CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 162

Implications of Anticipated Regret and Endogenous Beliefs for Equilibrium Asset Prices: A Theoretical Framework

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September 2006

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Abstract

This paper builds upon Suryanarayanan (2006a) and further investigates the implications of the model of Anticipated Regret and endogenous beliefs based on the Savage (1951) Minmax Regret Criterion for equilibrium asset pricing. A decision maker chooses an action with state contingent consequences but cannot precisely assess the true probability distribution of the state. She distrusts her prior about the true distribution and surrounds it with a set of alternative but plausible probability distributions. The decision maker minimizes the worst expected regret over all plausible probability distributions and alternative actions, where regret is the loss experienced when the decision maker compares an action to a counterfactual feasible alternative for a given realization of the state. We first study the Merton portfolio problem and illustrate the effects of anticipated regret on the sensitivity of portfolio rules to asset returns.We then embed the model in a version of the Lucas (1978) economy. We characterize asset prices with distorted Euler equations and analyze the implications for the volatility puzzles and Euler pricing errors puzzles.

Acknowledgement: I acknowledge the financial support from a Marie Curie Fellowship of the European Community programme *Improving Human Potential* (contract number HPMD-CT-2001-00069).

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Contents

1 Introduction

This paper builds upon Suryanarayanan (2006a) and further investigates the implications of the model Anticipated Regret and endogenous beliefs for portfolio allocation problems and equilibrium asset prices.

The competitive equilibrium price of a tradeable asset is commonly computed as the expected value of its future discounted payoffs. Asset payoffs and discount factors are contingent upon the realization of an exogenous state of nature. Expectations are measured with respect to the probability distribution of the state and economically plausible stochastic discount factors are interpreted as the outcome of market interactions between individual investors. Assumptions about investors' beliefs about the distribution of the state, their preferences and constraints are then crucial in the effort to link asset prices to economic fundamentals.

This paper assumes that investors cannot precisely assess the true probability distribution of the state and form instead their beliefs endogenously by minimizing their lifetime regrets with the Anticipated Regret model.

We first study the asset allocation and consumption decisions of a typical investor within a discrete time but infinite horizon Merton portfolio problem. In particular, we show with numerical examples how Anticipated Regret substantially dampens the sensitivity of the investor's portfolio policy and consumption/wealth ratio to asset returns. This result carries crucial implications for equilibrium asset pricing.

Next we set a general framework for analyzing the implications of the Anticipated Regret and endogenous beliefs for asset pricing by applying the results in Suryanarayanan (2006a) to a Lucas (1978) economy. We characterize the equilibrium asset prices with distorted Euler equations where expectations are measured with endogenously distorted beliefs instead of the prior. We then analyze the model's ability to explain the Shiller (1981) volatility puzzle, Hall (1978) and Grossman-Shiller (1981) martingale puzzle, and the ability to reduce Euler pricing equation errors and α-pricing errors in the implied factor model. The main mechanism is the implied certainty equivalent and the distorted endogenous beliefs.

2 Portfolio choice implications

2.1 Two period examples

In this Section, we consider an investor who must choose in period 1 the fraction of her wealth α to be invested in a risky asset with random return R, to be realized in the subsequent period 2, the remaining fraction $(1-\alpha)$ being invested in a riskless asset R^f. The exogenous set of states of nature is $Z = \{L, H\}$ and the risky return R is a random variable on Z and can take only two values R_L and R_H. We normalize the initial wealth to 1 and the investor derives the utility $u(\alpha(R_z - R^f) + R^f)$ in period 2 when state z is realized.

We assume that the investor has a prior $\pi^* = 0.5$ about the probability of the low state L but doubts about it and believes that the true probability of the low state lies in the interval $[\pi_L, \pi_H]$. When $\pi_L = \pi_H = \pi^*$, the investor perceives no ambiguity about her prior and chooses the portfolio which maximizes her expected utility:

$$\max_{\alpha \in A} \left\{ E_{\pi^*} u(\alpha(R_z - R^f) + R^f) \right\}$$

where A is a constraint on the portfolio share α, assumed to be convex.

When the investor doubts about her prior π^*, we assume that the investor uses the Anticipated Regret model of Suryanarayanan (2006a) to choose her portfolio:

$$\max_{\alpha \in A} \min_{\pi \in [\pi_{\inf}, \pi_{\sup}]} \min_{\alpha^* \in A} \left\{ E_\pi (u(\alpha(R_z - R^f) + R^f) - u(\alpha^*(R_z - R^f) + R^f)) \right\}$$

For each probability π and pair of portfolio shares (α, α^*), the expected regret is defined as the difference between the expected utility derived from portfolio α and that derived from portfolio α^*. The investor minimizes her worst expected regret across all possible counterfactual alternatives α^* and plausible probabilities π. As shown in Suryanarayanan (2006a) from a more general viewpoint, Anticipated Regret is different from Ambiguity Aversion (or Maxmin) where the investor would care about the worst expected utility and not the worst expected regret and would solve instead:

$$\max_{\alpha \in A} \min_{\pi \in [\pi_{\inf}, \pi_{\sup}]} \left\{ E_\pi u(\alpha(R_z - R^f) + R^f) \right\}$$

We first compare the implications for the choice of portfolio and its sensitivity to returns of expected utility, Anticipated Regret and Ambiguity Aversion models in the case of a risk-neutral investor (u is linear). As a second exercise, we introduce risk-aversion (curvature in u) and analyze the difference in the effects of risk-aversion and anticipated regret. In particular, while risk-aversion and anticipated regret may have similar effects on the choice of the portfolio, anticipated regret substantially further dampens the sensitivity of the portfolio weights to asset returns. Moreover the sensitivity to returns is a decreasing function of uncertainty as measured by the range of priors $|\pi_{\sup} - \pi_{\inf}|$. These implications will be crucial within the context of equilibrium asset pricing as a typical investor would require more premium to hold risky assets and will be a function of the level of uncertainty.

2.1.1 A comparison between Ambiguity Aversion, Expected Utility, and Anticipated Regret

We assume that the investor is risk-neutral (u is linear) in order to illustrate the effect of ambiguity and anticipated regret on investment decisions abstracting from effects of risk aversion embedded in the curvature of the utility function.

- **The case when** $[\pi_{\inf}, \pi_{\sup}] = [0, 1]$

The investor does not know the probability occurence of state L and takes the interval $[0, 1]$ to identify the set of alternative distributions. The decision problem of the Anticipated Regret investor can then be formulated as follows (we constrain the portfolio weight to be in $[0, 1]$:

$$\max_{\alpha \in [0,1]} \min_{\pi \in [0,1]} \min_{\alpha^* \in [0,1]} (\alpha - \alpha^*)(E_\pi R - R^f)$$

We first solve for the innermost minimization:

$$\begin{cases} \text{If } E_\pi R < R^f \text{ then } \alpha^*(\pi) = 0 \\ \text{If } E_\pi R > R^f \text{ then } \alpha^*(\pi) = 1 \end{cases}$$

The problem is then to solve:

$$\max_{\alpha \in [0,1]} \min_{\pi \in [0,1]} (\alpha - \alpha^*(\pi))(E_\pi R - R^f)$$

The objective function is strictly concave in the probability π, this means that the candidate minimizing probabilities are the extremes 0 and 1 and the problem becomes equivalent to:

$$\max_{\alpha \in [0,1]} \min \left\{ \alpha(R_L - R^f), (\alpha - 1)(R_H - R^f) \right\}$$

When she is not able to precisely compare the expected returns to the risk-free rate, the investor willing to avoid the worst regret equalizes her regret across the two extreme probabilities and solves:

$$(\alpha - \alpha^*(1))(E_1 R - R^f) = (\alpha - \alpha^*(0))(E_0 R - R^f)$$

Which yields:

$$\alpha = \left(\frac{R_H - R^f}{R_H - R_L} \right) \in (0, 1)$$

Thus, the investor chooses a strictly positive holding in both assets.

Now, a risk-neutral subjective expected utility investor who solves

$$\max_{\alpha \in [0,1]} \alpha(E_\pi R - R^f)$$

will either be indifferent between any choice of portfolio α in $[0, 1]$ if she believes that the stock earns the same as the bond on average (her prior π is such that $E_\pi R = R^f$), or choose to invest fully in the risky asset if she believes that the stock will earn higher average return than the bond ($E_\pi R > R^f$), or to invest fully in the riskless asset if her prior π is such that $E_\pi R < R^f$.

The Maxmin Expected utility investor solves

$$\max_{\alpha \in [0,1]} \min_{\pi \in [0,1]} \alpha(E_\pi R - R^f)$$

and evaluates the expected risk-premium $(R^L - R^f)$ under the most pessimistic probability 1. She then invests fully in the riskless asset.

Hence, while the ambiguity averse investor behaves exactly like an expected utility maximizer with a prior equal to the worst probability 1 and has a $0-1$ investment decision rule, an investor anticipating her worst expected regret has a fundamentally different investment policy. In particular, she always chooses to invest a strictly positive amount in both assets[1].

- **Sensitivity to the level of uncertainty and returns**

We still consider a risk-neutral investor but we now study the sensitivity of the portfolio choice with respect to returns and the level of uncertainty as measured by $|\pi_{\text{sup}} - \pi_{\text{inf}}|$.

The problem of the investor is now:

$$\max_{\alpha\in[0,1]} \min_{\pi\in[\pi_{\text{inf}},\pi_{\text{sup}}]} \min_{\alpha^*\in[0,1]} (\alpha - \alpha^*)(E_\pi R - R^f)$$

The solution for α^* is:

$$\begin{aligned} \text{If } E_\pi R &> R^f \text{ then } a^*(\pi) = 1 \\ \text{If } E_\pi R &< R^f \text{ then } a^*(\pi) = 0 \\ \text{undefined if } E_\pi R &= R^f \end{aligned}$$

When u is linear, we need to distinguish between three cases. If $E_{\pi_{\text{inf}}} R < R^f$ then we also have that $E_{\pi_{\text{sup}}} R < R^f$ and

$$\begin{aligned} \alpha^*(\pi) &= 0 \text{ for all } \pi \in [\pi_{\text{inf}}, \pi_{\text{sup}}] \\ \pi^* &= \pi_{\text{sup}} \end{aligned}$$

and the optimal portfolio weight α is 0.

If $E_{\pi_{\text{sup}}} R > R^f$ then we also have that $E_{\pi_{\text{inf}}} R > R^f$

$$\begin{aligned} \alpha^*(\pi) &= 1 \text{ for all } \pi \in [\pi_{\text{inf}}, \pi_{\text{sup}}] \\ \pi^* &= \pi_{\text{inf}} \end{aligned}$$

and the optimal portfolio weight α is 1.

If $E_{\pi_{\text{sup}}} R < R^f$ and $E_{\pi_{\text{inf}}} R > R^f$ then the problem becomes:

$$\begin{aligned} & \max_{\alpha\in[0,1]} \min \left\{(\alpha - \alpha^*(\pi_{\text{inf}}))(E_{\pi_{\text{inf}}} R - R^f), (\alpha - \alpha^*(\pi_{\text{sup}}))(E_{\pi_{\text{sup}}} R - R^f)\right\} \\ = & \max_{\alpha\in[0,1]} \min \left\{(\alpha - 1)(E_{\pi_{\text{inf}}} R - R^f), \alpha(E_{\pi_{\text{sup}}} R - R^f)\right\} \\ = & (\alpha - 1)(E_{\pi_{\text{inf}}} R - R^f) = \alpha(E_{\pi_{\text{sup}}} R - R^f) \end{aligned}$$

[1] This example is to be related to the one studied by Manski (2005), which we discuss further in Suryanarayanan (2006a)

where

$$\begin{aligned}
\alpha &= \frac{E_{\pi_{\inf}} R - R^f}{E_{\pi_{\inf}} R - E_{\pi_{\sup}} R} \\
&= \frac{\pi_{\inf}(R_L - R_H) + R_H - R^f}{(\pi_{\sup} - \pi_{\inf})(R_H - R_L)} \\
&= \frac{1}{(\pi_{\sup} - \pi_{\inf})} \left(\frac{R_H - R^f}{R_H - R_L} - \pi_{\inf} \right)
\end{aligned}$$

is the optimal portfolio choice.

The attached panel illustrates the result in the case when $\pi_{\sup} = 0.5 + \lambda$ and $\pi_{\inf} = 0.5 - \lambda$. We vary the level of uncertainty λ from 0 to 0.5 as well as the level of the high return R_H. We see that Anticipated Regret substantially dampens the sensitivity of the portfolio choice to returns and that the sensitivity is further decreasing with the level of uncertainty. Recall that the investor is risk-neutral in the sense that u is linear. As opposed to Ambiguity Aversion, Anticipated Regret distinguishes risk, the fact the returns are stochastic, from uncertainty, the fact the probability distribution of returns cannot be precisely assessed.

