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Institutional Trades and Herd Behavior in Financial Markets

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Abstract
The article studies the impact of transaction costs on the trading strategy of informed institutional investors in a sequential trading market where traders can choose to transact a large or a small amount of the stock. The analysis shows that high transaction costs may induce informed investors to herd. Moreover, for low levels of transaction costs, informed investors trade both the large and the small quantity of the asset. Finally, if transaction costs are very low and the market width is large enough, informed traders prefer to separate from small liquidity traders.

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1 Introduction

Recent research on market microstructure has shown a renewed interest in imitative behavior in financial economies. Several exceptional events in asset markets in the past years seem to contradict the efficient market paradigm. For example, on 19 October 1987, US stock prices fell by 22 per cent on the one day. It is difficult to identify some relevant economic news at the time that could account for a fall of this size. Again, the overpricing of US technology stocks in the second half 1990s cannot be fully explained in terms of efficient market hypothesis framework. Herding behavior has often been cited as one of the factors that can generate serious asset price inefficiencies and misbehavior.

Traditionally, herding has been related to irrational decision makers. The recent theoretical literature has advanced new rational models of herding. Informational cascades belong to this strand of literature on rational herding.[1]

An informational cascade occurs where it is optimal for individuals, after observing the actions of previous agents, to ignore their own information and mimic the decisions of their predecessors. The standard models of informational cascades apply to frameworks where prices are fixed, and therefore they can hardly be applied to asset markets, where prices adjust continuously to reflect all the relevant information. However, some recent theoretical models have analyzed mechanisms that may lead to informational cascades in asset markets.

The most interesting results of this strand of literature are the following. In the standard setting proposed by Glosten and Milgrom [9], price adjustment prevents the occurrence of an informational cascade in equilibrium. Nevertheless, multidimensional uncertainty and, more generally, non-monotonic signals open the possibility of herd behavior that may lead to a significant, short-run mispricing of assets (Avery and Zemsky, [1]). Moreover, informational cascades may develop in the case of different risk aversion among traders and market makers (Decamps and Lovo [6]; Cipriani and Guarino [4]), when traders care about their reputation for ability (Dasgupta and Prat [5]), if informed market participants must pay a brokerage fee to trade (Lee [12]), or in the case of costly market making (Romano [13]).

Extant literature focuses on frameworks where traders are restricted to transact a fixed quantity of the stock. However, during the last decades the importance of block trading in common stocks has raised significantly. On the New York Stock Exchange, the fraction of large transactions represented just two percent of the volume of trading in 1960 and over one half in 2001.[2] This has underscored the importance of studying the impact of block trading on stock prices and on the occurrence of informational cascades. [3]

[1] Informational Cascades first appeared in Banerjee [2] and in Bikhchandani, Hirshleifer and Welch [3]. Banerjee [2] and Bikhchandani, Hirshleifer and Welch [3] independently show in different settings that informational cascades (or "herding", in Banerjee's terminology) will eventually occur with certainty.

[2] Gemmill and Lakonishok [8]; Keim and Madhavan [10].

[3] Keim and Madhavan [11]; Saar [14]; Seppi [15].

This paper aims at contributing to this research area by analyzing informational cascades arising from transaction costs increasing in the trade size, in capital markets with asymmetric information, sequential trading and competitive price mechanism.

To this purpose, we develop a sequential trading model where traders are allowed to transact different trade sizes. This ability to transact orders for large or small quantities introduces a strategic element in the trading game. Informed traders choose their optimal order size given the price schedule. If risk neutral informed traders wish to trade, they prefer to trade larger amount at any given price. Consequently, the market maker sets a larger spread for larger trades in every equilibrium with information based trading. The crucial assumption is that the market maker bears an exogenous cost for executing orders which is proportional to the trade size. Such cost gives informed traders an incentive to purchase small, rather than large quantity.

We show that, for any public belief about the true asset value, three types of equilibria can arise depending on the magnitude of the transaction cost. In the separating equilibrium, informed traders place only large orders. Hence, small orders are uninformative and the spread for small quantities only reflects the exogenous transaction cost. This outcome occurs if the transaction cost is very low and the difference between the size of large and small orders is sufficiently large. For intermediate values of the transaction cost, a pooling equilibrium prevails in the market. In this equilibrium, informed traders trade both the large and the small amounts. The large trade spread reduces with respect to the separating equilibrium, whereas the spread for small quantities increases because of information costs. Finally, if the transaction cost exceeds the informational advantage of traders observing private signals on the true asset value, all informed traders prefer to refrain from trading. Orders do not convey any information about the fundamental value and the exogenous transaction cost is the only source of the spread for both small and large quantities.

