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Macello D'Amato[♣] and Vincenzo Galasso[♦]

Abstract

In a stochastic two-period OLG model, featuring an aggregate shock to the economy, ex-ante optimality requires intergenerational risk sharing. We compare the level of time-consistent intergenerational risk sharing chosen by a benevolent government and by an office-seeking politician. In our political system, the transfer of resources across generations is determined as a Markov equilibrium of a probabilistic voting game. Low realized returns on the risky asset induce politicians to compensate the old through a PAYG system. This political system typically generates an intergenerational risk sharing scheme that is (i) larger, (ii) more persistent, and (iii) less responsive to the realization of the shock than the (time consistent) social optimum. This is because the current politician anticipates her transfers to the elderly to be compensated by future politicians through offsetting transfers, and hence overspends. Aging increases the optimal transfer, but surprisingly makes office-seeking politicians more conservative, by increasing the cost for future politicians to compensate the current young.

Keywords: Pension Systems, Markov equilibria, social optimum.

JEL Classification: H55, D72.

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1. Introduction

In times of financial troubles, public PAYG pension systems come back into fashion. Large reductions in housing prices, losses in private pension funds, and increased volatility in the stock market have large negative effects on the private wealth of the elderly – and ultimately on their consumption. Real annuities – such as public pension benefits – may instead guarantee stable old age consumption, albeit typically at the cost of lower average returns on the contributions paid during the working period. In other words, PAYG pension systems entail an important intergenerational risk sharing component that proves crucial in periods of high financial instability.

This paper focuses on the role of the intergenerational risk sharing as a crucial motivation for the existence of social security systems. We characterize the optimal time-consistent risk sharing mechanism introduced by a benevolent government that is unable commit to future policy, and compare it to the social security system designed by office-seeking politicians, who choose this risk sharing policy in order to win the elections – again in the absence of commitment on future policies. We show that election-minded politicians typically prefer more spending in social security and introduce more persistent policies. We then investigate the effects of population aging on these social security decisions. Surprisingly, politicians are more conservative than a benevolent government in coping with aging.

Since early contributions by Enders and Lapan (1982) and Merton (1983), PAYG social security systems have been recognized as an important instrument of intergenerational risk sharing. The demand for risk sharing stems from the uncertainty that is usually associated with endowments, wages and/or rate of returns. Individuals would typically like to insure against bad realizations during their lifetimes, before they are even born[1]. If there exists a long term player, such as a benevolent government that can commit to carry out future policies on the behalf of yet-to-be-born generations, intergenerational risk sharing may arise, e.g., through PAYG social security systems.

A parallel, but less sympathetic literature provides instead evidence on the inefficiencies and the costs of the existing, generous social security systems. Large reductions in the employment rate among middle aged and elderly workers, rising labour cost, and the crowding out effect on the stock of capital induced by the

[1] Even in a dynastic environment in which parents care about the well being of their kids, and thus would like to purchase some insurance on their account, private markets may fail to work, to the extent that legal contracts signed by the parents do not bind their offsprings.

reduction in private savings are only some of the by-products of these pension systems, which have been largely criticized. The upshot of this literature is that social security spending is inefficiently large.

Bohn (2003), and more recently Krueger and Kubler (2006), took a more comprehensive approach, and consider both these costs and benefits of PAYG schemes. They suggest that the crowding out effect on the private savings may be so severe as to overweight the positive role of intergenerational risk sharing. Storeletten et al. (1999) analyze the risk sharing properties of social security systems vis-à-vis idiosyncratic risks, such as wage fluctuations and mortality risk, and reach similar conclusions. Clearly, additional considerations on the labor market distortion induced by social security would only strengthen this argument. Yet, in a more general setting, which – together with capital – features a long-lived asset (such as land), Gottardi and Kubler (2006) provide more optimistic results on the existence of a (ex-ante) Pareto improving social security system. Hence, the jury is still out.

In this paper, we abstract from many distortionary aspects associated with social security and concentrate on the intergenerational risk sharing property of PAYG systems. We show that both a benevolent government and an office-seeking politician have an incentive to introduce an unfunded system if a financial market crisis that wipes out the private wealth of the elderly occurs. This observation is consistent with the experience of the US, where the introduction of the social security system followed the 1929 stock market crash, and of several countries, such as Belgium, France, Germany, and Italy, where episodes of hyperinflation wiped out the value of the bonds issued in nominal terms and called for government's intervention to transform the existing (funded) systems into PAYG schemes (see Flora, 1987).

We then turn to examine the main features of these risk sharing instruments, and we ask whether electoral constraints may lead politicians to choose "too much" social security spending, even in a setting in which social security entails no distortions on labour supply or capital accumulation. In fact, we consider a stochastic environment in which intergenerational risk sharing is (ex-ante) Pareto improving and derive the optimal level of social security that a time consistent benevolent government would choose. In our two-period stochastic overlapping generation economy, the risk comes from an aggregate shock to the stock market, which affects the net private wealth – and thus the consumption of the agents in their old age. A benevolent government that cares about the well-being of current and future generations determines a time consistent policy, after the realization of the shock to the economy. The (ad interim) optimal linear risk sharing policy features a transfer typically from the young to the old, which depends negatively on the realization of the net private wealth of the elderly.

To model the electoral incentives faced by the politicians, we consider a probabilistic voting environment in which politicians choose the current social security policy but are unable to commit to future transfers. This lack of commitment would undermine intergenerational risk sharing, since young voters may not trust to receive back a transfer in their old age. To capture the intertemporal link between current and future policies in absence of commitment, we concentrate on Markov perfect equilibria of this probabilistic voting game, in which the equilibrium policy depends only on the state of the economy. In this political setting, a PAYG system is introduced during an economic crisis, and persists in future periods. Its size depends crucially on the electoral strength – as measured by the relative share of swing (or undecided) voters among the elderly – of the old generation, who happens to face the crisis. In other words, after a financial crisis office-seeking politicians are urged by their electoral constraints to "bail out" the elderly through the provision of generous public pensions. The politicians' incentives to intervene in case of a negative shock effectively create a quasi asset – the PAYG social security – whose returns are negatively correlated to stock market returns. Interestingly, this policy turns out to be quite persistent, since less disposable income for the current young generation leads to lower net private wealth in their old age and thus to more future government intervention.

We show that this political mechanism typically produces more intergenerational risk sharing than the (time consistent) social optimum. Overspending stems from the strategic behavior of the politicians. They exploit the expectations by current young voters, who anticipate that their current transfers will be compensated by offsetting transfers provided by future politicians. This strategic effect lowers the electoral cost to the politicians. They hence have an incentive to overspend in social security to please the current elderly voters. In other words, politicians play strategically by "leaving" more risk sharing on the future generations than optimal. Furthermore, these transfers are more persistent and less responsive to the realization of the shock than the optimal policy would require.

How do these office-seeking politicians react to population aging? The effect predicted by our model is quite surprising, given the results in the current literature on the political economy of social security[2]. Aging increases the transfers that

[2]See Razin, Sadka and Swagel (2002), Razin, Sadka and Swagel (2007), Disney (2007), Galasso and Profeta (2007), and Simonovits (2007) for a recent symposium on this issue.

a benevolent government would provide, but reduces the transfers chosen by the office-seeking politicians; and makes the politicians less likely to overspend. This novel result depends on the fact that aging makes it more costly for future politicians to compensate the current young – and so reduces the room for strategic behavior by the current politicians.

This paper relates to different strands of the literature. Several contributions have established important results on the Pareto optimal intergenerational risk sharing properties of social security both ex-ante, i.e., behind a veil of ignorance, and ad interim, that is, when the realization of one (or many) shock has occurred. The structure of our economy is closely related to Gordon and Varian (1988). Additional key contributions carried out in partial equilibrium framework include Allen and Gale (1997), Shiller (1999), Demange (2002), Matsen and Thogersen (2004) and Ball and Mankiw (2007). Instead Bohn (2003), Krueger and Kubler (2006), Gottardi and Kubler (2006) and Olovsson (2004) consider also the general equilibrium effects arising from the introduction of a social security system, such as the crowing out of the private savings.

