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Should Speculators Be Welcomed in Auctions?

Marco Pagnozzi

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University of Naples Federico II | University of Salerno | Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance – UNIVERSITY OF SALERNO
84084 FISCIANO (SA) - ITALY
Tel. +39 089 96 3167/3168 - Fax +39 089 96 3167 – e-mail: csef@unisa.it

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Abstract

The possibility of resale after an auction attracts speculators (i.e., bidders who have no use value for the objects on sale). In a multi-object auction, a high-value bidder may strictly prefer to let a speculator win some of the objects and then buy in the resale market, in order to keep the auction price low for the object she wins in the auction. Therefore, although speculators increase competition in the auction, they also affect high-value bidders' incentives to "reduce demand." We show that the net effect on the seller's revenue of allowing resale to attract speculators depends on the heterogeneity of bidders' valuations. But the presence of speculators may increase the seller's revenue only if speculators are eventually outbid. We also analyze the effect on the seller's revenue of allowing resale in the absence of speculators, and we discuss the strategies that the seller can adopt to increase his revenue.

JEL Classification: D44 (auctions).

Keywords: multi-object auctions, speculation, resale, demand reduction.

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[*] University of Napoli "Federico II" and CSEF. Correspondance address: Department of Economics, Università di Napoli Federico II, Via Cintia (Monte S. Angelo), 80126 Napoli, Italy (email: pagnozzi@unina.it).

Contents

1. Introduction

When resale after an auction is allowed, speculators — i.e., players who have no use value for the object on sale — may participate in the hope of winning the auction and then reselling to a bidder who has a high use value for the object on sale.[1] However, it is not clear why a bidder who has a high use value should let a speculator win the auction, only then to buy from him in the resale market. Indeed, in a single-object auction with complete information, a bidder is exactly indifferent between buying in the resale market and winning the auction (at the same price at which he can buy in the resale market).[2] And if there is an arbitrarily small cost to trade in the resale market or if players discount by even an arbitrarily small amount the future surplus from resale, then the bidder strictly prefers to outbid the speculator and win the auction.[3]

This paper analyzes the role of speculators in multi-object auctions and the effects of the possibility of resale and the presence of speculators on the seller's revenue. We show that, in contrast to single-object auctions, in multi-object auctions bidders with positive use values may *strictly* prefer to let a speculator win some of the objects on sale and then purchase from the speculator in the resale market. The reason is that letting the speculator win allows bidders to keep the auction price low, and hence pay a lower price for those objects that they acquire in the auction.[4] So it can be common knowledge that speculators will win an auction, even when competing against bidders with higher use values, and even with (not too large) resale costs and discounting.

Of course, speculators will win the auction when bidders with positive use values are not allowed to participate in the auction and can only acquire the objects in the resale market

[1] A speculator can also be defined as a player who has a positive but low use value, and only participates in the auction to resell to a player with a higher use value (because, for example, his own use value is lower than the opportunity cost of participating in the auction).

[2] Consider, for example, an ascending auction with one bidder who has value v for the object on sale and one speculator who has value 0. If the speculator wins the auction and players equally share the gains from trade in the resale market, the speculator resells to the bidder at price $\frac{1}{2}v$. Therefore, the speculator is willing to bid up to $\frac{1}{2}v$ in the auction. Similarly, the bidder is also willing to bid up to $\frac{1}{2}v$, because this is the price at which she can buy in the resale market. So there are multiple equilibria but the bidder has no compelling reason to let the speculator win the auction. (And if the bidder bids up to her willingness to pay, the speculator should expect to obtain no surplus and has no reason to participate in the auction in the first place.)

[3] However, Garratt and Tröger (2006) show that, if a bidder is privately informed about her use value, in second-price auctions there are also equilibria in undominated strategies in which a speculator wins by bidding a very high price (that he expects not to pay) and inducing the bidder to bid zero. But these equilibria rely upon the speculator bidding a price higher than the maximum price he would be happy to pay, even after taking into account the surplus he can obtain in the resale market. Pagnozzi (2007) shows that a high-value bidder may prefer to let a low-value bidder win the auction and then buy in the resale market if the auction price affects bargaining in the resale market because of wealth effects. Resale can also take place if the order of bidders' valuations change after the auction (Haile, 2000, 2003), if additional buyers appear after the auction (Milgrom, 1987), and in first-price auctions with asymmetric bidders (Gupta and Lebrun, 1999; Hafalir and Krishna, 2006).

[4] In our analysis, we explicitly consider uniform-price auctions, but our results apply to any multi-object auction in which players face a trade-off between winning more units and paying lower prices.

(Bikhchandani and Huang, 1989; Bose and Deltas, 2002).[5] But we show that speculators may win even if all bidders with positive use values can and do participate in the auction.

As an example, consider the UK 3.4GHz simultaneous ascending auction for 15 licenses to offer broadband wireless services in UK regions. Pacific Century Cyberworks (PCCW), a Hong-Kong telecom company, was widely considered the highest-value bidder and was expected to win all 15 licenses. Red Spectrum and Public Hub were two small companies created explicitly to participate in the auction. It was not clear they had a genuine interest in providing wireless services and, at the beginning of the auction, they chose to be eligible to bid for only one license each. As soon as PCCW, Red Spectrum and Public Hub were the only three bidders left in the auction, PCCW reduced its demand to 13 licenses, thus allowing Red Spectrum and Public Hub to win one license each and preventing the auction price from rising any further. The auction ended in June 2003. By March 2004, PCCW obtained the two licenses it had not won in the auction by taking over Red Spectrum and Public Hub.[6]

But what is the effect of the presence of speculators on the seller's revenue? It is often argued that speculators always increase the seller's revenue, because the effect of their presence is to increase the number of bidders, and hence to increase competition in the auction. So, it is argued, the seller should always allow resale after the auction and welcome speculators.

However, as our previous example suggests, attracting speculators also affects bidders' strategies in the auction. On the one hand, speculators can induce bidders with positive use values to bid more aggressively in order to beat speculators and win the auction. On the other hand, in multi-object auctions the possibility of resale also affects bidders' incentive to "reduce demand" — i.e., to bid for fewer objects than they actually want, in order to pay a lower price for the objects they win — which typically reduces the seller's revenue (Wilson, 1979).[7] Firstly, because

[5] Bikhchandani and Huang (1989) and Bose and Deltas (2002) analyze the role of speculators as intermediaries in auctions in which the final consumers are not allowed to participate, as with treasury bill auctions and large real-estate auctions.

[6] Since explicit resale of a license was not allowed, PCCW had to take over the original winners to obtain their licenses. Red Spectrum was actually taken over only 3 months after the end of the auction. For more details, see "3.4GHz Broadband Ready for Action" by Graeme Wearden, ZDNet UK, available at `http://news.zdnet.co.uk/communications/wireless/0,39020348,39149852,00.htm`.

The failure by PCCW to win all 15 licenses was described as a "surprise," a "gaffe," and "a costly mistake [that] may have cost [PCCW] the chance of offering a nationwide service" (see "UK Operators Miss Out in Wireless Broadband Auction" by Graeme Wearden, ZDNet UK, available at `http : //news.zdnet.co.uk/communications/0, 39020336, 2136110, 00.htm`). Given the subsequent events, it appears that PCCW's strategy was not a mistake after all.

[7] The literature on demand reduction also includes Ausubel and Cramton (1998), Back and Zender (1993), Engelbrecht-Wiggans and Kahn (1998a), Klemperer and Meyer (1989), Kremer and Nyborg (2004), and Noussair (1995). Kagel and Levin (2001) and List and Lucking-Reiley (2000) provide experimental evidence of demand reduction; Weber (1997), Wolfram (1998), and Wolak (2003) argue that demand reduction affected several FCC spectrum auctions, as well as the UK and the California electricity markets; Cassady (1967) reports of "collusive" demand reduction in many auctions of divisible commodities, such as fish in England, timber in the US and wool in Australia. See also Klemperer (2004). Milgrom (2004) provides an excellent exposition of the literature on multi-object auctions.

resale can correct an inefficient allocation, it makes demand reduction much less costly for the higher-value bidders (who have the strongest incentive not to reduce demand without resale). The reason is that, even if they lose some of the objects on sale to a speculator in the auction, higher-value bidders still have a chance to buy those objects in the resale market. Secondly, resale makes it more costly for a bidder to outbid lower-value competitors, because these competitors bid more aggressively in the auction if they can resell the objects they acquire. These two effects make demand reduction more profitable for bidders. Thirdly, in contrast to the previous two effects, by attracting speculators, resale may make demand reduction less profitable for some bidders with positive use values, because they may have to share the objects on sale with speculators if they reduce demand to keep the auction price low.

So speculators do indeed increase competition in the auction. But to attract speculators, the seller has to allow resale, and this may induce an accommodating strategy by bidders with positive use values and thus reduce the seller's revenue.[8]

The net effect on the seller's revenue of allowing resale and attracting speculators depends on the bidders' relative valuations: (*i*) if bidders' valuations are relatively similar (i.e., clustered), speculators increase the seller's revenue; (*ii*) if bidders' valuations are sufficiently different (i.e., dispersed), speculators reduce the seller's revenue. The reason is that, if bidders' valuations are similar, after winning an object in the auction, bidders with lower valuations have a much higher outside option (relatively to the highest valuation) than speculators in the resale market, and hence they obtain a much higher profit than speculators from winning the auction and reselling. Therefore, in this case, the presence of speculators induces lower-value bidders to bid more aggressively and has a strong competitive effect. By contrast, if bidders' valuations are far from each other, the main effect of allowing resale and attracting speculators is to induce all bidders with positive use values to reduce demand, because in this case it is too costly for lower-value bidders to outbid speculators.

In order to increase his revenue, the seller should design an auction to induce bidders to compete aggressively and not reduce demand. So the seller should allow resale to attract speculators if he knows bidders are relatively symmetric. However, provided he can credibly do so, the seller should forbid resale to increase his revenue if he knows bidders are asymmetric, even though this excludes speculators and reduces the number of competitors in the auction. In fact, attracting speculators by allowing resale when bidders are asymmetric only induces high-value bidders to reduce demand.[9]

[8] For an analysis of the effects of resale on the seller's revenue in single-object auctions with a fixed number of players see Garratt and Tröger (2006) and Pagnozzi (2007). Hafalir and Krishna (2006) show that, with resale and asymmetric bidders, a first-price auction yields a higher seller's revenue than a second-price auction. Ausubel and Cramton (1999), Calzolari and Pavan (2006), and Zheng (2002) analyze optimal selling mechanisms when resale is allowed.

