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An Approximate Consumption Function

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Abstract

This paper proposes an approximation to the consumption function in the buffer-stock model. The approximation is based on the analytic properties of the consumption function in the buffer-stock model. In such model, the consumption function is increasing and concave and its derivative is bounded from above and below. We compare the approximation with the consumption function obtained using the endogenous grid point algorithm and show that using the former or the latter for estimating the Euler equation leads to very similar results.

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1 Introduction

Solving realistic versions of stochastic dynamic models of consumer choices requires employing numerical methods. The availability of large scale data-set makes it attractive to estimate such models. To pin down structural parameters, one needs to solve the models and bring the solution to the data. This requires being able to nest estimation with optimization. The challenge is often one of dimensionality and CPU time. This makes it particularly useful approximating the solution of the models. Using approximations involves errors, but there is not agreement on the effect of such errors on estimation. To the extent that the errors are small, one can still rely on approximations. Furthermore, approximations might turn to be useful in the specification search.

Approximating the consumption function has been a common exercise among economists since long time. The use of perturbation methods in precautionary saving models dates back to Leland (1968). Only recently, however, Feigenbaum (2005) has investigated the accuracy of second, third and higher approximations to the consumption function and provided some warning on the use of perturbation methods.

This paper does not use perturbation methods in that departing from the literature. We provide a class C^{∞} function to approximate the consumption function in the buffer stock model of saving. The approximation is derived for the Carroll's (1992) incarnation of the buffer stock model, but equally applies to the Deaton's (1991) version of such model. It relies on the monotonicity of

the consumption function, on concavity, and on the fact that the consumption function is bounded from above and below and so is its derivative.

The paper is organized as follows. Notation is lied down in Section 2. Section 3 reviews two methods for numerically solving the model: the standard method and the endogenous gridpoints algorithm. The approximation is derived and discussed in Section 4. Section 5 compares the approximate consumption function with the solution obtained using the endogenous grid-point algorithm in five economies, and section 6 explores the factors affecting the shape the approximate consumption function. Gains and losses from using the approximate consumption function are discussed in section 7, while section 8 concludes.

2 The notation

Consumers live from time 0 to time T. They maximize:

$$E_0 \sum_{t=0}^{T} \beta^t u\left(C_t\right)$$

with respect to consumption, C_t, under the dynamic budget constraint:

$$W_{t+1} = R[W_t + Y_t - C_t]$$

where β is the subjective discount factor, W_{t+1} and W_t are, respectively, non-human wealth at time $t+1$ and t, R the interest factor and Y_t labor income at time t. The utility function is assumed to be of the CRRA type, i.e.:

$$u\left(C_t\right) = \frac{C_t^{1-\rho}}{1-\rho}$$

where ρ is the coefficient of relative risk aversion. Labor income shifts due to transitory and permanent shocks:

$$Y_t = P_t \Xi_t$$

$$P_t = G P_{t-1} \Psi_t$$

where P_t is permanent income, Ξ_t is the transitory and Ψ_t the permanent income shocks, G is the growth factor of permanent income. Income is zero with a small probability p, i.e.:

$$\Xi_{t+n} = \begin{cases} 0 & \text{with probability} \quad p > 0 \\ \frac{\Theta_{t+n}}{q} & \text{with probability} \quad q \equiv 1 - p \end{cases}$$

Furthermore, following Carroll (1992) we assume that transitory and permanent shocks are dawn from a log-normal distribution and that: $E_t[\Theta_{t+n}] = 1$ for $n > 0$, that $var\left(\log \Theta_{t+n}\right) = \sigma_\theta^2$, that $E_t[\Psi_{t+n}] = 1$ and that $var\left(\log \Psi_{t+n}\right) = \sigma_\psi^2$. Finally, it is assumed that consumer cannot die in debt, i.e.

$$C_T \leq W_T + Y_T$$

This last assumption naturally leads to use the dynamic programming principle.[1] The Bellman equation for the consumer problem is:

$$V_t\left(W_t, P_t\right) = \max_{C_t} \left\{ u\left(C_t\right) + \beta E_t V_{t+1}\left(W_{t+1}, P_{t+1}\right) \right\}$$

s.t.

$$P_{t+1} = G P_t \Psi_{t+1}$$

$$W_{t+1} = R[W_t - C_t + Y_t]$$

[1] The non-stationary nature of the problem is not an issue in this context, thanks to the homogeneity of the objective function.

In order to exploit the homogeneity of the utility function, one can define cash-on-hand as:

$$M_t = W_t + Y_t$$

This allows to rewrite the Bellman equation as:

$$\begin{aligned} & v_t\left(m_t\right) = \max_{c_t} \left\{ u(c_t) + \beta E_t G^{1-\rho} \Psi_{t+1}^{1-\rho} v_{t+1}(m_{t+1}) \right\} \\ & \text{s.t.} \\ & m_{t+1} = \tfrac{R}{G\Psi_{t+1}}[m_t - c_t] + \Xi_{t+1} \end{aligned} \tag{1}$$

where $m_t = M_t/P_t$ and $c_t = C_t/P_t$. Carroll (2004) shows that (1) defines a contraction mapping under three restrictions: (i) $G < R$; (ii) $(R\beta)^{\frac{1}{\rho}} < R$; (iii) $R\beta E_t[G\Psi_{t+n}^{-\rho}] < 1$.

The first condition guarantees that human capital does not explode in perfect foresight models; the second that consumers are not too patient; the third that consumers are impatient enough for cash-on-hand not to go to infinity. Carroll (2004) also shows that the consumption function is increasing, concave and that it is bounded from above and from below; moreover, that there exists a unique and stable level of cash-on-hand, the target m^*, such that $E_t m_{t+1} = m_t$ if $m_t = m^*$.

3 Standard solution methods and the endogenous grid-point algorithm

Problem (1) has not a closed form solution. This means that its solution requires employing numerical methods. The problem is naturally characterized as a recursive one, which means that the solution can be found by value or policy

function iteration or using projection methods.[2]

A common solution strategy amounts to iterate Euler Equation for consumption, starting from $c_T = m_T$:

$$u'(c_t) = \beta R E_t \left\{ G^{-\rho} \Psi_{t+1}^{-\rho} v'_{t+1} [\frac{R}{G\Psi_{t+1}} (m_t - c_t) + \Xi_{t+1}] \right\}$$

where, from the envelope condition, $v'_t(m_t) = u'(c_t)$. In order to iterate the Euler equation, one needs to discretize the state-space. This amounts to define a grid for m_t, i.e. $\{\mu_1, \mu_2, \cdots, \mu_I\}$, to discretize the distribution of permanent and transitory income shocks and solve:

$$\begin{aligned}
u'(\chi_1) &= \beta R E_t \left\{ G^{-\rho} \Psi_{t+1}^{-\rho} v'_{t+1} [\tfrac{R}{G\Psi_{t+1}} (\mu_1 - \chi_1) + \Xi_{t+1}] \right\} \\
u'(\chi_2) &= \beta R E_t \left\{ G^{-\rho} \Psi_{t+1}^{-\rho} v'_{t+1} [\tfrac{R}{G\Psi_{t+1}} (\mu_2 - \chi_2) + \Xi_{t+1}] \right\} \\
&\vdots \\
&\vdots \\
u'(\chi_I) &= \beta R E_t \left\{ G^{-\rho} \Psi_{t+1}^{-\rho} v'_{t+1} [\tfrac{R}{G\Psi_{t+1}} (\mu_I - \chi_I) + \Xi_{t+1}] \right\}
\end{aligned} \tag{2}$$

with respect to $\{\chi_1, \chi_2, \cdots, \chi_I\}$. The consumption function is then obtained by interpolating the couples $\{(\chi_1, \mu_1), (\chi_2, \mu_2), \cdots, (\chi_I, \mu_I)\}$. Solving system (2) requires evaluating the expected value of the marginal utility of consumption at each of the grid points. This entails a substantial amount of computer time, for fine enough state-space grids.

The endogenous gridpoints algorithm improves on standard solution methods (see Carroll, 2006). Instead of defining a grid for m_t, the algorithm requires

[2]For an introductory treatment of the topic see Adda and Cooper, 2003; more advanced readers might want to look at Judd, 1998.

discretizing $m_t - c_t$, i.e. the end of period asset, and solving:

$$\begin{aligned}
\chi_1 &= u'^{-1}\left\{\beta R E_t\left[G^{-\rho}\Psi_{t+1}^{-\rho} v'_{t+1}(\tfrac{R}{G\Psi_{t+1}}\alpha_1 + \Xi_{t+1})\right]\right\} \\
\chi_2 &= u'^{-1}\left\{\beta R E_t\left[G^{-\rho}\Psi_{t+1}^{-\rho} v'_{t+1}(\tfrac{R}{G\Psi_{t+1}}\alpha_2 + \Xi_{t+1})\right]\right\} \\
&\vdots \\
&\vdots \\
\chi_I &= u'^{-1}\left\{\beta R E_t\left[G^{-\rho}\Psi_{t+1}^{-\rho} v'_{t+1}(\tfrac{R}{G\Psi_{t+1}}\alpha_I + \Xi_{t+1})\right]\right\}
\end{aligned} \tag{3}$$

where $\{\alpha_1, \alpha_2, \cdots, \alpha_I\}$ is the grid for the end of period asset. The endogenous gridpoints algorithm is more efficient than other standard methods since it evaluates expectations only for points used in the interpolating functions. This translates into non-negligible savings in the amount of computer time needed to solve the consumers problem. We thus compare our approximate consumption function with that obtained using the endogenous grid-point method.

4 The approximate consumption function

In order to derive our approximate consumption function, we exploit the analytic properties of the marginal propensity to consume out of cash-on-hand (MPC). This is known to be decreasing in a model with precautionary saving (see Carroll and Kimball, 1996). Furthermore, Carroll (2004) shows that the MPC is bounded from above and from below, namely that:

$$\lim_{m\to 0} c'(m) = \overline{\kappa}$$

$$\lim_{m\to \infty} c'(m) = \underline{\kappa}$$

where $c'(m)$ is the MPC, $\overline{\kappa} > \underline{\kappa} > 0$ and:

$$\begin{aligned} \overline{\kappa} &= 1 - R^{-1}(R\beta p)^{\frac{1}{\rho}} \\ \underline{\kappa} &= 1 - R^{-1}(R\beta)^{\frac{1}{\rho}} \end{aligned}$$

This suggests to approximate the MPC with the following family of functions:

$$\frac{(1+e^{-ba})(\overline{\kappa}-\underline{\kappa})}{1+e^{b(m-a)}} + \underline{\kappa} \tag{4}$$

where a is non-negative and b is positive real.[3]

Given $\overline{\kappa}$ and $\underline{\kappa}$, each couple of parameters a and b identifies a different member within the family of functions (4). It is immediate to verify that the approximate MPC increases with a and that the larger b the faster the MPC goes to its bounds.[4] Figure 1 shows the approximate MPC obtained by varying a and for $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$ and $p = 0.005$ and $b = 1.5$. The approximate MPC is decreasing, and bounded between $\overline{\kappa}$ and $\underline{\kappa}$. Furthermore, it is concave for $m < a$, convex for $m > a$, and therefore posses an inflexion point at a. Figure 2 shows the approximate MPC for various values of b, for $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$ and $p = 0.005$ and $a = 1.5$. The Figure helps to visualize that the higher b the faster the approximate MPC goes to its bounds.

