CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 169

Are Disadvantaged Bidders Doomed in Ascending Auctions?

Marco Pagnozzi

November 2006

CSEF - Centre for Studies in Economics and Finance – UNIVERSITY OF SALERNO
84084 FISCIANO (SA) - ITALY
Tel. +39 089 96 3167/3168 - Fax +39 089 96 3167 – e-mail: csef@unisa.it

WORKING PAPER NO. 169

Are Disadvantaged Bidders Doomed in Ascending Auctions?

Marco Pagnozzi *

Abstract

A bidder is said to be advantaged if she has a higher expected valuation of the auction prize than her competitor. When the prize has a common-value component, a bidder competing in an ascending auction against an advantaged competitor bids especially cautiously and, hence, the advantaged bidder wins most of the time. However, contrary to what is often argued, a disadvantaged bidder still wins with positive probability, even if his competitor.s advantage is very large and even if the disadvantaged bidder has the lowest actual valuation ex-post. Therefore, the disadvantaged bidder has an incentive to participate in the auction, and the presence of a bidder with a small advantage does not have a dramatic effect on the seller's revenue.

JEL Classification: D44 (auctions).

Keywords: common-value auctions, asymmetric bidders.

Acknowledgements: I would like to thank George Konaris, Evelyn Goh, Marco LiCalzi and, especially, Paul Klemperer for comments and extensive discussions.

* University of Napoli "Federico II" and CSEF

Corresponding address: Department of Economics, Università di Napoli Federico II, Via Cintia (Monte S. Angelo), 80126 Napoli, Italy (email: pagnozzi@unina.it).

Contents

1. Introduction

In an ascending auction, the presence of a bidder with even a small advantage (i.e., a slightly higher valuation than her opponents)[1] is commonly believed to be deleterious for the seller's revenue. When the prize has a common-value component, a standard bidder competing against an advantaged opponent bids especially cautiously to avoid the "winner's curse", while the advantaged bidder bids especially aggressively.[2] Various papers argue that, as a consequence, the advantaged bidder should *always* win the auction (Bikhchandani, 1988, and Klemperer, 1998).[3] And with an arbitrarily small bidding cost, disadvantaged bidders do not participate in the auction at all because they know they would lose.[4] So even a slight asymmetry between bidders should drive the seller's revenue to zero.

We analyze a simple example of an auction where bidders receive both private and common signals on the prize's value, and we argue that the above extreme conclusion does not hold when a bidder is advantaged *ex-ante* (i.e., before the auction), but does not necessarily have the highest valuation *ex-post* (i.e., given the signals received by all bidders). We show that an advantaged bidder does win the auction with relatively high probability and, when she wins, pays a relatively low price. However, a disadvantaged bidder still wins the auction with strictly positive (although small) probability, even if his opponent's advantage is extremely large. Hence, the presence of an advantaged bidder does harm her opponents, but not to the extent that they never manage to overbid her. And since disadvantaged bidders have an incentive to participate in the auction, the expected seller's revenue is not reduced to zero. This appears consistent with anecdotal evidence.

As an example, consider the FCC mobile-phone license auction in 1995, where incumbents and national bidders were considered advantaged with respect to their competitors (Salant, 1997).[5] In markets where the number of licenses on sale was equal to the num-

[1] We adopt the convention of using feminine pronouns for an advantaged bidder and masculine pronouns for a disadvantaged bidder.

[2] Because of the winner's curse, a bidder is most likely to win when he has overestimated the prize's value. But a disadvantaged bidder knows that, if he beats an opponent with an advantage, he may have overestimated this value by even more, so he must bid very cautiously. Then the advantaged bidder can be less cautious, since winning need not imply she has overestimated the prize's value. And hence the disadvantaged bidder will bid even more cautiously, since he knows he can only beat the advantaged bidder if she has extremely bad information about the prize's value.

[3] However, de Frutos and Pechlivanos (2006) argue otherwise. We discuss this paper below. Levin and Kagel (2005) analyze the effect of increasing the number of disadvantaged bidders.

[4] This point does not require common-value elements (but it is reinforced by them). In an ascending auction, even if she is initially outbid, the bidder with the highest valuation will be the eventual winner because she can, and will, eventually top any competitor. (However, as argued by Pagnozzi, 2004, if resale is permitted a disadvantaged bidder can win against an opponent who is known to have a higher valuation.)

[5] Incumbents provided mobile-phone services before the auction, and hence had a higher valuation because

ber of advantaged bidders, fewer disadvantaged bidders participated, and those who did bid very cautiously, thus yielding lower prices (Klemperer and Pagnozzi, 2004). But disadvantaged bidders still participated and advantaged bidders did not always outbid them — in 12 out of 48 markets, national bidders were outbid by disadvantaged rivals.[6] Similarly, in the Netherlands 3G mobile-phone license auction in 2000, the number of incumbent firms, which were advantaged over potential entrants, was exactly equal to the number of licenses on sale. Nonetheless Versatel, a weak entrant that was believed to be strongly disadvantaged compared to incumbents, participated in the auction, and then dropped out at a very low price (Klemperer, 2002).

Auctions are typically analyzed assuming that bidders' valuations are either private or common. In a private-value auction, each bidder knows his own valuation, which is independent of other bidders' valuations; while in a common-value auction, the prize is worth the same to all bidders, who have different estimates of its value. But, in the real world, bidders' valuations are not exclusively private or common.[7] So we assume valuations consist of both a private- and a common-value component.[8]

We define a bidder to be advantaged if *ex-ante* it is common knowledge that, on average, her valuation has a higher private component than her competitor's valuation and hence that, before the auction, she is expected to have a higher actual valuation than her competitor.[9] This allows us to analyze the effects of the mere presence of an advantaged bidder on players' bidding strategies, when the bidder's ex-post advantage is not certain.

This paper is closely related to the analyses of auctions with common-value elements by Bikhchandani (1988), Klemperer (1998) and de Frutos and Pechlivanos (2006). Bikhchandani (1988) and Klemperer (1998) assume one bidder *always* has a (slightly) higher valuation than

of their brand-name advantages and their lower costs of building a network. National bidders competed for licenses all over the United States and, hence, could benefit of economies of scale and scope.

[6] In particular, this happened in Atlanta, Tampa, Houston, Cincinnati, Puerto Rico, Memphis, Birmingham, Portland, Des Moines, Columbus, Oklahoma and Honolulu where at least one of the strong national bidders (AT&T, PCS PrimeCo or WirelessCo) lost to a disadvantaged opponent.

[7] For example, even if a painting is typically described as having a private value, its value also depends on the price at which it can be resold, and this adds a common-value element. And even if the value of oil drilling rights mainly depends on the quantity of oil contained in a field, which is the same for all firms, a firm's valuation is also affected by its efficiency, and this adds a private-value element.