Yet, the closer literature is perhaps the one on the political support for intergenerational risk sharing. As Orszag and Stiglitz (2001, see myth 9), we in fact recognize that, if a negative shock occurs, office-seeking politicians may decide to "bail out" the elderly. Rangel and Zeckhauser (2001) present several arguments suggesting that neither the market nor politicians are typically able to provide the optimal level of intergenerational risk sharing. Our political environment is similar to Gonzales-Eiras and Niepelt (2008), who study the effect of demographic transition on Markov equilibrium social security transfers using a probabilistic voting model[3]. In the presence of demographic risks, but no uncertainty on the assets' returns, their political equilibrium features some degree of intergenerational risk sharing, which they compare to the Ramsey allocation (i.e., with commitment). Our contribution considers instead uncertainty in the financial markets and focuses on time consistent policies. Finally, Demange (2007) characterizes the conditions for a PAYG social security system to have political support in absence of commitment on future policies. She finds that – besides the redistributive elements often embedded in these pension systems – political support depends on the degree of risk aversion of the decisive voter and on the availability of financial assets. Our results are in line with her sustainability conditions. In addition, we are able

[3]Political environments that combine probabilitisc voting and Markov perfect equilibrium are also in Hassler et al. (2003 and 2007) and Song (2008) among others. For time consistent policies chosen by a benevolent government see Klein et al. (2008) and references therein.

to characterize and to compare the risk sharing policy chosen by a benevolent government and by the politicians.

The paper is organized as follows: section 2 describes the model and the ex-ante Pareto improving role of intergenerational risk sharing. Section 3 and 4 analyze respectively the time consistent policy chosen by a benevolent government, who cares about the current and future generations, and by office-seeking politicians in a probabilistic voting model. Section 5 compares these results and provides some comparative statics. Section 6 analyzes how the aging process affects these choices. Section 7 concludes. All the proofs are in the appendix.

2. A Simple Stochastic Economy

We consider a two period overlapping generations model of a small open economy. Every period two generations are alive: young and old. Population grows at a constant rate n. Agents are endowed with one unit of labor in youth, which they supply inelastically to receive a wage, w. Agents evaluate old age consumption only, according to an increasing and concave utility function: $U\left(c_{t+1}^{t}\right)$, with $U'(.) > 0$, $U''(.) < 0$, where c_{t+1}^{t} represents consumption at time $t+1$, i.e., in old age, by an agent born at time t.

Output in the economy is given by

$$y_t = wL_t + R_t K_t \tag{2.1}$$

where K_t represents the stock of capital, i.e., the amount of savings, in the economy. Capital fully depreciates at every period. The return on capital is stochastic. Claims to capital represent the only (risky) asset in this economy, which pays a real return R_t distributed according to a cumulative function $G(R_t)$, with mean $E[R_t] = R$, variance $Var[R_t] = \sigma^2$ and no serial correlation $E(R_t R_{t+1}) = R^2$. Limited liability applies in the stock market to the risky asset, which also features an upper bound $\bar{R}$ on its returns, $R_t \in [0, \bar{R}]$ $\forall t$. The wage is deterministic and assumed to be unitary, $w = 1$.

The distribution of the stochastic returns represents a crucial element in a model that analyzes intertemporal risk sharing. Instead of recurring to a specific distribution function, we choose to consider distributions that obey to two criteria. First, we assume that the average return from the risky asset is higher that the return from a PAYG social security system, $R > 1 + n$. Second, we assume that the distribution is rather spread out, so that the coefficient of variation, σ/R, is greater than one. Thus,we have the following assumption.

Assumption 1 $\sigma > R > (1+n)$.

Agents save their entire net endowment for old age consumption using the risky asset, so the budget constraint of an individual born at time t is

$$c_{t+1} = R_{t+1}\left(1 - T_t\right) + P_{t+1} \tag{2.2}$$

where $T_t\,(< 1)$ is the amount of taxes paid by the young, which is used to provide a transfer to the current old – as in a PAYG social security system, and $P_t = (1+n)\,T_t$ is the amount received by the old. It is also convenient to define the net private wealth of the elderly at time $t+1$ as $\omega_{t+1} = R_{t+1}\left(1 - T_t\right)$.

In most of the paper, agents are assumed to have quadratic preferences, which gives rise to a mean variance representation:

$$U\left(c_{t+1}\right) = -\frac{1}{2}\left(c_{t+1} - \gamma\right)^2 \tag{2.3}$$

where the parameter γ plays a double role. In the deterministic formulation that applies to the consumption of the elderly, once the shock is realized, γ determines the marginal utility of consumption. In the expected utility formulation that applies instead to the young, the parameter γ measures the degree of risk aversion: a lower γ characterizes a more risk averse individual.

For this utility function to feature positive marginal utility, we need to have that $\gamma > c_t\ \forall t$. A sufficient condition is that $\gamma > \bar{R} + (1+n)$. This amounts to require that the marginal utility of consumption is positive even when individuals pay no contributions in youth, obtain the largest possible return, $\bar{R}$, from their savings – consisting on their entire labor income – and receive the largest possible pension transfer, $P_{t+1} = (1+n)$, in old age[4]. We choose to impose an additional restriction on γ to guarantee that a policy consisting of a positive contribution rate imposed on the young at time t, with no corresponding pension benefit at $t+1$, is associated with a negative expected marginal utility for the young. This occurs for $\gamma > S/R$ where $S = \sigma^2 + R^2$. We thus have our next two assumptions.

[4] In what follows, we will allow the benevolent government and the politicians to choose a negative transfer – that is, a transfer of resources from the elderly to the young – at least for some realization of the shock. If this occurs, the savings of the young may exceed their labor income. However, it is straightforward to show that the consumption level implied by the equilibrium policy chosen by the benevolent government and the politicians (see sections 3 and 4) never exceeds $\overline{R} + (1+n)$. The sufficient condition at assumption 2 hence holds also for the following sections.

Assumption 2 $\gamma > \bar{R} + (1 + n)$.

Assumption 3 $\gamma > S/R$.

Notice that in this economy, in absence of a social security system, consumption in old age is simply equal to the realization of the return on the risky asset: $c_{t+1} = R_{t+1}$. Hence, the expected consumption at time t corresponds to the average return $E_t[c_{t+1}] = E_t[R_{t+1}] = R$, and similarly for the variance $Var_t[c_{t+1}] = Var_t[R_{t+1}] = \sigma^2$. Clearly, the coefficient of variation for the consumption is $\sigma/R > 1$.

2.1. Ex-Ante Pareto Efficient Intergenerational Risk Sharing

In this simple economic setting, individuals have no mechanism to ensure against the risk of a negative stock market shock. In absence of intergenerational transfers, a low realization of the return on their assets affects their private wealth and hence their consumption. Agents may hence be willing to incur in a tax on their labor supply when young in order to receive a transfer from the next generation of individuals when old. As a simple benchmark to discuss ex-ante Pareto efficiency, consider the decision of a young generation that uses a constant intergenerational transfer scheme to maximize its expected utility. In other words, we assume the existence of a commitment device that allows the young to set up a PAYG pension system by contributing a share, α, of their labor income today in exchange for the same share of the young labor income in the next period. As largely discussed in the literature, in order for this arrangement to arise, some long term player – such as the government – who is able to commit to future policy has to exist. The optimization problem is:

$$\begin{aligned} & \underset{\{\alpha\}}{Max} - \frac{1}{2} E_t \left(c_{t+1} - \gamma \right)^2 \qquad (2.4) \\ c_{t+1} &= R_{t+1} \left(1 - \alpha \right) + \alpha \left(1 + n \right) \end{aligned}$$

As the following first order condition shows, this amounts to a simple portfolio decision problem in which the agents have to choose how to divide their saving between a safe asset that provides a return $(1 + n)$, and a risky asset with a stochastic return R_{t+1}:

$$E_t \left(c_{t+1} - \gamma \right) \left(R_{t+1} - (1 + n) \right) = 0 \qquad (2.5)$$

The share of savings allocated to the intergenerational risk sharing is

$$\alpha = \frac{\sigma^2 + (R - (1+n))(R - \gamma)}{\sigma^2 + (R - (1+n))^2}.$$

Depending on their degree of risk aversion, the young may choose some intergenerational risk sharing. In particular, $\alpha > 0$ for a large enough degree of risk aversion, $\gamma < \frac{\sigma^2 + R[R-(1+n)]}{R-(1+n)} = \frac{S-(1+n)R}{R-(1+n)}$, where $S = \sigma^2 + R^2$. To consider an environment in which intergenerational risk sharing plays a role, we thus set our next assumption.

