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The Effects of Monetary Policy on Unemployment Dynamics Under Model Uncertainty. Evidence from the US and the Euro Area

Carlo Altavilla, Matteo Ciccarelli

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University of Naples Federico II

University of Salerno

Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unisa.it

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The Effects of Monetary Policy on Unemployment Dynamics Under Model Uncertainty. Evidence from the US and the Euro Area

Carlo Altavilla[♥], Matteo Ciccarelli[♠]

Abstract
This paper explores the role that the imperfect knowledge of the structure of the economy plays in the uncertainty surrounding the effects of rule-based monetary policy on unemployment dynamics in the euro area and the US. We employ a Bayesian model averaging procedure on a wide range of models which differ in several dimensions to account for the uncertainty that the policymaker faces when setting the monetary policy and evaluating its effect on real economy. We find evidence of a high degree of dispersion across models in both policy rule parameters and impulse response functions. Moreover, monetary policy shocks have very similar recessionary effects on the two economies with a different role played by the participation rate in the transmission mechanism. Finally, we show that a policy maker who does not take model uncertainty into account and selects the results on the basis of a single model may come to misleading conclusions not only about the transmission mechanism, but also about the differences between the euro area and the US, which are on average essentially small.

JEL Classification Codes: C11, E24, E52, E58

Keywords: Monetary policy, Model uncertainty, Bayesian model averaging, Unemployment gap, Taylor rule

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[♥] University of Naples "Parthenope" and CSEF. Address: University of Naples "Parthenope", Via Medina, 40 - 80133 Naples (Italy). E-mail: altavilla@uniparthenope.it; Phone: (+)39 0815474733, fax (+)39 0815474750.

[♠] European Central Bank, Kaiserstrasse 29, - 60311 Frankfurt am Main (Germany). E-mail: matteo.ciccarelli@ecb.europa.eu; Phone: (+)49 6913448721, fax (+)49 6913446575.

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1 Introduction

The pervasive uncertainty that central banks face precludes monetary policy from fine tuning the level of economic activity. This paper explores the role that the imperfect knowledge of the structure of the economy plays in the uncertainty surrounding the effects of rule-based monetary policy on unemployment dynamics in the euro area and the US.

An extended (empirical and theoretical) literature has described how central banks should take uncertainty into account in their decision-making process. A large part of this literature has focused on the robustness of policy actions. Since the seminal papers of Hansen and Sargent (e.g. 2001 and 2007), many researchers (e.g. Giannoni 2002; Onatski and Stock 2002; Brock al. 2003) have studied monetary policy uncertainty with a 'robust control methodology.' Within this framework, the uncertainty surrounding the effect of policy actions is measured by first constructing a model space where each model is obtained as a *local* perturbation to a given baseline model and then applying a minimax rule. As a result, a policy with the smallest possible maximum risk is preferred.

Other studies (e.g. Levin and Williams 2003; Brock et al. 2007) have recently accounted for model uncertainty with a Bayesian model averaging approach that, unlike the robust control methodology, usually considers a model space with theoretically distinct models. The idea is that, given considerable uncertainty about the true structure of the economy, policymakers aim at identifying measures that perform well across a wide range of *non-local* models. Results are then obtained as weighted averages across models, with weights given by the relative marginal likelihood of the models.

Our paper follows the latter approach. Moreover, unlike most of the literature which only focuses on how monetary policy should systematically react to changes in unemployment and inflation (i.e. the policy rules), we go further and also analyze how the uncertainty about the policy rule translates into the uncertainty surrounding the responses of the economy (and in particular of unemployment) to policy shocks. We assume that the monetary authority minimizes expected losses of a social loss function subject to the economy, and sets up a policy rule. In turn, the economy is alternatively summarized by a wide range of multivariate models that differ in the assumptions regarding the persistence of inflation and unemployment, the measurement of the natural rate of unemployment, the number and types of variables entering the model, and the lag structure.

The perspective adopted in this paper is Bayesian, meaning that a complete model involving unobservables (e.g. parameters), observables (e.g. data) and variables of interest (e.g. policy rule, impulse response functions) is identified by a joint distribution of these elements. If M denotes a model, θ_M denotes unobservable parameters, D denotes the observables, and ω is a vector of interest, then the model M specifies the joint distribution

$$p(\theta_M, D, \omega \mid M) = p(\theta_M \mid M)\, p(D \mid \theta_M, M)\, p(\omega \mid D, \theta_M, M) \tag{1}$$

The object of inference, then, is expressed as the posterior density of ω:

$$p(\omega \mid D, M) = \int p(\omega \mid D, \theta_M, M)\, p(\theta_M \mid D, M)\, d\theta_M \tag{2}$$

which is the relevant density for the decisionmakers. In this framework, two sources of uncertainty are considered. Model uncertainty is accounted for with the incorporation of several competing models $M_1, M_2, ..., M_J$ which might have generated the available sample of data. Parameter uncertainty is reflected in a series of informative priors on the unobservables $p\left(\theta_{M_j} \mid M_j\right)$. We evaluate the degree of dispersion of $p(\omega \mid D, M_j)$ between models and quantify the effects which policy prescriptions coming from different models have on unemployment.

The paper can be considered as an extended application of the methodological approach suggested, for instance, by Brock et al (2007). As in their work, all models are equally likely a priori; unlike their assumption, we specify informative priors for the model parameters and compare models on the basis of their marginal likelihoods.

Using data for the US and the euro area, we show that simple linear autoregressive models which differ in several dimensions may produce a significant degree of uncertainty in the distribution of optimal policy parameters, expected losses and impulse responses.

Cross-country comparison corroborates the findings of Sauch and Smets (2008) and Smets and Wouters (2005) that the differences in the monetary policy reaction function in the US and the euro area are small. Moreover, although a monetary policy shock might be less important than other structural shocks to explain unemployment dynamics, we show that on average it has a stable recessionary effect in both economies. We also find that the average unemployment responses are qualitatively and quantitatively very similar in the two economies, with results for the euro area being more dispersed than those for the US. The analysis of the transmission mechanism also indicates that other labor market variables, such as participation rate, play an important distinctive role in the two economies.

Our results have significant policy implications. The high degree of dispersion across models suggests that the effects of a given policy measure are model dependent, and therefore policy decisions should be based on a wide range of possible scenarios about the structure of the economy in order to overcome policy mistakes. We show that a policy maker who selects the results on the basis of a single model may come to misleading conclusions not only about the transmission mechanism – picking up models where, for instance, the price puzzle is more marked or the response of unemployment has a wrong sign – but also about the differences between the euro area and the US – which may only result as an outcome of model selection. A combination procedure, instead, helps dampen out this uncertainty. By taking into account model uncertainty and averaging across models, results are more consistent with the economic theory and provide the policymaker with a robust environment to calibrate interventions in a less distorting way for the economy.

The remainder of the paper is structured as follows. Section 2 describes the general framework with the model space and the solution to the central bank's problem. Section 3 reports the empirical findings in terms of expected loss and policy parameters. Section 4 discusses the effects of a monetary policy shock on unemployment in the designed uncertain environment. Section 5 summarizes the paper's main findings and provides conclusive remarks. A technical appendix presents the model space and derives the posterior distributions for the Bayesian inference.