2.1.2 Risk and Ambiguity with $CRRA$ current period utility

We extend the analysis of the previous Section by introducing risk-aversion. We consider a $CRRA$ utility:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

and we study the effect of an increase in the curvature in the utility function as measured by γ and we compare the effect of risk-aversion and ambiguity on the portfolio choice and the sensitivity of the portfolio to returns.

We start by solving the portfolio problem:

$$\begin{aligned}
&\max_{\alpha} \min_{\pi \in [\pi_{\inf}, \pi_{\sup}]} \min_{\alpha^*} \\
&\left\{ E_\pi (u(\alpha(R_z - R^f) + R^f) - u(\alpha^*(R_z - R^f) + R^f)) \right\}
\end{aligned}$$

We do not constrain the portfolio choice in this example. From Suryanarayanan (2006a) we know that the solution to the problem is such that:

$$\begin{aligned}
& E_{\pi_{\inf}} (u(\alpha(R_z - R^f) + R^f) - u(\alpha^*(\pi_{\inf})(R_z - R^f) + R^f)) \\
= \; & E_{\pi_{\sup}} (u(\alpha(R_z - R^f) + R^f) - u(\alpha^*(\pi_{\sup})(R_z - R^f) + R^f))
\end{aligned}$$

where $\alpha^*(\pi)$ is the solution to the a standard expected utility problem with prior π.

Define by $F(\alpha)$:

$$F(\alpha) = u(\alpha(R_H - R^f) + R^f) - u(\alpha(R_L - R^f) + R^f)$$

F is strictly increasing and continuous, it is then invertible. The optimal portfolio choice is then given by:

$$\alpha = \frac{1}{\pi_{\text{sup}} - \pi_{\text{inf}}} \times F^{-1}(u(\alpha^*(\pi_{\text{inf}})(R_z - R^f) + R^f)) - u(\alpha^*(\pi_{\text{sup}})(R_z - R^f) + R^f)))$$

The attached panel illustrates the result in the case when $\pi_{\text{inf}} = 0.5 - \lambda$ and $\pi_{\text{sup}} = 0.5 + \lambda$. While both risk aversion and the level of uncertainty have similar effects on the portfolio choice, the sensitivity of the portfolio choice to returns is much more dampened with increases in the level of uncertainty than with increases in the level of risk-aversion. The reason is that the level of uncertainty directly affects the implied certainty equivalent of the risky returns whereas we need substantial increases in the curvature of utility function γ to affect the certainty equivalent because $1/\gamma$ affects intertemporal substitution as well.

Implications of Anticipated Regret for equilibrium asset pricing (and to a further extent for macroeconomics of savings and investment, see Chamberlain and Wilson (1984)) are driven by the ability of the model to substantially lower the implied certainty equivalent, which is difficult to achieve with expected utility models.

2.2 Infinite horizon

In this Section, we apply the infinite horizon version of Anticipated Regret developped in Suryanarayanan (2006a) and extend the main results of the two-period framework with a numerical simulation.

2.2.1 The portfolio problem

- **The choice environment**

We consider an infinitely lived investor who must choose a lifetime consumption plan $(c_t)_t$ and a lifetime asset allocation plan $(\phi_t)_t$ where ϕ_t in each period t is the fraction of accumulated wealth W_t invested in a risky asset, the remaining fraction being invested in a riskless asset.

The risky asset's one period return is a random process $R(z_t)$ driven by an exogenous Markovian state of nature $(z_t)_t$. We assume that the realizations of z_t lie within the compact metric space Z. The investor has a prior $p^*(z_{t+1}|z_t)$ for the conditional probability of future state z_{t+1} given z_t, but she cannot precisely assess it and doubts that her prior is misspecified. In practice, estimating stationary Markovian models of asset returns from time series data is a hard task which provides only a rough approximation of the "true" Markovian model, even assuming the hypothesis that returns are Markovian is true. Acknowledging plausible misspecifications, she believes that the true conditional distribution in state z_t lies in $P(z_t)$, a compact and convex set of conditional measures absolutely continuous with respect to $p^*(\cdot|z_t)$. For any compact subset M of $P(z)$, we denote by $\Lambda_M(z)$ the set of all probability measures on M.

The riskless asset provides one-period return R^f.

- **The investor's problem: investing without regret**

The investor wishes to minimize her lifetime regrets in the sense of Suryanarayanan (2006a) when making her investment decisions and we apply the framework developped in Suryanarayanan (2006a) to define the investor's portfolio allocation problem. The relevant endogenous and exogenous state variables are the accumulated wealth W_t and the state of nature z_t :

$$
\begin{aligned}
& R\,(W,z) \\
= \; & \max_{(c,\phi)\in D(W,z)} \min_{\pi\in P(z)} \min_{(c^*,\phi^*)\in D(W,z)} \\
& \left\{u(c) - u(c^*) + \beta E_\pi \left(V(W',z') - V(W^{*\prime},z')\right)\right\}
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
D(W,z) &= \{(c,\phi) \mid 0 \le c \le W \text{ and } \phi \in \Re\} \\
W' &= (W-c)\left(\phi R_{zz'} + (1-\phi)\,R^f_{z'}\right) \\
W^{*\prime} &= (W-c^*)\left(\phi^* R_{zz'} + (1-\phi^*)\,R^f_{z'}\right) \\
V(W,z) &= u(c(W,z)) + \beta E_{\pi^{\exp}(W,z)} V(\overline{W}', z') \\
\overline{W}' &= (W - c(W,z))\left(\phi(z) R_{zz'} + (1-\phi(z))\,R^f_{z'}\right)
\end{aligned}
$$

and where $\pi^{\exp}(W,z)$ is the endogenous conditional distribution defined by:

$$
\begin{aligned}
\beta &< 1 \text{ and } \pi^{\exp}(W,z) = \int_{\pi\in M} \pi d\lambda(\pi) \\
M &= \left\{\pi \in P(z) \mid \psi(c,\pi) = \arg \min_{\pi\in P(z)} \psi(c,\pi)\right\} \\
\psi(c,\pi) &= u(c_t) - u(c^{*\pi}) + \beta E_\pi (V(W',z') - V(W'^{*\pi},z')) \\
(c^{*\pi},\phi^{*\pi}) &= \arg \min_{(c^*,\phi^*)\in D(W,z)} \\
& \quad \{u(c) - u(c^*) + \beta E_\pi (V(W',z') - V(W'^*,z'))\} \\
\lambda &= \arg \min_{\lambda\in\Lambda_M(z)} \max_{\substack{(c,\phi) \\ (c,\phi)\in D(W,z)}} \int_{\pi\in M} \psi(c,\pi) d\lambda(\pi)
\end{aligned}
$$

- **Implications of the Anticipated Regret model and the endogenous belief** $\pi^{\exp}(W,z)$

We know from Suryanarayanan (2006a) that there exists a unique solution to the above portfolio problem. We further show below how to solve the problem when u is $CRRA$. In this paragraph, we discuss the implications of the

Anticipated Regret model for the above Merton-type portfolio problem and the role of the endogenous belief $\pi^{\exp}(W, z)$.

In general, the model will imply a consumption policy $c(W, z)$ and an asset allocation policy $\phi(W, z)$ that depend on both the endogenous wealth W and the exogenous state z. This dependence is construed via the endogenous conditional belief $\pi^{\exp}(W, z)$. As shown in Suryanarayanan (2006a), the policy functions $c(W, z)$ and $\phi(W, z)$ are implicit functions of $\pi^{\exp}(W, z)$ as $((c, \phi), \pi^{\exp})$ is the saddle-point solution of an equivalent zero-sum game problem. This will in general be true even when the current period utility displays homogeneity property like the $CRRA$ utility. While in the Merton portfolio model with $CRRA$ utility, the consumption-wealth ratio and the portfolio share do not depend on wealth, this will no longer be true for the Anticipated Regret investor. Furthermore, the dependence on wealth is obtained through the endogenous beliefs.

It is difficult at this stage to fully understand the usefulness of endogenous wealth dependence for portfolio choice and further work needs to be undertaken to confront the empirical realities to the model. On the other hand, the implications appear more transparent when we consider embedding the portfolio model in models of consumption-savings (Chamberlain and Wilson (1984)) and models of equilibrium asset pricing, as it is usually done in practice. Having consumption-wealth ratios and portfolio weights which are sensitive to wealth may improve the Chamberlain and Wilson savings model and may generate state dependent and higher volatility in equilibrium asset returns.

2.2.2 Solution to the problem with $CRRA$ current period utility and $Z = \{L, H\}$

We provide a solution to investor's portfolio problem when the current period utility u is $CRRA$:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

We now also assume that the returns follow a two-state Markov chain. We denote with $R_{zz'}$ the vector of all possible returns, i.e the future return in state z' conditional on state z in the previous period for all z and z'. The prior transition probability matrix is in the form

$$P^* = \begin{pmatrix} p_L^* & 1 - p_L^* \\ 1 - p_H^* & p_H^* \end{pmatrix}$$

but the investor expresses doubts about P^* and believes that the true transition matrix defined by (π_L, π_H) lie in the set:

$$P = \{(\pi_L, \pi_H) \in [p_L^* - \varepsilon_L, p_L^* + \varepsilon_L] \times [p_H^* - \varepsilon_H, p_H^* + \varepsilon_H]\}$$

where ε_L and ε_H measures uncertainty in the low and the high state respectively.

Given the homogeneity property of the $CRRA$ utility, we conjecture that the value function is of the form:

$$V(W, z) = A_z \frac{W^{1-\gamma}}{1-\gamma}$$

This implies that the expost conditional probability distribution is not dependent on wealth anymore, we denote it with $\pi_z^{\exp}$. We can solve for $A_z, \pi_z^{\exp}$ and the consumption wealth ratios c_z/W_z and the allocation rule ϕ_z recursively as follows.

1. Start with an initial guess for A_L, A_H
2. Solve for the consumption-wealth ratios c_z/W_z, the portfolio weights ϕ_z and the expost distribution $\pi_z^{\exp}$ given A_z
3. Update A_z with A'_z:

$$A'_z = \left(\frac{c_z}{W_z}\right)^{1-\gamma} + \beta E_{\pi_z^{\exp}} A_z (\left(R_{zz'} - R^f\right) \phi_z + R^f)^{1-\gamma}$$

4. Iterate until convergence is reached in A_z and $A_{z'}$

2.2.3 Numerical simulations

In order to get results regarding the consumption-wealth ratio and the portfolio weight as well as their sensitivities to returns and the level of uncertainty, we first simplify the above set-up to an *iid* returns setting. In which case we solve the model without exogenous state dependence and the Markov chain exhibits the same conditional probabilities in both states.

We set the parameters and the values of returns for the baseline model to:

$$\begin{aligned} \beta &= 0.97 \\ \gamma &= 1.5 \\ R^f &= 1.03 \\ R^L &= 0.88 \\ R^H &= 1.24 \end{aligned}$$

The above numbers are chosen so that the optimal decisions span a sufficiently wide range of values when conducting the different sensitivity analysis.

When there is no uncertainty, the investor is an expected utility maximizer and we assume that her subjective belief π^* is equal to 0.5. We take $\varepsilon_L = \varepsilon_H = \varepsilon$ to measure uncertainty.

- **The effect of uncertainty**

Please refer to Panels A for the plots and figures related to this section.

The optimal portfolio weight (ϕ), the consumption/wealth ratio (c_t/W_t) and the regret consumption/wealth ratio (c_t^*/W_t) are all decreasing with the level of uncertainty. Thus, as the perceived uncertainty increases, the investor forms more pessimistic beliefs about the future state of the risky return, always chooses the highest probability value for the realization of the low state of returns and shifts her portfolio towards the risk-free asset. This is due to the ability of the Anticipated Regret model to lower the implied certainty equivalent of wealth. Especially, we see how we get lower sensitivities of consumption relative to wealth which may offer a way to encompass the problems experienced by Chamberlain and Wilson (1984) when conducting similar experiements. In their experiments, they consider an incomplete markets set-up without a risky asset, the allocation to the risk-free asset would then increase ad-infinitum over time! In this simple example, the increase would be substantially dampened with the Anticipated Regret model, and we may actually get converging risk-free asset allocation in a set-up where the endogenous belief becomes wealth dependent. One way to produce wealth dependence is to increase the state space Z and consider probability set based on the Kullback-Leibler distance or the Prohorov metric.

