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# *Early Retirement and Social Security: A Long Term Perspective*

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**Abstract**

We provide a long term perspective on the individual retirement behavior and on the future of retirement. In a Markovian political economic theoretical framework, in which incentives to retire early are embedded, we derive a political equilibrium with positive social security contribution rates and early retirement. While aging has opposite economic and political effects on social security contributions, it may lead to postponing retirement -- by reducing the generosity of pension benefits -- unless the political effect leads to a large increase in contribution and hence higher benefits. Economic slowdowns, captured by a reduction in wage income in youth, will also induce workers to postpone retirement and to vote for less social security.



**Keywords**: pensions, income effect, tax burden, politico-economic Markovian equilibrium

**JEL Classification**: H53, H55, D72

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# 1 Introduction

Retirement decisions represent one of the hottest issues in the current social security debate. Several studies (see Blondal and Scarpetta, 1998, and Gruber and Wise, 1999 and 2003) suggest that the individual retirement behavior is strongly affected by the design of the social security system. Retirement is concentrated at early and normal retirement age, as most individuals tend to retire as soon as this opportunity is available. Moreover, most social security systems provide strong incentives, such as large implicit taxes on continuing to work, to anticipate retirement. However, the individual retirement behavior is also largely influenced by wealth or income effects. Several recent studies (see Costa, 1988, Coronado and Perozek, 2003, Buetler, Huguenin and Teppa, 2005, and Euwals, Van Vuuren and Wolthoff, 2006) in fact show that both expected and unexpected increase in workers' income or wealth induce them to retire early.

The massive use of early retirement provisions and their generosity have contributed to the deterioration of the financial sustainability of the system, already under stress because of population aging. In fact, several international organizations - such as the European Union at the 2001 Lisbon Meetings - have advocated an increase in the effective retirement age, or - analogously - the increase in the activity rate among individuals aged above 55 years, as a key policy measure to control the rise in social security expenditure. In a nutshell, postponing the retirement age has become a common element to all social security reform's proposals. Yet, whether these policy prescriptions will actually be adopted depends on the politics of early retirement (see Fenge and Pestieau, 2005, for a detailed discussion of early retirement issues, and Galasso and Profeta, 2002, for a survey of the political economy of social security).

This paper provides a political economy framework that features a double link between the retirement decisions and the political determination of the social security contribution rates. The design of the social security system – and in particular the contribution rate and the generosity of the pension benefits – has the usual effect on the economic individual retirement decisions. In the political arena, the mass of early retirees helps to determine the evolution of the social security system as individuals condition their voting decisions on this relevant information. This Markovian politico economy model that characterizes the political equilibrium sequences of social security tax rates and the corresponding use of early retirement provisions suggests a positive link between social security contributions and mass of early retirees.

This paper's main contribution is to analyze the political role of (negative) income effects, for instance driven by aging or economic slowdowns, on the social security contribution rates and retirement age. In line with the existing literature (see Galasso and Profeta, 2002), our theoretical framework suggests that aging has two opposite effects on the contribution rates: it tends to decrease them, since it makes the public pension system less profitable, but it makes the median voter poorer, and thus induces higher social security. Despite this ambiguous result, however, aging may lead to less early retirement by reducing the generosity of the pension benefits, unless the social security contribution rate increases dramatically due to the political effect of aging. persistant economic slowdowns, as captured by a decrease of the income of young people, will induce all young agents to postpone retirement and to prefer less social security.

There exists a vast literature on retirement decisions. Already two decades ago, Feldstein (1974) and Boskin and Hurd (1978) analyzing the determinants of the decline in the labor force participation of elderly workers pointed at two key parameters of social security systems: the income guarantee and the implicit tax on earnings. Endogenous retirement decisions have been analyzed by showing how pension systems introduce distortions in the labor supply choice (see among others Diamond and Mirrless, 1978, Hu, 1979, Crawford and Lilien, 1981, and Michel and Pestieau, 1999). A new literature has lately emerged on the political economy of early retirement (see Fenge and Pestieau, 2005, Lacomba and Lagos, 2000, Casamatta et al., 2002, Cremer and Pestieau 2000, Cremer et al. 2002, Conde Ruiz and Galasso, 2003 and 2004), but has generally neglected the role of income effects. Markovian politico-economic models of social security have been recently studied by Azariadis and Galasso (2002), Hassler et al (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005). These models however focus on social security and abstract from the role of retirement. They also neglect the political impact of income effects, which are instead empirically significant.

The paper is structured as follows. In the next section, we present a Markovian politico economic model. Section 3 analyzes the impact of aging and of a negative income effect on the steady state level of early retirement and social security. Section 4 concludes.

# 2 A Politico-Economic Model

## 2.1 The Economic Environment

We introduce a simple two -period overlapping generations model. Every period, two generations are alive, we call them young and old. We consider a continuum of individuals heterogeneous in young and old wage income. The wage income of a type-$\delta$ individual is $w_t^y = \delta \overline{w}_t^y$ in youth, and $w_t^o = \delta \overline{w}_t^o$ in old age, where $\overline{w}_t^y$ and $\overline{w}_t^o$ are respectively the average income of young and old workers. Individual types $\delta$ are distributed according to some density function $f(\delta)$ over an interval $\left[\underline{\delta}, \overline{\delta}\right]$ with an average equal to 1 and cumulative density function $F(\delta)$.

Young individuals work: they receive a wage, $w^y$, pay a payroll tax, $\tau$, on labour income and save all their disposable income for old age consumption. There exists a storage technology that transforms a unit of today's consumption into $1+r$ units of tomorrow's consumption. All private intertemporal transfers of resources into the future are assumed to take place through this technology. Old individuals decide what fraction, $z$, of the second period to spend working; in other words, they decide when to retire. An old individual who works a proportion $z$ of the second period receives a net labor income equal to $w^o(1-\tau)$, for the fraction $z$ of the period, while he receives a pension $p$ for the remaining fraction $(1-z)$. Population grows at a non-negative rate, $n$; and we abstract from wage growth, so that $\overline{w}^j_{t+1} = \overline{w}^j_t \quad j = y, o$. Moreover, we assume that the economy is dynamically efficient, $r > n$.

