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Alberto Bennardo[♥] **and Salvatore Piccolo**[♦]

Abstract

We study a general equilibrium model where agents' preferences, productivity and labor endowments depend on their health status, and occupational choices affect individual health distributions. Efficiency typically requires agents of the same type to obtain different expected utilities if assigned to di¤erent occupations. Under mild assumptions, workers with riskier jobs must get higher expected utilities if health a¤ects production capabilities. The same holds if health affects preferences and health enhancing consumption activities are sufficiently effective, so that income and health are substitutes. The converse obtains when health a¤ects preferences, but health enhancing consumption activities are not very effective, and hence income and health are complements. Competitive equilibria are first-best if lottery contracts are enforceable, but typically not if only assets with deterministic payoffs are traded. Compensating wage differentials which equalize the utilities of workers in different jobs are incompatible with ex-ante efficiency. Finally, absent asymmetric information, there exist deterministic cross-jobs transfers leading to ex-ante efficiency.

JEL Classification: D5, D61, D80, I18

Keywords: compensating wage differentials, competitive markets, individual health risks, Pareto efficiency

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Alberto Bennardo[†] and Salvatore Piccolo[‡]

January 24, 2007

Abstract

We study a general equilibrium model where agents' preferences, productivity and labor endowments depend on their health status, and occupational choices affect individual health distributions. Efficiency typically requires agents of the same type to obtain different expected utilities if assigned to different occupations. Under mild assumptions, workers with riskier jobs must get higher expected utilities if health affects production capabilities. The same holds if health also affects preferences and health enhancing consumption activities are sufficiently effective. The converse obtains if health mainly affects preferences and health enhancing consumption activities are *not* very effective. Competitive equilibria are first-best if lottery contracts are enforceable, but typically not if only assets with deterministic payoffs are traded. Compensating wage differentials which equalize the utilities of workers in different jobs are incompatible with ex-ante efficiency. Finally, absent asymmetric information, there exist *deterministic* cross-jobs transfers leading to ex ante efficiency. We fully characterize a class of simple policies implementing these transfers.

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1 Introduction

The paper studies a simple competitive environment where the aggregate distribution of health is endogenous, and is determined jointly with the allocation of labor and consumption goods. The model has the following key features. First, health affects agents' preferences, productivity and their labor endowments,

[*]We thank Pierre Andrè Chiappori, Dimitris Christellis, Jim Dana, Elena Del Mercato, David Levine, Annamaria Menichini, Joe Ostroy, Marco Pagano, Grazia Romano, Duncan Thomas, Bill Rogerson, Antonio Villanacci, Bill Zame and Lucy White for very useful discussions and comments. All remaining errors are ours.

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namely their consumption and production capabilities. Second, the health distribution of each worker depends on his occupational choice. Third, occupational choices are indivisible, that is each occupation is defined by an indivisible set of tasks, and each worker can choose at most one occupation together with the associated health distribution.[1] These features capture some of the most significant real-life effects of individual health status, and are intended to illustrate the determinants of occupational choices in the presence of endogenous health risks. The effects of health on workers' productivity, labor endowment, and preferences are largely documented by the empirical literature (see Rosen, 1986, and Viscusi, 1993, among others). Indeed, occupational choices generally have both direct and indirect effects on individual health prospects. By influencing the likelihood of work-related injuries and diseases, they directly affect the distribution of future health states. Moreover, they may also change workers' health risks indirectly by determining their location choices, for instance by inducing them to locate in less safe areas (i.e., more crime-ridden or with poorer health facilities). Finally, an important real-world feature of most health risks associated to production activities is that they are diversifiable only to a limited extent. This is due to a non-convexity associated to the specialization of labor, leading most workers to choose a single occupation.

Our analysis encompasses both direct and indirect effects of occupational choices on health risks. Specifically, we consider a production economy with a continuum of agents, where different distributions of health are associated to different occupations and health status affects individual behavior. Workers can also undertake loss-reduction activities (health enhancing consumption activities), and use competitive markets to exchange goods and transfer income across individual states.

At a more abstract level, we analyze a set-up where agents (workers) choose among indivisible risky assets (occupations) paying either monetary or non-pecuniary random returns (wages and health, respectively), whereas the latter are only imperfectly transferable (health status cannot be separated from individuals, and can be modified only within certain limits).[2] Hence, our conclusions have a broader scope, and although for the sake of clarity throughout we shall interpret individual risks as health shocks, they can shed light on the determinants of individual choices of indivisible risky assets, such as education, clubs' memberships, entrepreneurial activities, to name only a few.[3]

By illustrating the specific efficiency trade-offs generated by individual risks on preferences and productivity in the presence of occupational indivisibilities, the paper provides a novel and fairly general characterization of efficient allocations. We demonstrate that cross-jobs transfers are typically necessary for ex ante efficiency, and identify the determinants of these transfers. We then show that the direction of these transfers depends in a precise way on the relative magnitude of health effects on consumption

[1] This assumption is imposed for simplicity; in our setting it is sufficient that a worker cannot choose an arbitrarily large number of jobs and offer a small amount of labor in each of them.

[2] Noteworthy, individual risks and occupational indivisibilities are both key ingredients of our model. In a setting with indivisibilities, but state independent preferences and production funcions, the problem at hand would become much more standard and lose much of its appeal.

[3] Other examples of assets with these characteristics include occupations requiring human capital, jobs with unpleasant characteristics, as well as memberships to clubs and organizations. Several of the results of our analysis hold in settings where those assets are traded.

choices, labor choices, and on the effectiveness of available medical treatments. Finally, we investigate either the efficiency properties of alternative financial markets structures or the issue of implementation of efficient allocations through "simple policy schemes".

The paper is related to a vast literature[4] on work-related health risks and non pecuniary job attributes, focusing on wage premia commanded by risky, or otherwise unpleasant, jobs.[5] Such literature characterizes and estimates competitive wage differentials, under the "equilibrium condition" that workers of the same type assigned to different occupations obtain equal utility. This condition is generally derived through a partial equilibrium labor market analysis, or directly imposed as part of the definition of equilibrium. Moreover, the conventional wisdom within the literature on non pecuniary job's attributes is that *utility equalizing* wage differentials lead to market efficiency.[6] By contrast, we demonstrate that Pareto optimality typically requires workers of the same type to obtain different (expected) utility levels when assigned to different occupations. As a consequence, efficient allocations are not *budget balancing*, and require cross-jobs transfers; while wage differentials equalizing utilities across occupations typically do not implement first-best outcomes.

These findings hinge upon the imperfect transferability of health[7], which makes consumption and occupational (production) decisions interdependent. More precisely, they rely upon the following optimality argument. Because health risks are specific to occupations, and both preferences and productivity are state-dependent, ex-ante identical workers under different occupations will generally feature different expected utility functions and budget constraints, and hence different indirect utility functions. For this reason, the equalization of (expected) marginal (indirect) utilities of contingent goods (income) across agents, which is a standard ex-ante efficiency condition, typically prevents either interim efficiency with *equal treatment* (i.e., equalization of the utility of agents of the same type assigned to different jobs), or budget balancing.

The inconsistency between ex ante efficiency and budget balancing, besides being a novel theoretical contribution, raises a number of important theoretical and policy issues, which concern the properties of optimal cross-transfers and their implementation. Addressing these issues is one of our main goals.

The first part of the paper characterizes the Pareto frontier of the economy, and identifies the main determinants of cross-jobs transfers. By ordering the health risk of different occupations according to first-order stochastic dominance, we first show that efficiency requires either compensating wage differentials (occupations associated to worse health distributions command higher wages) or cross-sectoral transfers. Then, we demonstrate that the direction of optimal cross-transfers across occupations depends

[4]This literature goes back to Adam Smith (see Evans and Viscusi, 1993, Lucas, 1974, Rosen, 1986, Viscusi, 1990 and 1993, among many others).

[5]This literature formalizes the Smithian idea that "*the whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighborhood, be either perfectly equal or continually tending toward equality*".

[6]See, for instance, the textbooks of Ehrenberg and Smith (2003) and Viscusi et al. (2000). In a general equilibrium analysis, however, Cole and Prescott (1997), which study a moral hazard model, take a different perspective. The present paper has several connections with this article and with the asymmetric information literature.

[7]This imperfect transferability invalidates the separability result between individual consumption and production choices which is standard in welfare analysis (see Mas Colell et al., 1995).

on the impact of health on production choices and health enhancing consumption activities. The formal analysis relies upon supermodularity arguments, (i.e., upon the relationships of complementarity, resp. substitutability, between health, labor and consumption goods).[8]

Specifically, efficiency requires workers with riskier jobs to get higher expected utilities and positive transfers under mild conditions, if negative health shocks have a relatively strong effect on labor supply choices (on individual productivity or disutility of labor). The same holds if health enhancing commodities are sufficiently valuable in reducing health losses, so that health shocks substantially influence loss reduction investment activities. Conversely, workers with riskier jobs must obtain lower utilities and negative transfers if health mainly affects preferences (i.e., marginal utility of consumption), and has relatively negligible effects either on production capabilities or on the value of loss-reduction consumption activities.

The second part of the paper develops the competitive analysis. We study two alternative contractual regimes, one where lottery contracts (i.e., contracts with random payoffs) are enforceable and the other where they are unenforceable, possibly because of limited liability (debt) constraints. In the former, there exist competitive insurance markets to cope with all idiosyncratic risks[9] but only financial contracts with deterministic returns are enforceable. In the latter regime, agents can also "trade" lottery contracts, i.e., assets with random payoffs. The complete contracts' regime where lotteries are assumed to be enforceable turns out to be the natural benchmark for understanding the welfare properties of complete competitive markets. The analysis of the case of unenforceable lotteries, though, is warranted by both empirical and theoretical reasons. First, in real markets the use of lottery contracts (or that of other financial instruments making allocations obtainable through random contracts attainable) does not appear to be very widespread.[10] And, in line with real world circumstances, most of the literature on non-pecuniary job attributes, which is a natural reference point for the problem at hand, has only considered contracts with deterministic payoffs. On a theoretical ground, the use of random contracts may result severely restricted by moral hazard problems, driven by limited liability, *or by* the imperfect verifiability of characteristics and outcomes of the random devices needed for their implementation.

We show that an equilibrium exists in both contractual regimes. Moreover, equilibria are generically efficient if and only if lottery contracts are enforceable. Efficiency requires the expected marginal utility of income to be the same for ex ante identical agents employed in different sectors. Satisfying this condition, generically does call for an individual not to be indifferent over occupations to which he is assigned with positive probability. In the absence of lottery contracts, competition does not deliver ex ante efficiency. Equalizing the expected utilities of workers of the same type employed in different sectors, indeed, creates a wedge between marginal utilities.

Finally, we show that understanding the nature of the inefficiencies determined by the missing market

[8] Testable implications of different assumptions on preferences for consumption and health are derived by Rey and Rochet (2004), see also on this issue Evans and Viscusi (1990).

[9] See also Malinvaud (1973) and Cass, Chichilnisky and Wu (1996) among others.

[10] Kehoe, Levine and Prescott (2001) show that, if there exists a sufficient number of assets paying units of numeraire in sunspot states of the world, competitive equilibria are first-best efficient. In our setting, however, efficient trades of financial instruments leading to random allocations are typically such that workers must take possibly large *short positions* in the asset markets. This is often impossible in real-life markets also because of incentive problems.

problem at hand within our general equilibrium setting, permits to go considerably beyond the accomplishment of an efficiency test of competitive equilibria. By relying upon our characterization of Pareto optimal allocations, we demonstrate that, in the absence of asymmetric information, Pareto optima can be implemented through simple deterministic cross-transfers policies. These policies display two key features: They implement cross-subsidies among insurance contracts designed for workers choosing different occupations, and impose minimal wages aimed at ensuring a natural non-manipulability requirement of the policy scheme. The transfer received by a worker at the optimum is then determined by the difference between the (shadow) value of his consumption and that of his production and endowment, both calculated at the optimal shadow prices.

2 Related Literature

Our results are related to the general equilibrium literature on indivisibilities, to the literature on optimal taxation and to some important contributions of the literatures on asymmetric information and on clubs.

Rogerson (1988), who studies the general equilibrium effects of production indivisibilities, is the contribution to which our work is closest. This article, indeed, provides an example where random contracts implement transfers across workers, which could be reinterpreted as a form of unemployment insurance. Rogerson's results are derived for a very specific class of preferences, and assuming that a completely indivisible labor supply (agents can either work a fixed amount of time or remain unemployed) generates a positive unemployment rate in equilibrium. In our environment, we prove that random contracts are *almost always* necessary to achieve efficiency through the market, even if the amount of labor a worker must supply within an occupation is perfectly divisible. Moreover, differently from Rogerson, lotteries implement cross-transfers across occupations in our setting. And the determination of the sign of these transfers is at the core of our paper.

Our setting also shares some features with those analyzed in the literature on the optimal design of public health and education policies. The welfare effects of subsidies to education and health insurance are two important issues in this literature. Henriet and Rochet (2004) illustrate, within an optimal taxation framework, the role of public health insurance as a redistributive tool. In a close spirit, Diamond and Sheshiniski (1995) analyze the effects on labor supply of health disabilities and retirement benefits. They focus on the optimal structure of disabilities benefits and consider the problem of optimal evaluations of disabilities evidence in the presence of asymmetric information. De Fraja (2002) analyzes optimal education policies and uses supermodularity arguments to show that missing capital markets and externalities may lead to elitist (regressive) utilitarian cross-transfers policies. Similarly to De Fraja our work relies on monotone comparative statics techniques to characterize cross transfers, and shows that optimal policy may increase ex post inequality. However, there are two fundamental differences, among others, between De Fraja's contribution and our work. First we focus on ex ante efficiency instead of taking the utilitarian viewpoint. Second, we show that, under mild conditions, optimal policies entail positive transfers to the agents who are more likely to be ex post less productive (i.e., incurring in adverse health shocks with larger probabilities). Thus, on an applied ground, our results provide a new rationale for policy interventions in the health insurance market, which has not been identified by the previous literature and can be helpful in evaluating actual health insurance markets policies.

The general equilibrium asymmetric information literature, starting with the seminal contribution of Prescott and Townsend (1984) has focused on establishing appropriate versions of the classical Welfare Theorems in the space of lottery contracts.[11] Several examples have also been developed to show that lotteries can be welfare beneficial, either in the presence of adverse selection or moral hazard, because of their convexifying effects on incentive constraints.[12] With the exception of Bennardo and Chiappori, and Cole and Prescott, this literature however mainly focuses on ex post lotteries. More importantly, it is not aimed at characterizing efficient cross jobs transfers.[13]

Finally, the recent literature on clubs and firms (Cole and Prescott, 1997, Ellickson, Grodal, Scothmer and Zame, 1999, Makowski and Ostroy, 2005, Zame, 2005) deals with the complex issue of pricing institutions, firms, and occupations in general equilibrium settings. An important connection of our paper with this literature is that agents' occupational choices (choices of firms' memberships) directly affect their utility and hence their consumption choices. In our setting, we assume away complementarities between agents working in the same firm, opportunistic behavior, and the related externality problems. These simplifications allow to focus on the characterization of efficiency trade-offs and on the beneficial role of cross-transfers across occupations, which are generated by idiosyncratic uncertainty and indivisible occupations alone; two issues which are not addressed in the club literature. Our conjecture, based on the analysis of the present paper, is that the generic inconsistency between ex ante and interim optimality continues to hold in most of the settings studied in the clubs' and in the asymmetric information literature. A result in this spirit is obtained by Bennardo (2005), which characterizes optimal transfers in a moral hazard set-up where health effects are not considered, but occupations affect agents' consumption choices via incentive constraints.

3 The Economy

Demography, consumption goods and preferences: A continuum of measure 1 of consumers-workers produce C consumption goods. There exists a finite set, $I = \{1, ..., I\}$, of agents' types, and μ_i is the total fraction (measure) of *type i* agents. Agents face health risks that may affect their preferences, endowments and productivity. The set of possible health states, $\Theta = \{\theta_1, ..., \theta_N\}$, with $\theta_{n+1} \geq \theta_n$ for all n, is assumed to be finite, and $\theta \in \Theta$ represents a typical health state. In the economy there are $C + 1$ consumption goods, C produced goods and leisure. *Type i* agents have an endowment $e_i \in \Re^C_+$ of produced goods which

[11] For the analysis of competitive and efficient random allocations see also Allen and Gale (2003), Bennardo and Chiappori (2003), Bisin and Gottardi (2000), Kehoe, Levine and Prescott (2001), Rustichini and Siconolfi (2003), and Bennardo (2005) among others.