2 Model uncertainty and optimal monetary policy: the macroeconometric framework

In this section we illustrate the empirical framework, which comprises: (i) a set of monetary policy rules; (ii) a monetary policymaker who chooses the parameters of the rules by minimizing a loss function; (iii) a set of models which summarize the constraints faced by the policymaker in the minimization problem.

A wide set of models is used to account for the uncertainty surrounding the representation of the economy. As described in Brock et al. (2007) model uncertainty results from sources as different as economic theory, specification conditional on theory, and heterogeneity regarding the data generating process. We will generate the model space by limiting the analysis to multivariate dynamic linear models (VARs) which entail policy and non-policy variables, with different prior assumptions on both sets of variables, as well as on the lag structure.

The structural behavior of the non-policy variables is assumed to be given by the estimates of the model. Using this estimated structure, the solution to the minimization problem yields the values of the loss function under alternative policy parameters. A given set of these parameters will then minimize the expected loss for each model. The interest rate policy which results from this optimization problem can be of two types: (i) a linear optimal feedback rule (OFR) where the nominal interest rate depends on all observable variables included in the model and which appear to have a closed-form solution; and (ii) an optimized Taylor rule (TR) where the interest rate only reacts to the current value of the unemployment gap and the inflation rate, similarly to the original work of Taylor (1993), and where the weights attached to both variables are obtained with a grid search procedure.

Finally, the optimal or optimized rule becomes part of the interest rate equation in a structural VAR, and its disturbance is used to quantify the uncertainty surrounding the effect of a monetary policy shock on the unemployment gap using a standard Impulse Response Function (IRF) analysis as, e.g., in Stock and Watson (2001).

In the following, we detail these elements backwards, starting from the model and then turning to the policymakers and the rules.

2.1 The model space

We start by specifying a comprehensive range of multivariate linear dynamic models which span the model space. The class of simultaneous equation models considered here takes the following general VAR form:

$$\begin{aligned} Z_t &= \sum_{j=1}^{p} \mathbf{A}_j Z_{t-j} + \sum_{j=0}^{p} \mathbf{b}_j i_{t-j} + \epsilon_t^z \\ i_t &= \sum_{j=0}^{p} \mathbf{c}_j' Z_{t-j} + \sum_{j=1}^{p} d_j i_{t-j} + \epsilon_t^i \end{aligned} \tag{3}$$

where Z_t is a vector of non-policy variables; i_t is the policy variable; $\mathbf{A}, \mathbf{b}, \mathbf{c}, d$ are conformable matrices and vectors; ϵ_t^Z and ϵ_t^i are vectors of serially uncorrelated structural disturbances. In section 3 we will explain in more details the estimation algorithm and the impulse response analysis. For the purpose of this section, it is sufficient to remark here that the structural coefficients can be easily recovered with some identification scheme. We will use the same scheme throughout the paper (both for the optimal policy derivation and for the impulse response analysis) and impose the timing assumption that the central bank reacts contemporaneously to all variables in the economy, whereas the policy rate does not contemporaneously affect the rest of the economy. In terms of the above VAR, this assumption imposes a Choleski scheme by setting $\mathbf{b}_0 = 0$.[1]

The non-policy block Z_t contains at least the inflation rate (π_t) and the (negative) unemployment gap (u_t), calculated as the difference between the natural rate of unemployment (u_t^*) and its actual value ($\widetilde{u}_t$). Other non-policy variables enter the specification in the form we will explain below.

Four broad sets of prior beliefs shape the dimensions of model uncertainty that characterize the model space.

[1] The set up is similar to the one used e.g. by Sack (2000) in a different context.

2.1.1 Priors on inflation dynamics

In the first set of priors, we deal with assumptions about the way the inflation rate is modelled. Concretely, four general prior assumptions are made according to whether inflation is (i) left unrestricted, or whether it is treated in the system as (ii) a random walk, (iii) an autoregressive process of order p, or (iv) a white noise. In all cases we take a Bayesian perspective and place the exclusion restrictions through the allocation of probability distributions to the model's coefficients. The starting point is always a Minnesota-type of prior: in the unrestricted case we complement the autoregressive representation with the specification of a vague prior distribution and a loose tightness on all coefficients; in the other three setups, instead, we assume that inflation follows one of the three processes by setting the mean of own-lag coefficients, and allow for a much tighter precision placed on all coefficients of the inflation equation as compared to the precision placed on the coefficients of other equations. In other words, priors are always informative and differ in the relative tightness placed on the coefficients in the equation for π_t.[2]

2.1.2 Priors on labor market variables

The second set of priors reflects different assumptions on the dynamics of the labor market variables. We distinguish two types of prior, according to (i) the degree of persistence of the variables and (ii) the computation of the natural rate of unemployment.

Analogously to the treatment of inflation, we model the degree of persistence of unemployment (and participation rate, when included in the specification) either in an unrestricted way – by placing a general unit root Minnesota prior and a loose tightness – or restricting the variables to have a lower degree of persistence. In the latter case, as for the inflation dynamics, we set the mean of own-lag coefficients to a value lower than one, while allowing for a much tighter precision placed on the variance of these coefficients.

Regarding the uncertainty about the natural rate of unemployment, there has been an extensive debate in the literature on the implications of natural rates mismeasurement for monetary policy. Staiger et al. (1997a,b) and Laubach (2001) found that estimates of a time-varying natural rate of unemployment are considerably imprecise. The same results are documented by Orphanides and van Norden (2005) when analyzing the output gap. Finally, Orphanides and Williams (2007) suggest that policymakers should consider policy rules that react to changes in economic activity either than

[2] Note that while the Random Walk and the Autoregressive hypotheses are relatively standard in the VAR literature (see e.g. Doan et al., 1984; Stock and Watson, 2007), the White Noise (WN) assumption has been recently validated in studies on inflation persistence that cover especially the last 10-15 years of sample observations. Benati (2008), for instance, shows that on recent samples the WN assumption might have become a reasonable one in several countries, including UK and the euro area, the latter especially after the creation of EMU. Our sample choice for the empirical analysis is consistent with this prior (see section 3.1).

reacting to the uncertain estimates of the natural rate. We do not pretend to be exhaustive here and limit the scope of our analysis to two types of detrending methods: (i) a "statistical" approach which uses the Baxter and King (1999) band pass filter; and (ii) a robust alternative that measures the natural rate with a Phillips-curve method and incorporates some "economic" content. The details of both approaches are given in the section on data transformation (Section 3.1).

2.1.3 Priors on other variables

In the third set of priors, we enlarge the model space by changing the model specification of the non-policy block, and considering all combinations of three additional endogenous variables: the labor force participation rate (pr_t); the exchange rate (e_t), and a commodity price inflation rate (cp_t).

The inclusion of the participation rate is motivated by the possibility of shaping more comprehensive dynamics of the labor market, as a negative impact of an increase in the nominal interest rate on output may have diverse effects on the labor force and, ultimately, on the unemployment rate. The inclusion of the participation rate would account for these effects and provide a cleaner picture of the transmission mechanism. As observed above in the description of the first set of priors, when the participation rate is included in the specification, it enters either with a vague Minnesota (unit root) prior, or with a lower degree of persistence.