By integrating back (4) and recalling that the consumption function goes to zero and infinity for cash-on-hand going to zero and infinity, respectively, one

[3]The approximate MPC is obtained form the Fermi-Dirac distribution by multiplying it by $(1+e^{-ba})(\overline{\kappa}-\underline{\kappa})$ and adding $\underline{\kappa}$. The Fermi-Dirac distribution describes the probability of a fermion occupying a given energy level. In the Fermi-Dirac distribution a is the so-called Fermi-energy and b the inverse of temperature.

[4]The first derivative of the approximate MPC with respect to a is $\frac{b\,e^{ab}\left(e^{bm}-1\right)(\overline{\kappa}-\underline{\kappa})}{\left(e^{ab}+e^{bm}\right)^2}$.

obtains the approximate consumption function, i.e.:

$$(1+e^{-ba})(\overline{\kappa}-\underline{\kappa})\left\{m-\frac{1}{b}[\log(1+e^{b(m-a)})-\log(1+e^{-ba})]\right\}+\underline{\kappa}m \qquad (5)$$

The approximate consumption function is continuous, increasing and concave.[5] Figure 3 plots the approximate consumption function for various values of a, for $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$ and $p = 0.005$ and $b = 1.5$. The effect of changing b is shown in Figure 4, which plots the approximate consumption function for various values of b.

In order to select a member in the class of the approximate MPC, one needs to chose a and b for given growth factor of income, interest factor, discount factor, relative risk aversion, standard deviation of permanent and transitory income shocks and probability of unemployment. Since in the buffer-stock model the most travelled region of the state space is around the target level of cash-on-hand, it seems natural to search for criteria that enhance the performance of the approximation around the target. We therefore set a and b to minimize the squared Euler equation errors at the target. Euler equation errors are a standard measure of the quality of an approximation: the better an approximation, the lower (in absolute value) Euler equation errors. As in Judd (1992) and Arouba, Fernández-Villaverde and Rubio-Ramírez (2006), Euler equation error are standardized by consumption.

The next section compares the approximate consumption with the consumption function obtained employing the endogenous grid point algorithm to solve

[5]Notice that one can easily allow for life-cycle effects by making G to vary with age.

the consumers problem. For simplicity of exposition, we call the latter the actual consumption function, but we will show that in some region of the state space the approximate involves lower errors than the actual consumption function.

The comparison between the actual and the approximate consumption function is made in five economies, which differ among them for the assumptions on labor income uncertainty and risk aversion. This exercise will help to understand how the quality of the approximation varies with uncertainty.

5 Five examples

This section uses the endogenous grid point algorithm to solve the consumer problem for five economies and compares the solution with the approximate consumption function.

Our first example assumes $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$. These are the values used in Carroll (2004). In such parametrization of the model, a and b, are found to be equal to 0.8982 and to 1.0941.

Figure 5 displays the actual and the approximate consumption function. The approximate is very close to the actual consumption function in the 0 to 2 range of cash-on-hand. The target level of cash-on-hand is equal to 1.45 and to 1.41 if one uses, respectively, the actual and the approximate consumption function. This implies that the two functions are close in the relevant area, around the target, where consumption spends most of time.

To judge the quality of the approximation we compute Euler equation errors. Since both our solution methods are in the end approximations, the Euler equation cannot be expected to hold exactly. The Euler equation errors are divided by consumption, to get a unit free number. Figure 6 plots the log 10 of the absolute value of normalized Euler Equation errors for both consumption functions. The Figure reveals that the Euler equation errors incurred by using the endogenous grid-point algorithm are generally lower, except around zero (between 0 and 0.36) and around the target.[6] The quality of the approximation is, therefore, better for very small values of cash-on-hand and around the target. This is not surprising since the approximation exploits the analytic properties of the consumption function at its bounds, while the endogenous grid-point algorithm does not, and the parameters a and b are set to minimize the Euler equation errors at the target.[7]

The comparison between the actual and the approximate consumption function is carried on by simulating 10,000 times the model. The correlation between actual and approximate consumption ranges between 0.90 and 0.99 across simulation runs and is equal on average to 96 percent. Figure 7 plots the actual and the approximate simulated consumption for the first simulation run, together

[6]The Euler equation errors for the approximate consumption function are also lower for cash-on-hand between 5 and 6, where there is a local minimum, and for very large values of cash-on-hand.

[7]This suggests modifying the endogenous grid-point algorithm to account for the limiting properties of the consumption function. This can be done by employing the approximate consumption function for points outside the grid. While this requires more computer time, the precision gains depends on the particular parametrization chosen.

with income. The approximate consumption path is smoother that the actual, as expected.

To shed further light on the quality of the approximation, we compute the first four moments of the consumption paths generated from the actual and the approximate consumption function. The across-simulation averages of these moments are reported in Table 1. Under the actual consumption function, the first, second, third and fourth moments of consumption average to 1.007 (with standard error equal to 0.014), 1.018 (0.026), 1.033 (0.039) and 1.051 (0.052) across simulation runs; under the approximate, to 1.004 (0.013), 1.014 (0.025), 1.029 (0.037), 1.049 (0.049), respectively. The differences between the actual and the approximate consumption function show up at the third decimal place for the first two moments of consumption, and at the second decimal place for the third and fourth moments, but they are never statistically significant. We also investigate the time dependency in the simulated data and compute two more moments, the expected value of time t consumption multiplied by $t-1$ consumption ($E(c_t c_{t-1})$) and by $t-2$ consumption ($E(c_t c_{t-2})$). Using the endogenous grid-point algorithm, $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$ are equal respectively to 1.02 and 1.021, using the approximate to 1.012 and 1.012. Again the differences shows up at the second decimal place but are not statistically significant.

Taking the averages across simulation runs helps to compare the approximate with the actual consumption function, but hides potential differences within each simulation run between the approximate and the endogenous grid point solution. We therefore compute in each simulation run the relative error for the first,

second, third and fourth moments of consumption paths, and for $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$ as:

$$\frac{\|\mu_i^{app} - \mu_i^{act}\|}{\|\mu_i^{act}\|}$$

where μ_i^{app} and μ_i^{act} are, respectively, the relevant moments of approximate and actual consumption computed in the i^th simulation run. Figure 8 plots the across-simulations kernel density of the relative error for the first four moments of approximate consumption and for $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$. The kernel densities are spiked nearby zero. Moreover, the Figure shows that for the first moment replacing the actual with the approximate consumption function causes an error at the third decimal place, for the other moments at the second.

The second example differs from the first by setting $\sigma_\theta = \sigma_\psi = 0.05$. Under this parametrization, a is found to be 1.07, and b 0.92. Figure 9 shows the actual and the approximate consumption function and Figure 10 reports the Euler equation error. As above, the quality of the approximation improves for small values of cash-on-hand and around the target, which is equal to 1.35 and to 1.31, if one uses the actual or the approximate consumption function, respectively.

We simulate the model and find that the correlation between actual and approximate consumption ranges between 96% and 99%. Table 1 reports the across simulation averages of the first four moments of consumption, and of $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$. Under the actual consumption function these are 1.002 (0.009), 1.008 (0.015), 1.016 (0.021), 1.025 (0.027), 1.009 (0.018), and 1.011 (0.018), under the approximate 1.001 (0.009), 1.005 (0.015), 1.013 (0.020), 1.022 (0.026), 1.004 (0.017), and 1.004 (0.018). The differences across solution methods show

up at the third decimal place for all moments except $E(c_t c_{t-2})$ and are never statistically significant. The kernel density of the relative errors for the first four moments of consumption and for $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$ are plotted in Figure 11. The Figure reveals that for all moments the most frequent errors are in the order of 10^{-3}.

In the third example, we consider the case of a high income risk economy and therefore set $\sigma_\theta = \sigma_\psi = 0.12$. [8] The optimal a is smaller and b larger than in the baseline case and are equal, respectively, to 0.7766 and 1.2232. The actual and the approximate consumption function for this economy are displayed in Figure 12, and the Euler equation errors in Figure 13. Again the approximation performs relatively better in the vicinity of the target, equal, in this case, to 1.57 under the actual and to 1.53 under the approximate consumption function. The correlation between actual and approximate consumption ranges between 88% and 98%, while the differences across the two solution methods show up at the most at the second decimal place and are never statistically significant, as shown in Table 1. Using the endogenous grid-point algorithm, the first for moments of consumption paths and $E(c_t c_{t-1})$ and $E(c_t c_{t-2})$ are 1.011 (0.017), 1.027 (0.034), 1.048 (0.052), 1.073 (0.070), 1.03 (0.035), 1.031 (0.036); using the approximate 1.007 (0.016), 1.021 (0.031), 1.041 (0.046), 1.067 (0.063), 1.02

[8] Values of σ_ψ larger than 0.12 are not compatible with this parametrization of the buffer-stock model of saving. In order for the solution to exhibit a unique target level of cash-on-hand, it must happen that $R\beta E_t[G\Psi_{t+n}^{-\rho}] < 1$ and it is easy to verify that LHS of this inequality increases with σ_ψ.

(0.032), 1.02 (0.032). Figure 14 shows the kernel densities of relative errors. For all moments most mass is around zero, and the distribution features a small probability of errors of order larger than 10^{-2}.

The last two experiments study how risk aversion affects the quality of the approximation. We set the relative risk aversion in turn to 4 (high risk-aversion case) and to 1.5 (low risk-aversion case). In the former case we find that a and b are equal to 1.067 and 0.919, in the latter to 0.962 and 1.080, respectively. Figure 15 plots the actual and the approximate consumption function for $\rho = 4$, and 16 for $\rho = 1.5$. The figures reveal that for both values of the relative risk aversion the approximate is close to the actual consumption function. To assess the degree of similarity between actual and approximate consumption function, we compute the Euler equation errors. These are plotted in Figure 17 for the high-risk aversion case and in Figure 18 for low risk-aversion case. The approximate features smaller errors than the actual consumption function around the target (1.824 for high and 1.314 for low risk-aversion).