- **The effect of the risky return**

Panel B and C illustrate the effects of the risky return for a given level of uncertainty. We see how implicit and endogenous beliefs acts as to counterbalance the effects of returns. If the investor perceives increases in returns, the endogenous beliefs tends to place less weight on the high state to signal the possibility that returns may still revert to the low state and makes the investor more pessimistic. And likewise, when the investor perceives a decrease in returns, the endogenous beliefs tends to place more weight on the high state to signal the possibility of an increase and makes the investor less pessimistic. Minimizing the worst regret then implies a balanced portfolio, depending on the returns, nor overly pessimistic nor overly confident.

In turn, the model of Anticipated Regret further dampens the sensitivity of asset allocations and the consumption-wealth ratio to asset returns. As already stated, this effect will carry crucial implications for equilibrium asset pricing.

- **Comparison with Gilboa and Schmeidler maxmin expected utility**

Last, we draw a simple comparison with the Gilboa and Schmeidler maxmin expected utility. In PANEL E, we conduct the experiment where we increase the level of uncertainty. We see that the ambiguity averse investor will tend to short the risky and uncertain asset whereas the Anticipated Regret investor will still continue to hold the risky asset, even when the level of uncertainty is at its maximum.

- **Markovian returns**

Panel F illustrates the Markovian case. One can view the policy functions as those obtained by solving the problem associated with Epstein-Zin recursive utility with both risk-aversion and intertemporal substitution parameters equal to γ provided we replace the certainty equivalent operator with $E_{\pi^{\exp}}(\cdot)$. Thus, we only represent the implicit endogenous belief.

The important observation is that the implicit transition matrix becomes endogenously asymmetric, i.e the probability of remaining in the low state given the low state is much higher than that of remaining in the high state given the high state. This shows the Anticipated Regret model's potential to generate endogenous heteroscedasticity in asset returns once we embed the portfolio problem in an equilibrium model. This in turn would help to generate time varying volatility with higher volatility in the low state relative to the high state.

3 Equilibrium asset pricing with Anticipated Regret

We consider a representative investor in the Lucas (1978) economy who minimizes her worst expected lifetime regrets. We characterize equilibrium asset prices with distorted Euler equations. In particular, asset prices embed both a premium for risk, to compensate for the stochastic nature of asset returns, and a premium for uncertainty, to compensate for not knowing the true probability distribution of asset returns.

3.1 The model economy

3.1.1 The problem of the representative investor

We adapt the framework developped in Suryanarayanan (2006a). Uncertainty in the economy is driven by an exogenous state of nature $(z_t)_t$ assumed to be Markovian with realizations in a compact metric state space Z with Borel $\sigma-$algebra $B(Z)$. We denote by $\Pi(Z)$ the set of all Borel probability measures on Z. Under the weak-convergence topology, $\Pi(Z)$ is also a compact metric space.

We extend the state space Z to Z^{∞} the product space $\left(\prod_{t=1}^{+\infty}(Z)\right)$ with associated Borel $\sigma-$algebra $B(Z^{\infty})$. Let z^t be the vector of histories of the realizations of the exogenous state in period t, an element of the product set Z^t with Borel $\sigma-$algebra $B\left(Z^t\right)$, induced by $B(Z^{\infty})$ on Z^t. A consumption plan is a real-valued process $(c(z^t))_t$ which is positive, $B\left(Z^t\right)-$adapted and continuous. Likewise, an asset allocation plan is an $\Re^n$ valued process $\left(\theta\left(z^t\right)\right)_t$, $B\left(Z^t\right)-$adapted and continous.

As in the previous Section, a representative investor has a prior $p^*(z_{t+1}|z_t)$ for the conditional probability of future state z_{t+1} given z_t, but she cannot precisely assess it and doubts that her prior is misspecified. She believes that the true conditional distribution in state z_t lies in $P(z_t)$, a compact and convex set of conditional measures absolutely continuous with respect to $p^*(\cdot|z_t)$. For any compact subset M of $P(z)$, we denote by $\Lambda_M(z)$ the set of all probability measures on M.

The representative investor must choose a lifetime state-contingent consumption plan $(c_t)_t$ and an allocation plan $(\theta_t)_t$ of n assets with state-contingent dividends $(d_i(z^t))_{i=1,..n}$, given her endowment $(e(z^t))_t$. The n assets are assumed, without loss of generality, to be available in zero net supply at prices $(q_i(z^t))_{t,i=1..n}$.

The relevant state variables for the problem will be the accumulated wealth W_t and the state of nature z_t. The law of motion for wealth accumulation is:

$$W_{t+1} = W_t - c_t - q_t \cdot \theta_t + q_{t+1} \cdot \theta_t + e_{t+1}$$

where $q_t \cdot \theta_{t+1}$ is the inner product of the two $n-$dimensional vectors q_t and θ_{t+1} and for any process $(x(z^t))_t$, the notation x_t stands for $x(z^t)$.

The problem of the representative investor is the following:

$$\begin{array}{rcl} & \max\limits_{(c,\theta)} v(c, z_0) & \\ & \text{Subject to :} & \\ \forall t,\ (c_t, \theta_t) & \in & D\left(W_t, z_t\right) \\ D\left(W_t, z_t\right) & = & \{(c,\theta) \mid 0 \leq c \leq W_t \text{ and } \theta \in \Re^n\} \\ W_{t+1} & = & W_t - c_t - q_t \cdot \theta_t + q_{t+1} \cdot \theta_t + e_{t+1} \end{array}$$

where v is the intertemporal utility as in Suryanarayanan (2006b). v is defined recursively as follows:

$$\begin{array}{rcl} v(c, z_0) & = & v_0(c, z_0) \\ v_t(c, z^t) & = & u(c_t) + \beta E_{\pi_t} v_{t+1}(c, (z^t, z_{t+1})) \end{array}$$

where

$$\begin{array}{rcl} \beta & < & 1 \text{ and } \pi_t = \displaystyle\int_{\pi \in M_t} \pi d\lambda_t(\pi) \\ M_t & = & \left\{ \pi \in P(z_t) \mid \psi_t(c,\pi) = \arg \min\limits_{\pi \in P(z_t)} \psi_t(c,\pi) \right\} \\ \psi_t(c,\pi) & = & u(c_t) - u(c_t^{*\pi}) + \beta E_\pi (v_{t+1}(c, z^{t+1}) - v_{t+1}(c^{*\pi}, z^{t+1})) \\ (c^{*\pi}, \theta^{*\pi}) & = & \arg \min\limits_{\substack{(c_t^*, \theta_t^*)_t \\ (c_t^*, \theta_t^*) \in D(W_t^*, z_t) \\ W_{t+1}^* = W_t^* - c_t^* - q_t \cdot \theta_t^* + q_{t+1} \cdot \theta_t^* + e_{t+1} \text{ and } W_t^* = W_t}} \\ & & \{u(c_t) - u(c_t^{*\pi}) + \beta E_\pi (v_{t+1}(c, z^{t+1}) - v_{t+1}(c^{*\pi}, z^{t+1}))\} \\ \lambda_t & = & \arg \min\limits_{\lambda \in \Lambda_{M_t}(z_t)} \max\limits_{\substack{(c_t, \theta_t)_t \\ (c_t, \theta_t) \in D(W_t, z_t)}} \displaystyle\int_{\pi \in M_t} \psi_t(c,\pi) d\lambda(\pi) \end{array}$$

3.1.2 Equilibrium

An equilibrium is a consumption plan (c_t), an asset allocation plan (θ_t) and a price process $(q_{i,t})$ such that:

1. Given prices $(q_{i,t})$, $(c_t, \theta_t)_t$ solves the representative investor's problem
2. Consumption good market clears: $c_t = e_t$ for all t
3. Asset markets clear: $\theta_t = 0$ for all t

3.2 Characterization of equilibrium asset prices

In this Section, we show how to solve for the equilibrium prices which are characterized with distorted Euler equations.

As in Suryanarayanan (2006a), the optimality conditions for the representative agent's problem can be computed as:

$$\begin{aligned} \text{for all } i &= 1, ..., n, \\ q_{i,t} &= E_{\pi^{\exp}(W_t,z_t)} \beta \frac{u'(c_{t+1})}{u'(c_t)} (q_{i,t+1} + d_{i,t+1}) \end{aligned}$$

This follows directly by solving the zero-sum game problem which defines the expost probability distribution or endogenous belief π_t. Since the relevant state variables which fully characterize the choice environment at time t are (W_t, z_t), we denote the endogenous one-period ahead conditional distribution π_t with $\pi^{\exp}(W_t, z_t)$.

- **Distorted Euler pricing equations**

The following proposition characterizes equilibrium asset prices:

Proposition 1 *A necessary and sufficient condition for* $(q_{i,t})$ *to be an equilibrium asset price process is condition* (E) :

$$(E) : q_{i,t} = E_{\pi^{\exp}(W_t,z_t)} \beta \frac{u'(e(z^{t+1}))}{u'(e(z^t))} (q_{i,t+1} + d_{i,t+1})$$

The fact that (E) is a necessary condition for equilibrium prices follows from the optimality conditions for the representative investor's problem and also from market clearing conditions. (E) is also a sufficient condition as any price process $(q_{i,t})$ satisfying (E) is such that $c_t = e_t$ and $\theta_t = 0$ solves the representative investor's problem and (c_t, θ_t) thus clear the markets.

We see that the equilibrium condition (E) is similar to the standard Euler pricing equation except that we need to measure expectations relative to the endogenous belief instead of the prior p^*. We then use the term distorted Euler equations to define (E). Note that this does not mean that the Anticipated Regret asset pricing model is observationally equivalent to the Expected Utility asset pricing model provided we use the appropriate time-varying prior. Indeed, we need to keep in mind that the endogenous belief $\pi^{\exp}(W_t, z_t)$ not only depends on wealth in general but also implicitly depends on all current and one-period ahead prices $q_{i,t}$ and $q_{i,t+1}$ of all assets. While one could in principle always find a prior so that an expected utility maximizer would choose the same allocation of assets and the same consumption, this would not lead to the same equilibrium prices and there is therefore no observational equivalence result in terms of prices. As seen in the portfolio problem, this stems from the fact that Anticipated Regret affects the sensitivity of portfolio allocations and consumption-wealth ratios with respect to asset prices.

- **The state price of uncertainty**

Since $\pi^{\exp}(W_t, z_t)$ is absolutely continuous with respect to $p^*(\cdot|z_t)$, we can define its Radon-Nikodym derivative $\frac{d\pi^{\exp}}{dp^*}$ which we denote with $h(W_t, z_t, z_{t+1})$. The equilibrium condition (E) may then be rewritten as:

$$(E) : q_{i,t} = E_{p^*(\cdot|z_t)} \beta h(W_t, z_t, z_{t+1}) \frac{u'(e(z^{t+1}))}{u'(e(z^t))} (q_{i,t+1} + d_{i,t+1})$$

This alternative characterization allows to better assess the nature of the distortion. The fear that the prior p^* is misspecified and Anticipated Regret behavior induces an multiplicative adjustment for uncertainty embedded in $h(W_t, z_t, z_{t+1})$ in addition to the usual adjustment for risk embedded in the intertemporal marginal rate of substitution $\beta \frac{u'(e(z^{t+1}))}{u'(e(z^t))}$. We then use the Radon-Nikodym derivative h which measures the distortions of the endogenous belief with respect to the prior p^* to define the endogenous state price of uncertainty. As seen in Suryanarayanan (2006a), while the state price of risk $\beta \frac{u'(e(z^{t+1}))}{u'(e(z^t))}$ is unable to generate high and time-varying prices of risk consistent with observed asset prices, the Anticipated Regret may have the potential to explain fluctuations in asset prices within a consumption based framework as it may generate high fluctuations in the endogenous state price of uncertainty which help to explain the observed variations in asset prices. Consumption is not highly correlated to equity returns, nor is it risky nor volatile enough to generate the observed prices of risk. Fluctuations in the wealth dependent endogenous state price of uncertainty provide an alternative source of asset price variations.

3.3 Discussion

- **A numerical example**

We conduct a numerical exercise to show how the distortions in the endogenous beliefs affects equilibrium asset prices. We consider a two period version with two assets and two states L and H. Asset 1 is the claim to the state-contingent consumption in period 1 and asset 2 is a risk-free asset paying one unit of consumption in each state. We consider a Constant Relative Risk Aversion $(CRRA)$ utility function u with relative risk aversion $\gamma = 1.5$ and take for the endowment growths $\frac{e_{1L}}{e_0}$ and $\frac{e_{1H}}{e_0}$ the values 0.982 and 1.054, which were taken by Mehra and Prescott (1986) to calibrate aggregate consumption growth as a two-state Markov chain. Last we assume $\beta = 0.97$ and we assume that the investor has a prior probability $\pi^* = (0.5, 0.5)$ but doubts about it. She instead believes that the true probability lies in a symmetric interval around 0.5 for the probability distribution set $[0.5 - \varepsilon, 0.5 + \varepsilon]$. We vary the degree of ambiguity ε from 0 to 0.5.

