k^i(x_i; p_1, \ldots, p_k) = \mathbb{E}\left[V_i \left| \begin{array}{l} X_i = Y_{n-1} = \ldots = Y_{k+1} = x_i, \\ \alpha_{k-1}^i(Y_k; p_1, \ldots, p_{k-1}) = p_k, \ldots, \alpha_0^i(Y_1) = p_1 \end{array}\right.\right], \tag{3.2}$$

$k = 1, \ldots, n-2$, are equilibrium bidding strategies.

Notice that, when $l \leq k$ bidders dropped out at price zero, the bidding strategy (3.2) is equivalent to:

$$\alpha_{k,l}^i(x_i; y_{l+1}, \ldots, y_k) = \mathbb{E}\left[V_i \left| \begin{array}{l} X_i = Y_{n-1} = \ldots = Y_{k+1} = x_i, \\ Y_k = y_k, \ldots, Y_{l+1} = y_{l+1}, Y_l \leq \underline{x}, \ldots, Y_1 \leq \underline{x} \end{array}\right.\right],$$

where $y_{l+1}, \ldots, y_k$ are the realizations of the random variables $Y_{l+1}, \ldots, Y_k$. Therefore, in equilibrium each bidder bids up to the price at which he is just indifferent between winning and losing, if all remaining bidders have his same signal, given the information revealed by bidders who dropped out of the auction. If this price is negative, then the bidder drops out at price 0, which can be interpreted as dropping out at the reserve price.

[22] In the pure common-value example of Section 2, signals are independent (and hence affiliated), and the prize value is "symmetrically" increasing in each signal.

[23] This assumption is not necessary for our results, but it simplifies the analysis since it ensures that the bidder with the highest signal is also the one with the highest valuation.

To update their estimate of the object's value, bidders use the quitting prices of their competitors to infer their information. Intuitively, the bidding strategy $\alpha^i_{k,l}(x_i; y_{l+1}, \ldots, y_k)$ is (strictly) increasing in x_i, is (strictly) increasing in the competitors' signals, and is (strictly) decreasing in l (Milgrom and Weber, 1982, Theorem 5). This is the feature that can be exploited by a ring to mislead outsiders and modify the outcome of the auction to its advantage.

We assume m randomly chosen bidders join a ring, $2 \le m < n$, and at least two of them have signals higher than $\underline{x}$.[24] Let $W_1, \ldots, W_m$ be respectively the lowest, ..., highest signal received by ring members, and let $Z_1, \ldots, Z_{n-m}$ be respectively the lowest, ..., highest signal received by non-ring bidders. We denote the realizations of W_i and Z_i by w_i and z_i respectively.

Following the literature, we assume that non-ring bidders do not know that they are facing a ring (see, e.g., Assumption 3 in Graham and Marshall, 1987). We believe this is a reasonable assumption, since rings usually attempt and manage to conceal their existence from competitors and auctioneers, in order to avoid being denounced and prosecuted by antitrust authorities.[25] We also assume that non-ring bidders remain unaware of the ring's presence after they observe a number of bidders drop out of the auction at low prices. This can be interpreted as non-ring bidders adopting the following strategy. After observing l bidders drop out at prices lower than p, a non-ring bidder follows the strategy described in Proposition 1 if the probability of l non-colluding bidders having signals that induce them to drop out at prices lower than p is higher than a threshold θ. Otherwise the non-ring bidder disregards all information embodied in her competitors' strategies (and hence cannot be fooled into believing that a ring bidder has a low signal). We assume that θ is small enough so that it is optimal for the ring to have all but one bidder drop out at the lowest possible price, since this does not reveal the ring's presence. In Section 6.2, we relax this assumption in a simple model of an almost common value auction, and consider non-ring bidders who know they are facing a ring.

4. Collusive Mechanism

There is a risk-neutral *ring center* who acts as mediator and banker for the ring, and designs a mechanism to regulate ring bidders' behavior. We will construct a mechanism that results in all ring bidders revealing their true signals and that allows the ring to increase its probability of winning the auction and its expected profit.