While the inclusion of an exchange rate might not be suitable for the US (e.g. Taylor, 2001), it might nonetheless be appropriate for the Euro area (e.g. Peersman and Smets, 2003; Altavilla, 2003). In any case, its inclusion is intended to reflect the external environment, as well as its conditionality role for monetary policy, as it is an important part of the monetary transmission mechanism in an open economy. Moreover, some researchers provide empirical evidence that exchange rates are statistically significant in monetary policy rules summarizing the reaction functions of several major central banks (e.g. Clarida et al., 1998; Svensson, 2000).

Finally, we include a commodity price inflation rate which should control for the expected future inflation, as it has become customarily in recent applied works on the transmission mechanism of monetary policy shocks (see e.g Sack 2000.)

2.1.4 Priors on the lag structure

In the last set of prior assumptions, the dynamics of the system is described by alternative lag structures. The Wold theorem implies that VAR residuals must be white noise. Sometimes this feature happens to be verified with a parsimonious representation of the lag structure, perhaps with a rich number of endogenous variables. The VAR, however, easily becomes overparametrized, since the number of coefficients grows as a quadratic function of the number of variables and proportionately to the number of lags. To trade-off between parsimonious and realistic assumptions, we combine dogmatic with flexible priors and consider models with p lags, where $p = 1, 2, 3$ or 4. Then, for models where $p > 1$, a tight Minnesota prior on coefficients different from the own lag is used.

Summing up, should we account for all possible combinations of the features described above, we would be dealing with a very large number of models. The model space would in fact be composed of 1024 models, as a result of the product of 4 priors on inflation persistence, 2 priors on the persistence of unemployment, 2 priors on the persistence of participation, 2 priors on the detrending methods, $2^3 = 8$ ways to combine variables in a model with a fixed block $[u, \pi, i]$ and three additional non-policy variables, and 4 lag assumptions.

We take a shortcut, instead, and restrict the analysis to a comprehensive subset spanned by 224 models. The composition of the models can be summarized as follows:

1. A group of models focuses on inflation dynamics and combines the three restrictive priors on inflation persistence with unrestricted labor market variables and a band-pass estimation of the natural rate of unemployment. This combination produces therefore 96 models given by the product of 3 alternative priors on inflation, 8 ways to add the other non-policy variables and 4 lag assumptions.

2. The remaining 128 models are characterized by assumptions on the labor market combined with unrestricted inflation dynamics, and are obtained from the product of 2 prior assumptions on the persistence of labor market variables, 2 detrending methods for the natural rate, 8 ways to add the other non-policy variables, and 4 lag assumptions.

Details of the model space are reported in the appendix (Table A1). The priors on other unknown of the system which have not been described above will be described in Section 3.

2.2 The Central Bank's Problem

The central bank minimizes an intertemporal loss function that has a positive relation with the deviation between the goal variables and their target levels:

$$L_t = E_t \left\{ \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\vartheta u_{t+\tau}^2 + \lambda \pi_{t+\tau}^2 + \gamma \left(i_{t+\tau} - i_{t+\tau-1} \right)^2 \right] \right\} \tag{4}$$

where E_t denotes the expectations conditional upon the available information set at time t; δ is a given discount factor, $0 < \delta < 1$; and ϑ, λ, and γ are non-negative weights.

The variable u_t has already been defined above as the gap between the natural rate of unemployment and its actual value. We also interpret here π_t as the deviation from a constant inflation target. As a benchmark for our analysis, we take $\vartheta = 4$, $\lambda = 1$, and $\gamma = 0.5$. Based on the Okun's law, the variance of the unemployment gap is about 1/4 of the variance of the output gap, so this choice of ϑ is consistent with an equal weight on inflation and output gap variability.[3]

As shown in Rudebush and Svensson (1999), for $\delta = 1$, the loss function can be written as the weighted sum of the unconditional variances of the target variables:

$$E\left[L_t\right] = \vartheta Var\left[u_t\right] + \lambda Var\left[\pi_t\right] + \gamma Var\left[i_t - i_{t-1}\right] \tag{5}$$

The aim is to minimize this function subject to

$$X_{t+1} = \Xi X_t + \Psi i_t + \eta_{t+1} \tag{6}$$

which is the State space representation of the VAR (Eq.3). The dynamics of the state are governed by the matrix Ξ and the vector Ψ, whose values are given by the point estimates of the corresponding VAR coefficients, and depend on the particular model considered in the estimation. As a consequence, we have 224 state-space representations for each country. For example, in a model with 4 non-policy variables and two lags, the state space has the following representation:

$$X_t = \begin{bmatrix} u_t \\ u_{t-1} \\ pr_t \\ pr_{t-1} \\ e_t \\ e_{t-1} \\ \pi_t \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix}, \Xi = \begin{bmatrix} a_{11}^1 & a_{11}^2 & a_{12}^1 & a_{12}^2 & a_{13}^1 & a_{13}^2 & a_{14}^1 & a_{14}^2 & b_{15}^2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}^1 & a_{21}^2 & a_{22}^1 & a_{22}^2 & a_{23}^1 & a_{23}^2 & a_{24}^1 & a_{24}^2 & b_{25}^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31}^1 & a_{31}^2 & a_{32}^1 & a_{32}^2 & a_{33}^1 & a_{33}^2 & a_{34}^1 & a_{34}^2 & b_{35}^2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ a_{41}^1 & a_{31}^2 & a_{42}^1 & a_{42}^2 & a_{43}^1 & a_{43}^2 & a_{44}^1 & a_{44}^2 & b_{45}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Psi = \begin{bmatrix} b_{15}^1 \\ 0 \\ b_{25}^1 \\ 0 \\ b_{35}^1 \\ 0 \\ b_{45}^1 \\ 0 \\ 1 \end{bmatrix}, \eta_t = \begin{bmatrix} \eta_t^u \\ 0 \\ \eta_t^{PR} \\ 0 \\ \eta_t^e \\ 0 \\ \eta_t^\pi \\ 0 \\ 0 \end{bmatrix}$$

For the policy rules, we follow Rudebush and Svensson (1999) and consider a general linear feedback instrument rule

$$i = f X_t \tag{7}$$

where f is a conformable row vector.

The problem of minimizing in each period the loss function in (4) subject to (6) is standard and results in an optimal linear feedback rule (OFR) which, under the limit assumption of $\delta = 1$, converges to a closed-form solution for the vector f (e.g. Rudebush and Svensson, 1999, *p.240*).

[3] We also checked how sensitive are results to alternative settings. In particular we were able to confirm the previous findings of the literature that the posterior distribution of the policy reaction to both unemployment and interest rate shifts monotonically with the values of these parameters in a reasonable range. These changes in the policy rules, however, do not seem to have a significant effect on the shape or the magnitude of the impulse response functions.

This rule is less restrictive than a classical Taylor rule, as the interest rate is a function of all current and lagged values of the non-policy variables and lagged values of the interest rate.

We also derive results under an optimized classical Taylor rule (TR) that allows the interest rate to react only to current values of unemployment gap and inflation, that is:

$$\begin{aligned} i_t &= f \cdot \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} \\ f &= \left[f_u\left(\Xi, \Psi\right) \quad f_\pi\left(\Xi, \Psi\right) \right] \end{aligned} \tag{8}$$

In this case the parameters of the rule depend on the VAR coefficients in an open form, and need to be recovered with an optimization routine.

In our empirical exercise we also allow for the presence of a lagged interest rate, capturing an interest rate smoothing (e.g. Clarida et al. 2000), or other relevant but omitted macroeconomic variables (e.g. Sack 2000).