The informational content of orders reaches its maximum level in the separating equilibrium. It reduces when a pooling equilibrium prevails in the market and tends to zero in the no-trading equilibrium, as an informational cascade develops. As in Romano [13], trading volume gradually is shown to decrease before a cascade occurs, and to reach its lowest value as the cascade develops.

The remainder of this paper is structured as follows. In section 3 we describe the model. In section 3 we define and derive the market equilibrium and discuss the impact of transaction costs on the learning process. In section 4 we conclude. All the proofs of propositions are in the appendix.

2 The Economy

We consider a sequential trading model *à* la Glosten and Milgrom [9], modified to account for different trade sizes.[4]

The market is for a risky asset which is exchanged among a sequence of risk neutral traders and competitive market makers who are responsible for quoting prices. The value $\widetilde{V}$ of the asset can be low ($\widetilde{V} = \underline{V}$) or high ($\widetilde{V} = \overline{V}$). The ex-ante probability of $\widetilde{V} = \overline{V}$ is $\pi_0 \in (0,\, 1)$.

Trades occur sequentially, with one trader allowed to transact at any point of time. The trader whose turn is to transact may either buy a small or a large quantity, or sell a small or a large quantity, or refrain from trading. We denote by Q_S and Q_L, with $Q_S < Q_L$, the size, respectively, of small and large orders, and by $A \equiv \{SQ_L,\ SQ_S,\ BQ_S,\ BQ_L,\ NT\}$ the traders' action space, with $SQ_i$and BQ_i indicating, respectively, a sell and a buy order for quantity Q_i, with $i = S, L$, and NT indicating a zero trading order.

There are two types of traders: liquidity traders (fraction $1-\mu$) and informed traders (fraction μ). To simplify the analysis, we assume that liquidity traders choose to submit large or small sell and buy orders and to refrain from trading with equal probability $\gamma = \frac{1-\mu}{5}$.

Informed traders privately observe a signal θ correlated with the asset value. The set of private signals is $\Theta = \{\underline{\theta}, \overline{\theta}\}$; the signal $\underline{\theta}$ indicates $\underline{V}$ and the signal $\overline{\theta}$ indicates $\overline{V}$. We assume that signals are symmetric and we let be $p = Pr(\underline{\theta}|\underline{V}) = Pr(\overline{\theta}|\overline{V}) > \frac{1}{2}$.

The expected asset value of an informed trader at time t is denoted by $E_t[\widetilde{V} \mid \theta]$ and the market maker's expectation by $E_t[\widetilde{V}]$. Finally, we denote by π_t the probability that market maker attaches to $\overline{V}$ at time t.[5]

Before a trader arrives, the market maker sets bid and ask prices at which he is willing to trade each asset quantity. We suppose that any transaction costs to the market maker $c \cdot Q$ euros, where $Q \in \{Q_S,\, Q_L\}$.

3 The Equilibrium Strategies and Prices

At the beginning of any trading round t, the market maker sets competitive prices. We denote P_t the price schedule at time t. Clearly:

$$P_t = \{B_{L,\,t},\ B_{S,\,t},\ A_{S,\,t},\ A_{L,\,t}\}$$

where $B_{L,\,t}$ is the bid price for the large quantity, $B_{S,\,t}$ the bid price for the small quantity, $A_{S,\,t}$ the ask price for the small quantity, and $A_{L,\,t}$ the ask price

[4] The approach taken involves a sequential trade model similar to that of Easley and O'Hara [7].

[5] By the Bayes rule:

$$E_t[\widetilde{V} \mid \theta] = \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)\frac{Pr(\theta|\underline{V})}{Pr(\theta|\overline{V})}} \cdot (\overline{V} - \underline{V}),$$

which exceeds $E_t[\widetilde{V}]$ if $\theta = \overline{\theta}$, and which is lower than $E_t[\widetilde{V}]$ if $\theta = \underline{\theta}$.

for the large quantity.

After prices are set, a trader, randomly selected to trade, observes the price schedule and plays his strategy.