Consider a mechanism that requires each ring bidder to report his private information.

[24] We do not analyze bidders' choice to participate in a ring. We assume that it is not possible for all bidders to join the ring because, for example, legal considerations force the ring to limit membership in order to avoid detection. Moreover, in contrast to standard models of collusion, colluding bidders have no incentive to allow outsiders to join the ring, when outsiders are not aware of the presence of the ring, as we assume.

[25] For example, it took over 15 years for a non-ring dealer to denounce a ring of stamp dealers operating in North America, even if non-ring dealers were strongly damaged by collusion (Asker, 2010).

Given the reports, the mechanism must determine: (*i*) the strategy of each bidder in the auction, (*ii*) the *designated* bidder who receives the prize if it is won by the ring, and (*iii*) the payments each ring bidder makes/receives. The mechanism is *incentive-compatible* if it is an equilibrium for each ring bidder to report his private information truthfully and to follow the bidding strategy set by the ring. The mechanism is (ex-ante) *budget-balanced* if side-payments sum to zero in expectation.

The following mechanism M — a pre-auction knockout — generalizes the one proposed by Graham and Marshall (1987) that considered the special case of independent private values.

1. Each ring bidder receives from the ring center a fixed side-payment of:

$$\frac{1}{m}\mathbb{E}\left[\pi_C^m\left(W_m = W_{m-1}\right)\right],$$

that is, an equal share of the expected collusive profit of the ring bidder with the highest signal, if he has a signal equal to the expected second-highest signal among ring bidders. (This expected profit is described in Section 5.)

2. Each ring bidder reports his signal to the ring center. Let $w_1, \ldots, w_m$ be respectively the lowest, ..., highest reported signal.

3. The ring member who reported the highest signal (and, hence, the highest valuation) is the designated bidder. He pays the ring center $\mathbb{E}\left[\pi_C^m\left(W_m = w_{m-1}\right)\right]$ (his expected collusive profit if he had a signal equal to the second-highest reported signal) and retains the prize if he wins the auction. The other $m-1$ ring bidders drop out of the auction at price zero.

4. The designated bidder bids according to the strategy $\beta = \left(\beta_0, \ldots, \beta_{n-m-1}\right)$ where β_k is the price at which he drops out given that k non-ring bidders have dropped out. This strategy is (implicitly) defined by:[26]

$$\beta_k = \mathbb{E}\left[V_m \,\middle|\, Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}\left(\beta_k\right);\ w_m, \ldots, w_1, z_k, \ldots, z_1\right], \tag{4.1}$$

$k = 0, \ldots, n-m-1$, where:

$$\psi_k\left(x_i\right) = \alpha_{k,m-1}^i\left(x_i; z_1, \ldots, z_k\right), \quad k = 0, \ldots, n-m-1.$$

Proposition 2. *Mechanism M is incentive-compatible and (ex-ante) budget-balanced.*

[26] In the proof of Lemma 1, we show that, when the ring adopts mechanism M, a non-ring bidder bids up to ψ_k after k non-ring bidders have dropped out of the auction, and that β is an equilibrium bidding strategy for the designated bidder. Notice that strategy β calls on the designated bidder to remain active up to the price at which he would be just indifferent between winning and losing the auction — i.e., up to his expected valuation conditional on winning, given the signals of all ring members and the information he can infer from the prices at which non-ring bidders drop out.

In mechanism M, before the auction each ring bidder receives from the ring center an equal share of the expected payment by the designated bidder to the ring center. In addition, the designated bidder retains the auction prize if he wins it and any additional profit (or losses) he obtains during the auction. As shown in the proof of Proposition 2, the mechanism is incentive compatible because the side payments made to and received from the ring center do not depend on the signal reported and, if other ring bidders report their signals truthfully, a bidder obtains positive expected profit by being chosen as the designated bidder if and only if he has the highest signal among ring bidders.