3 From the models to the data

In this section, we apply our framework to US and euro area data, describe the estimation technique and characterize the model space discussing its properties.

3.1 Data and transformations

The data are quarterly values of inflation, interest rate, unemployment rate, exchange rate, labor force participation rate, and a commodity price index for the euro area and the US, covering 1970:1 to 2007:4. The first part of the sample (from 1970:1 to 1990:4) is used as a training sample to derive the prior hyperparameters. The sample 1991:1 to 2007:4 is used for estimation and inference. Main sources for the data are Datastream and the Area Wide Model (AWM) database (Fagan et al., 2001).

The inflation rate is calculated as the four-quarter percentage change of CPI. The US interest rate is the Federal Funds rate; the euro area interest rate is the short-run rate of the AWM database. The unemployment gap is calculated as the difference between the natural rate of unemployment (u_t^*) and its actual value ($\widetilde{u}_t$). To account for some model uncertainty about the natural rate, as said in section 2, we compute (u_t^*) using both a "statistical" and an "economic" approach. For the former, the national unemployment series were detrended using the Baxter and King (1999) band pass filter. We extract cycles of length comprised between 6 and 32 quarters along with a truncation of 12 lags. As the filter uses a centered moving average method, we pad the series at the start and at the end with observations derived from AR(4) backcasts and forecasts.

The other approach, which incorporates some economic content, is based on a system of equations which comprises a Phillips curve, an Okun law and a set of equations defining the stochastic law of motions of the unobservable variables included in the system, namely potential output and the natural rate. For the Phillips curve we use a simple relationship between CPI inflation and lagged inflation, the state of aggregate demand as summarized by the unemployment gap, and a supply-side shock as summarized by import prices. As inflation is assumed to depend only on nominal factors in the long run, the coefficients of lagged inflation are constrained to add up to one. The Okun law relates output gap to unemployment gap. The system is estimated with standard Kalman-filter techniques.[4]

Exchange rates and commodity price are used in standardized four-quarter growth rates. The exchange rate is defined as the price of foreign currency in terms of domestic currency, therefore an exchange rate increase is a depreciation. Finally, the participation rate enter all models in gap form, with the trend computed using the Baxter and King filter. All series are demeaned to omit the constant term and ease the computations.

3.2 Estimation algorithm

The reduced form of (3) is estimated using Bayesian techniques and informative priors. If β denotes the vector of all VAR coefficients and Σ denotes the variance-covariance matrix of the reduced form disturbances, then $\theta_{M_j} = (\beta, \Sigma \mid M_j)$. Given the data as summarized by the likelihood $p\left(D \mid \theta_{M_j}, M\right)$, and a prior distribution $p\left(\theta_{M_j} \mid M_j\right)$, the Bayesian algorithm implies obtaining the posterior $p\left(\theta_{M_j} \mid D, M_j\right)$. In turn, given the estimated dynamic behavior of the non-policy variables as summarized by the latter posterior distribution, we solve the minimization problem and recover the distribution of the parameters of the rule that minimize the loss function.[5] If we denote with ω_1 the vector of such parameters, its posterior distribution $p\left(\omega_1 \mid D, M_j\right)$ is

$$p\left(\omega_1 \mid D, M_j\right) = \int f \cdot p\left(\theta_{M_j} \mid D, M_j\right) d\theta_{M_j} \tag{9}$$

where f is given by the OFR or the TR.[6] Finally, given the posterior mean of ω_1, we compute the distribution of the unemployment response to a monetary policy shocks. The algorithm is applied to each model M_j, each country and each policy rule.

[4] For more technical details the reader can refer to Steiger et al. (1997 a and b), and Fabiani and Mestre (2004). The latter generously shared with us their RATS codes on the Kalman filter approach to estimating the natural rate. We have used their baseline specification (see cit., p.320 and appendix A) for both euro area and US data.

[5] Following Sack (2000), the reaction function estimated from the VAR is ignored when solving the central bank's minimization problem.

[6] Note that the policy rule is assumed to be deterministic. Therefore its posterior uncertainty fully derives from the uncertainty of the VAR coefficients.

The following independent prior assumption is specified for each model (now omitting M_j):

$$\begin{aligned} p(\theta) &= p(\beta)\, p(\Sigma) \\ p(\beta) &= N\left(\underline{\beta}, \underline{V_\beta}\right) \\ p\left(\Sigma^{-1}\right) &= W\left(S^{-1}, \nu\right) \end{aligned}$$

where $W\left(S^{-1}, \nu\right)$ denotes a Wishart distribution with scale matrix S^{-1} and degrees of freedom ν; and $N\left(\underline{\beta}, \underline{V_\beta}\right)$ denotes a Normal distribution with mean $\underline{\beta}$ and variance-covariance matrix $\underline{V_\beta}$.

The general form of $p(\beta)$ in all models is the one of a Minnesota-type, where the prior mean of coefficients for the first own lag is equal to one and the others are set equal to zero; individual components of β are independent of each other, i.e. $\underline{V_\beta}$ is a diagonal matrix; and the diagonal elements of $\underline{V_\beta}$ have the structure:

$$v_{ij,l} = \begin{cases} (\gamma_1/l)^2 & \text{if } i = j \\ (\gamma_1\gamma_2\sigma_i/l\sigma_j)^2 & \text{if } i \neq j, \end{cases} \tag{10}$$

where $v_{ij,l}$ is the prior variance of $\beta_{ij,l}$ (coefficient in equation i relative to variable j at lag l), γ_1 is the general tightness, γ_2 is the tightness on "other coefficients", and l is the lag.

For all models we assume $\gamma_1 = 0.1$ and $\gamma_2 = 1$, and estimate the variances σ_i and σ_j from AR(p) regressions on the training sample. In all models where we restrict the persistence of inflation or labor market variables, the own-lag coefficients of the prior mean $\underline{\beta}$ are set accordingly, and the corresponding tightness is set to $10^{-3}\gamma_1$. For the AR assumptions of both inflation and labor variables, the own-lag coefficients of the prior mean $\underline{\beta}$ are estimated on the training sample with univariate AR(p) regressions.

Regarding the prior for Σ, the prior scale matrix S is set equal to $10^{-1}I$, and the degrees of freedom ν equal $n+3$, thus ensuring an informative but relatively vague prior assumption for Σ.

All in all, the prior assumptions on the unrestricted coefficients are sufficiently general and not too tight in order to ensure that the posterior mean of the first own lag of variables like exchange rate and commodity price will not necessarily be as persistent as the prior assumption.

Given the independent structure of the prior, a closed form solution for the posterior distribution of the parameters of interest is not available. It is easy to show, however, that a Gibbs sampler can be employed because the full conditional distributions $p(\beta \mid \Sigma, D)$ and $p(\Sigma \mid \beta, D)$ are easily derived (see Appendix). The sampler is initialized using the ML estimate of Σ on the training sample. For each draw of $\theta = (\beta, \Sigma)$, then, the parameters of the rule are derived from the minimization problem. This algorithm provides the posterior distribution (9).[7] For the optimized Taylor Rules, we use a

[7] Note that the Σ in the Gibbs sampler includes terms from the reduced form interest rate equations which are then zeroed out in the optimal policy computation.

grid search procedure to solve for the values of f that minimize the criterion function (5). Because the computation with high-order models is cumbersome, we solve the optimization problem by using the posterior median of θ, instead of grid-searching for each of its draws.