[8] The literature on auctions with private and common values includes Milgrom and Weber (1982), Maskin (1992), Jehiel and Moldovanu (2000), Goeree and Offerman (2003), Compte and Jehiel (2002), Pagnozzi (2003) and de Frutos and Pechlivanos (2006).

[9] The literature on auctions with asymmetric bidders includes Avery and Kagel (1997), Bikhchandani (1988), Bulow et al. (1999), Bulow and Klemperer (2002), Cantillon (1999), Klemperer (1998), and Maskin and Riley (2000). In general, these papers argue that the seller's revenue is lower when bidders are asymmetric. Section 2 discusses the differences between these papers and our model.

her opponent, and prove that she always wins an ascending auction.

De Frutos and Pechlivanos (2006) prove that, when before the auction it is uncertain which bidder has the highest actual valuation, there always exists a bidder who *ex-post* has a lower actual valuation than his opponent and wins the auction with positive probability.[10] However the authors do not determine whether or not this bidder could also have been disadvantaged *ex-ante* (i.e., whether ex-ante he could have had a lower expected valuation than his opponent). Moreover, in de Frutos and Pechlivanos (2006), a bidder with a large disadvantage *ex-ante* (in fact, any bidder whose ex-ante probability of having a strictly lower valuation than his opponent exceeds 1/2) never wins the auction when he also has the lowest valuation ex-post. By contrast, we prove the much more extreme result that a disadvantaged bidder may win an ascending auction even if ex-ante he has probability arbitrarily close to 1 of having a lower valuation than his opponent and ex-post he actually has the lowest valuation. So even very disadvantaged bidders are not doomed.

However, though our model can obtain this more extreme result, our quantitative results support the general view of Bikhchandani (1988) and Klemperer (1998) that asymmetries in auction with common-value elements have substantial effects: an ex-ante advantaged bidder, even if he does not necessarily have the highest valuation ex-post, wins the auction with a probability much higher than the probability of her having the highest valuation, and pays a relatively low price when she wins. Finally, our limit results are also consistent with Bikhchandani (1988) and Klemperer (1998) because the probability of the advantaged bidder winning tends to one as the probability of her having the actual highest valuation ex-post tends to one.

Our analysis suggests that, if there are only negligible bidding costs, we should expect a disadvantaged bidder to participate in the auction even against a bidder who has a large advantage ex-ante. So, in this case, the effect of asymmetries between bidders on the seller's revenue should not be dramatic. But if bidding costs are relatively high, a disadvantaged bidders may choose not to participate because of his lower expected profit, and the seller's revenue may be reduced to zero — as Bikhchandani (1988) and Klemperer (1998) suggest.

Moreover, in addition to the strategic effect on her competitors' behavior, the presence of an advantaged bidder also has a direct effect on the seller's revenue. Indeed, the advantaged bidder has a higher valuation of the auction prize, and so is willing to pay more for it. This

[10] De Frutos and Pechlivanos (2006) assume that a bidder's valuation is given by the product between a common value and a private-value signal drawn from a discrete distribution. The authors consider a bidder to be *ex-post* disadvantaged if ex-post he has a low actual valuation, regardless of his opponent's valuation.

tends to increase the seller's revenue. The net effect of the presence of an advantaged bidder depends on the magnitude of her advantage — we show that introducing a bidder with a slightly higher valuation in an otherwise symmetric auction increases the seller's revenue.

The next section presents the model and Section 3 derives equilibrium bidding strategies. Sections 4 and Section 5 analyze the effects of the presence of an advantaged bidder on players' probability of winning the auction and on the seller's revenue respectively. In order to obtain closed form solutions and quantitative results, we assume bidders' signals are uniformly distributed, but in Section 6 we prove that our qualitative results hold for any distribution. The last section concludes. All proofs are contained in the Appendix.

2. The Model

A prize (e.g., a mobile-phone license) is sold by an ascending auction with two bidders, called A and D. Bidder i's valuations is given by:[11]

$$V_i = x_i + x_j + t_i; \quad i, j = A, D; \quad i \neq j;$$

where x_i, x_j and t_i are, respectively, the realizations of the random variables X_i, X_j and T_i. We call x_i the *common* signal and t_i the *private* signal of bidder i. Both signals are private information to bidder i.

So bidders' valuations are partly private and partly common: the signals x_A and x_D affect the valuations of both bidders alike, while the signal t_i only affects the valuation of bidder i. We can interpret t_i as expressing bidder i's production cost (or efficiency level); or the financial cost that bidder i has to pay in order to raise money to bid, which depends on the particular credit condition he can obtain from financial institutions. On the other hand, x_A and x_D are signals of the intrinsic characteristic of the prize — for example the future level of demand for mobile-phone services — which affects the profitability of the license for all bidders.

We assume that signals are independent and that X_A, X_D, and T_D are uniformly distributed on $[0, 1]$, while T_A is uniformly distributed on $[\theta, 1 + \theta]$, for $0 \leq \theta \leq 1$. Therefore, $V_A \in [\theta, 3 + \theta]$ and $V_D \in [0, 3]$. So bidder A is advantaged with respect to bidder D since her private signal is drawn from a "higher" support and, hence, it is common knowledge before the auction that bidder A has a higher expected valuation than bidder D.[12] Note, however,

[11] We use an additive value function as in the "wallet game" of Klemperer (1998) and Bulow and Klemperer (2002), and as in Compte and Jehiel (2002).

[12] The valuation of bidder A "first-order stochastically dominates" the valuation of bidder B.

that bidder A is not necessarily advantaged *ex-post,* because she may not actually have the highest valuation.

The constant θ captures the intensity of bidder A's advantage. The higher is θ, the higher is bidder A's ex-ante advantage over bidder D (because the higher is bidder A's expected valuation and the higher is the probability of bidder A having a higher actual valuation than bidder D). When $\theta = 1$ bidder A always has the highest valuation, and so she is also advantaged ex-post for sure. Notice that, by contrast with Klemperer (1998) where the advantaged bidder's valuation is only marginally higher than the disadvantaged bidder's one, in our model bidder A's actual valuation can be much higher than bidder D's one — even for $\theta = 0.5$, bidder A's expected private signal is twice bidder D's one. On the other hand, when $\theta = 0$ the auction is a purely symmetric one and no bidder has any advantage.