Simulating the model, we discover that the correlation between actual and approximate consumption ranges between 0.85 to 0.97 in the high risk-aversion, and between 0.908 and 0.987 in the low risk-aversion case. The differences between moments are larger in the high risk aversion than in the low risk-aversion cases, but are never statistically significant in both cases, as shown in Table 1. The kernel densities plotted in figures 19 and 20 confirm this pattern: relative errors are larger in the more non-linear case. This is perhaps not surprising and leads to wonder how a and b depend on the deep parameters of the consumer's

problem. The next section is devoted to answer such question.

6 The choice of a and b

This section explores how the choice of a and b depends on the deep parameters of the consumer's problem. Table 2 computes a and b in several experiments for the discount factor, the relative risk aversion, the growth and the interest factor, the probability of unemployment and the standard deviation of the logarithm of permanent and transitory income shocks.

Letting the discount factor to vary between 0.95 and 0.99, we find a to decrease from 0.99 to 0.48 and b to increase from 1.05 and 1.40. To give economic content to the effect of changing the discount factor on a and b, Table 2 also reports the target cash-on-hand and the marginal propensity to consume out of cash-on-hand at the target (MPC). As shown in the third and fourth rows of the Table, the target increases and the MPC decreases as β increases, which accords with expectations.

Varying the relative risk aversion from 1.2 to 4 causes the target to increase from 1.24 to 1.83 and the MPC to decrease from 0.57 to 0.32, and a and b varying, respectively, from 1.02 to 1.06 and from 1.015 to 0.919. Again the effect of ρ on the target and the MPC has the expected sign.

The parameters a and b also vary with the income growth factor: a increases and b decreases. Accordingly, the target decreases from 2.2 to 1.41 and the MPC increases from 0.11 to 0.45. Raising the interest factor from 1.04 to 1.06 has a

positive impact on b and on the target and a negative impact on a and MPC. The same is true for the probability of unemployment, and for the standard deviations of log permanent and transitory shocks. Raising the probability of unemployment from 0.001 to 0.10 makes the a to decrease from 1.17 to 0.83, b to increase from 0.85 to 1.13, the target to more than double, and the MPC at the target to shrink from 0.62 to 0.11. The effect of changing the standard deviations of log permanent and transitory income has the same sign but is smaller in magnitude. As one expects from the theory, increasing uncertainty makes the precautionary motive for saving more intense and accordingly causes the target wealth to increase.

In summary, a ranges between 0.65 and 1.17, and b between 0.85 and 1.38 across all experiments. The optimal values of a and b are found for several different combinations of the model parameters and are reported in Table 3 and 4. Beyond a and b the tables also compute the target level of cash-on-hand and the MPC at the target. The results accord with the intuition: the target increases with the discount factor, with the interest rate, with relative risk aversion and with uncertainty and decreases with income growth. Having found the values of a and b under several parametrization of the model, one wanders what one gains (and loses) from using the approximate consumption function. The next section answers this question.

7 Gains and losses from using the approximate consumption function

In order to quantify gains and losses from using the approximate consumption function, two exercises are performed. First, we estimate the log-linearized Euler Equation (LLEE) on the data generated by the actual and the approximate consumption function. The second exercise simulates the actual and the approximate consumption function and investigates what error comes from replacing the actual with the approximate consumption function.

To estimate the LLEE, we simulate a 100 time periods consumption model using the actual and the approximate consumption function for 1000 consumers, each differing for the interest factor and the realizations of the income shocks. We will therefore exploit the across-consumers variation in the interest factor to estimate the coefficients of the LLEE. Since the expectation error in the Euler equation is correlated with the interest rate across-consumers, the exercise will not provide a consistent estimate of the intertemporal elasticity of substitution (IES). But, this equally applies to the estimation on the data obtained using the actual and the approximate consumption function, in line with the indirect inference approach to the estimation of the IES.[9]

[9]The indirect inference approach prescribes that one estimates the IES in two steps. In the first an auxiliary model is estimated on real and simulated data. The simulated data are obtained fixing to a given value the IES, and the other deep parameters of the consumer problem. The auxiliary model is typically a version of the Euler Equation, that delivers biased estimate of the IES. In the second step, the estimated coefficients of the auxiliary model on the real

We estimate a log-linearized version of the Euler equation, i.e.:

$$\Delta \ln C_{it+1} = \alpha_0 + \alpha_1 r_i + \varepsilon_{it+1}$$

where $r_i = \ln R_i$.

The results are shown in Table 5 and refer to five different configurations of the parameters' set. The first column of the Table refers to the baseline configuration, which sets the discount factor to 0.96, the relative risk aversion to 2, the growth factor to 1.03, the probability of unemployment to 0.005, and the standard deviation of log transitory and permanent income to 0.10. The Table shows that the estimated α_0 and α_1 do not statistically differ if one uses data generated from the actual or the approximate consumption function. The results are similar in the second column, which assumes that the standard deviation of the log transitory and permanent income shocks is 0.05 and leaves the other parameters unchanged, in the third, where the standard deviation of the log transitory and permanent income shocks is set to 0.12, in the fourth, where the relative risk aversion is set to 4, in the fifth where it is set to 1.5. Therefore, estimating the log-linearized Euler equation on data generated from the actual and the approximate consumption function delivers very similar results.

Replacing the actual with the approximate consumption function entails an approximation error. One might wonder what are the consequences of such error

data are compared with those obtained from the estimation of the auxiliary model in simulated data. If the two set of coefficients are close enough, this means that the IES is the one used to simulate the model. If they are not close, one goes back to the first step and sets a new value for the IES (see Allen and Browning, 2003).

for the estimation of the log-linearized Euler equation. To answer this question, we regress the approximation error on the interest rate. The results are reported at the bottom of Table 5 and show that for all parameters configurations, except for the high variance and the high risk aversion configurations (columns 3 and 4), the approximation error is orthogonal to the interest rate. This suggests that the error entailed by replacing the actual with the approximate consumption function is generally inconsequential for the estimation of the log-linearized Euler equation, but in the case when the consumption function is highly concave, due to high risk faced by the consumers or high risk aversion.

Quite often researchers are interested in solving and simulating models with heterogenous agents. Agents typically have different preferences or different beliefs. One way to describe such differences is to assume that the interest factor varies between agents. Therefore, as second exercise, we run the model for 100 time periods and 1000 agents, each differing by the the interest factor ranging from 1.025 to 1.055. One question that might arise when dealing with such an economy is what error entails replacing the heterogenous agents economy with a single agent economy. Accordingly, we investigate what happens if one replaces the actual consumption function with the approximate consumption function computed for a unique interest factor, say equal to 1.04. For interest factors different from 1.04, this amounts to ask how large is the error from replacing the right actual consumption function with the wrong approximate consumption function. This leads to the second exercise.

Table 6 computes the error that one incurs, on average, by replacing the right

actual with the wrong approximate consumption function. The Table focuses on two extreme cases. In the top panel, we show the average error when the actual consumption function obtained with the interest factor set to 1.025 is replaced with the approximate consumption function obtained with the interest factor set to 1.04. The error is small, but statistically significant. Exploring the parameter space along the interest factor dimension, we find that 1.025 is the only case in which the error from replacing the right actual with the wrong approximate consumption function is statistically different from zero. For brevity, we only report in the bottom panel of Table 6 the average error incurred by replacing the actual consumption function computed with the interest factor 1.055 and the approximate consumption with interest factor 1.04. The Table shows that the error is small and never statistically different from zero.

8 Conclusions

This paper has provided an approximate consumption function for the Carroll's (1997) buffer stock model of saving. The approximation is derived by exploiting the asymptotic behavior of the consumption function and of the marginal propensity to consume out of cash-on-hand. Using the restrictions implied by such asymptotic behavior, we proposed to approximate the marginal propensity to consume by a linear transformation of the Fermi-Dirac distribution. The transformation is made to depend explicitly on the interest factor, on the unemployment probability, on the discount factor and the constant relative risk

aversion. Moreover, the distribution depends on a couple of parameters, a and b, which control for the degree of concavity of the consumption function, the speed at which the marginal propensity to consume goes to its bounds and are therefore related to the income risk parameters.

We simulate the actual and the approximate consumption function model under five alternative configurations of the parameters space and show that one cannot statistically distinguish simulated consumption moments from the actual and the approximate consumption function. We then investigate how a and b change with the deep parameters of the consumer problem and show that in the several different parameter experiments a ranges between 0.65 and 1.17, and b between 0.85 and 1.38.

We finally show that replacing the actual with the approximate consumption function in the estimation of the log-linearized Euler equation is generally inconsequential, except for high risk or high risk aversion economies.

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TABLE 1. Actual and approximate consumption

Actual	Approximate	Actual	Approximate	Actual	Approximate	Actual	Approximate	Actual	Approximate	Actual	Approximate
1st moment		*2nd moment*		*3rd moment*		*4th moment*		$E(c_t c_{t-1})$		$E(c_t c_{t-2})$	
					$\rho = 2$, $\sigma_\psi = \sigma_\theta = 0.1$						
1.007	1.004	1.018	1.014	1.033	1.029	1.051	1.049	1.02	1.012	1.021	1.012
(0.014)	(0.013)	(0.026)	(0.025)	(0.039)	(0.037)	(0.052)	(0.049)	(0.028)	(0.026)	(0.028)	(0.026)
					$\rho = 2$, $\sigma_\psi = \sigma_\theta = 0.05$						
1.002	1.001	1.008	1.005	1.016	1.013	1.025	1.022	1.009	1.004	1.011	1.004
(0.009)	(0.009)	(0.015)	(0.015)	(0.021)	(0.020)	(0.027)	(0.026)	(0.018)	(0.017)	(0.018)	(0.018)
					$\rho = 2$, $\sigma_\psi = \sigma_\theta = 0.12$						
1.011	1.007	1.027	1.021	1.048	1.041	1.073	1.067	1.030	1.020	1.031	1.020
(0.017)	(0.016)	(0.034)	(0.031)	(0.052)	(0.046)	(0.070)	(0.063)	(0.035)	(0.032)	(0.036)	(0.032)
					$\rho = 4$, $\sigma_\psi = \sigma_\theta = 0.1$						
1.013	1.007	1.032	1.022	1.056	1.042	1.084	1.068	1.038	1.022	1.041	1.023
(0.017)	(0.015)	(0.034)	(0.030)	(0.052)	(0.045)	(0.071)	(0.061)	(0.034)	(0.030)	(0.035)	(0.031)
					$\rho = 1.5$, $\sigma_\psi = \sigma_\theta = 0.1$						
1.005	1.003	1.016	1.012	1.029	1.028	1.045	1.047	1.016	1.010	1.017	1.009
(0.013)	(0.013)	(0.025)	(0.024)	(0.037)	(0.035)	(0.050)	(0.047)	(0.027)	(0.025)	(0.027)	(0.026)

Note. The table shows the across simulation averages of moments of consumption paths. In each row, β, G, R and p are set to 0.96, 1.03, 1.04, and 0.005, respectively. Standard errors are reported in parenthesis.