Given the homogeneity property of the utility function u, the shadow price p_0^1 of the risky asset is proportional to the endowment at time 0 so that $p_0^1 = \mu e_0$

where μ is the constant price-consumption ratio. The distorted Euler equation then becomes:

$$(DE1) \quad : \quad \mu = E_{\pi^*} \frac{\pi^{\exp}(p_0)}{\pi^*} \beta \left(\frac{e_1}{e_0} \right)^{1-\gamma}$$

$$(DE2) \quad : \quad p_0^2 = E_{\pi^*} \frac{\pi^{\exp}(p_0)}{\pi^*} \beta \left(\frac{e_1}{e_0} \right)^{-\gamma}$$

In particular, we compare the equilibrium prices for the case when there is no ambiguity ($\varepsilon = 0$) to the case when there is full ambiguity ($\varepsilon = 0.5$) :

	p_0^1/e_0	p_0^2	R_L	R_H	R_f	Risk Premium
$\varepsilon = 0$	0.946	0.962	2.10%	9.58%	5.64%	0.20%
$\varepsilon = 0.5$	0.971	0.975	1.10%	8.51%	2.61%	2.19%

where $R_z = e_{1z}/p_0^1$ for $z \in \{L, H\}$ is the shadow return to the risky asset in state z, $R_f = 1/p_0^2$ is the shadow return on the riskless asset.

Markably, anticipated regret generates an implied risk premium of an order of magnitude 10 times greater than the standard expected utility model when the ambiguity level is at its highest ($\varepsilon = 0.5$). This illustrates the importance of ambiguity and anticipated regret behavior for generating higher prices of risk. The channel is the distorted endogenous belief. Indeed, the implicit and endogenous belief of the investor for the low growth state L is given by $\pi^{\exp} = 0.78$, with a likelihood ratio relative to the prior equal to $\frac{\pi^{\exp}}{\pi} = 1.56$. This means that the investor implicitly discounts low endowment states 1.56 times more than would a standard expected utility maximizer who would trust his prior. This in turn implies a lower certainty equivalent as measured by the low risk-free rate (2.61% compared to 5.64%) together with a higher price for the risky asset, and a higher risk premium to compensate the investor for holding the risky asset with ambiguous returns.

- **Volatility puzzle**

Since Shiller (1981), Leroy and Porter (1981) and Grossman and Shiller (1981), the observed patterns in aggregate stock index volatility are hard to explain with the existing asset pricing models. Indeed, the main source of risk that is emphasized is linked to aggregate consumption and dividends, which are much less risky and volatile compared to asset returns. An asset pricing model based on Anticipated Regret may have the potential to explain why asset prices may fluctuate while their fundamentals do not. Suryanarayanan (2006a) shows that endogenous distortions in the representative agent's beliefs are an important source of variations in asset prices and allow to match the implied stochastic discount factor in the Mehra and Prescott (1985) economy. In turn, the model generates time varying and higher levels of volatility. Improving the ability to match the observed volatility levels can be achieved by inducing higher sensitivity to wealth in the state price of uncertainty h. One possibility is to extend the state space, including an additional asset with dividends that match observed dividends and using the Kullback-Leibler distance as in Hansen's and Sargent's applications of Robust Control to measure uncertainty.

- **CAPM revisited**

The distorted Euler equations (E) allows us to derive a CAPM representation of asset returns. In turn, this would allow us to better assess the performance of the model.

We define the return on asset i as:

$$R_{i,t+1} = \frac{q_{i,t+1} + d_{i,t+1}}{q_{i,t}} - 1$$

Equation (E) can then be rewritten as

$$1 = E_{p^*(\cdot|z_t)} \beta h(W_t, z_t, z_{t+1}) \frac{u'(e(z_{t+1}))}{u'(e(z_t))} (R_{i,t+1} + 1)$$

We assume that there exists a risk-free security with return R^f_{t+1}. Thus:

$$E_{p^*(\cdot|z_t)} \beta h(W_t, z_t, z_{t+1}) \frac{u'(e(z^{t+1}))}{u'(e(z^t))} (R_{i,t+1} - R^f_{t+1}) = 0$$

This can be rewritten as:

$$\begin{aligned}
E_t(R_{i,t+1} - R^f_{t+1}) &= -cov_t(m_{t+1}, R_{i,t+1} - R^f_{t+1}) R^f_{t+1} \\
m_{t+1} &= \beta h(W_t, z_t, z_{t+1}) \frac{u'(e(z_{t+1}))}{u'(e(z_t))} \\
R^f_{t+1} &= \frac{1}{E_t(m_{t+1})}
\end{aligned}$$

This gives the modified CAPM consistent with Anticipated Regret. In the standard CAPM, the stochastic discount factor m_{t+1} is the state price of risk $\beta \frac{u'(e(z^{t+1}))}{u'(e(z^t))}$. We show that the CAPM that would be consistent with Anticipated Regret behavior would involve an adjustment for uncertainty by inflating the traditional m_{t+1} with the endogenous state price of uncertainty $h(W_{t+1}, z_t, z_{t+1})$. We see how relaxing the assumption of rational expecations (i.e the investors fully trust their prior and know it is the true probability) may induce, under Anticipated Regret, endogeneity biases in traditional CAPM regressions. With Anticipated Regret, m_{t+1} which defines the CAPM Beta would itself depend on returns and need to be jointly estimated with the risk-premium.

- **Euler pricing errors**

We now discuss the potential of the Anticipated Regret model to generate lower pricing errors. Lettau and Ludvigson (2006) show that leading models of asset pricing while improving the ability to match moments of asset returns do not do well in generating lower pricing errors, i.e the difference between the observed asset price and the one implied by the model. They investigate the puzzle by looking directly at the Euler equations in difference and point to the fact that the rational expectations hypothesis, i.e investors know the true probability distribution which could be perfectly estimated from the data, may be too restrictive.

Let $\varepsilon_{i,t}$ be the Euler error associated with asset i :

$$\varepsilon_{i,t} = E_{p^*(\cdot|z_t)}\beta h(W_t, z_t, z_{t+1})\frac{u'(e(z^{t+1}))}{u'(e(z^t))}(R_{i,t+1} - R^f_{t+1})$$

a measure of pricing errors may be the average empirical squarred Euler errors over time and across assets:

$$\varepsilon = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\sum_{i,j}\varepsilon_{i,t}\varepsilon_{j,t}}$$

In order to see why the Anticipated Regret model has the potential to generate lower pricing errors, we may decompose the Euler errors into a risk component and an uncertainty component:

$$\begin{aligned}
\varepsilon_{i,t} &= \varepsilon^R_{i,t} + \varepsilon^U_{i,t}E_{p^*(\cdot|z_t)}\beta\frac{u'(e(z^{t+1}))}{u'(e(z^t))}(R_{i,t+1} - R^f_{t+1}) \\
\varepsilon^R_{i,t} &= E_{p^*(\cdot|z_t)}\beta\frac{u'(e(z^{t+1}))}{u'(e(z^t))}(R_{i,t+1} - R^f_{t+1}) \\
\varepsilon^U_{i,t} &= E_{p^*(\cdot|z_t)}(h(W_t, z_t, z_{t+1}) - 1)\beta\frac{u'(e(z^{t+1}))}{u'(e(z^t))}(R_{i,t+1} - R^f_{t+1})
\end{aligned}$$

While Lettau and Ludvigson (2006) find that $\varepsilon^R_{i,t}$ is way too far from zero, $\varepsilon^U_{i,t}$ may compensate the existing gap between observed and predicted prices through the state price of uncertainty h. Findings in Suryanarayanan (2006) suggests that Anticipated Regret may indeed help to generate lower pricing errors in general settings since the model is able to match the stochastic discount factor implied by observing the first two moments of asset returns. Since the pricing kernel characterizes Euler pricing equations, a model generating sensible pricing kernels will in turn also generate lower Euler pricing errors.

Euler pricing errors are in turn related (with a one-to-one mapping) to the α pricing errors of factor models. The Anticipated Regret model may then explain why Fama and French (1984) conjectured that the pricing errors in factor models were non-systemic and unrelated to risk. Including additional factors since then has only merely improved the predicting powers of the model. We suggest an alternative source of errors and show how to construct uncertainty factors $\varepsilon^U_{i,t}$ to help reduce the α pricing errors.

- **Martingales and efficient markets**

Since Harrison and Pliska (1981) proposed a characterization of competitive equilibrium prices in terms of the equivalent martingale measure, i.e there exists a probability measure under which asset prices are the expected value of their payoffs, financial economists and macroeconomists have attempted to link the implied martingale measures to economic fundamentals. In leading models of asset pricing, the martingale measure is linked to the intertemporal rate of substitution of consumption. Hall (1978) concluded from his observations that consumption was a martingale. This would imply in turn a constant intertemporal marginal rate of substitution and in turn constant asset prices as well in standard asset pricing models with expected utility maximizing investors. Grossman and Shiller (1981) in particular find the observed variations in asset prices are at odds with Hall's conjectures although consumption growth is roughly stationary which does only slightly contradict Hall's observations. Either Hall's observations are erroneous or asset prices have an alternative source of variation.

As in Hansen's and Sargent's applications of Robust Control, let us define recursively

$$\begin{aligned} H_{t+1}(z') &= h(W_t, z_t, z')H_t(z_t) \\ H_0 &= 1 \end{aligned}$$

where h is the endogenous Radon-Nikodym derivative. Since

$$\int_{z_{t+1}\in Z} h(W_t, z_t, z_{t+1})dp^*(z_{t+1}|z_t) = 1$$

we have that (H_t) is a martingale. And the expost distribution is an equivalent martingale measure unadjusted for risk. Moreover, asset prices can be expressed as:

$$(E) : q_{i,t} = E_{p^*(\cdot|z_t)}\beta\frac{H_{t+1}}{H_t}\frac{u'(e(z^{t+1}))}{u'(e(z^t))}(q_{i,t+1} + d_{i,t+1})$$

Thus Hall's conjecture and Harrison and Pliska's equivalent martingale measure theory may be reconciled with the observed magnitude of fluctuations in asset prices. Indeed the marginal utility consumption may be almost constant over time while the state price of uncertainty h (which is the rate of growth in H_t) varies substantially over time.

4 Concluding remarks

This paper investigates the ability of the Anticipated Regret model to help us understand important asset pricing puzzles. Hall's (1978) investigations and conclusions that consumption is roughly a martingale are still commonly thought to be at odds with the observed fluctuations in asset prices. Indeed, as pointed by Grossman and Shiller (1981) and Shiller (1981), when investors are expected utility maximizers, Hall's observations lead to the conclusion that the marginal rate of substitution must be approximately constant as well which would lead to essentially constant asset prices. Further, Fama and French (1984) and subsequent studies lead to the conjecture that the pricing errors in factor models are not related to fundamental risk factors and Lettau and Ludvigson (2006) show that leading asset pricing models are unable to generate low pricing errors. The analysis in this paper suggests a way to reconcile these apparently puzzling and disconnected facts. Fluctuations in asset prices are not only due to fluctuations in fundamentals such as equity dividends and consumption growth but mostly due to endogenous fluctuations in investors' beliefs. Indeed, equilibrium asset prices in the Anticipated Regret model embed both the usual premium for risk through the intertemporal marginal rate of substitution and an uncertainty premium through the distorted wealth dependent endogenous beliefs which lowers the implied certainty equivalent. This enables us to define an endogenous state price of uncertainty which would help to explain why pricing errors in factor models and Euler equations, although substantial, are not related to omitted risk factors but to omitted uncertainty factors.

The investigations in Suryanarayanan (2006a) conclude to a first successful empirical study within the Mehra and Prescott two-point Markov chain economy driven by aggregate consumption growth. Further research should first study how to best model the probability sets to maintain tractability in the infinite horizon model and yet deliver interesting implications regarding the dependence of the endogenous beliefs on wealth. From an empirical viewpoint, the second step would extend the asset pricing exercise to include a continuous state space with more assets to disentangle the correlation between equity dividends growth and consumption growth. This would allow to better assess the performance of the model regarding the volatility puzzles and the Euler pricing equations puzzles.