[9] Bose and Deltas (2002) prove that, when speculators can resell to additional bidders who do not participate

Our analysis also shows that the presence of speculators can increase the seller's revenue only if they are eventually outbid by bidders with positive use values. Otherwise, if bidders accommodate speculators and allow them to win by reducing demand, the seller's revenue is (weakly) reduced. So observing speculators win the auction is bad news for the seller, because it shows that bidders with high use values have chosen to allow speculators to win in order to obtain a higher profit in the auction.

However, if the seller cannot prevent resale, the presence of speculators always weakly increases the seller's revenue.[10] This is because, if resale is possible, even when no speculator participates in the auction, low-value bidders can themselves resell to high-value bidders. Therefore, high-value bidders have a strong incentive to reduce demand and buy in the resale market even without speculators. In this case, the only effect of the presence of speculators is to increase competition in the auction.

Finally, when there is no speculator who may participate in the auction, allowing resale has only two contrasting effects on the seller's revenue. On the one hand, it increases the price that lower-value bidders — those who resell in the aftermarket — are willing to pay in the auction. On the other hand, allowing resale unambiguously facilitates demand reduction, because it makes it more costly for all bidders to deviate from demand reduction and less costly for higher-value bidders to reduce demand. (And, without speculators, allowing resale has no countervailing effect on the number of competitors in the auction.) When this second effect prevails, allowing resale reduces the seller's revenue.

A compelling reason to allow winners to resell after an auction is that resale favours an efficient final allocation of the objects on sale. But our analysis suggests that a revenue-maximizing seller may want to commit to prevent resale in order to increase the auction price, even if this may reduce efficiency.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 defines the types of equilibria of the auction. Section 4 analyzes bidding strategies and shows that speculators may win an auction against high-value bidders. The effects of resale and speculators on the seller's revenue and the strategies that the seller can adopt to increase his revenue are analyzed in Section 5. Section 6 analyzes how resale affects the seller's revenue when there is no speculator, and the last section concludes. All proofs are in the appendix.

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in the auction anyway, the seller may want to exclude bidders, rather than speculators, from the auction.

[10] In the UK 3.4 GHz auction bidders could, in practice, resell the licenses won and, arguably, the seller's revenue would have been even lower without the participation of Red Spectrum and Public Hub.

2. The Model

Consider a (sealed-bid) uniform-price auction for k units of the same good. Each player who participates in the auction submits k non-negative bids, one for each unit. The k highest bids are awarded the units, and the winner(s) pay for each unit won a price equal to the $(k+1)^{\text{th}}$-highest bid.[11] The reserve price is normalized to zero.[12]

We analyze a uniform-price auction for simplicity, because this is the auction mechanism in which the incentive to reduce demand arises more clearly (Ausubel and Cramton, 1998). But many of our qualitative results also hold for simultaneous ascending auctions (in which bidding remains open on all units on sale until no one wants to bid any more on any unit), for discriminatory-price auctions (in which winners pay the price they bid for each unit they are awarded),[13] and indeed for any mechanism to allocate multiple units in which players face a trade-off between winning more units and paying lower prices.

Players

There are n (female) bidders, called $B_1, ..., B_n$, who have positive use values for the units on sale. To make the model interesting, we assume that $n < k$. If the number of bidders is greater than or equal to the number of units on sale, competition among bidders crowds out speculators, who hence have no chance of winning the auction (as in a single-unit auction).[14] We let v_i be the use value of bidder B_i for each of the units on sale — i.e., we assume bidders have flat demand.[15] Without loss of generality, we assume that $v_1 > v_2 > ... > v_n$.

There are also $(k - n)$ identical (male) speculators who have no use value for the units on sale.[16] The speculators participate in the auction if the seller allows them to do so and if resale

[11] Uniform-price auctions are often used to allocate multiple identical objects. For example, uniform-price auctions are used for on-line IPOs (including the one of Google in August 2004), electricity markets, markets for emission permits, and by the US Treasury Department to issue new securities.

[12] Footnote 27 and Section 5.3 discuss the effects of increasing the reserve price in our model.

[13] In a discriminatory-price auction, a bidder always bids less than her valuation in order to make a profit. Engelbrecht-Wiggans and Kahn (1998b) show that, because the incentive to shade one's bid is stronger for the first units, also a discriminatory-price auction may be inefficient. See also Ausubel and Cramton (1998).

[14] As it will be clear from the analysis, if $n \geq k$ the auction price is at least equal to the highest price the lowest-valued bidder is willing to pay. At this price, speculators cannot possibly make a positive profit.

[15] Assuming flat demand simplifies the analysis because, as we are going to show, it implies that, when players bid in the auction, they know whether they will buy or sell in the resale market, and they know the price at which they will trade. When bidders have non-flat demand (i.e., use values for additional units are decreasing or there are complementarities), how bidders will trade in the resale market and the price at which they will trade depend on the identity of the auction winners, the number of units each of them wins, and the details of the bargaining process. This would complicate the analysis, but would yield qualitative results similar to the ones we obtain.

[16] It is possible to make the number of speculators endogenous by assuming that a speculator enters the auction if and only if he expects to make positive profit. In particular, suppose speculators arrive at the auction one after the other and observe only the number of bidders and the number of speculators who have entered. At this point, a speculator can pay an arbitrarily small cost and enter the auction, or walk away and obtain zero profit. If he enters, the speculator learns bidders' use values. Bidding is costless, so a speculator who enters the auction always bids.

after the auction is allowed.[17] We generically refer to a "player" when we want to indicate either a bidder or a speculator. Specifically, player S indicates a speculator and player i indicates bidder B_i, $i = 1, ..., n$.

We make the following assumption on valuations, which is standard in the literature on demand reduction (e.g., Wilson, 1979).

Assumption 1. *Use values are common knowledge among bidders and speculators, but the seller does not know bidders' use values.*

This assumption implies that players know the ex-post efficient allocation of the units on sale before the auction. Therefore, in our model resale is not caused by uncertainty in valuations, or by a change in the order of bidders' valuations after the auction (as in Haile, 2000, 2003).

In Section 5.3, in order to investigate the strategies that the seller can adopt to increase his revenue, we analyze how the seller's behavior depends on whether he knows if bidders' valuations are relatively similar to each other or different from each other,[18] and on whether he can prevent resale or not. We also analyze whether the seller can benefit from being able to distinguish bidders from speculators and to prevent the participation of speculators in the auction.

Resale Market

When resale is allowed, players always trade in the resale market if there are gains from trade obtainable. For simplicity, we assume that a unit can be traded only once in the resale market and that, if two players start bargaining on the terms of trade, they cannot then trade with any other player. (In other words, we assume that two players have to commit to trade with each other before bargaining on the terms of trade.) So when players bargain to trade a unit in the resale market, the outside option of the player who has won the unit in the auction is equal to his use value, while the outside option of the player who is trying to acquire the unit is

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It follows that a speculator enters the auction if and only if he has some positive probability of winning (before knowing bidders' use values). From the analysis, it will become clear that, if the number of speculators who have entered the auction is greater than $(k - n)$, no speculator can possibly obtain positive profit (see footnote 30). So once the number of speculators is equal to $(k - n)$, no other speculator pays the entry cost. While if there are no more than $(k - n)$ speculators, speculators win the auction and obtain positive profit for some bidders' use values (Proposition 1). Therefore, exactly $(k - n)$ speculators pay the entry cost and participate in the auction.

[17] For our analysis, the relevant difference between speculators and low-value bidders is that speculators can *only* obtain a positive profit by reselling, while low-value bidders can also obtain a positive profit by keeping a unit, even if they prefer to resell to a high-value bidder. Therefore, low-value bidders participate in the auction even if resale is not allowed, but speculators do not. See also footnote 33.

[18] Even if the seller does not know the exact bidders' valuations, he may have an estimate of bidders' *relative* valuations. For example, the seller may know whether the valuation of the highest bidder is more or less than k times the valuation of the second-highest bidder, even if he does not know the exact valuation of any of these bidders.

normalized to zero.[19] It follows that the gains from trade between two players are equal to the difference between their use values and, if any player apart from bidder B_1 wins a unit in the auction, to maximize the gains from trade he always resells to bidder B_1, the player with the highest use value.

We make the following assumption on the sharing of the gains from trade.

Assumption 2. *When two players trade a unit in the resale market, they equally share the gains from trade.*

Therefore, the outcome of bargaining between two players in the resale market is given by the Nash bargaining solution, where the disagreement point is represented by players' outside options.[20] And the resale price at which two players trade is "half way" between the two players' use values.

This assumption is made for simplicity, but all our qualitative results hold for any given sharing of the gains from trade in the resale market (in which the auction winner obtains some of the surplus). Indeed, the highest-value bidder may strictly prefer to reduce demand and allow other players to win the auction even if the auction winner makes a take-it-or-leave-it offer in the resale market, and hence obtains all gains from trade. This is because, even if the highest-value bidder can obtain no surplus in the resale market, reducing demand still allows her to pay a lower price for the units she wins in the auction.[21]

"Willingness to Pay"

When resale is allowed, a player's "willingness to pay" for a unit in the auction — which we define as the highest auction price that a player is happy to pay for a unit — is represented by the price at which he can buy or sell a unit in the resale market (e.g., Milgrom, 1987).

If a speculator wins a unit in the auction, he resells to bidder B_1 at price $\frac{1}{2}v_1$; and if bidder B_i, $i \neq 1$, wins a unit in the auction, he resells to bidder B_1 at price $\frac{1}{2}(v_1 + v_i)$. Therefore, during the auction, the highest price a speculator is happy to pay for one unit is $\frac{1}{2}v_1$ — i.e., the

[19] We choose to model the bargaining procedure in the resale market in the simplest possible way. But our results are robust to many alternative assumptions about bargaining. For example, we can assume that, while bargaining with one bidder in the resale market, the auction winner can still threaten to trade with a different bidder. In this case, the outside option of the auction winner is higher than his use value and he obtains a larger share of the gains from trade in the resale market. So players are willing to pay a higher price to win a unit in the auction. This alternative assumption changes the specific equilibrium conditions we derive in Sections 4 and 5, but not their qualitative interpretation, and it actually reinforces our results on the effects of allowing resale and attracting speculators on bidders' incentive to reduce demand and on the seller's revenue (because it makes demand reduction even more attractive for bidders when resale is allowed).

[20] This can be interpreted as the limit, as the length of the bargaining periods goes to zero, of a strategic model of alternating offers where players face a small exogenous risk of breakdown of negotiations, that induces them to take their outside options (Binmore et al., 1986; Sutton, 1986).