[12] See Arnott and Stiglitz (1986), Cole (1990), Garrett (1995), Kehoe, Levine and Prescott (2002).

[13] See however Bennardo (2005) for related results on cross-jobs transfers in a multicommodity production economy with moral hazard.

is constant across individual states, and an amount L of time which is allocated between work, l, and leisure, x_L. The maximal fraction of time that each agent can devote to work, $L(\theta)$, may depend on his health state; and $L(\cdot)$ is weakly increasing in θ.[14] Agents' preferences are assumed to be state (health) dependent and are represented by the utility function $U_i(x, \theta) : \Re_+^C \times [0, L] \to \Re$. We shall assume that U_i is n times differentiable, strictly concave and weakly increasing for all θ.

To clearly see that our utility representation allows for a subset of commodities to have health enhancing effects, one can formally introduce a variable, $\hat{\theta}$ representing *actual health conditions*; the impact of consumption choices on health conditions can then be captured by the real valued function $\hat{\theta} = \rho(x, \theta)$ of consumption, x, and initial health, θ. The utility representation of agents' preferences consistently becomes $U_i\left(x, \theta\right) \equiv \hat{U}_i(x, \rho(x, \theta))$. Throughout, we shall use such a representation whenever convenient.

Finally, let $\hat{C}(\theta) \subseteq C$ denote the subset of commodities whose consumption provides strictly positive (marginal) utility in the state θ. In proving existence, we shall assume that the indifference surfaces of U_i have no intersection with the axes of the Euclidean space $\Re_+^{\hat{C}(\theta)}$ corresponding to the subset of commodities $\hat{C}(\theta)$. We shall also occasionally impose $D_c U_i(x, \theta) > K$ as $x_c \to 0$, with $K > 0$ sufficiently large, and $D_c U_i(x, \theta) < k$, as $x_c \to \infty$, with $k > 0$ and small, for $c \in \hat{C}(\theta)$.

Technologies and uncertainty: Competitive firms produce goods by employing workers, and labor is the only production factor. Firms can hire positive measure of agents, while each worker can supply labor in at most one firm, as specialization prevents workers from performing different jobs. There are $T = C$ production sectors, and only one type of occupation within each sector. The productivity of each single worker is measurable and may depend on his health. Precisely, a *type i* worker who is employed in *sector* t and supplies l_i^t units of labor produces $y_i^t\left(\theta\right) = a_i^t(\theta) l_i^t$ units of commodity t in the health state θ, with $a_i^t(.)$ weakly increasing in θ.

The distribution of health of a *type i* agent working in *sector t* is $\left\langle p_i^t, \Theta \right\rangle$, with $p_i^t = (p_i^t(\theta_1), ..., p_i^t(\theta_N))$. Finally, health shocks are identically and independently distributed across *type i* workers in the same occupation, and independently distributed across sectors. The endogeneity of the health distribution can be seen as a consequence of the direct effects of labor activities on prospective workers' health; but it can also result from localization choices induced by labor activities.

Timing: The economy lasts two periods, $\tau = 0, 1$; at $\tau = 0$, agents trade in financial and labor markets. At $\tau = 1$, health shocks are realized; subsequently agents supply labor, and consumption goods are traded and consumed. The space of enforceable contracts will be defined in Section 5. For notational simplicity, we restrict attention to economies where all agents work in equilibrium, and use the following notation: $x_i^t(\theta)$ is a generic state contingent consumption vector of a *type i* agent employed in *sector t*, with $x_i^t = (x_i^t(\theta))_{\theta \in \Theta}$, and $x = \left(x_1^t\left(\theta\right), ..., x_I^t\left(\theta\right)\right)_{\theta \in \Theta}^{t \in T}$; $l_i^t = \left\{l_i^t(\theta)\right\}_{\theta \in \Theta}$ is the vector of state contingent

[14] This assumption is intended to capture real-life situations where a worker can perform with an appropriate quality standard a labor activity only for a limited amount of time. And the length of this time interval depends on his health state. For instance aircraft pilots, in order to guarantee appropriate safety standards, cannot fly more than a pre-specified number of hours per week. Similarly, a driver, a sportsman or a miner, who typically suffer of overuse syndromes cannot safely perform certain risky activities more than a certain number of hours in a year.

labor for a *type* i agent occupied in *sector* t. Finally, let $\alpha_i = (\alpha_i^1, ..., \alpha_i^T)$, with $\sum_{t=1}^T \alpha_i^t = 1$, represent an assignment of *type* i workers to production sectors, and $\alpha = (\alpha_i)_{1=1}^I$.

4 Ex-ante and Interim Pareto Optimality

Ex-ante Pareto Optimality: Let $u_i^t(x_i^t) = \sum_{\theta\in\Theta} p_i^t(\theta)\, U_i(x_i^t(\theta), \theta)$ and $\bar{x}_{ic}^t = \sum_{\theta\in\Theta} p_i^t(\theta) x_{ic}^t(\theta)$. By the law of large numbers, a *feasible allocation* of consumption goods and workers, $\langle x, \alpha\rangle$, is defined by the following constraints:

(1)
$$\sum_{i\in I}\mu_i \sum_{t\in T} \alpha_i^t \bar{x}_{ic}^t \leq \sum_{i\in I}\mu_i (e_{ic} + \alpha_i^c y_i^c), \quad \forall\, c \in C$$

(2)
$$l_i^t(\theta) + x_{iL}^t(\theta) = L, \;\; l_i^t(\theta) \leq L(\theta), \forall\, \theta \in \Theta, \; t \in T; \qquad \sum_{t\in T} \alpha_i^t = 1, \quad \forall\, i \in I$$

where $y_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta) l_i^t(\theta)$ for all t and i. Denote F the set of feasible allocations, and:

$$U = \left\{ \bar{u} = (\bar{u}_2, ..., \bar{u}_I) \in \Re^{I-1} : \exists\, (x,\alpha) \in F, \; s.t. \; \sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i, \quad \forall\; i = 2, .., I \right\}$$

A (ex-ante) Pareto optimum maximizes $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$, subject to $\langle x, \alpha\rangle \in F$, and $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for $i = 2, ..., I$ and $\bar{u} \in U$.

According to this definition, all *type* i agents face the same probability of being assigned to each occupation; however, they do not necessarily get the same expected utility if assigned to different occupations. Such a condition typically holds in the optima of convex economies; in our setting, though, there is no reason to impose it as part of the definition of first-best allocations. Finally, the definition of a Pareto optimum above rules out the possibility that an agent obtains a random consumption vector in the optimum conditionally on being assigned to a given occupation. Risk-aversion makes this assumption unrestrictive.

Interim Pareto Optimality: The following definition of interim Pareto optimality will play a central role in the welfare analysis of equilibria with unenforceable lottery contracts.

An *interim optimal allocation with equal treatment* maximizes $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$ subject to : $\langle x, \alpha\rangle \in F$, $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for $i = 2, ..., I$, $\bar{u} \in U$, and to the additional set of constraints $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for all pairs (t, t'), with $t \neq t'$, such that $\alpha_i^t > 0, \; \alpha_i^{t'} > 0$.

5 Competitive Equilibria

We shall now define competitive equilibria by assuming that there exist markets for all consumption goods, as well as financial markets for insuring *all* risks through assets with *deterministic* payoffs. We study either the case where only deterministic contracts (assets with random payoffs) are *enforceable* or that in which agents can also sign lottery contracts. Considering both cases is useful to fully understand either the beneficial role that random contracts may play in our economy, or the effects of a somewhat natural market friction that may prevent their use.

5.1 Competitive Equilibrium with *Deterministic Contracts*

Following the approach taken in several contributions of the literature on individual risks (see Malinvaud, 1973, among others), we assume that competing, risk-neutral intermediaries offer securities paying in individual states.[15] Specifically, security payoffs may be contingent on agents' type, occupations and individual health. Let $h_{i\theta}^t$ be a security paying to a *type* i agent employed in the t-th sector one unit of numeraire in his individual health state θ, and zero otherwise. Denote $z_{i\theta}^t$ and $\hat{z}_{i\theta}^t$ the units of $h_{i\theta}^t$ purchased by *type* i agents employed in *sector* t, and the per capita units of this security offered in the market, respectively. Finally, define $\phi_i^t(\theta)$ the unit price of $h_{i\theta}^t$. Production firms and agents trade at linear prices. Let $w_i^t(\theta)$ denote the state contingent wage of *type* i workers in the t-th occupation, with $w_i^t = (w_i^t(\theta))_{\theta\in\Theta}$.[16] And denote $q = (..., q_c, ...) \in \Re_+^C$ a generic vector of spot prices.[17]

Because of labor supply indivisibilities, it is expositionally convenient[18] to consider the possibility that workers choose their occupation by using mixed strategies. To this end, let $\varphi_i = (\varphi_i^1, .., \varphi_i^t, ..., \varphi_i^T) \in \Delta^T$ a generic probability vector according to which a *type* i worker mixes on occupations. Then, by the law of large numbers, φ_i^t is also the fraction of *type* i agents who are employed in *sector* t in equilibrium.

A competitive equilibrium with deterministic contracts is an allocation $(x_i^{t*}, \varphi_i^{t*})_{i\in I}^{t\in T}$, a collection of vectors $(\hat{z}_i^{t*}, z_i^{t*})_{i\in I}^{t\in T}$ and a vector of state contingent prices $(q, \phi_i^t, w_i^t)_{i\in I}^{t\in T}$ satisfying the following conditions.

(I) *Type-i agents maximize utility:*

(3) $$\left(x_i^{t*}, \varphi_i^{t*}, z_i^{t*}\right)_{t\in T} \in \arg\max_{\varphi_i\in\Delta^T.} \sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$$

(4) $$s.t. \quad \sum_{c\in C} q_c(x_{ic}^t(\theta) - e_{ic}) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta), \quad \forall\, \theta \in \Theta,\ t \in T$$

(5) $$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \le 0, \quad l_i^t(\theta) \le L(\theta), \forall\, \theta \in \Theta,\ t \in T.$$

[15] As it is conventional, intermediaries' risk-neutrality may be justified by the law of large numbers.

[16] The introduction of individual risks in a competitive settings requires assets' payoffs to be contingent either on individual shocks or on types; this point has been clarified in the seminal contribution of Malinvaud (1973).

[17] In the absence of aggregate uncertainty, spot market prices are independent from the realizations of individual shocks, as these shocks wash-out in the aggregate.

[18] It should be clear in the following that for any given equilibrium in mixed strategies, there exists a payoff equivalent equilibrium with pure strategies.

where (4) and (5) are the spot market and initial period budget constraints.

(II) *Production firms and intermediaries set:*

$$l_i^{t*} \in \arg\max \sum_{\theta\in\Theta} p_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta)\, l_i^t(\theta)\right) \quad \text{s.t.} \quad y_i^t(\theta) \le a_i^t(\theta)\, l_i^t(\theta), \quad \forall\, \theta \in \Theta,\ t \in T \tag{6}$$

$$\widehat{z}_i^{t*} \in \arg\max \sum_{\theta\in\Theta} (\phi_i^t(\theta) - p_i^t(\theta))\widehat{z}_i^t(\theta) \quad \text{s.t.} \quad \sum_{\theta\in\Theta} p_i^t(\theta)\,\widehat{z}_i^t(\theta) \ge 0, \quad \forall\, t \in T,\ i \in I \tag{7}$$

(III) *Consumption, labor and financial markets clear:*

$$\sum_{i\in I}\mu_i \sum_{t\in T}\varphi_i^{t*}\bar{x}_{ic}^{t*} = \sum_{i\in I}\mu_i(e_{ic} + \varphi_i^{c*}y_i^c), \quad \forall\, c \in C \tag{8}$$

$$x_{iL}^{t*}(\theta) = L - l_i^{t*}(\theta); \qquad z_i^t(\theta) = \widehat{z}_i^{t*}(\theta), \quad \forall\, \theta \in \Theta,\ t \in T \text{ and } i \in I \tag{9}$$

5.2 Competitive Equilibrium with *Lottery Contracts*

We now introduce lottery contracts. We shall assume that agents buy lotteries (assets with random payoffs) from financial intermediaries before making any other market trade. Following Arnott and Stiglitz (1987), these lotteries will be referred to as *ex-ante random contracts*. Formally, a lottery contract, $\mathcal{C} = ((\gamma, G), \rho(\gamma, G))$, is: *(i)* a finite distribution (γ, G) with probabilities $\gamma = (\gamma^1, ..., \gamma^M) \in \Delta^M$ and payoff support $G = (g^1, ..., g^M) \in \Re^M$, with M finite; and *(ii)* a price $\rho(\gamma, G) \in \Re$. The interpretation is that an agent signing $\mathcal{C}$ pays the price $\rho(\gamma, G)$ to the intermediary, and obtains the right to receive the payoff g^m with probability γ^m. A random devise, whose characteristics are publicly verifiable, is used to determine the contractual obligations of the parties signing $\mathcal{C}$. Such a device chooses an *artificial state of the world* by selecting a positive integer $m \in \{1, ..., M\}$ with probability γ^m. Subsequently, the intermediary pays (receives) g^m to the agent whenever the integer m is selected. The expected profit an intermediary earns from $\mathcal{C}$ is $\rho(\gamma, G) - \sum_{m\in M}\gamma^m g^m$.

A general formulation of the competitive equilibrium in the space of random allocations would require all possible lottery contracts (an infinite set) to be priced in equilibrium (see Rustichini and Siconolfi, 2003) and should take into account the possibility that an agent signs several lottery contracts. In order to avoid the technical difficulties arising in working with an infinite dimensional commodity space, as well as a more complex notation, we impose the following unrestrictive assumptions: (i) only the set of fair lottery contracts with payoff support of dimension $M = T$ are offered in the market[19]; (ii) each agent can sign at most one lottery contract; and (iii) will offer labor in *sector* t if and only if he receives the t-th payoff of his lottery contract.

[19] Consistently with the definition of lottery contracts, some or even all of its payoffs may be zero.

A standard arbitrage argument justifies (i). Assumption (ii) is unrestrictive since any finite distribution of net payoffs obtainable by means of N fair lottery contracts can also be achieved through a single fair contract;[20]and moreover, by risk aversion, it is always individually optimal to choose a contract with at most $M = T$ payoffs, different from zero. Intuitively, this is because a risk averse agent, conditionally on being assigned to a given production sector, will always prefer a certain payoff, $\hat{g}$, to a non-degenerate lottery, (γ, G), with an expected payoff equal to $\hat{g}$.[21] Finally, (iii) amounts to be a convenient notational convention once (ii) is imposed.

A competitive (Walrasian) equilibrium with lottery contracts is then an allocation $(\tilde{x}_i^t)_{i\in I}^{t\in T}$, a collection of vectors $(\tilde{z}_i^t, \tilde{z}_i^t)_{i\in I}^{t\in T}$, a vector of lottery contracts $(\mathcal{C}_i)_{i\in I}$, and a vector of prices $(\tilde{q}, \tilde{\phi}_i^t, \tilde{w}_i^t)_{i\in I}^{t\in T}$ satisfying the following conditions:

(I) *Type-i agents maximize utility:*

$$(\tilde{x}_i^t, \tilde{z}_i^t, \mathcal{C})_{t\in T} \in \arg\max_{\mathcal{C}_i\in\Gamma} \sum_{t\in T} \gamma^t u_i^t(x_i^t) \tag{10}$$

$$s.t. \quad \sum_{c\in C} q_c(x_{ic}^t(\theta) - e_{ic}) = w_i^t(\theta)\,(L - x_{iL}^t(\theta)) + z_i^t(\theta) + g^t - \rho(\gamma, G), \quad \forall\, \theta \in \Theta,\ t \in T \tag{11}$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \leq 0,\ \forall\, t \in T \tag{12}$$

where (11)-(12) are the first and second period budget constraints, and

$$\Gamma = \left\{ ((\gamma, G), \rho(\gamma, G)) : \rho(\gamma, G) = \sum_{t\in T} \gamma^t g^t \right\}$$

is the set of all fair lottery contracts.

(II) *Production firms and intermediaries solve programs (6) -(7), respectively.*[22]

(III) *Consumption, financial and labor markets clear:*

$$\sum_{i\in I} \mu_i \sum_{t\in T} \widetilde{\gamma}^t \widetilde{\widetilde{x}}_{ic}^t p_i^t(\theta) = \sum_{i\in I} \mu_i (e_{ic} + \widetilde{\gamma}^c \widetilde{y}_i^c), \quad \forall\, c \in C \tag{13}$$

[20]Precisely, such a contract is defined by a vector of probabilities and a vector of payoffs which are linear combinations of the probabilities and the payoffs of the N fair lottery contracts.