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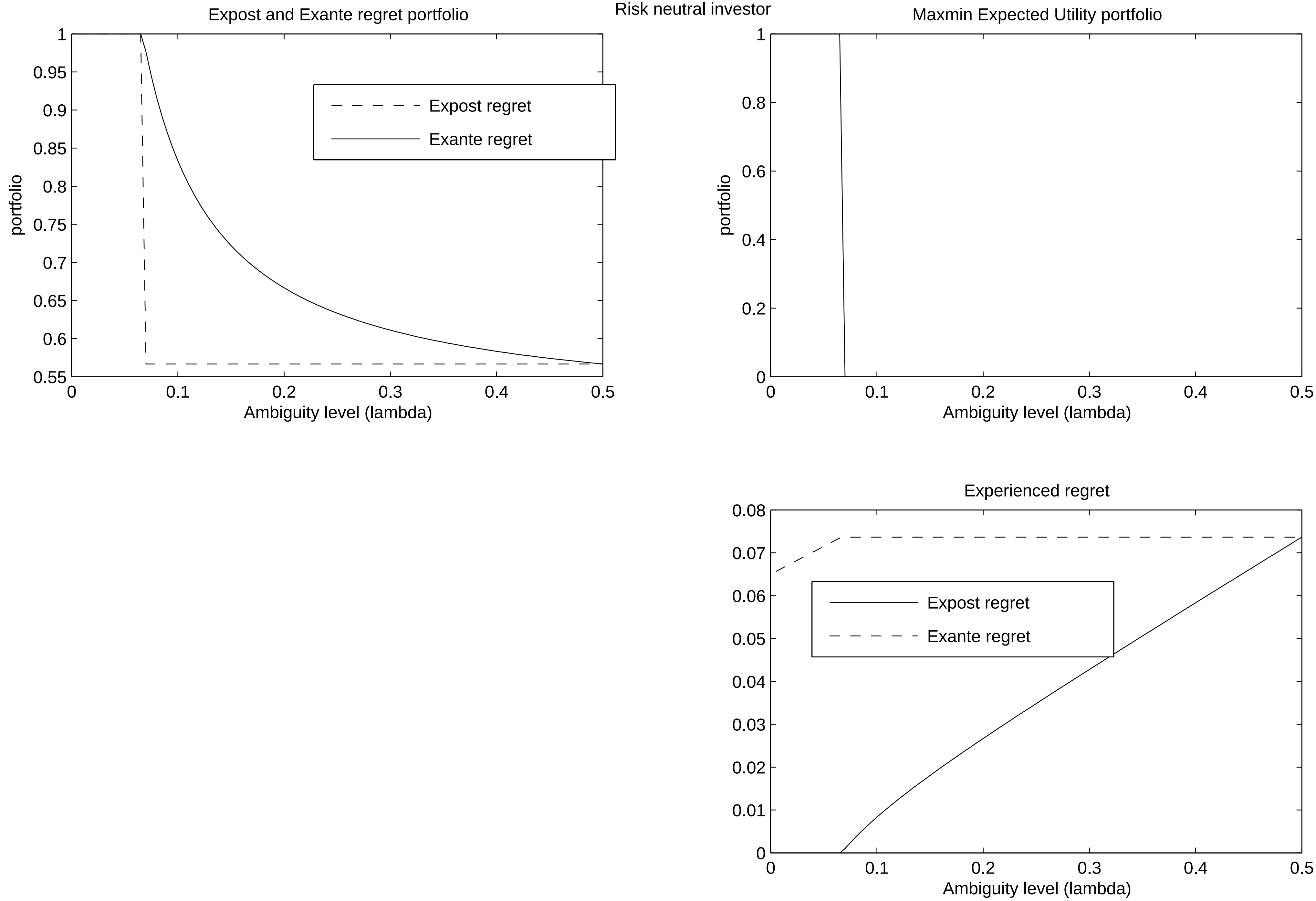
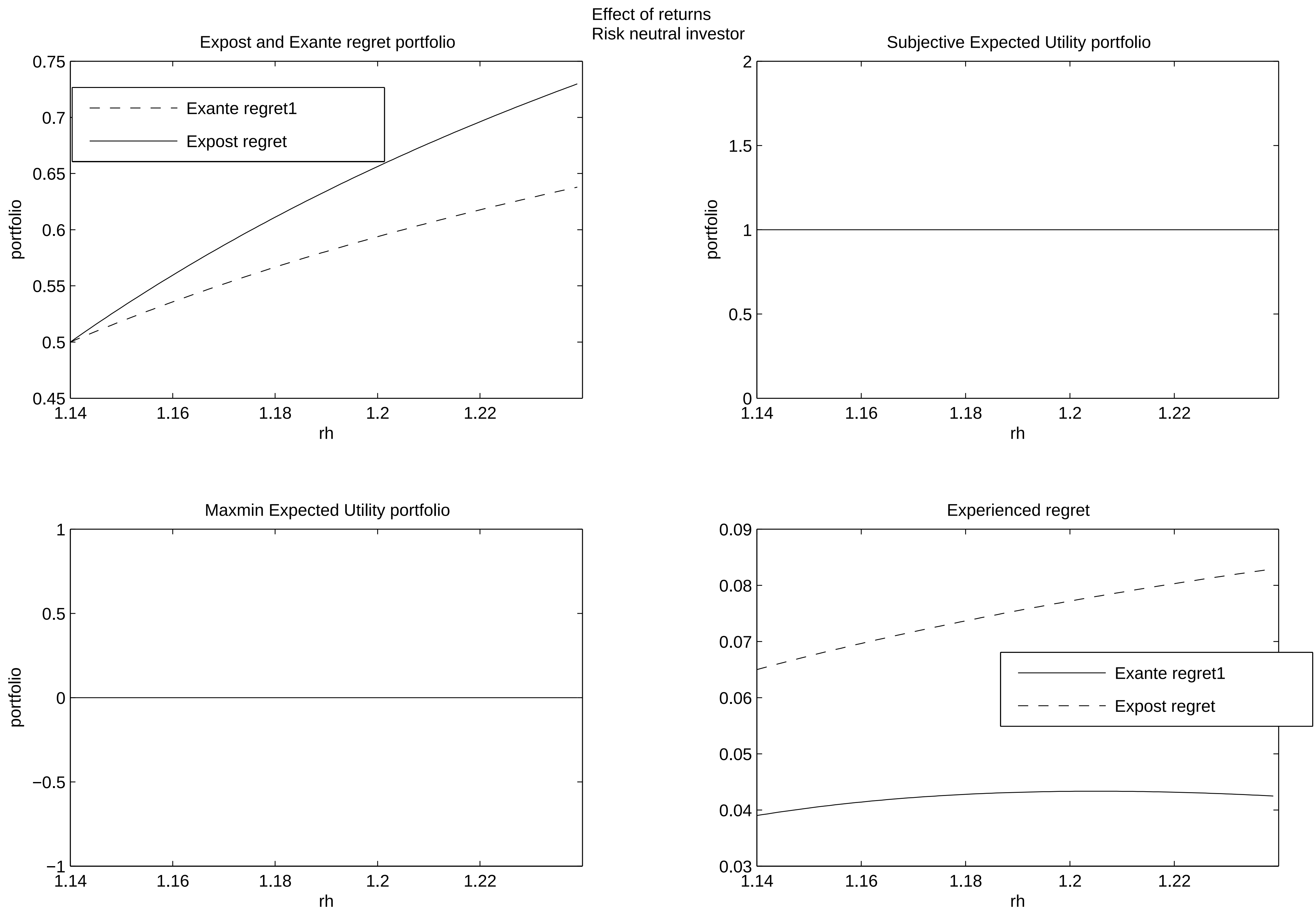
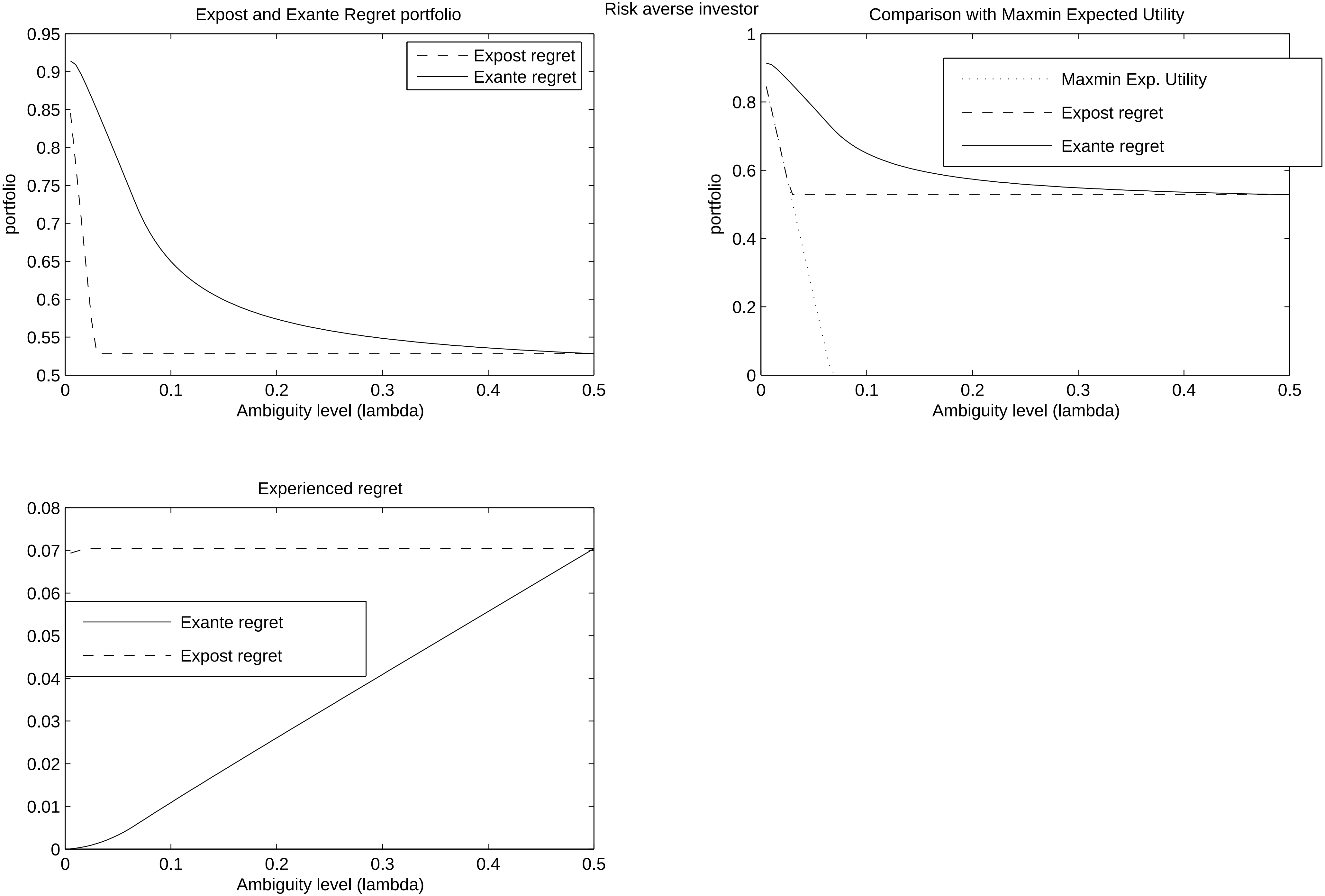
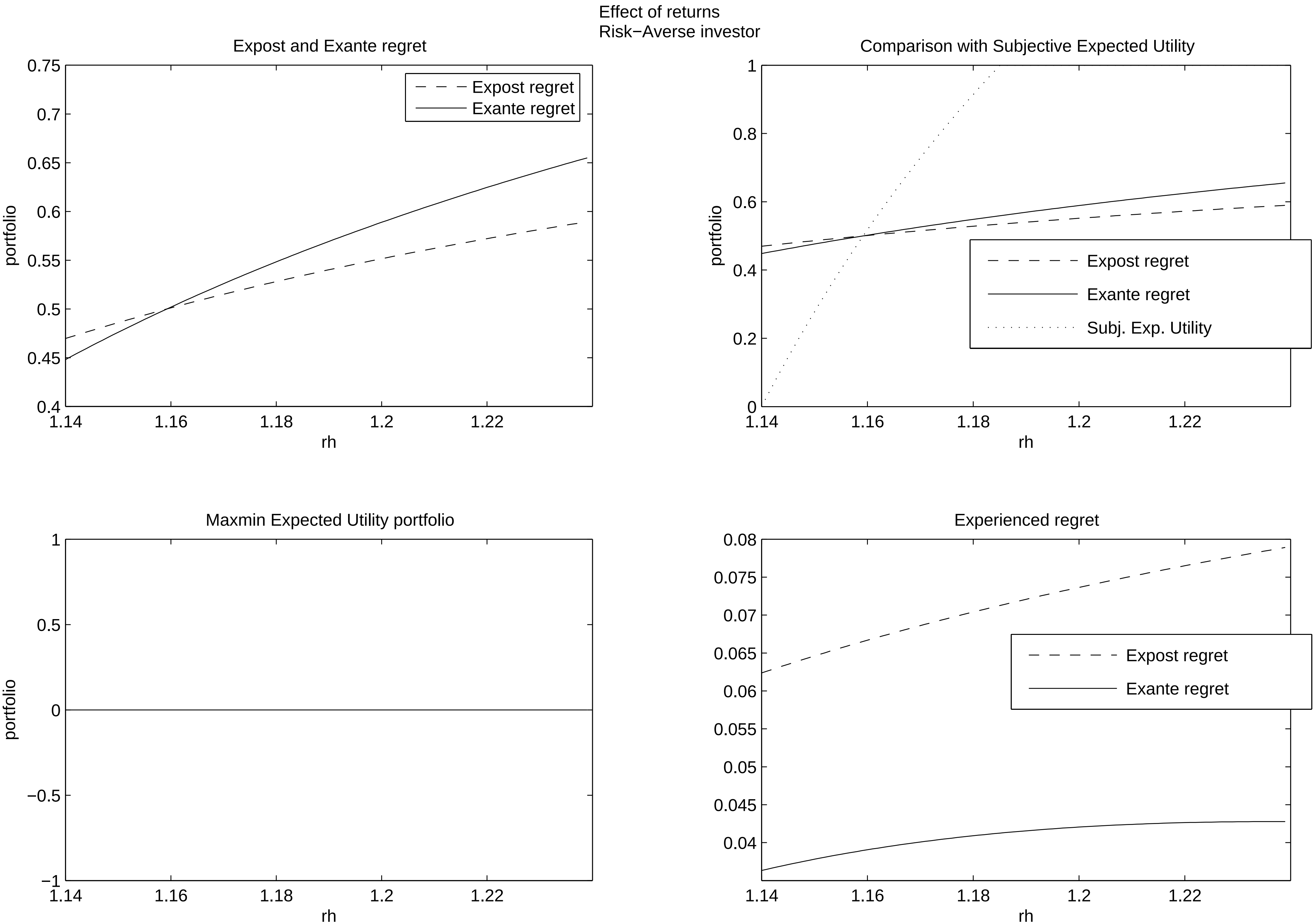
Effect of Ambiguity
Risk neutral investor