5. Effects of Collusion

Since the ring can design a mechanism to make each bidder truthfully report his signal, it can be assumed that the ring knows its members' signals. In this section, we analyze bidding strategies when the ring adopts mechanism M and show that collusion allows the designated bidder to win more often and pay a lower price. We say that a bidder bids more (less) aggressively in auction A than in auction B if the price at which he drops out is higher (lower) in auction A than in auction B.

Lemma 1. *When the ring adopts mechanism M, non-ring bidders bid less aggressively and the designated bidder bids more aggressively than in an auction without collusion.*

The intuition for this result is straightforward. When non-ring bidders who are unaware of collusion observe potential buyers dropping out at price zero, they infer that their signal is at most equal to $\underline{x}$. This induces them to reduce their estimate of the prize value and bid less aggressively. Given that non-ring bidders bid less aggressively, the designated bidder suffers a lower winner's curse if he wins the auction; hence, he can bid more aggressively.

Lemma 2. *Compared to an auction without collusion, when the ring adopts mechanism M: (i) the probability that the designated bidder wins the auction is higher, and (ii) conditional on winning the auction, the designated bidder pays a lower price.*

Without collusion, the designated bidder wins the auction if and only if he has the highest valuation. By contrast, the designated bidder can win even against a bidder who has a higher signal, and hence a higher valuation. Therefore, collusion may lead to an inefficient allocation of the auction prize.

The total profit ring bidders expects to obtain by adopting mechanism M is:

$$\mathbb{E}\left[\pi_C^m\right] = \mathbb{E}\left[\left(V_m - \psi_{n-m-1}\left(Z_{n-m}\right)\right) \cdot \mathbf{1}_{\{\beta_{n-m-1} > \psi_{n-m-1}(Z_{n-m})\}} \middle| w_1, \ldots, w_m\right], \tag{5.1}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. From Lemma 2, it follows that the ring increases its expected profit both by increasing the probability of winning the auction and by reducing the price paid.

Proposition 3. *By adopting mechanism M, the ring increases its expected profit (compared to an auction without collusion).*

The extra profit obtained by collusion depends on two different effects:

1. The *reduced competition* effect due to the fact that $m-1$ ring bidders do not bid positive prices.
2. The *signalling* effect due to the strategic behavior of ring bidders who drop out at price zero, making non-ring bidders bid less aggressively and the designated bidder bid more aggressively.[27]

The signalling effect only arises in an English auction with affiliated valuations. In fact, in other auction mechanisms bidders cannot observe their competitors' bid and hence infer their information. Moreover, when bidders' valuations are independent, bidders' strategies are not affected by their competitors' information. In both cases, bids lose their signalling content.[28]

The reduced competition effect does not affect the probability that the designated bidder wins the auction, it only increases his payoff, given that he wins. The previous literature on collusion in auctions concentrated on this first effect and neglected the potential advantage for ring bidders of strategically manipulating their bids. Moreover, by contrast to standard analysis that suggest that all players benefit from (or at least are not hurt by) collusion (because collusion reduces competition), in our model non-ring bidders who are unaware of the ring's presence are made worse off by collusion, because they are induced to bid less aggressively and this reduces their probability of winning the auction.

The actual (ex-post) extra profit of the ring is the given by the extra profit the designated bidder obtains by collusion, which depends on bidders' signals. When the designated bidder has the highest signal among all potential buyers, the ring gains by reducing the price paid for the object; when the designated bidder does not have the highest signal, the ring gains by giving him a chance to win the auction anyway.

6. Extensions

6.1. Seller's Strategy

Collusion reduces the efficiency when the prize is won by the designated ring bidder but he does not have the highest valuation. Moreover, with independent signals, collusion also

[27] The first effect reduces the expected price paid by the designated bidder from $\mathbb{E}\left[\alpha_{n-2}^{i}\left(Y_{n-1}\right)\right]$ to $\mathbb{E}\left[\alpha_{n-m-1}^{i}\left(Z_{n-m}\right)\right]$; while the second effect further reduces it from $\mathbb{E}\left[\alpha_{n-m-1}^{i}\left(Z_{n-m}\right)\right]$ to $\mathbb{E}\left[\psi_{n-m-1}\left(Z_{n-m}\right)\right]$.