The life time budget constraint for an agent born at time $t$ is equal to:

$$c^o_{t+1} = (1-\tau_t)\, w^y_t\, (1+r) + (1-\tau_{t+1})\, z_{t+1} w^o_{t+1} + p_{t+1}(1-z_{t+1}) \qquad (1)$$

where $c^o_{t+1}$ is old age consumption at time $t+1$, $\tau_t$ and $\tau_{t+1}$ are the payroll taxes respectively at periods $t$ and $t+1$, $r$ is the exogenous interest rate, and subscripts indicate the calendar time.

Every individual's pension benefit depends on her contribution in youth and in old age. In particular, we assume the individual pension to be earning-related for the contributions paid in old age, but flat for the contributions paid in youth. This combination induces an element of within-cohort redistribution, from high to low income individuals. As in Tabellini (2000) and in Conde-Ruiz and Galasso (2005), this feature is crucial to ensure the political sustainability of social security system, through the support of the low ability young[1]. The use of an old age earning related component allows also to model the incentive effect created by current contributions and pension benefits in the retirement decision. The total pension benefits obtained by an individual in her old age at time $t+1$ is thus:

[1] Alternatively, a 3-period OLG could be introduced in which social security is supported by a voting coalition of old and middle aged individuals. See Galasso and Profeta (2002) for a discussion of different elements leading to the political sustainability of social security.

$$p_{t+1}(1-z_{t+1}) = \tau_{t+1} w^o_{t+1} z_{t+1} + \overline{p}_{t+1} \tag{2}$$

Since we concentrate on budget balanced PAYG social security systems, the fixed component is

$$\overline{p}_{t+1} = \tau_{t+1}(1+n_{t+1})\overline{w}^y_{t+1} \tag{3}$$

In the aggregate, a balanced budget pay as you go (PAYG) social security system requires total pension transfers to equal total contributions:

$$\int_{\underline{\delta}}^{\overline{\delta}} p_{t+1}(1-z_{t+1}) f(\delta) d\delta = \tau_{t+1} \left( \int_{\underline{\delta}}^{\overline{\delta}} \delta \overline{w}^o_{t+1} z_{t+1} f(\delta) d\delta + (1+n_{t+1})\overline{w}^y_{t+1} \right) \tag{4}$$

Agents maximize a logarithmic utility function, which depends on old age consumption and leisure:

$$U(c_{t+1}, z_{t+1}) = \ln c^o_{t+1} + \phi \ln(1-z_{t+1}) \tag{5}$$

where $\phi < 1$ measures the relative importance of leisure to the individuals.

An old agent at time $t+1$ hence maximizes eq.5 with respect to $z_{t+1}$ subject to the budget constraints at eq.1 and 3, since agents internalize the effect of their retirement behavior on the pension benefits via the earning related component.

The solution of the maximization problems yields the following optimal individual labor supply decision:

$$\widehat{z}_{t+1} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1-\tau_t) w^y_t (1+r) + \overline{p}_{t+1}}{w^o_{t+1}} \tag{6}$$

or, equivalently,

$$\widehat{z}_{t+1} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1-\tau_t)(1+r)\overline{w}^y_t}{\overline{w}^o_{t+1}} - \frac{\phi}{1+\phi} \frac{\tau_{t+1}(1+n_{t+1})\overline{w}^y_{t+1}}{\delta \overline{w}^o_{t+1}} \tag{7}$$

This individual retirement decision displays standard properties. A positive income effect, such as an increase in the net labor income in youth, induces all agents to retire early; while an increase in the net labor income in old age, or a decrease in the pension benefits, would lead them to postpone retirement –

due mainly to a positive substitution effect. A generalized increase in wages – both in youth and in old age – would combine income and substitutions effects. If individuals appreciate the impact that an increase in wages has on their pension benefits (see eq.7), in this simple formulation, the income and substitution effects perfectly compensate one another, thus leaving the retirement decision unaffected. In the remaining of the paper, we will hence concentrate on changes in the wages in youth to analyze income effects and in the wages in old age to study incentive effects.

To ensure that no type-$\delta$ agent will end up either working the entire old age or retiring at the end of youth – that is, to avoid corner solutions in the individual labor supply decision – some conditions need to be imposed. In particular, with no social security system in place, i.e., for $\tau_t = 0\ \forall t$, no agent will ever want to work for the entire old age, and all agents will work for some period if $\phi < \overline{w}_t^o / \overline{w}_t^y (1+r)$. We shall hence assume that this condition holds. For positive contribution rates, the condition that individual labor supply decisions lead to interior solutions, i.e., $\widehat{z}_t \in (0,1)\ \forall t$, amounts to impose some restrictions on the dynamics of the contribution rates. In particular, we have that

$$\tau_{t+1} < \frac{\delta \left( \overline{w}_t^o - \phi \overline{w}_t^y (1+r)(1-\tau_t) \right)}{(1+n_{t+1}) \overline{w}_t^y}. \tag{8}$$

The mass of employed elderly in the economy[2] at time $t+1$ can easily be obtained by aggregating all individuals' retirement decisions:

$$Z_{t+1} = \int_{\underline{\delta}}^{\overline{\delta}} \hat{z}_{t+1} f(\delta) d\delta \tag{9}$$

which can also be written as

$$Z_{t+1} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1-\tau_t)(1+r)\overline{w}_t^y}{\overline{w}_{t+1}^o} - \frac{\phi}{1+\phi} \frac{\tau_{t+1}(1+n_{t+1})\overline{w}_{t+1}^y \widehat{\delta}}{\overline{w}_{t+1}^o} \tag{10}$$

[2] Clearly, $1-Z$ defines the mass of (early) retirees.

with

$$\widehat{\delta} = \int_{\underline{\delta}}^{\overline{\delta}} \frac{1}{\delta} f(\delta) d\delta \tag{11}$$

Since individuals with different income display different retirement behaviors, the mass of retirees will depend on the distribution of income in the economy. In particular, due to the incentive effect embedded in the model, high income elderly workers will be induced to retire later than low income workers. Yet, this effect is not linear, but tends to magnify the importance of the agents who enjoy very low income in old age and hence have an incentive to retire very early. The parameter $\widehat{\delta}$ captures this aspect by weighting the mass of these low-income elderly with their retirement behavior. The larger – for instance – the share of low-income elderly, the larger $\widehat{\delta}$ will be; and hence the larger the mass of (early) retirees $(1 - Z)$.