[21] And giving different bargaining powers to different players (e.g., by assuming that speculators can obtain a smaller share of the gains from trade than low-value bidders when trading with bidder B_1, or vice versa) also does not affect any of our qualitative results.

resale price he can obtain in the aftermarket — and the highest price bidder B_i, $i \neq 1$, is happy to pay for one unit is $\frac{1}{2}(v_1 + v_i)$ — i.e., the resale price she can obtain in the aftermarket. And since bidder B_1 can buy a unit in the resale market at a price no higher than $\frac{1}{2}(v_1 + v_2)$, this is the highest price she is happy to pay in the auction.

Hence, taking into account the resale market, bidder B_1 is willing to pay a lower price in the auction because of the possibility of purchasing the units she loses from the auction winners in the aftermarket, while all other bidders and speculators are willing to pay a higher price in the auction because of the possibility of reselling in the aftermarket to bidder B_1.[22] (But notice that a player's bid in the auction does not affect the outcome of the resale market.)

When resale does not take place, the profit of a player who wins the auction is given by the difference between the use value of the unit(s) he acquires and the auction price he pays. When resale takes place, the profit of a player who wins the auction and resells the unit(s) is given by the difference between the price he receives in the resale market and the auction price he pays; while the profit of a player who buys in the resale market is given by the difference between the use value of the unit(s) he acquires and the price he pays in the resale market. The profit of a player who loses the auction is normalized to zero.

Bidding Strategies

In the auction, a strategy for player i is a k-element vector:

$$\mathbf{b}_i = \left(b_i^1;\ b_i^2; ...;\ b_i^k\right), \quad i = 1, ..., n, S,$$

where b_i^j is player i's bid for the j^{th} unit. Bids must be such that $b_i^j \geq b_i^{j+1}$ (i.e., a player's demand must be non-increasing in price). There is *demand reduction* if a player's bid is lower than the highest price he is happy to pay for a unit. We assume players do not play weakly dominated strategies. As we will show, this implies that b_i^j cannot be higher than the highest price player i is happy to pay for the j^{th} unit, and that players do not reduce demand for the first unit.

We say that an equilibrium is Pareto dominated by another equilibrium from the players' point of view if in the second equilibrium at least one player is strictly better off and no player is worse off than in the first equilibrium. We make the following assumption on players' equilibrium strategies.

Assumption 3. *Players do not play an equilibrium that is Pareto dominated, from the players' point of view, by another equilibrium.*

[22] In the terminology of Haile (2003), bidder B_1 is willing to pay a lower price in the auction because of the "resale buyer effect" and the other players are willing to pay a higher price in the auction because of the "resale seller effect."

This assumption allows us to select among multiple equilibria. (In our game, Assumption 3 is consistent with the "coalition proofness" concept of Bernheim, Peleg and Whinston, 1987.)

We also make the following assumption to simplify the analysis.

Assumption 4. *If two players submit the same bid for a unit, the unit is assigned to the player with the highest use value.*

This assumption allows us to simplify the description of equilibrium strategies but our results do not hinge on it.

3. Definition of Equilibria

It is well known that, in a uniform-price auction, it is a weakly dominant strategy for a player to bid his willingness to pay for the first unit (see, e.g., Milgrom, 2004).[23] Moreover, bidding more than your willingness to pay for any unit is also a weakly dominated strategy.[24] But players may find it profitable to reduce demand and bid less than their willingness to pay for some units other than the first one, in order to pay a lower price for the units they win and so obtain a higher profit (Wilson, 1979; Ausubel and Cramton, 1998). The logic is the same as the standard textbook logic for a monopsonist withholding demand: buying an additional unit increases the price paid for the first, inframarginal, units.

We consider two types of equilibria (in undominated strategies) of the auction. In one type of equilibria, which we call Demand Reduction equilibria, speculators win if they participate; in the other type of equilibria, which we call Positive Price equilibria, all speculators and possibly also some of the bidders lose. Notice that, in order for speculators to win the auction, it is necessary that bidders reduce demand, because bidders have higher willingness to pay for all units than speculators and speculators do not bid more than their willingness to pay.

Definition 1. *A (zero-price) Demand Reduction (DR) equilibrium is a Nash equilibrium of the auction in which each player who participates in the auction wins at least one of the units on sale and the auction price is zero.*

Of course, there can be other equilibria in which each player wins at least one of the units on sale but the auction price is strictly positive. However, any such equilibrium with a positive auction price is Pareto dominated for players by the zero-price DR equilibrium in which each

[23] A player's first-unit bid affects the auction price only when it is the $(k+1)^{\text{th}}$-highest bid, in which case the player wins no unit and the price is irrelevant to him. Therefore, exactly as in a single-unit second-price auction, the first-unit bid is chosen to allow the player to win whenever it is profitable for him to do so — i.e., when his willingness to pay is no lower than the auction price.

[24] Excluding weakly dominated strategies allows us to exclude, in our model, demand reduction equilibria in which a speculator bids for the first unit a price that he does not want to pay (and that even a bidder does not want to pay), expecting not to pay for it (see Levin, 2005).

player wins exactly the same number of units as in the first equilibrium (because the final allocation is the same in both equilibria but the auction price that players pay is lower in the zero-price DR equilibrium). And whenever the auction has an equilibrium in which each player wins at least one unit and the auction price is positive, it also has a zero-price DR equilibrium.[25] So it is natural to expect that players will never choose an equilibrium in which they all win but the auction price is positive (and Assumption 3 excludes these equilibria).[26]

Definition 2. *A Positive Price (PP) equilibrium is a Nash equilibrium of the auction in which the auction price is strictly positive and:* (i) *if speculators participate in the auction, no speculator wins any unit;* (ii) *if speculators do not participate in the auction, at least one bidder does not win any unit.*

Although in a PP equilibrium some players do not win any unit and the auction price is different from zero, bidders do not necessarily bid the highest price they are willing to pay for all units — i.e., there may still be demand reduction. There are many possible types of PP equilibria in which speculators lose when they participate in the auction. For example, bidders may outbid speculators and share the units on sale (and there are many different ways in which they can share the units on sale), so that each bidder wins at least one unit. Or some higher-value bidders may outbid all other players (including lower-value bidders) and win all units. Similarly, when speculators do not participate in the auction, there are many possible types of PP equilibria in which some of the bidders lose.

We choose to distinguish only between Demand Reduction and Positive Price equilibria because we are interested in analyzing under which conditions bidders may reduce demand and allow speculators to win. This is also the only case in which the auction price can be zero when speculators participate in the auction and, as it will become clear from the analysis, in this case the possibility of resale and the presence of speculators can reduce the seller's revenue.

4. Successful Speculators

When resale is allowed and speculators participate in the auction, in a zero-price DR equilibrium each player bids the highest price he is happy to pay for one unit and zero for all other units. Precisely, bidder B_1 bids:

$$\mathbf{b}_1 = \left(\tfrac{1}{2}\left(v_1 + v_2\right);\underbrace{0; ...;\ 0}_{k-1}\right),$$

[25] However, as we will show, the converse is not true — i.e., for some bidders' use values, a zero-price DR equilibrium is the only equilibrium in which each player wins at least one unit.

[26] In other words, we assume that if players are able to tacitly coordinate to reduce demand, they do so on the lowest possible price.

bidder B_i bids:

$$\mathbf{b}_i = (\tfrac{1}{2}(v_1 + v_i);\underbrace{0;...;\ 0}_{k-1}), \quad i = 2, ..., n,$$

and each speculator bids:

$$\mathbf{b}_S = (\tfrac{1}{2}v_1;\underbrace{0;...;\ 0}_{k-1}).$$

So each player wins exactly one unit and the seller's revenue is zero. All bidders, apart from B_1, and all speculators resell the units they buy in the auction to bidder B_1 in the aftermarket.

We investigate under which condition there is a DR equilibrium in which speculators win the auction.[27] Define the following $(n-1)$ conditions "Dispersed Top Values" (DTV):

$$\begin{cases} v_1 + (k+2-i)\,v_i > (k+1-i)\,v_2, & i = 3, \ldots, n, \\ v_1 > (k-n)\,v_2. \end{cases} \tag{DTV}$$

Lemma 1. *When resale is allowed and speculators participate, the auction has a (zero-price) DR equilibrium if and only if conditions DTV are satisfied. Moreover, if conditions DTV are satisfied, the (zero-price) DR equilibrium is the unique Pareto dominant equilibrium for players. If instead one or more of conditions DTV is not satisfied, the auction only has PP equilibria.*

As shown in the proof of Lemma 1, if conditions DTV are satisfied, no players wants to deviate from a DR equilibrium. In other words, when his competitors reduce demand to one unit, each player prefers to win a single unit in the auction at price zero, rather than try to obtain more units by outbidding his competitors. (And deviating by winning less than one unit is clearly not profitable.) We now provide intuition for why this is the case.

Bidder B_1 and the speculators have no incentive to deviate from demand reduction. To see this notice that, in order to win more than one unit, a speculator has at least to outbid another speculator. But this would raise the auction price to at least $\frac{1}{2}v_1$, a price at which the speculator can obtain no profit. Similarly, if bidder B_1 wins more than one unit in the auction, she only increases the auction price that she pays for the first unit (since after reducing demand to one unit, she still buys all other units in the resale market, at the same prices she would have to pay to win them in the auction).

Bidder B_2 is the one who has the highest incentive to deviate from demand reduction, because she gains the most from outbidding lower-value players and winning more units to resell in the aftermarket. So if bidder B_2 prefers not to deviate from demand reduction, all other lower-value

[27] If the auction has a positive reserve price, it is less profitable for players to reduce demand, because they have at least to pay the reserve price to win a unit. In this case, an equilibrium in which all players reduce demand requires more restrictive conditions than the ones we derive for a (zero-price) DR equilibrium. However, our qualitative results on the effects of resale and speculators hold even with a positive reserve price.

bidders also prefer not to deviate. And bidder B_2 does not deviate from the DR equilibrium if and only if conditions DTV hold.[28]

Conditions DTV require that the use values of the two highest-value bidders are sufficiently dissimilar. For example, all conditions DTV are satisfied if $v_1 > (k-2)\, v_2$. The reason is that, if bidder B_1's use value is sufficiently higher than bidder B_2's use value, it is too costly for bidder B_2 to outbid lower-value bidders and/or speculators. Precisely, if v_1 is high, lower-value bidders and speculators can resell at a high price to bidder B_1, and thus they bid a high price for at least one unit in the auction. And if v_2 is low, bidder B_2 has a low outside option in the resale market; hence, she can only resell at a relatively low price and she is not willing to pay a high price in the auction.