Assumption 4 $\gamma < \frac{S-(1+n)R}{R-(1+n)}$.

How does consumption in old age evolves in the presence of this intergenerational risk sharing transfer? The expected consumption at time t is thus equal to $E_t[c_{t+1}] = R - \alpha (R - (1+n))$. For $\alpha > 0$, this is lower than the expected consumption in absence of risk sharing, $E_t[c_{t+1}] < R$. Yet, also the variance decreases, $Var_t[c_{t+1}] = (1-\alpha)^2 \sigma^2 < \sigma^2$. Finally, it is easy to see that the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing.

To summarize, in this simple economic environment, even an uncontingent, constant policy can be ex-ante Pareto improving, if individuals are sufficiently risk averse[5]. This constant policy however rests on the assumption that current individuals can commit to future policies. In the remaining of the paper, we will allow for more complex policies, which relate the degree of risk sharing to the realization of the shock, but will drop the assumption of commitment. In choosing the intergenerational policies, a benevolent government and the office-seeking politicians will have to consider that future policies cannot be committed upon; quite on the opposite, they can be modified at any time, if convenient.

3. Intergenerational Risk Sharing by a Benevolent Government

In this section, we consider the intergenerational risk sharing decision of a benevolent government which cares about the well being of current and future generations, and has to choose a time consistent policy. In every period, after the

[5]It is worth noticing that assumption 4 is always consistent with assumption 3 and is compatible with assumption 2 for distribution functions that have a sufficiently high variance, that is, if $\sigma^2 > (R - (1+n)) \left(\overline{R} - R + (1+n)\right)$.

realization of the shock has occurred, and hence the net private wealth of the elderly has become known, a benevolent government decides whether to transfer resources from the young to the elderly (or viceversa). We assume no commitment, so that the policy can be modified over time, if it is optimal to do so[6].

The benevolent government optimization problem at time t is thus

$$\underset{\{T_{t+i}\}_{i=0}^{\infty}}{Max}\ U\left(c_t\right)+\delta\left(1+n\right)E_tU\left(c_{t+1}\right)+\delta^2\left(1+n\right)^2E_tU\left(c_{t+2}\right)+.... \tag{3.1}$$

subject to the budget constraint at eq. 2.2, where $\delta < 1/\left(1+n\right)$ represents the benevolent government's discount rate, and the utility function is at eq. 2.3. Individual agents take no economic decisions, while the government decision variable is the policy, T_t. The state variable is $\omega_t = R_t\left(1-T_{t-1}\right)$, which characterizes the net private wealth of the elderly at time t. We can thus use the following recursive formulation:

$$V\left(\omega_t\right)=\underset{\{T_t\}}{Max}\left\{U\left(\omega_t,T_t\right)+\delta\left(1+n\right)E_tV\left(\omega_{t+1}\right)\right\}. \tag{3.2}$$

The first order condition of this optimization problem is:

$$-\left(\omega_t+\left(1+n\right)T\left(\omega_t\right)-\gamma\right)+\delta E_t\left(\omega_{t+1}+\left(1+n\right)T\left(\omega_{t+1}\right)-\gamma\right)R_{t+1}=0. \tag{3.3}$$

The former term represents the marginal utility for the elderly of an increase in their consumption due to the intergenerational transfer, whereas the latter represents the reduction in marginal utility for the young from lower future consumption. To solve this optimization problem, we guess a linear time consistent policy, $T\left(\omega_t\right)=A+B\omega_t$, and verify that it satisfies eq. 3.3. Recall that $S=\sigma^2+R^2$. The next proposition characterizes the optimal interior time-consistent linear policy function.

Proposition 3.1. *If* $\delta\in\Lambda=\left(\frac{1}{R},\frac{1}{1+n}\right)$*, there exists a time-consistent linear policy function* $T^G\left(\omega_t\right)=A+B\omega_t$*, that solves the benevolent government problem at*

[6]For time consistent policies chosen by a benevolent government in a more general, yet deterministic, environment in which individuals take savings (and labor) decisions, see Klein et al. (2008).

eq. 3.1, with $T^G(\omega) < 1 \ \forall\omega$, and $T^G(\omega) > 0$ for some ω, where

$$\begin{aligned} B &= -\frac{1}{\delta S} \\ A &= \frac{\delta\left[S - R\gamma\right] + (\gamma - (1+n))}{\delta\left[S - R(1+n)\right]} \end{aligned}$$

This proposition suggests that if a benevolent government cares sufficiently about the future generations, i.e., if $\delta \in \Lambda$, it will implement a linear interior intergenerational risk sharing mechanism, which provides to the elderly a transfer consisting of a constant share, A, which is reduced of a proportion B according to the realization of the state of the world, ω. In the worst case scenario, in which the elderly have zero private wealth, $\omega_i = 0$ – for instance because of a very bad stock market shock, $R_i = 0$ – the benevolent government imposes a positive, large transfer on the young, $T^G(\omega_i) = A \in (0,1)$. Better realizations of the rate of return, and hence higher private wealth for the elderly, are associated with lower transfers from the young.

Clearly, a lower weight on the future generations, i.e., $\delta < 1/R$, may lead, in the occurrence of a particularly negative shock on the returns, to a complete transfer of resources from the young to the elderly, $T^G(\omega_i) = 1$. These equilibria with full expropriation have the unpleasant feature of representing an absorbing state. In fact, the young generation on which a 100% tax rate is imposed reaches old age with zero private wealth, $\omega = 0$, which will in turn command a 100% tax rate on the young and so on. In the remaining of the paper, we will disregard these full expropriation equilibria and concentrate on interior equilibrium solution, thereby assuming that $\delta > 1/R$. On the contrary, an extremely large weight on the future generations could lead to a transfer from the old to the young even in the worst case scenario, in which $\omega = 0$. In the appendix, we show that for $\delta < 1/(1+n)$, this does not occur. The next proposition further characterizes this interior-equilibrium risk sharing policy by presenting the results of some comparative statics.

Proposition 3.2. *For $\delta \in \Lambda$, an increase in (i) the discount factor, δ; or in (ii) average rate of return, R, reduces the fixed component, A, of the time-consistent linear policy function $T^G(\omega_t)$, and makes the transfer less responsive to the shock. An increase in the variance of the shock, σ^2, increases the time-consistent linear policy function $T^G(\omega_t)$.*

The intuition is straightforward. Recall that for any given realization of the shock, providing a transfer to the current elderly comes at the cost of lower expected utility for the young generations, because of the opportunity cost of using a PAYG system for risk sharing – given that $R > 1 + n$. Hence, the higher the weight placed on these future generations, or the higher the average return of the risky asset – and hence the opportunity cost – the lower the fixed component of the transfer, which becomes also flatter. On the other hand, higher volatility of the returns clearly requires more risk sharing, and hence $T^G(\omega_t)$ increases.

4. Intergenerational Risk Sharing by Office-seeking Politicians

In the political system, intergenerational risk sharing may arise because office-seeking politicians choose to transfer resources from the young to the old (or viceversa) in order to improve their electoral perspective. Politicians act after the stock market shock has occurred – and hence the return on the risky asset, R_t, is realized.