[21]More precisely, it is never optimal for a risk averse agent to choose a lottery contract such that: (i) he receives the payoffs g^m and $g^{m'}$, with $g^m \neq g^{m'}$, with positive probabilities γ^m and $\gamma^{m'}$ respectively, and (ii) he chooses to work in *sector t* either when he receives g^m or $g^{m'}$. By convexity, indeed, there exists another fair contract, say $\mathcal{C}'$, which pays $\gamma^m g^m + \gamma^{m'} g^{m'}$ with probability $\gamma^m + \gamma^{m'}$, which, conditionally on working in *sector t*, is strictly preferred to $\mathcal{C}$.

[22]This is exactly as in the competitive equilibrium with deterministic contracts.

$$(14) \qquad \tilde{x}_{iL}^{t}(\theta) = L - \tilde{l}_{i}^{t}(\theta); \qquad \tilde{z}_{i}^{t}(\theta) = \hat{z}_{i}^{t}(\theta), \quad \forall\, \theta \in \Theta,\ t \in T \text{ and } i \in I$$

6 Pareto Optimal Allocations

This section characterizes first-best allocations. Let $\lambda = (\lambda_2, ..., \lambda_I)$ and $\eta = (\eta_1, ..., \eta_C)$ be the vectors of Lagrange multipliers associated to the utility constraints, $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for $i = 2, ..., I$, and the feasibility constraints, respectively. Setting $\lambda_1 = 1$, the first-order conditions with respect to $(x_i^t(\theta), x_{iL}^t(\theta), \alpha_i^t)$ of the (ex-ante) Pareto program are:

$$(15) \qquad \lambda_i D_c U_i(x_i^t(\theta), \theta) - \eta_c \mu_i \leq 0, \quad \forall\, c \in C,\ \theta \in \Theta,\ t \in T \text{ and } i \in I$$

$$(16) \qquad \lambda_i U_{ix_L}(x_i^t(\theta), \theta) - \eta_t a_i^t(\theta) \mu_i \leq 0, \quad \forall\, \theta \in \Theta,\ t \in T \text{ and } i \in I$$

$$(17) \qquad \lambda_i(u_i^t(x_i^t) - u_i^{t'}(x_i^{t'})) - \mu_i(Z_i^t - Z_i^{t'}) = 0,\ \forall\ (t, t') \text{ such that } (\alpha_i^t, \alpha_i^{t'}) > 0 \text{ and } i \in I$$

where (15) and (16) hold with equality whenever $x_{ic}^t(\theta) > 0$ and $x_{iL}^t(\theta) > 0$[23]; and where:

$$Z_i^t = \sum_{c\in C, \theta\in\Theta} \eta_c \left(p_i^t(\theta) x_{ic}^t(\theta) - e_{ic}\right) - \eta_t \sum_{\theta\in\Theta} p_i^t(\theta) a_i^t(\theta) \left(L - x_{iL}^t(\theta)\right), \quad \forall\, t \in T,\ i \in I$$

is the difference between the value of the consumption of a *type* i worker employed in *sector* t and that of the sum of his endowment and his production, both evaluated at the vector of shadow prices η. In other words, Z_i^t represents the value of the net transfer received in the optimum by a *type* i agent assigned to *sector* t.

As standard, (15) and (16) imply the equality of marginal rates of substitution between state contingent commodities across types. The first-order conditions with respect to α in equation (17) are less standard, and play a crucial role in our analysis. They indicate that the differences in expected utilities across occupations, $\Delta u_i(t, t') = u_i^t(x_i^t) - u_i^{t'}(x_i^{t'})$, are proportional to $\Delta Z_i(t, t') = Z_i^t - Z_i^{t'}$. Noteworthy, only if $\Delta Z_i(t, t') = 0$ for all i, all workers assigned to the occupations t and t', respectively, will get the same utility, and ex-ante and interim optima coincide.

Let $\mathcal{F}(.) = 0$ denote the system of equations (15)-(17). The next proposition shows that $\Delta u_i(t, t')$ typically differs from zero at the solution of $\mathcal{F}(.) = 0$, implying that interim efficiency is generally incompatible with ex ante efficiency. This is, indeed, the distinguishing feature of our environment.

In order to prove the result, we need to introduce some notation. Let $t_i = \left(\left\langle p_i^t, \Theta\right\rangle, A_i^t\right)_{t\in T}$, with $A_i^t = \left\{a_i^t(\theta_1), ..., a_i^t(\theta_N)\right\}$, be the *sector* t technology available to *type* i workers. And let $\varepsilon = \langle e, \mathbf{t}, U\rangle$ represent

[23] For simplicity we neglect the case where $x_{iL}^t(\theta) = L$ in stating the first-order conditions. Implicitly, we assumed $D_{x_L} U_i$ sufficiently small at $x_{iL}^t(\theta) = L$.

a specific economy defined by an aggregate endowment $e \in \Re^C_{++}$, a vector of production technologies $\mathbf{t} = (t_1, ..., t_I)$ and a profile of utility functions $U = (U_i, ..., U_I)$. The set of possible economies is then defined as $\mathcal{E} = \Re^C_{++} \times \mathcal{T} \times \mathcal{U}$, where $\mathcal{T}$ is the set of all possible technologies, and $\mathcal{U} = \prod_{i=1}^{I} \mathcal{U}_i$, where $\mathcal{U}_i$ is the set of *type i* admissible utility functions, which will be precisely defined in the Appendix.

Proposition 1 *A unique Pareto optimum is associated to each vector of reservation utilities,* $\bar{u}$*. Moreover, the subset of economies* $\mathcal{S} \subset \mathcal{E}$ *such that ex ante and interim Pareto optima are disjoint is generic in* $\mathcal{E}$*, if the number of produced goods is larger than the number of agents' types and* $p_i^t(\theta) \neq p_i^{t'}(\theta)$ *for at least a type i worker, an health state* θ *and a pair* (t, t')*.*

The proof of this result as well as all the subsequent ones are provided in the Appendix. It uses a transversality argument in order to prove that the set of solutions of $\mathcal{F}(.)$ typically does not satisfy the interim efficiency constraints.[24]

Intuitively, ex ante efficiency mandates the equalization of marginal rates of substitution across all workers. Typically, however, any pair of ex-ante identical workers with different occupations generally feature different expected utility functions and technological constraints, since health distributions are occupation specific. As a consequence, for agents of the same type assigned to different jobs, the equalization of the marginal utilities of contingent goods (*margins*) prevents that of the that of expected utilities (*levels*).

Proposition 1 has two simple but very important corollaries. First, ex ante efficiency typically requires transfers of resources across workers assigned to different occupations, since the first order conditions of the Pareto programs imply that budget balancing obtains only when utility levels are equalized. Moreover, theses transfers are implemented through a random allocation of workers across occupations. Second, compensating wage differentials which equate (expected) utilities of workers assigned to different sectors are typically incompatible with first-best efficiency.

The next proposition shows that the Pareto shadow wages, $\eta_t^P a_i^t(\theta)$, associated to technologies inducing riskier health distributions, in the sense of First-Order Stochastic Dominance (FOSD), are relatively higher in the optimum. This an important result since it unveils that efficiency still commands higher wages to be paid in riskier occupations, even though wage differentials do not equalize utilities.

Definition *For any pair of health distributions,* $\langle p_i^t, \Theta \rangle$ *and* $\left\langle p_i^{t'}, \Theta \right\rangle$*, we shall say that* $\langle p_i^t, \Theta \rangle$ *FOSD* $\left\langle p_i^{t'}, \Theta \right\rangle$ *if* $\sum_{\theta \leq \theta_n} p_i^t(\theta) \leq \sum_{\theta \leq \theta_n} p_i^{t'}(\theta), \forall\, \theta_n \in \Theta$*, with at least one strict inequality.*

Proposition 2 *If* $\langle p_i^t, \Theta \rangle$ *FOSD* $\left\langle p_i^{t'}, \Theta \right\rangle$*, then* $\eta_t^P < \eta_{t'}^P$ *for all Pareto optima such that* $\alpha_i^{tP} > 0$ *and* $\alpha_i^{t'P} > 0$ *for at least one type* $i \in I$*.*

While its proof is not immediate, the proposition has a clear intuition. Were shadow wages, and thus Lagrange multipliers, equal across two sectors whose associated health distribution are ordered according

[24] Notice that the inconsistency between ex ante and interim efficiency stated in Proposition 1 holds only under the assumption that health distributions are endogenous.

to FOSD, the marginal utilities of consuming the goods produced in those sectors would also be equal for all agents in all states. It would then be welfare enhancing to move a fraction of workers from the riskier to safer sector, and to marginally increase (resp. decrease) the consumption of the good produced in the safer (resp. riskier) sector for all agents. Roughly, this would leave unaltered the utility agents can get from consumption but would increase their ex ante expected health.

Finally, Proposition 3 below, which is a direct corollary of Charateodory Theorem, states that all efficiency gains obtainable by random assignments of workers to occupations can also be achieved through a randomization involving only two occupations for each worker. This finding allows to simplify the efficiency analysis performed in the next section.

Proposition 3 *Given any Pareto optimal allocation* $\langle x^P, \alpha^P \rangle$ *such that* $\sum_{t \in T} \alpha_i^{tP} u_i^t(x_i^{tP}) = u_i$*, there exists a pair* (t, t') *such that* $\hat{\alpha}_i^{t,t'} u_i^t(x_i^t) + (1 - \hat{\alpha}_i^{t,t'}) u_i^{t'}(x_i^{t'}) = u_i$*, with* $\hat{\alpha}_i^{t,t'} = \alpha_i^t / (\alpha_i^t + \alpha_i^{t'})$*.*

Proposition 3 permits to restrict attention without loss of generality to *two-sectors* economies in the analysis of the determinants of wage and utility differentials across sectors, and of Pareto optimal cross-jobs transfers. This is because, for each worker, the first-order optimality condition $\lambda_i(u_i^t(x_i^t) - u_i^{t'}(x_i^{t'})) - \mu_i(Z_i^t - Z_i^{t'}) = 0$ imposes restrictions on $\Delta u_i(t, t')$ and $\Delta Z_i(t, t')$ for at most a pair (t, t').

6.1 Ex-Ante Efficiency and Optimal Cross-Jobs Transfers

In this section we study how the effects of health shocks on preferences, endowments, and productivity contribute to determine optimal cross-jobs transfers, as well as the differences between the utilities obtained by workers of the same type assigned to different technologies. For this purpose, we shall assume that occupations differ in their *health riskiness,* and impose that the health distributions associated to different occupations are ordered by the FOSD criterion. Moreover, merely for expositional purposes, we shall study a simplified setting where two goods are produced by a representative agent. Accordingly, we assume that *occupation 1* is safer than *occupation 2* in the sense that $\langle p^1, \Theta \rangle$ FOSD $\langle p^2, \Theta \rangle$, meaning that the likelihood of better health states is higher in occupation 1 relative to occupation 2.[25] Finally, in order to distinguish the effects of health status on the utility of produced consumption goods from that on the disutility of labor, we shall use the following separable utility representation:[26]

$$U(x, \theta) = \hat{U}(x, \rho(x, \theta)) = f(x_1, x_2, \theta) - \psi(l, \theta),$$

where $U(x, \theta)$ satisfies all the assumptions in Section 2, and where $f(x_1, x_2, \theta)$ and $\psi(l, \theta)$ represent the utility of consumption commodities and the disutility of labor, $l = L - x_L$, respectively. According

[25] Focusing on an economy with only two occupations and a representative agent is without loss of generality for the purpose of investigating the determinants of cross transfers and utility differentials across jobs. Indeed, as proved in Proposition 3, all efficiency gains obtainable by random assignments of workers to occupations can also be achieved through a randomization involving only two occupations for each worker.

[26] Separability is imposed only for the sake of tractability. Most of our results extend immediately to the case where consumption goods affect the labor disutility ψ, and these effects are bounded relatively to the health effects on marginal utility of consumption goods and leisure.

to this formulation, both f and ψ may possibly depend on θ. Moreover, in order to derive a sharper characterization of Pareto optima we also impose the following restrictions:

(A1) All derivatives of $U(.)$ are bounded.

(A2) $a^t(\theta) = a(\theta)$ for all t.

(A3) $U(., \theta)$ is supermodular in x for all θ, and $\psi_{l\theta} \leq 0$ for all (x, θ).

A1 is actually almost unrestrictive since the bounds on the derivatives of U are allowed to be arbitrarily large. Assuming that $a^t(\theta)$ is invariant across sectors is an innocuous normalization whenever $a^t(\theta_1) = 0$ for all t (i.e., whenever workers are unproductive in the worst health state). Supermodularity is a simplifying assumption.[27] Finally, as health is typically an input for production, imposing $\psi_{l\theta} \leq 0$ seems a really sensible restriction.

For the sake of readability, in the next sections we shall consider separately how health effects on pure consumption choices, treatments' decisions and labor choices determine the properties of optimal allocations and transfers. We study first a pure consumption economy where neither health nor consumption goods can be "produced", so that adverse health shocks only affect preferences by reducing their marginal utility of consumption. We then turn to the case where agents can undertake health enhancing consumption activities (i.e., can "produce health"). Finally, we consider the case where agents produce consumption goods and analyze *the health effects on agents' labor choices,* including the effects of health on disutility of labor, labor endowment, and productivity.

Distinguishing these three cases allows us to illustrate what "transfers" across occupations one should actually observe in efficient, possibly regulated, competitive markets. In the real world, indeed one observes either situations where the effects of treatments and other consumption activities may affect substantially agents health conditions or cases where these effects are relatively negligible. Moreover, it is also possible to distinguish occupations for which physical or mental health are important prerequisites for productive activities, from jobs requiring only a minimal level of health to be performed satisfactorily. Intuitively, health effects on production should determine the sign of cross transfers for jobs of the former type; while health effects on consumption decisions should be more important otherwise.

The super (sub) modularity properties of utility and production functions, as determined by complementarity (substitutability) relationships between health consumption goods and labor, will be the fundamental ingredients of our analysis. In this respect, it is worthy to note that **A1-A3** do not impose any restriction on the sign of the cross-derivatives $U_{c\theta}$. The reason is that the sign of $U_{c\theta}$ depends on the relative magnitude of two effects, which may generally go in opposite directions given the specific nature of health services. First, the marginal utility of most consumption activities increases (at least weakly) in better health status, since health can be used as an input for these consumption activities. At worst, a better health should not reduce the marginal utility of consuming any good. Were this the only

[27] This assumption can be easily relaxed. In order to derive our characterization results, we only need U_{cx_L} to be *not too negative* for all c.

channel through which health impacts consumption choices, one should have $U_{c\theta} > 0$ for all c. However, whenever health "can be produced" by the agents or, more precisely, whenever agents can influence their health by devoting resources to medical treatments or health enhancing consumption activities, a possibly counterbalancing effect may arise. In the next section we will in fact show that in this case the agents' marginal utility of medical treatments or health enhancing consumption activities may well be larger in bad health states and hence one may have $U_{c\theta} < 0$ at least for some consumption goods.

6.1.1 Health Effects on Pure Consumption Choices

This section studies how the properties of Pareto optima are influenced by the impact of their health status on agents' preferences for consumption goods. To this end, we begin by focusing on a consumption economy where agents cannot modify their health conditions by devoting resources to treatments or other consumption activities, i.e., $\rho(x, \theta) = \theta$.[28] Moreover, to concentrate on the impact of health on *pure* consumption choices, we shall assume for the time being that workers supply labor inelastically, while either productivity or labor endowments are independent from health status, i.e., $a(\theta) = a$ and $l^t(\theta) = L(\theta) = L$ for all θ and $t = 1, 2$. As we explained above, under these assumptions health can simply be seen as an input for consumption activities, for this reason it is natural to assume $U_{c\theta}(x, \theta) > 0$ for all c. The next proposition illustrates that the (positive) effect of health on the (marginal) utility of consumption and the direct health effect on well being, measured by $U_\theta(x, \theta)$, determine optimal cross transfers and the sign of (expected) utility differential.

Let $\Delta u^P = \sum_{\theta\in\Theta} p^1(\theta)U(x^{1P}(\theta), \theta) - \sum_{\theta\in\Theta} p^2(\theta)U(x^{2P}(\theta), \theta)$ and $\Delta Z^P = Z^{1P} - Z^{2P}$ be differences in expected utilities and transfers, respectively, received by the agents assigned to the two sectors. Since the first-order conditions with respect to α of the Pareto program imply $\Delta u^P \geq 0$ (resp. $<$) if and only if $\Delta Z^P \geq 0$ (resp. $<$), from hereafter we shall only study the sign of Δu^P.

