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Abstract

When the government provides public services necessary to production tax evasion results in some degree of income redistribution which may imply an higher or a lower level of aggregate income in the long-run. The outcome mainly depends on the burden of fiscal pressure. If the tax administration is harmed by corruptibility of some agents then the performance of the economy is also affected by the diffusion of corruption, its impact depending upon the cost of detecting a bribe agreement. When such cost varies with the stage of development, as it happens if the latter determines the level of transparency, then poverty traps may emerge and the steady state level of income will depend on the initial condition. Some implications of the model are in line with recent empirical evidence.

JEL Classfication: D73, H26, H41, K42, O41
Keywords: Tax evasion, corruption, endogenous deterrence, public spending, poverty traps

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1 Introduction

This paper develops a simple model where tax evasion, corruption, and their deterrence are jointly determined with capital accumulation, in a framework where the government provides public services necessary to production. In general, tax evasion and corruption are found to have ambiguous effects on the level of economic development and to result in some degree of income redistribution. Tax evasion increases the amount of resources some entrepreneurs may retain for private investment but it also reduces the amount of public services supplied by the government, thus inducing a negative externality to the entire economy. The net effect on capital accumulation and welfare depends on the burden of fiscal pressure. When the government tends to heavily distorts the composition between public and private inputs of production, in favor of the public one, tax evasion mitigates the ensuing inefficiency by reducing the effective level of taxation and thus the size of the public sector. The peculiar effect of bribery in tax administration rests on recognizing that its detection is related to the level of trasparency in the economy, that is on the difficulty to detect the bribes. At an initial stage of development the level of trasparency can be very low, thus determining a very high cost of detecting bribery. In this case, a poverty trap due to the pervasiveness of corruption may emerge.

Barreto (2000) and Ellis and Fender (2003) also investigate the implications of misbehavior on economic growth driven by public capital accumulation. In both papers public agents who have discretionary power in *supplying* public goods may appropriate of rents by determining a lower amount of the goods.[1] In the present paper misbehavior is instead related to entrepreneurs and public agents, whose choices determine the amount of resources the government can *raise through taxation.* Thus, the present analysis may be regarded as complementary to previous ones. The necessity of raising funds in order to supply public services provides the rationale for the government of taxing entrepreneurs profits. Rational entrepreneurs, however, may decide to underreport the tax base in order to save on taxes. Thus, the government assigns to some individuals the task of auditing a

[1] Barreto (2000) assumes that self-seeking public agents, acting as representatives of the government, exploit the monopoly power that the government has over the provision of public goods, thus determining that less public goods are provided at higher prices. Moreover, by assuming that the public sector is subject to bureaucratic red-tape and that corruption can alleviate red-tape, it follows that the corruption equilibrium may be Pareto superior to the bureaucracy plagued competitive equilibrium: corruption implies the transfer of resources that would have otherwise been lost to bureaucracy. Ellis and Fender (2003) assume that it cannot be verified until a later date whether the government has applied tax revenues to public capital accumulation or simply consumed them. The higher is the time lag in public capital production the higher is the corruption-output ratio.

subset of tax reports. However, since any auditor may conceal the evidence of evasion, he might decide to take a bribe from the entrepreneur in return for not reporting his evasion. Thus, corruptibility in tax administration provides the rationale for the existence of some individuals who investigate on bribery.[2]

The main assumptions of the model are as follow. First, I consider the behavior of taxpayers-entrepreneurs facing the probability that evasion is documented, once an audit takes place, positively related to the amount of evasion. As noted by Slemrod and Yitzhaki (2000), this feature of the verification technology characterizes most tax systems. Second, in order to deter tax evasion the government is assumed to set a maximal fine, that is a fine equal to the disposable income of the taxpayer. This assumption is consistent with one of the most popular conclusion of the economic literature on crime deterrence, which implies that maximal fine is an important policy instrument to maximize welfare (Becker, 1968; Polinsky and Shavell, 2001). The main conclusions of the paper, however, remain true even relaxing such assumption, the crucial feature being an inverse relationship between the expected income in case of evasion and the tax rate. Third, in the case of corruptibility the probability of detecting corruption is endogenously determined. In particular, the government relates the incentive to detect a bribe agreement to the fine for evasion. This seems a natural assumption as corruption in tax administration implies that such fine does not accrue to the public revenue. Moreover, it recognizes that the fine structure set by a legislator usually affects both the incentive to misbehave and the incentive to monitoring misbehavior. The resulting deterrence structure implies that an higher level of fiscal pressure induces more evasion and may favor the diffusion of corruption and that a lower rate of corruption, determined by an higher fine, could entail as side-effect a larger size of evasion. Fourth, the production-side of the economy is based on the work of Barro (1990); that is, there is a single composite good which is a function of private capital and public services, the latter produced from public revenues. Finally, each individual in the economy makes his choice taking as given others actions; in particular, each entrepreneur does not internalize the effect of his decision on the level of public services supplied by the government.

In the present framework, the share of taxable income available for consumption and saving depends on whether the entrepreneur is audited and, in case of auditing, on whether the evasion is detected. In the latter case, such share depends upon the diffusion of corruption. Since the total amount of resources raised by the government determines the quantity of public services necessary to production, for any given level of taxation the decision of the group of entrepreneurs about the size of evasion affects the aggregate

[2]This simplified two-levels hierarchy resembles that in Kofman and Lawarrée (1993).

level of output and in turn the level of taxable income itself. Thus, when the government provides public services necessary to production tax evasion results in some degree of income redistribution which may imply either an higher or a lower level of economic development. In particular, if the government tends to heavily distorts the composition between public and private inputs of production, tax evasion might mitigate such inefficiency by reducing the amount of public resources. Moreover, the long-run performance of the economy is also affected by corruption, its impact depending upon the cost of detecting the bribes. In particular, when such cost varies with the stage of economic development, history matters and poverty traps may emerge.

The rest of the paper is as follows. Section 2 presents the model economy. In section 3 I derive the equilibrium levels of evasion and corruption, for a given level of capital, and provide some comparative statics results. Section 4 illustrates the impact of tax evasion and corruption on capital accumulation. Section 5 concludes.

2 The structure of the model

Consider a two period overlapping generations economy consisting of risk-neutral individuals with constant size. In each period the young consist of a large number J of private entrepreneurs and of N public agents engadged by the government. Both N and J (with $N < J$) are exogenously given. The young divide the *disposable income* resulting from their activity between first-period consumption and saving. The saving in period t contributes to generate the capital stock in period $t+1$. Apart from the level of wealth, there are not relevant differences among individuals when old. The old simply consume all their wealth, that is the saving and any interest rate they earn on saving held from period t to period $t+1$. There is some private capital stock K_0 that is owned equally by all individuals of the initial old generation.