If the selected trader is a liquidity trader, he acts in the ex-ante specified probabilistic way. If the selected trader is informed, he chooses the strategy which maximizes his expected profit given the price schedule.

The market maker anticipates the strategies of informed traders and announces his price schedule. Bertrand competition restricts the market maker to earn zero expected profit from each trade. Hence, the trader arriving at t faces a price schedule which satisfies:

$$\begin{aligned} B_{i,\,t} &= E_t[\widetilde{V}\,|SQ_i] - c \quad \forall\, i \in \{S,\ L\} \\ A_{i,\,t} &= E_t[\widetilde{V}\,|BQ_i] + c \quad \forall\, i \in \{S,\ L\}. \end{aligned} \tag{1}$$

Following Easly and O'Hara [7], we are interested in an equilibrium price schedule P_t^* in which bid prices $B_{S,\,t}^*$ and $B_{L,\,t}^*$ and ask prices $A_{S,\,t}^*$ and $A_{L,\,t}^*$ straddle the unconditional expected asset value $E_t[\widetilde{V}]$. Given a such equilibrium, traders observing a good signal will never prefer to sell the asset, and traders with a bad signal will never prefer to buy the asset. However, depending on the parameters of the model, different outcomes may prevail. If informed traders prefer to trade only the large quantity, they are separated from small liquidity traders. We call this a separating equilibrium. If informed traders submit either small or large orders with positive probability, a pooling equilibrium occurs. If they prefer to refrain from trading, a no trading equilibrium occurs.

It is useful to stress that the equilibrium on one side of the market may differ from the equilibrium on the other side. For example, traders observing $\underline{\theta}$ may prefer to sell only large quantity and traders observing $\overline{\theta}$ may choose to buy with positive probability either small or large quantity. In the next sections we show how different outcomes can occur by changing the transaction cost c.

3.1 The no-trading equilibrium

In the no-trading equilibrium, no sell or buy order arises from informed traders. Since transactions are not information based, they do not affect the probability that the market maker attaches to $\overline{V}$, and the expected asset value, conditional on a trading order, is always equal to the unconditional expectation. Therefore, the competitive price schedule is $P^{ne} = \{B_L^{ne},\, B_S^{ne},\, A_S^{ne},\, A_L^{ne}\}$[6] such that:

$$\begin{aligned} B_L^{ne} = \quad & B_S^{ne} = \quad E[\widetilde{V}] - c = \quad B^{ne} \\ A_S^{ne} = \quad & A_L^{ne} = \quad E[\widetilde{V}] + c = \quad A^{ne}. \end{aligned}$$

P^{ne} is an equilibrium price schedule when the expected profit from trading for informed is strictly negative whatever quantity of asset they choose to sell or

[6] For brevity we omit the time subscript.

buy. In particular, the no-trading equilibrium prevails in the ask side of market when $E[\widetilde{V}\,|\overline{\theta}] - A^{ne} < 0$, that is true only if:

$$c > \left(\frac{\pi}{\pi + (1-\pi)\frac{1-p}{p}} - \pi\right) \cdot (\overline{V} - \underline{V}) = c^{ne}(\pi,\, \overline{\theta}).$$

Similarly, the bid side of market is in the no-trading equilibrium when $B^{ne} - E[\widetilde{V}\,|\underline{\theta}] < 0$, that is true only if:

$$c > \left(\pi - \frac{\pi}{\pi + (1-\pi)\frac{p}{1-p}}\right) \cdot (\overline{V} - \underline{V}) = c^{ne}(\pi,\, \underline{\theta}).$$

These results are summarized in the following proposition.

Proposition 1 *For any public belief $\pi \in (0,\, 1)$ on $\widetilde{V} = \overline{V}$, a no-trading equilibrium prevails in the bid side of market if, and only if, $c > c^{ne}(\pi,\, \underline{\theta})$. It prevails in the ask side of market if, and only if, $c > c^{ne}(\pi,\, \overline{\theta})$.*

$c^{ne}(\pi,\, \theta)$ is equal to the informational advantage of traders observing θ. Hence, the no-trading equilibrium prevails if the marginal transaction cost exceeds the informational advantage of informed traders.