Formally, we consider a probabilistic voting model. Two political candidates compete in a majoritarian election. Each candidate determines her political platform, which is represented by the contribution, T, in order to maximize her probability of winning the election. The candidate who wins the election becomes the policy-maker, and implements the proposed policy. Elections take place every period, after the realization of the stochastic return on the assets of the current old. Hence, political candidates can condition their intergenerational risk sharing policy on the realized state of the world.

At every election, individual's voting decisions depend on the policy chosen by the political candidates – and thus on how this affects their utility, on the individual's political ideology towards the two candidates, and on a common popularity shock that may hit the candidates before the election. Political candidates will use the intergenerational risk sharing policy to target the young and/or the old, in an attempt to increase their probability of winning the election, but they cannot affect the voters' ideology or their own popularity. Within each age group, all individuals share the same economic preferences, thereby being equally affected by the candidates' platforms. Elderly care only about the current transfer. Instead, the preferences of the current young – and thus their voting behavior – depend also on the expected future policy. If the young were myopic, they would only consider the direct effect of the current payroll tax on their savings and thereby on their future consumption. Young workers however realize that a current tax makes them more likely to be poorer tomorrow, and this may modify the future politicians' behavior. Current young electors hence need to understand and evaluate how the decisions of the current politicians may affect the future politicians' policy choice. We choose to consider a Markov policy, in which intergenerational risk sharing transfers depend only on the current state of the economy, in order to emphasize the absence of commitment to future policies by the politicians. This also justifies our previous decision of concentrating on time consistent policy determined by a benevolent government (see section 2).

Besides the utility provided by the economic policy, individuals care also about the political ideology, with some people feeling ideologically closer to one candidate or another. The distribution of ideology within each age group affects the candidate policy decision by determining the size of the swing voters, i.e., of the non-ideological voters who can be convinced to vote for a candidate if targeted with the appropriate policy. It is convenient to assume that each age group has a uniform distribution of ideology across agents.

In this environment, the two political candidates face the same optimization problem, and thus their political platforms converge, i.e. both candidates choose the same contribution, T (see Persson and Tabellini, 2000, and Hassler et al., 2003, for a framework with dynamic voting). Maximizing the probability of winning the election at time t is equivalent to maximizing the following expression, which may also be interpreted as the welfare function of the policy-maker at time t:

$$W_t = \phi_o U(c_t) + (1+n)\,\phi_y E_t U(c_{t+1}) \tag{4.1}$$

where ϕ_o and ϕ_y represent the density of the uniform ideology distribution function in the two groups, respectively old and young. We normalize $\phi_o = 1$ and define $\phi = \phi_y \geq 0$ as the relative importance of non-ideological, or swing, voters among the young generation. This can be interpreted as a measure of how fiercely the young generations pursue their interests in the political arena.

Eq. 4.1 shows that political competition, as modelled in this probabilistic voting framework, entails a trade off between providing state contingent transfers (and utility) to current retirees and providing current negative transfers, but expected positive transfers (and utility), to current workers. Hence, the voting behavior of the young depends on the policy chosen by the current politician, as well as on its impact on tomorrow's policy. To model this intertemporal link, we focus on stationary Markov perfect equilibria, in which each politician's policy decision is contingent on the current state of the economy. At any time t, the state variable is the amount of old age consumption that can be financed out of the private assets ω_t. This clearly depends on the young' savings (or net income) and on the outcome of the stock market. Thus, past policies directly contribute to defining the state of the economy. Clearly, each politician anticipates that its current choice will affect the incentive faced by the future politicians and, therefore, the future level of the intergenerational risk sharing.

The optimization problem of a policy-maker at time t is thus

$$\underset{\{-\omega\leq T(\omega)\leq 1\}}{Max} U\left(c_t\right)+\phi\left(1+n\right)E_tU\left(c_{t+1}\right) \tag{4.2}$$

where the Markov strategy is $T_t = T(\omega_t)$, the state variable is defined as $\omega_t = R_t\left(1-T_{t-1}\right)$. Consumption can be written as $c_t = \omega_t + (1+n)\,T(\omega_t)$. Notice that $c_{t+1} = R_{t+1}\left(1-T_t\right)+(1+n)\,T_{t+1}$ where T_{t+1} is the expected strategy played by future governments.

We can now formally define the linear Markov policy analyzed in this section.

Definition 4.1. *A policy $T^P(\omega) = \theta + T'\omega$, where θ and $T' = \frac{\partial T^P(\omega_{t+1})}{\partial\omega_{t+1}}$ are constant parameters, is a linear Markov perfect equilibrium of the intergenerational risk sharing game if it is a fixed point of the mapping from $T^e(.)$ to $T^P(.)$, where $T^e(.)$ is the expected policy function, $T^P(\omega_t) \in \underset{T(\omega_t)}{\arg\max} U\left(\omega_t+(1+n)\,T(.)\right) + \phi\left(1+n\right)E_tU\left((1-T(\omega_t)R_{t+1}+(1+n)\,T^e(\omega_{t+1})\right)$ and $T^P(\omega_t) = T^e(\omega_t)$.*

In what follows, we will characterize this equilibrium policy outcome for any well behaved utility function with $U' > 0$ and $U'' < 0$. We will return to the quadratic utility function later in this section.

The first order condition for the politician's problem is

$$U'\left(c_t\right)+\phi\frac{\partial E_tU\left(c_{t+1}\right)}{\partial T^P(\omega_t)}=0 \tag{4.3}$$

where – for $T_t^P > 0$ – the former term represents the marginal utility of increasing the consumption of the current old, while the latter defines the expected marginal disutility to the current young from imposing this risk sharing policy. This marginal cost can be decomposed as follows:

$$\frac{\partial E_tU\left(c_{t+1}\right)}{\partial T^P(\omega_t)} = E_tU'\left(c_{t+1}\right)[1+(1+n)\frac{\partial T^P(\omega_{t+1})}{\partial\omega_{t+1}}]\frac{\partial\omega_{t+1}}{\partial T^P(\omega_t)}.$$

Notice that the impact of today's policy on tomorrow net private wealth is $\frac{\partial \omega_{t+1}}{\partial T^P(\omega_t)} = -R_{t+1}$ and define $\frac{\partial T^P(\omega_{t+1})}{\partial \omega_{t+1}} = T'$. The first order condition of the maximization problem at eq. 4.2 becomes:

$$U'(c_t) - \phi[1 + (1+n)T']E_t U'(c_{t+1}) R_{t+1} = 0. \tag{4.4}$$

Thus, if an interior Markov equilibrium policy $T^P(\omega_t)$ exists, it must satisfy $-1/(1+n) < T'(\omega_t) \leq 0$. The above equation provides a first insight on this political intergenerational risk sharing. This policy is shaped by the political trade off between bailing out the current old from negative stock market shocks and imposing an expected cost on current young. In an interior Markov equilibrium, the intergenerational risk sharing agreement[7] features a transfer from the young to the old that is inversely related to the outcome of the stock market. It is important to notice that the political discretion by policy-makers in setting an intergenerational transfer policy creates a quasi-asset, whose returns are negatively correlated to stock market returns. Furthermore, by increasing T^P, the current politician reduces, for any future realization of the stock market R_{t+1}, the level of private wealth of the current young, thereby requiring larger future intervention. This property creates a strategic effect that induces persistence in the policy. In this model, a large current political intervention creates its own constituency for future large political interventions. Although this is commonly thought as the root of the persistence of inefficient policies (see Coate and Morris, 1999, and Conde-Ruiz and Galasso, 2003), in our context the tension between persistence and efficient allocation of risk is more subtle. The essence of intergenerational risk sharing is to spread current shocks on to future generations (see also Gordon and Varian, 1988), i.e., persistence is a crucial ingredient of an efficient risk sharing policy. By transferring the burden of current negative shock to current workers, the politician triggers a reaction by all future politicians, who keep transferring this shock into the infinite future.