Output Y is produced by entrepreneurs using private capital K, rented from old individuals, and free public productive services G, supplied by the government. In particular, following Barro (1990) among others I concentrate on nonrival and nonexcludable type of public goods and assume a Cobb-Douglas production function. Hence, in period t a young entrepreneur j (with $j = 1, ..., J$) produces output according to

$$Y_{j,t} = AK_{j,t}^{\beta} G_t^{\gamma} \tag{1}$$

with $\beta \leq 1 - \gamma$.[3] Market for capital is perfectly competitive so that capital

[3] A more standard economy structure with working households in each generation and both labor as well as capital production inputs is easy to accomodate.

rented is paid its marginal product. It follows that the gross profit $M_{j,t}$ of the j-th entrepreneur is

$$M_{j,t} = Y_{j,t} - r_t K_{j,t} = (1 - \beta)\, Y_{j,t}. \tag{2}$$

To raise resources for supplying G, the government levies the entrepreneurs profits at rate τ, which is assumed to be constant over time. The tax system of the economy relies on voluntary compliance in that each entrepreneur pays taxes based on the level of reported profit. The tax base, however, is not easily observable by the government tempting the rational entrepreneur to underreport the level of profit in order to reduce the tax liability. Thus, the government sets up a fine for evasion and assigns to N_a auditors, a subset of N, the task of auditing a subset of the tax reports. The auditors, however, are able to conceal evidence of evasion in return of bribes paid by the entrepreneurs. Therefore, the government also copes with the problem of corruptibility allowing for monitoring of auditors by N_i inspectors, a subset of N. The composition of the public sector work-force, that is N_a and N_i, will be determined within the model. There is no coordination issue here so each auditor verifies honesty of a single entrepreneur and may be inspected by a single inspector; an entrepreneur never gets inspected by two auditors. The government pays the net of taxes wage W_t to all public agents.

By denoting with R the total revenue of the government and assuming that at any period t the government runs a balanced budget, it follows

$$R_t = G_t + NW_t \tag{3}$$

where R_t includes the taxes voluntary paid by the entrepreneurs and the fines accruing to the government, and $N = N_a + N_i$.

In order to concentrate on the effects of tax evasion and corruption on capital acumulation, a simple behavioral rule is assumed for the consumption-saving choice: all individuals in the economy share a constant saving rate, s. Thus, total saving in period t is given by

$$S_t = s\left(\sum_{j=1}^{J} V_{j,t} + NW_t\right) \tag{4}$$

where V_j denotes the level of disposable income of the j-th entrepreneur.

Finally, to simplify the analysis I assume that capital depreciates fully from one period to another; thus, the capital stock in period $t+1$ is the amount saved by the young in period t:

$$K_{t+1} = S_t.$$

3 Tax evasion, corruption and deterrence

Three types of individuals characterize the young of any generation: entrepreneurs, auditors and inspectors. The present section deals with the strategic choices of a single entrepreneur, facing with the decision of how much tax base to report to the government, a single auditor, facing with the decision of whether or not to take a bribe for concealing evidence of evasion, and a single inspector, who has to decide if investigating on the possibility of bribery.

The basic framework in studying tax evasion rests on the assumption that the true tax base of the taxpayer is costly observable by the tax authority, tempting a rational taxpayer to underreport the tax base in order to reduce the tax liability (Allingham and Sandmo, 1972; Graetz, Reinganum and Wilde, 1986). The private cost of exploiting this opportunity is related to the pecuniary and nonpecuniary penalty the taxpayer faces if the evasion will be detected. The original contributions have been extended along a number of directions. In particular, Chander and Wilde (1992), Hindricks, Keen and Muthoo (1999), and Sanyal, Gang and Goswami (2000) evaluate the consequences of assuming that tax auditing can be performed by a corruptible official, who takes a bribe for not reporting the detected evasion.[4] In the following I depart from previous work by modelling the probability of detecting corruption which was otherwise considered exogenous.

3.1 The taxpayer decision

Consider a single taxpayer-entrepreneur whose level of taxable profit is M. (Index j is omitted to simplify the exposition.) Let denote the fine for tax evasion with Φ_e and the probability of detecting underreport with q. Moreover, let assume the taxpayer expects that, with a given probability χ, the public agent eventually in charge for controlling his tax report will accept a bribe b for not reporting the evidence of tax evasion and that, with a given probability m, he also expects that the bribe agreement would be detected. The entrepreneur's problem amounts to determine the fraction of profit, α, to report to the tax authority.

As regards to the assumptions, the probability of detecting evasion is assumed inversely related to the share of reported profit: $q(\cdot) = ap(\alpha)$, with $0 \leq p(\alpha) \leq 1$, $p_\alpha < 0$, $p_{\alpha,\alpha} \geq 0$, $p(1) = 0$, and $0 < a < 1$. The probability q recognizes that in a real economy only a fraction of tax reports is effectively audited, since the auditing activity is costly, and that underreport is

[4] The possibility of corruption turns out to affect the auditing probability settled by a profit maximizing tax agency: corruption may either induce the agency to dismantle auditing altogether or, given that some auditing takes place, may induce it to audit evasion more aggressively (Chander and Wilde, 1992).

difficult to be detected even when an evader is audited. In other words, the inverse relationship which characterizes $p(\alpha)$ draws on the objective difficulty of demonstrating that the level of reported income is different from the true tax base and admits that the possibility of detecting evasion is as easier as greater the share of unreported income.[5] Moreover, the fine for evasion is assumed to equal the maximum feasible fine: $\Phi_e = M - \tau\alpha M$. This assumption is consistent with one of the most popular conclusion of the economic literature on crime deterrence, which implies that maximal fine is an important policy instrument to maximize welfare (Becker, 1968). If the fine were not maximal, it could be raised and the probability of detection lowered without affecting deterrence, but saving enforcement costs (Polinsky and Shavell, 2001). Finally, following Chandler and Wilde (1992) and Acemoglu and Verdier (2000), among many others, I also assume that the bribe is proportional to the fine for evasion, $b = (1 - \lambda)\Phi_e$, and that the taxpayer minimizes the expected tax-related cost, being indifferent whether the cost is due to tax, bribe, or fine. Thus, the taxpayer problem is equivalent to maximizing the expected disposable income, EV, where

$$EV \equiv \{1 - [1 - \lambda(1 - m)\chi]\, ap(\alpha)\}(1 - \tau\alpha) M. \tag{5}$$

Given χ and m, the share of profit reported to the tax authority satisfies the following equation

$$a[1 - \lambda(1 - m)\chi][\tau p(\alpha) - (1 - \tau\alpha) p_\alpha(\alpha)] = \tau \tag{6}$$

provided that $0 \leq \alpha \leq 1$.[6]

If $\chi = 0$, it is straightforward to conclude from (6) that the fraction of unreported profit (i) increases with the tax rate τ, (ii) decreases with the probability of auditing a, and (iii) depends on the shape of $p(\alpha)$, that is on the difficulty of demonstrating the underreport of the tax base.[7] When $\chi > 0$, corruption may arise. The presence of corruption in the economy seriously affects the taxpayer's problem, since the expected profit in case of evasion is increasing in the probability of being audited by a corruptible

[5] Alternatively, one can assume that the tax authority just knows the average profit of the class to which the taxpayer belongs to, for example on the basis of his working activity, and that the more is the difference between reported and average profit the more is the effort of the auditor in verifying the taxpayer's report.