When conditions DTV are satisfied, there may also be PP equilibria. However, as shown in the proof of Lemma 1, when a bidder prefers not to deviate from a DR equilibrium, she also obtains a higher profit in the DR equilibrium than in any equilibrium with a positive auction price. Therefore, when conditions DTV are satisfied, the zero-price DR equilibrium is the Pareto-dominant equilibrium for players and, by Assumption 3, players select the zero-price DR equilibrium. When one or more of conditions DTV is not satisfied, at least one bidder (bidder B_2) prefers to outbid speculators (by bidding at least $\frac{1}{2}v_1$ for $(k-n+1)$ units). So speculators cannot win the auction and there are only PP equilibria with an auction price of at least $\frac{1}{2}v_1$.[29]

By Lemma 1, all bidders with positive use values may strictly prefer to let speculators win some of the units on sale in equilibrium. So we have the following result.

Proposition 1. *If and only if conditions DTV are satisfied, in equilibrium each speculator wins one of the units on sale and resells to bidder B_1 in the aftermarket.*

Therefore, speculators may successfully participate in a multi-unit uniform-price auction, and resell in the aftermarket to the highest-value bidder, thus obtaining a strictly positive profit.[30] But speculators are not always able to win. The reason is that the presence of speculators in the auction has two effects on bidders:

(i) *Competition Effect*: speculators increase competition with low-value bidders;

(ii) *Demand Reduction Effect*: resale and speculators affect bidders' incentive to reduce demand.

[28] For example, bidder B_2 prefers to win one unit at price 0 rather than overbid bidder B_3 (together with lower-value bidders and speculators) and win $(k-1)$ units to resell to bidder B_1 if and only if $\frac{1}{2}(v_1+v_2) > (k-1)\left[\frac{1}{2}(v_1+v_2) - \frac{1}{2}(v_1+v_3)\right] \Leftrightarrow (k-1)\, v_3 + v_1 > (k-2)\, v_2$. Similarly, bidder B_2 prefers to win one unit at price 0 rather than overbid speculators and win $(k-n+1)$ units if and only if $\frac{1}{2}(v_1+v_2) > (k-n+1)\left[\frac{1}{2}(v_1+v_2) - \frac{1}{2}v_1\right] \Leftrightarrow v_1 > (k-n)\, v_2$.

[29] An analysis of the different types of PP equilibria is contained in the working paper version of this article.

[30] Clearly, if more than $(k-n)$ speculators participate in the auction, competition among speculators drives their profit to zero. This is because each speculator bids $\frac{1}{2}v_1$ for at least one unit (and bidders bid even higher prices for at least one unit) and, therefore, the auction price is no lower than $\frac{1}{2}v_1$, which is the highest profit a speculator can obtain by reselling a unit in the aftermarket.

The demand reduction effect always induces bidder B_1 to bid less aggressively and accommodate speculators. The reason is that bidder B_1 can always buy from speculators in the resale market, and hence obtain the units she loses in the auction. Therefore, bidding aggressively in the auction has the only effect of increasing the auction price that bidder B_1 pays for the units she wins in the auction. Moreover, the demand reduction effect may also induce other bidders to bid less aggressively, in order to pay a lower price in the auction (even at the cost of winning less units).[31]

On the other hand, the competition effect may induce bidders different from B_1 to bid more aggressively in order to beat speculators. The reason is that, during the auction, these bidders are directly competing with speculators for a chance to resell to bidder B_1 in the aftermarket. And if these bidders lose a unit in the auction, they have no chance of buying it in the aftermarket. Speculators win the auction if the demand reduction effect is stronger than the competition effect.

In other words, even when resale is allowed, it is the possibility that bidders reduce demand that really attracts speculators to the auction. But speculators win the auction only if bidders actually prefer to reduce demand rather than outbid them.[32]

5. Seller's Revenue

In this section, we analyze how the presence of speculators in an auction affects the seller's revenue. Notice that the two effects we have described of speculators on bidders may affect the seller's revenue in opposite directions: the competition effect tends to increase the seller's revenue, while the demand reduction effect may reduce the seller's revenue.

There are two different reasons why speculators may not participate in an auction: (*i*) resale is not allowed, and so speculators have no incentive to participate in the auction, (*ii*) the seller prevents speculators from participating, even if resale is allowed and so speculators would like to participate in the auction. Therefore, to analyze the effect of the presence of speculators in each of these cases, we consider the seller's revenue in three different scenarios:

(1) Resale is allowed and speculators are allowed to participate in the auction.

(2) Resale is not allowed and hence speculators do not participate in the auction.

(3) Resale is allowed but speculators are not allowed to participate in the auction.

The first scenario (in which speculators participate in the auction) was analyzed in Section

[31] Bidders have an incentive to reduce demand even without speculators. But allowing resale and attracting speculators in the auction can increase this incentive. See Section 5.

[32] The reason why in single-unit auctions it is unclear why speculators may win is that there is no scope for profitable demand reduction by a bidder with a positive use value: if the bidder loses the single unit on sale, she obtains no profit in the auction.

4. In the next two sections, we analyze the seller's revenue in the other two scenarios (in which, for different reasons, speculators do not participate in the auction) and compare it with the seller's revenue in the first scenario.

5.1. Should Resale Be Allowed to Attract Speculators?

Assume resale is not allowed, so that speculators do not participate in the auction. In this case, we have a standard auction with a fixed number of bidders,[33] and the highest price a bidder is happy to pay in the auction is equal to her use value.

Even without speculators, there may be a zero-price DR equilibrium in which bidder B_1 bids her valuation for $(k-n+1)$ units and zero for all other units — i.e.:

$$\mathbf{b}_1 = (\underbrace{v_1; ...; \ v_1}_{k-n+1}; \underbrace{0; ...; \ 0}_{n-1}),$$

and each other bidder bids her valuation for one unit and zero for all other units — i.e.:

$$\mathbf{b}_i = (v_i; \underbrace{0; ...; \ 0}_{k-1}), \quad i = 2, ..., n.$$

In this equilibrium, bidder B_1 wins $(k-n+1)$ units, all other bidders win one unit each, and the auction price is zero.

There are many other possible DR equilibria in which each bidder wins at least one unit and the auction price is zero. For example, there may be DR equilibria in which a bidder wins any number of units between 1 and $(k-n+1)$. We focus on the DR equilibrium in which bidder B_1 wins $(k-n+1)$ units because the conditions that have to be satisfied in order for this to be an equilibrium are less restrictive than the conditions for any other DR equilibrium. The reason is that bidder B_1 has the strongest incentive to deviate from a DR equilibrium and, in the equilibrium described, she wins the highest number of units consistent with each bidder winning at least one unit.[34]

We investigate under which condition there is a DR equilibrium, and therefore the seller can obtain no revenue from the auction. Define the following $(2n-3)$ conditions "Clustered Values" (CV):

$$\begin{cases} (k+2-i)\, v_i > (n+1-i)\, v_1, & i = 2, \ldots, n, \\ (n+2-i)\, v_i > (n+1-i)\, v_2, & i = 3, \ldots, n. \end{cases} \tag{CV}$$

[33] In contrast to speculators, low-value bidders have an incentive to participate in the auction even when resale is not allowed, because they can win and obtain their use value if their higher-value competitors reduce demand. And bidders participate even if there is a small entry cost that they have to pay to enter the auction and learn the other bidders' use values. The reason is that, ex-ante, all bidders have a positive probability of winning if their opponents' reduce demand, and hence they expect to earn positive profit. (Once a bidder enters the auction, we assume she always bids because bidding is costless.)

[34] So the conditions for this DR equilibrium are the "minimum" conditions that have to be satisfied for having a DR equilibrium. More details can be found in the proof of Lemma 2.

Lemma 2. *When resale is not allowed, the auction has a (zero-price) DR equilibrium if and only if conditions CV are satisfied. Moreover, if conditions CV are satisfied, the (zero-price) DR equilibrium Pareto dominates, from the bidders' point of view, any other equilibrium with a positive auction price. If instead one or more of conditions CV is not satisfied, the auction only has PP equilibria.*

Each bidder may have an incentive to deviate from a DR equilibrium, because she may prefer to outbid lower-value bidders and win more units. As shown in the proof of Lemma 2, the first $(n-1)$ conditions CV imply that bidder B_1 does not want to deviate from the DR equilibrium described (in which she wins $(k-n+1)$ units at price 0), while the last $(n-2)$ conditions CV imply that bidder B_2 does not want the deviate from the DR equilibrium described.[35] And bidder B_2 has an higher incentive to deviate from demand reduction than all lower-value bidders (because she has a higher use value and hence obtains a higher profit from winning more units). So if it is not profitable for bidder B_2 to deviate from demand reduction, it is also not profitable to deviate for lower-value bidders. Summing up, if conditions CV are satisfied, each bidder prefers to reduce demand and maintain the auction price at zero, rather than obtain more units by outbidding her competitors.

Conditions CV require that the use values of the two highest-value bidders are not too much higher than all other bidders' use values. For example, conditions CV are all satisfied if $kv_n > (n-1)\,v_1$ and $(n-1)\,v_n > (n-2)\,v_2$.[36] The intuition is that, if the two highest use values are sufficiently close to the other lower use values, lower-value bidders can resell at a relatively high price in the aftermarket, and so they are willing to pay a relatively high price in the auction. Therefore, it is too costly for bidders B_1 and B_2 to outbid their competitors and they prefer to keep the auction price low by reducing demand.

Moreover, as shown in the proof of Lemma 2, the fact that bidders do not want to deviate from a DR equilibrium also implies that they obtain a higher profit in the DR equilibrium than in any other equilibrium with a positive auction price. Therefore, when conditions CV are satisfied, equilibria with a positive auction price are Pareto dominated by the DR equilibrium and, by Assumption 3, they are never chosen by bidders.

Now consider the seller's revenue. Allowing resale increases the maximum price that low-value bidders are willing to pay in the auction and, by attracting speculators, increases the number of competitors. These effects tend to increase the seller's revenue. However, allowing

[35] For example, bidder B_1 prefers not to deviate from the DR equilibrium by outbidding bidder B_2 (and all lower-value bidders) and winning all units if and only if $(k-n+1)\,v_1 > k\,(v_1 - v_2) \Leftrightarrow kv_2 > (n-1)\,v_1$; and bidder B_2 prefers not to deviate from the DR equilibrium by outbidding bidder B_3 (and all lower-value bidders) and winning $(n-1)$ units if and only if $v_2 > (n-1)\,(v_2 - v_3) \Leftrightarrow (n-1)\,v_3 > (n-2)\,v_2$.