[28] However, in Section 6.3 we show that with sequential auctions bids have a signalling content that can be exploited by colluding bidders even if valuations are independent.

reduces the expected auction price and the expected seller's revenue. To see this, notice that, with independent signals (and downward sloping *marginal revenues*),[29] an English auction with an appropriate reserve price maximizes the seller's revenue if bidders bid independently, because it sells to the bidder with the highest *marginal revenue* (Myerson, 1981; Bulow and Klemperer, 1996). But collusion among bidders modifies the allocation achieved by the auction since the prize need not be assigned to the bidder with the highest *marginal revenue*, and this reduces the expected seller's revenue.[30]

So a seller who wants to achieve an efficient allocation and maximize revenue should try to prevent bidders from joining a ring and, if he cannot do so, he should try to prevent colluding bidders from signalling to their opponents. For example, the seller could choose an auction mechanism in which bids are unobservable, like a second-price sealed-bid auction.

6.2. Non-Secret Rings with Almost Common Values

In this Section, we consider a simple model that allows us to analyze the effects of relaxing the assumption that non-ring bidders do not know they are facing a ring. In the pure common-value example of Section 2, if bidder 3 knows that bidders 1 and 2 collude, then she knows that she bids against a ring who shared its members' information on the value of the object and bids accordingly. So this is like an auction with two bidders who have signals x_3 and $x_1 + x_2$ respectively. The problem is that, in a pure common-value auction, there is a continuum of equilibria and, typically, a single equilibrium is only pinned down by assuming symmetry among bidders (Bikhchandani and Riley, 1991). But when a bidder knows she is facing a ring, there is an intrinsic asymmetry between the ring's information and bidder 3's information on the value of the object.[31] For instance, when $c = 0$, bidder 3 bidding up to tx_3 and a ring bidder bidding up to $\frac{t}{t-1}(x_1 + x_2)$ is an equilibrium of the auction, for every $t > 1$. However, there is a natural way to select a unique equilibrium by slightly perturbing this example.

Consider an *almost* common-value auctions with three bidders, in which bidders 1 and 2 join a ring and learn each other's signals. As in Bulow and Klemperer (2002), bidders' valuations are:

$$\begin{cases} V_1 = V_2 = (1+\varepsilon)(x_1 + x_2) + x_3, \\ V_3 = (1+\varepsilon)x_3 + x_1 + x_2, \end{cases}$$

where $\varepsilon \approx 0$. This represents a situation where a bidder places a slightly higher weight to a signal he knows before the auction starts.

[29] Letting $h_i(x_i)$ be bidder i's hazard rate, the *marginal revenue* of bidder i is defined as $V_i - \frac{1}{h_i} \cdot \frac{\partial V_i}{\partial x_i}$.

[30] For example, if signals are uniformly distributed on $[0, 1]$ and $c = 0$, the expected seller's revenue of the pure common-value auction of Section 2 is equal to $\frac{5}{4}$ without collusion, while it is equal to $\frac{11}{12}$ when two bidders collude.

[31] Levin (2004) analyzes joint bidding in a second-price auction by symmetric groups of bidders (i.e., groups composed by the same number of bidders).

An interpretation of these value functions is that information known before the auction starts is more valuable than information obtained during or after the auction (like a competitor's signal), because bidders are better able to exploit information they obtain earlier, and act upon it in order to earn higher profit. Another interpretation is that bidders actively collect information before starting the auction. In this case, when bidders choose what particular type of information to collect after joining a ring, they can focus on information that is better suited to their own specific use of the auction prize, and hence is more valuable than their competitor's information. For example, before an auction for a mobile-phone license, telecom firms usually conduct surveys of costumers in order to forecast future demand, and each firm's survey is also valuable for its competitors. But firms with different business plans conduct different surveys and attach different weights to their competitors' surveys: a firm that plans to focus on business customers will conduct a survey of those customers and will attach a lower weight to a survey made by another firm focused on residential customers.