[6] The second order condition for a maximum is always satisfied, given the assumptions on $p(\alpha)$. Note also that $ap(0) = 1$ and $p_\alpha(1) = 0$ are sufficient conditions for an interior solution.

[7] When $\chi = 0$, the condition for the taxpayer to report the true tax base is $\tau \leq \frac{-ap_\alpha(1)}{1-\alpha[p(1)-p_\alpha(1)]}$. For instance, when $p(\alpha) = 1 - \alpha$, the share of unreported profit becomes: $1 - \alpha = \frac{1}{2}(1 + \frac{1}{a} - \frac{1}{\tau})$. In this case, it is easy to verify that when $\frac{a}{1+a} < \tau < \frac{a}{1-a}$ an interior solution emerges, while if either $\tau \leq \frac{a}{1+a}$ or $\tau \geq \frac{a}{1-a}$ then truthful report or full evasion, respectively, holds.

auditor. The possibility of avoiding the tax payment, even when the evasion is detected, further stimulates tax evasion. In particular, the size of evasion increases with the probability χ and decreases with the probability m of detecting corruption. The same conclusions reported above about changing τ or a hold.[8] Finally, envelope theorem implies that an increase of the tax rate determines a reduction of the taxpayer's expected profit.

3.2 Corruption in tax administration

Consider now an auditor *who holds evidence of an entrepreneur's evasion.* A bribe agreement between the entrepreneur and the auditor may be settled such that the latter does not reveal the evasion and receives the bribe b from the former. The auditor, however, is aware that the agreement can be detected with a given probability m, which would imply he will have to pay the fine Φ_b.[9] Following Rose-Ackerman (1999), among others, I assume that the fine for corruption is proportional to the bribe: $\Phi_b = \phi_b b$, with $\phi_b > 0$.

The auditor chooses between the two possible strategies, taking the bribe and concealing the evidence of evasion or revealing the evasion, according to the value of the expected profit. In particular, *given* m the auditor's best strategy is taking the bribe if and only if

$$(1-m)\,b - m\Phi_b \geq 0$$

or, equivalently, if and only if

$$m \leq \frac{1}{1+\phi_b}.$$

3.3 The incentive to deter corruption

Bribery entails two main effects. On one side, it dilutes tax evasion deterrence because it results in a lower payment by a taxpayer than the fine for evasion (Polinsky and Shavell, 2001). On the other side, it determines a lower level of public services G in part because the government does not get the fine for evasion Φ_e. Since public services are necessary to production, the bribe agreement between an auditor and an entrepreneur determines a negative externality for the rest of the economy, whose relevance depends indeed on the amount Φ_e. Given that, in order to mitigate the effects of bribery the government is assumed to set at Φ_e the revenue for an inspector who detects a bribe agreement.[10]

[8] When $p(\alpha) = 1-\alpha$ an equivalent conclusion to that reported in note 7 holds, with $\widehat{a} \equiv a\,[1-\lambda(1-m)\chi]$ replacing a.

[9] Following Polinsky and Shavell (2001), in this case the bribe is given back to the taxpayer.

[10] Of course, the main results of the paper carry out even if it was assumed that the inspector's gain is a share of the fine for evasion.

The activity of investigation entails a cost Z which depends on the difficulty to provide evidence of a bribe agreement. In particular, it is assumed that the higher is the level of profit M, more complex is the investigation activity, and the higher is its cost: $Z = zM$. Thus, the expected profit due to the investigation activity is given by $\Phi_e p\chi - Z$, where $p\chi$ denotes the probability of facing a corrupt auditor.[11] It follows that *given the size of evasion α and the probability χ*, the inspector's best strategy is to investigate if and only if

$$\Phi_e p\chi - Z \geq 0.$$

3.4 The equilibrium

Let the function $p(\cdot)$ and the set of parameters $\{\tau, \lambda, \phi_b\}$ define the institutional framework which characterizes our economy. In the following I will concentrate on an interior equilibrium (α^*, m^*, χ^*), with $0 < \alpha^* < 1$, $0 < m^* < 1$, and $0 < \chi^* < 1$ such that:

- The rate of evasion maximizes the expected disposable income of the taxpayer-entrepreneur, given M, m^* and χ^*. This implies
$$a\left[1 - \lambda\left(1 - m^*\right)\chi^*\right]\left[\tau p\left(\alpha^*\right) - \left(1 - \tau\alpha^*\right)p_\alpha\left(\alpha^*\right)\right] = \tau.$$
- The auditor — who detects the taxpayer's evasion — is indifferent between taking the bribe or revealing the evasion. This implies
$$m^* = \frac{1}{1 + \phi_b}. \tag{7}$$
- The inspector is indifferent between investigating on the possibility of a bribe agreement or not. This implies
$$\chi^* = \frac{zM}{\Phi_e\left(\alpha^*\right)p\left(\alpha^*\right)}. \tag{8}$$

An higher level of the fine for evasion, which implies an higher level of the bribe, both rises the incentive to investigate on bribery and the incentive to conceal evidence of evasion. In equilibrium, the interdependence between corruption and monitoring implies that the conditional probability that the auditor takes the bribe is inversely correlated to the fine for evasion and to the probability of detecting evasion.[12] At the same time, the model implies that the probability of detecting a bribe agreement is inversely related to the fine for corruption.

[11] If either the auditor does not detect evidence of evasion or he correctly reveals the true tax base, then his payoff equals zero. At the same time, the inspector payoff equals $-Z$ if he chooses to investigate when the auditor is honest.

[12] Note that this solution holds for values of the investigation cost sufficiently low. If $Z > \Phi_e p$ a corner solution emerges with $\chi = 1$ and $m = \Phi_e p/Z$.