To gain an intuition about the link between the public belief on the true asset value and the threshold costs, that is the informational advantage of informed traders, let's consider the limit case of perfect signals ($p = 1$), and suppose that $\widetilde{V} = \overline{V}$. Since signals are perfect, the expected asset value of informed traders is always $\overline{V}$, whatever public belief. The informational advantage of informed traders is equal to $(\overline{V} - \underline{V})(1 - \pi)$. It is low if the probability the market maker attaches to $\overline{V}$ is high, and it grows as the public belief about $\overline{V}$ approaches to 0. Then, the more the valuation of the market differs from $\overline{V}$, the higher the cost that informed are willing to pay in order to transact.

In a similar way, when signals are not perfect ($p < 1$), the informational advantage of traders observing θ is low if the public belief is consistent with θ, and it grows as the valuation of the market maker moves in the opposite direction with respect to the signal. But, unlike the case of perfect signals, if the market maker attaches a very low probability to the asset value consistent with θ, then the informational advantage of traders observing θ decreases. This occurs because, when signals are not perfect, the expectation of informed traders depends not only on the private signal, but also on the public belief. If the past history of trades strictly indicates a value inconsistent with θ, traders observing this signal attach a low weight to their private information respect to the public belief. However, if the market attaches a greater probability to $\underline{V}$ ($\pi < \frac{1}{2}$), the informational advantage of traders observing $\underline{\theta}$ is greater than that of traders observing $\overline{\theta}$; if the market attaches a greater probability to $\overline{V}$ ($\pi > \frac{1}{2}$), the reverse is true.

As a consequence, the threshold cost for the no trading equilibrium in the ask side of market exceeds that in the bid side when the market maker attaches a greater probability to $\underline{V}$ ($\pi < \frac{1}{2}$) and it is lower otherwise.

In the no-trading equilibrium, no new information reaches the market because all informed traders choose to refrain from trading. Therefore, the economy is in an informational cascade.

It is interesting to notice that, the occurrence of an informational cascade does not depend on the asset quantities that agents can sell or buy. This result follows from the interaction between the risk neutrality of agents and the assumption of constant marginal transaction costs.

3.2 The separating equilibrium

A separating equilibrium prevails in the market when the competitive price schedule, $P^{se} = \{B_L^{se}, B_S^{se}, A_S^{se}, A_L^{se}\}$, is such that informed traders place only large orders. Thus, small trades are not information-based and do not affect the public belief about the true asset value, while the information content of large trades is very strong.

This implies that, the equilibrium prices for the small orders are given by:

$$\begin{aligned} B_S^{se} = &\ \underline{V} + \pi \cdot (\overline{V} - \underline{V}) - c \\ A_S^{se} = &\ \underline{V} + \pi \cdot (\overline{V} - \underline{V}) + c, \end{aligned}$$

and the equilibrium prices for the large orders are given by:

$$\begin{aligned} B_L^{se} = &\ \underline{V} + \tfrac{\pi}{\pi + (1-\pi)\lambda_{SQ_L}^{se}} \cdot (\overline{V} - \underline{V}) - c \\ A_L^{se} = &\ \underline{V} + \tfrac{\pi}{\pi + (1-\pi)\lambda_{BQ_L}^{se}} \cdot (\overline{V} - \underline{V}) + c, \end{aligned}$$

where $\lambda_{SQ_L}^{se} \equiv \frac{\gamma + \mu p}{\gamma + \mu(1-p)}$ and $\lambda_{BQ_L}^{se} \equiv \frac{\gamma + \mu(1-p)}{\gamma + \mu p}$ are, respectively, the likelihood ratio of a large sell and buy order. Clearly, $\lambda_{SQ_L}^{se} > 1$ because a large selling order could be submitted by a trader with a bad signal and $\lambda_{BQ_L}^{se} < 1$ to reflect the probability of trading with a trader observing a good signal.

A separating equilibrium prevails only if, given the price schedule P^{se}, informed traders prefer to trade large. For the bid side of the market this means:

$$(B_L^{se} - E[\widetilde{V} \,|\, \underline{\theta}])Q_L \geq (B_S^{se} - E[\widetilde{V} \,|\, \underline{\theta}])Q_S \geq 0, \tag{2}$$

and for the ask side:

$$(E[\widetilde{V} \,|\, \overline{\theta}] - A_L^{se})Q_L \geq (E[\widetilde{V} \,|\, \overline{\theta}] - A_S^{se})Q_S \geq 0. \tag{3}$$

A separating equilibrium occurs in the market when the expected profit from trading for informed traders is strictly positive, and the advantage due to the large quantity exceeds the better price available for the small trades.