Effect of returns
Risk neutral investor
Expost and Exante regret portfolio
Exante regret1
Expost regret
portfolio
0.75
0.7
0.65
0.6
0.55
0.5
0.45
1.14
1.16
1.18
1.2
1.22
rh
Subjective Expected Utility portfolio
portfolio
2
1.5
1
0.5
0
1.14
1.16
1.18
1.2
1.22
rh
Maxmin Expected Utility portfolio
portfolio
1
0.5
0
−0.5
−1
1.14
1.16
1.18
1.2
1.22
rh
Experienced regret
Exante regret1
Expost regret
0.09
0.08
0.07
0.06
0.05
0.04
0.03
1.14
1.16
1.18
1.2
1.22
rh

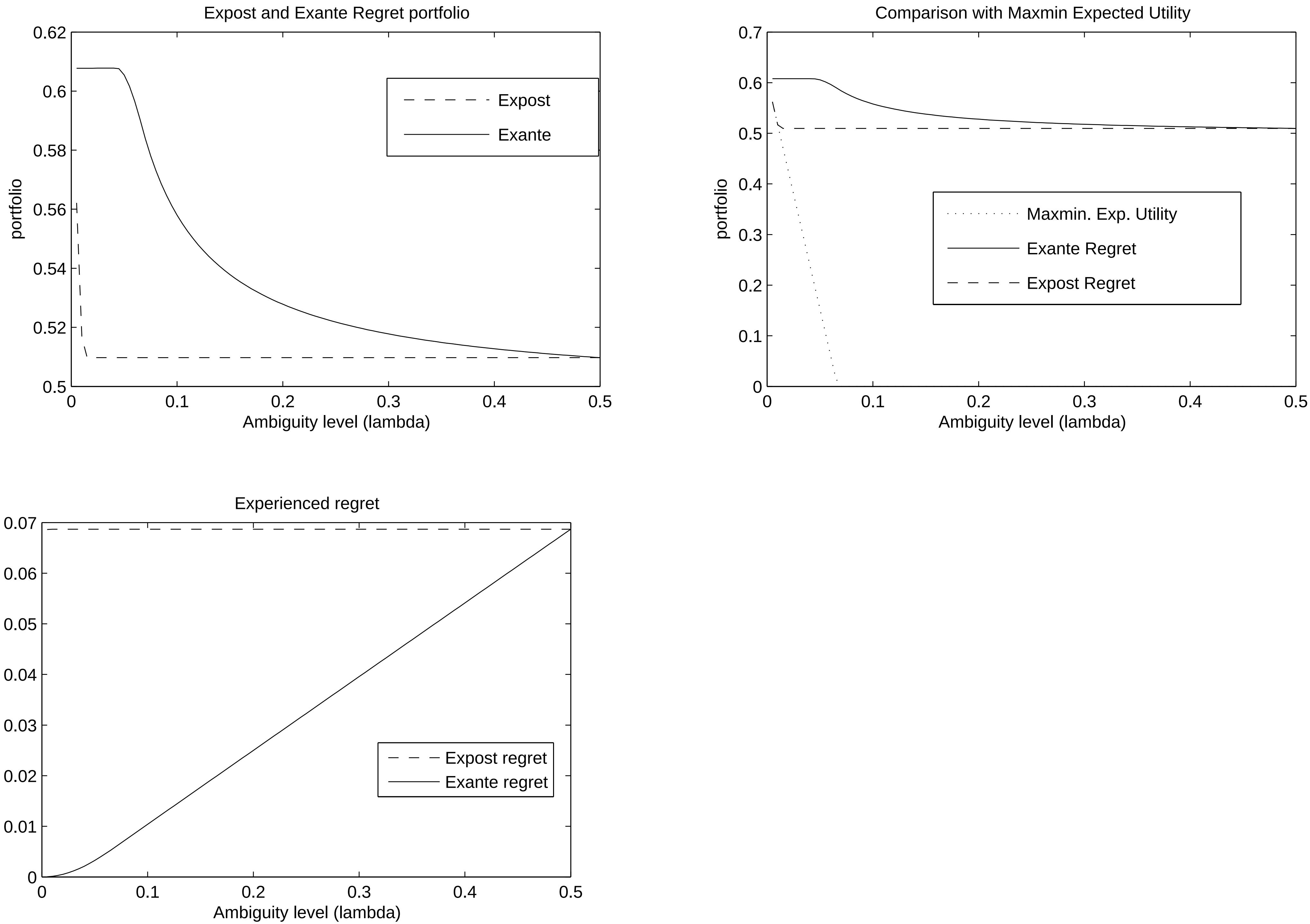
Effect of Ambiguity
Risk averse investor