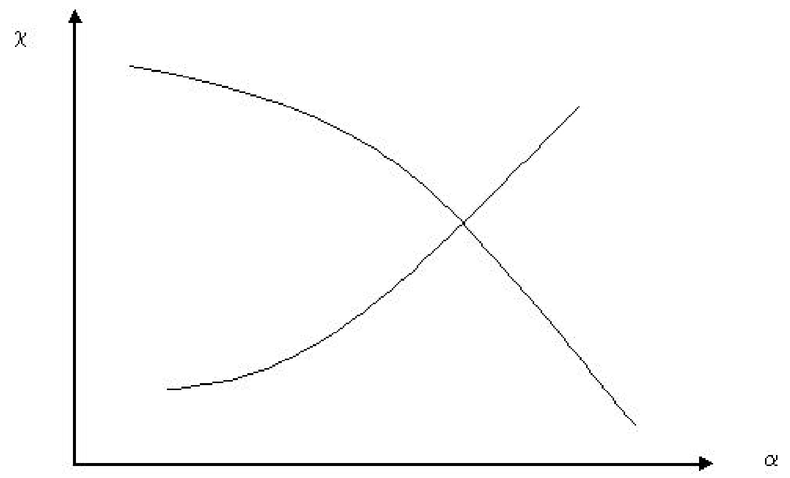

Figure 1: Tax Evasion and Corruption

3.4.1 Comparative statics

The simple expression for m makes possible to further investigate on the properties of the equilibrium simply looking at the following two equations:

$$a\left[1-\lambda\left(1-\frac{1}{1+\phi_b}\right)\chi\right]\left[\tau p\left(\alpha\right)-\left(1-\tau\alpha\right)p_\alpha\left(\alpha\right)\right]=\tau \tag{9}$$

and

$$\left(1-\tau\alpha\right)p\left(\alpha\right)\chi=z. \tag{10}$$

Given the assumptions on $p(\alpha)$, equation (9) implies a function $\alpha\left(\chi\right)$ which is continuous and decreasing in χ while equation (10) implies a function $\chi\left(\alpha\right)$ which is continuous, convex, and increasing in α (Figure 1).[13]

Let consider the effect of an increase in the tax rate τ. Given χ and m, an higher τ induces the taxpayer to increase the share of income unreported (in terms of Figure 1 the curve $\alpha(\chi)$ moves on the left). At the same time, for any level of α a raise in τ lowers the fine for evasion which, in turn, reduces the incentive of the inspector to investigate on bribery. This implies a raise in the level of corruption (the curve $\chi(\alpha)$ moves on the top). It follows that an higher tax rate induces larger evasion in equilibrium:

[13] Note that the shapes of the two functions imply that there exists a set of parameters determining an interior equilibrium which is unique.

$$\frac{\partial \alpha^*}{\partial \tau} < 0.$$

Moreover, as an increase in the tax rate has an ambiguous effect on the equilibrium level of the fine for evasion, in general the effect of higher τ on χ is ambiguous. However, it is straightforward to show that

$$\text{if } \left|\varepsilon^*_{\alpha,\tau}\right| \geq 1 \longrightarrow \frac{\partial \Phi^*_e}{\partial \tau} \geq 0 \longrightarrow \frac{\partial \chi^*}{\partial \tau} < 0.$$

where $\varepsilon_{\alpha,\tau}$ denotes the elasticity of the share of reported income with respect to the tax rate. When the elasticity is greater than, or equal to 1 an increase in τ determines an increase in the fine for evasion which implies stronger incentive for the inspector to deter corruption. In equilibrium, the latter effect determines a reduction in the probability of corruption. Thus, an inverse relationship between the size of evasion and that of corruption could emerge.

As regards the impact of a change in τ on the expected disposable income of the entrepreneur, for a given M, it follows

$$\frac{\partial EV^*}{\partial \tau} = \frac{\partial EV}{\partial \chi}\frac{\partial \chi^*}{\partial \tau} + \frac{\partial EV}{\partial \tau}$$

where the first term on the right-hand side refers to the effect of τ on EV^* which goes through χ^* while the second term refers to the direct effect. Since $\frac{\partial EV}{\partial \chi} > 0$ and $\frac{\partial EV}{\partial \tau} < 0$ while the sign of $\frac{\partial \chi^*}{\partial \tau}$ is ambiguous, in principle it could be the case that an increase in τ increases EV^*. However, it can be shown that endogenous deterrence implies that the overall effect of an increase in the tax rate on the expected disposable income is negative. In other words, when the tax base is exogenously given the diffusion of corruption does not change the sign of the relationship between public revenue and tax rate:

$$\frac{\partial EV^*}{\partial \tau} < 0.$$

Proposition 1 *Given the level of the tax base, a raise in the fiscal pressure causes an increase in the taxpayer' evasion and a reduction of his expected disposable income. Moreover, it may favor the diffusion of corruption.*

Let now assume an increase of the fine rate for corruption ϕ_b. The analysis of the taxpayer highlighted that a change in ϕ_b does not determine any direct effect on the decision of evading, since an increase in ϕ_b does not change neither the fine for evasion nor the cost of paying a bribe.[14]

[14] Note that, under some circumstances, this conclusion holds true even without the assumption of maximal fine.

However, the rise in ϕ_b reduces the incentive of the auditor to take a bribe thus determining a reduction of the equilibrium probability of monitoring in the economy and no effect at all on χ. The reduction in m implies an increase in the rate of evasion: In terms of Figure 1 the curve $\alpha(\chi)$ moves on the left. It follows that an increase in the fine for corruption lowers the probability of corruption and it raises the rate of evasion.

Proposition 2 *In an economy where the diffusion of corruption hinges on taxpayers' evasion, it is possible that the reduction of corruption, obtained through higher fines, realizes at the cost of larger evasion.*

Proof. See appendix A. ■

4 Aggregate implications of tax evasion and corruption

Previous section illustrated the equilibrium of the model for a given level of M. The notion of equilibrium implies that each entrepreneur takes into account the impact of his decision about the tax report on the probability $p(\alpha)$. Conversely, having only a marginal effect with respect to the whole economy, each entrepreneur takes as given the values of m and χ and he does not internalize the effect of his decision on the level of public capital supplied by the government. Similarly, in deciding on whether to take a bribe a single auditor does not take into account if other auditors have been monitored by the inspectors, as if the inspectors investigate on bribe agreements simultaneously. Thus, *for given K and G* on aggregate previous results may be interpreted as follows.[15]

- $N_a = aJ$ entrepreneurs are audited.
- A fraction $1-\alpha$ of the entrepreneurs' total profit is concealed; $p(\alpha) N_a$ entrepreneurs are detected to have underreported the tax base.
- Among all auditors detecting tax evasion, a fraction $\chi = \frac{Z}{\Phi_e(\alpha)p(\alpha)}$ is corrupt.
- N_i^* inspectors investigate on bribery, with $N_i = \frac{1}{1+\phi_b} N_a$, which implies that on aggregate the auditors have zero profit from bribery.
- The size of corruption — that is the number of auditors who take bribes, $\frac{Z}{\Phi_e(\alpha)} N_a$ — is such that each inspector is indifferent between investigating and not investigating and that the aggregate profit from the activity of detecting bribery is equal to zero.