Finally, by substituting the individual decision at eq.6 and the social security budget constraint, we can easily derive the indirect utility respectively of a type-$\delta$ young and old individual at time $t$, which we denote by $v_t^y(\tau_t, \tau_{t+1}, \delta)$ and $v_t^o(\tau_{t-1}, \tau_t, \delta)$.

## 2.2 The Political Equilibrium

The purpose of this paper is to propose a theoretical framework in which to analyze the link between early retirement provision and the size of the social security system. As already showed at eq.6, early retirement behavior may be induced by specific features of the social security system, such as the size of contribution rates and pension benefits. Here, we study the political determination of this social security contribution rate. Every year, elections take place in which the current social security contribution rate is determined. All young and old agents participate at the elections. Their preferences over the contribution rate may differ – typically according to their income ($\delta$ type) and age. We follow a well established tradition in political economics by concentrating on the median voter decision. Moreover, due to the intergenerational nature of the system, we allow for some interdependence between current and future political decisions. In particular, we analyze Markov perfect equilibrium outcomes[3] of a repeated voting game over the social security contribution rate. Since we want to examine the possible link between the use of early retirement provisions and the size of the social security system, we base our notion of Markov equilibrium on the idea that current voters – in taking their policy decisions – expect future policy-makers to base their political decisions on social security on the mass of early retirees – or employed elderly – in the economy. These expectations will be validated in equilibrium.

More specifically, at every period $t$, the median voter in each generation of voters – typically a young individual[4] – decides her most favorite social security system (i.e., the tax rate $\tau_t$). In taking her decision, she expects her current decision to have an impact of future policies. In particular, her expectations about the future social security tax rate – and hence about her pension benefits – depend on the current level of employed elderly, according to a function $\tau_{t+1} = q^e(Z_t)$. Hence, future contribution rates depend on the current level of labor force participation by the elderly, which is in turn affected by the current voter's decision over the social security contribution rate. The median voter's optimal decision can thus be obtained by maximizing her lifecycle utility with respect to $\tau_t$, given expectations on the next period policy function $\tau_{t+1} = q^e(Z_t) = Q(Z_t(\tau_t, \tau_{t-1}))$:

$$\max_{\tau_t} v_t^y \left(\tau_t, \tau_{t+1,} \delta\right) = \max_{\tau_t} v_t^y \left(\tau_t, Q\left(Z_t\left(\tau_t, \tau_{t-1}\right)\right), \delta\right) \tag{12}$$

We can now define the Markov political equilibrium as follows

[3]For examples of Markov equilibria, see Krusell et al.(1996), Grossman and Helpman (1998), Bassetto (1999), Azariadis and Galasso (2002), Hassler et al. (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005).

[4]It is easy to show that, in this setting, an elderly voter will support a 100% contribution rate. For a positive population growth rate, $n > 0$, the median voter will hence be young.

**Definition 1** *A Markov political equilibrium is a pair of functions* $(Q, Z)$*, where* $Q : [0,1] \to [0,1]$ *is a policy rule,* $\tau_t = Q(Z_{t-1})$*, and* $Z : [0,1] \times [0,1] \to [0,1]$ *is an aggregation of private decision rules,* $Z_t(\tau_t, \tau_{-1}) = \int_{\underline{\delta}}^{\overline{\delta}} \hat{z}_t f(\delta) d\delta$*, such that the following functional equations hold:*

*i)* $Q(Z_{t-1}) = \arg\max_{\tau_t} v_t^y(\tau_t, \tau_{t+1}, \delta^m)$ *subject to* $\tau_{t+1} = Q(Z_t(\tau_t, \tau_{-1}))$*.*

*ii)* $Z_t(\tau_t, \tau_{t-1}) = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1-\tau_{t-1})(1+r)\overline{w}_{t-1}^y}{\overline{w}_t^o} - \frac{\phi}{1+\phi}\frac{\tau_t(1+n_t)\overline{w}_t^y\widehat{\delta}}{\overline{w}_t^o}$

*iii)* $\delta^m$ *identifies the median voter's type among the young.*

The first and last equilibrium conditions require that $\tau_t$ maximizes the objective function of the median voter – a type-$\delta^m$ young individual – taking into account that the future social security system tax rate, $\tau_{t+1}$, depends on the current social security tax rate, $\tau_t$, via the mass of elderly employed and thus the private labor supply decision of the elderly. Furthermore, it requires $Q(Z_{t-1})$ to be a fixed point in the functional equation in part i) of the definition. In other words, if agents believe future benefits at any time $t + j$ to be set according to $\tau_{t+j} = Q(Z_{t+j-1})$, then the same function $Q(Z_{t-1})$ has to define the optimal voting decision today. The second equilibrium condition requires that all old individuals choose their labor supply optimally.

In order to compute the Markov political equilibrium, we have to consider the optimal social security tax rate chosen by the median voter at time $t$ who maximizes the indirect utility function with respect to $\tau_t$ and subject to $\tau_{t+1} = Q(Z_t(\tau_t, \tau_{t-1}))$.

The corresponding first order condition is:

$$-w_t^y(1+r) + \frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\overline{w}_{t+1}^y = 0 \tag{13}$$

where the first element represents the current cost to the median voter in terms of higher contributions, while the second term represents the future benefits corresponding to a higher pension, if a higher current contribution leads to a higher contribution rate also tomorrow: $\partial\tau_{t+1}/\partial\tau_t > 0$. The redistributive design of the social security system yields the usual result that – with perfect financial markets – the most preferred contribution rate of a young individual is decreasing in her income, since the contribution cost depends on the wage income while (part of) the benefits do not. The elderly most preferred social security contribution rate does not depend on their type and is always larger than any young's. These features command the usual distribution of social security preferences among the voters, which is displayed in the next proposition.

The solution of the maximization problem of the median voter yields the equilibrium fiscal policies, as summarized in the following proposition.