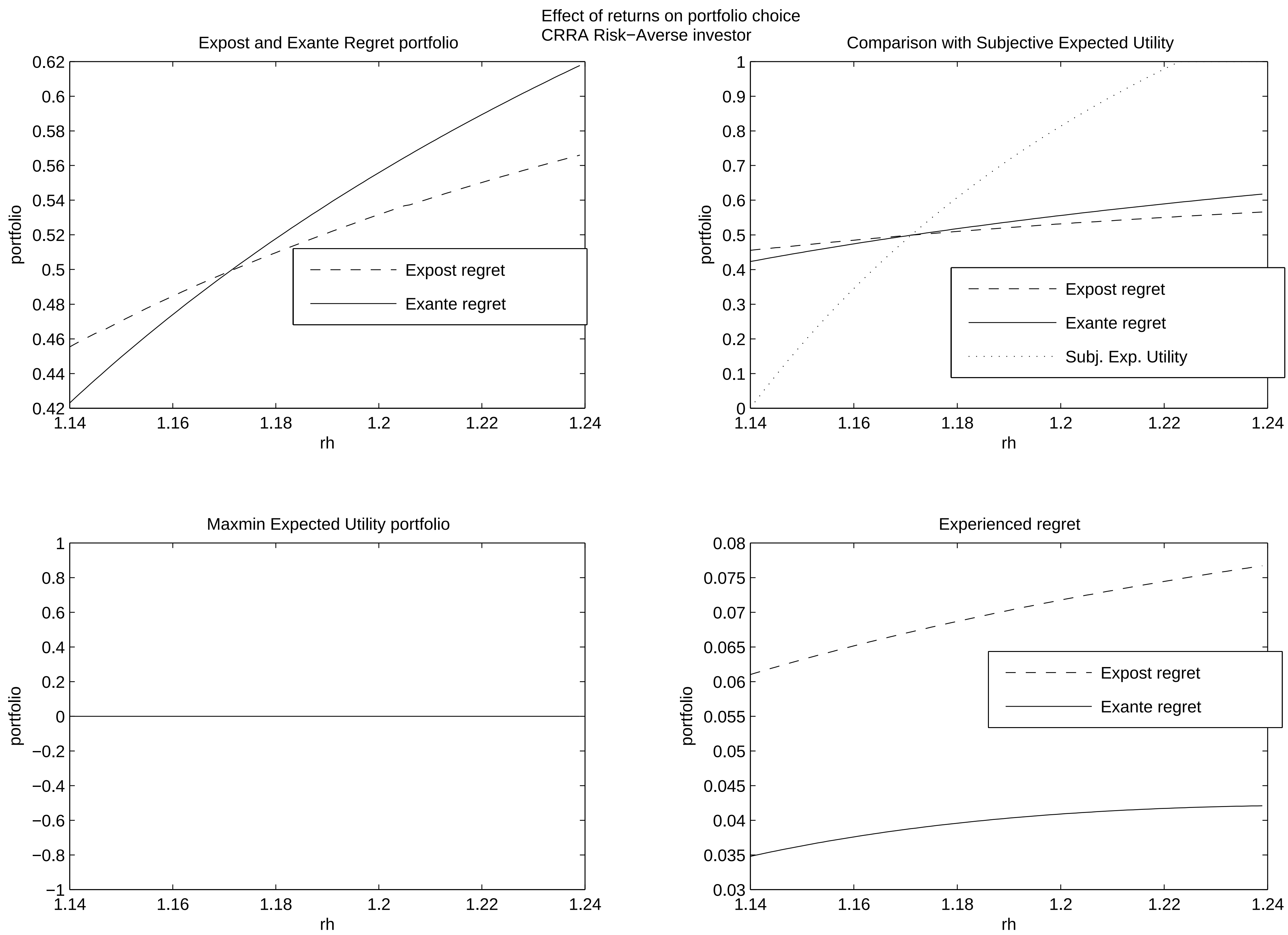




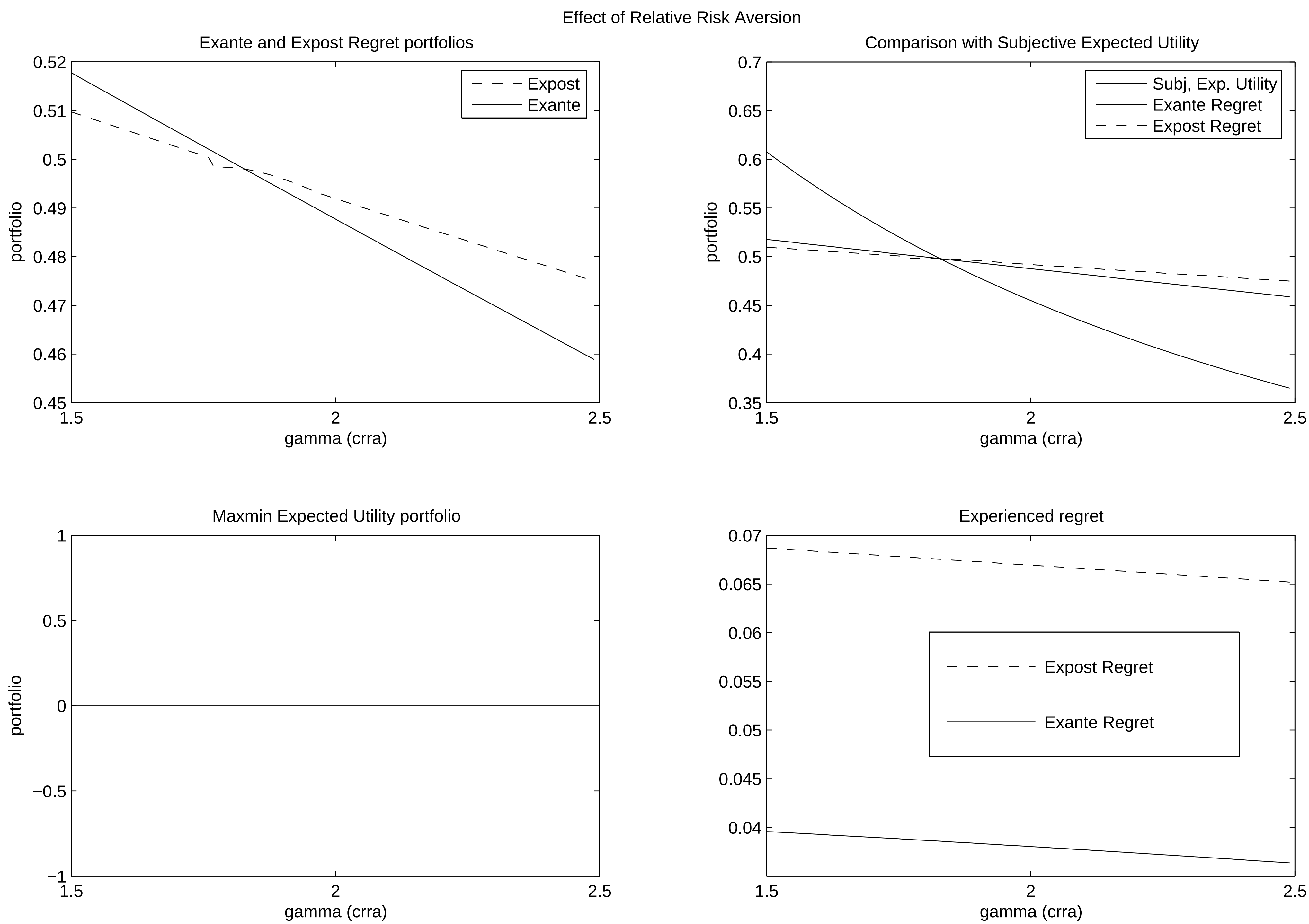
Effect of returns
Risk−Averse investor
Expost and Exante regret
Expost regret
Exante regret
portfolio
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
1.14
1.16
1.18
1.2
1.22
rh
Comparison with Subjective Expected Utility
Expost regret
Exante regret
Subj. Exp. Utility
portfolio
1
0.8
0.6
0.4
0.2
0
1.14
1.16
1.18
1.2
1.22
rh
Maxmin Expected Utility portfolio
portfolio
1
0.5
0
−0.5
−1
1.14
1.16
1.18
1.2
1.22
rh
Experienced regret
Expost regret
Exante regret
0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
1.14
1.16
1.18
1.2
1.22
rh

Effect of Ambiguity
CRRA Risk−Averse investor



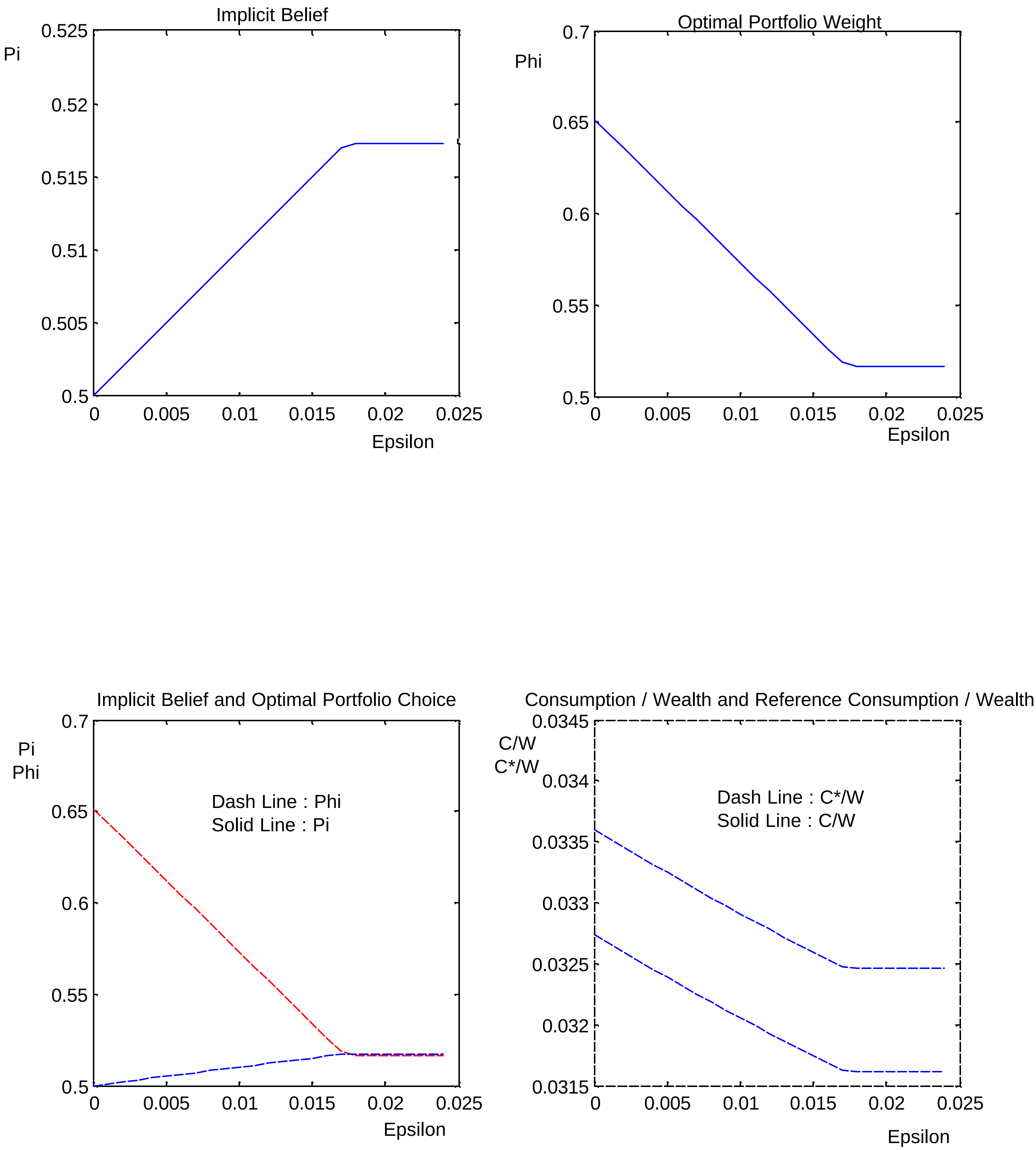


Effect of returns on portfolio choice
CRRA Risk−Averse investor
Expost and Exante Regret portfolio
Comparison with Subjective Expected Utility
Maxmin Expected Utility portfolio
Experienced regret
portfolio
rh
Expost regret
Exante regret
Subj. Exp. Utility

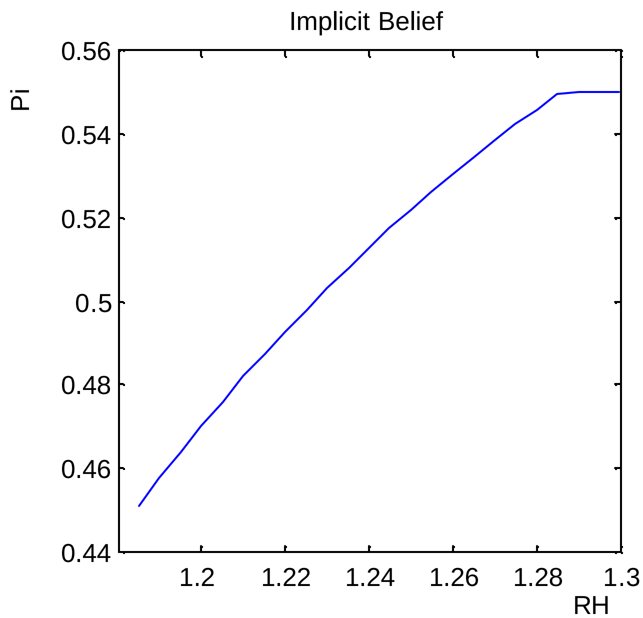


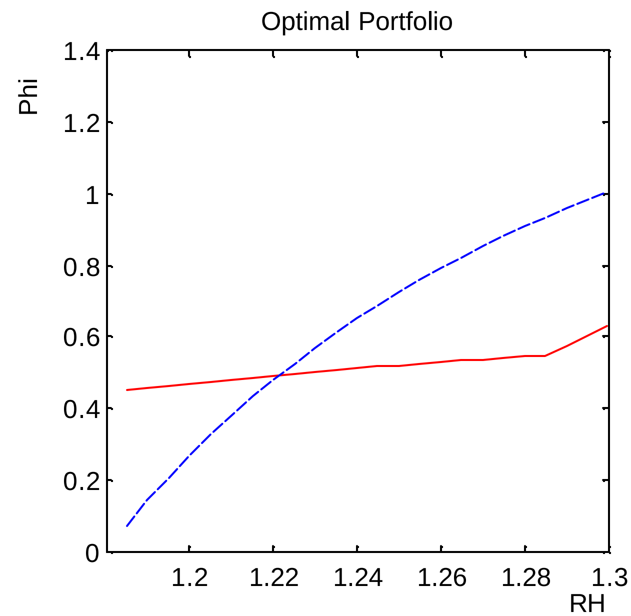
Effect of Relative Risk Aversion
Exante and Expost Regret portfolios
Expost
Exante
portfolio
0.52
0.51
0.5
0.49
0.48
0.47
0.46
0.45
1.5
2
2.5
gamma (crra)
Comparison with Subjective Expected Utility
Subj, Exp. Utility
Exante Regret
Expost Regret
portfolio
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
1.5
2
2.5
gamma (crra)
Maxmin Expected Utility portfolio
portfolio
1
0.5
0
−0.5
−1
1.5
2
2.5
gamma (crra)
Experienced regret
Expost Regret
Exante Regret
0.07
0.065
0.06
0.055
0.05
0.045
0.04
1.5
2
2.5
gamma (crra)

PANEL A
Effect of Uncertainty in the "Best" expost reference portfolio model

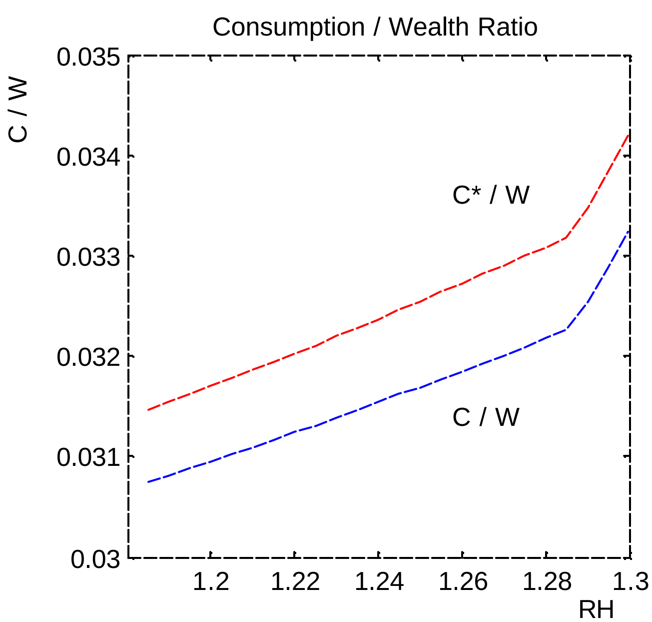


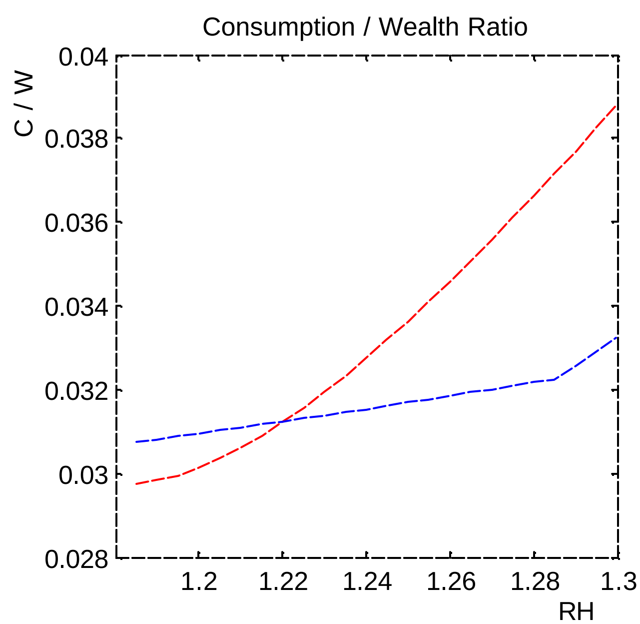
PANEL B
Effect of the Risky and Uncertain Return