[15] In the following the time index t and the symbol $*$ are omitted when not necessary.

- In period t, the distribution of disposable income among entrepreneurs is given by

$$V_{j,t} = \begin{cases} \overline{V}_t \equiv (1 - \tau\alpha) M_t & J - N_a p(\alpha) \\ \lambda \overline{V}_t & N_a p(\alpha) \chi (1 - m) \\ 0 & N_a p(\alpha) (1 - \chi + \chi m) \end{cases}$$

where the two columns refer to, respectively, the level of income and its frequency. Thus, the average disposable income of the entrepreneurs is equal to the expected disposable income of each entrepreneur:

$$\frac{\sum_{j=1}^{J} V_{j,t}}{J} = \Psi(\tau, z) M_t \tag{11}$$

where $\Psi(\tau, z) \equiv \{1 - a[1 - \lambda\chi^* (1 - m^*)] p(\alpha^*)\} (1 - \tau\alpha^*)$.

- the allocation of talent condition holds: the wage of public officials W_t equals the average disposable income of the entrepreneurs

$$W_t = \Psi(\tau, z) M_t. \tag{12}$$

In other words the aggregate implications relative to the size of tax evasion, the diffusion of corruption and the strength of deterrence mainly rest on interpreting the probabilities p, m and χ as frequencies. Given that, since the tax base is not easily observable by the government and investigating on bribery is costly, it follows that only some bribe agreements are detected and only some auditors decide of being corrupt.

4.1 Tax evasion and capital accumulation

Let now consider the supply of public capital by the government, in order to investigate on the overall impact of tax evasion and corruption in the economy.

The revenue of the government can be expressed as

$$R \equiv J\tau\alpha M + N_a p(\alpha)(1 - \chi)\Phi_e + N_a p(\alpha)\chi m \Phi_b$$

where $J\tau\alpha M$ measures taxes voluntary paid by the entrepreneurs, $N_a p(\alpha)(1 - \chi)\Phi_e$ denotes the amount of fines for evasion — that is what the entrepreneurs would pay without corruptibility less the fines which do not accrue to the public budget because of corruption —and $N_a p(\alpha)\chi m \Phi_b$ is the total amount of fines for corruption.[16] Substituting out the expression for R in equation

[16]Note that the expression for public revenue amounts to assume that the fines paid by those auditors who make a false reports and by the corresponding entrepreneurs are confiscated.

(3), after some algebra the public budget can be written as

$$JM - \sum_{j=1}^{J} V_j - N_i Z = G + NW$$

where the left-hand side is the difference between the total amount of funds raised by the government and the total cost of monitoring corruption.[17] Given (2), (5) and (12), and normalizing $J = 1$, it follows

$$G = [1 - (1 + N)\,\Psi(\tau, z) - N_i z]\,(1 - \beta)\,Y. \tag{13}$$

Follwing Barro (1990), equations (1) and (13) can be used to get an expression for G:

$$G = \{[1 - (1 + N)\,\Psi(\tau, z) - N_i z]\,A\,(1 - \beta)\}^{\frac{1}{1-\gamma}}\,K^{\frac{\beta}{1-\gamma}}.$$

Substituting out this equation for G in (1), then I get the level of output in terms of the level of capital

$$Y = \left\{[1 - (1 + N)\,\Psi(\tau, z) - N_i z]\,A^{\frac{1}{\gamma}}\,(1 - \beta)\right\}^{\frac{\gamma}{1-\gamma}}\,K^{\frac{\beta}{1-\gamma}}$$

For a given K, the amount of output produced in the economy depends on $\Psi(\tau, z)$. If the entrepreneurs would truthfully report the tax base then it holds $\Psi = 1 - \tau$. Tax evasion implies instead that $\Psi(\tau, z) > 1 - \tau$ and thus that a lower level of resources are available for supplying the public services G. The following lemma holds.

Lemma 3 *In any period* t*, tax evasion depresses the level of aggregate output.*

Substituting out the expression for Y in that for W, it follows

$$W = [A\,(1 - \beta)]^{\frac{1}{1-\gamma}}\,H\,[\Psi(\tau, z)]\,K^{\frac{\beta}{1-\gamma}} \tag{14}$$

where

$$H\,[\Psi(\tau, z)] \equiv \Psi(\tau, z)\,[1 - (1 + N)\,\Psi(\tau, z) - N_i z]^{\frac{\gamma}{1-\gamma}}.$$

Apart from depressing the amount of output and the taxable income, tax evasion also implies on average an higher share of disposable income.

[17] If the uncovering of a bribe agreement is considered as a risk that any corrupt auditor faces, the total cost of deterrence activity for the economy only includes the auditing cost. Under this assumption, Chandler and Wilde (1992) argue that one of the effects of corruption consists in the possibility of determining a reduction of fiscal revenue as a consequence of an increase in the tax rate. (On this point, see also Sanyal, Gang and Goswami, 2000). Endogenous monitoring of corruption implies, instead, that the public budget also takes into account the cost of detecting bribery.

Thus, its overall effects on the average level of the disposable income of the entrepreneurs and on the wage of the public agents are ambiguous. Moreover, since total saving depends on the average disposable income in the economy the effect of tax evasion on capital accumulation is ambiguous, too. In particular, given (4), (11) and (14) dynamic equilibria in this economy satisfy the following first order difference equation in the stock of private capital

$$K_{t+1} = \overline{s} H \left[\Psi \left(\tau, z \right) \right] K_t^{\frac{\beta}{1-\gamma}} \tag{15}$$

where $\overline{s} \equiv s\left(1+N\right)\left[A\left(1-\beta\right)\right]^{\frac{1}{1-\gamma}}$. If $\beta = 1-\gamma$ the model implies endogenous growth at positive rate provided that $\overline{s}H\left(\cdot\right) > 1$; if $\beta < 1-\gamma$ then an unique positive steady state level of capital K^{ss} exists with

$$K^{ss} = \left(\overline{s}H\right)^{\frac{1-\gamma}{1-\gamma-\beta}} .$$

Moreover, in this case it follows $W^{ss} = \left[A\left(1-\beta\right)\right]^{\frac{1}{1-\gamma}} \overline{s}^{\frac{\beta}{1-\gamma-\beta}} H^{\frac{1-\gamma}{1-\gamma-\beta}}$.