Proposition 4 *If U has increasing differences in (x, θ) then $\Delta u^P > 0$.*

In the optimum, risk-averse workers assigned to different occupations must get the same consumption in each individual health state (i.e., $x^1(\theta) = x^2(\theta) = x^P(\theta)$ for all θ). As consumption goods and health are complements, optimality also imposes agents' consumption to be larger in better health states; and for this reason $U\left(x^P(\theta), \theta\right)$ is increasing in θ. Furthermore, since workers using less risky technologies are more likely to experience better health states, they obtain larger utility levels with larger probabilities. Thus, they also obtain a larger expected utility level.

6.1.2 Health Effects on Loss Reduction Activities

We now characterize Pareto optima in environments where agents can *produce* health by devoting resources to medical treatments or health-enhancing consumption activities. Again, we assume that agents supply

[28] These assumptions allow to characterize optimal allocations and transfers for situations where the productivity of health henancing activities is relatively low and health shocks play a relatively minor role in production decisions.

labor inelastically and that their labor endowment is not affected by the health status. This is done in order to isolate the effects of health enhancing consumption activities on utility differentials and cross-jobs transfers.

To begin with, consider the utility representation $U(x,\theta) = \hat{U}(x, \rho(x,\theta))$ where $\rho(.)$ satisfies $\rho_\theta(x,\theta) > 0$ and $\rho_c(x,\theta) \geq 0$. By differentiating $\hat{U}$ one has:

$$\hat{U}_{c\theta} = \hat{U}_{c\rho}\rho_\theta + \hat{U}_\rho \rho_{c\theta} + \hat{U}_{\rho\rho}\rho_c\rho_\theta,$$

hence $U_{c\theta}$ is negative for $\rho_{c\theta} < 0$ and sufficiently small (i.e., large in absolute value).[29] In fact, assuming $\rho_{c\theta} < 0$ seems completely natural in most real-world situations involving health enhancing consumption activities, and in particular medical treatments. Consider, for instance a generic treatment c; by its own nature, the treatment is beneficial only in *relatively bad* health states, hence $\rho_{c\theta}$ must be negative for at least a subset of Θ. Moreover, $\rho_{c\theta}$ must also be small (large in absolute value) if the treatment is (marginally) very effective in that subset of Θ. Finally, assuming that $\rho_{c\theta}$ does not change sign, hence that $U_{c\theta}$ is negative for those goods whose consumption contributes substantially to increase health conditions, seems a sensible assumption in many real-life cases. It becomes even more appropriate whenever, as it is often convenient for both theoretical and practical purposes, c can be interpreted as a total amount of certain types of medical expenses, i.e., a composite good.

Similar considerations hold with regard to health enhancing consumption activities, ranging from those aimed at satisfying nutritional and housing needs to physical activities. To provide an example, consider workers who spend a significant fraction of their income for nutritional and housing needs. It is very sensible to assume that higher levels of consumption improve their health conditions especially in lower health states. This again amounts to impose $\rho_{c\theta} < 0$.[30]

For the sake of clarity, we consider first the simplest situation where both consumption goods are substitutes with health, turning, subsequently to the more general case where one good is complement with health while the other one (i.e., the treatment) is substitute.

Proposition 5 *If U has decreasing differences in (x,θ) there exist a pair of real numbers (k,K) with $k < K < 0$ such that:*

(i) $\Delta u^P < 0$ whenever $U_{c\theta}/U_\theta < k$ for at least one good c;

(ii) $\Delta u^P > 0$ whenever $U_{c\theta}/U_\theta > K$ for all $c = 1, 2$.

[29] The first term of this sum represents the effect of θ on the marginal utility of consumption activities, which should be positive as discussed before; the second term represents the effect of health-enhancing consumption activities on $\hat{\theta}$; while the third addendum captures a second-order effect which reinforces that of health-enhancing consumption activities.

[30] As an example, consider the case of a worker, living in a low-income country which experiences high diffusion rates of a contagious disease. Contracting the disease generally impairs his consumption and working aptitudes and reduces his utility. However, the more adequately this worker can satisfy his basic consumption and housing needs the smaller should be the effects of the disease on his health conditions. Making this assumption amounts imposing $\rho_{c\theta} < 0$.

The economic intuition behind this result is that the optimal consumption allocation $x^P(\theta)$ is smaller in better health states, where consumption goods and health are substitutes. If such an effect is sufficiently large to compensate the impact of U_θ, the function $U\left(x^P(\theta), \theta\right)$ is decreasing in θ. Thus, workers using riskier technologies obtain a larger utility at the optimum. The converse is true otherwise.

Consider now the case where, besides consuming a treatment (or devoting resources to an health enhancing activity), agents can also consume another good whose impact on health is negligible, which is $U_{c\theta} > 0$ and $U_{c'\theta} < 0$ for $c \neq c'$. In this situation, a careful continuity argument permits to derive the sign of optimal transfers and utility differentials as an extension of the results stated in Propositions 4 and 5. It then remains uncovered the case where one good is substitute with health, the other is complement, and none of these effects is negligible relatively to the other. In such a case, one can easily verify that the direction of optimal transfers depends not only on the magnitude of second cross derivatives, but also on the marginal utility of consumption commodities (which, in turn, is affected by initial endowments). The main issue then becomes whether there exists a synthetic measure having empirical correlates, that one can use to determine which one of the two effects prevails. We conclude the analysis of this section by showing that the cross derivative of the indirect utility with respect to income and health is, indeed, the appropriate measure. Consider an economy where health is complement with good c, $U_{c\theta} > 0$, and substitute with good c', $U_{c'\theta} < 0$. Define $V(q, I(q), \theta) \equiv \max_{x \in \Re_+} \{U(x, \theta) \ s.t., \ qx \leq I(q)\}$ the state dependent indirect utility associated to the vector of prices q and total wealth $I(q)$. Corollary 6 below is a direct implication of Propositions 4 and 5; it shows that the sign of Δu^P is determined by the sign and the magnitude of $V_{I\theta}(q, I(q), \theta)$.

Corollary 6 *Assume $V_{I\theta}$ has constant sign for all q, I, and θ. Then, the following properties hold:*

(i) *if $V_{I\theta} > -k$ with k positive and sufficiently small, $\Delta u^P > 0$;*

(ii) *if $V_{I\theta} < -K$ with K positive and sufficiently large, $\Delta u^P < 0$.*

The proof follows from straightforward comparative statics and it is left to the reader.[31] It simply consists in verifying that $U\left(x^P(\theta), \theta\right)$ is increasing (resp. decreasing) whenever $V_{I\theta}$ is sufficiently large and positive (resp. negative), thereby health and income are *strong* complements (resp. substitutes). By FOSD the slope $U\left(x^P(\theta), \theta\right)$ implies, as for previous propositions, the sign of Δu^P.

Summarizing, Corollary 6 states that efficiency requires workers assigned to riskier jobs to get lower expected utilities if health enhancing consumption activities have relatively negligible effects on health, while the converse will often be true otherwise.[32]

[31] Note that $U_{c\theta}(x, \theta) > 0$ (resp. < 0) for all c implies $V_{I\theta} > 0$ (resp. < 0), hence Proposition 6 generalizes the results stated in Propositions 4 and 5.

[32] Note, however, that if one restricts attention to the case where no health henancing activity provides utility directly (as it is the case for medical treatments) Δu^P is negative only if $\rho\left(x^P(\theta), \theta\right)$ is decreasing at least in some subset of Θ. This is not anymore true, tough, as soon as some health enhancing consumption activities increase directly agents utility.

6.1.3 Health Effects on Labor Choices

This section illustrates the health effects on labor choices. We begin by considering the case where health conditions affect labor endowment. This case is the simplest to analyze and permits to illustrates a key effect for the determination of optimal cross-jobs transfers. We shall turn subsequently to study the basic trade-off which arises whenever health affects labor supply indirectly by influencing also agents' disutility of labor and productivity and hence optimal labor choices.

Health Effects on Labor Endowment: In order to focus on the effects of health risks on labor endowment, we now assume that health does not affect neither utility nor productivity (formally, $U_\theta(.) = 0$ for $c = 1, 2$, $\psi_\theta = 0$, $a(\theta) = a$ for all θ). Moreover, as before it is also convenient to impose that labor supply is completely inelastic (i.e., the marginal disutility of labor is sufficiently low) so that $l^t(\theta) = L(\theta)$ for all θ and t. Under this assumption, the effects of labor endowment's shocks (due to health realizations) result magnified, since labor supply is completely unaffected by state contingent shadow prices and wages. The next proposition demonstrates that agents employed in the sector yielding the worst health distribution (i.e., in sector 2) obtain a larger utility (and a positive transfer) in the optimum.

Proposition 7 *Assume $L(\theta_n) \geq L(\theta_{n-1})$ with strict inequality for at least one n, then $\Delta u^P < 0$.*

Either the proof, which is left to the reader, or the intuition for the result of Proposition 7 are similar to those of Propositions 4 and 5. More precisely, in the optimum agents work more and obtain a lower utility in better health states, where their labor endowment, and hence their labor supply, are larger. As a consequence, workers in safer occupations, who supply more labor on average, obtain a lower expected utility.

Health Effects on the Disutility of Labor and Individual Productivity: We now study the more general case where labor supply is not inelastic and health risks mainly affect agents' disutility of labor and their individual productivity.[33] In such a case, health affects labor choices both directly, by influencing the disutility of labor and the productivity or indirectly by affecting shadow prices (wages). These two effects may influence the sign of optimal utility differentials and transfers in opposite directions. This is for the following reasons. Analogously to the previous case, efficiency requires agents in each sector to work more in better health states, where the disutility of labor is lower or individual productivity is higher. This may lead to positive utility differentials in favor of workers using less safe technologies since better health is more likely in the safer sector. However, now labor supply in the two sectors is also influenced by shadow prices, and Pareto optimality imposes compensating wage differentials in favor of the riskier occupation, as established in Proposition 2. Optimality then imposes that *in each individual state* agents assigned to this occupation, obtain larger shadow wages, and for this reason work more in the optimum. The next proposition makes a first step toward the analysis of this trade-off, by showing

[33] As we shall formally show, this direct effect is qualitatively analogous to the effects of health on endowments. For the sake of readability, throughout we shall then assume that the Pareto program has only internal solutions and labor contingent endowments are health independent, i.e., $L(\theta) = L$ for all θ, so that the health effect on endowment does not appear.

that workers in the riskier sector obtain positive transfers and utility differentials whenever health has a sufficiently strong impact either on productivity, or on marginal disutility of labor, and the marginal disutility of labor is sufficiently increasing. Let $\sigma_\psi = \psi_{ll}/\psi_l$.

Proposition 8 *Assume $L(\theta) = L$, $\partial a(\theta)/\partial\theta \geq 0$ for all θ, the following properties hold:*

(i) if $|\psi_{l\theta}| + (\partial a(\theta)/\partial\theta)/a(\theta)$ is sufficiently large relatively to $|\psi_\theta|$ for all (l, θ), then $\Delta u^P < 0$ whenever $\sigma_\psi > K$, with K sufficiently large for all (l, θ);

(ii) if $|\psi_\theta|$ is sufficiently large relatively to $|\psi_{l\theta}| + (\partial a(\theta)/\partial\theta)/a(\theta)$ for all (l, θ) then $\Delta u^P > 0$.

The economic intuition rests upon recognizing the determinants of the magnitude of the price effects on labor supply. Specifically, since the elasticity of labor with respect to shadow wages is smaller for larger values of σ_ψ[34], assuming that the marginal disutility of labor is sufficiently increasing amounts to impose that labor supply is not very responsive to shadow wages. As a consequence, for any given θ, $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ cannot be too large. This implies, in turn, that the magnitude of the price effect is relatively small when health affects substantially either productivity or the marginal disutility of labor. Hence, workers in the riskier sector must obtain a higher utility if the direct effect of health on productivity and marginal disutility of labor is sufficiently large.

Finally, it is worth to notice that σ_ψ large is a necessary condition for the elasticity of labor with respect to the wage to be small. There exists a quite large empirical literature (see the seminal contribution of Abowd and Card 1989, among others) indicating that this elasticity is not large in actual labor markets, and may even be quite close to zero.

We conclude the section by showing that the assumption imposing a lower bound on σ_ψ in Proposition 8 can be relaxed under mild assumptions which are often used in the applied literature. In particular, if one restricts to the case where the impact of health on the marginal disutility of labor is sufficiently large (in Proposition 8 this was not guaranteed by the assumptions), the next proposition holds. For the sake of simplicity, in proving the result we shall impose the standard Inada condition $\psi_l(0, \theta) = 0$ for all θ.

Proposition 9 *Assume $\psi_{lll}(l, \theta) > 0$ for all (l, θ), then $\Delta u^P < 0$ if $\psi_{l\theta}$ is sufficiently large relatively to ψ_θ for all (l, θ).*

Imposing $\psi_{lll} > 0$ is common in most applications of the principal-agent literature, and in the (theoretical and applied) literatures studying the effects of multiple risks. Intuitively, this assumption guarantees that the labor schedules are strictly concave in the shadow wage, while $\psi_{l\theta}$ large ensures that the disutility of labor becomes relatively small for sufficiently good health states. These effects again imply that $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ cannot be too large neither under bad health states, where the disutility of labor is large and hence actual labor supply is mall, nor in favorable health state, where agents labor supply become less responsive to the wage, as the marginal disutility of labor is large. As a consequence, workers

[34] This immediately follows from optimality conditions with respect to labor.

in the riskier sector must obtain a higher utility if the direct effect of health on the marginal disutility of labor is sufficiently large.

Finally, the assumption imposing a lower bound on σ_ψ can also be relaxed in the case where health only affects productivity, i.e., $\psi_\theta(l, \theta) = 0$ for all (l, θ), and the distribution of health in the safer sector is concentrated around the healthier state θ_N. The next proposition shows that in this case, which is often studied in the applied labor literature, optimal transfers and utility differentials can be derived by imposing very mild conditions on the sensitivity of labor with respect to the shadow wage.

Denote $w_\theta = a(\theta)\eta$ and let $l(w_\theta, \theta)$ be the contingent labor supply schedule implicitly defined by optimality conditions; finally, define $\zeta_{l,w} = dl(w_\theta, \theta)/dw_\theta)/(l(w_\theta, \theta)/w_\theta)$ the measure of the sensitivity of the optimal labor schedule with respect to the shadow wage, the following result holds:

Proposition 10 *Assume $p^1(\theta_N) = 1$, then $\Delta u^P < 0$ (resp. $\geq$) if $\partial\zeta_{l,w}/\partial w_\theta < 0$ (resp. $\geq$) for all θ.*

This result indicates that efficiency requires agents using riskier technologies to get an higher utility in the optimum whenever $\zeta_{l,w}$ is non-decreasing in the shadow wage. Again, this assumption is in line with the empirical findings of the labor supply literature; its interpretation is indeed that agents who are already "working a lot" react less to wage increases.

7 Characterization of Competitive Equilibria

In this section, we characterize competitive equilibria for economies with enforceable and unenforceable lottery contracts. We begin by proving the existence of a competitive equilibrium. The proof exploits the convexifying effect of large numbers.

Proposition 11 *A competitive equilibrium exists either under enforceable or under unenforceable lottery contracts.*

Next proposition states the First Welfare Theorem for economies where lottery contracts are enforceable.

Proposition 12 *Under enforceability of lottery contracts, competitive equilibria are first-best.*

The proof is standard and is omitted.

The logic of the First Welfare Theorem is also used to show that competitive equilibria are interim efficient allocations with equal treatment if only deterministic contracts are enforceable.

Proposition 13 *Under unenforceability of lottery contracts, competitive equilibria are interim efficient allocations with equal treatment.*

This result together with Proposition 1 has the following important corollary.

Corollary 14 *Competitive equilibria with deterministic contracts are generically not first-best.*

Next proposition states that agents trade individual securities at fair prices in both contractual regimes, and that state-contingent wages equal the value of state-contingent labor productivity for each type of worker. Furthermore, occupations associated to riskier health distributions command relatively higher contingent wages. Finally, when lotteries are enforceable, the value of consumption for agents of the same type assigned to different occupations typically differs from the sum of the values of their endowment and production. By using lottery contracts, indeed, wealth is optimally transferred across occupations in such a way that agents obtaining the higher (resp. lower) expected utility get a positive (resp. negative) transfer.