[36] If the first inequality holds, the first $(n-1)$ conditions CV are satisfied; if the second inequality holds, the last $(n-2)$ conditions CV are satisfied.

resale also has contrasting effects on bidders' incentive to reduce demand. On the one hand, compared to a DR equilibrium without resale and speculators, in a DR equilibrium with speculators bidders can win less units in the auction (because speculators have to win too). This makes demand reduction less attractive for some bidders. But, on the other hand, when resale is allowed demand reduction is more attractive for bidder B_1, because she can buy in the resale market the units she does not win in the auction; and deviating from demand reduction may also be less attractive for all other bidders, because they have to pay a higher auction price to outbid their competitors (whose bids for the first unit are higher due to the option to resell). So allowing resale may facilitate demand reduction and reduce the seller's revenue.

Specifically, allowing resale and attracting speculators reduce the seller's revenue if this induces players to choose a DR equilibrium (when there is no DR equilibrium without resale). By contrast, the presence of speculators increases the seller's revenue if it eliminates a DR equilibrium or if it induces bidders to bid more aggressively in a PP equilibrium.

Proposition 2. *Allowing resale and hence attracting speculators to the auction:* (i) *reduces the seller's revenue if conditions DTV are satisfied and one or more of conditions CV is not satisfied;* (ii) *increases the seller's revenue if conditions CV are satisfied and one or more of conditions DTV is not satisfied;* (iii) *increases the seller's revenue only if one or more of conditions DTV is not satisfied.*

Proposition 2 shows that the effect of resale and speculators on the seller's revenue depends on bidders' relative valuations. If bidders are (very) asymmetric — i.e., if bidders' use values are sufficiently dispersed — conditions DTV are satisfied and conditions CV are not satisfied. Therefore, when resale is allowed all bidders bid for less units than they actually want, in order to maintain the auction price low, even if their strategy accommodates speculators. This induces a DR equilibrium and reduces the seller's revenue. In this case, the demand reduction effect prevails when resale is allowed.

On the other hand, if bidders are (more) symmetric — i.e., if the use values of the two highest-value bidders are sufficiently similar — one or more of conditions DTV is not satisfied and, when resale is allowed, some bidders bid more aggressively than when resale is not allowed in order to outbid speculators. When conditions CV are satisfied — i.e., when bidders' use values are sufficiently clustered — this eliminates a DR equilibrium (compared to an auction without resale and speculators) and increases the seller's revenue. In this case, the competition effect prevails when resale is allowed.[37]

[37] There are two other possible cases. Firstly, when both conditions DTV and conditions CV are satisfied, allowing resale and attracting speculators does not affect the seller's revenue, because the auction has a DR equilibrium both with and without resale. Secondly, when one or more of conditions DTV is not satisfied and

Moreover, by Proposition 2, speculators can increase the seller's revenue only if the auction has no (zero-price) DR equilibrium when resale is allowed, and hence speculators are eventually outbid by bidders with positive use values.

Corollary 1. *Allowing resale and attracting speculators can increase the seller's revenue only if speculators do not win any unit in the auction.*

So the presence of speculators in the auction is not *per se* good news for the seller, because speculators may be accommodated by bidders in order to keep the auction price low. It is the fact that speculators participate in the auction but eventually lose that indicates an actual increase in competition, and therefore a higher seller's revenue.[38]

5.2. Should Speculators Be Allowed to Participate?

Assume now that resale is allowed but that the seller prevents speculators from participating in the auction. In this case, the highest price a bidder is happy to pay in the auction is equal to the price at which she can trade in the resale market.

Consider a possible DR equilibrium in which bidder B_1 bids the highest price she is happy to pay for $(k-n+1)$ units and zero for all other units — i.e.:

$$\mathbf{b}_1 = (\underbrace{\tfrac{1}{2}(v_1+v_2);...;\ \tfrac{1}{2}(v_1+v_2)}_{k-n+1};\underbrace{0;...;\ 0}_{n-1}),$$

and each other bidder bids the highest price she is happy to pay for 1 unit and zero for all other units — i.e.:

$$\mathbf{b}_i = (\tfrac{1}{2}(v_1+v_i);\underbrace{0;...;\ 0}_{k-1}), \quad i = 2, ..., n.$$

In this equilibrium, bidder B_1 wins $(k-n+1)$ units, all other bidders win one unit each, and the seller's revenue is zero. All bidders apart from B_1 resell the units they win in the aftermarket.

There are many other possible DR equilibria in which each bidder wins at least one unit and the auction price is zero. We focus on the equilibrium in which bidder B_1 wins $(k-n+1)$ units because the conditions that have to be satisfied in order for this to be an equilibrium can be

one or more of conditions CV is not satisfied either (i.e., when there is no DR equilibrium regardless of whether resale is allowed or not), the effect of allowing resale on the seller's revenue depends on whether resale induces more bidders to reduce demand or to compete more aggressively in a PP equilibrium.

[38] In our model, speculators never win the auction at a positive price. Of course, in the real world speculators may win the auction at a positive price in the presence of uncertainty because, for example, in a simultaneous ascending auction bidders may reduce demand only after raising the auction price some distance to test the credibility of speculators, or since they do not know at the beginning of the auction that their opponents are only trying to win to resell. In this case, the seller's revenue in principle may be higher if resale is allowed even when bidders allow speculators to win. But our point is that, in general, observing bidders accommodate speculators rather than compete aggressively to outbid them should not be considered good news for the seller.

readily compared to conditions DTV and, therefore, this DR equilibrium is directly comparable to the DR equilibrium when speculators participate in the auction.

Moreover, the conditions that have to be satisfied for the DR equilibrium described are sufficient but not necessary for a DR equilibrium, because these conditions are *more* restrictive than the conditions that have to be satisfied for any other DR equilibrium. The reason is that, in the equilibrium described, bidder B_1 wins the highest number of units consistent with each bidder winning at least one unit. But, when resale is allowed, bidder B_1 has no incentive to deviate from demand reduction, and other bidders would have a weaker incentive to deviate from a DR equilibrium in which they win more units.

We investigate when the seller can obtain no revenue from the auction. Define the following $(n-2)$ conditions "Dispersed Top Values or Clustered Bottom Values" (DCV):

$$v_1 + (n+2-i)\, v_i > (n+1-i)\, v_2, \quad i = 3, \dots, n. \qquad \text{(DCV)}$$

Lemma 3. *When resale is allowed but speculators are not allowed to participate, the auction has a (zero-price) DR equilibrium if conditions DCV are satisfied. Moreover, if conditions DCV are satisfied, the (zero-price) DR equilibrium Pareto dominates, from the bidders' point of view, any other equilibrium with a positive auction price.*

As shown in the proof of Lemma 3, if conditions DCV are satisfied, no players wants to deviate from the DR equilibrium described. Exactly as with speculators, bidder B_1 has no incentive to deviate from demand reduction, because trying to win more units in the auction would only increase the auction price she has to pay. Bidder B_2 is the one who has the strongest incentive to deviate from demand reduction, because she gains the most from outbidding her competitors and reselling in the aftermarket. So if bidder B_2 does not deviate from the DR equilibrium, the other lower-value bidders do not deviate either.

Conditions DCV require that bidder B_1's use value is sufficiently higher than bidder B_2's use value or, alternatively, that the use values of lower-value bidders are sufficiently high. For example, conditions DCV are satisfied if $v_1+2v_n > (n-2)\, v_2$. The intuition is that, when bidder B_2's use value is closer to lower-value bidders' use values than to bidder B_1's use value, it is too costly for bidder B_2 to outbid lower-value bidders (whose bids for the first unit are increasing in bidder B_1's use value and their own use values). So bidder B_2 prefers to keep the auction price low by reducing demand, rather than win more units by outbidding her competitors. Moreover, by Assumption 3, if conditions DCV are satisfied players do not play an equilibrium with a positive auction price, because such an equilibrium is Pareto dominated by the DR equilibrium.

Now consider the seller's revenue. If there is no DR equilibrium when speculators participate in the auction, the presence of speculators only increases the number of competitors; hence it cannot reduce the seller's revenue. So speculators can reduce the seller's revenue if and only if they induce bidders to choose a DR equilibrium — i.e., if and only if demand reduction is not an equilibrium without speculators, but it is an equilibrium if speculators participate in the auction. But, as the next proposition shows, this never happens.

Proposition 3. *If the seller cannot prevent resale, the presence of speculators in the auction (weakly) increases the seller's revenue.*

As shown in the proof of Proposition 3, when resale is allowed there is a DR equilibrium when speculators participate in the auction only if there is a DR equilibrium also when speculators do not participate in the auction.[39] Therefore, the presence of speculators never reduces the seller's revenue. And if conditions DCV are satisfied but one or more of conditions DTV is not satisfied, the presence of speculators strictly increases the seller's revenue because it eliminates a DR equilibrium.

The intuition for this result is that, if resale is possible, bidders have a stronger incentive to reduce demand when speculators do not participate in the auction. Firstly, because any player who wins a unit in the auction wants to resell it to bidder B_1, bidder B_1 has an incentive to reduce demand even when speculators do not participate. Secondly, lower-value bidders have a stronger incentive to reduce demand when speculators do not participate, because in a DR equilibrium without speculators they can win more units than in a DR equilibrium with speculators. And even if they win the same number of units, they pay a (weakly) higher price when speculators participate. Hence, by contrast to the case in which the seller can prevent resale, if resale is always possible demand reduction cannot possibly be easier when speculators participate in the auction. And so the only effect of the presence of speculators is to increase competition in the auction.

5.3. Seller's Strategy

Summing up the results of the last two sections, if the seller cannot prevent resale, he should always welcome speculators in the auction. But if the seller can credibly forbid resale and knows bidders' relative valuations, he should forbid resale if bidders are asymmetric (i.e., if their use values are dispersed), while he should allow resale and attract speculators to the auction if

[39] We show this by proving that conditions DTV (the necessary and sufficient conditions for a DR equilibrium when speculators participate in the auction) imply conditions DCV (the sufficient conditions for a DR equilibrium when resale is allowed but speculators do not participate in the auction). Therefore, if resale is allowed, whenever demand reduction is an equilibrium with speculators, it is also an equilibrium without speculators.

bidders are symmetric (i.e., if their use values are clustered).[40,41]

Therefore, knowing bidders' relative valuations and being able to prevent resale can help the seller to increase his revenue. By contrast, being able to distinguish speculators from bidders is not useful for the seller, because excluding speculators from the auction can increase the seller's revenue only if it is achieved by forbidding resale. And if the seller wants to induce enough speculators to participate, he can simply allow resale and not restrict entry in the auction.

When resale is allowed, the final allocation of the units is always efficient, even if bidders reduce demand during the auction (and so the allocation at the end of the auction is not efficient). But if the seller prevents resale to increase his revenue, the highest-value bidder may still prefer to reduce demand and let other bidders win, in which case the final allocation of the units is inefficient.[42] So the seller may face a trade-off between increasing his revenue and maximizing efficiency.