Our model differs from other models in the literature that analyze the effects of asymmetries in the distributions of bidders' signals. Cantillon (1999) and Maskin and Riley (2000) consider the effects of asymmetries in signals' distributions in private value auctions. By avoiding any common-value element in bidders' valuation, the authors exclude the winner's curse effects that we are instead interested in here. As argued above, de Frutos and Pechlivanos (2006) define a bidder to be advantaged from an ex-post point of view. Finally, Bikhchandani (1988), Klemperer (1998) and Bulow et al. (1999) assume that one of the bidders is always advantaged, both ex-ante and ex-post, and obtain results that can be interpreted as a limit case of our model (see also Avery and Kagel, 1997).

3. Bidding Strategies

The next lemma describes equilibrium bidding strategies.[13]

Lemma 1. *It is an equilibrium of the ascending auction for bidder A to bids up to:*

$$\beta_A(x_A, t_A; \theta) = \frac{3}{2}(x_A + t_A) + \frac{\theta}{2}, \tag{3.1}$$

and for bidder D bids up to:

$$\beta_D(x_D, t_D; \theta) = \frac{3}{2}(x_D + t_D) - \theta. \tag{3.2}$$

If no bidder is advantage (i.e., if $\theta = 0$), then the symmetric equilibrium bidding function is $\beta_i = \frac{3}{2}(x_i + t_i)$. For $\theta > 0$, bidder D bids up to a strictly lower price (than in the symmetric

[13] As it is standard in the literature on common-value auctions, there are multiple equilibria. The equilibrium described in Lemma 1 is the unique equilibrium in linear strategies.

equilibrium), while bidder A bids up to a strictly higher price. This depends on the enhanced winner's curse effect: when bidder D wins against an advantaged competitor, he expects the prize to be less valuable than in a symmetric context, because it is relatively more likely that bidder A is bidding aggressively due to a high private signal rather than a high common signal. Hence, bidder D has to bid less aggressively in order to avoid paying more than the prize is worth. This, in turns, allows bidder A to bid more aggressively since, whenever she wins, the prize is more valuable to her than in a symmetric context, and so on. Summing up, we have the following result.

Proposition 1. *Compared to an auction with symmetric bidders, when a bidders has a higher expected valuation than her competitor, the disadvantaged bidder bids less aggressively, and the advantaged bidder bids more aggressively.*

Notice that $\partial\beta_A/\partial\theta > 0$ and $\partial\beta_D/\partial\theta < 0$: as θ increases, bidder A can bid more aggressively while bidder D needs to bid less aggressively. This depends on the common-value element in bidders' valuations: the more aggressively a bidders bids in equilibrium, the less aggressively the other bidder has to bid in order to avoid the winner's curse. So the higher the advantage of bidder A, the higher the winner's curse suffered by bidder D (since, for a given auction price, the expected common signal of bidder A is lower), and hence the lower bidder D bids.

When $\theta = 1$, bidder D never bids more than 2 while bidder A never bids less than 2. So when the advantaged bidder has a higher valuation for sure, she always wins the auction, and this is consistent with the results in literature (Bikhchandani, 1988 and Klemperer, 1998). However, even if bidder A is advantaged for sure ex-post and her expected valuation is much higher than her competitor's valuation, she does not always win at price 0. Indeed, even if $\theta = 1$, a bidder D with signals $x_D + t_D > \frac{2}{3}$ bids up to a strictly positive price.

4. Are Disadvantaged Bidders Doomed?[14]

Even if, in equilibrium, bidder D has to bid cautiously while bidder A can bid aggressively, bidder D still wins the auction with strictly positive probability, regardless of bidder A's advantage (provided $\theta \neq 1$). In particular:

$$\Pr\left[D \text{ wins the auction}\right] = \begin{cases} \frac{1}{6}\left(3 - 8\theta + 16\theta^3 - 12\theta^4\right) & \text{for } \theta < \frac{1}{2}, \\ \frac{2}{3}\left(1-\theta\right)^4 & \text{for } \theta > \frac{1}{2}. \end{cases} \tag{4.1}$$

[14] The Appendix contains the derivation of all the expressions in Sections 4 and 5.

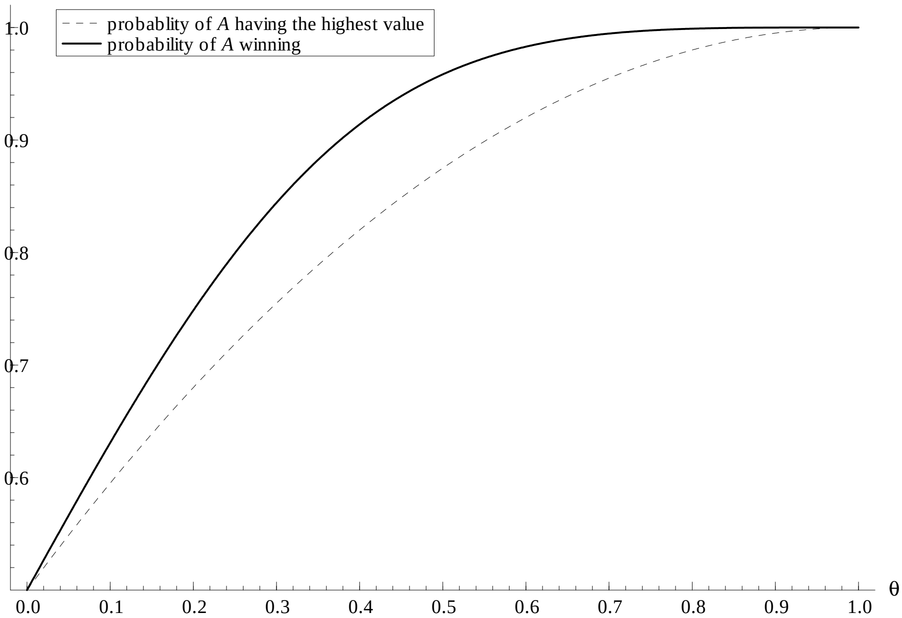

Figure 4.1: Probability of bidder A having the highest value and probability of bidder A winning the auction.

When $\theta = 0$ bidders are symmetric and, as expected, bidder D wins the auction with probability $1/2$. When $\theta > 0$, bidder D wins the auction with probability lower than $1/2$. Hence, when competing against an advantaged bidder, bidder D wins less often than in a symmetric situation. Nonetheless, even for $\theta = 0.25$ (in which case bidder A's private signal is on average 50% higher than bidder D's private signal), bidder D still wins the auction about 20% of times.[15] Finally, as $\theta \to 1$, the probability of bidder D winning the auction tends to 0.

Figure 4.1 represents the probability of bidder A winning the auction and the probability of her having the highest valuation, as a function of the intensity of A's advantage.[16] So bidder A wins with a higher probability than bidder D, and also more often than she has the highest valuation.[17] (For example, when $\theta = 0.4$, bidder A has the highest valuation with probability 0.82 but wins the auction with probability $\simeq 0.914$.) Summing up, we have the following result, which is consistent with the anecdotal evidence presented in the introduction.