Assume first that bidder 3 does not know that she is facing a ring. It is straightforward that, in equilibrium, bidder 3 starts bidding up to $(3+\varepsilon)\,x_3$ and, after a bidder drops out at price p, she bids up to $(2+\varepsilon)\,x_3 + \frac{p}{3+\varepsilon}$ (because she expects the bidder who dropped out to have signal $\frac{p}{3+\varepsilon}$). Therefore, if the ring adopts mechanism M and a ring bidder drops out at price zero, then bidder 3 bids up to $(2+\varepsilon)\,x_3$ while the remaining ring bidder bids up to $(2+\varepsilon)\,(x_1+x_2)$.[32] For $\varepsilon \to 0$, these bidding strategies converge to the equilibrium bidding strategies of the pure common-value example of Section 2.[33]

Suppose now that bidder 3 knows that her opponents joined a ring.

Lemma 3. *When bidder* 3 *knows she is facing a ring, in the unique linear equilibrium of the almost common-value auction bidder* 3 *bids up to* $(2+\varepsilon)\,x_3$ *and one ring bidder bids up to* $(2+\varepsilon)\,(x_1+x_2)$ *(while the other ring bidder does not participate in the auction).*

Notice that the equilibrium involves exactly the same bidding strategies as in the case in which bidder 3 does not know she is facing a ring. For $\varepsilon \to 0$, the almost common-value model selects a "natural" equilibrium for the pure common-value case.[34]

The intuition for this result is the following. If bidder 3 does not know that she is facing a ring, then after a bidder drops out at a low price she believes that bidder has a low signal,

[32] To see that this is an equilibrium, notice that if bidder 3 bids up to $(2+\varepsilon)\,x_3$, then when a ring bidder wins the auction at price p he knows the prize is worth $(1+\varepsilon)\,(x_1+x_2) + \frac{p}{(2+\varepsilon)}$; hence he is willing to stay in the auction up to price p^* such that $p^* = (1+\varepsilon)\,(x_1+x_2) + \frac{p^*}{(2+\varepsilon)}$.

[33] From the seller's point of view, even if allowing bidders to join a ring in this almost common-value setting slightly increases their valuation, it can still reduce revenue because it induces a non-ring bidder to bid less aggressively. Moreover, for $\varepsilon \approx 0$ the auction is always (almost) efficient, regardless of which bidder wins it.

[34] In this equilibrium, bidder 3 bids relatively cautiously and the ring bidder can bid quite aggressively. An interpretation is that, when the presence of a ring is common knowledge, bidder 3 knows she is competing against a bidder who is "advantaged" (since, on average, his valuation is $\varepsilon \cdot \mathbb{E}[x]$ higher than bidder 3's valuation) and, hence, has to bid cautiously to avoid the winner's curse.

which is bad news about the prize value. However, bidder 3 also believes that the other remaining active bidder is choosing to stay in the auction notwithstanding the fact that he also knows that the bidder who dropped out has a low signal. This means that the remaining active bidder has a high signal, and this is good news for bidder 3 about the prize value. On the other hand, if bidder 3 knows she is facing a ring, she makes none of the two inferences.[35] In our simple model, the bad and good news exactly cancel out, so that the outcome of the auction is the same whether bidder 3 believes there is a ring with probability 0 (but a ring is active) or with probability 1. Basically, when bidder 3 does not know she is facing a ring, the ring profits from misleading her strategy; while when bidder 3 knows she is facing a ring, the ring profits from bidder 3 knowing that her opponents shared information about the prize value.

Therefore, in this almost common-value auction, the ring can do just as well when bidder 3 knows she is facing a ring as when bidder 3 does not suspect that her opponents are colluding. Even if bidder 3 places some positive, but different from 1, probability on the existence of a ring in the auction, bidder 1 and 2 can credibly signal that they are colluding and obtain the same outcome as under our assumption.