**Proposition 2** *The set of feasible fiscal policies* $\{\tau_t^*\}_{t=s}^{\infty} \in [0,1]$ *which can be supported by a Markovian politico-economic equilibrium satisfies:*

$$\tau_{t+1} = Q(Z_t) = A - \frac{(1+\phi)\,\delta_t^m(1+r)\overline{w}_t^o}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y} Z_t$$

*where* $\delta_t^m$*, the identity of the median voter at time* $t$*, solves the following equation*

$$1 + (1+n_t)\,F(\delta_t^m) = 1 + n_t/2$$

*and* $A$*, the free parameter pinned down by the first median voter's expectation of future policies, is restricted to the support* $A \in [\frac{\overline{w}_t^o\delta_t^m}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y}, 1 - \frac{(1+r)\delta_t^m}{(1+n_{t+1})}\left(1 + \frac{1}{(1+n_t)\widehat{\delta}}\left(1 + r - \frac{\overline{w}_t^o}{\phi\overline{w}_t^y}\right)\right)]$

**Proof.** See Appendix.

The result in the above proposition points to the existence of a positive political link between the current use of the early retirement provisions – that is, the mass of (early) retirees $(1-Z)$ – and the future social security contribution rate. This link complements the economic channel running from the social security contribution rate to the current labor supply decision of the elderly, as described at eq.6. In particular, a current increase in the social security contribution rate – by reducing the opportunity cost of retirement – leads to more current retirees, which in turn creates expectations of higher future social security contributions – and hence more early retirees in the future.

By exploiting this double link between contribution rate and mass of retirees, the dynamics for the equilibrium policy function can be described as follows:

$$\begin{aligned}\tau_{t+1} \quad = \quad & \frac{\delta_t^m(1+r)}{(1+n_t)(1+n_{t+1})}\tau_t + \frac{\delta_t^m(1+r)^2}{(1+n_t)(1+n_{t+1})\widehat{\delta}}(1-\tau_{t-1}) + \\ & +A - \frac{\overline{w}_t^o \delta_t^m (1+r)}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y} \end{aligned} \tag{14}$$

Interestingly, the dynamics of the contribution rate involves more than just one period, as the contribution rate at time $t+1$ depends – positively – on the tax rate at time $t$; but negatively on the tax rate at time $t-1$. This is due to the impact that the contribution rates at time $t$ and $t-1$ have on the retirement decision at time $t$.

It is now convenient to consider a constant demographic process, with $n_t = n_{t+i} = n \ \forall i$, and to define $\alpha = (1+r)/(1+n)$ as the performance of the saving (storage) technology relatively to the PAYG social security system; since we assumed the economy to be dynamically efficient, then $\alpha > 1$.

The next proposition examines the dynamic properties of the sequence of contribution rates.

**Proposition 3** *If $\alpha\delta^m < 1$ and $\alpha < \widehat{\delta}$, the Markovian politico-economic equilibrium path converges to a stable steady state corresponding to*

$$\overline{\tau} = \frac{\overline{w}^y \phi (1+n) \widehat{\delta} A - \overline{w}^o \delta^m \alpha + \alpha^2 \delta^m \phi (1+n) \overline{w}^y}{\overline{w}^y \phi (1+n) \widehat{\delta} \left(1 - \alpha\delta^m + \alpha^2 \frac{\delta^m}{\widehat{\delta}}\right)}.$$

*At this steady state, the mass of employed elderly is*

$$\overline{Z} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1+r)\overline{w}^y}{\overline{w}^o} - \frac{\phi(1+n)}{1+\phi}\frac{\overline{w}^y}{\overline{w}^o}\left[\widehat{\delta} - \alpha\right]\overline{\tau}$$

**Proof.** See Appendix.

The above proposition suggests that – even in this dynamically efficient economy – a stable steady state with a positive level of the social security contribution rate may emerge as an equilibrium of the Markovian political game, if two conditions are satisfied. The first condition, $\alpha\delta^m < 1$, is relatively standard in the social security literature and requires the young type-$\delta^m$ median voter to obtain a better deal from social security than from alternative assets, due to the redistributive nature of the social security system. In order for this condition to be satisfied, together with a highly redistributive social security system, the economy has to feature a high level of income inequality[5], as measured by the density function $f(\delta)$. Yet, unlike most systems analyzed in this literature (see Conde-Ruiz and Galasso, 2003), we here allow the agents to choose their retirement age. The second condition, $\alpha < \widehat{\delta}$, amounts to assume that a large number of individuals retire early. In particular, as shown by the equation in the proposition above, if this condition is satisfied, the impact of the (steady state) contribution rate on the (steady state) mass of employed elderly is negative, since higher taxes lead to more early retirees; thereby validating – also at steady state – the negative impact of the current contribution rate (see eq. 10). In the remaining sections, we will assume these two conditions to be always satisfied.

# 3 The future of Social Security and Early Retirement

## 3.1 The role of Aging

The equilibrium policy function obtained in the previous section allows us to analyze the effects of aging on the social security tax rate and on the use of early retirement. In line with standard political economy models of social security

[5] A necessary but not sufficient condition is that the income distribution is skewed in the standard direction, and thus $\delta^m < E(\delta) = 1$.

(for a survey, see Galasso and Profeta, 2002), in our model, aging has opposite economic and political effects on the steady state social security tax rate. Among the former, aging reduces the profitability of the PAYG pension system with respect to alternative savings; and may convince the median voter to downsize the system – in order to increase her private provision of retirement income through alternative private assets. Moreover, for a given contribution rate, an increase in the share of elderly in the population reduces the pension benefits, thereby inducing the elderly to postpone retirement. This reduction in the mass of (early) retirees will then lead to a decrease in the contribution rate. Among the latter, aging tends to change the identity of the median voter, who becomes poorer, and hence keener on increasing the contribution rate.

The next proposition summarizes the impact of the former, economic effects, by addressing the effect of aging on the steady state social security contribution rate, for a given median voter type.

**Proposition 4** *For a given median voter type, $\delta^m$, if $\widehat{\delta} \in (0, \overline{w}^o / (\overline{w}^y \phi (1+n)))$, aging (corresponding to a reduction in the population growth rate) decreases the steady state social security contribution rate, $\partial \overline{\tau} / \partial n > 0$.*

**Proof.** See Appendix.

Hence, aging has the expected economic impact on the social security contribution rate at steady state, provided that the mass of (early) retirees is not too large, i.e., $\widehat{\delta}$ is below a threshold. As the population growth rate drops, the implicit return from a PAYG social security system decreases. Median voters will modify the policy function by making it more responsive – in absolute terms – to the mass of employed elderly.