Proposition 2. *The probability of the advantaged bidder winning the auction is higher than*

[15] For $\theta = 0.5$, bidder A's private signal is on average twice bidder D's private signal and bidder D wins the auction about 4% of times.

[16] The probability that bidder A has the highest valuation is $\Pr[t_A > t_B] = \frac{1}{2} + \theta - \frac{1}{2}\theta^2$.

[17] As argued by Maskin (1992), Jehiel and Moldovanu (2000), and Goeree and Offerman (2003), if bidders receive multi-dimensional signals auctions can be ex-post inefficient.

the probability of her having the highest valuation. However, the disadvantaged bidder still wins with strictly positive probability (unless the advantaged bidder has the highest valuation for sure).

Therefore, the presence of an advantaged competitor does reduce the probability of a standard bidder winning the auction, but still leaves him with some chance of winning, and hence an incentive to participate.

It may be expected that bidder D wins the auction only when he is not disadvantaged ex-post, and hence he has the highest actual valuation. But this is not the case.

Lemma 2. *If $x_D - x_A - \theta > t_A - t_D > 0$, then bidder D has the lowest valuation and wins the auction.*

So provided bidder D's common signal is sufficiently higher than bidder A's common signal, he can win even if he has a lower private signal and, hence, a lower valuation than bidder A (and even if he faces an enhanced winner's curse). And this is true for every θ and, hence, even if the probability of bidder D having the lowest valuation is arbitrarily close to 1. Therefore, bidder D can beat an opponent who is advantaged both ex-ante and ex-post.

5. Seller's Revenue

We now analyze the effect on the seller's revenue of the presence of an advantaged bidder in the auction. Consider a symmetric benchmark, in which no bidder is advantaged (i.e., $\theta = 0$). Then the expected auction price is equal to:

$$\begin{aligned} \mathbb{E}\left[\text{Revenue in symmetric auction}\right] &= \tfrac{3}{2}\mathbb{E}\left[X_i + T_i \,|X_i + T_i > X_j + T_j\right] \\ &= \tfrac{3}{2}\tfrac{23}{30} = 1.15. \end{aligned} \tag{5.1}$$

It is often argued that, with common-value elements, even a small asymmetry between bidders drives the seller's revenue to zero, because a disadvantaged bidder has no chance of winning against an advantaged competitor and, hence, does not participate in the auction if there is an arbitrarily small bidding cost. In our model, this does not happen, because the disadvantaged bidder wins with strictly positive probability and, hence, has an incentive to participate. Furthermore, as we are going to show, the presence of a bidder with only a slight advantaged compared to her competitor actually increases the seller's revenue.

In the asymmetric case, when the advantaged bidder wins the auction, she expects to pay a relatively low price since she wins against an opponent who is bidding cautiously:

$$\begin{aligned}\mathbb{E}\left[\text{price}\right| A \text{ wins}] &= \mathbb{E}\left[\max\left\{\tfrac{3}{2}\left(X_D+T_D\right)-\theta;\ 0\right\}|A \text{ wins}\right] \\ &= \begin{cases} \frac{432\theta^5+2880\theta^4-4240\theta^3-360\theta^2+1620\theta+621}{180\left(12\theta^4-16\theta^3+8\theta+3\right)} & \text{for } \theta<\frac{1}{2}, \\ \frac{36\theta^5+180\theta^4-1100\theta^3+1800\theta^2-990\theta-81}{90\left(2\theta^4-8\theta^3+12\theta^2-8\theta-1\right)} & \text{for } \theta>\frac{1}{2}. \end{cases}\end{aligned} \tag{5.2}$$

This is lower than the expected price in a symmetric auction.

On the other hand, when the disadvantaged bidder wins the auction, he expects to pay a relatively high price since he wins against an opponent who is bidding aggressively:[18]

$$\begin{aligned}\mathbb{E}\left[\text{price}\,|D \text{ wins}\right] &= \mathbb{E}\left[\tfrac{3}{2}\left(X_A+T_A\right)+\tfrac{1}{2}\theta\,|D \text{ wins}\right] \\ &= \begin{cases} \frac{192\theta^5+80\theta^4-480\theta^3+200\theta^2+120\theta-69}{20\left(12\theta^4-16\theta^3+8\theta-3\right)} & \text{for } \theta<\frac{1}{2}, \\ \frac{2}{5}\left(3+2\theta\right) & \text{for } \theta>\frac{1}{2}. \end{cases}\end{aligned} \tag{5.3}$$

This is higher than the expected price in a symmetric auction.

Whether the expected auction price, and hence the seller's revenue, is lower or higher than in the symmetric case depends on how large bidder A's advantage is. Indeed, when bidders A is advantaged, the expected seller's revenue is:

$$\begin{aligned}\mathbb{E}\left[\text{Revenue}\right] &= \mathbb{E}\left[\text{price}\,|A \text{ wins}\right]\cdot\Pr\left[A \text{ wins}\right]+\mathbb{E}\left[\text{price}\,|D \text{ wins}\right]\cdot\Pr\left[D \text{ wins}\right] \\ &= \begin{cases} \frac{1}{2}\theta-2\theta^2+\frac{2}{27}\theta^3+2\theta^4-\frac{6}{5}\theta^5+\frac{23}{20} & \text{for } \theta<\frac{1}{2}, \\ \theta-4\theta^2+\frac{110}{27}\theta^3-2\theta^4+\frac{2}{5}\theta^5+\frac{11}{10} & \text{for } \theta>\frac{1}{2}. \end{cases}\end{aligned} \tag{5.4}$$

Figure 5.1 represents the expected seller's revenue as a function of the intensity of bidder A's advantage. For $\theta=0$, bidders are symmetric and, as expected, the expected revenue is the same as in (5.1). For $\theta=1$, bidder A is advantaged ex-post for sure and so the expected revenue is much lower. And if θ is sufficiently high, the larger the advantage of bidder A, the lower the expected revenue, because the advantaged bidder wins the auction most of the time and pays a lower price. Hence, an auction with very asymmetric bidders yields a very low revenue.

However, for θ close to 0, bidder D wins almost 50% of the time and, when he wins, pays a high price because bidder A bids aggressively due to her higher valuation and her lower winner's curse.[19] For a relatively low advantage, the higher price paid by bidder D when he

[18] Compared to a symmetric case, bidder A bids more aggressively for two reasons. Firstly, she competes against a cautious bidder (and, hence, she increases her bid by $\theta/2$.) Secondly, on average she has a higher valuation because $\mathbb{E}\left[T_A\right]\geq\frac{1}{2}$.