Let $\tilde{Z}_i^t = \sum_{c\in C,\theta\in\Theta}(q_c(p_i^t(\theta)x_{ic}^t(\theta) - e_{ic}) - q_t\sum_{\theta\in\Theta}p_i^t(\theta)a_i^t(\theta)(L - x_{iL}^t(\theta)),$

Proposition 15 *(i) under both contractual regimes $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ for some $g_i^t \in \Re_+$ and $w_i^t(\theta) = q_t a_i^t(\theta)$, for all i, t and θ; (ii) assume $a_i^t(\theta) = a_i(\theta)$ for all t, $w_i^t(\theta) < w_i^{t'}(\theta)$ if $\langle p_i^t, \Theta\rangle$ FOSD $\left\langle p_i^{t'}, \Theta\right\rangle$ and strictly positive measures of type i agents are assigned to sectors t and t' under both contractual regimes; (iii) in any equilibrium with lottery contracts such that positive measures of type i agents are employed in sectors t and t', then $u_i^t(x_i^t) - u_i^{t'}(x_i^{t'}) \geq 0$ (resp. $<$) if and only if $\tilde{Z}_i^t - \tilde{Z}_i^{t'} \geq 0$ (resp. $<$).*

Part *(i)* of Proposition 15 follows from the linearity of the intermediaries and production firms maximization programs; *(ii)* indicates that *compensating wage differentials* are paid to riskier occupations and it follows from first-order conditions; *(iii)* is a corollary of the optimality analysis performed in Section 5.

8 Second Welfare Theorem and Decentralization

This section focuses on the implementation of competitive equilibria with transfers. As unenforceability prevents competitive markets from achieving efficiency[35] we shall consider a situation where lottery contracts are unenforceable and a policy authority can implement cross-jobs transfers.

In the real-world, transfers across workers with different health prospects are implemented through a variety of institutions and policy schemes. Transfers' systems across health insurance policies and occupations on the one hand, and subsidies to health enhancing activities, such as medical treatments, health care etc. on the other hand, are in particular largely diffused. In our set-up, these two types of policy instruments play similar roles. For the sake of brevity, we shall study the effects of cross-subsidies across insurance policies, and discuss only briefly and informally the welfare effects of subsidies to health enhancing consumption activities.

Accordingly, we introduce a class of policy instruments based on deterministic transfers across insurance contracts and on minimal wages. To simplify the definition of transfers policy, in the following we

[35] The proof that the Second Welfare Theorem holds, under standard assumptions, when random contracts are enforceable may be showed to follow standard arguments.

shall assume, without loss of generality, that in equilibrium each agent trades with only one intermediary. Under this assumption an individual vector of assets' trades can be interpreted as an insurance contract.

Let s_i^t the (possibly negative) monetary transfer[36] received by a *type* i agent who signs a health insurance contract designed for *sector* t workers; and denote $f_i^t(\theta)$ the monetary transfer received by a *sector* t firm for each *type* i worker in state θ which it employes. Finally, let $\hat{w}_i^t(\theta)$ the minimal state contingent wage that firms must pay to *type* i workers employed in *sector* t who experiences the health state θ.

A transfers' policy, $\wp = (s, f, \hat{w})$, is a vector $s = (s_i^t)_{i\in I}^{t\in T}$ of subsidies to the workers; a vector $f = (f_i^t(\theta))_{i\in I,\theta\in\Theta}^{t\in T}$ of transfers to production firms and a vector $w = (\hat{w}_i^t(\theta))_{i\in I,\ \theta\in\Theta}^{t\in T}$ of state contingent minimal wages.

Feasible policies must be budget-balancing. Hence,

$$\wp \in \mathcal{P} \equiv \left\{ \wp : \sum_{t\in T, i\in I} \mu_i \varphi_i^t \left(s_i^t + \sum_{\theta\in\Theta} p_i^t(\theta) f_i^t(\theta) \right) = 0 \right\},$$

where φ_i^t represents the measure of *type* i workers who are effectively assigned to *sector* t in an equilibrium with transfers.

Minimal wages may well induce rationing, and for this reason market clearing rules must now be carefully specified. Throughout we assume that, in any equilibrium with transfers, all commodity as well as asset markets clear at "walrasian" prices without rationing (i.e., exactly as in the absence of transfers), and that firms' labor demand is not rationed as well. Differently, as transfers and minimal wages can generally make some occupations more attractive than others, a rule according to which workers are assigned to each occupation must be specified. We shall assume that whenever *type* i agents receive a larger utility in *sector* t than in *sector* t', for some $t' \neq t$, the probability that a *type* i agent is assigned to *sector* t in equilibrium, is equal to α_i^t, which is precisely the measure of *type* i workers assigned to *sector* t. The motivation for the clearing rule of consumption and assets markets is the usual one: namely, were firms or agents rationed, they would have an incentive to manipulate prevailing prices.[37]The same type of argument justifies the assumption that labor demand is never rationed in the equilibrium. Differently, our workers' assignment rule can be seen as the outcome of a *decentralized* job search process where: in a first stage, workers simultaneously apply for occupations; subsequently, applications are randomly selected whenever the number of workers applying for a job is larger than the number of posted vacancies, and firms offer jobs to the workers; finally, in a third stage, workers, who have possibly received more than one offer, pick an offer in the set containing their most preferred ones. Noteworthy, while this type of assignment mechanism introduces a randomization on agents' labor demand, the transfers policies we consider are completely deterministic, and hence their implementation does not rely on random devices.

[36] We will use monetary transfer as a synonimus of "transfer in units of numeraire".

[37] See, for instance, Mas Colell and others (pp. 315, 1995) for a justification of the walrasian equilibrium notion along these lines.

Consistently with the above definition of the policy scheme and with the description of market clearing rules, a *rational expectation equilibrium with transfers,* $\{\varphi, x, \alpha, z, p, w, \phi, \wp\}$, is formally defined by the following conditions:

(**I**) *type i* consumers' choose $\left(x_i^t, \varphi_i^t, z_i^t\right)_{\theta\in\Theta}^{t\in T}$ by maximizing $\sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$ subject to the budget constraints

$$\sum_{c\in C} q_c(x_{ic}^t(\theta) - e_{ic}) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta) + s_i^t,\ \forall\ \theta \in \Theta,\ t \in T$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \le 0\ \forall\ t \in T$$

and to a set of *rationing* constraint of the type

$$\varphi_i^t \le \alpha_i^t\ \forall\ t \in T$$

indicating that a *type i* agent who offers labor in *sector t* will be assigned to that sector with probability lower or equal to α_i^t, which the measure of *type i* workers effectively assigned to *sector t* in the equilibrium;

(II) production firms' labor demand, l_i^t, and intermediaries assets' supply, $\widehat{z}_i^t$, satisfy the same conditions as in the competitive equilibrium with deterministic contracts (i.e., conditions (6) and (7)) except that, because of the presence of transfers, the *sector t* production firms' objective function is now $\sum_{\theta\in\Theta} p_i^t(\theta)\, l_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta) + f_i^t(\theta)\right)$;

(III) the minimal wages' constraints, $w_i^t(\theta) \ge \hat{w}_i^t(\theta)$ for all θ, are satisfied;

(IV) all feasibility conditions hold.

The next proposition shows that all Pareto optimal allocations can be implemented as equilibria with transfers provided that agents' types are public information. Optimal policy schemes generally hinge on state and sector contingent minimal wages, but do not require transfers across firms. However, in the case of inelastic labor supply, uniform minimal wages suffice to implement Pareto optima, if appropriate transfers across sectors are also implemented.

Proposition 16 *All Pareto optimal allocations can be implemented as equilibria with transfers by policy schemes such that $f_i^t(\theta) = 0$ for all θ, i and t. Moreover, if workers' labor supply is inelastic for any positive wage, Pareto optima are implementable through policy schemes such that $\hat{w}_i^t(\theta) = \hat{w}_i$ and $f_i^t(\theta) = \hat{w}_i - \eta_t^P a_i^t(\theta)$ for all θ, t and i.*

Intuitively, the proposition shows that contingent monetary transfers allow to equalize, at the Pareto optimal shadow prices, the marginal utilities of contingent goods and wealth across occupations. Minimal wages prevent firms from manipulating the transfers' scheme by undercutting wages in the sectors where, at the Pareto shadow prices, workers obtains higher utility levels and labor supply is rationed.

As a remark, note also that, by continuity, Pareto improving policy schemes relying only on uniform minimal wages exist whenever the elasticity of labor supply is sufficiently small.[38]

A decentralization result similar to the one stated in the previous proposition can be proved if one considers alternative policy schemes based on (possibly negative) *non-linear* subsidies to health-enhancing consumption activities. The logic of the proof remains the same as the one of the previous proposition since *non-linear* subsidies to the purchase of health services turn out to be substantially equivalent to cross subsidies.[39] However, it is noteworthy that non-linear consumption subsidies are necessary for the implementation of Pareto optima. This is because *linear* consumption subsidies would distort individual consumption choices, thereby preventing the equalization of marginal rates of substitution to relative prices.[40] Finally, it is worthwhile mentioning that robust examples can be constructed where simple deterministic cross-transfers policies, that do not discriminate across types, allow to improve upon competitive allocations (see Bennardo and Piccolo, 2005). It may be showed that these policies are based on: (i) a uniform, public or regulated insurance scheme implementing cross transfers; and (ii) an opt-out clause allowing agents who prefer to buy insurance at market rates to exit the regulated insurance scheme.

9 Extensions

In the previous sections we made two simplifying assumptions: we assumed away prevention activities and aggregate uncertainty. As we now explain, both these restrictions can be removed.

Introducing aggregate uncertainty does not involve any analytical complication. All the results of the paper, as well as the analytical arguments extend to the more general case, provided that the number of aggregate states is finite.

Introducing prevention behavior requires some carefulness. Prevention is naturally described as workers' investments which allow obtaining, at a positive cost, a first-order stochastic shift of the health distributions associated to his occupations. If a pair of health distributions are initially ordered by the FOSD criterion, prevention activities may determine three possible scenarios. In the *first*, prevention technologies are such that the ordering of the two health distributions is preserved after prevention is undertaken. This is the case, for instance, whenever prevention activities are *very* costly, or have a similar impact on the two health distributions. In the *second scenario*, the ordering of the two distributions is reversed after prevention activities are performed. This may happen whenever prevention is relatively much more effective under the riskier health distribution. Finally, there also exists a *third* possible scenario where, once prevention activities are undertaken, health distributions cannot be anymore ordered by the FOSD criterion. As for the *first* case, introducing prevention leaves unaltered the results derived in the paper. In the second case, all our analysis still applies but must be appropriately reinterpreted.

[38] Uniform minimal wages and sector dependent minimal wages are both observed in developed countries.

[39] The formal proof of this claim are available upon request .

[40] Similarly, policies that do not discriminate across types (either cross subsidies on insurance or subsidies to health services purchases), generally do not allow to equalize, for all possible types, the marginal utility of expected wealth of agents assigned to different occupations.

Precisely, the ordering of the distributions determining optimal cross transfers and utility differentials is the ex-post one (i.e., the one emerging in equilibrium as a result of prevention activities), and not that holding ex ante. Only in the third case our characterization, which relies on the FOSD criterion, does not anymore apply.[41]

10 Concluding Remarks

The endogeneity of individual health distributions generates specific "cost-benefit trade-offs" involving agents' occupational choices and their consumption and production capabilities. We studied how these trade-offs shape either the Pareto frontier of the economy or agents' competitive choices. We showed that the relative magnitude of health effects on production and consumption choices determines the sign of Pareto optimal utility differentials across workers who use different technologies as well as that of optimal cross-jobs transfers. Moreover, we proved that competitive equilibria are ex-ante efficient if lottery contracts are enforceable, but not otherwise. As a consequence, the unenforceability of lotteries may justify the introduction of policy schemes implementing cross-transfers across occupations.

From a theoretical perspective, our results suggest that cross-jobs transfers may result necessary for efficiency, in any setting where consumption and production choices are interdependent because of complementarities between consumption and production activities, or asymmetric information. Our conjecture, based on the analysis of the present paper, is that the generic inconsistency between ex ante and interim optimality, determining the need for cross-transfers, continues to hold in most of the settings studied in the general equilibrium literature on clubs and in the asymmetric information literature. A result in this spirit is obtained by Bennardo (2005), which characterizes optimal transfers in a moral hazard set-up where health effects are not considered, but occupations affect agents' consumption choices via incentive constraints.

From an applied perspective these results raise doubts about the conventional evaluation of the empirical evidence that wage differentials usually appear too small within the class of low skill jobs (i.e., those performed by "uneducated people"), which include some of the dirtiest and riskiest occupations. While the standard interpretation is that this seeming failure of wages to compensate reflects some omitted variables, determining for example the extent to which certain occupations are more dangerous than others, our analysis suggests that *too small* wage differentials can be the natural consequence of specific market imperfections, such as missing markets, driving competitive allocations to be inefficient. Finally, another reason why our results are interesting from a policy perspective is that real-life insurance markets for work-related health risks are often heavily regulated, and health insurance policy highly debated, albeit the rationales for policy interventions (specific insurance markets failure) do not seem to be satisfactorily figured out.

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[41] All the results mentioned in this section are formally proved in a more extended version of this paper. Their proofs are available on request.

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11 Appendix

Proof of Proposition 1

The proof of the uniqueness part relies on a standard convexity argument. In order to prove the genericity result, we need to formally define the utility space $\mathcal{U}_i$. Following the literature[42] assume that, beyond all assumptions stated in Section 2, agents' preferences satisfy the following property: a sequence $U_{ik}(x_i, \theta)$ in $\mathcal{U}_i$ converges to $U_i(x_i, \theta) \in \mathcal{U}_i$ if and only if $U_{ik}(x_i, \theta)$, $DU_{ik}(x_i, \theta)$ and $D^2U_{ik}(x_i, \theta)$ uniformly converge to $U_i(x_i, \theta)$, $DU_i(x_i, \theta)$ and $D^2U_i(x_i, \theta)$, respectively, for all θ, on any compact subset of $\Re^C_+ \times [0, L]$.[43]

Let $\xi = (x, \alpha, \eta, \lambda)$ define the vector of variables in the Pareto program. We consider first the case where the solution of the Pareto program is internal, that is $D_c U_i(x, \theta) > 0$ for all i, c and θ and $\alpha_i^t \in (0, 1)$ for all i and t. A Pareto optimum solves:

$$\mathcal{F}(\xi, \varepsilon, \bar{u}) = \begin{pmatrix} \lambda_i D_c U_i(x_i^t, \theta) - \eta_c \mu_i \quad \forall\, c \in C \\ -\lambda_i U_{ix_L}(x_i^t, \theta) + \eta_t a_i^t(\theta)\mu_i \\ \lambda_i(u_i^t(x_i^t) - u_i^T(x_i^T)) - \mu_i(Z_i^t - Z_i^T) \quad \forall\, t \neq T \\ \sum_{i \in I} \mu_i (\overline{x}_i - e_i) - \sum_{i \in I} \mu_i y_i \\ \sum_{t \in T} \alpha_i^t u_i^t(x_i^t) - \bar{u}_i \quad \forall\, i \neq 1 \end{pmatrix}_{\theta \in \Theta, t \in T, i \in I} = \mathbf{0}$$

for some vector of Pareto weights, $\bar{u} = (\bar{u}_i)_{i=2}^I$. Now for any arbitrary economy $\varepsilon \in \mathcal{E}$, define the extended system of equations $\mathcal{G}(\xi, \varepsilon, \bar{u}) \equiv (\mathcal{F}(\xi, \varepsilon, \bar{u}),\ (u_1^1(x_1^1) - u_1^T(x_1^T))) = 0$, which is obtained by adding one interim efficiency condition to $\mathcal{F}(.) = 0$. Finally, let $\mathcal{S}_{\bar{u}} = \{\varepsilon \in \mathcal{E} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\}$ be the subset of economies where a solution $\xi(\varepsilon, \bar{u})$ of $\mathcal{G}(.)$ exists for a given $\bar{u}$. We will show that the sets of ex ante and interim Pareto optima are generically disjoint, by proving the equivalent statement that the complement of $\mathcal{S}_{\bar{u}}$ is open and dense. The space, $\mathcal{E}$, of economies is infinite dimensional. However, as *density* is a local property, one may restrict attention to a properly defined subset of $\mathcal{E}$. Specifically, we will consider the linear subspace of $\mathcal{U}$ defined as follows. Given an utility profile $\hat{U} \in \mathcal{U}$, we shall consider the perturbed utility functions $U_i(x_i, \theta) = \hat{U}_i(x_i, \theta) + \kappa_i(\theta) + \beta_i(\theta)\,(x_i - x_i^P(\theta|\bar{\varepsilon}, \bar{u}))$ where, for all θ and i, $\kappa_i(\theta)$ is a scalar and $\beta_i(\theta)$ denotes a $(C+1)$ dimensional vector. Assume $|\kappa_i(\theta_n)|$, $\|\beta_i(\theta_n)\|$, $|\kappa_i(\theta_{n+1}) - \kappa_i(\theta_n)|$ and $\|\beta_i(\theta_{n+1}) - \beta_i(\theta_n)\|$ sufficiently small for all (i, n). This class of utility functions clearly satisfies all the assumptions stated in Section 2 and defines a finite dimensional, linear subspace of $\mathcal{U}$. We shall prove density on $\widehat{\mathcal{E}} = \mathcal{E} \times \mathcal{T} \times \widehat{\mathcal{U}}$.