The political effect of aging is captured by the change in the identify of the median voter, who becomes poorer[6]. The next proposition shows that – in

[6] From the expression for the median voter at proposition 2, it is easy to see that $\frac{\partial \delta^m}{\partial n} =$

line with the existing literature – a poorer median voter will prefer more social security provided that the mass of (early) retirees is not too large, i.e., $\widehat{\delta}$ is small – i.e., below the same threshold as in the previous proposition.

**Proposition 5** *If* $\widehat{\delta} \in (0, \overline{w}^o / (\overline{w}^y \phi (1+n)))$, *the equilibrium steady state social security contribution rate depends negatively on the income type of the median voter:* $\partial \overline{\tau} / \partial \delta^m < 0$.

**Proof.** See Appendix.

These two propositions suggest that the economic and political effects of aging push in opposite directions. Which effect will dominate represents an empirical question that remains to be settled[7].

How does aging affect the individual retirement decisions and hence the mass of elderly workers? Our political economy model indentifies two channels for the aging process to influence the retirement decisions. Aging – as characterized by a reduction in the population growth rate, $n$ – reduces the profitability of the social security system. For a given level of the contribution rate, $\tau$, pension benefits will decrease (see eq.3). This negative substitution (and income) effect hence leads to postponing retirement. Yet, aging also modifies the political determination of the social security contribution rate, as discussed in the previous two propositions. If contributions increase, individuals will retire early – and viceversa. If is useful at this point to define $\varepsilon_{\overline{\tau}n} = \frac{\partial \overline{\tau}}{\partial n} \frac{1+n}{\overline{\tau}}$ as the elasticity of the social security contribution rate to the population growth. Clearly, $\varepsilon_{\overline{\tau}n} < 0$ if aging increases the contribution rate, that is, if the political effect dominates, and viceversa.

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$\frac{f(\delta^m)}{2(1+n)^2} > 0$.

[7] For instance, Galasso and Profeta (2004) simulate the political effect to prevail, whereas Razin's et al (2002) empirical analysis leads the opposite results. See also Disney (2007), Simonovits (2007) and Galasso and Profeta (2007) for empirical and theoretical contributions on this debate.

**Proposition 6** *Aging increases the mass of employed elderly at steady state,* $\partial \overline{Z}/\partial n < 0$, *if* $\varepsilon_{\overline{\tau} n} > -1$ .

**Proof.** See Appendix.

This proposition hence suggests that aging will indead help to reduce the widespread early retirement phenomenon, by forcing less generous pension benefits; unless the political effect – characterized by the median voter becoming poorer – is so strong as to lead to a large increase in the social security contribution, and hence to an increase also in the pension generosity.

## 3.2 The Role of the Income effects

In this section we highlight the role of income effects on retirement decisions and thus on the social security equilibrium tax rate. While most studies concentrate on the role of the incentives (substitution effect) in the retirement behavior, available empirical evidence (see for instance Costa, 1998, and Coronado and Perozek, 2003) suggest that income effects do play a crucial role in the labor supply decisions of elderly workers.

The political economy model presented in section 2 may help to understand how changes in the individual retirement decisions induced by income effects modify the political determination of social security and hence the equilibrium mass of early retirees. To focus on the role of the income effect, we consider a permanent variation in the wage income at youth only. A reduction in the wage income at youth decreases the agents' lifetime income both directly and through a reduction in the pension benefits[8], which are financed by a contribution on the wage income at youth (see eq. 3). If, however, we were to consider a generalized permanent change in the wage income both in youth and in old age, an additional (strong) substitution effect would arise. As discussed in section

[8] Clearly, this reduction in pension benefits creates also a substitution effect, since agents have an incentive to postpone retirement.

2.1, in this case income and substitution effects would perfectly compensate each other.

**Proposition 7** *A decrease of the average wage income in youth,* $\overline{w}^y$*, leads to a reduction of the steady state social security contribution rate* $\partial\overline{\tau}/\partial\overline{w}^y > 0$ *and to an increase in the steady state mass of employed elderly* $\partial\overline{Z}/\partial\overline{w}^y < 0$*.*

**Proof. See Appendix.**

A reduction in the wage income at youth induces individuals to postpone retirement, due to a negative income effect – since their lifetime income decreases – but also to a substitution effect – since the pension benefits, which are financed by the workers' wage, decrease. Although at steady state neither the overall profitability of the social security system nor the identity of the median voter is affected by a permanent drop in the wage income in youth, the steady state social security contribution rate decreases, since most individuals tend to postpone retirement, thereby reducing the share of (early) retirees. Interestingly, the magnitude of the adjustment in the retirement age driven by a change of the young income is decreasing in income, thus implying that low income workers will react more actively to a negative income effect. In our political equilibrium, this increase in the overall fraction of employed elderly will command a lower contribution rate. The reduction of early retirees at steady state depends instead both on the direct negative income and substitution effects caused by the reduction of $\overline{w}_t^y$ and on an indirect substitution effect due to the decrease of the social security tax rate, which leads to fewer (early) retirees.

# 4 Conclusions

This paper concentrates on the long term determinants of the retirement decisions and on the future evolution of social security system and early retirement provisions. In our politico-economic Markovian environment, every period a young low-income median voter determines the social security contribution by considering the evolution of the early retirement behavior. We emphasize the role of substitution and income effects in these retirement decisions. The incentive effects have been analyzed by a large empirical literature, which shows how (at the margin) non-actuarially fair pension systems may induce rational agents to retire early, by reducing the opportunity cost of leisure. Income effects have instead generally been neglected in models of retirement and social security, despite the empirical evidence suggesting that variation in income and wealth may modify individual retirement decisions.

In line with the existing political economy literature (see Galasso and Profeta, 2002), in our model, aging has opposite economic and political effects on social security; and the overall impact depends on which effect dominates.

Yet, aging may lead to a reduction in the widespread use of early retirement provisions. By commanding less generous pension benefits (for a given level of contribution rates), aging induces workers to postpone retirement. If the political effect is not so overwelming as to determine a sizable increase in social security contributions, and thus also in pension benefits, at steady state aging societies will be associated with less early retirement.

Morevoer, our model also suggests that a decrease in the wage income in youth leads to lower social security tax rate and fewer early retirees. To the extent that this change in young wage income may proxy for a drop in the life-time labor income, this may prove a crucial result to understand the future evolution of the early retirement provision. Societies characterized by economic stagnation or raise in lifetime inequality that increase the share of low-income individuals may thus be associated with a less pervasive use of these early retirement provisions.