To obtain further insights on the intergenerational risk sharing policy chosen

[7] The structure of the problem faced by these office-seeking politicians resambles the problem of optimal bequests strategies in altruistic economies where the current generation cares about the utilities of their immediate successors (see Phelps and Pollak, 1968, Bernheim and Ray, 1989, and, more recently, Nowak, 2006, and references therein). The main difference is that in our political environment the weight on different generations depends on the relative share of non ideological (swing) voters in each age group; whereas in the former class of models the relative weight between (state contingent) utility to ancestors and expected utility to descendants is dictated by altruism and other ethical considerations.

by office-seeking politicians, we continue our analysis using the quadratic utility function described at eq. 2.3. The first order condition of the maximization problem at eq. 4.2, which describes the stationary Markov policy chosen by the politician a time t, becomes:

$$-[\omega_t+(1+n)\,T^P\,(\omega_t)-\gamma]+\phi(1+(1+n)\,T')E_t[\omega_{t+1}+(1+n)\,T^P(\omega_{t+1})-\gamma]R_{t+1}=0. \tag{4.5}$$

subject to the linear policy $T^P(\omega)=\theta+T'\omega$. The next proposition characterizes the optimal linear policy function.

Proposition 4.2. *If* $\phi\in\Phi=\left(\frac{S/R}{S-R(1+n)},\frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}\right)$, *there exists a linear Markov perfect policy function* $T^P\,(\omega_t)=\theta+T'\omega_t$, *with* $T^P\,(\omega)<1\ \forall\omega$, *and* $T^P\,(\omega)>0$ *for some* ω, *where*

$$\begin{aligned}T' &= -\frac{1}{2\,(1+n)}\left(1-\sqrt{1-\frac{4\,(1+n)}{\phi S}}\right)\\ \theta &= \frac{2\,(\gamma-(1+n))-\phi\,(\gamma R-S)\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}{\phi\,[S-R\,(1+n)]\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}\end{aligned}$$

This proposition characterizes the behavior of the sequence of office-seeking politicians under a Markov perfect equilibrium when individual preferences have a mean-variance representation. Analogously to the benevolent government case, the conditions on ϕ makes sure that the young generation has sufficient relative electoral weight to avoid equilibrium sequences featuring full expropriation of the young, in the occurrence of negative stock market shocks. Also in this case, in fact, full expropriation would become an absorbing state, since it would lead the young to have zero private wealth in old age, $\omega=0$, and thus trigger further full expropriation by future generations.

For a higher electoral weight of the young, office-seeking politicians will still introduce a linear intergenerational risk sharing scheme, featuring a positive constant component, θ, which is reduced by a share T' according to the realization of the state of the world, ω. In the worst case scenario, $\omega=0$, the elderly obtain the largest transfer $T^P(\omega)=\theta\in(0,1)$. Higher levels of private wealth, ω, command lower transfers. Finally, if the relative electoral weight of the young is large enough, that is, if ϕ is above the upper limit of Φ, politicians would refrain from introducing a (PAYG) intergenerational risk sharing system, even when the worst case, $\omega = 0$, occurs.

The next proposition provides some results on the comparative statics.

Proposition 4.3. *For $\phi \in \Phi$, an increase in the average rate of return, R, or in political weight of the young, ϕ, modifies the time-consistent linear policy function $T^P(\omega) = \theta + T'\omega$ by decreasing its fixed component θ and by making it less responsive to the realization of the state variable, ω.*

The intuition is straightforward. A lower average return reduces the (opportunity) cost – recall that $R > 1 + n$ – of using a PAYG system to provide risk sharing, while a decrease in the political weight of the young, ϕ, increases the electoral cost of introducing risk sharing. In both cases, the fixed component of the system thus shrinks, and the system becomes less responsive to the shocks. In other words, the young prefer less risk sharing with a flatter schedule.

5. How well do politicians do?

Both office-seeking politicians and a benevolent government would provide intergenerational risk sharing in the stochastic environment introduced at section 2. Moreover, their linear time-consistent equilibrium policies share similar properties. As characterized at propositions 3.1 and 4.2, both policies consist of a constant component (A for a benevolent government and θ for the politicians), which is transferred to the elderly in the worst case scenario, i.e., for $\omega = 0$, and of a proportion – respectively B and T' – which reduces the maximum transfer in accordance to the realization of the state variable ω. Propositions 3.2 and 4.3 push these similarities even further, as they suggest that the steady state properties of the two policy functions are comparable.

We now examine under which conditions office-seeking politicians behave exactly as a benevolent government. In other words, when is the interior linear Markov equilibrium policy chosen by politicians aiming to be elected optimal? The next proposition characterizes when the interior linear Markov equilibrium policy $T^P(\omega)$ coincides with the time consistent optimum, $T^G(\omega)$. A graphic representation is at figure 1.

Proposition 5.1. *For $\delta \in \Lambda$ and $\phi \in \Phi$, if $\phi = f(\delta) = \delta \left[1 - \frac{1+n}{\delta S}\right]^{-1}$, the interior linear Markov equilibrium policy chosen by office-seeking politicians correspond* *to the time consistent optimum, i.e.,* $T^P(\omega) = T^G(\omega)\ \forall\omega$*. For* $\phi < f(\delta)$*,* $T^P(\omega) > T^G(\omega)\ \forall\omega$*, with* $T' > B$ *and* $\theta > A$*. For* $\phi > f(\delta)$*,* $T^P(\omega) < T^G(\omega)\ \forall\omega$*, with* $T' < B$ *and* $\theta < A$*.*

According to this proposition, in a Markov game among successive generations of office-seeking politicians deliver the (time consistent) optimum if the relative electoral weight of the young is larger than the weight assigned by a benevolent government to the future generations. In fact, for equal weights, i.e., for $\phi = \delta$, office-seeking politicians will provide more transfer than the social optimum. This transfer is characterized by a larger than optimal fixed component, $\theta > A$; and by a lower than optimal reduction associated to the state of nature, $T' > B$. In other words, political intergenerational risk sharing policy is too generous, and too persistent, that is, not enough responsive to the state variable. These distortions come from the politicians' strategic behavior. In fact, in their decisions over the transfer policy, current politicians anticipate that future politicians will compensate the current young in their old age for their current social security contributions. This stems from the fact that higher taxes on today environment lead to a lower private wealth in old age – that is, to a lower state variable in the following period – thereby triggering more transfers by the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly and leads to overspending – unless the young enjoy an unusually large political power, i.e., $\phi > f(\delta)$.

These two intergenerational risk sharing policies have different implications for the consumption in old age. In both cases, old age consumption depends on the realization of the shock to the returns of the risky assets. However, for $\phi < f(\delta)$, that is, when the politicians are more generous than optimal in their risk sharing policy, they guarantee a higher than optimal expected consumption in old age, but at the cost of introducing also a higher than optimal variance of consumption. By transferring too much resources to old age, and by failing to have these transfers depending more on the realization of the state variable, the politicians fail to provide the optimal risk sharing policy.

6. Aging

One of the most severe challenges for the public social security systems in developed economies is given by population aging. Aging decreases the returns from a PAYG system – thereby requiring either a reduction in pension benefits or an increase in contribution rates (or both), or alternative policy measures, such as a raise in the retirement age. Besides these economic effects, aging has also an important political effect. As suggested in a recent political economy literature, electoral constraints become more binding under aging, since elderly individuals, who are highly supportive of pension systems, are increasingly more relevant in the electorate (see references in footnote 2).

How does aging affect the functioning of a PAYG pension system designed by a benevolent government to achieve intergenerational risk sharing? And how does aging affect the pension policy decisions of politicians who face electoral constraints? The next proposition summarizes the results for both a benevolent government and the politicians.