TABLE 2. The optimal values of a and b: ranges

	β $[0.95-0.99]$	ρ $[1.2-4]$	G $[1.014-1.03]$	R $[1.04-1.056]$	p $[0.001-0.097]$	σ_ψ $[0.06-0.14]$	σ_θ $[0.06-0.14]$
a	0.994-0.477	1.024-1.066	0.431-0.898	0.898-0.742	1.174-0.835	1.022-0.653	0.952-0.837
b	1.047-1.403	1.015-0.919	1.380-1.094	1.094-1.226	0.850-1.127	1.016-1.307	1.070-1.184
m^*	1.363-2.161	1.245-1.827	2.205-1.411	1.411-1.555	1.221-2.574	1.351-1.667	1.386-1.471
MPC	0.486-0.118	0.570-0.322	0.110-0.446	0.446-0.335	0.624-0.114	0.504-0.267	0.469-0.393

Note. In the first column, β varies between 0.95 and 0.99 and the other parameters are set to their value in the baseline specification; in the second column, ρ between 1.2 and 1.4 and the other parameters are set to their value in the baseline specification; in the third, G between 1.014 and 1.03 the other parameters are set to their value in the baseline specification; in the fourth, R between 1.04 and 1.056 the other parameters are set to their value in the baseline specification; in the fifth p between 0.001 and 0.097 and the other parameters are set to their value in the baseline specification; in the sixth and seventh σ_ψ and σ_θ, respectively, vary between 0.06 and 0.14 the other parameters are set to their value in the baseline specification. In the baseline specification $\beta = 0.96$, $\rho = 2$, $G = 1.03$, $R = 1.04$, $p = 0.005$, $\sigma_\psi = \sigma_\theta = 0.1$.

TABLE 3. The optimal values of a and b: Preferences and Income.

β	a	b	m^*	MPC	ρ	a	b	m^*	MPC	G	a	b	m^*	MPC	R	a	b	m^*	MPC
0.95	0.9944	1.047	1.363	0.4864	1.2	1.024	1.015	1.245	0.5703	1.014	0.4309	1.38	2.205	0.1103	1.04	0.8982	1.094	1.411	0.4458
0.9525	0.9677	1.06	1.375	0.4757	1.375	0.953	1.055	1.286	0.5304	1.015	0.5612	1.411	1.981	0.1541	1.041	0.8932	1.106	1.418	0.4395
0.955	0.9444	1.078	1.389	0.4635	1.55	0.9149	1.084	1.325	0.4977	1.016	0.6181	1.374	1.82	0.205	1.042	0.8919	1.122	1.424	0.4331
0.9575	0.9398	1.111	1.401	0.4516	1.725	0.8939	1.099	1.363	0.4708	1.017	0.606	1.319	1.76	0.2319	1.043	0.8744	1.121	1.431	0.4277
0.96	0.8982	1.094	1.411	0.4458	1.9	0.9311	1.151	1.398	0.4465	1.018	0.6359	1.299	1.703	0.2569	1.044	0.8557	1.119	1.438	0.4224
0.9625	0.8885	1.13	1.429	0.4298	2.075	0.9074	1.105	1.428	0.436	1.019	0.6648	1.264	1.643	0.2877	1.045	0.8621	1.144	1.446	0.4145
0.965	0.8637	1.152	1.449	0.4139	2.25	0.8977	1.09	1.466	0.4187	1.02	0.7062	1.293	1.63	0.2909	1.046	0.8527	1.152	1.454	0.4079
0.9675	0.8421	1.178	1.47	0.397	2.425	0.9122	1.091	1.504	0.4019	1.021	0.7222	1.262	1.594	0.3125	1.047	0.8399	1.156	1.462	0.4018
0.97	0.8018	1.182	1.494	0.3812	2.6	0.939	1.092	1.54	0.3878	1.022	0.7565	1.248	1.559	0.3331	1.048	0.8387	1.173	1.47	0.3945
0.9725	0.7666	1.197	1.521	0.3628	2.775	0.9447	1.071	1.577	0.376	1.023	0.7676	1.214	1.531	0.3528	1.049	0.8372	1.19	1.478	0.3873
0.975	0.7612	1.249	1.557	0.3369	2.95	0.9665	1.056	1.612	0.3662	1.024	0.8293	1.239	1.507	0.3657	1.05	0.8134	1.182	1.487	0.3818
0.9775	0.7087	1.255	1.599	0.3134	3.125	0.9963	1.046	1.647	0.3567	1.025	0.8061	1.176	1.488	0.3835	1.051	0.8068	1.193	1.496	0.3747
0.98	0.6757	1.255	1.626	0.2998	3.3	1.002	1.015	1.683	0.3491	1.026	0.8479	1.185	1.471	0.3944	1.052	0.7947	1.199	1.506	0.3676
0.9825	0.6414	1.298	1.697	0.2626	3.475	1.032	1.01	1.723	0.3378	1.027	0.8536	1.154	1.454	0.4088	1.053	0.7937	1.217	1.517	0.359
0.985	0.6129	1.339	1.777	0.2263	3.65	1.055	0.9923	1.76	0.33	1.028	0.8545	1.118	1.438	0.4233	1.054	0.7814	1.225	1.529	0.3509
0.9875	0.5745	1.355	1.854	0.1984	3.825	1.064	0.9533	1.79	0.3276	1.029	0.8933	1.124	1.425	0.433	1.055	0.7548	1.218	1.541	0.344
0.99	0.4768	1.404	2.161	0.1179	4	1.066	0.9194	1.827	0.3221	1.03	0.8982	1.094	1.411	0.4458	1.056	0.7417	1.226	1.555	0.3349

Note. β varies between 0.95 and 0.99 and the other parameters are set to their value in the baseline specification in the first five columns; from column six to ten, ρ varies between 1.2 and 1.4 and the other parameters are set to their value in the baseline specification; from column eleven to fifteen G between 1.014 and 1.03 and the other parameters are set to their value in the baseline specification; from column sixteen to twenty R between 1.04 and 1.056 and the other parameters are set to their value in the baseline specification. In the baseline specification $\beta = 0.96$, $\rho = 2$, $G = 1.03$, $R = 1.04$, $p = 0.005$, $\sigma_\psi = \sigma_\theta = 0.1$.

TABLE 4. The optimal values of a and b: Uncertainty.

p	a	b	m^*	MPC	σ_ψ	a	b	m^*	MPC	σ_θ	a	b	m^*	MPC
0.001	1.174	0.8503	1.221	0.6239	0.06	1.022	1.016	1.351	0.5044	0.06	0.9518	1.07	1.386	0.469
0.007	0.8086	1.112	1.484	0.397	0.065	0.9993	1.02	1.358	0.4976	0.065	0.9307	1.064	1.391	0.4651
0.013	0.7945	1.209	1.63	0.3141	0.07	0.9933	1.021	1.36	0.4957	0.07	0.9463	1.071	1.388	0.4672
0.019	0.7677	1.214	1.736	0.2722	0.075	0.9632	1.03	1.371	0.4852	0.075	0.9242	1.073	1.396	0.4602
0.025	0.7625	1.224	1.83	0.2401	0.08	0.9681	1.06	1.378	0.4766	0.08	0.9156	1.073	1.398	0.4582
0.031	0.7563	1.223	1.918	0.2151	0.085	0.9517	1.061	1.383	0.4722	0.085	0.915	1.082	1.401	0.455
0.037	0.7543	1.217	1.996	0.1963	0.09	0.9189	1.069	1.396	0.4605	0.09	0.9037	1.082	1.405	0.4517
0.043	0.7613	1.21	2.061	0.183	0.095	0.9087	1.09	1.406	0.4502	0.095	0.9166	1.103	1.408	0.4477
0.049	0.7654	1.203	2.131	0.1695	0.1	0.8982	1.094	1.411	0.4458	0.1	0.8982	1.094	1.411	0.4458
0.055	0.7725	1.198	2.199	0.1574	0.105	0.8504	1.1	1.431	0.4295	0.105	0.8988	1.121	1.421	0.436
0.061	0.8021	1.207	2.259	0.1468	0.11	0.8582	1.143	1.445	0.4153	0.11	0.8919	1.125	1.425	0.4324
0.067	0.7667	1.16	2.31	0.1437	0.115	0.8503	1.181	1.464	0.3981	0.115	0.9219	1.162	1.425	0.4291
0.073	0.8296	1.192	2.366	0.1329	0.12	0.7904	1.167	1.486	0.3836	0.12	0.6752	0.9839	1.438	0.4337
0.079	0.8461	1.186	2.417	0.1268	0.125	0.7552	1.203	1.522	0.3564	0.125	0.8649	1.136	1.439	0.4202
0.085	0.8535	1.174	2.47	0.1213	0.13	0.7559	1.253	1.55	0.3345	0.13	0.846	1.153	1.454	0.4076
0.091	0.8446	1.149	2.521	0.1176	0.135	0.7428	1.308	1.594	0.3043	0.135	0.8493	1.175	1.462	0.4001
0.097	0.8354	1.127	2.574	0.1136	0.14	0.6528	1.307	1.667	0.2672	0.14	0.8371	1.184	1.471	0.3927

Note. p varies between 0.001 and 0.097 and the other parameters are set to their value in the baseline specification in the first five columns; from column six to ten, σ_ψ varies between 0.06 and 0.14 and the other parameters are set to their value in the baseline specification; from column eleven to fifteen σ_θ varies between 0.06 and 0.14 and the other parameters are set to their value in the baseline specification. In the baseline specification $\beta = 0.96$, $\rho = 2$, $G = 1.03$, $R = 1.04$, $p = 0.005$, $\sigma_\psi = \sigma_\theta = 0.1$.

TABLE 5. The log-linearized Euler equation

			Actual		
r	0.013	0.010	0.018	0.005	0.018
	(0.008)	(0.007)	(0.008)*	(0.006)	(0.008)*
constant	0.026	0.030	0.024	0.028	0.026
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
			Approximate		
r	0.015	0.009	0.025	0.009	0.019
	(0.006)*	(0.006)	(0.006)***	(0.006)	(0.007)**
constant	0.025	0.029	0.023	0.026	0.025
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***
			Error		
r	-0.002	0.001	-0.006	-0.004	-0.001
	(0.002)	(0.001)	(0.002)**	(0.001)**	(0.002)
constant	0.001	0.001	0.001	0.001	0.001
	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***

Note. The table shows the estimated coefficients of the log-linearized Euler equation. In each column β, G, and p are set to 0.96, 1.03, and 0.005, respectively. In the first, second and third column, ρ is set to 2, in the fourth to 4 and in the fifth 1.5. The standard deviation of log income shocks is set to 0.1 in the first, the fourth, and the fifth, to 0.05 in the second and to 0.12 in the third. Standard errors are reported in parenthesis.