Consider the ask side of market and suppose that, given the price schedule P^{se}, the expected profit from buying the large quantity of a trader observing $\overline{\theta}$ is strictly positive. The difference between the expected profit from buying the large and the small quantity can be written as follows: $\Delta(\Pi) = [(E[\widetilde{V} \mid \overline{\theta}] - A_L^{se})(Q_L - Q_S)] - [(A_L^{se} - A_S^{se})Q_S]$. The first term represents the expected gain due to the greater quantity of asset bought, and the second term is the loss due to the higher price paid to purchase the first Q_S units of asset. An informed trader endowed with $\overline{\theta}$ chooses to place a large order if this difference is positive. Clearly, the separating equilibrium is more likely to prevail when the distance between the large and the small quantity is greater. The following proposition states a necessary condition on Q_L and Q_S for a separating equilibrium to occur on each side of market. Denote with $\lambda_{\underline{\theta}}$ and $\lambda_{\overline{\theta}}$ the informational content of signals $\underline{\theta}$ and $\overline{\theta}$, where: $\lambda_{\underline{\theta}} \equiv p/\left(1-p\right)$ and $\lambda_{\overline{\theta}} \equiv \left(1-p\right)/p$.

Proposition 2 *Given the public belief π, if a separating equilibrium prevails in the bid side of market, then:*

$$\frac{Q_L}{Q_S} > \frac{\lambda_{\underline{\theta}} - 1}{\lambda_{\underline{\theta}} - \lambda_{SQ_L}^{se}}(\pi + (1-\pi)\lambda_{SQ_L}^{se}), \tag{4}$$

and if it prevails in the ask side of market, then:

$$\frac{Q_L}{Q_S} > \frac{1 - \lambda_{\overline{\theta}}}{\lambda_{BQ_L}^{se} - \lambda_{\overline{\theta}}}(\pi + (1-\pi)\lambda_{BQ_L}^{se}). \tag{5}$$

Proof. See Appendix.

A separating equilibrium can prevail in the market only if Q_L is sufficiently larger than Q_S. From proposition 2, it follows that the minimum distance between the large and the small quantity to observe a separating equilibrium reduces as signals become more informative. Moreover, it is easy to verify that in the absence of transaction costs, if Q_S and Q_L satisfy conditions 4 and 5, the market equilibrium is separating. On the other hand, if the distance between Q_L and Q_S is too small in relation to the informational advantage of informed traders, a separating equilibrium never prevails in the market whatever π is and apart from the existence of the transaction cost c. This result is illustrated by the following proposition.

Proposition 3 *If $\frac{Q_L}{Q_S} < \frac{1-\lambda_{\overline{\theta}}}{\lambda_{BQ_L}^{se} - \lambda_{\overline{\theta}}}$, a separating equilibrium never occurs in the market.*

Proof. See Appendix.

The minimum size of the relative distance between the large and the small quantity depends on the probability of a large liquidity order. If the fraction of liquidity traders is small, the information that the market maker can infer from a large trading order is very accurate ($\lambda^{se}_{SQ_L}$ is close to $\lambda_{\underline{\theta}}$). This, in turn, implies that the difference between B^{se}_S and B^{se}_L and the difference between A^{se}_L and A^{se}_S are significant and then the losses due to the worse prices in trading Q_L rather than Q_S are high. Hence, Q_L has to be very large with respect to Q_S in order to encourage informed traders to separate from small liquidity traders. In the following we assume $\frac{Q_L}{Q_S} > \left(1 - \lambda_{\overline{\theta}}\right) / \left(\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}\right)$. Next Proposition illustrates conditions on c for the occurrence of a separating equilibrium on the bid and on the ask side of market.

Proposition 4 *For any public belief π such that condition 4 is satisfied, a separating equilibrium prevails in the bid side of market if, and only if: $c \leq c^{se}(\pi, \underline{\theta})$, where $c^{se}(\pi, \underline{\theta}) = c^{ne}(\pi, \underline{\theta}) - \frac{Q_L}{Q_L - Q_S}(\pi - \frac{\pi}{\pi + (1-\pi)\lambda^{se}_{SQ_L}})(\overline{V} - \underline{V})$. Analogously, for any public belief π such that condition 5 is satisfied, a separating equilibrium prevails in the ask side of market if, and only if: $c \leq c^{se}(\pi, \overline{\theta})$, where $c^{se}(\pi, \overline{\theta}) = c^{ne}(\pi, \overline{\theta}) - \frac{Q_L}{Q_L - Q_S}(\frac{\pi}{\pi + (1-\pi)\lambda^{se}_{BQ_L}} - \pi)(\overline{V} - \underline{V})$.*

Proof. See Appendix.