[19] Of course, the higher price partially depends on the advantaged bidder having a higher valuation and, hence, being willing to pay more. (See also Levin and Kagel, 2005.) Nonetheless, the literature has argued that the presence of a bidder with a higher valuation reduces the seller's revenue, even though the advantaged bidder is willing to pay a higher price.

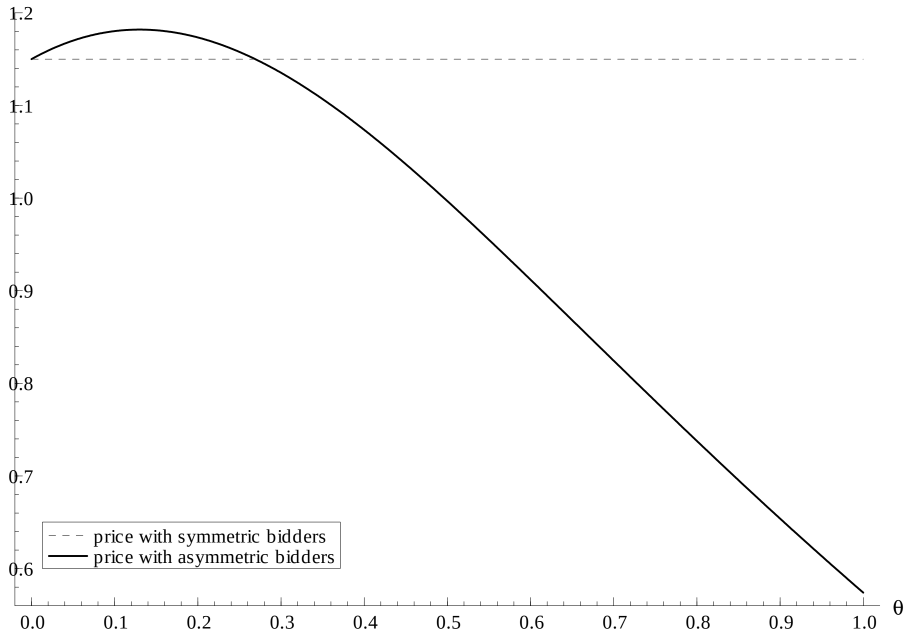

Figure 5.1: Expected seller's revenue with an advantaged bidder

wins more than compensates for the lower price paid by bidder A when she wins (even if the former event is less likely than the latter). Therefore, introducing in an otherwise symmetric auction a bidder with a slightly higher expected valuation (and keeping the number of bidders constant) increases the seller's revenue. Summing up:

Proposition 3. *Compared to an auction with symmetric bidders, in an auction with an advantaged bidder: the advantaged bidder pays a lower price when she wins, and the disadvantaged bidder pays a higher price when he wins. The net effect of the presence of an advantaged bidder on the seller's revenue depends on the intensity of the bidder's advantage: revenue is much lower than in a symmetric auction only if the bidder's advantage is relatively large.*

Therefore, in line with what is argued in the literature, our model suggests that an auction with very asymmetric bidders yields a very low revenue for the seller. So the seller should try to reduce large asymmetries between bidders, for example favoring disadvantaged bidders, in order to induce disadvantaged bidders to participate and bid aggressively.[20]

[20] For example, in the Italian 3G auction in 2000, the government offered an additional block of spectrum at a very favorable price to winning firms that were new entrants in the Italian market, and hence disadvantaged

However, our model also suggests that if a bidder's advantage is relatively small and not certain, the seller's revenue is not dramatically reduced. Therefore, when bidders are asymmetric, the seller should try to evaluate the type and intensity of a bidders' advantage in order to choose the appropriate auction's rules: the presence of a bidder with only a small ex-ante advantage compared to her competitor (i.e., a slightly higher expected valuation) may even increase, rather than reduce, the seller's revenue.

6. Extension

We assumed bidders' signals to be uniformly distributed in order to be able to obtain quantitative results by deriving closed form solutions for bidders' strategies and the seller's revenue. But our results are more general. In this section we show that, regardless of the distributions of bidders' signals, in our model the disadvantaged bidder always wins with strictly positive probability, because (as long as $\theta < 1$) there are always types of bidder D who win the auction against some types of bidder A. And the disadvantaged bidder can win even if he has a lower actual valuation than his opponent. Therefore, ex-ante, a disadvantaged bidder always expects to win with strictly positive probability and has an incentive to participate in the auction.

Restricting the analysis to undominated strategies (and absent bidding costs), a disadvantaged bidder always bids up to at least $x_D + t_D$ (i.e., his lowest possible valuation), and an advantaged bidder never bids more than $x_A + t_A + 1$ (i.e., her highest possible valuation).[21] Therefore, a bidder D with signals $x_D = t_D = 1$ bids at least up to 2, and wins against a bidder A with signals $x_A = 0$ and $t_A = \theta$, who does not bid above $1 + \theta$. So a disadvantaged bidder with the highest possible signals always wins against an advantaged bidder with the lowest possible signals (and, hence, before the auction the disadvantaged bidder does not expect to lose with probability 1).[22]

But the disadvantaged bidder does not win only when he has the highest valuation — as the following lemma shows, bidder D may win the auction even if ex-post he has a lower

compared to incumbents. The government expected the higher auction price achieved thanks to increased competition to more than compensate for the lower price paid by new entrants for the additional blocks. A rule in favor of potential entrants was also adopted in the UK 3G auction in 2000 (Klemperer, 2004). Bidding credits and set-asides were used to help weak bidders in the US FCC spectrum auctions (Ayres and Cramton, 1996).

[21] This argument is similar to the argument of Proposition 1 in de Frutos and Pechlivanos (2006), which refers to bidders who are ex-post disadvantaged.

[22] Note also that all types of bidder D who bid more than $1+\theta$ (i.e., at least all types with signals $x_D + t_D > 1 + \theta$), will surely win against at least a bidder A of the lowest type.

valuation than bidder A.

Lemma 3. *In any equilibrium in continuous strategies, there always exists a disadvantaged bidder who ex-post has a lower valuation than the advantaged bidder and wins the auction.*

7. Conclusions

In ascending auctions, an advantaged bidder, who on average has a higher valuation for the prize than her competitor, wins most of the time and pays a relatively low price. However, a disadvantaged bidder, even if he has to bid less aggressively to avoid an enhanced winner's curse, still expects to win with positive probability and, hence, has an incentive to participate in the auction. So the seller's revenue is reduced by the presence of the advantaged bidder, but not to the extent sometimes argued in the literature — a small asymmetry between bidders does not have a dramatic effect on the seller's revenue when bidding costs are negligible (unless one bidder has a higher valuation than her opponent with absolute certainty).