Proposition 6.1. *For $\delta \in \Lambda$ and $\phi \in \Phi$, population aging, i.e., a reduction in n, increases the time-consistent linear policy transfer chosen by a benevolent government, $T^G(\omega_t)$. For $T^P(\omega_t) > T^G(\omega_t)$, aging tilts the linear Markov equilibrium policy transfer chosen by office-seeking politicians, by reducing the fixed component, θ, and the responsiveness to the shocks. Moreover, population aging reduces the function $f(\delta) = \delta\left[1 - \frac{1+n}{\delta S}\right]^{-1}$.*

These results are quite surprising. In an intergenerational risk sharing framework, aging increases the cost of using the main risk sharing instrument – the PAYG pension system; but it also increases the relative importance of the current generation of elderly both to a benevolent government and to office-seeking politicians. For the benevolent government, the latter effect dominates, and thus aging leads to higher transfers. Surprisingly, for $T^P(\omega_t) > T^G(\omega_t)$, that is, when pension transfers are already very generous – politicians display a more conservative behavior. Aging reduces the transfer level, θ, provided to the elderly in the worst case scenario, i.e., for $\omega = 0$, but it also decreases the responsiveness to the shocks – that is, in good time the transfer level is reduced by a lower amount.

Aging thus affects the time-consistent (interior) linear intergenerational policy chosen by a benevolent government and by office-seeking politicians in opposite direction. As a result, aging has also an effect on the comparison between the choices by a benevolent government and by the politicians analyzed at Proposition 5.1. As shown in figure 2, aging moves downward the function $f(\delta) = \delta\left[1 - \frac{1+n}{\delta R}\right]^{-1}$, which characterizes the equivalence between the benevolent government and the politicians' decision. Politicians thus become less likely to "overspend" – that is, the range of parameters over which they provide more transfers than optimal is reduced. This, perhaps surprising, result stems from the fact that aging increases the cost of large current transfers in terms of tomorrow's adjustments. In other words, it becomes more costly for current politicians to provide "too much" risk sharing, since the PAYG system is less efficient and future politicians will become less keen on accommodating the previous policies with provision of large transfers. Unlike most of the results in the current political economy literature, Proposition 6.1 thus suggests that aging will limit social security spending by reducing the room for the politicians' strategic behavior.

7. Concluding Remarks

The risk sharing properties of social security have long been recognized in the literature. In several stochastic environments, individuals would benefit from insuring against aggregate shocks before they are even born. Clearly, this is not possible. Yet, once they are born – and uncertainty is realized, there is no more room for risk sharing. Establishing a PAYG system thus seems to require the existence of a long term player, who can bind future, yet-to-be-born generations to carry out the risk sharing policy.

In absence of commitment, we show that a benevolent government and office-seeking politicians choose to adopt a state contingent social security system. The amount of resources transferred to the elderly by the working generation depends *negatively* on the elderly private wealth – and therefore on the realization of the aggregate shock to the returns of the risk asset. This state contingent social security thus constitutes a quasi asset, whose returns are negatively related to the market returns. This result is in line with Ball and Mankiw (2007) who propose an optimal intergenerational risk sharing plan with a negative correlation between social security benefits and asset returns.

However, the intergenerational risk sharing schemes proposed by a benevolent government and by the politicians may differ. Office-seeking politicians are more likely to provide generous transfers that are less responsive to the aggregate shock, and hence more persistent. While persistence is typically at the root of efficient intergenerational risk sharing policies, since it allows to spread the risk over time and hence over several generations, office-seeking politicians have an incentive to overplay this feature. In fact, politicians are willing to tax more heavily current workers and to provide generous transfers to the current elderly, because they anticipate that future politicians – facing elderly individuals with low net wealth, due to the large contributions they had to pay in their youth – will compensate them with generous pension transfers. This mechanism thus reduces the electoral cost among the young voters of providing large transfers and leads to generous, persistent pension systems. This result is consistent with Bohn (2003) findings that the current US social security system does not provide the optimal level of risk sharing, since it is too generous with the elderly and shifts most of the burden of risk sharing on to future generations.

Perhaps surprisingly, our model suggests that the aging process needs not to exacerbate the generous treatment that office-seeking politicians provide to the elderly. In fact, due to population aging, electoral-minded politicians will indeed become less likely to "overspend". This is because aging makes the strategic behavior by current office-seeking politicians more costly. In fact, aging reduces the return from social security, and hence the amount of transfers that future politicians will be willing to provide to elderly individuals with low net wealth – thereby increasing the electoral cost for the current politicians of higher transfers.

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Appendix

Proof of Proposition 3.1

Consider the optimization problem of the benevolent government at eq. 3.1. Its recursive formulation yields the following first order condition:

$$\frac{\partial U(c_t)}{\partial T_t} + \delta E_t \frac{\partial U(c_{t+1})}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial T_t} = 0$$

where $\omega_t = R_t(1 - T_{t-1})$ defines the state variable at time t. Using the utility function at eq. 2.3, the above expression can be written as eq. 3.3. To solve for interior equilibrium policies, we guess a linear solution: $T^G(\omega_t) = A + B\omega_t$. Using simple algebra, from eq. 3.3 we obtain the following expression:

$$T^G(\omega_t) = -\frac{\omega_t}{1 + n + \delta S(1 + B(1 + n))} + \frac{\gamma(1 - \delta R) + \delta S(1 + B(1 + n)) - \delta(1 + n) AR}{1 + n + \delta S(1 + B(1 + n))}.$$

Hence, we have

$$B = -\frac{1}{1 + n + \delta S(1 + B(1 + n))} \tag{7.1}$$

$$A = \frac{\gamma(1 - \delta R) + \delta S(1 + B(1 + n)) - \delta(1 + n) AR}{1 + n + \delta S(1 + B(1 + n))} \tag{7.2}$$

For $1 + B(1 + n) \neq 0$, we obtain

$$B = -\frac{1}{\delta S} \text{ and} \tag{7.3}$$

$$A = \frac{\gamma - (1 + n) + \delta(S - \gamma R)}{\delta(S - (1 + n) R)} \tag{7.4}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, that is, $T^G(\omega) < 1$ $\forall \omega$. To guarantee that this condition holds, we need to impose $T^G(\omega) < 1$ for $\omega = 0$, i.e., $A < 1$. Simple algebra yields $\delta > 1/R$. Additionally, we require some risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The most favorable case for this transfer to occur is for $\omega = 0$. We hence need to have $A > 0$. Simple algebra shows that, since $\gamma > S/R$ (assumption 3), this occurs for $\delta < \frac{\gamma-(1+n)}{R\gamma - S}$. However, it is easy to see that $\frac{\gamma-(1+n)}{R\gamma-S} > \frac{1}{1+n}$, thus $\delta \in \Lambda = \left(\frac{1}{R}, \frac{1}{1+n}\right)$. Q.E.D.

Proof of Proposition 3.2

For $\delta \in \Lambda$, recall that the time-consistent linear policy function is $T^G(\omega_t) = A + B\omega_t$ with A and B defined in proposition 3.1.

(i) consider a change in the discount factor, δ. Simple algebra shows that $\frac{\partial B}{\partial \delta} = \frac{1}{\delta^2 S} > 0$ and $\frac{\partial A}{\partial \delta} = -\frac{\gamma-(1+n)}{\delta^2(S-(1+n)R)} < 0$.

(ii) consider a change in the average rate of return, R. We have that $\frac{\partial B}{\partial R} = \frac{2R}{\delta S^2} > 0$ and $\frac{\partial A}{\partial R} = -\frac{(\gamma-(1+n))\left[\delta\left(\sigma^2-R^2\right)+2R-(1+n)\right]}{\delta(S-(1+n)R)^2} < 0$, since $\sigma^2 > R^2$ (assumption 1).

Finally, (iii) consider a change in the variance of the shock, σ^2. It is easy to see that $\frac{\partial B}{\partial \sigma^2} = \frac{1}{\delta S^2} > 0$ and $\frac{\partial A}{\partial \sigma^2} = \frac{(\delta R-1)[\gamma-(1+n)]}{\delta(S-(1+n)R)^2} > 0$. Hence, $\frac{\partial T^G(\omega)}{\partial \sigma^2} > 0$. Q.E.D.