6.3. Sequential Private-Value Auctions

In sequential private-value auctions, a ring of bidders can adopt a strategy similar to the one we have described for a single auction with affiliated values. Consider, as a simple example, a sequence of two English auctions for two identical prizes with three bidders. Each bidder i demands exactly one prize and has a privately known valuation v_i for each prize, $i = 1, 2, 3$.

Suppose there is no collusion. In the second auction, it is a dominant strategy for the two bidders who did not win the first auction to bid up to their valuation. In the first auction, bidders start bidding up to their valuation. After a bidder drops out at price p, the two remaining bidders learn their opponent's value and know they can win the second auction at price p. So they both drop out immediately of the first auction (and the prize is assigned randomly to one of them).

Suppose now that bidders 1 and 2 join a ring and that bidder 3 does not know they do. Moreover, assume that ring bidders know each other valuations and, without loss of generality, that $v_1 > v_2$. If bidder 1 drops out at price zero in the first action, this induces bidder 3 to bid less aggressively, because she expects to win the second auction at price zero if she loses the first one. So bidder 3 drops out immediately after bidder 1, and bidder 2 wins the first auction at price 0. In the second auction, bidder 2 does not participate and it is a dominant strategy for bidder 1 and bidder 3 to bid up to their valuations (even if bidder 3 is

[35] Of course, in both cases bidder 3 is worse off than in an auction without a ring, since in this last case bidder 3 makes the two inferences described (after a bidder drops out) *and* the remaining active bidder bids less aggressively than he does when he is part of a ring.

then "surprised" to see bidder 1 bidding more than zero).[36]

Therefore, the collusive strategy induces a competitor who is not aware of the presence of the ring to bid less aggressively in the first auction, as in a single-object auction with affiliated values, and this increases the probability that the ring wins the first auctions and reduces the price it pays. In our simple example, the ring always wins the first auction at price 0. Moreover, the ring also wins the second auction with a strictly higher probability than without collusion, since bidder 1, rather than bidder 2, competes with bidder 3 in the second auction. Indeed, for the ring to win both auctions it is sufficient that one of its members has a higher valuation than bidder 3.

In contrast to an auction with affiliated values, in sequential auctions a ring bidder who drops out at a low price sends a misleading signal about the intensity of competition in later auctions, rather than about the prize value.[37] As in our main model, collusion reduces efficiency and the seller's revenue. Finally, our analysis suggests that prices should increase in sequential auctions, when some (but not all) bidders collude.

7. Conclusions

Collusive behavior in auctions is arguably the main concern of auction designers and sellers. We have described how colluding bidders may strategically use bids to mislead their competitors (and the auctioneer) into believing that their valuation of the prize is very low. Collusion hurts outsiders and reduces the efficiency of an English auction.

During recent European 3G auctions, some bidders managed to convince governments and competitors that the licenses on sale were not profitable by bidding extremely low prices or by failing to participate altogether. Perhaps firms were trying to reduce competition in future auctions, improve their bargaining power with sellers, or induce more favorable trading conditions with suppliers or a more benevolent attitude from regulators. Many telecom firms have then tried to induce governments to relax rules that prevent them from owning two licenses or from sharing a 3G network.

But when bidders drop out of an auction at a very low price, they may not necessarily do it because they believe the prize is not worth it.

[36] Of course, a similar collusive strategy can be used even if the objects on sale are not identical, and their values are either positively or negatively correlated.

[37] Sequential (private-value) auctions have a common-value element given by the value of losing the first auction and winning the second one. In sequential auctions, as in our main model, a ring bidder who drops out of the first auction signals to his opponent that the value of losing the first auction is high.

A. Appendix

Proof of Proposition 2. We need to prove that the truthful revelation of their signals is an equilibrium for ring bidders. Notice that the side-payment received by a ring bidder from the ring center does not depend on the signal he reports and, hence, cannot affect incentives. Therefore, a ring bidder's report depends only on his expected payment to the ring center and his expected profit if he is chosen as the designated bidder.