We extend the analysis of de Frutos and Pechlivanos (2006) by showing that a disadvantaged bidder may win the auction even if his competitor has a very large and almost certain advantage ex-ante and the disadvantaged bidder has the lowest valuation ex-post. Our results also complement the analyses of Bikhchandani (1988) and Klemperer (1998) and suggest that what really harms standard bidders and hence the seller is the presence of a bidder with an ex-post certain, although small, advantage rather than an ex-ante large, but uncertain, one.

Appendix

A. Proof of Lemma 1

We are going to prove that strategy (3.2) is a best response to (3.1), and vice-versa. First, suppose bidder A bids according to (3.1). Then, if bidder D wins the auction at price p, he expects A's common signal to be equal to:[23]

$$\begin{aligned}\mathbb{E}\left[X_A \left| \tfrac{3}{2}\left(x_A+t_A\right)+\tfrac{\theta}{2}=p\right.\right] &= \mathbb{E}\left[X_A \left| x_A+t_A=\tfrac{2}{3}p-\tfrac{\theta}{3}\right.\right] \\ &= \tfrac{1}{3}\left(p-2\theta\right).\end{aligned}$$

In an ascending auction, a bidder bids up to the expected value of the prize, conditional on winning. Hence, bidder D bids up to price p_D such that:

$$p_D=\mathbb{E}\left[V_D \left| D \text{ wins at } p_D\right.\right]=x_D+t_D+\tfrac{1}{3}\left(p_D-2\theta\right)$$

$$\Leftrightarrow \quad p_D=\tfrac{3}{2}\left(x_D+t_D-\tfrac{2}{3}\theta\right).$$

Notice that, when $x_D+t_D<\theta$, bidder D cannot obtain positive profit by winning the auction. The reason is that, when bidder A bids according to (3.1), conditional on winning at a price equal to or higher than $\frac{1}{2}\theta$ (which is bidder A's lowest possible bid), bidder D's expected valuation is lower than the auction price. Therefore, any bid lower than $\frac{1}{2}\theta$ is a best reply by bidder D when $x_D+t_D<\theta$. For simplicity, we allow a negative bid by bidder D and we interpret it as a bid of 0.

Now assume bidder D bids according to (3.2). Then, if bidder A wins the auction at price p, she expects D's common signal to be equal to:

$$\begin{aligned}\mathbb{E}\left[X_D \left| \tfrac{3}{2}\left(x_D+t_D\right)-\theta=p\right.\right] &= \mathbb{E}\left[X_D \left| x_D+t_D=\tfrac{2}{3}p+\tfrac{2}{3}\theta\right.\right] \\ &= \tfrac{1}{3}\left(p+\theta\right).\end{aligned}$$

Hence, bidder A bids up to price p_A such that:

$$p_A=\mathbb{E}\left[V_A \left| A \text{ wins at } p_A\right.\right]=x_A+t_A+\tfrac{1}{3}\left(p_A+\theta\right)$$

$$\Leftrightarrow \quad p_A=\tfrac{3}{2}\left(x_A+t_A+\tfrac{1}{3}\theta\right).$$

Notice that, when $x_A+t_A>2-\theta$, bidder A always obtains positive profit by winning the auction. The reason is that, when bidder D bids according to (3.2), conditional on winning at a price equal to or lower than $3-\theta$ (which is bidder D's highest possible bid), bidder A's expected valuation is higher than the auction price. Therefore, any bid higher than $3-\theta$ is a best reply by bidder A when $x_A+t_A>2-\theta$.

It is straightforward to show that the equilibrium described is the unique linear equilibrium. ■

[23] Let $X\sim U\left[0,1\right]$ and $T\sim U\left[\theta,1+\theta\right]$ and let $k=x+t$ where x and t are realizations of the random variables X and T respectively. Then $\mathbb{E}\left[X\left|k=x+t\right.\right]=\frac{1}{2}\left(k-\theta\right)$.

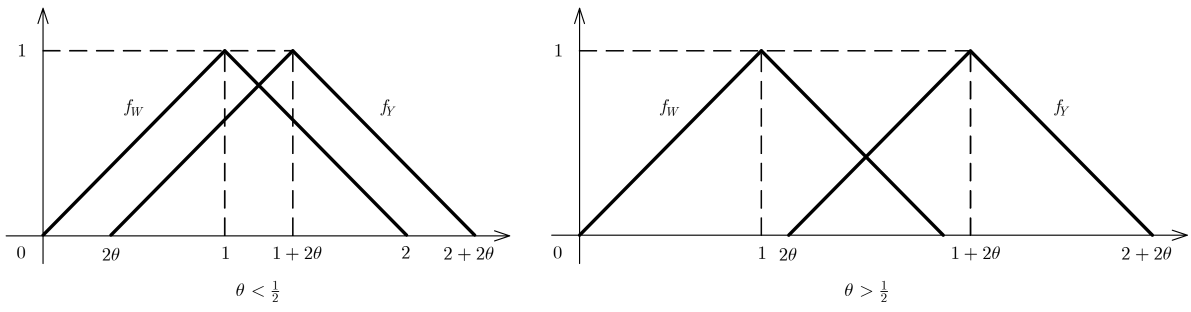

Figure 7.1: Densities of W and Y

B. Proof of Lemma 2

Bidder D wins the auction if $\beta_D(x_D, t_D; \theta) > \beta_A(x_A, t_A; \theta) \Leftrightarrow x_D + t_D > x_A + t_A + \theta$, and bidder D has the lowest valuation if $t_A > t_B$. Rearranging yields the statement. ■

C. Derivation of expressions (4.1), (5.2), (5.3), and (5.4)

Let $W \equiv X_D + T_D$ and $Y \equiv X_A + T_A + \theta$.[24] The two random variables W and Y have densities (which are represented in Figure 7.1):

$$f_W(w) = \begin{cases} w & \text{for } 0 \leq w < 1, \\ 2 - w & \text{for } 1 \leq w \leq 2; \end{cases}$$

and

$$f_Y(y) = \begin{cases} y - 2\theta & \text{for } 2\theta \leq y < 1 + 2\theta, \\ 2 + 2\theta - y & \text{for } 1 + 2\theta \leq y \leq 2 + 2\theta; \end{cases}$$

and distributions:

$$F_W(w) = \begin{cases} \frac{1}{2}w^2 & \text{for } 0 \leq w < 1, \\ 2w - \frac{1}{2}w^2 - 1 & \text{for } 1 \leq w \leq 2; \end{cases}$$

and

$$F_Y(y) = \begin{cases} \frac{1}{2}(y - 2\theta)^2 & \text{for } 2\theta \leq y < 1 + 2\theta, \\ 2y - 1 - 4\theta - \frac{1}{2}(y - 2\theta)^2 & \text{for } 1 + 2\theta \leq y \leq 2 + 2\theta. \end{cases}$$

The probability of bidder D winning the auction is given by:

$$\begin{aligned} \Pr[D \text{ wins}] &= \Pr[x_D + t_D > x_A + t_A + \theta] \qquad (C.1) \\ &= 1 - \Pr[w < y] \\ &= 1 - \int_{2\theta}^{2+2\theta} F_W(y) f_Y(y)\, dy. \end{aligned}$$

[24] W is the sum of two independent draws from a uniform distribution on $[0, 1]$, while Y is the sum of two draws from a uniform distribution on $[0, 1]$ plus a constant equal to 2θ.