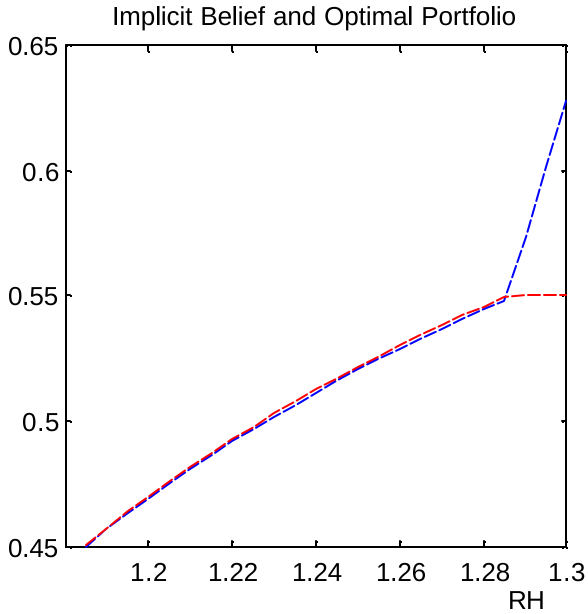
Dash line : Subjective Expected
Utility, pi = 0.5
Solid line :
Reference Point Utility



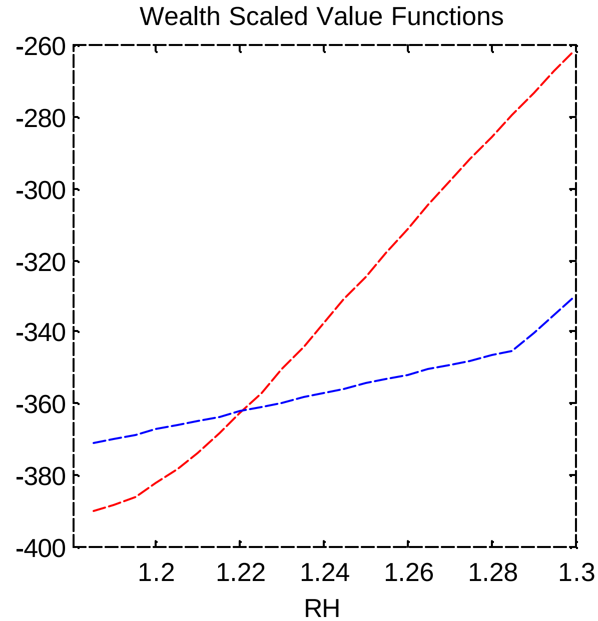


Dash line : Subjective Expected
Utility, pi = 0.5
Solid line :
Reference Point Utility

PANEL C
Effect of the Risky and Uncertain Return

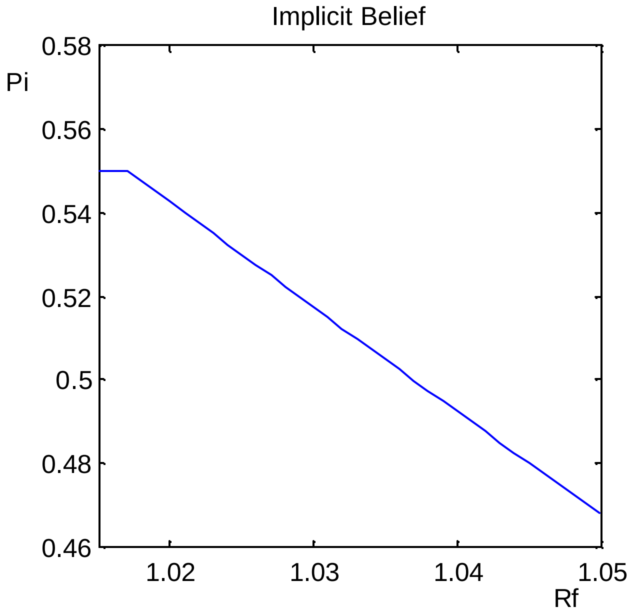


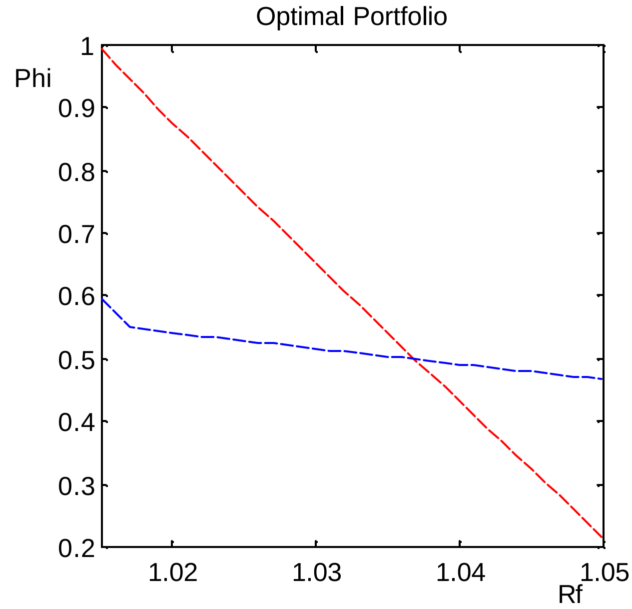
Dash line : Phi
Solid line : Pi



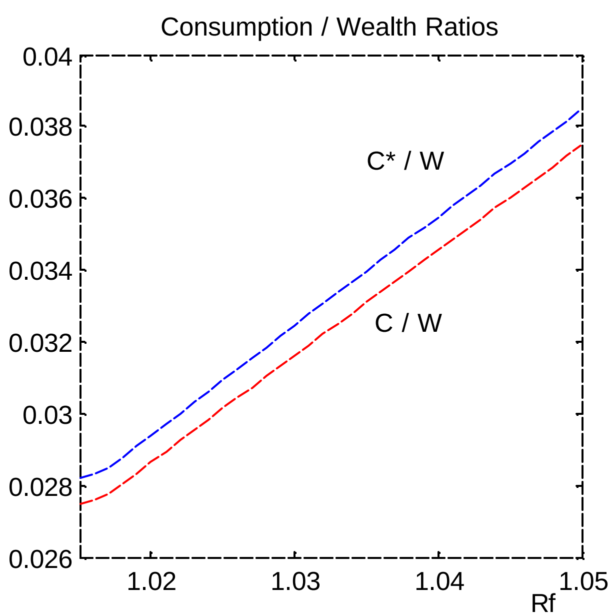
Dash line : Subjective Expected
Utility with pi = 0.5
Solid Line : Reference Point Utility

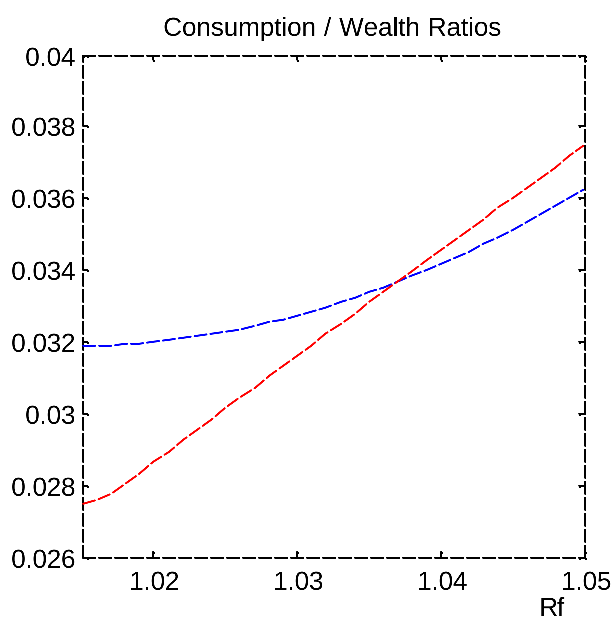
PANEL D
Effect of the risk-free rate





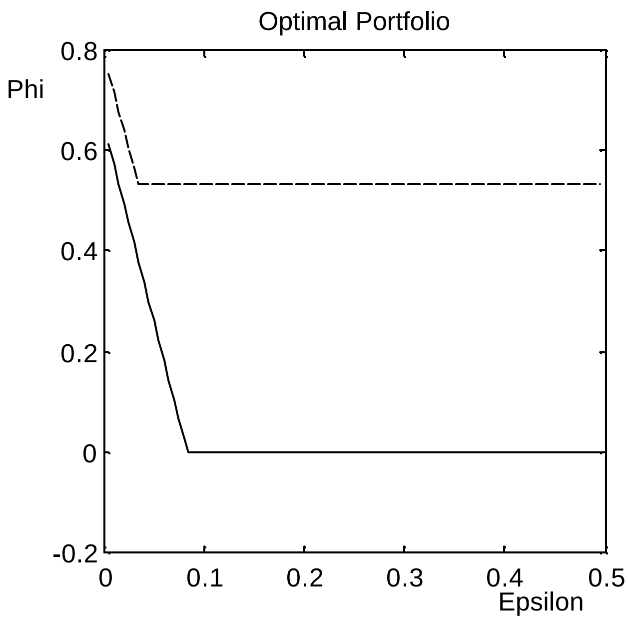
Dash line : Subjective Expected
Utility, pi = 0.5
Solid line : Reference Point Utility



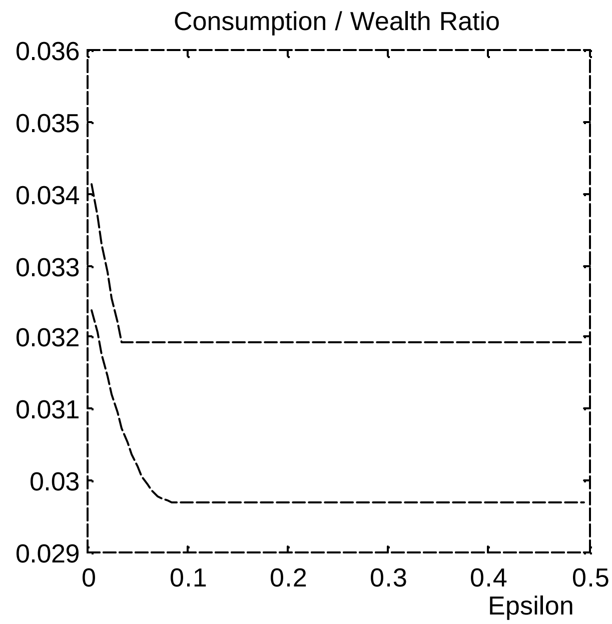


Dash line : Subjective Expected
Utility, pi = 0.5
Solid line : Reference Point Utility

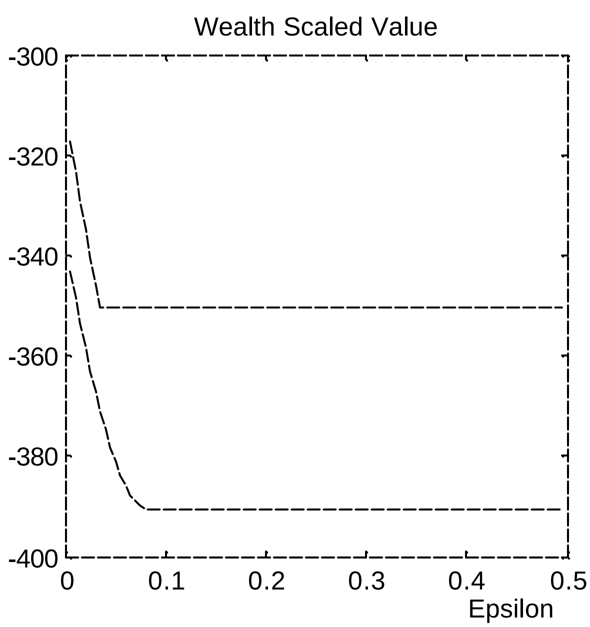
PANEL E
Comparison with Minmax Expected Utility



Dash line : Reference Point Utility
Solid line : Minmax Expected Utility



Dash line : Reference Point Utility
Solid line : Minmax Expected Utility



Dash line : Reference Point Utility
Solid line : Minmax Expected Utility