In the case of the optimal feedback rule, instead, the computational burden is not so heavy, for the optimal values of f and of the loss function are straightforward to compute. However, in order to ensure that the parameters of the rule have meaningful signs, we restrict the prior to be

$$q(\theta) = p(\theta) \cdot \Im(\omega_1 \in \mathcal{F})$$

where $\Im(\omega_1 \in \mathcal{F})$ is the indicator function that equals 1 if $\omega_1 \in \mathcal{F}$ and 0 otherwise, and $\mathcal{F}$ is the relevant region. The corresponding posterior distribution is therefore $q(\theta \mid D) = p(\theta \mid D) \cdot \Im(\omega_1 \in \mathcal{F})$. Strictly speaking, an importance sampling algorithm should be used instead of the Gibbs sampling, and an importance function elicited. It is easy to show, however, that if the importance function is the unrestricted posterior distribution we can still use the Gibbs sampling, drawing from the unrestricted posterior and discarding draws which violate the restrictions.[8]

Finally, an equal prior probability $p(M_j) = 1/J$ is assigned to each model, therefore the posterior probability of the models is proportional to their marginal likelihood, i.e.

$$\begin{aligned} p(M_j \mid D) &= \frac{p(M_j)\, p(D \mid M_j)}{\sum_j p(M_j)\, p(D \mid M_j)} \\ &= \frac{p(D \mid M_j)}{\sum_j p(D \mid M_j)} \end{aligned} \tag{11}$$

where $p(D \mid M_j) = \int p\left(D \mid \theta_{M_j}, M\right) p\left(\theta_{M_j} \mid M\right) d\theta_{M_j}$ is the marginal likelihood of model M_j. An analytical evaluation of this integral is not possible given our prior assumptions. Therefore we simulate it from the Gibbs output using the harmonic mean of the likelihood values at each draw of θ (Newton and Raftery, 1994). Note that the marginal likelihood comparisons and averaging require the set of left-hand side variables to be the same across models. In the computation of the harmonic mean, therefore, all marginal likelihoods have been computed on the basis of equations for the same three endogenous variables, namely unemployment, inflation and interest rate.[9]

Results (discussed in the next subsections) are based on 10000 iterations of the Gibbs sampling, after discarding an initial 5000 burn-in replications and using the remaining 5000 for inference.

[8] In particular we assign a zero weight to negative values of the parameters attached to the negative unemployment gap, the inflation gap and the lagged interest rate. Note that a similar approach has been used by Cogley and Sargent (2005) and Benati (2008) in different contexts, to rule out explosive autoregressive roots in VARs with time-varying parameters.

[9] If the VAR is written as a linear regression model, $y = X\beta + \Sigma^{1/2}\varepsilon$, under the normality assumption a linear transformation of y, Ry, is also normal. In all models, therefore, the matrix R selects always the same endogenous variables when computing the likelihood values.

3.3 Properties of model space and rules

The properties of the model space can be briefly described by focusing on the Marginal Likelihood, the parameters of the rules, and the expected losses.

In Figure 1 we plot the Relative Marginal Likelihood (RML) of the models, defined as in (11), where j goes from 1 to 224. Given an equal prior model probability, $p(M_j)$, the RML measures how likely the data believes a given model is the most appropriate one. Models are ordered according to scheme described in appendix A (Table A1), in ascending number of lags.

Figure 1 about here

The RML turn out to be substantially different across models, as shown by the difference between the highest and the lowest values, and by the fact that, especially for the euro area, only for few models the RML is greater than the equal weight (EW).

The data support relatively parsimonious models, and the best models are clustered around specifications with three and four variables, particularly the specifications which include 3 lags for the US and 4 lags for the euro area. More interestingly, there is clear evidence that a specification which includes (either jointly or alternatively) the participation rate and the exchange rate is highly supported by both the US and the euro area data, meaning that the inclusion of these variables in an otherwise standard VAR model may be important to obtain an appropriate inference on the effects of policy on unemployment. Data also support models with moderate persistence in the labor market variables, and with an economic-based and a statistical-based detrending of the unemployment rate for Euro area and US, respectively.

The posterior distributions of the optimal policy parameters and the associated expected losses across models are summarized in Figure 2 and 3. Figure 2 reports the posterior distributions of the relevant parameters and of the losses for the OFR and each model. The solid black line that goes through the areas is the posterior median of each model. The shaded areas comprise the 95 percent of the posterior distribution around it, as in a fan chart representation: there are an equal number of bands on either side of the central band. The latter covers the interquartile range and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on until the 95% of the distribution is covered. Models on the x-axes are organized according to two layers of complexity: they are first sorted in ascending lag length order and then by number of variables.

Figure 2 about here

In Figure 3 we summarize instead the distribution of the optimal policy parameters and expected losses by only taking the posterior median across models. In this way, we can visually compare results also across the two rules.[10] The box plots report the extreme values and the interquartile ranges computed using the posterior medians across the 224 models in a given class (OFR or TR) of the relevant policy parameters and the expected losses. For the TR, where $i_t = f_u u_t + f_\pi \pi_t + f_i i_{t-1}$, the coefficient on interest rate is simply f_i, whereas the unemployment and inflation long-run reaction coefficients are computed as $f_u / (1 - f_i)$ and $f_\pi / (1 - f_i)$, respectively. For the OFR, where the policy rate depends also on the lags of the variables, i.e., $i_t = \sum_{j=0}^{p-1} f_u^j u_{t-j} + \sum_{j=0}^{p-1} f_\pi^j \pi_{t-j} + \sum_{j=1}^{p-1} f_i^j i_{t-j} + f'_{\mathcal{Z}} \mathcal{Z}$, and $\mathcal{Z}$ contains all other non-policy variables, the respective coefficients are$\sum_{j=1}^{p-1} f_i^j$, $\sum_{j=0}^{p-1} f_u^j / \left(1 - \sum_{j=1}^{p-1} f_i^j\right)$, and $\sum_{j=0}^{p-1} f_\pi^j / \left(1 - \sum_{j=1}^{p-1} f_i^j\right)$, where p is the order of autoregression of the estimated model**.** The dark squares in the box plot are the weighted averages of the results, where the weights are given by the RML. The empty circles represent instead results associated with the best models (i.e. Model 196 for the euro area and Model 117 for US as described in Table A.1).

Figure 3 about here

Some considerations emerge from the charts. The first immediate feature is the high degree of uncertainty, as measured by the dispersion of the results both within and between models. The average ranges of results are, however, consistent with previous literature, as the bulks of the distributions are concentrated on values in line both with the theory and with previous empirical findings, for both classes of rules. The dispersion across models seems to be only marginally larger for the TR than for the OFR in both countries, and results seem more volatile across models for the euro area than for the US.