Let $D_{(\xi,\varepsilon)}\mathcal{F}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ and $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$, the matrices associated to the Jacobian of $\mathcal{G}(.)$ and $\mathcal{F}(.)$ evaluated at $(\xi^P(\bar{u}), \varepsilon(\bar{u}))$, respectively. In proving the density result one could proceed in two steps by proving first that $\mathcal{F}(.)$ is differentiable in a neighborhood of the Pareto optimum. For the sake of brevity we shall skip this step; it will be straightforward in the following that if $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ has full rank, then also $D_{(\xi,\varepsilon)}\mathcal{F}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ has full rank.

(i) *Density*

Define $\widehat{\mathcal{S}}_{\bar{u}} = \left\{\varepsilon \in \widehat{\mathcal{E}} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\right\}$ and let $(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ a generic point such that $\mathcal{G}(.) = 0$. We

[42] See Geanakoplos and Polemarchakis (1986) and Citanna, et al. (1994) for a detailed discussion.

[43] In words, we assume that $\mathcal{U}_i$ is endowed with the subspace topology of the C^2 uniform convergence topology on compact sets. Notice also that $\mathcal{U} = \prod_{i=1}^I \mathcal{U}_i$ is endowed with product topology.

will show that the complement of $\widehat{\mathcal{S}}_{\bar{u}}$ is dense by proving that $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}),\varepsilon(\bar{u}))$ has full row rank (i.e., $\mathcal{G}(.)$ is transversal to zero). Let $e_c = \sum_{i\in I}\mu_i e_{ic}$ for all c, and $e = (...,e_c,...) \in \Re^C$; moreover, define $a_i = (a_i^1(\theta_{n_i}),..,a_i^{T-1}(\theta_{n_i})) \in \Re_+^{T-1}$ for a generic $\theta_{n_i} \in \Theta$.

As a preliminary step we show that the rank of $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}),\varepsilon(\bar{u}))$ is equal to the rank of the following matrix:

$$\mathbf{A} = \left(\begin{array}{cccccccccccc}
\text{equat.\textbackslash variab.} & x_1 & a_1 & \kappa_1(\theta_1) & \beta_{1x_L}(\theta_{n_1}) & x_2 & a_2 & \kappa_2(\theta_2) & \beta_{2x_L}(\theta_{n_2}) & ... & e \\
\text{FOCs}(x_1) & \mathbf{H}_1 & \mathbf{C}_1 & \mathbf{0} & \mathbf{D}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\text{FOCs}(\alpha_1) & \mathbf{0} & \mathbf{B}_1 & * & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
u_1^1(x_1^1) - u_1^T(x_1^T) = 0 & * & \mathbf{0} & \rho_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\text{FOCs}(x_2) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_2 & \mathbf{C}_2 & \mathbf{0} & \mathbf{D}_2 & \mathbf{0} & \mathbf{0} \\
\text{FOCs}(\alpha_2) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_2 & * & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\text{PC}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & * & \mathbf{0} & \rho_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
. & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & ... & \mathbf{0} \\
. & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & ... & \mathbf{0} \\
. & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & ... & \mathbf{0} \\
\text{FEAS} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}
\end{array}\right)$$

Where $\mathbf{A}$ is a $T(I+1)+I(C+1)N$ dimensional square matrix which is obtained by differentiating the extended system $\mathcal{G}(\xi,\varepsilon,\bar{u})$ with respect to $(x_i, a_i, \kappa_i(\theta_i), \beta_{ix_L}(\theta_{n_i}), e)_{i=1}^I$.[44] $\mathbf{H}_i$ denotes the agent-i's Hessian submatrix for all $i = 1,..,I$; $\mathbf{C}_i$ is a $(T-1)$ dimensional square matrix having all entries equal to zero, except for the ones corresponding to the first-order conditions (FOCs) with respect to $x_{iL}^t(\theta_{n_i})$ which are equal to $\eta_t\mu_i$ for all $i = 1,..,I$ and $t = 1,..,T$; $\mathbf{B}_i$ is a $(T-1)$ dimensional square matrix with all null entries, but the ones of the principal diagonal which are equal to $p_i^t(\theta_{n_i})l_i^t(\theta_{n_i})$ for all i and t; $\mathbf{D}_i$ has all null entries except for the elements corresponding to FOCs with respect to $x_{iL}^t(\theta_i)$, which are equal to 1 for all i and t. Moreover, $\rho_i = \sum_{t\in T}\alpha_i^t p_i^t(\theta_i)$ for all $i \neq 1$, $\rho_1 = p_1^1(\theta_1) - p_1^T(\theta_1)$ and the symbol "$*$" denotes all submatrices whose rank does not influence the rank of $\mathbf{A}$.

Indeed, simple elementary operations allow to obtain $\mathbf{A}$ from $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}),\varepsilon(\bar{u}))$. First, one obtains the null matrices appearing in the rows corresponding to FEAs by summing the columns corresponding to e (multiplied by appropriate scalars) to the ones corresponding to a_i and $\kappa_i(\theta_i)$ for all i, respectively. Second, by using the first-order conditions with respect to x_i of the Pareto program one shows that the elements corresponding to FOCs(α_i) and x_i are zero for all i. Now we shall prove that $\mathbf{A}$ has full rank.

[44] Note that in the matrix $\mathbf{A}$ the equations associated to FOCs(x_i) indicate the first-order conditions of the Pareto program with respect to x_i for each i, those associated to FOCs(α_i) indicate the first-order conditions with respect to α_i for each i, PC$_i$ indicates the participation constraint $\sum_{t\in T}\alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for each agent $i = 2,..,I$, and the equations corresponding to FEAs are the feasibility conditions.

To this end, define

$$\mathbf{A}_i = \begin{pmatrix} \text{equat./variab.} & x_i & a_i & \kappa_i(\theta_i) & \beta_{ix_L}(\theta_{n_i}) \\ \text{FOCs}(x_i) & \mathbf{H}_i & \mathbf{B}_i & \mathbf{0} & \mathbf{D}_i \\ \text{FOCs}(\alpha_i) & \mathbf{0} & \mathbf{C}_i & * & \mathbf{0} \\ \bar{u}_i\text{-CONS.} & * & \mathbf{0} & \rho_i & \mathbf{0} \end{pmatrix},$$

for all $i = 2, .., I$, and:

$$\mathbf{A}_1 = \begin{pmatrix} \text{equat.\textbackslash variab.} & x_1 & a_1 & e & \kappa_1(\theta_1) & \beta_{1x_L}(\theta_{n_1}) \\ \text{FOCs}(x_1) & \mathbf{H}_1 & \mathbf{C}_1 & \mathbf{0} & \mathbf{0} & \mathbf{D}_1 \\ \text{FOCs}(\alpha_1) & \mathbf{0} & \mathbf{B}_1 & \mathbf{0} & * & \mathbf{0} \\ \text{FEAs} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ u_1^1(x_1^1) - u_1^T(x_1^T) = 0 & * & \mathbf{0} & \mathbf{0} & \rho_1 & \mathbf{0} \end{pmatrix}.$$

One can easily check that if all submatrices $\mathbf{A}_i$ have full rank so does $\mathbf{A}$, and thus $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ has full rank.

To begin with we show that $\mathbf{A}_1$ is nonsingular. Using the columns corresponding to $\beta_{1x_L}(\theta_{n_1})$ and those corresponding to a_1 (multiplied by appropriate scalars) simple elementary operations allow to obtain the matrix $\mathbf{A}'_1$ from $\mathbf{A}$:

$$\mathbf{A}'_1 = \begin{pmatrix} \text{equat.\textbackslash variab.} & x_1 & a_1 & e & k_1(\theta_1) & \beta_{1x_L}(\theta_{n_1}) \\ \text{FOCs}(x_1) & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_1 \\ \text{FOCs}(\alpha_1) & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \text{FEAs} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ u_1^1(x_1^1) - u_1^T(x_1^T) = 0 & * & * & \mathbf{0} & \rho_1 & \mathbf{0} \end{pmatrix}$$

where $\rho_1 \neq 0$ $p_i^t(\theta) \neq p_i^{t'}(\theta)$ for at least a type i worker with health status θ, and a pair (t, t') with $t \neq t'$. Hence, as the Hessian $\mathbf{H}_i$ has full rank for all $i = 1, ..I$ because preferences are strictly convex, it follows immediately that $\mathbf{A}'_1$ has full rank, and so does $\mathbf{A}_1$. Showing that $\mathbf{A}_i$ has also full rank follows exactly the same kind of logic and thus it is omitted. Hence the matrix $\mathbf{A}$ has full rank. Thus $\mathcal{G}(.)$ is transversal to zero and $\widehat{\mathcal{S}}_{\bar{u}}$ is dense whenever the solution of the Pareto program is internal.

Finally, the proof extends to the case where in the optimum there are corner solutions such that $\alpha_i^{tP} \in \{0, 1\}$ for some (i, t). Suppose, for instance, that $\alpha_i^{t'P} = 1$ for some $t' \in T$ and $i = 1$. Let $\gamma_1^{t'P}$ be the multiplier associated to the constraint $\alpha_1^{t'} \leq 1$. In order to prove density we must consider either the case where $\gamma_i^{Pt'} > 0$ or that where $\gamma_i^{Pt'} = 0$. When $\gamma_i^{Pt'} > 0$, the proof follows exactly the same kind of logic used for the case of interior solutions. Indeed, one only needs to add the equation $\alpha_1^{1P} = 1$ to the system $\mathcal{G}(\xi, \varepsilon, \bar{u}) = 0$, and then differentiate the extended system $\mathcal{G}'(\xi, \varepsilon, \bar{u}) = (\mathcal{G}(\xi, \varepsilon, \bar{u}) = 0,$ $\alpha_1^{1P} = 1)$ also with respect to $\alpha_1^{t'}$ to show that $D_{(\xi,\varepsilon)}\mathcal{G}'(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ has full rank. When $\gamma_i^{Pt'} = 0$, the proof is done in two steps whose details are left to the reader for the sake of brevity. The argument is as follows: first, one shows that the extended system $\mathcal{F}'(.) = (\mathcal{F}(.), \alpha_i^{t'P} = 1, \gamma_i^{Pt'} = 0)$ has generically no solutions as the matrix $D_{(\xi,\varepsilon)}\mathcal{F}'(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ has full rank. Then it follows that in an optimum where $\alpha_i^{tP} = 1$ typically $\gamma_i^{Pt'} > 0$. The same kind of strategy can be immediately used to show the result in the case where $\alpha_i^{tP} \in \{0, 1\}$ for more than one agent. □

(ii) *Openness*

Let $\mathcal{P}_{\bar{u}} = \{(\xi, \varepsilon) : \mathcal{F}(\xi, \varepsilon, \bar{u}) = 0\}$ denote the Pareto optimal manifold for $u = \bar{u}$, and consider the natural projection $\pi : \mathcal{P}_{\bar{u}} \to \mathcal{E}$, $\pi(\xi, \varepsilon, \bar{u}) = \varepsilon$. As proper mappings take closed sets into closed sets, $\mathcal{S}_{\bar{u}}$ is open if the natural projection is *proper*. Hence we need to prove that for any sequence $(\xi_k(\bar{u}), \varepsilon_k(\bar{u}))_{k=1}^{\infty}$ such that $\mathcal{F}(\xi_k, \varepsilon_k, \bar{u}) = 0$ for all k, and $\varepsilon_k \to \varepsilon$ as $k \to \infty$, there exists a converging subsequence of $(\xi_k(\bar{u}))_{k=1}^{\infty}$ with limit $\xi(\bar{u})$ such that $\mathcal{F}(\xi, \varepsilon, \bar{u}) = 0$. To this end, note first that $\{\alpha_k(\bar{u})\}_{k=1}^{\infty}$ must converge, say to α, as it belongs to the compact set $[0, 1]^{T \times I}$. Moreover, $D_c U_i(x, \theta) > K$ for all $c \in \hat{C}(\theta)$ as $x_c \to 0$,with K large, imply $\{x_{ik}(\bar{u})\}_{k=1}^{\infty} \gg 0$ for all i; while since $D_c U_i(x, \theta) < k$, with k small, as $x_c \to \infty$, there exists a positive vector G such that $x_{ik}(\bar{u}) < G$, hence $\{x_k(\bar{u})\}_{k=1}^{\infty}$ must converge, say to x. Given the assumptions on U_i, $U_{ik}(x_i, \theta) \to U_i(x_i, \theta)$ implies $DU_{ik}(x_i, \theta) \to DU_i(x_i, \theta)$ uniformly on compact sets for all (x_i, θ), then this must also hold at $x_i = x_{ik}(\bar{u})$ for all i. Finally, from (15)-(17) one gets $(\lambda_k(\bar{u}), \eta_k(\bar{u})) \to (\lambda(\bar{u}), \eta(\bar{u}))$. □

Proof of Proposition 2

For the sake of brevity we provide the proof only for the case where $U_\theta > 0$ at least in some interval $d\theta$. The proof for the case where $U_\theta = 0$ for all θ and $\partial L(\theta)/\partial\theta > 0$ in some interval $d\theta$ follows exactly the same logic; while the result for the case where $U_\theta = 0$, $\partial L(\theta)/\partial\theta = 0$ and $\partial a_i^t(\theta)/\partial\theta > 0$ can be obtained through simple algebraic manipulations of first-order conditions of the Pareto program.

Assume without loss of generality that $\langle p_i^1, \Theta \rangle$ FOSD $\langle p_i^2, \Theta \rangle$ and that $\alpha_i^1 > 0$, $\alpha_i^2 > 0$ and let $\langle x^P, \alpha^P \rangle$ a generic Pareto optimal allocation. Let $\mathbf{\Pi}$ be an $N \times N$ matrix, and denote $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m)$ be the element in the n-th row and the m-th column of $\mathbf{\Pi}$ satisfying the following conditions: (i) $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = 0$ for all pairs (n, m) such that $n > m$; (ii) $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$ for all pairs (n, m) with $n = m$, and (iii)

$$\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = \min \left\{ p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m),\ p_i^2(\theta_n) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_k) \right\}$$

for all (n, m) with $n < m$. As a preliminary step, we show that $\mathbf{\Pi}$ is a stochastic matrix satisfying the following properties:

(I) $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) \geq 0$ and $\sum_{n=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m)$;
(II) $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) \leq p_i^2(\theta_n)$ for $n = m$;
(III) $\sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^2(\theta_n)$;
(IV) $u_i^2(x_i^{2P}) < \sum_{m=1}^{N} \sum_{n=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U_i(x_i^{2P}(\theta_n), \theta_m)$.

Part (I) By construction $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) \geq 0$, for all (n, m). By (i) one has

$$\sum_{l=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) + \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m),$$

thus, (ii) in turn implies

$$\sum_{l=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) + p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = p_i^1(\theta_m).$$

Part (II) We can restrict attention to the case of $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) > 0$ for $n = m$. By construction, in this case $\tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = p_i^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_k)$ for all $l < n$. Indeed, were $\tilde{\pi}(x_i^{2P}(\theta_{\hat{l}}), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$ for some $\hat{l} < n$, it would follow from part (I) that $\tilde{\pi}(x_i^{2P}(\theta_{\hat{l}+t}), \theta_m) = 0$ for all $t > 0$, contradicting $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) > 0$ for $n = m$. Therefore, for all (n, m) such that $n = m$ one must have $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} \left(p_i^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_k) \right)$. As for $n = m$, $\sum_{k=1}^{m-1} \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_k) = \sum_{k=1}^{m-1} p_i^1(\theta_k)$ by part **(I)**, we obtain $\tilde{\pi}(x_i^{2P}(\theta_{\hat{l}}), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} p_i^2(\theta_l) + \sum_{k=1}^{m-1} p_i^1(\theta_k)$, which implies $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) < p_i^2(\theta_n)$ by FOSD.