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# 5 Appendix

## 5.1 Proof of proposition 2

The first order condition of the median voter is:

$$-w_t^y(1+r)+\frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\overline{w}_{t+1}^y=0 \tag{15}$$

or, equivalently

$$-\delta_t^m\overline{w}_t^y(1+r)+\frac{\partial \tau_{t+1}}{\partial \tau_t}(1+n_{t+1})\overline{w}_t^y=0 \tag{16}$$

where

$$\frac{\partial \tau_{t+1}}{\partial \tau_t}=Q'\frac{\partial Z_t}{\partial \tau_t} \tag{17}$$

with

$$Q'=\frac{\partial Q}{\partial Z_t} \tag{18}$$

and

$$\frac{\partial Z_t}{\partial \tau_t}=-\frac{\phi}{1+\phi}\frac{(1+n_t)\overline{w}_t^y}{\overline{w}_t^o}\int_{\underline{\delta}}^{\overline{\delta}}\frac{1}{\delta}f(\delta)d\delta=-\frac{\phi}{1+\phi}\frac{(1+n_t)\overline{w}_t^y}{\overline{w}_t^o}\widehat{\delta} \tag{19}$$

Substituting eq. 19 and eq. 18 into eq. 17 and using it into the first order condition at eq. 16 we obtain

$$Q'=-\frac{(1+\phi)\,\delta_t^m(1+r)\overline{w}_t^o}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y} \tag{20}$$

Integrating the above equation with respect to $Z_t$ we obtain

$$\tau_{t+1}=Q(Z_t)=A-\frac{(1+\phi)\,\delta_t^m(1+r)\overline{w}_t^o}{\phi(1+n_t)(1+n_{t+1})\widehat{\delta}\overline{w}_t^y}Z_t \tag{21}$$

where $A$ is a constant of integration.

Since $\tau_{t+1}=Q(Z_t)$ represents a tax rate, we impose that $Q\in[0,1]$. Using eq. 10 for $Z_t$, it is easy to see that a sufficient condition for $\tau_{t+1}$ to be positive is that $\tau_{t+1}$ is positive for $\tau_{t-1}=1$ and $\tau_t=0$ which requires

$$A>\underline{A}=\frac{\overline{w}_t^o\delta_t^m}{\phi\,(1+n_t)\,(1+n_{t+1})\,\widehat{\delta}\overline{w}_t^y}>0. \tag{22}$$

Moreover, it is easy to see that a sufficient condition for $\tau_{t+1} < 1$ is to have $\tau_{t+1} < 1$ for $\tau_t = 1$ and $\tau_{t-1} = 0$, which requires

$$A < \overline{A} = 1 - \frac{(1+r)\,\delta_t^m}{(1+n_{t+1})}\left(1 + \frac{1}{(1+n_t)\,\widehat{\delta}}\left(1 + r - \frac{\overline{w}_t^o}{\phi \overline{w}_t^y}\right)\right) \tag{23}$$

Given the assumption $\phi < \frac{\overline{w}_t^o}{(1+r)\overline{w}_t^y}$, it is easy to show that $\underline{A} < \overline{A}$ if $\frac{1+r}{1+n_{t+1}}\delta^m < 1$, which is a condition required by the next proposition 3 for the stability of the system.

Finally, to determine the identity of the median voter, notice that – by equation 16 – the most preferred social security contribution rate among the young is weakly decreasing in their income; and that the old always command a higher tax rate than the any young. For non-negative population growth rates, the median voter is among the young and has a type $\delta_t^m$, which divides the distribution of preference in halves: $1 + (1+n_t)\,F(\delta_t^m) = 1 + n_t/2$.

## 5.2 Proof of proposition 3

Eq. 14 can be rewritten as

$$\tau_{t+1} - \alpha\delta_t^m \tau_t + \alpha^2 \frac{\delta_t^m}{\widehat{\delta}}\tau_{t-1} = A - \frac{\overline{w}_t^o \delta_t^m \alpha}{\phi\,(1+n)\,\widehat{\delta}\overline{w}_t^y} + \alpha^2 \frac{\delta_t^m}{\widehat{\delta}} \tag{24}$$

It is easy to see that three cases arise in the solution of this second order differential equation, depending on the sign of the determinant of the associated characteristic equation

$$b^2 + a_1 b + a_2 = 0 \tag{25}$$

with

$$a_1 = -\alpha\delta^m,\ a_2 = \alpha^2 \frac{\delta^m}{\widehat{\delta}}. \tag{26}$$

In fact, we obtain

$$b_1, b_2 = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} = \frac{\alpha\delta^m \pm \sqrt{\alpha^2\,(\delta^m)^2 - 4\alpha^2 \frac{\delta^m}{\widehat{\delta}}}}{2} = \frac{\alpha\delta_t^m}{2} \pm \alpha\sqrt{\delta^m\left(\frac{\delta^m}{4} - \frac{1}{\widehat{\delta}}\right)} \tag{27}$$

Depending on the sign of $\Delta^2 = \delta^m \left( \frac{\delta^m}{4} - \frac{1}{\widehat{\delta}} \right)$, there are 3 possible cases: (i) real and distinct roots $b_1$ and $b_2$; (ii) real and equal roots $b_1 = b_2$; and (iii) complex roots $b_1$ and $b_2$. If $\alpha\delta^m < 1$ and $\alpha < \widehat{\delta}$, the conditions for a stable convergence towards the steady state are always satisfied.

Case i) A sufficient condition to guarantee stability is that $b_1 < 1$ and $b_2 < 1$. $b_2 = \frac{\alpha\delta^m + \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2 \frac{(\delta^m)}{\widehat{\delta}}}}{2} < 1$ if $\alpha\delta^m + \sqrt{\alpha^2 (\delta^m)^2 - 4\alpha^2 \frac{\delta^m}{\widehat{\delta}}} < 2$. If $\alpha\delta^m < 1$, this is satisfied since both terms are less than 1. Moreover, $b1 = \frac{\alpha\delta^m - \sqrt{\alpha^2(\delta^m)^2 - 4\alpha^2 \frac{\delta^m}{\widehat{\delta}}}}{2} < 1$ because $0 < b_1 < b_2 < 1$.