A closer look shows that the interquartile range of the optimal long-run reaction of unemployment is $[1.7 - 3.5]$ for the US and $[0.7 - 2.8]$ for the euro area; the long run reaction of inflation is in the range $[1.1 - 2.5]$ for the US and $[1.3 - 2.6]$ for the euro area; and the lagged interest rate coefficient is in the range $[0.1 - 0.65]$ for both countries, with the variance of the distribution of coefficients derived from the OFR significantly smaller than the one obtained from the TR. The weighted averages and the results associated with the best models are very much similar to the median values. These findings indicate that in both countries the policies have on average been marginally more aggressive than the original Taylor rule, and that interest rate smoothing is a robust feature of the policy. Very similar results have been found by, for instance, Brock et al. (2007),

[10] Recall that due to the complexity of the grid search in the TR, we simulate the posterior distribution of parameters and losses only for the OFR, whereas for the TR we compute the estimates of f using the posterior mean of $\theta = (\beta, \Sigma)$.

Levin and Williams (2003), and Clarida et al (2000), for the US; and by Smets and Wouters (2005), and Gerlach and Schnabel (2000), among others, for the euro area.

Comparing the two economies, the euro area policy rate reacts on average relatively more to inflation than to unemployment, whereas the opposite seems to be true for the US policy rate (on this see also Sahuc and Smets 2008). Another interesting finding is the negative relationship between the optimal policy parameters and the model complexity as the median values in Figure 2 are clearly decreasing by lags and coefficients spike up with the first prior and short lag length. This pattern is more evident for the euro area than for the US, and partly confirms previous results which relate model complexity and optimal parameters (see e.g. Brock et al. 2007).

Finally, posterior expected losses are also consistent with the existing literature using similar values for the weights in the loss function. If anything, our estimates seem to be on the lower side (see e.g. Brock et al., 2007; and Rudebush and Svensson, 1999 for a comparison) and become notably similar to those obtained by previous studies only under the autoregressive prior for inflation. Interestingly, the posterior losses associated with the best models are overall lower than the average (except in the Taylor Rule for the US).

In sum, the evidence provided above confirms that simple linear autoregressive models may give rise to a significant degree of uncertainty in the distribution of optimal policy parameters and expected losses. Simple or weighted averages across models help dampen this uncertainty and provide a reasonable representation of the policy rules. Our results would also suggest the choice of a relatively parsimonious representation of the economy, regardless of the country and the policy rules.

4 Effects of policy on unemployment

The successful conduct of monetary policy requires policymakers not only to specify a set of objectives for the performance of the economy but also to understand the effects of policies designed to attain these goals. In this section, therefore, we will answer the following questions: Given the set of objectives and rules, what are the effects of policy prescriptions that come from different models on the unemployment gap? What is the role of model uncertainty and what are the consequences for policymakers of allowing for it?

The estimation algorithm directly follows from the one described in Section 3. Using the structural VAR in Eq. (3), we assume that the central bank sets the policy variables i_t according to the two policy rules OFR and TR as estimated in the previous step. The estimated equation error ϵ_t^i can be interpreted as a monetary policy shock, as also discussed e.g. by Stock and Watson (2001), or Sack (2000). The shock is identified by (i) replacing the parameters of the policy equation with the posterior means of the f estimated above, while leaving unrestricted all the other parameters of the VAR; and (ii) imposing the timing assumption that the central bank reacts contemporaneously to all variables in the economy, whereas the policy rate does not contemporaneously affect the rest of the economy. The former restriction is placed in the form of a normal distribution with a very tight variance. The latter restriction is a pure zero-restriction. A relatively vague Minnesota prior is assumed on the rest of parameters in the two blocks. Results are reported in terms of the probability distributions of the responses to the identified monetary policy shock (Figure 4 and Tables 1-2); in terms of variance decomposition (Figure 5); and in terms of the transmission mechanism (Figure 6).

4.1 Impulse response dispersion

Figure 4 reports the responses of unemployment gap to a 100-basis-point contractionary monetary policy for both countries and rules. Since the unemployment gap has been computed as the difference between the natural rate of unemployment (u_t^*) and its actual value (u_t), a slowdown correspond to a negative response.

To jointly visualize the "average" effect and the dispersion *within* and *between* models we report the posterior distribution of the IRF obtained from the Markov Chain Monte Carlo (MCMC) simulation by 'fan-charting' separately three quantiles of such distributions – the median responses, the $16th$ percentile and the $84th$ percentile – for all models. Therefore, in the charts with the title 'median', for instance, we plot the distribution across models of the median responses. In each chart, the shaded areas represent the dispersion across models. The principle has already been described for Figure 2: there is an equal number of bands on either side of the central band. The latter covers the interquartile range across models and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on up to the 95%. The solid black line that goes through the areas is the weighted average of each quantile (median, $16th$ and $84th$ percentile) across models, where the weights are given by the RML of each model.

A detailed quantification of the responses is also reported in Table 1, which displays the impacts computed from the median of Figure 4 and reports the $10th$ and the $90th$ percentile, the median and the weighted average across the 224 models.

Figure 4 and Table 1 about here

Four preliminary comments are in order.

First, impulse responses look reasonably well behaved and give rise to the usual hump-shaped dynamics. Their pattern is fairly robust across models, countries and rules. One dimension of such robustness is that, although model responses are very much dispersed – and therefore any statement on statistical significance would require some caution, especially for the euro area – the 68% posterior probability intervals do not include the 0 at the horizons of the peak effects, and this, on average, appears to be a stable feature.

Second, regarding the dynamics, most of the significant economic slowdown occurs in the first two years after the rate hike, when the *cumulative* impact on the unemployment gap is between -0.2 and -0.3 percentage points, on average across models, rules and countries. Measured on the weighted average response across models (the dark line in the charts), the (negative) unemployment gap reaches a maximum decline of around 5 basis points 5-6 quarters after the contractionary monetary policy shock for the US, and of 4 basis points 4-5 quarters after the rise in interest rate for the euro area. Half of the maximum effect on the gap disappears after about 9 to 11 quarters for both economies.

It is important to note that, the timing of the peak effect obtained by the previous literature is very consistent with our results (see e.g. Christiano et al, 1996; Stock and Watson, 2001; Bernanke et al. 2005). The size of our effects appears to be more subdued than in other studies, most likely because our responses are measured on the unemployment gap and not on the unemployment rate. Intuitively, as the natural rate of unemployment is not constant in the measurement of the gap, a contractionary monetary policy shock might lead to an increase in the natural rate itself, after an initial increase in the actual unemployment rate, and this in turn would explain the muffled effect on the gap.[11]

Third, impulse responses are only marginally sensitive to the policy rule used in the identification of the structural VAR. Visual inspection, however, seems to show that results based on the TR are to some extent less dispersed than those based on OFR, and also that with a TR the average peak effects might be delayed of one or two quarters with respect to the OFR, in both economies. These results do not come entirely as a surprise for, even if both rules are backward-looking, the OFR is less restrictive than the TR, being a function of all current and lagged values of the non-policy variables beside the lagged values of the interest rate.

Fourth, there is a substantial degree of uncertainty across models, for a given rule or country. The dispersion is significant for both economies and regardless of the policy rules in particular

[11] A more extensive analysis of this point goes clearly beyond the scope of this paper. We have, however, run a subset of models with the (demeaned) actual unemployment rate instead of the unemployment gap. *Coeteris paribus*, the average responses were doubled, thus confirming our intuition that the transformation used is in part responsible for the result. In a companion paper (Altavilla and Ciccarelli, 2007), where we use the actual rate instead of the gap, our impulse responses are the same as, e.g., those obtained by Stock and Watson (2001).

around the peak values of the responses, between one and two years. Nonetheless, overall results for the US are much less dispersed than those for the euro area where some models can even show puzzling positive effects of monetary policy on the (negative) unemployment gap at the crucial horizons. Moreover, for the euro area the weighted average provides more muted responses at the peak than a simple average, meaning that models which receive more support by the data – and therefore are weighted more in the average – tend to dampen the response of unemployment to a monetary policy shock relatively to the other models.