For $\theta < \frac{1}{2}$, (C.1) equals to:

$$\begin{aligned}
& 1 - \int_{2\theta}^{1} \tfrac{y^2}{2}(y-2\theta)\,dy - \int_{1}^{1+2\theta} \left(2y - \tfrac{y^2}{2} - 1\right)(y-2\theta)\,dy + \\
& - \int_{1+2\theta}^{2} \left(2y - \tfrac{y^2}{2} - 1\right)(2+2\theta-y)\,dy - \int_{2}^{2+2\theta} (2+2\theta-y)\,dy \\
= \; & \tfrac{1}{6}\left(3 - 8\theta + 16\theta^3 - 12\theta^4\right).
\end{aligned}$$

For $\theta > \frac{1}{2}$, (C.1) equals to:

$$\begin{aligned}
& 1 - \int_{2\theta}^{2} \left(2y - \tfrac{y^2}{2} - 1\right)(y-2\theta)\,dy - \int_{2}^{1+2\theta} (y-2\theta)\,dy - \tfrac{1}{2} \\
= \; & \tfrac{2}{3}(1-\theta)^4.
\end{aligned}$$

This yields expression (4.1).

The expected auction price when bidder A wins the auction is given by:

$$\begin{aligned}
\mathbb{E}[\text{price}\,|A \text{ wins}] &= \mathbb{E}\left[\max\left\{\tfrac{3}{2}(X_D + T_D) - \theta;\ 0\right\}|A \text{ wins}\right] \qquad \text{(C.2)} \\
&= \tfrac{3}{2}\mathbb{E}\left[\max\left\{W;\ \tfrac{2}{3}\theta\right\}|y > w\right] - \theta \\
&= \tfrac{3}{2}\left[\int_0^{\frac{2}{3}\theta} \tfrac{2}{3}\theta \frac{[1-F_Y(w)]\, f_W(w)}{\Pr[y>w]}dw + \int_{\frac{2}{3}\theta}^{2} w \frac{[1-F_Y(w)]\, f_W(w)}{\Pr[y>w]}dw\right] - \theta \\
&= \frac{3}{2\Pr[y>w]}\left[\begin{array}{c} \frac{4}{27}\theta^3 - \frac{2}{3}\theta\int_0^{\frac{2}{3}\theta} F_Y(w)\, f_W(w)\,dw \\ +1 - \frac{8}{81}\theta^3 - \int_{\frac{2}{3}\theta}^{2} wF_Y(w)\, f_W(w)\,dw \end{array}\right] - \theta.
\end{aligned}$$

For $\theta < \frac{1}{2}$, (C.2) equals to:

$$\begin{aligned}
& \tfrac{9}{\left(3+8\theta-16\theta^3+12\theta^4\right)}\left[\begin{array}{c} 1 + \frac{4}{81}\theta^3 - \int_{2\theta}^{1} \frac{w}{2}(w-2\theta)^2\, wdw - \int_1^{1+2\theta} \frac{w}{2}(w-2\theta)^2 (2-w)\,dw \\ - \int_{1+2\theta}^{2} w\left(2w - 1 - 4\theta - \frac{1}{2}(w-2\theta)^2\right)(2-w)\,dw \end{array}\right] - \theta \\
= \; & \frac{432\theta^5 + 2880\theta^4 - 4240\theta^3 - 360\theta^2 + 1620\theta + 621}{180\left(12\theta^4 - 16\theta^3 + 8\theta + 3\right)}.
\end{aligned}$$

For $\theta > \frac{1}{2}$, (C.2) equals to:

$$\begin{aligned}
& \tfrac{9}{2[3-2(1-\theta)]^4}\left[1 + \tfrac{4}{81}\theta^3 - \int_{2\theta}^{2} \tfrac{w}{2}(w-2\theta)^2 (2-w)\,dw\right] - \theta \\
= \; & \frac{36\theta^5 + 180\theta^4 - 1100\theta^3 + 1800\theta^2 - 990\theta - 81}{90\left(2\theta^4 - 8\theta^3 + 12\theta^2 - 8\theta - 1\right)}.
\end{aligned}$$

This yields expression (5.2). For $\theta = 0$, this also yields expression (5.1).

The expected auction price when bidder D wins the auction is given by:

$$\begin{aligned}
\mathbb{E}\left[\text{price}\,|D\text{ wins}\right] &= \mathbb{E}\left[\tfrac{3}{2}\left(X_A+T_A\right)+\tfrac{1}{2}\theta\,|D\text{ wins}\right] \\
&= \tfrac{3}{2}\mathbb{E}\left[Y\,|w>y\right]-\theta \\
&= \tfrac{3}{2}\int_{2\theta}^{2+2\theta} y\frac{\left[1-F_W\left(y\right)\right]f_Y\left(y\right)}{\Pr\left[w>y\right]}dy-\theta \\
&= \frac{3}{2\Pr\left[w>y\right]}\left[1+2\theta-\int_{2\theta}^{2+2\theta} yF_W\left(y\right)f_Y\left(y\right)dy\right]-\theta.
\end{aligned} \tag{C.3}$$

For $\theta<\frac{1}{2}$, (C.3) equals to:

$$\begin{aligned}
&\tfrac{9}{\left(3-8\theta+16\theta^3-12\theta^4\right)}\left[\begin{array}{c} 1+2\theta-\int_{2\theta}^{1}y\frac{y^2}{2}\left(y-2\theta\right)dy-\int_{1}^{1+2\theta}y\left(2y-\frac{y^2}{2}-1\right)\left(y-2\theta\right)dy+ \\ -\int_{1+2\theta}^{2}y\left(2y-\frac{y^2}{2}-1\right)\left(2+2\theta-y\right)dy-\int_{2}^{2+2\theta}y\left(2+2\theta-y\right)dy \end{array}\right]-\theta \\
=\;& \frac{192\theta^5+80\theta^4-480\theta^3+200\theta^2+120\theta-69}{20\left(12\theta^4-16\theta^3+8\theta-3\right)}.
\end{aligned}$$

For $\theta>\frac{1}{2}$, (C.3) equals to:

$$\begin{aligned}
&\tfrac{9}{4(1-\theta)^4}\left[\begin{array}{c} 1+2\theta-\int_{2\theta}^{2}y\left(2y-\frac{y^2}{2}-1\right)\left(y-2\theta\right)dy+ \\ -\int_{2}^{1+2\theta}y\left(y-2\theta\right)dy-\int_{1+2\theta}^{2+2\theta}y\left(2+2\theta-y\right)dy \end{array}\right]-\theta \\
=\;& \tfrac{2}{5}\left(3+2\theta\right).
\end{aligned}$$

This yields expression (5.3).