The seller may also want to impose a higher reserve price in order to make demand reduction less attractive for bidders and increase his revenue. While it is typically argued that a reserve price reduces efficiency because it may lead to no sale, in our model a higher reserve price can increase the efficiency of the initial allocation achieved by the auction, because it can eliminate a DR equilibrium and crowd out speculators if resale is allowed. Moreover, if resale is not allowed, a reserve price that eliminates a DR equilibrium also increases the efficiency of the final allocation of the units on sale.

6. Resale without Speculators

What is the effect on the seller's revenue of allowing resale, when the number of competitors in the auction is fixed and there is no speculator who is willing to participate in the auction? To answer this question, we compare an auction with only n bidders and no speculator in which resale is not allowed, to an auction with the same number of bidders and no speculator in which resale is allowed.[43] (So we basically compare the second and third scenarios of Section 5.)

To simplify the analysis, in this section we assume $n = k$, but our qualitative results also hold for $n \neq k$. Define the following $(n-1)$ conditions "Clustered Values 1" (CV1), which are

[40] The seller may know that there are different "types" of bidders in the auction, and he may have a good estimate of how heterogeneous their valuations are. For example, in mobile-phone license auctions incumbents typically have substantially higher use values than new entrants, and the seller often knows how far apart their values are, even though he does not know the exact amount of any of their values.

[41] Of course, the uniform-price auction that we consider is not an optimal selling mechanism, neither with resale nor without resale.

[42] However, if resale is allowed and speculators win some of the units on sale, this may reduce efficiency (compared to an auction in which the units are directly assigned to bidder with the highest use value), because it can delay the productive use of the units by the bidder with the highest use value.

[43] Pagnozzi (2006) analyzes the combined effect that bundling the units on sale and allowing resale have on the seller's revenue in multi-unit auctions without speculators.

equivalent to conditions CV when $n = k$:

$$(n + 2 - i)\, v_i > (n + 1 - i)\, v_1, \quad i = 2, \ldots, n. \tag{CV1}$$

Lemma 4. *Assume there is no speculator. If resale is not allowed, the auction has a (zero-price) DR equilibrium if and only if conditions CV1 are satisfied. If resale is allowed, the auction has a (zero-price) DR equilibrium if and only if conditions DCV are satisfied. In both cases, when the auction has a (zero-price) DR equilibrium, this is also the unique Pareto dominant equilibrium for bidders.*

The effect on the seller's revenue of allowing resale depends on whether it induces bidders to reduce demand to one unit each (compared to an auction without resale) or it induces bidders to bid more aggressively when they do not reduce demand to one unit.

Proposition 4. *When there is no speculator who may participate in the auction, allowing resale: (i) reduces the seller's revenue if conditions DCV are satisfied and one or more of conditions CV1 is not satisfied; (ii) increases the seller's revenue if all conditions DCV are not satisfied.*

Notice that, without resale, there is no DR equilibrium if bidders are asymmetric — i.e., if bidder B_1's use value is much higher than the other bidders' use values — because in this case bidder B_1 prefers to win more units in the auction, even at the cost of paying a higher price (which depends on her competitors' use values). But allowing resale makes bidders' willingness to pay in the auction closer to each other, because bidder B_1 is willing to pay a price lower than her use value due to the option to buy in the resale market, while all other bidders are willing to pay a price higher than their use values due to the option to sell in the resale market. This makes demand reduction more attractive for bidders.[44]

Specifically, allowing resale makes it more likely that the auction has a DR equilibrium, because: (1) resale makes it more profitable for bidder B_1 to reduce demand (since she can buy in the resale market the units she loses in the auction when she reduces demand), (2) resale makes it less profitable for all other bidders to deviate from a DR equilibrium by outbidding their competitors (whose bids for the first unit are higher due to the option to resell),[45] and (3) when there is no speculator, resale has no countervailing effect on the number of competitors in the

[44] In the proof of Proposition 4, we show that conditions CV1 imply conditions DCV, but the converse is not true. Therefore, whenever demand reduction is an equilibrium without resale, it is also an equilibrium with resale.

[45] Consider, for example, bidder B_{n-1}. Without resale, she does not deviate from a DR equilibrium if and only if outbidding bidder B_n is not profitable — i.e., if and only if $v_{n-1} > 2\,(v_{n-1} - v_n)$. By contrast, with resale bidder B_{n-1} does not deviate from a DR equilibrium if and only if $\frac{1}{2}\,(v_1 + v_{n-1}) > 2\left[\frac{1}{2}\,(v_1 + v_{n-1}) - \frac{1}{2}\,(v_1 + v_n)\right]$, a condition that is always satisfied. (Moreover, with resale, no bidder ever wants to deviate from a DR equilibrium by outbidding the lowest-value bidder.)

auction (which could otherwise make demand reduction less profitable for bidders).[46] Therefore, when conditions DCV are satisfied but one or more of conditions CV1 is not satisfied, bidders choose a DR equilibrium only when resale is allowed. In this case allowing resale reduces the seller's revenue.

However, allowing resale also increases the highest prices that all bidders, apart from B_1, are willing to pay for the first unit in the auction. When bidders never choose a DR equilibrium — i.e., when there are only PP equilibria both when resale is allowed and when resale is not allowed — resale induces all bidders, apart from B_1, to bid more aggressively for at least one unit and may increase the seller's revenue (even though resale still makes it relatively more profitable for bidders to reduce demand). For example, allowing resale increases the seller's revenue when all conditions DCV are not satisfied, because then bidder B_2 has a much higher use value than lower-value bidders and prefers to outbid all her competitors rather than reduce demand to one unit, even if resale is allowed. And, as shown in the proof of Proposition 4, in this case the equilibrium auction price is always higher with resale.

Allowing resale ensures an efficient final allocation of the units on sale. By contrast, if resale is not allowed and bidders reduce demand, the final allocation of the units is inefficient. Therefore, as when there are speculators who may participate in the auction, in choosing whether to allow resale the seller may face a trade-off between increasing his revenue and maximizing efficiency.

7. Conclusions

Although speculators are attracted by the possibility of resale, in single-object auctions it is unclear why high-value bidders should let speculators win and then buy in the resale market, rather than simply outbid speculators during the auction.

We have made three main points. Firstly, we have shown that, in multi-object auctions, high-value bidders may *strictly* prefer to let speculators win, in order to keep the price low and acquire some of the objects on sale more cheaply in the auction. Arguably, this is what happened in the UK 3.4GHz license auction.

Secondly, it is not true that the only effect of allowing resale to attract speculators is to increase competition in the auction, and therefore that resale and speculators always increase the seller's revenue. We have shown that allowing resale, even though it attracts speculators, may induce an accommodating strategy by high-value bidders, and hence it may reduce the seller's revenue. In fact, when high-value bidders allow speculators to win the auction, they do so to avoid raising the auction price. So resale and speculators increase the seller's revenue

[46] The first two effects of resale are present both with and without speculators and always facilitate demand reduction; while the third effect of resale on the number of competitors in the auction is only present when there are speculators willing to participate in the auction.

only if their effect on competition is stronger than their effect on bidders' incentives to reduce demand.

Thirdly, when it does not attract speculators, it is even more likely that resale reduces the seller's revenue, because allowing resale increases the incentive for all bidders to reduce demand, and without speculators it has no countervailing effect on the number of competitors in the auction.

It is often argued that resale after an auction should never be forbidden because, by allowing bidders to exploit gains from trade in the aftermarket, resale ensures an efficient final allocation of the units on sale.[47] While our analysis does not dispute this claim, it shows that the possibility of resale, through its effect on bidding strategies during the auction, may yield a lower revenue for the seller, even when there are speculators who will participate in the auction when resale is allowed. Therefore, in a multi-object auction resale may be forbidden by a seller who favors revenue over efficiency.

[47] For example, the 2002 "Cave Report," which was commissioned by the UK Government to review its spectrum policies, recommends allowing trading of the spectrum licenses auctioned by the Government to increase efficiency. And since 2003, the US Federal Communications Commission allows leasing and trading of the spectrum licenses it sells by auctions.

A. Appendix

Proof of Lemma 1. We let π_i^* denote the profit that player i obtains in equilibrium. In a DR equilibrium, each player wins one unit in the auction at price zero. Therefore, each speculator obtains a profit equal to the price at which he resells in the aftermarket — i.e.:

$$\pi_S^* = \tfrac{1}{2}v_1,$$

bidder B_i obtains a profit equal to the price at which she resells in the aftermarket — i.e.:

$$\pi_i^* = \tfrac{1}{2}(v_1 + v_i), \quad i = 2, ..., n,$$

and bidder B_1 obtains a profit equal to the difference between her valuation for the k units, and the price she has to pay to acquire $(k-1)$ units in the aftermarket ($(n-1)$ units from other bidders and $(k-n)$ units from speculators) — i.e.:

$$\begin{aligned}\pi_1^* &= kv_1 - \tfrac{1}{2}\sum_{i=2}^{n}(v_1 + v_i) - (k-n)\tfrac{1}{2}v_1 \\ &= (k+1)\tfrac{1}{2}v_1 - \tfrac{1}{2}\sum_{i=2}^{n}v_i.\end{aligned}$$

First notice that no player has an incentive to deviate from a (zero-price) DR equilibrium by winning less units, because this does not affect the auction price, and hence can only reduce the player's profit. And speculators have no incentive to deviate from a DR equilibrium at all because, in order to win more than one unit, a speculator has to raise the auction price at least up to $\frac{1}{2}v_1$, in which case he obtains no profit.

Similarly, bidder B_1 has no incentive to deviate from a DR equilibrium because, after reducing demand, bidder B_1 still buys in the resale market all the units she does not win in the auction, at the same price she would have to pay to win them in the auction. So the only effect of winning more than one unit in the auction is to increase the auction price that bidder B_1 pays for the first unit she wins.

By contrast, other bidders may want to deviate from a DR equilibrium and outbid lower-value bidders and/or speculators. (It is clearly never profitable for a bidder to outbid the first-unit bid of a competitor with a higher value.) Consider bidder B_2. Clearly, if bidder B_2 overbids speculators, she overbids all of them. Bidder B_2 can outbid all speculators by bidding $\frac{1}{2}v_1$ for $(k-n+1)$ units.[48] In this case, she wins $(k-n+1)$ units in the auction that she can resell at price $\frac{1}{2}(v_1+v_2)$ to bidder B_1 in the aftermarket. This deviation is not profitable if and only if:

$$\pi_2^* = \tfrac{1}{2}(v_1 + v_2) > (k-n+1)\left[\tfrac{1}{2}(v_1+v_2) - \tfrac{1}{2}v_1\right] \quad \Leftrightarrow \quad v_1 > (k-n)v_2. \tag{A.1}$$

Bidder B_2 may also want to outbid lower-value bidders. Indeed, by bidding $\frac{1}{2}(v_1+v_j)$ for $(k-j+2)$ units and a lower price for all other units, she overbids bidders B_j,..., B_n (in addition to all speculators)[49] and wins $(k-j+2)$ units in the auction that she can resell to bidder B_1 in the aftermarket. This deviation is not profitable if and only if, for $j = 3, ..., n$:

$$\pi_2^* = \tfrac{1}{2}(v_1+v_2) > (k-j+2)\left[\tfrac{1}{2}(v_1+v_2) - \tfrac{1}{2}(v_1+v_j)\right]$$

[48] By Assumption 4, in order to beat a speculator, bidder B_2 only needs to bid exactly the same price that the speculator bids.

[49] By Assumption 4, in order to beat a lower-value bidder, bidder B_2 only needs to bid exactly the same price that the lower-value bidder bids.

$$\Leftrightarrow \quad (k-j+2)\, v_j + v_1 > (k-j+1)\, v_2. \tag{A.2}$$

Summing up conditions A.1 and A.2, bidder B_2 does not want to deviate from a DR equilibrium if and only if the following $(n-1)$ conditions are satisfied:

$$\begin{cases} v_1 + (k-1)\, v_3 > (k-2)\, v_2, \\ v_1 + (k-2)\, v_4 > (k-3)\, v_2, \\ \vdots \\ v_1 + (k-n+3)\, v_{n-1} > (k-n+2)\, v_2, \\ v_1 + (k-n+2)\, v_n > (k-n+1)\, v_2, \\ v_1 > (k-n)\, v_2. \end{cases} \tag{DTV}$$

If bidder B_2 does not want to deviate from a DR equilibrium, then no other lower-value bidder wants to deviate either. To see this, consider bidder B_i, $i \neq 1, 2$. Bidder B_i prefers to reduce demand and win one unit at price 0 rather than outbid bidder B_j, $j > i$, and win $(k-j+1)$ units if and only if:

$$\pi_i^* = \tfrac{1}{2}\,(v_1 + v_i) > (k-j+1)\left[\tfrac{1}{2}\,(v_1 + v_i) - \tfrac{1}{2}\,(v_1 + v_j)\right]$$

$$\Leftrightarrow \quad (k-j+1)\, v_j + v_1 > (k-j)\, v_i.$$

Clearly, this condition is implied by A.2, the condition for bidder B_2 not wanting to outbid bidder B_j. Similarly, the condition for bidder B_i not wanting to deviate from a DR equilibrium by outbidding speculators is implied by condition A.1.

In conclusion, if and only if conditions DTV are satisfied, all players prefer to reduce demand and win one unit at price zero when their opponents are reducing demand to one unit, rather then deviate and win more units. Hence, there is a zero-price DR equilibrium.

When conditions DTV are satisfied, there may be other equilibria with demand reduction in which each player wins one unit but the auction price is strictly positive.[50] However, these equilibria are Pareto dominated for all players by the zero-price DR equilibrium, because the final allocation is the same in all these equilibria and only the auction price that players pay is different. Moreover, it is straightforward to verify that equilibria with demand reduction and a strictly positive auction price require conditions that are more restrictive then conditions DTV. Therefore, whenever the auction has an equilibrium with demand reduction and a strictly positive price, it also has a zero-price DR equilibrium.

When conditions DTV are satisfied, there may also be PP equilibria. (An analysis of the various types of PP equilibria is contained in the working paper version of this article.) However, these equilibria are Pareto dominated for all players by the zero-price DR equilibrium. To see this, notice that in a PP equilibrium the auction price is at least $\frac{1}{2}v_1$ if only speculators lose, and is at least $\frac{1}{2}\,(v_1 + v_i)$ if bidder B_i loses. But, when conditions DTV are satisfied, each bidder prefers to win one unit at price zero, rather than win more units by outbidding speculators and paying a price equal to $\frac{1}{2}v_1$ or by outbidding a lower-value bidder and paying a price equal to her willingness to pay. Therefore, each bidder obtains a strictly higher profit in a DR equilibrium than in a PP equilibrium. And, clearly, also speculators are strictly better off in a DR equilibrium.

[50] For example, it may be an equilibrium for each player to bid the highest price he is happy to pay for one unit, and price p such that $0 < p < \frac{1}{2}v_1$ for all other units. In this case, each player wins one unit and the auction price is p.

When one or more of conditions DTV is not satisfied, at least one bidder wants to outbid all speculators, even if all other players reduce demand. So speculators cannot win the auction. And in order to outbid speculators, this bidder has to raise the auction price at least up to $\frac{1}{2}v_1$ (because each speculator bids $\frac{1}{2}v_1$ for the first unit). So the only possible equilibria of the auction are PP equilibria. ■

Proof of Proposition 1. The "if" part of the statement follows from Lemma 1. The "only if" part follows because, in order for all speculators to win in equilibrium, each bidder must prefer to reduce demand and bid less than $\frac{1}{2}v_1$ for all units apart from the first one, rather than overbid speculators in order to win more units. And, as shown in the proof of Lemma 1, if this is the case then the auction also has a (zero-price) DR equilibrium. ■

Proof of Lemma 2. In the DR equilibrium that we have defined, bidder B_1 wins $(k-n+1)$ units at price zero and obtains a profit equal to:

$$\pi_1^* = (k-n+1)\, v_1,$$

while each other bidder wins one unit at price zero and obtains a profit equal to her valuation — i.e.:

$$\pi_i^* = v_i, \quad i = 2, ..., n.$$

Bidders may prefer to deviate, outbid their competitors, and win more units. Consider bidder B_1 first. Bidder B_1 prefers to win $(k-n+1)$ units at price 0, rather than outbid bidder B_j, $j = 2, ..., n$ and win $(k-j+2)$ units by bidding v_j for $(k-j+2)$ units, if and only if:

$$\pi_1^* = (k-n+1)\, v_1 > (k-j+2)\,(v_1 - v_j) \quad \Leftrightarrow \quad (k-j+2)\, v_j > (n-j+1)\, v_1. \tag{A.3}$$

Now consider bidder B_2. It is never profitable for bidder B_2 to outbid bidder B_1. Moreover, bidder B_2 prefers to win one unit at price 0 rather than outbid bidder B_j, $j = 3, ..., n$, and win $(n-j+2)$ units if and only if:

$$\pi_2^* = v_2 > (n-j+2)\,(v_2 - v_j) \quad \Leftrightarrow \quad (n-j+2)\, v_j > (n-j+1)\, v_2. \tag{A.4}$$

Summing up conditions A.3 and A.4, bidders B_1 and B_2 do not want to deviate from the DR equilibrium if and only if the following $(2n-3)$ conditions are satisfied:

$$\begin{cases} kv_2 > (n-1)\, v_1, \\ (k-1)\, v_3 > (n-2)\, v_1, \\ \vdots \\ (k-n+2)\, v_n > v_1, \\ (n-1)\, v_3 > (n-2)\, v_2, \\ (n-2)\, v_4 > (n-3)\, v_2, \\ \vdots \\ 2v_n > v_2. \end{cases} \tag{CV}$$

It is straightforward to check that if bidder B_2 does not want to deviate from the DR equilibrium, then no other lower-value bidder wants to deviate either. Therefore, if and only if conditions CV are satisfied, all bidders prefer to reduce demand to one unit and bid zero for all other units when their opponents are reducing demand to one unit, rather then deviate and outbid any other bidder to win more units. And so, in this case, there is a zero-price DR equilibrium.

There are other possible strategies that involve demand reduction and in which bidders divide the units among themselves differently. But the conditions that have to be satisfied for these strategies to be an equilibrium are more restrictive than conditions CV. This is because bidder B_1 is the bidder with the strongest incentive to deviate from demand reduction (because she has the highest use value, and hence she gains the most from winning a unit in the auction). Therefore, the DR equilibrium in which bidder B_1 wins $(k - n + 1)$ units is the equilibrium with demand reduction that minimizes her incentive to deviate.[51]

Of course, the auction may also have PP equilibria in which some bidders do not win any unit and the auction price is positive. However, when conditions CV are satisfied, the DR equilibrium that we have defined Pareto dominates, from the bidders' point of view, all equilibria with a positive auction price. To see this, notice that in a PP equilibrium the auction price is at least equal to the valuation of the highest-value bidder who loses the auction. But, when conditions CV are satisfied, each bidder B_j, $j = 2, ..., n$, prefers to win one unit at price zero, rather than win more units by outbidding a lower-value bidder and paying a price equal to her valuation, and bidder B_1 prefers to win $(k - n + 1)$ units at price zero, rather than win more units by outbidding a lower-value bidder and paying a price equal to her valuation. Therefore, each bidder obtains a strictly higher profit in the DR equilibrium defined than in any equilibrium with a positive auction price.

When one or more of conditions CV is not satisfied, at least bidder B_2 wants to outbid a lower-value bidder and raise the auction price, even if all other bidders reduce demand. So there are only PP equilibria in which some bidders do not win any unit and the auction price is positive. ■

Proof of Proposition 2. First notice that conditions DTV and conditions CV are not necessarily mutually exclusive — i.e., the two sets of conditions are not disjoint and may be satisfied together.[52] Moreover, none of the two sets of conditions implies the other; and conditions in both sets may simultaneously not be satisfied. Whether resale reduces or increases the seller's revenue depends on whether demand reduction is an equilibrium when resale is allowed.

If conditions DTV are satisfied and one or more of conditions CV is not satisfied, there is a zero-price DR equilibrium if and only if resale is allowed. Therefore, allowing resale induces bidders to choose a DR equilibrium and strictly reduces the seller's revenue. This proves part (i) of the statement.

If one or more of conditions DTV is not satisfied and conditions CV are satisfied, there is a zero-price DR equilibrium if and only if resale is not allowed. Therefore, allowing resale induces

[51] This is true also for a (zero-price) DR equilibrium in which bidders equally share the units on sale. Assume, for example, that $k = 2n$ and consider a possible DR equilibrium in which each bidder wins 2 units and the auction price is zero. In order for this to be an equilibrium, it is necessary and sufficient that bidder B_1 does not want to deviate. Bidder B_1 prefers to win 2 units at price 0 rather than overbid bidder B_j and win $(2n - 2j + 4)$ units at price v_j if and only if:

$$2v_1 > (2n - 2j + 4)(v_1 - v_j) \quad \Leftrightarrow \quad (n - j + 2)\, v_j > (n - j + 1)\, v_1, \quad j = 2, ..., n, .$$

The above conditions imply conditions CV; hence they are more restrictive.