Clearly, both the average disposable income in the economy W_t and the steady state capital K^{ss} increase with H. At the same time, note that the function $H\left(\Psi\right)$ can be represented as an inverse U-shaped relation: it increases for relatively low levels of Ψ, attains a maximum when $\Psi = \frac{(1-N_i z)(1-\gamma)}{1+N}$, and then starts to decrease. Thus, the term Ψ characterizes all relevant expressions of the model and its value determines the long-run outcome of the economy.[18]

As shown above, the scale of government intervention and the hurdle of detecting a bribe, here exemplified respectively by the policy parameter τ and the cost z, are the two main elements affecting the value of Ψ. In particular, it emerged that $\Psi\left(\tau, z\right)$ decreases with τ and increases with z.

The economy without corruptibility. Consider for the moment the case without the potential of corruptibility, which implies that the economy does not suffer the cost of deterring corruption. In this case, all public agents are involved in auditing the tax reports and the maximum steady state capital stock is reached when $\Psi = \frac{1-\gamma}{1+N}$.[19]

[18] Of course, the main implications of the model about the impact of corruption and tax evasion on the steady state level of capital can be reinterpreted as implications on the growth rate if $\beta = 1-\gamma$.

[19] This condition corresponds to the efficiency condition for the size of the government which characterizes the model in Barro (1990). When evasion is excluded by assumption and $\beta = 1-\gamma$, the production side of the model collapses to the case considered in Barro (1990), and equation (15) represents an inverse U-shaped relation between the balanced growth rate of capital and the size of the government $\frac{G}{\gamma Y}$. In this case, the efficiency condition reduces to $\tau = \gamma$.

Similarly to Barro (1990), the effects of government on capital accumulation involve two channels: one relates to the negative effect of taxation on saving and thus on the level of investment of private agents; the other is due to the positive effect of public services on output.[20] In the present framework, however, these effects of government intervention on capital accumulation coexist with those due to income redistribution resulting from tax evasion and represented by the difference between $\Psi(\tau, \cdot)$ and $1-\tau$. The term $\Psi(\tau, \cdot)$, with $\Psi(\tau, \cdot) > 1-\tau$, takes into account the average positive effect of tax evasion in period t on the level of capital of the entrepreneurs in period $t+1$, for a given level of output produced in t; the term $[1-(1-N)\Psi(\tau, \cdot)]^{\frac{\gamma}{1-\gamma}}$ takes into account instead the negative effect of tax evasion on the supply of public services in period t, and thus on the level of income of all agents in the economy. Therefore, on a theoretical level a priori tax evasion is neither efficiency enhancing nor efficiency detracting with respect to the level of development, its impact depending upon the ratio between private capital and public services determined. In fact, when the mix between private and public inputs is severely biased towards the public one, respect to the efficient composition, then a higher level of the private input, due to tax evasion, more than compensate the reduction in the amount of the public input. In other words, although tax evasion always results in some degree of income redistribution, the strong negative effect on capital accumulation of an extremely high level of taxation might be diluted by tax evasion, which thus determines a better use of resources and an higher level of aggregate income.[21]

Let W^{ae} denote the disposable income of the public agents in the absence of tax evasion and note that without corruptibility the disposable income of the entrepreneurs is either $\overline{V}$ or 0. In general three cases of interest may be considered.

If

$$W^{ae} > \overline{V} > W$$

then all individuals in the economy suffer from the overall effect of tax evasion. This happens when the contribution of public services to production is very relevant and the probability $1-\overline{a}p(\alpha^{*})$ is sufficiently high, that is when $\lambda\chi^{*}(1-m^{*})$ is high enough. In this case, any entrepreneur would benefit of an higher level of disposable income if all other entrepreneurs truthfully reported the tax base. However, this outcome is not an equilibrium since for

[20]Note that if the entrepreneurs would truthfully report the tax base, then $\Psi = 1-\tau$.

[21]The possibility that the entire economy may benefit of the misbehavior of some agents is also suggested by Barreto (2000). The explanation, however, is different since Barreto points at the effect of lower bureaucratic red-tape eventually associated with corruption. Of course, by assuming that the economy is plagued by bureaucratic red-tape, which implies that some resources are lost within the public sector production process, the positive effect of tax evasion would be stronger and previous conclusion strengthened.

each entrepreneur is profitable to underreport the tax base when the other entrepreneurs truthfully report. Conversely, if

$$\overline{V} > W > W^{ae}$$

then the entire economy benfits from tax evasion of entrepreneurs. This result constitutes a further example of the general idea that misbehavior of some agents in the economy could promote allocative efficiency by alleviating the impact of distortions induced by government policy. Finally, if

$$\overline{V} > W^{ae} > W$$

then tax evasion is profitable for those entrepreneurs whose evasion is not detected but has a negative effect for the economic system as a whole, since it depresses the average level of disposable income and total saving. Tax evasion results in some degree of redistribution from wage earners to the lucky evaders. Moreover, since in this case aggregate saving in period t is depressed by tax evasion, the level of capital available at $t+1$ is lower than otherwise thus also implying a redistribution from all individuals of the next generation to the lucky evaders of period t.

Differences in the aggregate effects of tax evasion on capital accumulation and income mainly depend on the level of fiscal pressure. For instance, let assume the government sets the tax rate $\widehat{\tau} = 1 - \frac{1-\gamma}{1+N}$. If the entrepreneurs would truthfully report the tax base, then $\Psi = \frac{1-\gamma}{1+N}$ and the economy approachs the maximum steady state level of capital. However, if they realize to be favorable to underreport the tax base then $\Psi\left(\alpha^{*}\left(\widehat{\tau}\right)\right) > \frac{1-\gamma}{1+N}$. Thus, an higher level of capital may be approached in steady state if the tax rate be higher than $\widehat{\tau}$.[22] In this case the direct positive effect of tax evasion on the level of saving and thus on capital accumulation is overwhelmed by the negative indirect effect which goes through the level of output. Figure 2 depicts the function $H\left[\Psi\left(\tau,\cdot\right)\right]$ with truthful reports and tax evasion (dotted line). In general, if $\tau < \tau'$ then tax evasion determines a steady state level of capital lower than that under truthful reports and thus it is negative for the aggregate economy. Conversely, if $\tau > \tau'$ then tax evasion may alleviate the strong negative effect of fiscal pressure on private investment, thus determining an higher steady state level of capital.