TABLE 6. Approximation error

			Interest factor equal to 1.025		
Average Error	-0.028	-0.025	-0.032	-0.040	-0.025
	(0.012)*	(0.012)*	(0.012)**	(0.010)***	(0.014)
			Interest factor equal to 1.055		
Average Error	0.009	0.005	0.016	0.013	0.010
	(0.007)	(0.004)	(0.008)	(0.008)	(0.007)

Note. In the top panel the interest factor is 1.025, in the bottom 1.055. The approximate consumption function is obtained with 1.04 interest factor. The error is computed as the difference between actual and approximate consumption. In each column β, G, and p are set to 0.96, 1.03, and 0.005, respectively. In the first, second and third column, ρ is set to 2, in the fourth to 4 and in the fifth 1.5. The standard deviation of log income shocks is set to 0.1 in the first, the fourth, and the fifth, to 0.05 in the second and to 0.12 in the third. Standard errors are reported in parenthesis.

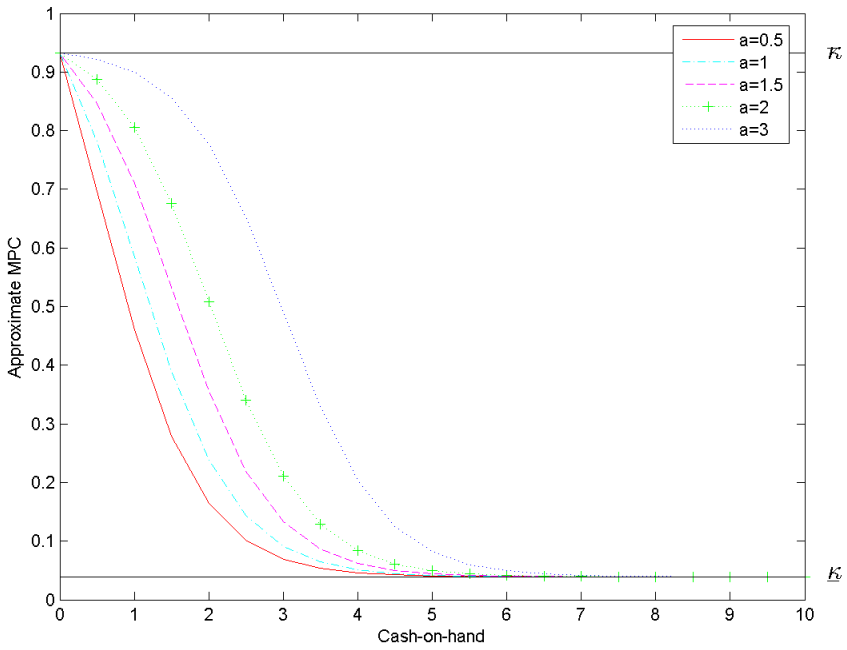

FIGURE 1. The effect of a on the marginal propensity to consume

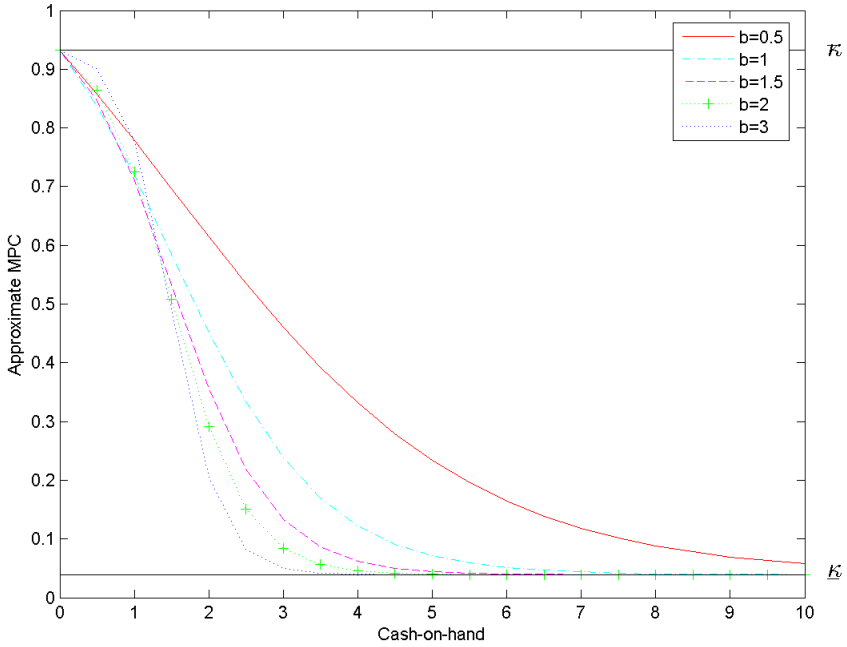

FIGURE 2. The effect of b on the marginal propensity to consume

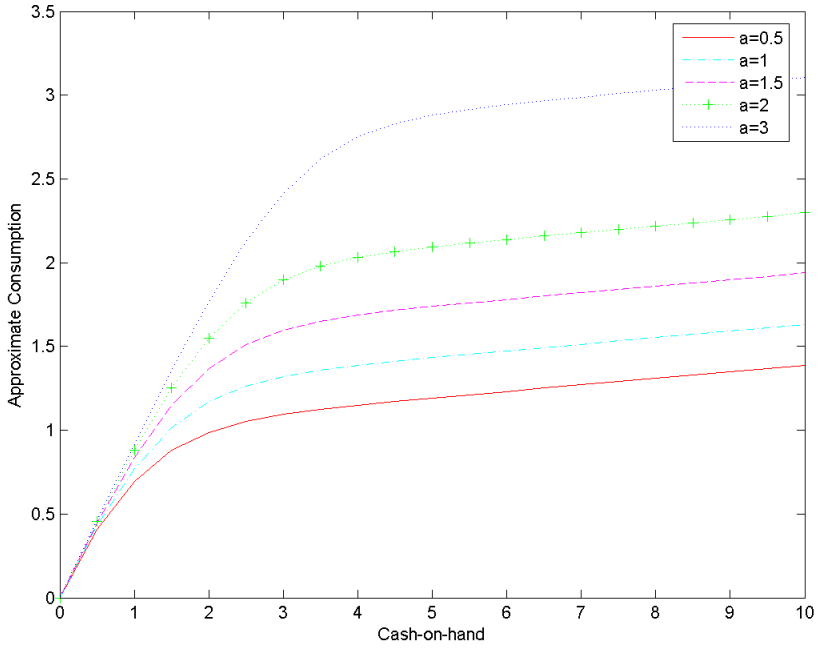

FIGURE 3. The effect of a on the approximate consumption function

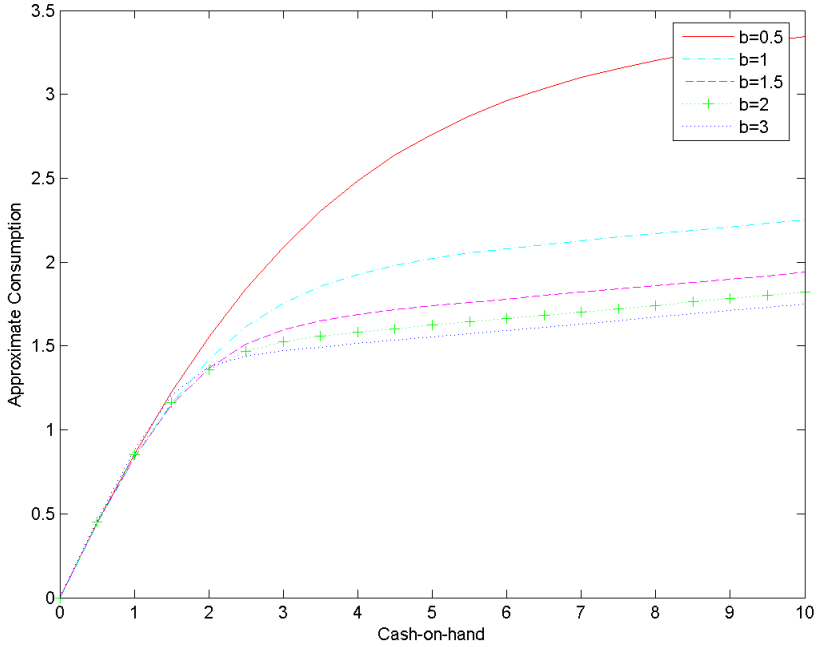

FIGURE 4. The effect of b on the approximate consumption function

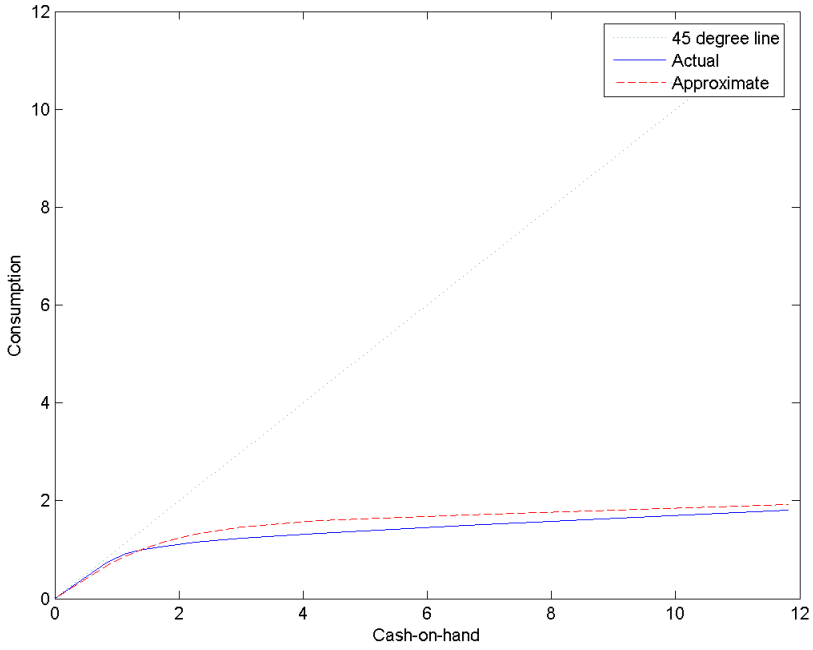

FIGURE 5. Approximate and actual consumption function. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