In mechanism M, ring bidders actually participate in a second-price sealed-bid knockout auction whose prize is the right to be chosen as the designated bidder and to retain the auction prize if it is won by the ring. So the value of winning the knockout for a ring bidder is the expected collusive profit if he is the designated bidder, given all signals reported by ring bidders (which affect his valuation). This expected profit is increasing in a bidder's signal since, other things being equal, a bidder with a higher signal has a higher valuation, and hence he expects to obtain a higher collusive profit. And if he wins the knockout, a bidder pays the expected collusive profit if he had a signal equal to the second-highest reported signal, which does not depend on his report. This payment is lower than his expected collusive profit as the designated bidder if and only if his actual signal is higher than the second-highest reported signal. Therefore, if other ring bidders report their true signals, a bidder is pleased to win the knockout if and only if he has the highest signal among all ring bidders. This implies that it is an equilibrium for each ring bidder to report his signal truthfully.

In Section 5 we are going to prove that it is an equilibrium for the designated bidder to bid in the auction according to the strategy β — i.e., up to his expected valuation conditional on winning, given the ring information and the information he infers from the behavior of non-ring bidders. Other ring bidders drop out of the auction at price zero and cannot gain by deviating because they cannot win at a price lower than the expected valuation of the designated bidder, which is higher than their valuation (since the designated bidder has the highest signal).

It follows that mechanism M is incentive-compatible. The fact that M is (ex-ante) budget-balanced in expectation follows from the definitions of the side-payments made and received by the ring center. ■

Proof of Lemma 1. Since non-ring bidders are unaware of the presence of a ring, their bidding strategy is defined by Proposition 1. Therefore, after the $m-1$ ring bidders with the lowest signals drop out at price zero and k non-ring bidders drop out at prices $p_m \leq \ldots \leq p_{k+m-1}$, a non-ring bidder with signal x_i bids up to:

$$\begin{aligned} \psi_k(x_i) &= \alpha^i_{k,m-1}(x_i; p_m, \ldots, p_{k+m-1}) \\ &= \mathbb{E}\left[V_i \left| \begin{array}{l} X_i = Y_{n-1} = \ldots = Y_{k+m} = x_i, \\ Y_{k+m-1} = z_k, \ldots, Y_m \leq \underline{x}, \ldots, Y_1 \leq \underline{x} \end{array} \right. \right]. \end{aligned}$$

This is lower than the price at which she drops out when there is no collusion, that is if $m-1$ ring bidders do not all necessarily drop out at price zero.

After k non-ring bidder dropped out, if the last $n-m-k$ non-ring bidders all drop out at price p and the designated bidder wins the auction, his expected valuation is:

$$\mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p);\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right. \right],$$

because each of the $n-m-k$ non-ring bidder has signal $\psi_k^{-1}(p)$. Therefore, after winning at price p^*, the designated bidder's profit is positive if and only if:

$$p^* \leq \mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p^*)\, ;\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right.\right].$$

By the definition of β_k in (4.1), the designated bidder stays in the auction as long as the above inequality holds. Hence, the strategy β is a best reply to the strategies $\psi_k(.)$ of non-ring bidders.

By Proposition 1, after $m+k-1$ bidders dropped out, without collusion the designated bidder bids up to:

$$p_{m+k} = \mathbb{E}\left[V_m \left| X_i = Y_{n-1} = \ldots = Y_{m+k} = w_m;\ y_{m+k-1}, \ldots, y_1 \right.\right]. \tag{A1}$$

With collusion, the designated bidder's expected valuation when the price is p_{m+k} (after k non-ring bidders dropped out) is no lower than:

$$\mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p_{m+k})\, ;\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right.\right]. \tag{A2}$$