Using the above results, the expected seller's revenue is equal to:

$$\begin{aligned}
\mathbb{E}\left[\text{Revenue}\right] &= \mathbb{E}\left[\text{price}\,|A\text{ wins}\right]\cdot\Pr\left[A\text{ wins}\right]+\mathbb{E}\left[\text{price}\,|D\text{ wins}\right]\cdot\Pr\left[D\text{ wins}\right] \\
&= \begin{cases} \frac{1}{2}\theta-2\theta^2+\frac{2}{27}\theta^3+2\theta^4-\frac{6}{5}\theta^5+\frac{23}{20} & \text{for } \theta<\frac{1}{2}, \\ \theta-4\theta^2+\frac{110}{27}\theta^3-2\theta^4+\frac{2}{5}\theta^5+\frac{11}{10} & \text{for } \theta>\frac{1}{2}. \end{cases}
\end{aligned}$$

This is expression (5.4).

D. Proof of Lemma 3

We will prove that there is always a bidder D who wins the auction against a bidder A when $t_A>t_D$ (regardless of the distributions of bidders' signals). In particular, we will show that, in any equilibrium in continuous and undominated strategies, a bidder D who receives the highest possible common signal wins against a bidder A who receives the lowest possible common signal and a private signal only marginally higher than bidder D's private signal.

First note that bidder D never bids less than x_D+t_D (i.e., his lowest possible valuation). It follows that, conditional on bidder A winning at price $p<2$, her expectation of bidder D's private signal is strictly lower than 1, because this expectation can be equal to 1 only if any bidder D with a private signal lower than 1 always drops out of the auction at a price lower than p (which is impossible since a bidder D with signals $x_D+t_D\geq p$ does not).

Assume now bidder A has signals $t_A = t \in (\theta, 1)$ and $x_A = 0$. If bidder A wins the auction at price $1 + t < 2$, her expected valuation is lower than the price she pays because:

$$\begin{aligned} \mathbb{E}\left[V_A \left| A \text{ wins at price } 1+t\right.\right] &= t_A + \mathbb{E}\left[t_D \left| A \text{ wins at price } 1+t\right.\right] \\ &< t+1. \end{aligned}$$

Hence, bidder A wants to drop out at a price which is strictly lower than $1 + t$.

Therefore, a bidder A with signals $t_A = t \in (\theta, 1)$ and $x_A = 0$ loses the auction against a bidder D with the same private signal $t_D = t$ and common signal $x_D = 1$, because this bidder D bids at least up to $1 + t$. But because of the continuity of bidders' strategies, there is an $\varepsilon > 0$ small enough such that bidder A's bid when she has signals $t_A = t + \varepsilon \in (\theta, 1)$ and $x_A = 0$ is also lower than $1 + t$. So a bidder D with signals $t_D = t$ and $x_D = 1$ also wins the auction against such a bidder A. ■

References

[1] Avery, C. and J. Kagel (1997), "Second-Price Auction with Asymmetric Payoffs: An Experimental Investigation," *Journal of Economics and Management Strategy*, 6, 573-603

[2] Ayres, I. and P. Cramton (1996), "Deficit Reduction Through Diversity: How affirmative Action at the FCC Increased Auction Competition." *Stanford Law Review*, 48(4), 761-815.

[3] Bikhchandani, S. (1988), "Reputation in Repeated Second-Price Auctions." *Journal of Economic Theory*, 46, 97-119.

[4] Bulow, J., M. Huang and P. Klemperer (1999), "Toeholds and Takeovers." *Journal of Political Economy*, 107, 427-54.

[5] Bulow, J. and P. Klemperer (2002), "Prices and the Winner's Curse." *RAND Journal of Economics*, 33(1), 1-21.

[6] Cantillon, E. (1999), "The Effect of Bidders' Asymmetries on Expected Revenue in Auctions." Mimeo, Harvard University.

[7] Compte, O. and P. Jehiel (2002), "On the Value of Competition in Procurement Auctions." *Econometrica*, 70(1), 343-355.

[8] De Frutos, M. A. and L. Pechlivanos (2006), "Second-Price Common-value Auctions under Multidimensional Uncertainty." *Games and Economic Behavior*, 55(1), 43-71.

[9] Goeree, J. and T. Offerman (2003), "Competitive Bidding in Auctions with Private and Common Values." *Economic Journal*, 113, 598-613.

[10] Jehiel, P. and B. Moldovanu (2000), "Efficient Design with Interdependent Valuations." *Econometrica*, 69, 1237-1259.

[11] Klemperer, P. (1998), "Auctions with Almost Common Values: The 'Wallet Game' and its Applications." *European Economic Review*, 42, 757-569.

[12] Klemperer, P. (2002), "How (Not) to Run Auctions: the European 3G Telecom Auctions." *European Economic Review*, 46, 829-845.

[13] Klemperer, P. (2004), *Auctions: Theory and Practice*. Princeton University Press.

[14] Klemperer, P. and M. Pagnozzi (2004), "Advantaged Bidders and Spectrum Prices: An Empirical Analysis." Mimeo, Oxford University.

[15] Levin, D. and J. Kagel (2005), "Almost Common Values Auctions Revisited." *European Economic Review*, 49(5), 1125-1136.

[16] MASKIN, E. (1992), "Auctions and Privatization." In *Privatization,* ed. by H. Siebert, Kiel: Institut Fuer Weltwirtschaft der Universitaet Kiel, 115-136.

[17] MASKIN, E. and J. RILEY (2000), "Asymmetric Auctions." *Review of Economic Studies*, 67, 413-438.

[18] MILGROM, P. and R. WEBER (1982), "A Theory of Auctions and Competitive Bidding." *Econometrica*, 50(5), 1089-1122.

[19] PAGNOZZI, M. (2003), "Sorry Winners." CSEF Working Paper n.108.

[20] PAGNOZZI, M. (2004), "Bidding to Lose? Auctions with Resale." CSEF Working Paper n.116.

[21] SALANT, D. (1997), "Up in the Air: GTE's Experience in the MTA Auctions for PCS Licenses." *Journal of Economics and Management Strategy*, 6(3), 549-72.