Proof of Proposition 4.2

Consider the first order condition at eq. 4.5, which describes the stationary Markov policy chosen by the politician a time t. Recall that the state variable is defined as $\omega_t = R_t\left(1 - T^P_{t-1}\right)$ $\forall t$, and that $T^P(\omega_t) = \theta + T'\omega_t$. Moreover, define $Q = 1 + (1+n)T'$. We need to obtain the value of the parameters T' and θ, which solve this FOC. Using simple algebra, from eq. 4.5 we obtain the following expression:

$$T^P(\omega_t) = -\frac{\omega_t}{1+n+\phi S Q^2} + \frac{\gamma + \phi S Q^2 - \phi Q R(\gamma - \theta(1+n))}{1+n+\phi S Q^2}.$$

Hence, we have

$$T' = -\frac{1}{1+n+\phi S Q^2} \tag{7.5}$$

$$\theta = \frac{\gamma + \phi S Q^2 - \phi Q R(\gamma - \theta(1+n))}{1+n+\phi S Q^2} \tag{7.6}$$

Since $Q = 1 + (1+n)T'$, we solve the expression at eq. 7.5 for T' to find two solutions:

$$T'_A = -\frac{1}{2(1+n)}\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right) \tag{7.7}$$

$$T'_B = -\frac{1}{2(1+n)}\left(1 - \sqrt{1 - \frac{4(1+n)}{\phi S}}\right). \tag{7.8}$$

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ω. That is, we require $T^P(\omega) < 1 \ \forall \omega$. To guarantee this condition for these two candidate solutions of T', we need to impose the first order condition of the politicians (see eq. 4.5) evaluated at $\omega = 0$ to be negative when $T = 1$. Substituting $\omega = 0$ and $T = 1$ in eq. 4.5, and imposing the expression to be negative yields the following inequality:

$$\phi(1 + (1+n)\,T')R < 1 \tag{7.9}$$

Let's begin investigating the candidate solution $T' = T'_A$. Substituting T'_A in eq. 7.9 yields the following inequality

$$\phi R \sqrt{1 - \frac{4\,(1+n)}{\phi S}} < \phi R - 2.$$

Clearly, for $\phi < 2/R$, the above inequality is not satisfied, and thus T'_A is not part of an interior equilibrium solution. For $\phi > 2/R$, we can elaborate on the above expression to obtain the following inequality: $\phi > \frac{S/R}{S-R(1+n)}$. Simple algebra shows that for $\sigma^2/R^2 > 1$ (see assumption 1), this inequality cannot hold for $\phi > 2/R$. Hence, candidate solution $T' = T'_A$ cannot be part of an interior equilibrium solution.

Let's now turn to the candidate solution $T' = T'_B$. Substituting T'_B in eq. 7.9 yields the following inequality

$$\phi R \sqrt{1 - \frac{4\,(1+n)}{\phi S}} > 2 - \phi R.$$

Clearly, for $\phi > 2/R$, the above inequality is always satisfied, and thus T'_B can be part of an interior equilibrium solution. For $\phi < 2/R$, the above inequality can be rewritten as $\phi > \frac{S/R}{S-R(1+n)}$. Notice that for $\sigma^2/R^2 > 1$ (see assumption 1) $\frac{S/R}{S-R(1+n)} < 2/R$. Hence, candidate solution $T' = T'_B$ is part of an interior equilibrium solution if $\phi > \frac{S/R}{S-R(1+n)}$.

With $T' = T'_B$, we can now solve the expression at eq. 7.6 for θ. Simple algebra shows that $\theta = \frac{2(\gamma-(1+n))-\phi(\gamma R-S)\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}{\phi[S-R(1+n)]\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)}$.

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, ω. That is, we require $T^P(\omega) < 1\ \forall\omega$. To guarantee that this condition holds, we need to impose $T^P(\omega) < 1$ for $\omega = 0$, i.e., $\theta < 1$. Simple algebra yields $\phi > \frac{S/R}{S-R(1+n)}$. Additionally, we require some risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The more favorable case for this transfer to occur is for $\omega = 0$. We hence need to have $\theta > 0$. Simple algebra shows that the denominator is always positive, while the numerator is positive for $\phi < \frac{S(\gamma-(1+n))^2}{(S-R(1+n))\gamma(\gamma R-S)}$. Q.E.D.

Proof of Proposition 4.3

For $\phi \in \Phi$, recall that the time-consistent linear policy function is $T^P(\omega) = \theta + T'\omega$ with θ and T' defined in proposition 4.2. Notice that we can write

$$\theta = \frac{2\left(\gamma - (1+n)\right)}{\phi\left(S - R\left(1+n\right)\right)\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)} - \frac{\gamma R - S}{S - R\left(1+n\right)}. \tag{7.10}$$

Consider an increase in the average rate of return, R. It is easy to see that $\frac{\partial T'}{\partial R} = \frac{2R}{\phi S^2\sqrt{1-\frac{4(1+n)}{\phi S}}} > 0$ and $\frac{\partial \theta}{\partial R} = -\frac{\theta(2R-(1+n))}{(S-R(1+n))} - \frac{\gamma-2R}{(S-R(1+n))} - \frac{8(1+n)R(\gamma-(1+n))}{\phi^2(S-R(1+n))\left(1+\sqrt{1-\frac{4(1+n)}{\phi S}}\right)^2} < 0$, since $\gamma R > S$ and $\sigma > R$ imply that $\gamma > 2R$.

Consider an increase in ϕ, it is easy to see that $\frac{\partial T'}{\partial \phi} = \frac{1}{\phi^2 S\sqrt{1-\frac{4(1+n)}{\phi S}}} > 0$. Notice that θ can be written as

$$\theta = \frac{\phi S Q^2 + \gamma\left(1 - \phi Q R\right)}{\phi S Q^2 + \left(1+n\right)\left(1 - \phi Q R\right)} \tag{7.11}$$

with $Q = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)$ and $\phi Q R > 1$. Using the above expression, we have

$$\frac{\partial \theta}{\partial \phi} = -2\frac{(\gamma - (1+n))\left[2(1+n) + S(1+\sqrt{\Delta})\sqrt{\Delta}\phi\right]}{S(S-(1+n)R)(1+\sqrt{\Delta})^2\sqrt{\Delta}\phi^3} < 0$$

with $\sqrt{\Delta} = \sqrt{1 - \frac{4(1+n)}{\phi S}}$. Q.E.D.

Proof of Proposition 5.1

Comparing the first order condition respectively for the benevolent government (eq. 3.3) and for the politicians (eq. 4.5), we have that, in an interior equilibrium, the benevolent government and the politicians will adopt the same policy if $\delta = \phi\left(1 + (1+n)\,T'\right)$. Recall that interior equilibrium policies, involving risk sharing at least when $\omega = 0$, require respectively, $\delta \in \Lambda$ and $\phi \in \Phi$. Using the expression for T' at proposition 4.2, the above expression can be rewritten as $\phi = f(\delta) = \delta \left[1 - \frac{1+n}{\delta S}\right]^{-1}$. Furthermore, it is trivial to see that for δ and $T^G(\omega)$ that solve the benevolent government problem (for an interior equilibrium), if $\phi > f(\delta)$, at $T^P(\omega) = T^G(\omega)$ the first order condition of the politicians is negative, so that $T^P(\omega) < T^G(\omega)$. And viceversa for $\phi < f(\delta)$.

Proof of Proposition 6.1

Recall that, for $\delta \in \Lambda$, the time-consistent linear policy function chosen by the benevolent government is $T^G(\omega_t) = A + B\omega_t$ with A and B defined in proposition 3.1. To see the effect of population aging, i.e., a reduction in n, notice that $\frac{\partial B}{\partial n} = 0$ and that $\frac{\partial A}{\partial n} = -\frac{(\delta R - 1)(R\gamma - S)}{\delta [S - R(1+n)]^2} < 0$. Hence, $\frac{\partial T^G(\omega_t)}{\partial n} < 0$.