Proposition 4 *Tax evasion implies a different allocation of resources relative to the case of truthful report. A priori, it is neither beneficial nor*

[22]When corruptibility is excluded, it follows:

$$\frac{\partial H}{\partial \tau} = \frac{\Psi_{\tau}\left[1-\gamma-(1+N)\,\Psi\right]}{1-(1+N)\,\Psi}.$$

Since $\Psi_{\tau} < 0$, if $\Psi > \frac{1-\gamma}{(1+N)}$ then $\frac{\partial H}{\partial \tau} > 0$.

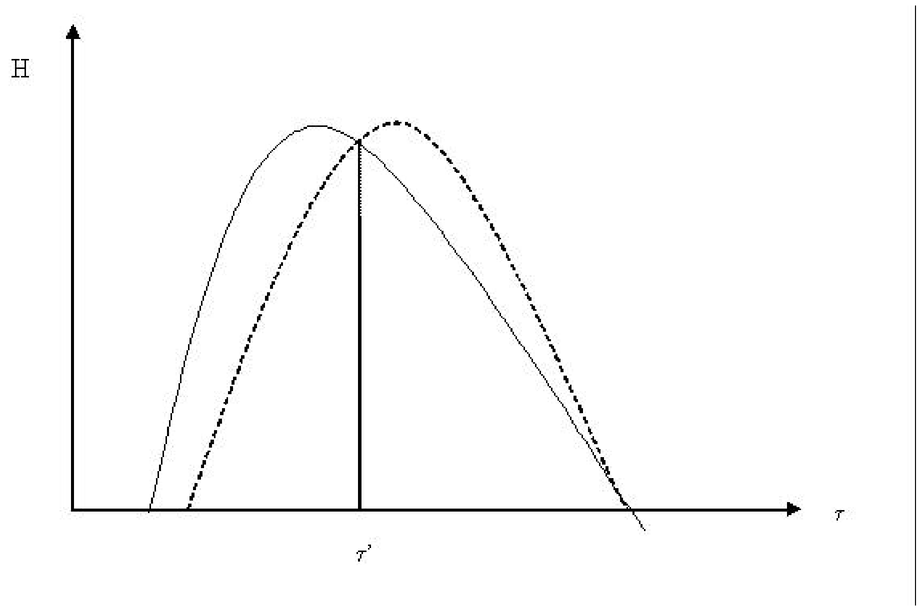

Figure 2: The long-run effect of tax evasion

detrimental for the aggregate level of income, its effect depending upon the burden of fiscal pressure.

The economy with corruptibility. The positive cost of detecting corruption determines a wedge between the total amount of taxes and fines paid by entrepreneurs and auditors and the amount of resources available to the government for public spending. It follows that corruption reduces the *maximum* steady state level of capital attainable, simply because some resources are wasted in deterring it. In particular, the higher is the cost z, the larger is the diffusion of corruption among auditors, and the lower is the steady state level of capital. Corruption, however, also dilutes tax evasion deterrence thus determining a larger size of evasion and (on average) an higher share of taxable income available for consumption and saving for the entrepreneurs. Thus, similarly to the analysis developed before, corruption affects the public-private inputs ratio and in principle it might determine a level of capital in steady state higher than that without coruptibility, if the tax rate is extremely high.[23]

[23] Again the misbehavior of some agents in the economy could promote allocative efficiency by alleviating the impact of distortions induced by government policy. Aidt (2003) reviewed some explanations which support the idea that corruption can enhance allocative efficiency.

In the present simple framework, the impact of corruption on the level of development depends on the relative magnitudes of the tax rate τ and the unit cost of bribe detecting z. In particular, as other models in the literature also the present one hinges on the idea that the diffusion of corruption is determined by the institutional design of the economy, here exemplified by the modelling of deterrence. In this case, once institutions have been determined, history and expectations do not have any significative role in determining the economic outcome. A main element in determining the diffusion of corruption, however, is related to the hurdle of detecting the reward an auditor receives when he conceals the evidence of an entrepreneur's evasion. Assuming that the level of difficulty is constant with respect to the stage of development may be a restrictive assumption. In fact, it seems more plausible to assume that the level of trasparency and thus the obstacle in detecting a bribe varies with the level of economic development in that the higher is the stage of development the higher is the level of trasparency. This consideration implies that the state of the economy affects a single agent decision and that a peculiar form of external increasing returns to scale in the bribe detection activity arises.

A simple way to formalize previous consideration consists in assuming that the unit cost z is a decreasing function of the aggregate level of economic activity.[24] In this case a rich set of dynamics including poverty traps and multiple equilibria due to corruption may arise. To be specific, let $z_t = z(K_t)$ with z a decreasing function of K that tends to zero as $K \to \infty$ and takes on a large value at zero, such that $\chi = 1$ for a low level of capital. Now the equilibrium patterns of capital stock are described by the following difference equation:

$$K_{t+1} = F(K_t) \equiv \overline{s} H \left[\Psi\left(\tau, z\left(K_t\right)\right)\right] K_t^{\frac{\beta}{1-\gamma}}.$$

Since $z(K_t)$ is decreasing in K the function H is increasing in K. Hence, the function F is increasing in K. Moreover, $F(K)$ is constrained between the two polar cases which characterize the equilibrium sequences of capital under absence of corruption, $\chi = 0$, and full corruption, $\chi = 1$. Now the economy may posses multiple steady states and corruption traps can emerge. A possible case is depicted in figure (3), where history matters: if $K_0 < K'''$ then the economy will asymptotically end up with a low level of capital; conversely, if the economy starts out above the unstable steady state K''' then it converges to K' after a period of sustained growth.[25]

[24] An alternative, more standard, explanation for the inverse relationship between the cost z and the stage of economic development is related to the idea that, because of learning by doing, the information about an activity is a by-product of production. Azariadis and Chakraborty (1999), for example, introduces a similar assumption in modelling the state