Case ii) The stability property depends on $\frac{-a_1}{2} = \frac{\alpha\delta^m}{2}$. For $\alpha\delta^m < 2$ the system converges to the steady state.

Case iii) The stability property depends on $R = \sqrt{a_2} = \alpha\sqrt{\frac{\delta_t^m}{\widehat{\delta}}} = \sqrt{\alpha\delta_t^m \frac{\alpha}{\widehat{\delta}}}$. If $R < 1$, the system converges to the steady state, although through fluctuations. Clearly, this is satisfied for $\alpha\delta^m < 1$ and $\alpha < \widehat{\delta}$.

The steady state value of the tax rate corresponds to the particular solution of eq.14 with $\tau_{t+1} = \tau_t = \tau_{t-1} = \overline{\tau}$:

$$\overline{\tau} - \alpha\delta^m\overline{\tau} + \alpha^2 \frac{\delta^m}{\widehat{\delta}} \overline{\tau} = A - \frac{\overline{w}^o \delta^m}{\phi\,(1+n)\,\widehat{\delta}\overline{w}^y} \alpha + \alpha^2 \frac{\delta^m}{\widehat{\delta}} \tag{28}$$

$$\overline{\tau} = \frac{A - \frac{\overline{w}^o \delta^m}{\phi(1+n)\widehat{\delta}\overline{w}^y} \alpha + \alpha^2 \frac{\delta^m}{\widehat{\delta}}}{1 - \alpha\delta^m + \alpha^2 \frac{\delta^m}{\widehat{\delta}}} \tag{29}$$

recalling that $\alpha = \frac{(1+r)}{(1+n)}$, we have

$$\overline{\tau} = \frac{\overline{w}^y \phi\,(1+n)\,\widehat{\delta} A - \overline{w}^o \delta^m \alpha + \alpha^2 \delta^m \phi\,(1+n)\,\overline{w}_t^y}{\overline{w}^y \phi\,(1+n)\,\widehat{\delta} \left( 1 - \alpha\delta^m + \alpha^2 \frac{\delta^m}{\widehat{\delta}} \right)} \tag{30}$$

Notice that the denominator of $\overline{\tau}$ is always positive since $1 > \alpha\delta^m(1 - \frac{\alpha}{\widehat{\delta}})$, while the numerator is positive because $A > \underline{A}$, as defined at Eq.22.

Finally, by imposing $\tau_{t+1} = \tau_t = \tau_{t-1} = \overline{\tau}$, after some simple manipulation, eq.10 leads to the expression of the steady state level of employed elderly:

$$\overline{Z} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1+r)\overline{w}^y}{\overline{w}^o} - \frac{\phi(1+n)}{1+\phi} \frac{\overline{w}^y}{\overline{w}^o} \left[ \widehat{\delta} - \alpha \right] \overline{\tau} \tag{31}$$

Simple algebra yields that $\overline{Z} \in [0,1]$.

## 5.3 Proof of proposition 4

In order to prove propositions 4 and 5, it is convenient to introduce the following lemma.

**Lemma 8** *If* $\widehat{\delta} \in (0, \frac{\overline{w}^o}{\overline{w}^y \phi(1+n)})$, $K = \frac{\overline{w}^o - \alpha\phi(1+n)\overline{w}^y}{\overline{w}^y \phi(1+n)\left(\widehat{\delta}-\alpha\right)} > \overline{A}$.

Proof of the lemma: Substituting $n_t = n_{t+1} = n$ and the definition of $\alpha$ into the expression for $\overline{A}$ at eq.23, we need to show that

$$K = \frac{\overline{w}^o - \alpha\phi\left(1+n\right)\overline{w}^y}{\overline{w}^y \phi\left(1+n\right)\left(\widehat{\delta}-\alpha\right)} > \overline{A} = 1 - \alpha\delta^m - \frac{\alpha\delta^m}{\widehat{\delta}}\left(\alpha - \frac{\overline{w}^o}{\phi(1+n)\overline{w}^y}\right)$$

After some algebra this condition can be rewritten as

$$(1-\alpha\delta^m)\left(\frac{\frac{\overline{w}^o}{\overline{w}^y\phi(1+n)} - \alpha}{\widehat{\delta}-\alpha} - 1\right) + \alpha^2\delta^m\left(\frac{\frac{\overline{w}^o}{\overline{w}^y\phi(1+n)} - \alpha}{\widehat{\delta}\left(\widehat{\delta}-\alpha\right)}\right) > 0 \qquad (32)$$

Since we assumed that $\frac{\overline{w}^o}{\overline{w}^y\phi(1+n)} - \alpha > 0$ and $\widehat{\delta} - \alpha > 0$ , a sufficient condition to guarantee the above inequality is that $\widehat{\delta} < \frac{\overline{w}^o}{\overline{w}^y\phi(1+n)}$.**Q.e.d.**

We can not turn to the sign of $\partial\overline{\tau}/\partial n$.

Define $\beta = \alpha(1+n)$, the steady state level of the tax rate becomes

$$\overline{\tau} = \frac{A(1+n)^2 - \frac{\overline{w}^o\delta^m\beta}{\overline{w}^y\phi\widehat{\delta}} + \beta^2\frac{\delta^m}{\widehat{\delta}}}{(1+n)^2 - \beta(1+n)\delta^m + \beta^2\frac{\delta^m}{\widehat{\delta}}}.$$

It is easy to see that the sign of $(\partial\overline{\tau}/\partial n)$ is equal to the sign of the following expression:

$$2A(1+n)((1+n)^2 - \beta(1+n)\delta^m + \beta^2\frac{\delta^m}{\widehat{\delta}}) - \left(A(1+n)^2 - \frac{\overline{w}^o\delta^m\beta}{\overline{w}^y\phi\widehat{\delta}} + \beta^2\frac{\delta^m}{\widehat{\delta}}\right)(2(1+n) - \beta\delta^m) \qquad (33)$$

which can be written as

$$\frac{A(1+n)^2}{\widehat{\delta}} B_1 + B_2 * B_3 \frac{1+n}{\overline{w}^y \phi \widehat{\delta}} > 0 \tag{34}$$

where $B_1 = 2\alpha - \widehat{\delta}$, $B_2 = \overline{w}^o - \alpha(1+n)\overline{w}^y \phi$, and $B_3 = 2 - \alpha\delta^m$. Notice that $B_3 > 0$ since $\alpha\delta < 1$ and $B_2 > 0$ since $\alpha < \frac{\overline{w}_t^o}{(1+n)\overline{w}_t^y \phi}$. Therefore we have two possible cases:

- If $B_1 \geq 0$, for $\alpha \geq \widehat{\delta}/2$ , $\frac{\partial \overline{\tau}}{\partial n} > 0$ if $A \geq -\frac{B_2 B_3}{B_1(1+n)\overline{w}^y \phi}$ $(< 0)$, which is always true since $A > \underline{A} > 0$.