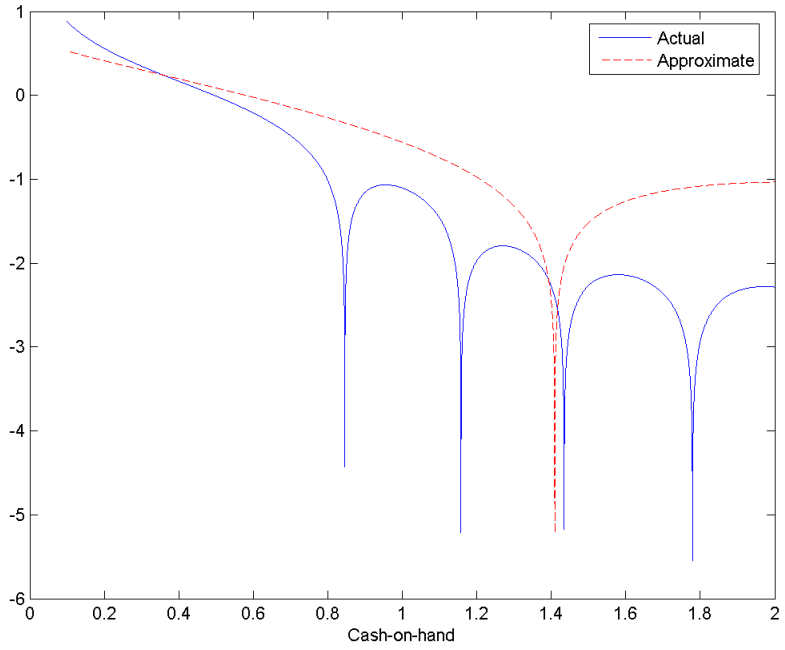

FIGURE 6. Log 10 of the absolute value Euler Equation errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

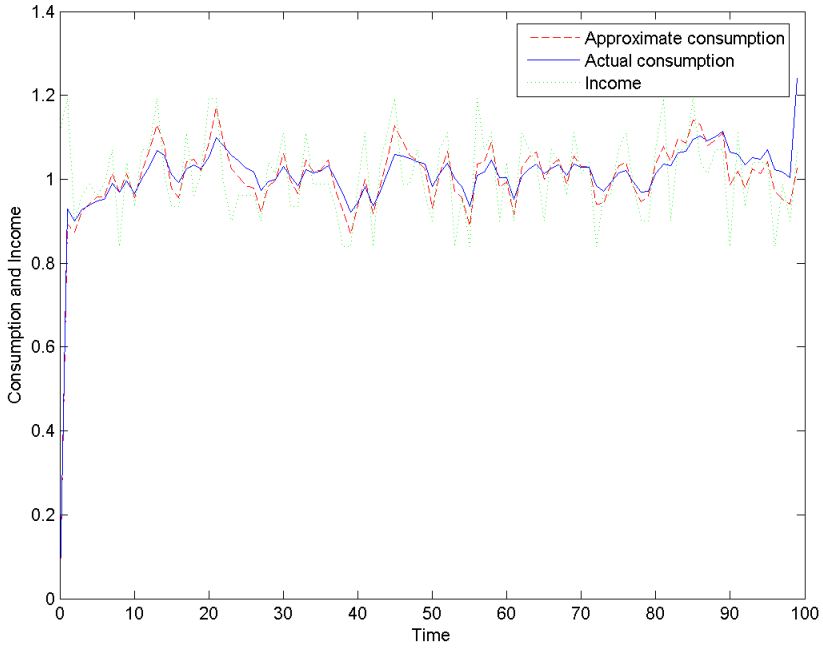

FIGURE 7. Consumption and Income. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

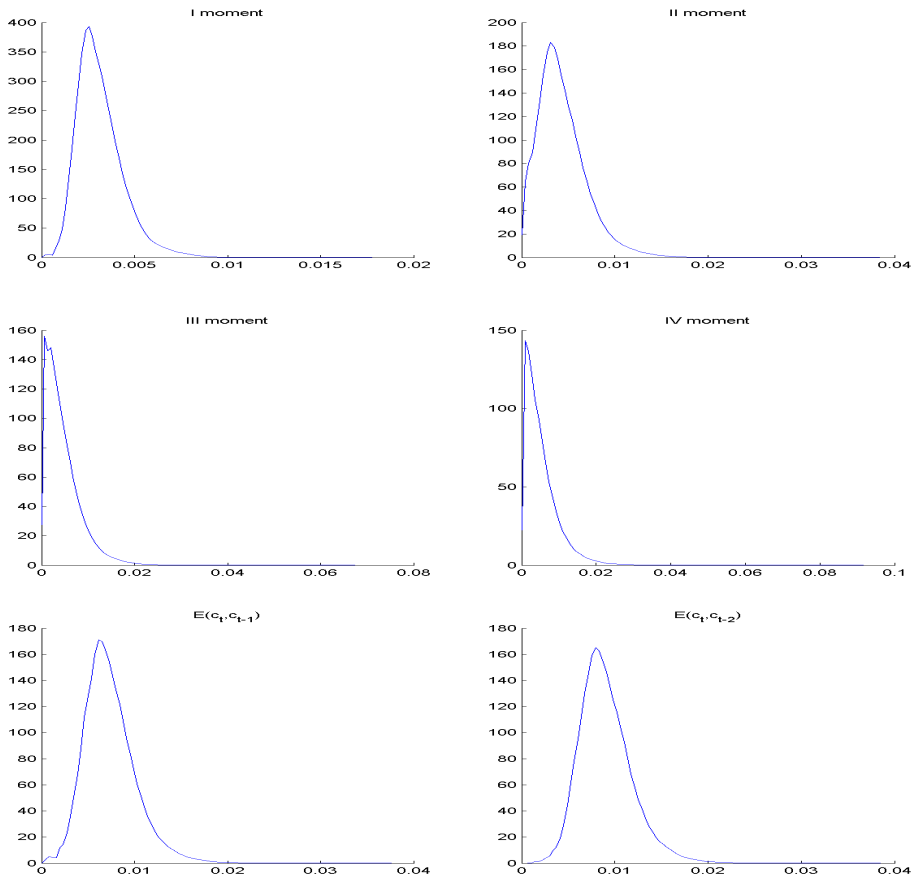

FIGURE 8. Across simulations distribution of relative errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

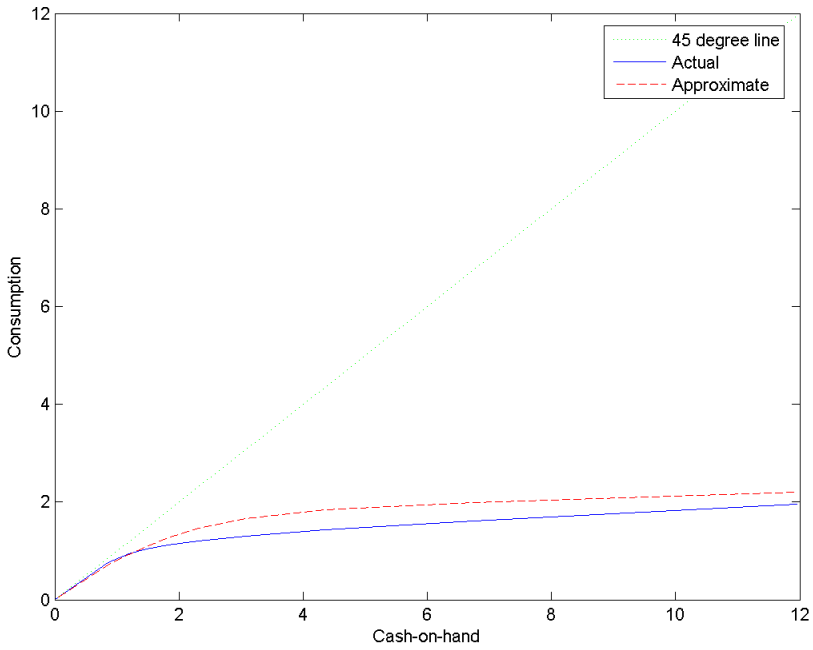

FIGURE 9. Approximate and actual consumption function. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.05$.

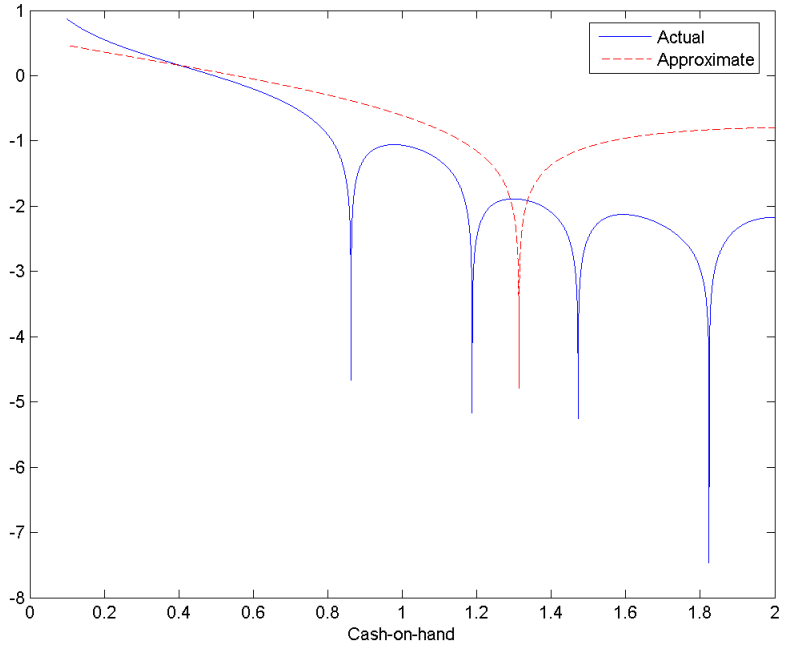

FIGURE 10. Log 10 of the absolute value Euler Equation errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.05$.