Part (III) The proof is by induction. We first prove that the equality holds for $n = 1$. Since $\sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = 0$ for $n = 1$, by (iii) this amounts to show that $p_i^1(\theta_m) > p_i^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_1), \theta_k)$ for some $m < N$. If this were not true, one should have $p_i^1(\theta_m) \leq p_i^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_1), \theta_k)$ for all $m \leq N$. However, for $m = N$ this is impossible; indeed by condition (iii) $\sum_{k=1}^{N-1} \tilde{\pi}(x_i^{2P}(\theta_1), \theta_k) = \sum_{k=1}^{N-1} p_i^1(\theta_k)$ whenever $p_i^1(\theta_m) \leq p_i^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_1), \theta_k)$ for all $m \leq N$. Suppose now that $\sum_{k=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_k) = p_i^2(\theta_n)$ for $n = 1, 2, ...\bar{n}$, but $\sum_{k=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_k) < p_i^2(\theta_{\bar{n}+1})$. In this case, $\sum_{k=1}^{m} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_k) < p_i^2(\theta_{\bar{n}+1})$ for all $m \leq N$ so that by condition (iii) $\tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$ $\forall\ m > n$. By summing over m it follows:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = \sum_{m=\bar{n}+1}^{N} \left(p_i^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) \right),$$

which, in turn, implies:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = 1 - \sum_{m=1}^{\bar{n}} p_i^1(\theta_m) - \sum_{m=1}^{N} \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) + \sum_{m=1}^{\bar{n}} \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m).$$

As we are assuming for $n = 1, 2, ...\bar{n}$, $\sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^2(\theta_n)$, the right-hand-side of this expression is equal to $1 - \sum_{m=1}^{\bar{n}} p_i^1(\theta_m) - \sum_{n=1}^{\bar{n}} p_i^2(\theta_n) + \sum_{m=1}^{\bar{n}} p_i^1(\theta_m) = 1 - \sum_{n=1}^{n} p_i^2(\theta_n)$. This proves the claim, since $\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = 1 - \sum_{n=1}^{n} p_i^2(\theta_n) > p_i^2(\theta_{\bar{n}+1})$ contradicts $\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) \leq p_i^2(\theta_n)$.

Part (IV) As $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = 0$ for all (n, m) with $n > m$ and $U_{i\theta} > 0$, for all n, we have:

$$\sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U_i(x_i^{2P}(\theta_n), \theta_m) > \sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U_i(x_i^{2P}(\theta_n), \theta_n) = p_i^2(\theta_n) U_i(x_i^{2P}(\theta_n), \theta_n)$$

by summing up over n one obtains **(IV)**.

We can now prove that $\eta_1^P < \eta_2^P$. The proof is again by contradiction. Assume first that $\eta_1^P > \eta_2^P$ and that, at least for one type, say *type* i, $0 < \alpha_i^{2P} \leq \mu_i$ (i.e., some type i workers are assigned to sector 2 in the optimum). Consider an allocation having the following features. A measure $\alpha_i^1 = \alpha_i^{1P} + d\alpha_i$ of the set of *type* i workers is assigned to *sector 1* while a measure $\alpha_i^2 = \alpha_i^{2P} - d\alpha_i$ is assigned to *sector 2*, with $d\alpha_i$ sufficiently small. All *type* i' agents for $i' \neq i$ receive $(x_{i'}^P, \alpha_{i'}^P)$. All *type* i workers in *sector* 2 and a measure $\alpha_i^{1P} - d\alpha_i$ of *type* i workers in *sector 1* obtain x_i^{2P} and x_i^{1P}, respectively; a set of measure $d\alpha_i$ of *type* i workers in sector 1 obtain $\tilde{x}_i^1 = (..., \tilde{x}_i^1(\theta_m), ...)$, with $\tilde{x}_i^1(\theta_m) = \sum_n \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) x_i^{2P}(\theta_n)$, while another set of measure $d\alpha_i$ of *type* i workers in sector 1, obtain the allocation $x_i^{1P} + \varepsilon = (..., x_i^{1P}(\theta_n) + \varepsilon(\theta_n), ...)$ where, for all n, $\varepsilon(\theta_n)$ is such that $\varepsilon_1(\theta_n) = \varepsilon$, $\varepsilon_2(\theta_n) = -\varepsilon$, with $\varepsilon > 0$ and sufficiently small, and $\varepsilon_c(\theta_n) = 0$ for all $c > 2$. By construction, this allocation is feasible; moreover the result proved in **Part (IV)** and strict convexity of preferences imply $\tilde{x}_i^1 \succ_i x_i^{2P}$, while $\eta_1^P > \eta_2^P$ implies $x_i^{1P} + \varepsilon \succ_i x_i^{1P}$. This contradicts the optimality of $\langle x^P, \alpha^P \rangle$; thus, $\eta_1^P \leq \eta_2^P$. Finally, a standard continuity argument implies $\eta_1^P \neq \eta_2^P$. □

Proof of Proposition 4

As a preliminarily step we state, without proving it, the following well known lemma, that we shall use several times subsequently. Let $P^t(\theta_n) = \sum_{\theta \leq \theta_n} p^t(\theta_n)$ for $n \in N$ and $t = 1, 2$,

Lemma 17 *For any map $g : \Theta \to \Re^+$, $\theta \to g(\theta)$, with $dg(\theta_{n+1}) = g(\theta_{n+1}) - g(\theta_n)$, the following identity holds:*

$$\sum_{\theta \in \Theta} \left(p^1(\theta) - p^2(\theta)\right) g(\theta) := \sum_{n \in N} \left(P^2(\theta_n) - P^1(\theta_n)\right) dg(\theta_{n+1}).$$

We can now prove the statement of the proposition. The first-order conditions with respect to x of the Pareto program, together with strict concavity of $U(x, \theta)$ in x imply $x^{1P}(\theta) = x^{2P}(\theta) = x^P(\theta)$ for all θ. Let $x^P : \Theta \to \Re_+^2$, $\theta \to x^P(\theta)$, be the map associating to each $\theta \in \Theta$ the optimal consumption vector $x^P(\theta)$. Assume $\theta_{n+1} - \theta_n = d\theta$ for all n, with $d\theta$ sufficiently small, and let $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) = U\left(x^P(\theta_{n+1}), \theta_{n+1}\right) - U\left(x^P(\theta_n), \theta_n\right)$, one then obtains:

$$dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) \approx \sum_{c=1,2} dx_c^P(\theta_{n+1}) \eta_c^P + U_\theta\left(x^P(\theta_{n+1}), \theta_{n+1}\right) d\theta. \tag{18}$$

By Lemma (17), $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $dU\left(x^P(\theta), \theta\right) \gtreqless 0$; hence (18) implies $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $\sum_{c=1,2} dx^P(\theta_{n+1}) \eta_c^P + U_\theta\left(x^P(\theta_{n+1}), \theta_{n+1}\right) d\theta \gtreqless 0$. For $d\theta$ small, the first-order conditions of the Pareto program with respect to x imply $dx_1^P(\theta_{n+1}) \approx (U_{1\theta} |U_{22}| + U_{2\theta} U_{21})/\Lambda) d\theta$ and $dx_2^P(\theta_{n+1}) \approx (U_{2\theta} |U_{11}| +$

$U_{1\theta}U_{12})/\Lambda)d\theta$, where from strict concavity of $U(x,\theta)$ in x we have $\Lambda = U_{11}U_{22} - (U_{12})^2 > 0$. Summing up, we obtain:

$$\sum_{c=1,2} dx_c^P(\theta_{n+1})\eta_c^P \approx U_1 \frac{U_{1\theta}\,|U_{22}| + U_{2\theta}U_{12}}{\Lambda} d\theta + U_2 \frac{U_{2\theta}\,|U_{11}| + U_{1\theta}U_{12}}{\Lambda} d\theta, \tag{19}$$

(18) and (19) then imply that $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}\left(U_1|U_{22}| + U_2U_{12}\right) + \frac{U_{2\theta}}{\Lambda}(U_2|U_{11}| + U_1U_{12}) + U_\theta \gtreqless 0, \tag{20}$$

equation (20) together with supermodularity in x (i.e., $U_{12} \geq 0$), increasing differences in (x,θ) (i.e., $U_{c\theta} \geq 0$ for $c = 1,2$) and $U_\theta > 0$, imply the result. □

Proof of Proposition 5

As showed in Proposition 4, $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}\left(U_1|U_{22}| + U_2U_{12}\right) + \frac{U_{2\theta}}{\Lambda}(U_2|U_{11}| + U_1U_{12}) + U_\theta \gtreqless 0.$$

Since we have assumed decreasing differences of U in (x,θ) (i.e., $U_{c\theta} \leq 0$ for $c = 1,2$), one can check that the first two terms in the above equation are negative, while the third term is positive. It is easy to show then that if $|U_{c\theta}|/U_\theta$ is large enough for at least one c one must have $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) < 0$ and hence $\Delta u^P < 0$, which proves part (i). Similarly, when $|U_{c\theta}|/U_\theta$ is small enough for each $c = 1,2$ then $dU\left(x^P(\theta_{n+1}), \theta_{n+1}\right) > 0$ so to have $\Delta u^P > 0$, thereby proving part (ii). □

Proof of Proposition 8

Part (i) Let $\Delta u^P = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^{2P}(\theta),\theta) - \sum_{\theta\in\Theta} p^1(\theta)\psi(l^{1P}(\theta),\theta)$ and define $\sigma_\psi(l,\theta) \equiv \psi_{ll}(l,\theta)/\psi_l(l,\theta)$. Summing by parts,

$$\Delta u^P = -\sum_{\theta\in\Theta}(p^1(\theta) - p^2(\theta))\psi(l^{1P}(\theta),\theta) + \sum_{\theta\in\Theta} p^2(\theta)(\psi(l^{2P}(\theta),\theta) - \psi(l^{1P}(\theta),\theta)).$$

Now, let η_t^P the value of the Lagrange multiplier, calculated in the optimum, and denote $l_t(\theta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = a(\theta)\eta_t^P$ for $t = 1,2$; and $l(\theta,\eta)$ that defined by $\psi_l(l(\theta),\theta) = a(\theta)\eta$. Let $\Delta P(\theta_n) = P^1(\theta_n) - P^2(\theta_n)$, we then have:

$$\Delta u^P \approx \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\frac{d\psi(l_1(\theta),\theta)}{d\theta}\right) d\theta + \sum_{\theta\in\Theta} p^2(\theta)a(\theta) \int_{\eta_1^P}^{\eta_2^P} \frac{1}{\sigma_\psi(l(\theta,\eta),\theta)} d\eta.$$

Let $\Phi(l_t(\theta),\theta) = |\psi_{l\theta}(l_t(\theta),\theta)| + \psi_l(l_t(\theta),\theta)((\partial a(\theta)/\partial\theta)/a(\theta))$ for $t = 1,2$, one can easily check that

$$\Delta u^P \approx \Delta_1^P + \Delta_2^P, \tag{21}$$

with

$$\Delta_1^P = \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\frac{\Phi(l_1(\theta),\theta)}{\sigma_\psi(l_1(\theta),\theta)} + \psi_\theta(l_1(\theta),\theta) \right) d\theta,$$

and

$$\Delta_2^P = \sum_{\theta\in\Theta} p^2(\theta)a(\theta) \int_{\eta_1^P}^{\eta_2^P} \frac{1}{\sigma_\psi(l(\theta,\eta),\theta)} d\eta,$$

where $\Delta_2^P > 0$ since $\eta_2^P > \eta_1^P$ by Proposition 2, and $\sigma_\psi \geq 0$.

Observe that from **A1** σ_ψ must be finite. This together with Lemma 17, and the continuity of preferences, imply that there exists $d \in \Re_{++}$ such that if $dl^P(\theta) = l_2^P(\theta) - l_1^P(\theta) \leq d$ for all θ, $\Delta u^P < 0$. It then remains to prove that $\Delta u^P < 0$ whenever $dl(\theta) > d$ for some θ. From (21) we have $\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (d\psi(l_1(\theta),\theta)/d\theta) d\theta < 0$ for ψ_θ sufficiently large. Since from **A1** $\Phi/\sigma_\psi > 0$ and we assumed σ_ψ large, $\Delta u^P < 0$ if there exists a strictly positive h such that $\Delta\eta^P = \eta_2^P - \eta_1^P < h$. In the following, we use an optimality argument to prove the existence of an upper bound on $\Delta\eta^P$. To this purpose, let

$$EU(x,\alpha) = \sum_{t=1,2} \alpha^t \sum_{\theta\in\Theta} p^t(\theta) U(x^t,\theta),$$

by definition $EU(x^P,\alpha^P) \geq EU(x',\alpha')$, for all feasible (x',α'). In particular, consider the consumption allocation $\check{x}$ such that $\check{x}_c^t = x_c^{tP}$ for $c = 1,2$; $\check{l} = \beta l^{1P} + (1-\beta) l^{2P}$, with $\beta \in (0,1)$. Since $l^{1P} < l^{2P}$ by Proposition 2, a continuity argument implies that for any β sufficiently small there exists a real number k such that $0 < \check{\alpha} = \alpha^P + k < 1$, and $(\check{x},\check{\alpha})$ satisfies the feasibility constraints (possibly as inequality).

Let $\Delta EU = EU(x^P,\alpha^P) - EU(\check{x},\check{\alpha}) \geq 0$. By adding and subtracting $EU(\check{x},\alpha^P)$ to ΔEU, and then using the first-order conditions of the Pareto program one gets $\Delta EU = \tilde{A}^P + \tilde{B}^P$ where $\tilde{A}^P = \sum_{t\in T,\theta\in\Theta} \alpha_t^P p^t(\theta) \Delta\psi(l^t(\theta))$, with $\Delta\psi(l^t(\theta)) = (\psi(\check{l}(\theta),\theta) - \psi(l^{tP}(\theta),\theta))$, and where

$$\tilde{B}^P \approx \beta\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\psi_l(\check{l}(\theta),\theta) \frac{\Phi(l_1(\theta),\theta)}{\psi_{ll}(l_1(\theta),\theta)} \right) d\theta +$$

$$+(1-\beta)\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\psi_l(\check{l}(\theta),\theta) \frac{\Phi(l_2(\theta),\theta)}{\psi_{ll}(l_2(\theta),\theta)} \right) d\theta +$$

$$+\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \psi_\theta(\check{l}(\theta),\theta) d\theta,$$

with $\Delta\alpha = (\check{\alpha} - \alpha^P)$. For β sufficiently close to 0,

$$\tilde{B}^P \approx \Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\psi_l(\check{l}(\theta),\theta) \frac{\Phi(l_2(\theta),\theta)}{\psi_{ll}(l_2(\theta),\theta)} + \psi_\theta(\check{l}(\theta),\theta) \right) d\theta \leq$$

$$\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\frac{\Phi(l_2(\theta),\theta)}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(\check{l}(\theta),\theta) \right) d\theta,$$

since $\check{l}(\theta) \le l_2(\theta)$ for all θ and $\psi_{ll} > 0$.

$\tilde{B}^P$ is bounded above as all derivatives of ψ are bounded by **A1**. Moreover, from the first-order conditions of the Pareto program and the convexity of ψ it follows $\tilde{A}^P < A' = \sum_{t\in T} \alpha_t^P \eta_t^P \sum_{\theta\in\Theta} a(\theta)p^t(\theta)\Delta l^t(\theta)$ where $\Delta l^t(\theta) = (\check{l}(\theta) - l^{tP}(\theta))$. Using the definition of $\hat{l}(\theta)$ we then get:

$$A' = \alpha_1^P \eta_1^P (1-\beta) \sum_{\theta\in\Theta} p^1(\theta)a(\theta)(l^{2P}(\theta) - l^{1P}(\theta)) - \alpha_2^P \eta_2^P \beta \sum_{\theta\in\Theta} p^2(\theta)a(\theta)(l^{2P}(\theta) - l^{1P}(\theta)).$$

As $(l^{2P}(\theta) - l^{1P}(\theta)) > d$, the above expression implies $\tilde{A}^P \to -\infty$ as $\eta_2^P - \eta_1^P \to +\infty$. We can conclude that $\Delta EU = \tilde{A}^P + \tilde{B}^P \ge 0$ implies $\eta_2^P - \eta_1^P < h$ for some positive h. Therefore, since ψ_l is bounded from **A1** it follows that $\Delta u^P < 0$ if $|\psi_{l\theta}| + (\partial a(\theta)/\partial\theta)/a(\theta)$ is sufficiently large relative to $|\psi_\theta|$ for all (l,θ).