Proposition 4 points out that the threshold cost for a separating equilibrium is lower than the informational advantage of informed traders, that is $c^{ne}(\pi, \theta)$. We can conclude that a separating equilibrium prevails in the market only if the large quantity is big enough with respect to the small quantity, and the marginal transaction cost is very low.

3.3 The pooling equilibrium

In a market in the pooling equilibrium, informed traders submit both small and large orders. For the competitive price schedule $P^{pe} = \{B^{pe}_L, B^{pe}_S, A^{pe}_S, A^{pe}_L\}$ to describe a pooling equilibrium, informed traders must be indifferent between trading the large and the small quantity. This condition requires:

$$(B^{pe}_L - E[\widetilde{V} \,|\, \underline{\theta}])Q_L = (B^{pe}_S - E[\widetilde{V} \,|\, \underline{\theta}])Q_S \geq 0 \tag{6}$$

$$(E[\widetilde{V} \,|\, \overline{\theta}] - A^{pc}_L)Q_L = (E[\widetilde{V} \,|\, \overline{\theta}] - A^{pc}_S)Q_S \geq 0. \tag{7}$$

For every price schedule satisfying condition 6, the optimal strategy of traders observing the bad signal is any mixed strategy defined on the simplex $\Delta(SL, SS)$ if the expected profit from selling is strictly positive, or any mixed strategy defined on the simplex $\Delta(SL, SS, NT)$ in the case of zero expected profit. Analogously, condition 7 implies that the optimal strategy of traders observing the good signal is any mixed strategy defined on the simplex $\Delta(BL, BS)$ if the expected profit from buying is strictly positive, or any mixed strategy defined on the simplex $\Delta(BL, BS, NT)$ in the case of zero expected profit.

It is immediate to see that if $B_L > B_S$, traders endowed with $\underline{\theta}$ never choose the small sale, and if $A_L < A_S$, traders observing $\overline{\theta}$ never buy the small quantity. Conditions 6 and 7 can be satisfied only if the price schedule is such that $B_L^{pe} \le B_S^{pe}$ and $A_L^{pe} \ge A_S^{pe}$.

More precisely, a pooling equilibrium with positive probability of no trading for traders observing $\underline{\theta}$ occurs only if: $E[\widetilde{V} \mid \underline{\theta}] = B_L^{pe} = B_S^{pe}$, and the pooling equilibrium with positive probability of no trading for traders observing $\overline{\theta}$ occurs only if: $E[\widetilde{V} \mid \overline{\theta}] = A_L^{pe} = A_S^{pe}$.

A pooling equilibrium with zero probability of no trading for informed traders requires on the bid side of market: $B_L^{pe} < B_S^{pe}$, and on the ask side of market: $A_L^{pe} > A_S^{pe}$. This, in tune, implies that informed traders are more likely to place a large order.

The next proposition dictates conditions on c for the occurrence of a pooling equilibrium on each side of market.

Proposition 5 *For any public belief $\pi \in (0,\ 1)$, a pooling equilibrium prevails in the bid side of market if, and only if, $c \in (c^{se}(\pi,\ \underline{\theta}),\ c^{ne}(\pi,\ \underline{\theta})]$ and $c \ge 0$. It prevails in the ask side of market if, and only if, $c \in (c^{se}(\pi,\ \overline{\theta}),\ c^{ne}(\pi,\ \overline{\theta})]$ and $c \ge 0$.*

Proof. See Appendix.

A pooling equilibrium prevails in the market when the marginal transaction cost is lower than the informational advantage of traders observing a private signal, but it is not low enough to induce informed to separate from small noise traders. In particular, if conditions 4 and 5 are not satisfied, $c^{se}(\pi,\ \underline{\theta})$ and $c^{se}(\pi,\ \overline{\theta})$ are negative and the market equilibrium is pooling every time the informational advantage of informed traders exceeds the marginal transaction cost.