Two conclusions can be drawn from the comparison between US and euro area results. First, the elevated dispersion across models implies that policy decisions based on few selected models – as opposed to a combination from several of them – may potentially give a twisted picture of the policy effects, and this, in turn, might lead to policy mistakes. Second, while the degree of uncertainty can differ considerably across countries, as Figure 4 shows, the average impacts hardly exhibit meaningful differences across the two economies, both in terms of timing and in terms of magnitude. We interpret this evidence as a warning for other comparative studies which may find significant differences in the reaction of unemployment to a monetary policy shock across the two economies. Our results suggest that major discrepancies could mainly arise as an outcome of conditioning the analysis on few specific models, instead of accounting for model uncertainty. Consequent policy decisions taken on the basis of presumed differences between the two economies could therefore lead to distorting effects.

4.2 Variance decomposition

So far the discussion seems to indicate that, albeit a mute one, monetary policy shocks play a similar recessionary role for unemployment fluctuations in both economies. In order to examine from a different perspective the relative importance of the identified shock for the volatility of the unemployment gap, we have also inspected the forecast error variance decomposition. Results – reported in Figure 5 using the same fan-chart approach – show that at short and long horizons only a small fraction of the forecast error variance of the unemployment gap is accounted for by the monetary policy shock which, beyond the one year horizon, is never contributing with more than 10 percent on average across countries and rules.

Interestingly, the dispersion across models of the percentage of variance explained by the identified shock is very tiny when compared with the dispersion of the portion of variance explained by other non-policy variables. This, in turn, leads to two additional considerations. On the one hand, it seems that there are important sources of variability in unemployment that are not identified by the monetary policy shock and are reflected in the portion of variance explained by the other variables of the model space. On the other hand, it suggests that such a muted contribution of the monetary policy shock would anyway be a robust feature, should we not account for model uncertainty.

Figure 5 about here

Clearly, any speculation about the role that other structural shocks might have played goes beyond the scope of this paper. It is nonetheless interesting to remark that the selected non-policy variables can help explain up to 40 percent of the movements in unemployment gap beyond the three-year horizon. Incidentally, this high percentage amply justifies our prior variable selection to construct the model space.

A significant role is played in particular by the labor force participation rate whose variability helps explain 20 to 25 percent of the variability of unemployment gap across countries, models and rules after a two-year horizon.[12] This is a remarkable result and reinforces the finding - previously discussed in Section 3 - that including labor force participation often increases the relative posterior probabilities (and lowers the value of the loss function), meaning that the data at hand support the importance of this variable to understand the effects of policy on unemployment.

One might want to ask, therefore, whether the impact of a monetary policy shock measured with our model space may change (and by how much) depending on the presence of the participation rate in the specification. An attempt to describe and quantify a plausible answer could be based on the same kind of inference discussed so far, only dividing the set of models in two groups, according to the presence of the labor force participation rate among the non-policy variables.

In table 2 we report the evidence on unemployment gap. Given the model space described in table A.1 we have 112 models including participation rate (denoted as "*with*" in table 2) and 112 models which do not include participation rate (denoted as "*without*" in table 2). The quantiles and the weighted averages have been computed from the median responses of all models as in Table 1. Although the differences might not be impressive, they point out that on average the monetary policy effect is slightly more muted in models that contain the participation rate. This is easily rationalized and it is in line with the evidence reported in table 1 showing that the simple average across models provides a deeper impact than the weighted average, as the models with the highest RML always contain the participation rate. Intuitively, in models with participation a contractionary monetary policy shock eventually has a negative impact on the participation rate (see below) and this, in turn, reduces the initial impact on unemployment.

[12] Note that the average variances explained by each variable as shown in Figure 6 cannot sum up to one as not all variables appear always in the same models. Therefore the variance attributed to the single variables refers to the fraction of the variance explained by these variables only in models whose specification contains them.

4.3 Transmission mechanism

The difference in impacts displayed in table 2 is similar in the two economies and seems only marginally more pronounced for the Euro area than for the US. This turns out to be related to a different transmission mechanism and in particular to different dynamics that the participation rate shows in the two countries in response to a monetary policy shock. To better analyze this point, Figure 6 reports the distributions (over all models) of the median responses of all variables.

Figure 6 about here

Average responses (the darkest areas of the charts or the black lines inside them) present the expected signs and patterns, and, except for the somewhat uncertain response of the exchange rate for the US in the OFR, which might depend on the subsequent dynamics of the interest rate after the initial hike, they are also qualitatively similar in the two economies.

More interestingly, in both countries the response function for the inflation rate obtained by combining all models with the Bayesian averaging scheme (the black line in all graphs) exhibits a small price-puzzle. This evidence might suggest that the initial positive reaction of the inflation rate to a monetary policy shock is a model-dependent phenomenon that tends to disappear when taking into account model uncertainty.

One of the main differences in the transmission mechanisms of the two economies is clearly related to the different responses of the participation rate. In the US the response of labor force participation follows with some lag the inverted U-shape of the unemployment response, has broadly the same magnitude, and peaks 8 to 9 quarters after the rise in interest rate. The fall in participation rate is therefore consequent to the initial impact on unemployment.

In the euro area, on the contrary, the participation rate not only reacts earlier than unemployment, but it also displays an initial positive response which, depending on the policy rule, may last more than one year. Afterwards the response becomes negative, with a maximum decrease reached only around 9 to 14 quarters after the initial increase in the interest rate. This pattern may help explain the greater uncertainty around the unemployment responses in the euro area, and is consistent with a lower degree of flexibility of the European labor market with respect to the one exhibited in the US, where a contractionary shock directly influences the unemployment gap without being transmitted through the participation (see, e.g. Blanchard and Katz, 1992; and Blanchard, 2006).

The initial positive response of participation in the euro area –which is also responsible for the slightly more persistent dynamics of the unemployment response and for some positive responses of unemployment right after the shock– is not necessarily unreasonable and can be rationalized from a theoretical perspective. After a contractionary monetary policy shock, unemployed workers may stop actively looking for a job and exit the labor force. This effect –that the literature has typically denominated "discouraged worker effect"– gives rise to a net reduction of the labor force participation rate. On the other hand, the same contractionary monetary policy can force workers who are currently outside the labor force to start actively looking for a job, and this, in turn, may result in a positive effect on the participation rate. In fact, secondary workers (women and youths) might start seeking employment because of drop in primary workers' wages and employment. The literature has typically referred to this phenomenon as "added work effect". Theoretically, models of family utility maximization indicate that a decrease in family income due to the earnings losses of one family member might be offset by increases in the labor supply of others (e.g. Stephens, 2002).

In the comparison between the US and the euro area the relative importance of the two effects, in combination with different degrees of flexibility in the labor market, provides a reasonable explanation for the different transmission mechanisms.

5 Conclusive remarks

We have shown that model uncertainty plays a crucial role in determining the effects of monetary policy shocks on unemployment dynamics in the euro area and the US.