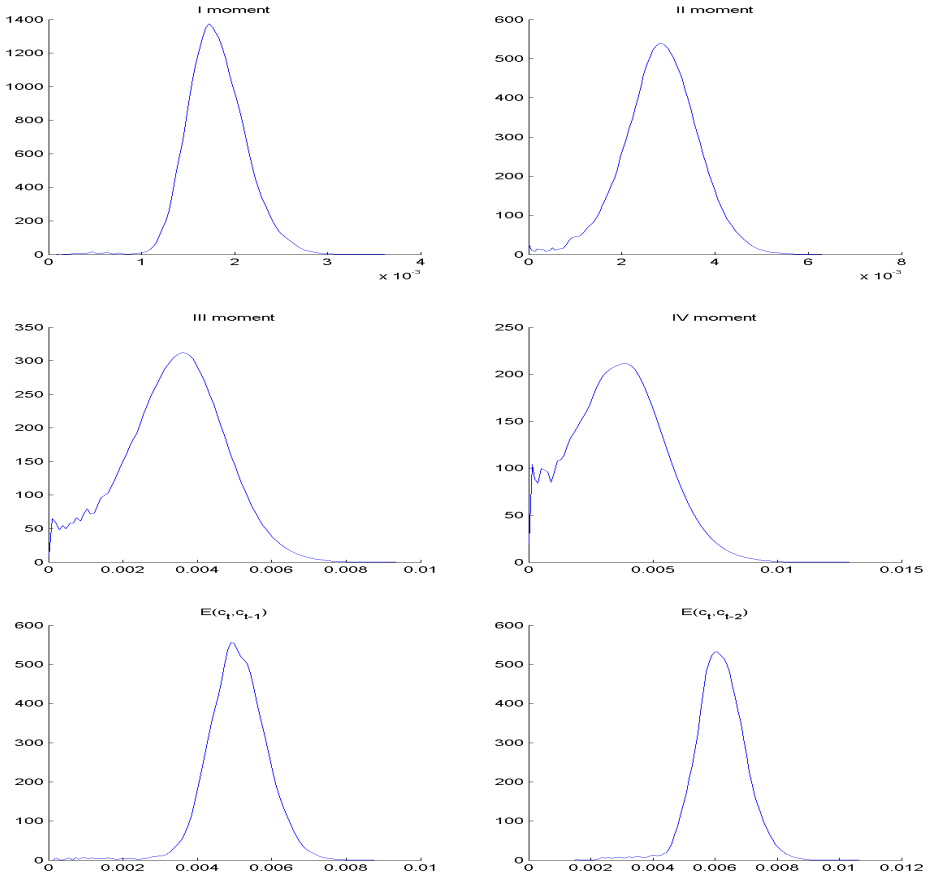

FIGURE 11. Across simulations distribution of relative errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.05$.

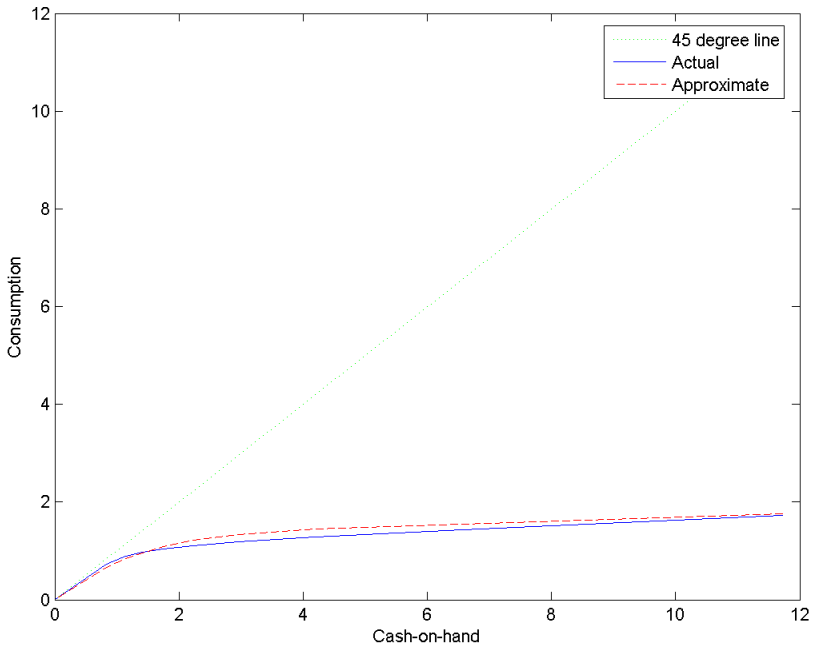

FIGURE 12. Approximate and actual consumption function. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.12$.

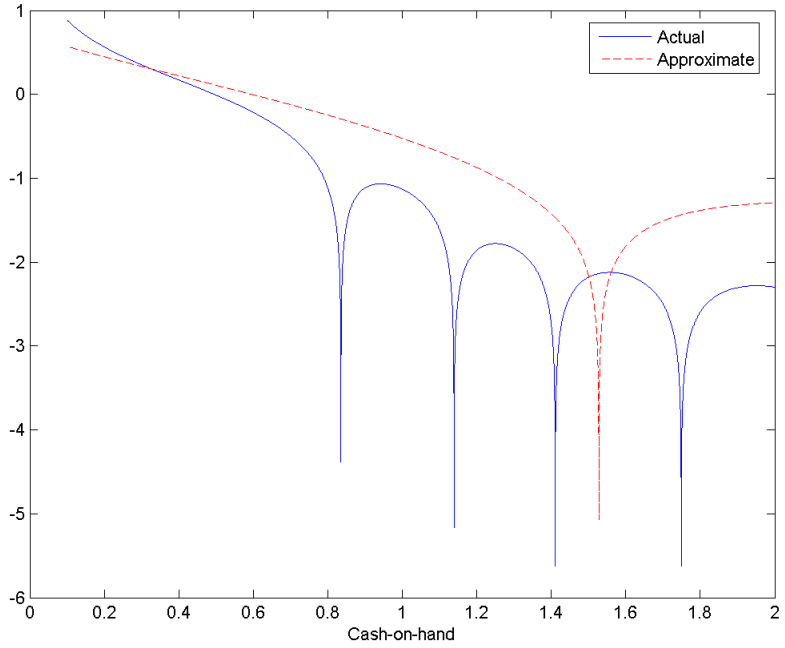

FIGURE 13. Log 10 of the absolute value Euler Equation errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.12$.

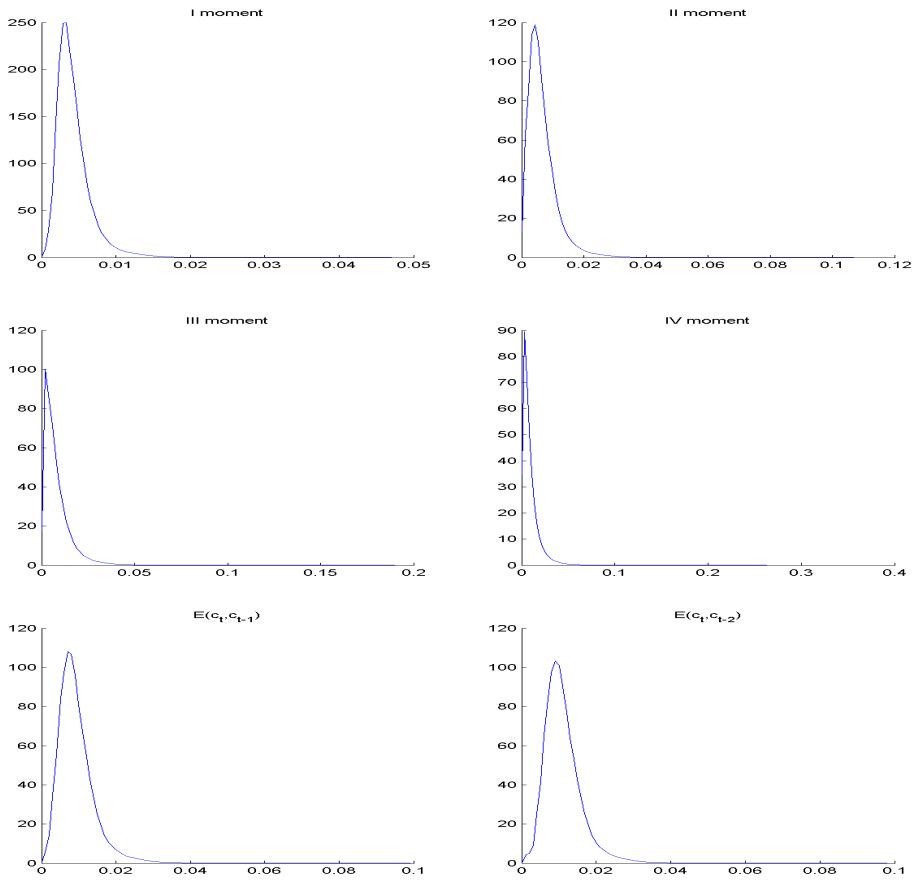

FIGURE 14. Across simulations distribution of relative errors. $G = 1.03$, $R = 1.04$, $\rho = 2$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.12$.

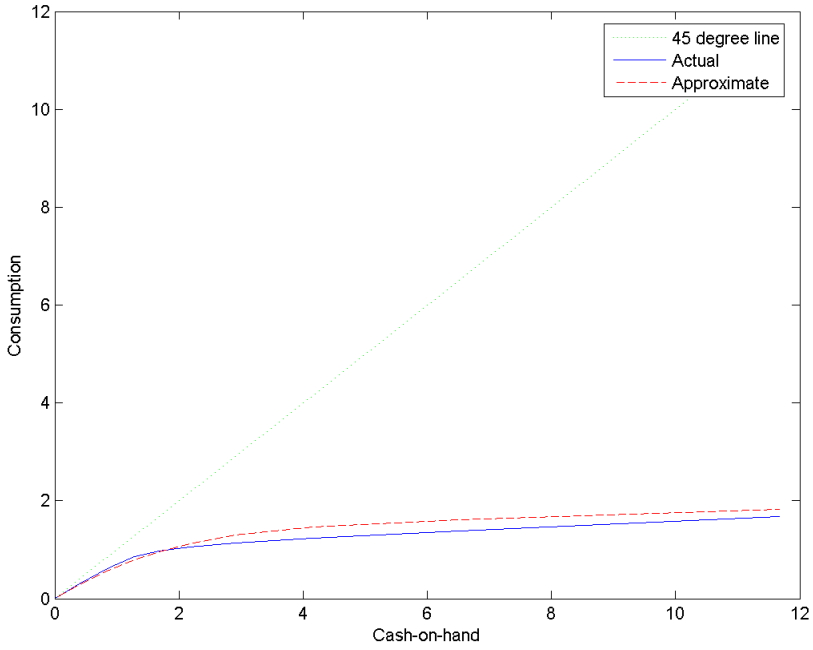

FIGURE 15. Approximate and actual consumption function. $G = 1.03$, $R = 1.04$, $\rho = 4$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