In a market in the pooling equilibrium, the learning process about the true asset value can be very slow. If the informational advantage of traders endowed with a private signal slightly exceeds the transaction cost, then at the equilibrium the probability of an information based trading order is low. Moreover, in a pooling equilibrium with positive probability of no trading for informed traders, the informational content of a large trading order is the same as the informational content of a small trading order. The information that the market can infer from a trading order increases if the transaction cost is small with respect to the informational advantage of traders observing a private signal. In this case, the probability that an informed trader refrains from trading goes to zero and the large orders become more informative than the small ones.

4 Comments and concluding remarks

In this paper we have examined the impact of trading costs proportional to the order size on price discovery. We have shown that the occurrence of an informational cascade does not depend on the amount of the asset traded. It develops as the marginal transaction cost exceeds the informational advantage of traders endowed with private information. Moreover, during a cascade no informed trader chooses to trade at the equilibrium.

If the marginal transaction cost is not too big with respect to the informational advantage of informed traders, two equilibria can arise: the separating equilibrium, where informed choose to trade only the large quantity, and the pooling equilibrium, where informed choose to submit both large and small orders with positive probability. The first outcome prevails in the market if two conditions are satisfied: the marginal transaction cost has to be very low, and the difference between the size of large and small orders must be sufficiently large. Since in the separating equilibrium the spread at the large quantity exceeds the spread at the small quantity, if the large quantity is not large enough to offset the better prices for small orders, a separating equilibrium never prevails whatever the transaction cost is.

Finally, the analysis has shown that as the public belief gets concentrated in the extreme tails of the asset value distribution, the upper threshold costs for both separating and trading equilibria reduce. Consequently, before an informational cascade develops, the trading volume gradually decreases.

Appendix

Proof. of Proposition 2

Assume that a separating equilibrium prevails in the ask side of market and suppose, by way of obtaining a contradiction, that

$$\frac{Q_L}{Q_S} \leq \frac{1-\lambda_{\overline{\theta}}}{\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}}(\pi + (1-\pi)\lambda^{se}_{BQ_L}). \tag{8}$$

Simple algebra shows that 8 implies:

$$(E[\widetilde{V}\,|\,\overline{\theta}] - E[\,\widetilde{V}\,|BQ_L])Q_L \leq (E[\widetilde{V}\,|\,\overline{\theta}] - E[\widetilde{V}])Q_S. \tag{9}$$

Since $A^{se}_L = E[\,\widetilde{V}\,|BQ_L] + c$ and $A^{se}_S = E[\widetilde{V}] + c$, 9 implies:

$$(E[\widetilde{V}\,|\,\overline{\theta}] - A^{se}_L)Q_L < (E[\widetilde{V}\,|\,\overline{\theta}] - A^{se}_S)Q_S$$

which contradicts the assumption that a separating equilibrium prevails in the ask side of market. The part of the proof concerning the bid side of market is analogous and it is omitted. □

Proof. of Proposition 3

Let be $f_{\underline{\theta}}(\pi) = \frac{\lambda_{\underline{\theta}}-1}{\lambda_{\underline{\theta}}-\lambda^{se}_{SQ_L}}(\pi + (1-\pi)\lambda^{se}_{SQ_L})$ and $f_{\overline{\theta}}(\pi) = \frac{1-\lambda_{\overline{\theta}}}{\lambda^{se}_{BQ_L}-\lambda_{\overline{\theta}}}(\pi + (1-\pi)\lambda^{se}_{BQ_L})$. The proposition is proved immediately by noting that $\min_{\pi\in[0,\,1]} f_{\overline{\theta}}(\pi) = \min_{\pi\in[0,\,1]} f_{\underline{\theta}}(\pi) = \frac{1-\lambda_{\overline{\theta}}}{\lambda^{se}_{BQ_L}-\lambda_{\overline{\theta}}}$. □

Proof. of Proposition 4

The proof is straightforward by plugging P^{se} into conditions 2 and 3, and by noting that condition 4 implies $c^{se}(\pi,\,\underline{\theta}) > 0$ and condition 5 implies $c^{se}(\pi,\,\overline{\theta}) > 0$. □

Proof. of Proposition 5

We prove the proposition for the ask side of market. The proof for the bid side is similar. First we consider the pooling equilibrium with positive probability of no trading for traders observing $\overline{\theta}$. This equilibrium requires that the ask price for both the large and the small quantity equates the expected asset value conditional to $\overline{\theta}$.