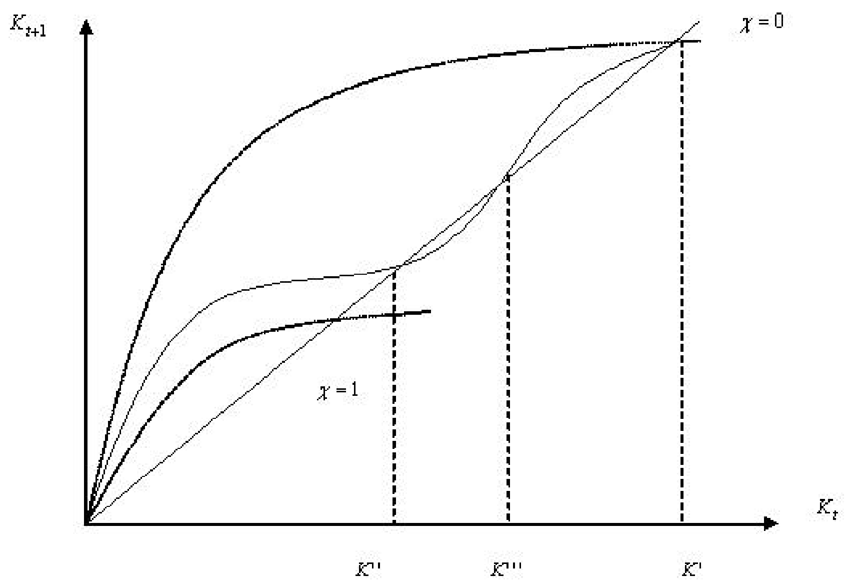

Figure 3: Dynamics with decreasing cost of detecting corruption

5 Conclusions

Many authors have recently pointed out to tax evasion and corruption as two main determinants of the long-term performance of many countries and regions. In this paper I dealt with the the long-run implications of tax evasion and corruption, by developing a simple model of economic growth with government intervention. The government is assumed to provide public services necessary to production and to employ public agents for auditing the entrepreneurs tax reports and for devising investigation on bribery. Tax evasion consists in not revealing the true tax base; corruption consists in taking a bribe from an entrepreneur in order to conceal evidence of his evasion. The case of endogenous deterrence is analyzed on the premise that this reflects in a reliable way how enforcement is actually delivered in the real world. In general, tax evasion and corruption are found to affect income distribution and to have ambiguous effects on the level of economic development. In fact, the model suggests that tax evasion and corruption may be either beneficial or detrimental for the aggregate level of income, the outcome de-

verification cost of bankruptcy.

[25]History dependency and multiple equilibria also arise when the incentive to corruption perceived by a single agent also depends on his expectation about how many individuals in the same organization are corrupt. See Rose-Ackerman (1999) and Aidt (2003) for a review of this topic.

pending upon the burden of fiscal pressure and the level of trasparency. The enhancing effect of corruption on tax evasion and its negative impact on economic development are consistent with some recent empirical evidence. In fact large size of unofficial activity and low level of tax revenue tends to be associated with high level of corruption (Friedman, Johnson, Kaufman, and Zoido-Lobaton, 2000; Tanzi and Davoodi, 2000). At the same time, corruption is found to be correlated negatively with investment and income growth rates (Mauro, 1995; Ehrlich and Lui, 1999; Mèon and Sekkat, 2005).

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Appendix A

Let denote

$$F^1 \equiv a\left(1-\lambda\chi^* + \frac{\lambda\chi^*}{1+\phi_b}\right)\left[\tau p\left(\alpha^*\right)-\left(1-\tau\alpha^*\right)p_\alpha\right]-\tau,$$

$$F^2 \equiv \left(1-\tau\alpha^*\right)p\left(\alpha^*\right)\chi^* - z$$

and

$$\overline{a} \equiv a\left(1-\lambda\chi^* + \frac{\lambda\chi^*}{1+\phi_b}\right).$$

It is straightforward to conclude that the determinant of the Jacobian matrix

$$|J| \equiv \begin{vmatrix} F^1_\alpha & F^1_\chi \\ F^2_\alpha & F^2_\chi \end{vmatrix} = \begin{vmatrix} \overline{a}\left[2\tau p_\alpha - \left(1-\tau\alpha^*\right)p_{\alpha\alpha}\right] & -\frac{\lambda\phi_b a\tau}{(1+\phi_b)\overline{a}} \\ -\left[\tau p\left(\alpha^*\right)-\left(1-\tau\alpha^*\right)p_\alpha\right]\chi^* & \frac{z}{\chi^*} \end{vmatrix}$$

evaluated at the optimum is strictly negative. Therefore, the sign of the derivative of α^* with respect to ϕ_b is the same as the sign of the following determinant

$$\begin{vmatrix} F^1_{\phi_b} & F^1_\chi \\ F^2_{\phi_b} & F^2_\chi \end{vmatrix} = \begin{vmatrix} -\frac{a\lambda\chi^*}{(1+\phi_b)^2\overline{k}} & -\frac{\lambda\phi_b a\tau}{(1+\phi_b)\overline{a}} \\ 0 & \frac{z}{\chi^*} \end{vmatrix}$$

which is negative. The sign of the derivative of χ^* with respect to ϕ_b is the same as the sign of the following determinant

$$\begin{vmatrix} F^1_\alpha & F^1_{\phi_b} \\ F^2_\alpha & F^2_{\phi_b} \end{vmatrix} = \begin{vmatrix} \overline{a}\left[2\tau p_\alpha - \left(1-\tau\alpha^*\right)p_{\alpha\alpha}\right] & -\frac{a\lambda\chi^*}{(1+\phi_b)^2\overline{a}} \\ -\left[\tau p\left(\alpha^*\right)-\left(1-\tau\alpha^*\right)p_\alpha\right]\chi^* & \frac{z}{(1+\phi_b)} \end{vmatrix}$$

which is negative. Thus, $d\chi^*/d\phi_\chi < 0$.

The sign of the derivative of α^* with respect to τ is the same as the sign of the following determinant

$$\begin{vmatrix} F^1_\tau & F^1_\chi \\ F^2_\tau & F^2_\chi \end{vmatrix} = \begin{vmatrix} \overline{a}\left[p+\alpha^* p_\alpha\right]-1 & -\frac{\lambda\phi_b a\tau}{(1+\phi_b)\overline{a}} \\ -\frac{\alpha^* z}{(1-\tau\alpha^*)} & \frac{z}{\chi^*} \end{vmatrix}$$

which is negative. Thus, $d\alpha^*/d\tau < 0$. The sign of the derivative of χ^* with respect to τ is the same as the sign of the following determinant

$$\begin{vmatrix} F^1_\alpha & F^1_\tau \\ F^2_\alpha & F^2_\tau \end{vmatrix} = \begin{vmatrix} \overline{a}\left[2\tau p_\alpha - \left(1-\tau\alpha^*\right)p_{\alpha\alpha}\right] & \overline{a}\left[p+\alpha^* p_\alpha\right]-1 \\ -\left[\tau p\left(\alpha^*\right)-\left(1-\tau\alpha^*\right)p_\alpha\right]\chi^* & -\frac{\alpha^* z}{(1-\tau\alpha^*)} \end{vmatrix}$$

which is ambiguous.