- If $B_1 < 0$, for $\alpha < \widehat{\delta}/2$ , $\frac{\partial \overline{\tau}}{\partial n} > 0$ if $A < -\frac{B_2 B_3}{B_1(1+n)\overline{w}^y \phi} = M$ $(> 0)$. Clearly, $A < M$ if $M > K = \frac{\overline{w}_t^o - \alpha\phi(1+n)\overline{w}_t^y}{\overline{w}_t^y \phi(1+n)\left(\widehat{\delta} - \alpha\right)}$, and $K > \overline{A}$, which is satisfied – according to the lemma above – if $\widehat{\delta} \in (0, \overline{w}_t^o / (\overline{w}_t^y \phi (1+n)))$. After simple algebra we have that $M > K$ if

$$\frac{(2 - \alpha\delta^m)}{(\widehat{\delta} - 2\alpha)} > \frac{1}{\left(\widehat{\delta} - \alpha\right)} \tag{35}$$

  or

$$\alpha^2 \delta^m + \widehat{\delta}\,(1 - \alpha\delta^m) > 0 \tag{36}$$

  which is always satisfied, since we assumed $1 > \alpha\delta^m$. Thus $\partial\overline{\tau}/\partial n > 0$.

## 5.4 Proof of proposition 5

Using the definition at Proposition 3, we can define the steady state social security contribution rate as follows:

$$\overline{\tau} = \frac{\overline{w}_t^y \phi\,(1+n)\,\widehat{\delta} A(1/\delta^m) - \overline{w}_t^o \alpha + \alpha^2 \phi\,(1+n)\,\overline{w}_t^y}{\overline{w}_t^y \phi\,(1+n)\,\widehat{\delta}\left((1/\delta^m) - \alpha + \alpha^2 \frac{1}{\widehat{\delta}}\right)} = g(1/\delta^m) \tag{37}$$

Thus,

$$\frac{\partial \overline{\tau}}{\partial \delta^m} = \frac{\partial g(1/\delta^m)}{\partial (1/\delta^m)}\left(-\frac{1}{(\delta^m)^2}\right) = -\frac{\phi\,(1+n)\left(A\alpha^2 - \widehat{\delta} A\alpha - \alpha^2\right) + \frac{\overline{w}^o}{\overline{w}^y}\alpha}{\phi\,(1+n)\,\widehat{\delta}\left((1/\delta^m) - \alpha + \alpha^2 \frac{1}{\widehat{\delta}}\right)^2 (\delta^m)^2} < 0 \tag{38}$$

if

$$\phi\,(1+n)\left(A\left(\widehat{\delta}-\alpha\right)+\alpha\right)-\frac{\overline{w}^{o}}{\overline{w}^{y}}<0 \tag{39}$$

with the above inequality being satisfied for

$$A<K=\frac{\overline{w}^{o}-\alpha\phi\,(1+n)\,\overline{w}^{y}}{\overline{w}^{y}\phi\,(1+n)\left(\widehat{\delta}-\alpha\right)}. \tag{40}$$

Using the lemma above, we know that $K>\overline{A}$, and thus $A<K$ always, if $\widehat{\delta}\in\left(0,\overline{w}_t^{o}/\left(\overline{w}_t^{y}\phi\,(1+n)\right)\right)$.

## 5.5 Proof of proposition 6

The impact of aging on the steady state level of early retirement depends on the direct effect of $n$ and on the change in $\overline{\tau}$ induced by $n$:

$$\frac{\partial\overline{Z}}{\partial n}=-\frac{\phi}{1+\phi}\frac{\overline{w}^{y}}{\overline{w}^{o}}\left[\widehat{\delta}-\alpha\right]\overline{\tau}-\frac{\phi}{1+\phi}\frac{\overline{w}^{y}(1+n)}{\overline{w}^{o}}\left[\widehat{\delta}-\alpha\right]\frac{\partial\overline{\tau}}{\partial n} \tag{41}$$

$$\frac{\partial\overline{Z}}{\partial n}=-\frac{\phi}{1+\phi}\frac{\overline{w}^{y}}{\overline{w}^{o}}\left[\widehat{\delta}-\alpha\right]\overline{\tau}\left(1+\frac{\partial\overline{\tau}}{\partial n}\frac{1+n}{\overline{\tau}}\right). \tag{42}$$

Thus $\frac{\partial\overline{Z}}{\partial n}<0$ if $\frac{\partial\overline{\tau}}{\partial n}\frac{1+n}{\overline{\tau}}=\varepsilon_{\overline{\tau},n}>-1$.

## 5.6 Proof of proposition 7

From the equation for $\overline{\tau}$ at proposition 3, it is immediate to see that $\partial\overline{\tau}/\partial\overline{w}^{y}>0$. From the equation for $\overline{Z}$ at proposition 3, we have that

$$\frac{\partial\overline{Z}}{\partial\overline{w}^{y}}=-\frac{\phi}{1+\phi}\frac{(1+r)}{\overline{w}^{o}}-\frac{\phi(1+n)}{1+\phi}\frac{1}{\overline{w}^{o}}\left[\widehat{\delta}-\alpha\right]\overline{\tau}-\frac{\phi(1+n)}{1+\phi}\frac{\overline{w}^{y}}{\overline{w}^{o}}\left[\widehat{\delta}-\alpha\right]\frac{\partial\overline{\tau}}{\partial\overline{w}^{y}}<0 \tag{43}$$

since all terms are negative, because $\widehat{\delta}-\alpha>0$ by assumption and $\partial\overline{\tau}/\partial\overline{w}^{y}>0$.