[52] For example, conditions DTV and conditions CV are all satisfied if $v_1 > (k-2)\, v_2$, $kv_n > (n-1)\, v_1$, and $(n-1)\, v_n > (n-2)\, v_2$ (which is true, for example, if $n = 2$ and $v_1 > (k-2)\, v_2 > (n-1)\, v_1 - 2v_2$).

bidders to choose an equilibrium with a positive auction price and increases the seller's revenue. This proves part *(ii)* of the statement.

If both conditions DTV and conditions CV are satisfied, there is a zero-price DR equilibrium both with and without resale. Therefore, allowing resale has no effect on the seller's revenue.

Finally, if one or more of conditions DTV is not satisfied and one or more of conditions CV is not satisfied either, there are only PP equilibria both when resale is allowed and when resale is not allowed. In this case, allowing resale induces all bidders apart from bidder B_1 to bid more aggressively for the first unit than when resale is not allowed, because of the possibility of reselling to bidder B_1, and induces some bidders (but not bidder B_1) to also bid aggressively for more than one unit, in order to outbid speculators and other lower-value bidders. This tends to increase the seller's revenue. However, allowing resale also affects bidders incentive to reduce demand, and there may be a PP equilibrium in which bidders reduce demand, even if they outbid all speculators. This may reduce the seller's revenue. So a PP equilibrium when resale is allowed may have both a higher price and a lower price then a PP equilibrium when resale is not allowed. Therefore, in this case, the effect of allowing resale on the seller's revenue is more ambiguous and depends on whether resale induces more higher-value bidders to reduce demand or to bid more aggressively to beat their competitors, compared to an auction without resale.

Nonetheless, in order for resale to strictly increase the seller's revenue, it is necessary that the auction has no (zero-price) DR equilibrium when resale is allowed; hence that one or more of conditions DTV is not satisfied. (Otherwise, the seller's revenue is equal to zero when resale is allowed.) This proves part *(iii)* of the statement. ■

Proof of Lemma 3. In the DR equilibrium that we have defined, bidder B_1 wins $(k-n+1)$ units in the auction at price zero. All other bidders win one unit each and resell to bidder B_1 in the aftermarket. Therefore, each bidder different from B_1 obtains a profit equal to the price at which she resells to bidder B_1 in the aftermarket — i.e.:

$$\pi_i^* = \tfrac{1}{2}\left(v_1 + v_i\right), \quad i = 2, ..., n,$$

and bidder B_1 obtains a profit equal to the difference between her valuation for the k units, and the price she has to pay to acquire $(n-1)$ units from the other bidders in the aftermarket — i.e.:

$$\begin{aligned}\pi_1^* &= kv_1 - \tfrac{1}{2}\sum_{i=2}^{n}\left(v_1+v_i\right) \\ &= \tfrac{1}{2}\left(2k-n+1\right)v_1 - \tfrac{1}{2}\sum_{i=2}^{n} v_i.\end{aligned}$$

First notice that bidder B_1 has no incentive to deviate from the DR equilibrium, because the only effect of outbidding another bidder and winning more than one unit in the auction is to increase the auction price that bidder B_1 pays for the first unit she wins.

Now consider bidder B_2. It is never profitable for bidder B_2 to outbid bidder B_1. Moreover, bidder B_2 prefers to win one unit at price 0 rather than outbid bidder B_j, $j = 3, ..., n$, and win $(n-j+2)$ units if and only if:

$$\pi_2^* = \tfrac{1}{2}\left(v_1+v_2\right) > \left(n-j+2\right)\left[\tfrac{1}{2}\left(v_1+v_2\right) - \tfrac{1}{2}\left(v_1+v_j\right)\right]$$

$$\Leftrightarrow \quad (n-j+2)\, v_j + v_1 > (n-j+1)\, v_2.$$

So bidder B_2 does not deviate from the DR equilibrium if and only if the following $(n-2)$ conditions are satisfied:

$$\begin{cases} v_1 + (n-1)\, v_3 > (n-2)\, v_2, \\ v_1 + (n-2)\, v_4 > (n-3)\, v_2, \\ \vdots \\ v_1 + 2v_n > v_2. \end{cases} \tag{DCV}$$

And it is straightforward to check that, if bidder B_2 does not want to deviate from the DR equilibrium, no other lower-value bidder wants to deviate either. Therefore, if conditions DCV are satisfied, all bidders prefer to reduce demand when their opponents reduce demand, rather then deviate and outbid any other bidder to win more units.

By an argument analogous to the one in the proof of Lemma 2, when conditions DCV are satisfied, the DR equilibrium that we have defined Pareto dominates, from the bidders' point of view, all other equilibria with a positive auction price. ■

Proof of Proposition 3. Conditions DTV imply conditions DCV, but the converse is not true, because, since $k > n$:

$$v_1 + (k-i+2)\, v_i > (k-i+1)\, v_2 \quad \begin{matrix} \Rightarrow \\ \not\Leftarrow \end{matrix} \quad v_1 + (n-i+2)\, v_i > (n-i+1)\, v_2.$$

Therefore, when resale is allowed, if there is a DR equilibrium when speculators participate in the auction, there is also a DR equilibrium when speculators do not participate in the auction. So the presence of speculators eliminates a DR equilibrium and increases the seller's revenue when conditions DCV are satisfied and one or more of conditions DTV is not satisfied.

When speculators are not allowed to participate in the auction, there are other possible (zero-price) DR equilibria in which each bidder wins one unit, but bidders share the units differently from the equilibrium we have defined. (For example, there may be an equilibrium in which bidder B_1 wins less than $(k-n+1)$ units and another bidder wins more than one unit.) But the conditions that have to be satisfied for these other DR equilibria are less restrictive than conditions DCV. This is because the equilibrium we have defined is the DR equilibrium in which bidder B_1 wins the highest possible number of units. But bidder B_1 never wants to deviate from a DR equilibrium anyway (regardless of the number of units he wins in the DR equilibrium), and when another bidder wins more units, this bidder has a lower incentive to deviate from a DR equilibrium. Therefore, considering other DR equilibria when speculators do not participate in the auction only reinforces our conclusion that it is easier to have a DR equilibrium without speculators than with speculators; hence that speculators cannot reduce the seller's revenue by inducing bidders to reduce demand.

Finally, if there is no DR equilibrium when speculators participate in the auction, the presence of speculators can never reduce the seller's revenue because, in this case, speculators only increase the number of competitors in the auction and can only induce bidders to bid more aggressively and raise the auction price. ■

Proof of Lemma 4. The first part of the statement is a special case of Lemma 2. The second part follows from Lemma 3. In contrast to Lemma 3, conditions DCV are also necessary for a DR equilibrium when resale is allowed because, if $n = k$, in a DR equilibrium each bidder has to win exactly one unit. By an argument analogous to the one in the proof of Lemma 1, whenever the auction has a (zero-price) DR equilibrium, this equilibrium is also the unique Pareto dominant equilibrium for bidders. ■

Proof of Proposition 4. Conditions CV1 are more restrictive than conditions DCV. Indeed, conditions CV1 imply conditions DCV, but the converse is not true, because:

$$(n-i+2)\,v_i > (n-i+1)\,v_2 \quad \underset{\not\Leftarrow}{\Rightarrow} \quad v_1 + (n-i+2)\,v_i > (n-i+1)\,v_2.$$

Hence, without speculators, if there is a DR equilibrium when resale is not allowed, there is also a DR equilibrium when resale is allowed. So the possibility of resale facilitates demand reduction and, therefore, it may reduce the seller's revenue. This happens when conditions DCV are satisfied, so that there is a zero-price DR equilibrium with resale, and one or more of conditions CV1 is not satisfied, so that there is no zero-price DR equilibrium without resale. This proves part (i) of the statement.

On the other hand, if both conditions DCV and conditions CV1 are satisfied, resale does not affect the seller's revenue, because there is a zero-price DR equilibrium both with and without resale.

If one or more of conditions DCV is not satisfied, there is no DR equilibrium when resale is allowed. And since one or more of conditions CV1 is not satisfied either, there is also no DR equilibrium when resale is not allowed. So the auction price is positive both with resale and without resale. But when resale is allowed all bidders apart from bidder B_1 bid more aggressively for the first unit than when resale is not allowed, because of the possibility of selling to bidder B_1 in the resale market, and some bidders (but not bidder B_1) bid aggressively for more than one unit, in order to outbid their competitors. This tends to increase the seller's revenue. However, allowing resale still affects bidders incentive to reduce demand "among themselves," because there may be a PP equilibrium in which bidders reduce demand, even if they outbid all speculators. Therefore, if one or more of conditions DCV is not satisfied, the effect of allowing resale on the auction price is more ambiguous.

But suppose that *all* conditions DCV are not satisfied. Then, when resale is allowed, bidder B_2 outbids all lower-value bidders, while bidder B_1 does not outbid bidder B_2 because she always prefers to reduce demand. Therefore, when resale is allowed, there is only one PP equilibrium with an auction price $p^* = \frac{1}{2}(v_1 + v_3)$.

By contrast, when resale is not allowed, there can be a PP equilibrium with an auction price at most equal to v_2, which is bidder B_2's bid for the first unit. If without resale the auction price is lower than v_2, then clearly allowing resale increases the seller's revenue. So suppose that, without resale, there is a PP equilibrium in which bidder B_1 wins all units and the auction price is equal to v_2, which may be higher than p^*. (This is the only case in which the price is equal to v_2 in equilibrium.) Then it is at least necessary that bidder B_1 prefers to outbid all other bidders and win n units at price v_2, rather than win $(n-1)$ units only at price v_3. Otherwise, by Assumption 3, an equilibrium in which bidder B_1 wins all units and the auction price is v_2 would not be played because it would be Pareto dominated by an equilibrium in which bidder B_1 reduces demand and wins $(n-1)$ units, bidder B_2 wins 1 unit, and the auction price is v_3. Therefore, in order for the auction price to be equal to v_2 when resale is not allowed it is necessary that:

$$n(v_1 - v_2) > (n-1)(v_1 - v_3) \quad \Leftrightarrow \quad v_1 + (n-1)v_3 > nv_2. \tag{A.5}$$

And rearranging inequality A.5, we have that:

$$v_1 + v_3 - (n-2)(v_2 - v_3) > 2v_2$$

$$\Rightarrow \quad v_1 + v_3 > 2v_2 \quad \Leftrightarrow \quad p^* > v_2.$$

Hence, if all conditions DCV are not satisfied, when resale is not allowed in equilibrium the auction price is lower than p^*, which is the auction price when resale is allowed. This proves part (ii) of the statement. ■

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