Part (ii) The proof is straightforward and follows directly from equation (21). □

Proof of Proposition 9

Let $l(\eta,\theta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = \eta$, and let $T(\eta,\theta) := \sigma_\psi^{-1}(l(\eta,\theta),\theta) - l(\eta,\theta)$, then $\psi_l(0,\theta) = 0$ for all θ together with $\psi_{ll}(l,\theta) > 0$ imply $T(0,\theta) = 0$ for all θ. As it can be easily checked that $\psi_{lll}(l,\theta) > 0$ implies $\partial T(\eta,\theta)/\partial\eta \le 0$, it follows that $l(\eta,\theta) \ge \sigma_\psi^{-1}(l(\eta,\theta),\theta)$ for all (η,θ). Moreover, consistently with the notation introduced before, let $l_2(\theta)$ denote the function implicitly defined by $\psi_l(l_2(\theta),\theta) = \eta_2^P$. Given the definition of Δu^P in equation (21) (appearing in the proof of Proposition 8), the inequality $l(\eta,\theta) \ge \sigma_\psi^{-1}(l(\eta,\theta),\theta)$ implies:

$$(22) \qquad \Delta u^P \le \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(l_2(\theta),\theta) \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta) d\eta \right).$$

As we want to study the sign of Δu^P, by using the first-order conditions of the Pareto program with respect to α we will rewrite the second addendum of the right hand side of (22) in such a way to compare it with the first addendum. This will allow us to sign the upper-bound of Δu^P defined in (22). To this end, let $h(l,\theta) := \psi_l(l,\theta)l$, by adding and subtracting $\sum_{\theta\in\Theta} p^1(\theta)\psi(l^{2P}(\theta),\theta)$ to the left-hand-side and $\sum_{\theta\in\Theta} p^1(\theta)h(l^{2P}(\theta),\theta)$ to the right-hand-side of (17), and using Lemma 17 one gets:

$$\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{d\psi(l_2(\theta),\theta)}{d\theta} \right) d\theta + \sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} \left(\psi_l(l(\theta,\eta),\theta) \frac{\partial l(\theta,\eta)}{\partial\eta} \right) d\eta =$$

$$\sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} \left(h_l(l(\theta,\eta),\theta) \frac{\partial l(\theta,\eta)}{\partial\eta} \right) d\eta + \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{dh(l_2(\theta)\,\theta)}{d\theta} \right) d\theta.$$

Simple algebraic manipulations allow to rewrite the above equality as:

$$(23) \qquad \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \psi_\theta(l_2(\theta),\theta)d\theta = \sum_{\theta\in\Theta} p^1(\theta) \int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta)d\eta,$$

Equations (22) and (23) then imply:

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + 2\psi_\theta(l_2(\theta),\theta) \right) d\theta.$$

Then since by FOSD $\Delta P(\theta_n) \leq 0$ with at least one strict inequality, it follows that $\Delta u^P \leq 0$ if $|\psi_{l\theta}| \geq 2\sigma_\psi |\psi_\theta|$ for all (l,θ). □

Proof of Proposition 10

To begin with, we show the following preliminary lemma which will be useful in proving the proposition. Let $h(l) = \psi'(l)l$, $\sigma_h(l) = h''(l)/h'(l)$, $\Delta h = \sum_{\theta\in\Theta} p^2(\theta)h\left(l^2(\theta)\right) - h(l^1(\theta_N))$, and $\Delta\sigma = (\sigma_\psi - \sigma_h)$.

Lemma 18 *(i)* $\partial\zeta_{l,w}/\partial w_\theta \gtreqless 0$ *for all* (l,θ) *if and only if* $\sigma_\psi(l) \gtreqless \sigma_h(l)$ *for all* l*; (ii)* $\Delta u^P = 0$ *implies* $sign\Delta h = -sign\Delta\sigma$.

Proof The proof of part (i) follows from straightforward manipulations of the FOCs of the Pareto program, and is omitted. In order to prove part (ii) denote $l^2(h) = \sum_{\theta\in\Theta} p^2(\theta)h\left(l^2(\theta)\right)$ the certainty equivalent under h of the distribution $\left\langle p^2, (l^2(\theta))_{\theta\in\Theta}\right\rangle$. Since $\hat{x}^{1P} = \hat{x}^{2P}$, $\Delta u^P = 0$ implies $\psi\left(l^1(\theta_N)\right) = \sum_{\theta\in\Theta} p^2(\theta)\psi\left(l^2(\theta)\right)$; therefore, $l^1(\theta_N)$ is the certainty equivalent, under ψ, of the distribution $\left\langle p^2, (l^2(\theta))_{\theta\in\Theta}\right\rangle$. Since $h(l)$ is an increasing function, and $l^1(\theta_N) \gtreqless l^2(h)$ whenever $\Delta\sigma \gtreqless 0$, it follows that $\Delta h \lesseqgtr 0$ if $\Delta\sigma \gtreqless 0$. □

We shall now prove the claim of the proposition beginning with the case $\sigma_\psi < \sigma_h$. To this end, we introduce an auxiliary program which maximizes $\sum_{t=1,2} \alpha^t u^t(x^t)$ under the feasibility constraints and the additional constraint:

$$(24) \qquad \Delta u = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) \leq 0.$$

The FOCs with respect to $l^t(\theta)$, $t = 1,2$, and α of this program are:

$$(25) \qquad \psi'(l^1(\theta_N)) = \hat{\eta}_1 a(\theta_N) + \frac{\varkappa}{\alpha}\psi'(l^1(\theta_N)),$$

$$(26) \qquad \psi'(l^2(\theta)) = \hat{\eta}_2 a(\theta) - \frac{\varkappa}{1-\alpha}\psi'(l^2(\theta)), \quad \forall\, \theta \in \Theta$$

$$(27) \qquad \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) = \hat{\eta}_2 \sum_{\theta\in\Theta} p^2(\theta)a(\theta)l^2(\theta) - \hat{\eta}_1 a(\theta_N)l^1(\theta_N),$$

where $\hat{\eta}_t$ for $t = 1, 2$ are the Lagrangian multipliers associated to the feasibility constraints of the auxiliary program, and $\varkappa$ is the multiplier associated with (24). Substituting (25) and (26) into (27) one gets:

$$(28) \qquad \Delta u = \Delta h + \varkappa \left(\frac{\sum_{\theta\in\Theta} p^2(\theta)h(l^2(\theta))}{1-\alpha} + \frac{h(l^1(\theta_N))}{\alpha} \right).$$

We can now verify that $\varkappa = 0$ and (24) holds as inequality whenever $\Delta\sigma < 0$. This immediately implies that $\Delta u^P < 0$ for $\Delta\sigma < 0$.

First we must have $\varkappa = 0$; indeed $\varkappa > 0$ and $\Delta u = 0$ would imply (28) $\Delta h < 0$; but this is impossible, as we showed above that $\Delta h > 0$ whenever $\Delta u = 0$ and $\Delta\sigma < 0$. Moreover, (24) must hold as inequality. Otherwise, one would have $\Delta u = 0$, and hence $\Delta h > 0$ whenever $\Delta\sigma < 0$, which contradicts (28).

The proof that $\Delta\sigma > 0$ implies $\Delta u^P > 0$ and that $\Delta\sigma = 0$ implies $\Delta u^P = 0$, requires exactly the same type of argument developed above and is left to the reader. □

Proof of Proposition 11

We begin with the case where lottery contracts are unenforceable. Consider the *auxiliary* program which maximizes $\sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$ within the compact set defined by the agents' budget constraints (4-5) and the additional constraints $x_i^t(\theta) \in \bar{X} \subset \Re^C \times [0, L]$, $z_i^t(\theta) \in \bar{Z}$ with $\bar{X}$ and $\bar{Z}$ finite but sufficiently large. A standard risk aversion argument implies that the set of solutions of program (3)-(5) and that of the auxiliary program coincide for $\bar{X}$ sufficiently large. As both production and intermediation technologies are linear, equilibrium prices must satisfy: $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ for some $g_i^t \in \Re_+$, and $w_i^t(\theta) = q_t a_i^t(\theta)$ for $i \in I$, $t \in T$ and $\theta \in \Theta$. At these prices assets' supply and labor demands are indeterminate. By using these conditions and normalizing prices appropriately, the budget correspondence can be rewritten as:

$$B_i^t(q) = \left\{ (x_i^t, \varphi_i^t) : \sum_{\theta\in\Theta, c\in C} p_i^t(\theta)\, q_c(x_{ic}^t(\theta) - e_{ic}) - q_t \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta)(L - x_{iL}^t(\theta)) \le 0,\ \varphi_i^t \in \Delta \right\}$$

$B_i^t(q)$ is continuous for all $q \gg 0$. Let $\zeta_i^t(q)$ and $\varphi_i^t(q)$ be respectively the individual demand correspondences for commodities and occupations. The continuity of $B_i^t(q)$ implies that both $\zeta_i^t(q)$ and $\varphi_i^t(q)$ are upper- hemicontinous for all $q \gg 0$. Only $\varphi_i^t(q)$ but not $\zeta_i^t(q)$, though, is convex valued. By construction, however, the per capita demand correspondence $\xi_i^t(q) = \sum_{t\in T} \varphi_i^t(q)\zeta_i^t(q)$ is upper-hemicontinuous and convex valued. Hence, a standard application of the Kakutani Fixed Point Theorem in the commodity space $\Re^L$ implies the existence result.

The existence proof for the case of enforceable lottery contracts is completely analogous. It only requires more carefulness in proving that the solution of standard argument to prove that the feasible set defined by the constraint of program (10)-(12) can be bounded without loss of generality. To show this, suppose by contrary that the auxiliary program defined by (10)-(12) and the additional constraints $x_i^t(\theta) \in \bar{X} \subset \Re^C \times [0, L]$, $z_i^t(\theta) \in \bar{Z}$, $g \in \bar{G}$, with $\bar{X}$, $\bar{Z}$, and $\bar{G}$ sufficiently large, have a boundary solution such that, for some pair (θ', t'), $\hat{x}_i^{t'}(\theta)$ belongs to the boundary of $\bar{X}$. Since γ belongs to the *t-dimensional* simplex, for (10) to be satisfied there must necessarily exist at least one t such that $\hat{x}_i^t(\theta)$ belongs to the interior of $\bar{X}$, for all θ. But then the multiplier associated to the initial period budget constraints must necessarily be positive and equal to $D_{x_1} U_i(\hat{x}_i^t(\theta), \theta)$ after prices' normalization. As a consequence, from the first order optimality conditions one obtains $D_{x_c} U_i(\hat{x}_i^{t'}(\theta'), \theta') \geq k D_{x_1} U_i(\hat{x}_i^t(\theta), \theta)$ for some finite number k. But, for this inequality to be satisfied, $\hat{x}_i^t(\theta') = (..., \hat{x}_{ci}^{t'}(\theta'), ...)$ must be contained in the interior of $\bar{X}$, for $\bar{X}$ sufficiently large, since all derivatives of $U_i(x, \theta)$ are assumed to be finite. This proves that the auxiliary, bounded program and program (10)-(12) have the same set of solutions (for all positive price vectors) for $\bar{X}$ sufficiently large. The rest of the proof follows exactly the same lines as the one for the deterministic case. □

Proof of Proposition 13

Competitive equilibria satisfy the fair treatment condition, so that if $(\varphi_i^t, \varphi_i^{t'}) >> 0$, then $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$. Indeed, if $u_i^t(x_i^t) > u_i^{t'}(x_i^{t'})$, $\varphi_i^{t'} > 0$ would not be optimal.

Now, let $(x^*, \varphi^*, q^*, z^*, \phi^*)$ be a competitive equilibrium such that $(\varphi_i^{*t}, \varphi_i^{*t'}) >> 0$. Suppose it is not interim efficient, there must exist a feasible allocation $(\widehat{x}, \widehat{\varphi}) \neq (x^*, \varphi^*)$ such that $u_i^t(\widehat{x}_i^t) = u_i^{t'}(\widehat{x}_i^{t'})$ for all i, t and t' with $(\widehat{\varphi}_i^t, \widehat{\varphi}_i^{t'}) >> 0$, and $(\widehat{x}_i, \widehat{\varphi}_i) \succeq_i (x_i^*, \varphi_i^*)$ with $(\widehat{x}_i, \widehat{\varphi}_i) \succ_i (x_i^*, \varphi_i^*)$ for at least one i. Then:

$$\sum_{t \in T} \widehat{\varphi}_i^t \sum_{\theta \in \Theta} p_i^t(\theta) \sum_{c \in C} q_c^* (\widehat{x}_{ic}^t(\theta) - e_{ic}) \geq \sum_{t \in T} \widehat{\varphi}_i^t \sum_{\theta \in \Theta} p_i^t(\theta) q_t^* a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)), \quad \forall\, i \in I$$

where the inequality must be strict for at least one i. Multiplying both sides by μ_i and summing up with respect to i, one obtains:

$$\sum_{c \in C} q_c^* \left(\sum_{i \in I} \mu_i \left(\sum_{t \in T, \theta \in \Theta} \widehat{\varphi}_i^t p_i^t(\theta) \sum_{c \in C} \widehat{x}_{ic}^t(\theta) - e_{ic} - \sum_{t \in T, \theta \in \Theta} \widehat{\varphi}_i^t p_i^t(\theta) a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)) \right) \right) > 0,$$

which immediately implies that the allocation $(\widehat{x}_i, \widehat{\varphi}_i)$ violates feasibility. □

Proof of Proposition 16

Let $\langle \alpha^P(\bar{u}), x^P(\bar{u}) \rangle$ be the Pareto optimal allocation associated to vector of Pareto weights, $\bar{u} = (\bar{u}_i)_{i=2}^I$. We show that there exists an equilibrium with transfer policy $\tilde{\wp}$ with $w_i^t = \hat{w}_i^t = \eta_t^P a_i^t$, $f_i^t = 0$ and:

$$s_i^t = \sum_{\theta \in \Theta, c \in C} p_i^t(\theta)(\eta_c^P(x_{ic}^{tP}(\theta) - e_{ic}) - \eta_t^P \sum_{\theta \in \Theta} p_i^t(\theta) a_i^t(\theta) l_i^{tP}(\theta)\,,$$

such that $\varphi_i^t = \alpha_i^{tP}$, $x_i = x_i^P$, $q_c/q_1 = \eta_c^P/\eta_1^P$, $\phi_i^t = p_i^t$, for $c \in C$, $t \in T$ and $i \in I$.

First, $\tilde{\wp}$ is budget balancing by construction. Moreover, $\langle \alpha^P(\bar{u}), x^P(\bar{u}) \rangle$ satisfies as equality all the budget constraints at the prices, wages and subsidies vectors defined above. Hence $(\alpha_i^P(\bar{u}), x_i^P(\bar{u}))$ must solve the *type i* agents' maximization program. Finally, all the market clearing conditions are satisfied at $\phi_i^t = p_i^t$ and $w_i^t = \hat{w}_i^t = \eta_t^P a_i^t$ for all $t \in T$. Indeed, at these prices the supply of all state contingent assets, as well as labor demand, are indeterminate.

Consider now an economy where $x_{iL}^t(\theta) = L - L(\theta)$ for all $w_i^t(\theta) > 0$, with $\theta \in \Theta$, $t \in T$ and $i \in I$. Take a Pareto optimum $\left\langle \alpha^P(\bar{u}), x^P(\bar{u}) \right\rangle$ of this economy and consider a policy $\tilde{\wp}$ such that: $\hat{w}_i^t(\theta) = \hat{w}_i = \max_{t \in T} \left\{ \eta_t^P a_i^t(\theta_N) \right\}$; $s_i^t = \sum_{c \in C, \theta \in \Theta} p_i^t(\theta) \eta_c^P (x_{ic}^{tP}(\theta) - e_{ic}) - \hat{w}_i^t \sum_{\theta \in \Theta} p_i^t(\theta) L(\theta)$; and $f_i^t(\theta) = \hat{w}_i - \eta_t^P a_i^t(\theta)$ for all θ. By using the same argument developed above, one verifies that $(\alpha_i^P(\bar{u}), x_i^P(\bar{u}))$ solves the *type i* agents' maximization program for $\phi_i^t = p_i^t$ given the transfer policy just defined, that the market clearing conditions are satisfied, and that $\tilde{\wp}$ is budget balancing. □