Recall that for $\phi \in \Phi$, the time-consistent linear policy function chosen by the politicians is $T^P(\omega) = \theta + T'\omega$ with θ and T' defined in proposition 4.2. To assess the effect of population aging, we need to determine $\frac{\partial T'}{\partial n}$ and $\frac{\partial \theta}{\partial n}$, and hence $\frac{\partial T^P(\omega)}{\partial n}$. We have

$$\frac{\partial T'}{\partial n} = \frac{2(1+n) - \phi S \left(1 - \sqrt{1 - \frac{4(1+n)}{\phi S}}\right)}{2(1+n)^2 \phi S \sqrt{1 - \frac{4(1+n)}{\phi S}}} < 0$$

since it is easy to show that the numerator is negative. To calculate the change in θ, it is convenient to use the expression at eq. 7.10. After simple algebra we have

$$\frac{\partial \theta}{\partial n} = \frac{2(1+\sqrt{\Delta})\sqrt{\Delta} S \phi [\gamma R - S] + 4[\gamma - (1+n)][S - (1+n)R] - [\gamma R - S] R \phi^2 (1+\sqrt{\Delta})^2 \sqrt{\Delta} S}{\phi^2 [S - (1+n)R]^2 (1+\sqrt{\Delta})^2 \sqrt{\Delta} S}$$

where $\sqrt{\Delta} = \sqrt{1 - \frac{4(1+n)}{\phi S}}$. Since the denominator is clearly positive, the sign of $\frac{\partial \theta}{\partial n}$ corresponds to the sign of the numerator. Recall that $Q = \frac{1}{2}\left(1 + \sqrt{\Delta}\right)$. Then $\frac{\partial \theta}{\partial n} > 0$ if

$$[\gamma - (1+n)][S - (1+n)R] > (\gamma R - S)\phi S Q \sqrt{\Delta}[\phi Q R - 1]. \tag{7.12}$$

Using the definition of θ at eq. 7.10, we have that $(\gamma R - S) = \frac{\gamma - (1+n)}{\phi Q} - \theta[S - R(1+n)] < \frac{\gamma - (1+n)}{\phi Q}$. Thus, a sufficient condition for 7.12 to hold is that $[\gamma - (1+n)][S - (1+n)R] > \frac{\gamma - (1+n)}{\phi Q} \phi Q S \sqrt{\Delta}[\phi Q R - 1]$, that is,

$$[S - (1+n)R] > S\sqrt{\Delta}[\phi Q R - 1]. \tag{7.13}$$

We now show that eq. 7.13 is satisfied when $T^P(\omega_t) > T^G(\omega_t)$. First, notice that $S\sqrt{\Delta}[\phi QR - 1]$ is increasing in ϕ for $\phi QR > 1$. Since $T^P(\omega_t) > T^G(\omega_t)$, for $\phi < f(\delta)$, with f being increasing in δ, a sufficient condition for eq. 7.13 to hold is that $[S-(1+n)R]$ is greater than $S\sqrt{\Delta}[\phi QR-1]$ evaluated at $\phi = f\left(\delta = \frac{1}{1+n}\right)$. This amounts to

$$S - (1+n)R > \frac{[S - 2(1+n)^2}{S} \frac{R - (1+n)}{(1+n)}$$

or rearrenging terms to

$(1+n)S^2 - [(1+n)^2 R + R - (1+n)]S + 2(1+n)^2[R - (1+n)] > 0$. Just by concentrating on the first two terms, it is easy to see that this condition holds, since $S > 2R^2$ and $R > (1+n)$.

Finally, population aging affects the function $f(\delta) = \delta\left[1 - \frac{1+n}{\delta S}\right]^{-1}$. In particular, $\frac{\partial f(\delta)}{\partial n} = \left(\frac{\delta}{\delta - \frac{1+n}{S}}\right)^2 \frac{1}{S} > 0$. Thus, for a given value of δ, aging reduces the value of ϕ such that the transfer policy chosen by the politicians is socially optimal. Since by proposition 5.1 we have that $T^P(\omega) > T^G(\omega)$ for $\phi < f(\delta)$ aging reduces the set of parameters (ϕ, δ) such that politicians "overspend", that is, such that $T^P(\omega) > T^G(\omega)$.

Figure 1: Comparing Intergenerational Risk Sharing
by a Benevolent Government (G) and by Politicians (P)

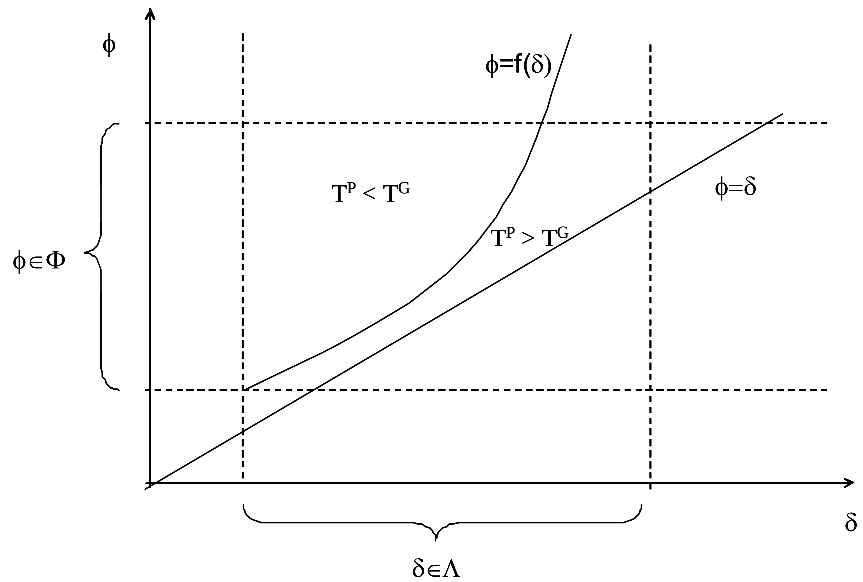

Figure 7.1:

Figure 2: Comparing Intergenerational Risk Sharing by a Benevolent Government (G) and by Politicians (P): the Effect of Aging

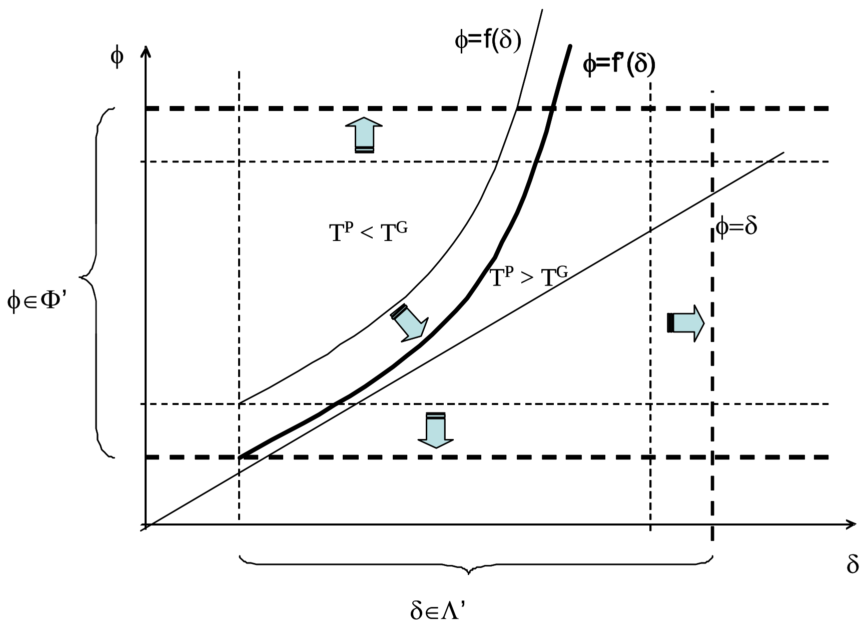

Figure 7.2: