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WORKING PAPER NO. 147

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and welfare effects of funding regulation

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November 2005

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Pension benefit default risk and welfare effects of funding regulation[*]

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Abstract

This paper analyzes the welfare effects of funding regulation for defined benefit pension plans subject to pension benefit default risk in an incomplete financial markets OLG-setting with aggregate uncertainty and idiosyncratic pension default risk. The financial market incompleteness arises from the inability to trade human capital claims. Using numerical methods to solve for equilibrium, we show first that default-free defined benefit pension plans are welfare-improving even in a dynamically efficient economy. Second, we show that in the presence of default risk funding regulations improve aggregate welfare by making larger size plans more attractive and that full funding is not necessarily the optimal policy. Our results provide a rationale for the widespread underfunding of defined benefit pension plans and might explain the decline of these plans after the introduction of stringent funding regulation in the US.

JEL Classification: H21, H31, H55

Keywords: generations, pension default, funding regulation

[*] I would like to thank Dimitris Christelis, Thomas Hintermaier, Tullio Jappelli, Marco Pagano, Paul Sengmueller and seminar participants at the University of Salerno and the University of Vienna for their comments. Any remaining errors are my own. Research funding from the European Commission's Research and Training Network "Financing Retirement in Europe" is acknowledged.

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1 Introduction

Employer-sponsored defined benefit pension plans have been an important part of the compensation package for dependent employees in the US in the last century. Since the beginning of the 80's the pension landscape is undergoing a dramatic change however. Most companies are phasing out wage-indexed defined benefit plans and the fraction of active workers covered by this type of plan has declined from 35.3% in 1975 to only 17.5% in 1999. This does not imply that employer-sponsored pension plans have declined in importance. The fraction of employees covered by such plans has continued to rise to 56.6% in 1999 from 49.8% in 1975. But most new plans are of a defined contribution nature, lacking indexation to future wages and carrying full investment risk like for example private savings plans. From a risk-sharing perspective, this development is puzzling, since pension economists have been claiming that precisely because of wage indexation, defined benefit pension plans provide unique risk-sharing opportunities for households and can help make financial markets more complete (Bodie, Marcus, and Merton (1985)). The underlying reason for these changes is a hotly debated issue and various explanations have been offered in the literature. One explanation is based on changes in the distribution of bargaining power between workers and firms, making it possible for firms to shift retirement income risk onto workers (Besley and Prat (2003)). The proponents of this view argue that firms profit from the move towards defined contribution plans because future profits are less exposed to longevity or wage risk now. Another argument made, claims that expected returns to stock market investment have increased relative to expected domestic wage growth and that households therefore prefer to invest a larger fraction of their wealth in claims on future capital income rather than future wage income. A third group of researchers (see Barnow and Ehrenberg (1979), Ledolter and Power (1984)) argues that excessive regulation by the government is at the root of the phenomenon, pointing towards the 1974 Employee Retirement Income Security Act (ERISA) which has introduced tight regulation of defined benefit pension plans and has created a governmental institution enforcing these regulations (the Pension Benefit Guarantee Corporation (PBGC)).

We try to answer the question under which conditions funding regulations for wage-indexed defined benefit pension plans can be "excessive" and reduce aggregate welfare. For such a situation to emerge, it is a necessary that these plans have the potential to improve welfare. We argue that incompleteness of financial markets is key to understanding why wage-indexed defined benefit pension plans are useful financial instruments for households. If claims on human capital income cannot be traded on the market[1], the wage indexation feature of defined benefit pension plans implies that these plans can partly substitute for the human capital asset. This is the source of welfare gains from additional risk-sharing in our economy. To model aggregate production risk, we assume that the factor share parameter in our overlapping generations economy is subject to iid shocks as suggested by Merton (1981). Since we are taking a general equilibrium perspective in our analysis, defined benefit pension plans are modeled primarily as an institution that provides intergenerational risk sharing, abstracting from the role of these plans as tools of corporate governance. Consequently, defined benefit pension plans are assumed to be contracts between finitely-lived households and infinitely-lived firms, paying a fixed fraction of the working generation's income as a benefit to the retired generation. We introduce pension default risk through exogenous default on pension payments and a corresponding reduction of contribution payments to the benefit of working generations[2]. Despite being relatively stylized, our setup is not tractable analytically due to the incomplete markets assumption and the heterogeneity among agents. This feature of OLG–models with aggregate risk and agents living for more than 2 periods is well-known since Huffman (1987). We

[1] It is beyond the scope of the paper to explore the reasons why human capital income cannot be traded on the market. Straightforward reasons would be legal restrictions or moral hazard (see Hart and Moore (1994)).

[2] Clearly, these modeling choices sweep under the rug many interesting and important issues at the firm-level such as profit-sharing arrangement between employees and shareholders or capital structure and firm default risk implications of pension plan investments. A general equilibrium perspective however, requires a relatively high level of abstraction and while being stylized, we feel that we still capture the main characteristics of real-world defined benefit pension plans.

therefore apply numerical methods to solve for equilibrium and evaluate welfare in the economy.

We show first that in the our model, the introduction of default-free wage-indexed defined benefit pension plans increases welfare even if the economy is dynamically efficient. This finding illustrates that in our economy defined benefit pension plans provide unique risk-sharing opportunities that allow households to reduce their consumption risk in retirement. Further, we find that upon introducing pension default risk and funding regulation, a positive relation between the average size of DB plans and the desirability of funding regulation emerges. Funding regulation makes larger size DB plans more attractive and in this sense can promote intergenerational risk-sharing while simultaneously reducing consumption risk in retirement. We also find that the optimal policy does not necessarily require full pre-funding of benefits however. This is because regulatory bodies deciding upon their policies should take into account aggregate welfare measures rather than the welfare of a subset of the population only. In our setting this implies that regulation should take into account the adverse effects on dynamic productive efficiency as well as the risk-reducing effects on retirement consumption. Clearly, any measures that regulators may take to decrease the probability of firms defaulting on promised pension benefits without affecting the welfare of future generations should be taken. Funding regulations however do affect savings and productive efficiency and should therefore be used more cautiously. Requiring that promised pension benefits are fully covered by long positions in tradable financial assets in all states of the world, turns out to reduce the expected rate of return on these assets in our model and distorts savings and investment decisions by households and firms. Rather than requiring full funding always, we find that the optimal rate of funding that should be required from companies providing wage-indexed defined benefit pension plans should depend on the average size of these plans and the probability of pension default, which might imply both, substantial underfunding or overfunding depending on preferences and technology.

The rest of the paper proceeds as follows: In the following section, we relate our methodology and results to the existing literature and then we present the model and describe our method to approximate the equilibrium. Our main results are presented in section 5, followed by concluding remarks.

2 Literature

We are not aware of other papers in the literature studying regulation of defined benefit pension plans in a general equilibrium context with heterogeneous agents, aggregate and idiosyncratic risk, and incomplete markets. In this sense, we also make a methodological contribution here. The idea that wage-indexed defined benefit pension plans can provide intergenerational risk-sharing by creating a new type of financial asset, originated in the work of Merton (1981) and Bodie, Marcus, and Merton (1985). Merton (1981) studies the complete markets case of a version of our GE-model and shows that a system of taxes and transfers resembling unfunded social security can reestablish the complete markets allocation even if claims to human capital are not traded on financial markets. In their thorough review of defined benefit pension plans in the U.S., Bodie, Marcus, and Merton (1985) claim that: "DB plans create implicit securities that can be welfare improving and which are not now available in capital markets and which might not be expected to be created in capital markets. Some examples of these "securities" are factor-share claims, price-indexed claims, and perhaps deferred life annuities at fair interest rates." Our paper can be seen as an attempt to make this point more rigorously and study the implications of funding regulations within such a model.

There is an extensive literature on the shift from DB to DC plans in the last twenty years. Ross and Wills (2002) discuss the theoretical issues involved in the choice between DB and DC plans from the point of view of both firms and employees. Barnow and Ehrenberg (1979) discuss the implications of stringent funding regulations for DB plans on firm behavior. They predict that stringent funding rules might lead to a decline of those plans. Cocco and Lopes (2004) and Huberman and Sengmueller (2004) study empirically the choice of pension plan type by employees. They find that households seem to self-select into the individually rational choice of pension plan, but that choices made are influenced also by options made available by employers. The economic literature has also produced arguments for the existence of defined benefit pension plans which we view as complementary to the argument for the existence of DB plans we use. Ippolito (1985) argues that DB plans emerge because they serve to decrease employee turnover when portability of pension rights is not ensured, while Lazear (1981) finds that they provide incentives which reduce moral hazard through their deferred compensation property.

The systematic "underfunding" of DB plans has also attracted quite some attention in the economic literature. Cooper and Ross (2002) argue that firms would underfund pension plans in a world of imperfect financial markets. Their credit constraints assumption provides a rationale for both, why DB plans exist and why they will be underfunded. We focus on missing markets rather than borrowing constraints and take a general equilibrium perspective, but our results concord with those obtained by Cooper and Ross (2002). The classical papers studying the implications of individual firm DB plans on sponsoring company finance and investment were written by Sharpe (1976), Black (1980) and Tepper (1981). Sundaresan and Zapatero (1997) and Webb (2004) analyze the optimal funding policy of a pension plan in dynamic stochastic models allowing for firm default. Besley and Prat (2003) apply the insights of capital structure theory to the design of single firm pension plans. All of these papers focus on the firm level and assume complete financial markets. Their predictions for the optimal funding of these plans is therefore not surprising: optimal behaviour of firms requires that they are fully funded in order to not expose employees to default risk. Given the empirical evidence on underfunding, we conclude that the assumption of perfect financial markets may be inappropriate here.

Our methodological approach is based on results from the literature on computation of equilibrium with incomplete financial markets. Krueger and Kubler (2004) show how equilibrium can be computed in economies with overlapping generations and aggregate risk. In Krueger and Kubler (2002) they evaluate the gains from intergenerational risk sharing in an incomplete markets context, applying their methods to the case of the introduction of social security in the United States in 1934.

3 Economy

Our economy is a simple overlapping generations economy in which each generation is of size 1 and lives for 3 periods. There is a single perishable good in the economy which can be used for both investment and consumption. There is no other technology for converting installed capital goods back to the output good other than the production technology.

3.1 Production

The single output good is produced by a large number of identical firms characterized by a standard Cobb-Douglas production function with constant returns to scale to capital and labor and no productivity growth[3].

$$F_t = i_{t-1}^{\theta_t} n_t^{(1-\theta_t)} \tag{1}$$

There are three arguments in the firms' production function: i_{t-1}, investment in period $t-1$ which equals the capital stock installed in period t, n_t, the labor input purchased by firms, and θ_t, the labor share realization in period t. The fact that i_{t-1} enters the production function directly implies that there is full depreciation in each period. The factor share parameter of the production function, θ_t, is subject to iid shocks. We assume that the random variable θ_t can take on a finite number of values and that the distribution of θ_t, $\Psi(\theta)$, is constant over time with

[3] The equilibrium allocations would be given by the same set of equations, if we would allow for productivity growth and scaled all variables by the current level of technology.

mean $E[\theta_t] = \alpha$. These assumptions define a finite-state Markov chain, Θ, which captures the stochastic process for θ_t. Product and labor markets are assumed to be competitive and the firm chooses labor inputs after having observed the shock realization. Investment becomes productive only with a one period delay. In period t, wages, w_t, and the return to capital, r_t, are therefore given by

$$r_t = \theta_t i_{t-1}^{(\theta_t - 1)} n_t^{(1-\theta_t)} \tag{2}$$

$$w_t = (1 - \theta_t)\, i_{t-1}^{\theta_t} n_t^{-\theta_t} \tag{3}$$

The fact that uncertainty enters the model through stochastic variation in aggregate factor shares needs some motivation, since the standard assumption in macroeconomics is that factor shares are constant over time. It turns out that recent empirical evidence shows that factor shares did fluctuate significantly over the last 40 years in developed countries. While the US and Canada had the most stable factor share series of all OECD-countries in the period 1960-2000 and even in those countries factor share varied by a few percentage points. This empirical fact is documented by Blanchard (1997). Jones (2005) proposes a model based on search and embodied technological change to explain the fluctuations, while Blanchard and Giavazzi (2003) argue for a wage bargaining model. We follow Merton (1981) and assume that some exogenous factor drives factor share changes directly which is consistent with both arguments. Since the period length in our OLG-model is about 20 to 30 years, we assume that these shocks are iid over time.

3.2 Households

All generations share the same time-separable preferences over the single consumption good, are endowed with h^s units of labor at age s and do not discount the future. Since all households within a generation are equal, consumption is indexed only by the time period in which it occurs t and the superscripted age of the household 0, 1, or 2. There is no bequest motive, lifetime is deterministic and households do not have a financial endowment when they are born. The objective function of a generation born in period t is therefore given by

$$U_t^0 = u\left(c_t^0\right) + E_t\left[u\left(c_{t+1}^1\right) + u\left(c_{t+2}^2\right)\right] \tag{4}$$

Age 0 households are born without assets and their budget constraint is simply

$$c_t^0 = h^0 w_t - s_{t+1}^0 \tag{5}$$

where s_{t+1}^0 denotes the savings of age 0 agents carried over from period t to period $t+1$. At age 1 households born in period t earn (pay) interest, r_{t+1}, on their initial savings (borrowings) and save some amount s_{t+2}^1 to finance future consumption. Their budget constraint therefore reads

$$c_{t+1}^1 = h^1 w_{t+1} + r_{t+1} s_{t+1}^0 - s_{t+2}^1 \tag{6}$$

At age 2, households do not have a labor endowment, they simply collect the interest on their savings and consume everything.

$$c_{t+2}^2 = r_{t+2} s_{t+2}^1 \tag{7}$$

Financial markets are incomplete in the sense that human capital cannot be traded in the market. Households are therefore not able to insure fully against the systematic risk originating from the factor share fluctuations. If human capital would be tradable, age 0 households could sell their future labor endowment, finance their consumption and invest the resulting savings optimally in both physical and human capital claims. Age 1 and 2 households would derive their income from both, human and physical capital, and as a result risk-sharing within the economy would be optimal (see Breeden (1979) for necessary conditions for optimal risk sharing). Apart from aggregate output risk, agents would be able to diversify all income risks and each generation would consume a fixed fraction of output. This result has been shown by Merton (1981) for the log-utility case. If claims on human capital are not tradable, the marginal rates of substitutions of all generations alive cannot be equalized through trade and risk-sharing is therefore suboptimal. All generations have to bear additional consumption risk.

3.3 DB pension plans and funding regulations

Once we have obtained the functions describing the equilibrium path of the aggregate economy, we ask whether there exist potential welfare gains from risk-sharing among households. In particular, we consider the case of firms offering a defined benefit pension plan to its employees. We introduce DB plans exogenously, assuming that firms and households agree on such an arrangement and treat the size of the DB plan as a parameter of the economy. The DB plan requires employees to pay a fraction τ of wages into a pension fund. Retirees receive benefits paid from the fund which are proportional to the going wage rate.

The consumption risk-reducing feature of our defined benefit plans is that they are indexed to wages since the missing financial market in our economy is the market for claims on human capital. When defined benefit pension plans link retirement benefits to current and future wages by defining benefits as a fraction of total wage income received adjusted by the change in wages from period t to period $t+1$, they provide a payoff that is equivalent to the payoff of a claim on human capital. The pension benefit formula in this case reads

$$b_{t+2} = \tau w_{t+1} \frac{w_{t+2}}{w_{t+1}} \tag{8}$$

where b_{t+2} are the benefits paid to the generation born in period t. If there is no default risk, this benefit rule implies that a completely unfunded plan is financially viable. The generations of age 0 and 1 always finance through their contribution payments exactly the benefits to be paid out to the current retiree generation. In the absence of funding regulations, we assume that firms and households prefer this financing arrangement.

If default is possible, defined benefit plans no longer provide only additional risk-sharing possibilities by acting like a claim on human capital, but also introduce additional risk to retirement consumption - pension default risk. We incorporate pension default risk by assuming that only a fraction δ of households actually receives the promised pension claims. For a fraction $1-\delta$ of households, the pension fund does not have any funds to pay out the pension benefit and retirement consumption has to be financed by private savings only. Households learn about the default realization only when they are about to claim their benefits, but they anticipate that default is a possibility and adjust their consumption policies accordingly. We assume that the pension benefits not paid out to workers are not lost, but are used to reduce the pension fund contributions by the working generations. Note that since there is a continuum of households, the default risk washes out in the aggregate.

One way to reduce the risk to retirement benefits created by the possibility of default and an accompanying reduction in contribution payments is to require the firms to pre-fund the expected retirement benefits. This route has been taken by the US government in the 1970's and it is a hotly debated issue in Europe how strict such funding regulations should be. While obviously the debate on optimal funding regulations is just one aspect of a larger problem - which is the optimal provision of retirement income in general - we focus on the funding issue since our model allows for a concise formulation of defined benefit pension plan funding and its impact on benefits, contributions and profits.

Our modeling of pre-funded pension plans assumes that each firm runs its own pension fund and that neither the firm nor the fund have problems in accessing the capital market. Funding regulation is modeled parametrically as the fraction of pension payments which is secured against pension plan default by pre-funding.

Our full funding benchmark is the amount of retirement benefits that each firm would have to pay in the worst-case scenario (the highest possible realization of retirement benefits). Given our assumptions on the production function and benefit rules, the maximum level of benefits in period $t+1$ is given by

$$\text{Full Funding Benchmark} = max\left(\frac{\tau w_{t+1}}{r_{t+1}}\right) = \frac{\tau\left(1-\bar{\theta}\right)}{\bar{\theta}} i_t \tag{9}$$

where $\bar{\theta}$ is the lowest possible realization of the profit share. Funding regulation chooses a parameter λ which requires each firm to pre-fund at least a fraction λ of this benchmark. The funding requirement f_t in period t therefore is

$$f_{t+1} = \lambda \frac{\tau\left(1-\bar{\theta}\right)}{\bar{\theta}} i_t \tag{10}$$

We implement this by assuming that the pension plan is funded only through contributions by the working generation and that firm profits are not affected directly. There is an indirect effect on firm profits in our formulation, because the assets of the fund are reinvested in the firms, increasing the capital available for production in the next period.

$$s^0_{t+1} + s^1_{t+1} + f_{t+1} = i_t \tag{11}$$

The positive effect of pre-funding is that in case of default the benefits of retirees are at least partially secured. We assume that the retirees actually have a claim on the entire fund in case of default which explains why they get the entire fund in case of default in 15. Contributions of the working generation are adjusted in each period to ensure that the pre-funding requirements are exactly met. They are reduced in period t by the amount of contributions that are not paid to retirees due to pension plan default, $(1-\delta)\,\tau w_t$ and they are increased by the difference between the required pre-funding for all plans and the amount of pre-funding available from surviving pension plans. This formulation implies that all contributors are treated equally and that no intertemporal linkages exist between pension plan defaults and the expected retirement benefit for a given household. After each period, all pension plans divide up the existing assets and demand contributions from each contributor which exactly fulfill the pre-funding requirement. The budget constraints with default risk and funding regulations read:

$$c_t^0 = h^0 \left(w_t - \delta \tau w_t + \left(f_{t+1} - \delta r_t f_t \right) \right) - s_{t+1}^0 \tag{12}$$

$$c_{t+1}^1 = h^1 \left(w_{t+1} - \delta \tau w_{t+1} + \left(f_{t+2} - \delta r_{t+1} f_{t+1} \right) \right) + r_{t+1} s_{t+1}^0 - s_{t+2}^1 \tag{13}$$

$$c_{t+2}^2 = b_{t+2} + r_{t+2} s_{t+2}^1 \tag{14}$$

where

$$b_{t+2} = \begin{cases} r_{t+2} f_{t+2} & \left| \; if \;\; \delta_i = 1 \right. \\ \tau w_{t+2} & \left| \; if \;\; \delta_i = 0 \right. \end{cases} \tag{15}$$

and the distribution of the random variable δ_i is iid across households and time and such that each household with probability δ receives the full benefit ($\delta_i = 0$) and with probability $1 - \delta$ it receives only the pre-funded portion ($\delta_i = 1$). Now, we define an equilibrium in this economy and describe our procedure to solve the model.

4 Equilibrium

Following Krueger and Kubler (2004) we define a recursive equilibrium of this OLG-economy which takes the distribution of capital holdings as the endogenous state space. In the economic literature, these equilibria are referred to as "Functional Rational Expectations Equilibria" (FREE), a terminology introduced by Spear (1988). We describe the endogenous state space by a two-dimensional box defined by lower and upper bounds on the aggregate capital stock, i_{t-1}, and the share of capital held by age 1 households, μ_t^0, in period t. We denote these bounds by $\left(\underline{i}, \overline{i}\right)$ and $\left(\underline{\mu}, \overline{\mu}\right)$, respectively.

The recursive equilibrium is defined by a set of policy functions for households,

$\left\{c_t^j\right\}^{j=0,1,2}$ and the representative firm, $\{n_t\}$, and a set of competitive prices, $\{r_t, w_t\}$, such that given an initial condition, $\left(i_{-1}, \mu_0^0, \theta_0\right)$, the following conditions hold at all periods $t = 0, ..., \infty$

1. for all generations alive, the choices $\left\{c_t^j\right\}^{j=0,1,2}$, maximize the household's utility subject to the budget constraints and given equilibrium prices $\{r_t, w_t\}$,

2. the representative firm maximizes profits,

$$n_t \equiv argmax_{n_t}\left[F_t - w_t n_t\right] \tag{16}$$

3. aggregate savings equals aggregate investment, $s_{t+1}^0 + s_{t+1}^1 + f_{t+1} = i_t$,

4. the labor market clears, $n_t = h^0 + h^1$,

5. and aggregate output equals aggregate expenditure,

$$c_t^0 + c_t^1 + c_t^2 + i_t = F_t \tag{17}$$

The Euler equations relevant for the computation of equilibrium which in this model are necessary and sufficient conditions for optimality of household's consumption choices read

$$u'\left(c_t^0\right) = E_t\left[r_{t+1} u'\left(c_{t+1}^1\right)\right] \tag{18}$$

$$u'\left(c_t^1\right) = E_t^d\left[r_{t+1} u'\left(c_{t+1}^2\right)\right] \tag{19}$$

where E_t is the expectations operator with respect to the factor share realizations and E_t^d is the expectations operator with respect to the joint realizations of factor share and pension plan default.

4.1 Solution method

An analytical solution for the equilibrium in this economy is not available. We therefore approximate the equilibrium by means of a computational procedure which solves the system of equations defining the equilibrium at a finite number of points and uses function approximation techniques to determine equilibrium choices off the grid. The recursive structure of the problem allows us to focus on a single system of equations despite the infinite number of periods for which this economy exists.

The method we use to solve the system of equations has been introduced by Judd (1992) and termed the computational method "a projection algorithm". The key idea in this approach which is extensively discussed in Judd (1997) approximate the equilibrium decision rules of the household by a finite-dimensional polynomial defined by a vector of unknown coefficients and solve the system of equations at a predetermined set of points. We denote the approximate policy rules for ages 0 and 1 by $\hat{c}_t^0\left(i_{t-1}, \mu_t^0, \theta_t; \xi\right)$ and $\hat{c}_t^1\left(i_{t-1}, \mu_t^0, \theta_t; \xi\right)$ where ξ is the vector of coefficients defining the approximation. The number of points at which the Euler equations are evaluated and solved is equal to the number of coefficients of the policy functions to be determined. In order to implement this procedure we need to define a finite grid on an appropriate endogenous state space on which the approximating functions are defined and the system of equations is solved. As mentioned above, we use a 2-dimensional box, B, as our endogenous state space and define G gridpoints on this space. The entire state space is just the product space of the grid on B and the state space of the finite-state Markov chain, Θ. The precise definition of the system of equations, Λ, that we use to solve for the functional rational expectations equilibrium is given by

$$u'\left(\hat{c}_t^0\left(i_{t-1}, \mu_t^0, \theta_t; \xi\right)\right) = E_t\left[r_{t+1}\left(i_t, h^0+h^1, \theta_{t+1}\right) u'\left(\hat{c}_{t+1}^1\left(i_t, \mu_{t+1}^0, \theta_{t+1}; \xi\right)\right)\right] \tag{20}$$

$$u'\left(\hat{c}_t^1\left(i_{t-1}, \mu_t^0, \theta_t; \xi\right)\right) = E_t^d\left[r_{t+1}\left(i_t, h^0+h^1, \theta_{t+1}\right) u'\left(c_{t+1}^2\left(i_t, \mu_{t+1}^0, \theta_{t+1}\right)\right)\right] \tag{21}$$

$$i_t\left(i_{t-1},\mu_t^0,\theta_t\right) = \frac{\bar{\theta}}{\bar{\theta}-\lambda\tau\left(1-\bar{\theta}\right)}\left(h^0+h^1\right)\left(\left(1-\delta\tau\right)w_t-\delta r_t f_t\right) + r_t\mu_t^0 i_{t-1} - \hat{c}_t^0\left(i_{t-1},\mu_t^0,\theta_t;\xi\right) - \hat{c}_t^1\left(i_{t-1},\mu_t^0,\theta_t;\xi\right) \tag{22}$$

$$c_{t+1}^2\left(i_t,\mu_{t+1}^0,\theta_{t+1}\right) = \tau w_{t+1} + r_{t+1}\left(h^1\left(\left(1-\delta\tau\right)w_t+\left(f_{t+1}-\delta r_t f_t\right)\right)+r_t\mu_t^0 i_{t-1}-c_t^1\right) \tag{23}$$

plus the functions defining f_t, w_t For the continuous approximation of the policy functions $\hat{c}_t^0\left(i_{t-1},\mu_t^0,\theta_t;\xi\right)$ and $\hat{c}_t^1\left(i_{t-1},\mu_t^0,\theta_t;\xi\right)$ an appropriate basis of functions must be chosen. Judd (1992) recommends orthogonal polynomials as an appropriate basis and we opt for using a tensor product base of Chebyshev polynomials. We choose the same degree of approximation k in both dimensions of the endogenous state space and therefore use the set of basis functions

$$T \equiv \left\{T_i\left(x\right)T_j\left(y\right)|0\leq i\leq k, 0\leq j\leq k\right\}$$

where $T_n\left(x\right)$ denotes the n-th degree Chebyshev polynomial evaluated at point x. The chosen gridpoints in each dimension of the endogenous state space correspond to the $k+1$ zeros of the degree $k+1$ Chebvyshev-polynomial.

Starting with an initial guess for the policy functions, one can solve the Euler equations at the grid points through the use of a non-linear equation solving procedure for multidimensional functions. The solution procedure determines the vector of coefficients for the policy functions at which the system of equations is exactly fulfilled. In this procedure it is important to start with a good initial guess, because convergence is not likely when the initial guess is far from the solution. The iterations over the approximation of the policy functions stop when the coefficients of the approximating functions do not change by much anymore. The resulting consumption rules define the equilibrium policy function of households at age 0 and 1. Together with the budget constraints and the market-clearing conditions they define the approximate equilibrium of the economy. The importance of the initial guess for the convergence of the algorithm described above requires a "continuation method" for solving the system of equations for an arbitrary set of parameter values. Starting from the solution of a simple case without uncertainty and log-utility for which an analytical solution exists, we move to the solution of the model by gradually increasing the amount of uncertainty and the degree of risk-aversion of the utility function. A single solution step with a good initial guess requires approximately 1-2 minutes computation time on standard Desktop-PC running MATLAB.

4.2 Calibration of the benchmark economy

To solve numerically for the equilibrium of the economy, we have to specify the functional form of the utility function and choose values for the parameters. We follow standard practice and choose power utility with the relative risk aversion parameter equal to 2. The only other parameters we need to specify are the labor endowments h_0 and h_1, the vector of possible realizations θ and the transition matrix M of the stochastic process of the factor share. We assume that labor endowments are constant and sum to 1, implying $h_0 = h_1 = \frac{1}{2}$. In order to limit the dimensionality of the system of equations to be solved, we set the number of possible theta realizations to 3 and derive the values of the transition matrix and the realization vector from a discretized normal distribution with mean $\alpha = 0.3$ and standard deviation $\sigma_\theta = 0.03$, choosing one standard deviation as the interval on the grid for θ. This results in a vector of possible realizations $\theta = (0.27, 0.3, 0.37)$ and a transition matrix with equal rows given by $m = (0.274, 0.452, 0.274)$. Approximation errors of the Euler equation are of the order $\exp(-7)$ already with a total of 25 basis functions ($k = 4$).

The dynamics of the capital stock do not display long-run fluctuations since there is complete depreciation in each period and the population of households is renewed after 2 periods. There is still considerable variation in the factor share, the capital stock and the consumption of households due to the factor share shocks however. The discrete nature of the factor share process and the absence of persistent dynamics imply that the capital stock realizations of the next period depend heavily on the current realization of the factor share shock. Figure 1 illustrates this

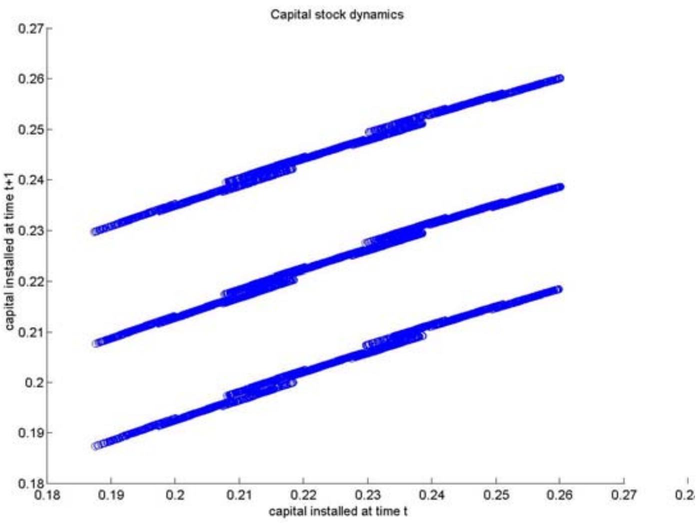

Figure 1: Capital stock dynamics

5 Results

All simulations reported in this section are based on the equilibrium policy functions that have been computed according to the procedure described above. In order to be able to do the welfare comparisons, we first fix a sequence of realizations, $\hat{\theta}$, of the random variable θ and an initial condition, $\left(\hat{\imath}_{-1}, \hat{\mu}_0^0, \hat{\theta}_0\right)$, that are used for all simulations.

5.1 Measuring welfare

Since the focus is on finding optimal policies in this paper, we need to compare equilibrium allocations across different policy regimes. To do this, we need a suitable measure of welfare and opt for a standard criterion: the certainty equivalent of consumption giving average expected utility at birth in the stationary equilibrium of the economy. This criterion is adequate for a long-run perspective of regulation, but does not take into account the transition to the stationary equilibrium, which might be an important issue in designing real-world policies. Formally, we compute the expected utility of the economy described by the parameter vector Φ as

$$EU^{\Phi} = \frac{1}{T-2}\sum_{s=1}^{T-2} u(c_s^0(\zeta)) + \sum_{\zeta'} \pi(\zeta'|\zeta)u(c_{s+1}^1(\zeta')) + \sum_{\zeta''} \pi(\zeta''|\zeta)u(c_{s+2}^2(\zeta''))$$

where ζ is a draw of the state variables out of the stationary distribution of the economy and $\pi(\zeta'|\zeta)$ and $\pi(\zeta''|\zeta)$ are the corresponding 1-step and 2-step transition probabilities from state ζ to states ζ' and ζ'', respectively. The we compute the certainty equivalent CE^{Φ} as $CE^{\Phi} = u^{-1}\left(EU^{\Phi}\right)$. To obtain the stationary distribution of our economy, we first draw a sequence $\left\{\tilde{\theta}\right\}_{j=1}^{T+100}$ of factor share realizations that we hold constant throughout all simulations. We initialize the simulation at the midpoint of the box B for the endogenous state variables and then compute the realizations of all variables for the entire simulation horizon $T + 100$. The transition to the stationary distribution is rapid in this economy because of the absence of serial correlation in the exogenous state variable and little persistence in the distribution of capital stocks due to the finite horizon and full depreciation. Therefore after cutting out the first 100 periods, we are almost sure to have reached the stationary distribution and expected utility is then calculated as the average of expected utility over $T - 2$ draws out of the stationary distribution.

5.2 Default-free pension plans

If financial markets were complete and agents were able to trade human capital claims, risk-sharing among generations would be perfect and consumption shares of output allocated to each generation would be constant. Default-free defined benefit pension plans in this setting could not improve risk-sharing anymore and would only affect welfare indirectly by affecting dynamic productive efficiency, not by changing the consumption allocation given available resources. This result was shown by Merton (1981), but breaks down, if financial markets are imperfect. The equilibrium consumption allocations show considerable variability, if financial markets are incomplete. Figure 2 displays the relative deviations from the mean share of total output allocated to consumption of each age group.The graph shows that

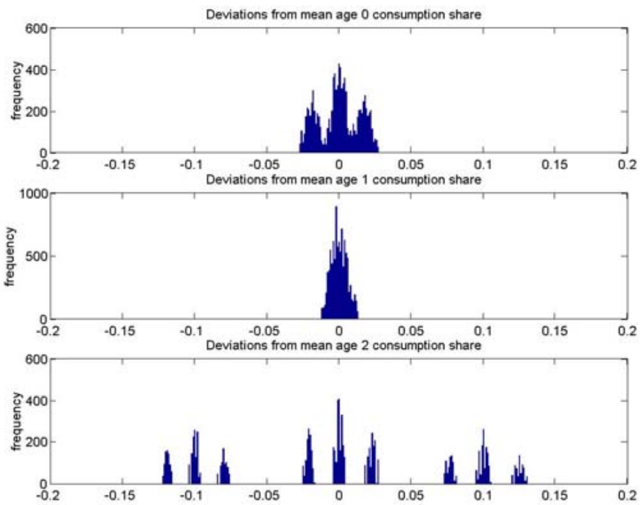

Figure 2: Deviations from mean consumption shares

the age group mostly affected by the inability to trade human capital is the generation of retirees. The consumption share allocated to them varies a lot more than those of the generations in working age. The middle generation is best insulated against factor share shocks, since this generation holds a diversified portfolio of human and physical capital. The young generation holds human capital only, but can self-insure against factor share risk by adjusting savings. The retired generation is exposed the most because they hold capital assets only and cannot adjust savings anymore. Quantitatively, these deviations can be quite large reaching up to 15% of the mean consumption share which amounts to about a $3-4\%$ share of total output.

Introducing default-free defined benefit plans in this setting reduces consumption share variability primarily for the retired generation and in this sense improves intergenerational risk-sharing. The defined benefit pension plan provides the age group of retirees with a substitute for human capital claims and enables them to hold a more balanced portfolio. The portfolios of the working generations are not affected that much, since the young still hold human capital only, and the middle ages already held relatively well-diversified portfolios in the previous solution. Figure 3 displays the variability of consumptions shares if the parameter governing the size of defined benefit pension plans, τ, is set to 0.15. The reduction in

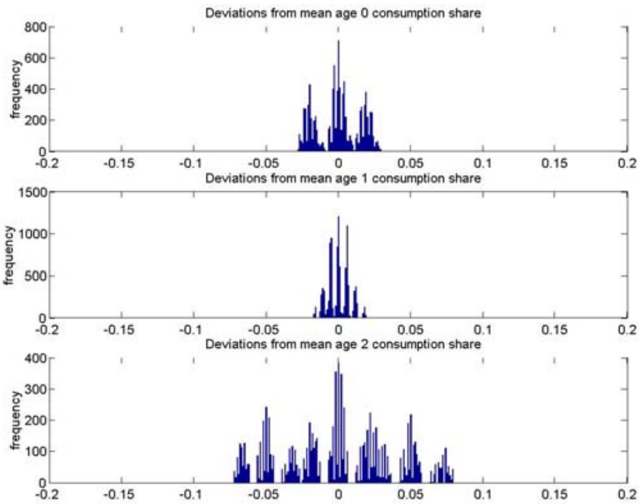

Figure 3: Consumption share variability with DB plans

consumption share variability for the old is clearly visible. from the graph, although the retired generation still remains the most affected by financial market incompleteness. In fact, there is an optimal size of default-free DB pension plans which induces optimal intergenerational risk sharing. The overall welfare effects of default-free defined benefit pension plans do not only depend on the variability of consumption shares, but also on how they affect dynamic productive efficiency of the economy. Krueger and Kubler (2004) claim their numerical results suggest that production effects often overcompensate the consumption variability effects and we also find evidence for this. In fact, the more inefficient the dynamic allocation of productive resources is, the more positive are the welfare effects of default-free pension plans. Financial market incompleteness does imply however, that even in an economy with optimal dynamic allocation of productive resources, welfare can be improved by introducing default-free pension plans. Now, we explore the consequences of allowing for default on pension benefits and discuss the welfare effects of funding regulation in this context.

5.3 Default risk and regulation

Allowing for default on the defined benefit pension plans must limit the welfare gains that can be obtained from introducing them, because additional consumption risk is introduced by the default risk. Although households can partially self-insure against the default by accumulating larger private savings, the overall effect must be negative. The surprising finding is that quantitatively, the reduction in welfare is not as large as one might expect. This finding is illustrated by Figure 4 which shows a welfare index, computed as the ratio of the certainty equivalent of a given parametrization with respect to the certainty equivalent of the no DB plan case, that varies with the average size of defined benefit pension plan and the default probability of these plans. Default-free plans with a contribution rate of about 5% increase welfare by about $1,3\%$ in our calibration. DB plans larger on average than about 10% of wages actually reduce welfare due to their negative effect on dynamic productive efficiency. Allowing for default on plans of the moderate sizes analyzed here does not affect these results qualitatively. Even with a 15% default probability, which seems unreasonably large considering the frequency of actual default on defined benefit pension plans, welfare is still improved by the introduction of these plans by about $1,1\%$. The optimal size of the plans is actually increasing in the default probability since households partially insure against the default risk by accumulating larger savings and this effect on dynamic productive efficiency is offset only by a larger plan size. These results suggest that in a standard calibrated

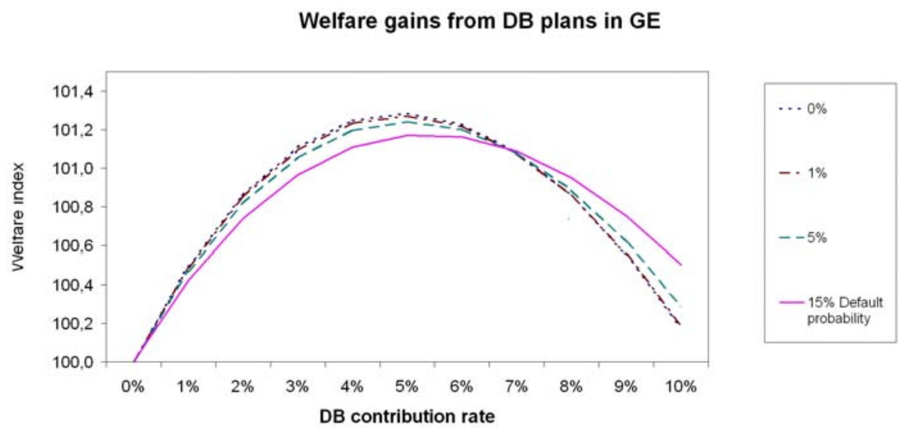

Figure 4: Default probability and welfare

OLG economy with imperfect intergenerational risk-sharing, wage-indexed defined benefit pension plans robustly improve welfare as long as they are of moderate size. It should be noted however, that an important restriction of our model is that there is no heterogeneity among households of one generation at all. Our results therefore only apply of the assumptions necessary for the representative individual to represent the whole population are valid within one generation. Generalizing the model to allow for substantial wealth or preference heterogeneity has so far not been possible for computational and theoretical reasons although some papers have made attempts to find approximate solutions for these types of models (see Krusell and Smith (1998) or Storesletten, Telmer, and Yaron (2004)).

Even if the welfare consequences of moderate default rates on defined benefit pension plans appear to be small, we now want to ask whether funding regulations are able to improve welfare in the presence of default risk. We try to answer that question by again comparing expected utilities across different economies, abstracting from transitory effects. The analysis uncovers a positive relation between the average size of DB pension plans and the desirability of stringent funding regulation documented by Figure 5. The figure contains graphs of our welfare index varying with the average size of DB plans, DB plan default probability and the required funding ratio. For small size plans, funding requirements do not increase

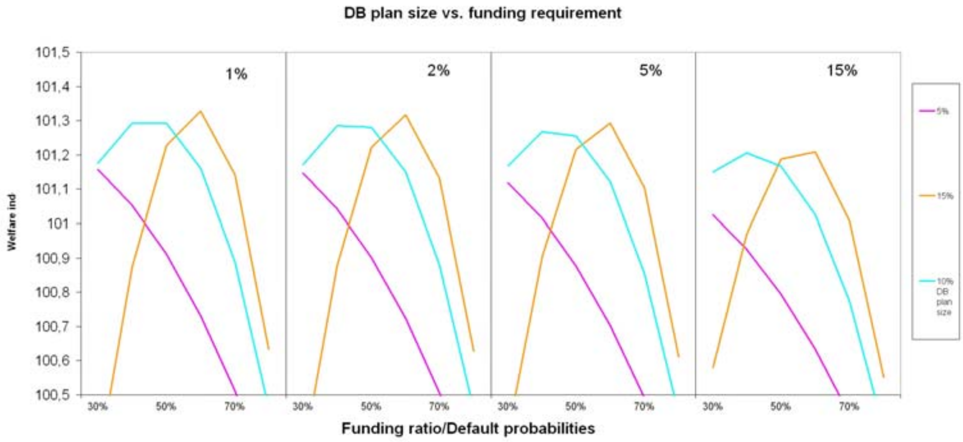

Figure 5: Welfare effects of funding regulation

welfare, since households are able to relatively efficiently self-insure against default of these plans and because the effect on dynamic productive efficiency is negative. As the default probability increases, welfare is somewhat reduced however. As the fraction of retirement consumption financed by DB plans grows, funding regulation becomes more desirable and can actually increase welfare substantially with respect to the unfunded case. If default probability is high, larger funded plans achieve higher utility than smaller unfunded plans. With a 5% default probability for example the increase in welfare by moving from an optimal size unfunded plan to an optimally funded plan twice as large amounts to a 0.1% increase in certainty equivalent consumption. Funding regulation therefore increases welfare by making larger size plans more attractive and households benefit from having larger size plans. There are two separate effects of funding regulations which make this possible. The first is that funding regulation reduces the risk in retirement income by providing insurance against default, the second effect is that in our incomplete markets economy, funding regulation affects also the dynamic productive efficiency of the economy. We do not find however, that requiring full pre-funding of defined benefit pension plans is the optimal choice of regulators. In fact, in our calibration substantial underfunding is optimal from a long-term welfare perspective. There are also cases however, in which overfunding would be optimal. The important consideration for regulators is that not only the insurance aspect of funding regulation, but also the production efficiency aspect needs to be considered.

6 Conclusion

We have analyzed the long-run welfare effects of funding regulation for wage-indexed defined benefit plans with positive exogenous default probability in an OLG-economy with incomplete financial markets and aggregate risk. The financial market incompleteness results from the assumption that claims on human capital cannot be traded among agents. Aggregate risk emerges from iid shocks to the factor share. In this economy, default-free DB plans provide a particular type of financial asset which improves intergenerational risk sharing and welfare even if the economy is dynamically efficient. The reason is that primarily the retiree generation cannot hold well-diversified wealth portfolios in the absence of tradable human capital claims. DB plans provide a substitute for such claims and improve the portfolio allocation of households. With positive default probability, DB plans also introduce additional risk to consumption however, and funding regulation is useful to offset these risks. A positive relation between the average size of DB plans and the optimal funding requirement for DB pension plans emerges. This relation emerges from two effects: first, the higher the fraction of consumption in retirement financed by the DB plan, the larger the benefit from funding regulation; second, the funding regulation affects dynamic productive efficiency positively, if DB plans are large. The optimal funding regulation however is not necessarily the one that requires full pre-funding of DB pension plans. In our calibration, substantial underfunding is actually the optimal policy for plans of moderate average size. In other cases, also overfunding might also emerge as the optimal policy. The important consideration for regulators is that not only the insurance aspect of funding regulation, but also the production efficiency aspect needs to be considered. Interpreting the results of our analysis and applying the model to recent

US experience, these findings might explain why a large part of defined benefit plans is underfunded and why defined benefit pension plans have declined in the US as stringent funding regulation affected negatively the welfare of households given the size of these plans. The analysis also suggests that if regulation is not able to affect the average size of DB plans, it should attempt to improve welfare by directly reducing the default probability of defined benefit pension plans and choose funding requirements appropriate for given average size of DB plans. If instead the government is able to regulate both the size of DB plans and the funding requirement, the optimal regulatory policy would be to have large size plans with substantial overfunding. This policy would eliminate both sources of inefficiency in the economy, the insufficient intergenerational risk-sharing and the dynamic inefficiency in production.

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