By combining $E[\widetilde{V}\,|\,\overline{\theta}] = A^{pe}_L = A^{pe}_S$ with the condition of zero market maker's expected profit, we obtain: $\sigma_{\ BQ_S} = \sigma_{\ BQ_L} = \frac{1-\sigma_{\ NT}}{2} \leq \frac{1}{2}$, where $\{\sigma_i\}_{i\in A}$, is the equilibrium mixed strategy of traders observing $\overline{\theta}$. Then, the ask side of market is characterized by a pooling equilibrium with no trading if there exists a $\sigma_{\ BQ_L} \leq \frac{1}{2}$ such that:

$$E[\widetilde{V} \,|\, \overline{\theta}] = \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\gamma+\mu(1-p)\sigma_{\ BQ_L}}{\gamma+\mu p\sigma_{\ BQ_L}}} \cdot (\overline{V} - \underline{V}) + c.$$

We define the function $f(x)$ defined on $[0,\ \frac{1}{2}]$ as follows:

$$f(x) = E[\widetilde{V} \,|\, \overline{\theta}] - \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\gamma+\mu x(1-p)}{\gamma+\mu x p}} \cdot (\overline{V} - \underline{V}).$$

It is easy to see that:

- $f'(x) < 0$ for all $x \in [0,\ \frac{1}{2}]$
- $\min_{x\in[0,\ \frac{1}{2}]} f(x) = f(\frac{1}{2})$
- $\max_{x\in[0,\ \frac{1}{2}]} f(x) = f(0)$

We can conclude that there exists a $\sigma_{\ BQ_S} \leq \frac{1}{2}$ such that $f(\sigma_{\ BQ_S}) = c$, if and only if $c \in (f(\frac{1}{2}),\ f(0)]$. This part of the proof is complete by noting that $f(0) = c^{ne}(\pi,\ \overline{\theta})$ and $f(\frac{1}{2}) > c^{se}(\pi,\ \overline{\theta})$.

Let's move to consider the pooling equilibrium with zero probability of no trading for traders observing $\overline{\theta}$. This equilibrium requires that $E[\widetilde{V} \,|\, \overline{\theta}] > A_L^{pe} > A_S^{pe}$. By combining $A_L^{pe} > A_S^{pe}$ with the condition of zero market maker's expected profit, we obtain: $\sigma_{\ BQ_L} = 1 - \sigma_{\ BQ_S} \in (\frac{1}{2},\ 1)$. So, the ask side of market is characterized by a pooling equilibrium with $\sigma_{NT} = 0$, if there exists a $\sigma_{\ BQ_L} \in (\frac{1}{2},\ 1)$ such that:

$$[(\frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{\pi}{\pi + (1-\pi)\frac{\gamma+\mu(1-p)\sigma_{\ BQ_L}}{\gamma+\mu p\sigma_{\ BQ_L}}}) \cdot (\overline{V} - \underline{V}) - c] \cdot Q_L =$$

$$[(\frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{\pi}{\pi + (1-\pi)\frac{\gamma+\mu(1-\sigma_{\ BQ_L})(1-p)}{\gamma+\mu(1-\sigma_{\ BQ_L})p}}) \cdot (\overline{V} - \underline{V}) - c] \cdot Q_S. \quad (10)$$

We define the function $g(\alpha)$ defined on $[\frac{1}{2},\ 1]$ as follows:

$$g(\alpha) = \frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{1}{Q_L - Q_S} \cdot$$

$$(\frac{\pi \cdot Q_L}{\pi + (1-\pi)\frac{\gamma+\mu\alpha(1-p)}{\gamma+\mu\alpha p}} - \frac{\pi \cdot Q_S}{\pi + (1-\pi)\frac{\gamma+\mu(1-\alpha)(1-p)}{\gamma+\mu(1-\alpha)p}})$$

Condition 10 is satisfied if and only if there exists $\alpha \in (\frac{1}{2},\ 1)$ such that $g(\alpha)(\overline{V} - \underline{V}) = c$. Since $g(\alpha)$ is a strictly decreasing function, this is true when $c \in (g(1)(\overline{V} - \underline{V}),\ g(\frac{1}{2})(\overline{V} - \underline{V}))$. The proposition for the ask side of market is proved by noting that $g(1)(\overline{V} - \underline{V}) = c^{se}(\pi,\ \overline{\theta})$, and $g(\frac{1}{2})(\overline{V} - \underline{V}) = f(\frac{1}{2})$. $\square$

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