Notice that, since w_m is the highest signal among ring bidders, even without collusion the $m-1$ bidders with signals $w_1, \ldots, w_{m-1}$ drop out of the auction before the designated bidder. It follows that the expectations in (A1) and (A2) are conditioned on the same signals. Moreover, $\psi_k^{-1}(p_{m+k}) \geq w_m$ since, after observing $m-1$ bidders quit at price 0, a non-ring bidder must have a signal at least as high as w_m to be willing to remain active up to the same price at which the designated bidder with signal w_m is willing to remain active. Therefore, (A2) is greater than (A1): with collusion, at price p_{m+k} the valuation of the designated bidder is greater than p_{m+k} and, hence, he does not drop out of the auction. ■

Proof of Lemma 2. The probability that a buyer with signal w_m wins an auction with n potential buyers and no collusion is:

$$\Pr\left[w_m > y_{n-1}\right] = \Pr\left[\alpha^i_{n-2,0}\left(w_m; y_1, \ldots, y_{n-2}\right) > \alpha^i_{n-2,0}\left(y_{n-1}; y_1, \ldots, y_{n-2}\right)\right].$$

The probability that the designated bidder wins the auction when the ring adopts mechanism M (i.e., the probability that he bids higher than the $n-m$ non-ring bidders) is:

$$\Pr\left[\beta_{n-m-1} > \psi_{n-m-1}\left(z_{n-m}\right)\right].$$

The latter probability is greater than the former because:

(*i*) $\psi_{n-m-1}\left(z_{n-m}\right) < \alpha^i_{n-2}\left(y_{n-1}; y_1, \ldots, y_{n-2}\right)$ by Lemma 1 and the fact that $z_{n-m} \leq y_{n-1}$;

(*ii*) $\beta_{n-m-1} > \alpha^i_{n-2}\left(w_m; y_1, \ldots, y_{n-2}\right)$ by Lemma 1.

The second part of the statement follows from Lemma 1 and the fact that $m-1$ ring bidders drop out at price zero. ■

Proof of Lemma 3. Consider a generic equilibrium in linear and increasing bidding functions. Let the price at which the designated ring bidder drops out of the auction in equilibrium be:

$$h\left(x_1, x_2\right) = a + b\left(x_1 + x_2\right),$$

where a and b are two constants. To determine the values of a and b, we use the fact that equilibrium bidding functions must be reciprocal best replies.

If bidder 3 wins the auction at price p, then she expects the sum of the two ring bidders' signals $(x_1 + x_2)$ to be equal to $h^{-1}(p) = \frac{p-a}{b}$. In equilibrium bidder 3 bids up to the expected value of the prize conditional on winning. Therefore, she bids up to price p_3 such that:

$$p_3 = (1+\varepsilon)\,x_3 + \frac{p_3 - a}{b} \quad \Leftrightarrow \quad p_3 = \frac{b\,(1+\varepsilon)}{b-1}x_3 - \frac{a}{b-1}.$$

It then follows that, if the designated ring bidder wins at price p, he expects bidder 3's signal x_3 to be equal to $\frac{b-1}{b(1+\varepsilon)}\left[p + \frac{a}{b-1}\right]$. And in equilibrium the designated bidder bids up to the expected value of the prize conditional on winning. So he bids up to price p_1 such that:

$$p_1 = (1+\varepsilon)\,(x_1 + x_2) + \frac{b-1}{b\,(1+\varepsilon)}\left[p_1 + \frac{a}{b-1}\right] \quad \Leftrightarrow \quad p_1 = \frac{b\,(1+\varepsilon)^2}{b\varepsilon + 1}\,(x_1 + x_2) + \frac{a}{b\varepsilon + 1}.$$

In order for the function $h(.)$ to be an equilibrium bidding function, it must consistent with the above expression for p_1. Therefore, it must be that:

$$a = \frac{a}{b\varepsilon + 1} \quad \text{and} \quad b = \frac{b\,(1+\varepsilon)^2}{b\varepsilon + 1}.$$

The unique meaningful solution to these two equations is $b = 2 + \varepsilon$ and $a = 0$ (the other solution being $b = 0$).

An identical argument holds for the bidding function of bidder 3. Finally, notice that the other ring bidder can do no better than abstain form the auction, since bidder 3 would not make any inference from his bidding behavior. ■

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