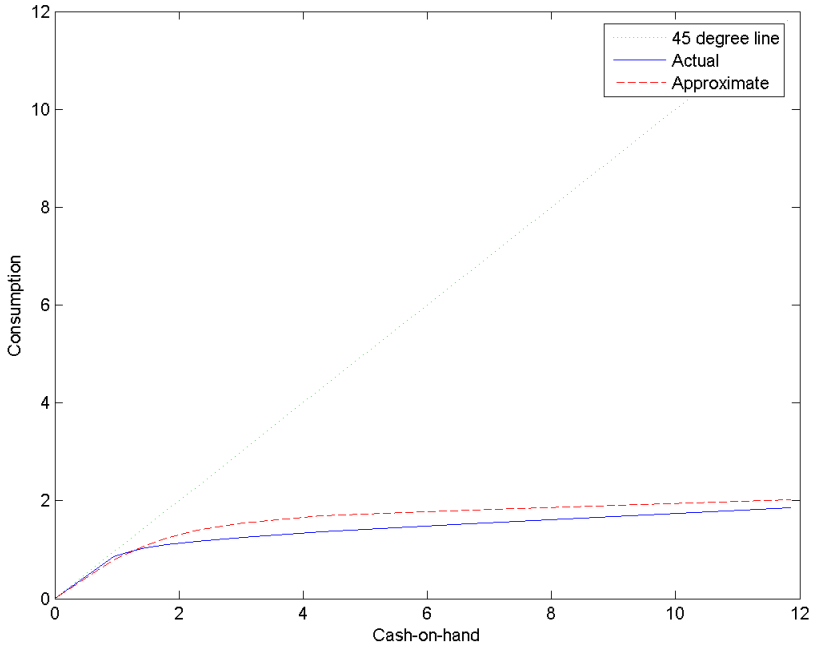

FIGURE 16. Approximate and actual consumption function. $G = 1.03$, $R = 1.04$, $\rho = 1.5$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

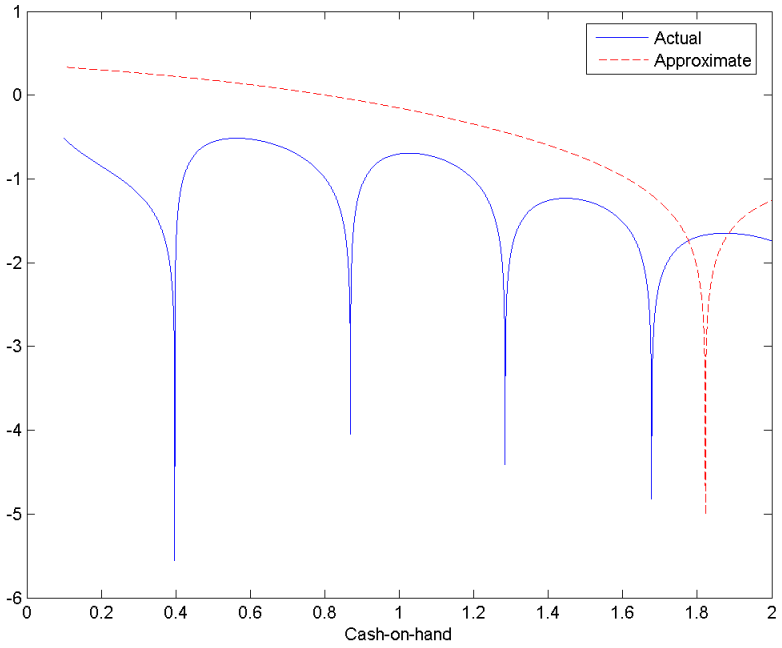

FIGURE 17. Log 10 of the absolute value Euler Equation errors. $G = 1.03$, $R = 1.04$, $\rho = 4$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

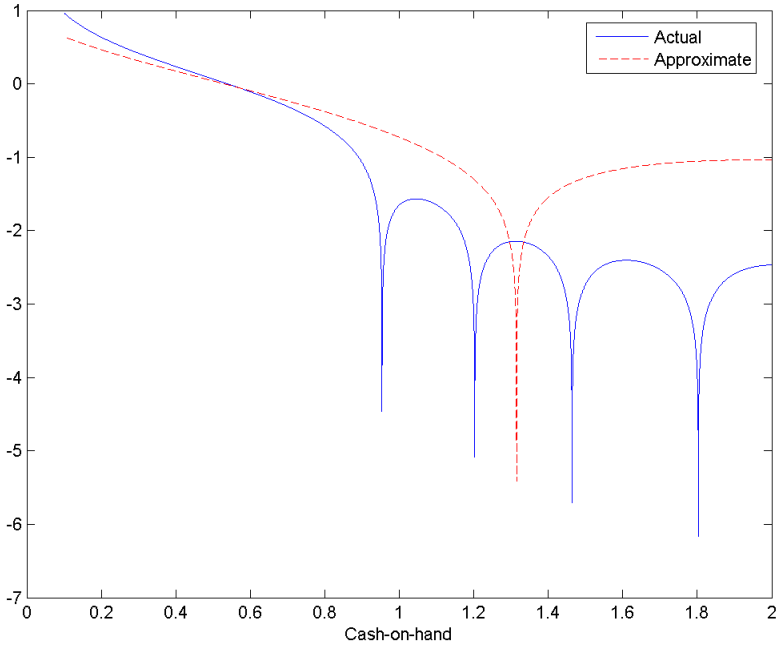

FIGURE 18. Log 10 of the absolute value Euler Equation errors. $G = 1.03$, $R = 1.04$, $\rho = 1.5$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

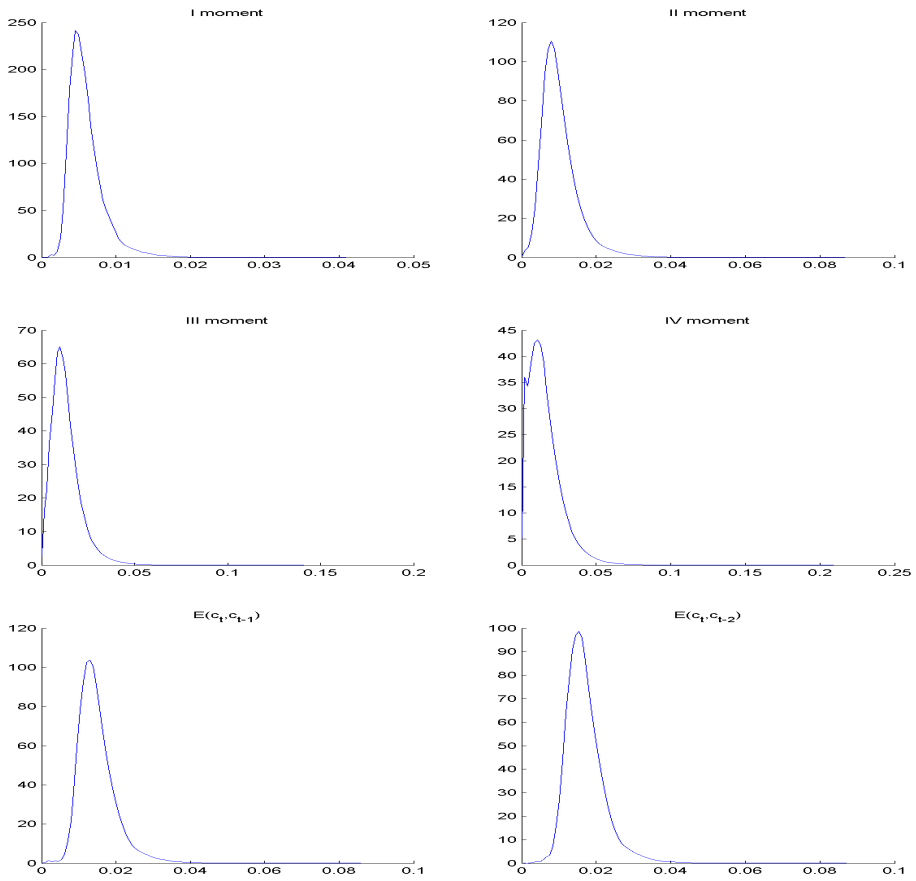

FIGURE 19. Across simulations distribution of relative errors. $G = 1.03$, $R = 1.04$, $\rho = 4$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.

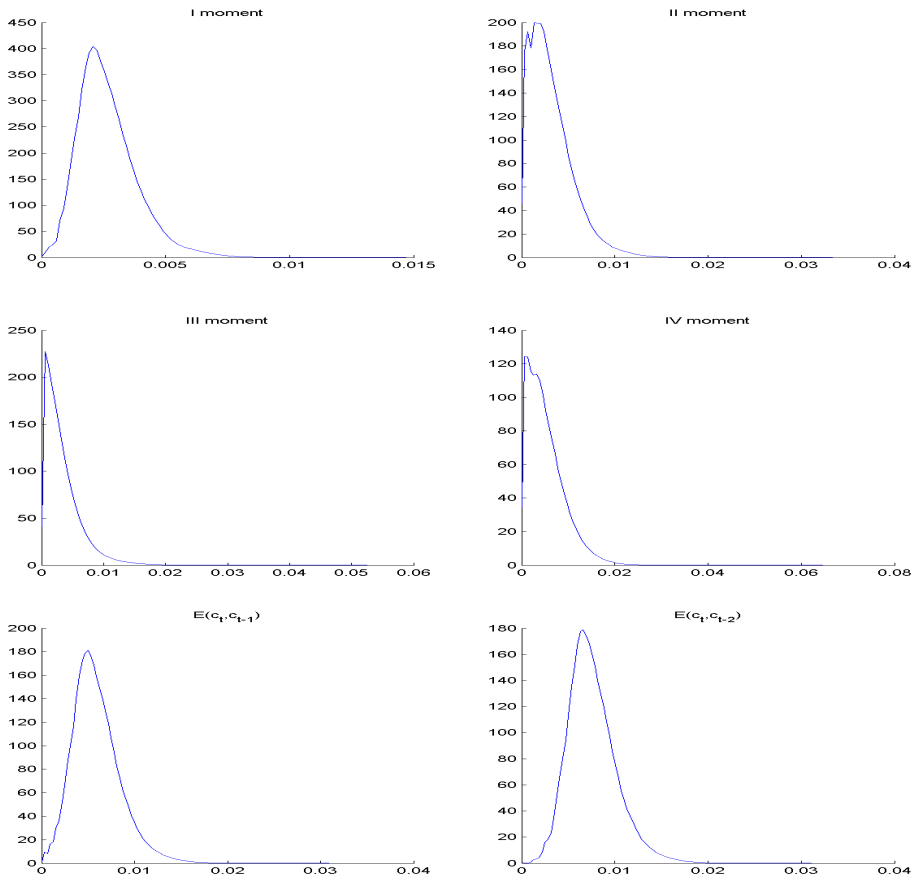

FIGURE 20. Across simulations distribution of relative errors. $G = 1.03$, $R = 1.04$, $\rho = 1.5$, $\beta = 0.96$, $p = 0.005$, $\sigma_\theta = \sigma_\psi = 0.1$.