Our findings support the view that in order to overcome severe policy mistakes, decisions could be based on a wide range of possible scenarios about the structure of the economy. As a result, when allowing for model uncertainty, policy advice may look significantly different from the one that would be optimal based on few selected models.

With the help of a Bayesian model averaging procedure to account for the uncertainty inherent to the model selection process, we have specified a range of 224 BVAR models that differ in several dimensions according to assumptions regarding inflation, persistence of labor market variables, measurement of the natural rate of unemployment, number of variables and lag structure. Each model represents a constraint for the central bank which sets the interest rate minimizing a social loss function. Given the solution in terms of policy rule, we have quantified the impact of a monetary policy shock on unemployment and measured the degree of uncertainty as represented by the dispersion of both the policy rule parameters and the impulse response functions across models.

The comparative evidence from the US and the euro confirms that simple linear autoregressive models that differ in several dimensions may give rise to a significant degree of uncertainty in the distribution of optimal policy parameters, expected losses and impulse response functions.

We have shown that, although a monetary policy shock might be less important than other structural shocks to explain unemployment dynamics, it has a stable recessionary effect. Moreover, the average unemployment responses for the US and the euro area are qualitatively and quantitatively very similar, with results for the euro area being more dispersed than those for the US. The analysis of the transmission mechanism also indicates that other labor market variables such as participation rate play a significantly different role in the transmission mechanism of the two economies.

One of the main policy implications of our results is that combining results from alternative representations of the structure of the economy represents a useful strategy to account for model uncertainty when assessing the risks for price stability or when deciding a given policy. In particular, our results show that a policymaker who selects the results on the basis of a single model may come to misleading conclusions not only about the transmission mechanism –picking up models where, for instance, the price puzzle is more marked or the effect on unemployment has a wrong sign– but also about the differences between the euro area and the US, which on average are tiny. By allowing for model uncertainty, instead, results are on average closer to what we expect from a theoretical point of view, and put the policymaker in a favorable position to calibrate the policy interventions in a more appropriate way, that is, more consistently with the economic theory and less distorting for the economy.

Some extensions that enrich the previous analysis are feasible in the same framework. Another dimension of uncertainty could be explored perhaps in a unified framework that considers model and data uncertainty. First-released data are often noisy, as incomplete or mismeasured initial information has been used in their construction and it may take several years of revisions before data are considered as final. All relevant information for monetary policy is, therefore, measured with error and the difference between the responses obtained with real-time vs final data might be sizable.

The model space can also be enlarged by considering several alternative economic models in the estimation of the natural rate of unemployment based on the Phillips curve. We have taken a shortcut and considered, instead, only one possible specification ignoring further sources of uncertainty.

Finally, the set of models could be further expanded by including additional labor market variables such as wages, which would provide a more complete dynamics of the labor market and a richer transmission mechanism. Wages would in fact reflect the conditions on which the equilibrium in the labor market is established and might, at the same time, give some indications on the price formation process or the existence of nominal pressures on the path of prices.

Appendix

A Models

The table A1 describes the 224 models that span the model space. The first column reports the model number. In the second column the models specification is detailed with the number and the type of variables used; the third column reports the codification. Each model is characterized by 4 elements: the number of variables (V), the number of lags (L), the type of prior (P), and the type of detrending method used in the calculation of the natural rate of unemployment (U). The estimated VAR can have three (3V) to six (6V) endogenous variables, and one (1L) to four (4L) lags. Five types of priors are possible. With the first prior (1P), both the inflation persistence and the persistence of the labor market variables are unrestricted, in the sense that a very loose prior assumption is assumed. With the second (2P), third (3P) and fourth (4P) prior, inflation is assumed to be a Random Walk, an Autoregressive process and a White Noise respectively, while the persistence of the labor market variables is unrestricted. With the fifth prior (5P), there is no restriction on the inflation persistence and the labor market variables are assumed to follow an Autoregressive process. Two types of detrending methods are used to compute the natural rate of unemployment. The first one (1U) uses the Baxter and King band pass filter. The second one (2U) is a Phillips-curve-based method estimated with Kalman Filter techniques. As an example, in model 117 (coded as 3V_3L_5P_1U) there are three variables (unemployment, inflation rate and interest rate), three lags, the prior on inflation and unemployment is the fifth one, and the natural rate of unemployment has been computed with Baxter and King's method.

Table A1 here

B Derivation of the posterior

By stacking appropriately variables and coefficients in the VAR (3), we can re-write it as:

$$y_t = (I_n \otimes W_t)\,\beta + \varepsilon_t \tag{12}$$

where, y_t is the $(n \times 1)$ vector of endogenous variables $[Z_t' \; i_t']'$, $W_t = \left(y_{t-1}', ..., y_{t-p}'\right)'$ is $k \times 1$, β is the $nk \times 1$ vectorization of all coefficients, ε_t is the $(n \times 1)$ vector of reduced form innovations, and $k = np$ is the number of parameters in each equation.

Because by assumption it is $p(\varepsilon_t) = N(0, \Sigma)$, the likelihood is proportional to

$$L(D \mid \beta, \Sigma) \propto |\Sigma|^{-T/2} \exp\left\{-\frac{1}{2}\sum_t \left[y_t - (I_n \otimes W_t)\,\beta\right]' \Sigma^{-1} \left[y_t - (I_n \otimes W_t)\,\beta\right]\right\} \tag{13}$$

where, as in the text, D represents the stacked data.

Given the joint prior distribution on the parameters, $p(\beta, \Sigma)$, the joint posterior distribution of the parameters conditional on the data is obtained through the Bayes rule

$$\begin{aligned} p(\beta, \Sigma \mid D) &= \frac{p(\beta, \Sigma) L(D \mid \beta, \Sigma)}{p(D)} \\ &\propto p(\beta, \Sigma) L(D \mid \beta, \Sigma), \end{aligned}$$

We have assumed an independent Normal-Wishart distribution for the prior, with

$$p(\beta) = N\left(\underline{\beta}, \underline{V_\beta}\right) \propto \left|\underline{V_\beta}\right|^{-1/2} \exp\left\{-\frac{1}{2}\left(\beta - \underline{\beta}\right)' \underline{V_\beta}^{-1}\left(\beta - \underline{\beta}\right)\right\} \tag{14}$$

and

$$p\left(\Sigma^{-1}\right) = W\left(S^{-1}, \nu\right) \propto |\Sigma|^{-(\nu - n - 1)/2} \exp\left\{-\frac{1}{2} tr\left(S\Sigma^{-1}\right)\right\} \tag{15}$$

As remarked above (Section 3), the chosen hyperparameters S and ν ensure a relatively vague prior assumption for Σ and therefore for most terms of the Cholesky decomposition. The joint posterior density for (β, Σ) is proportional to the product of (13), (14), and (15). Given the independency assumption, such posterior does not take the form of a standard distribution and cannot be directly used for inference. A Gibbs sampling algorithm is instead available, for the conditional posterior of both β and Σ are simple to derive. The conditional posterior of β is derived by multiplying (13) and (14), and ignoring the terms that in the product do not involve β. It is given by

$$\begin{aligned} p(\beta \mid D, \Sigma) &= N\left(\bar{\beta}, \bar{V}_\beta\right) \\ &\propto \exp\left\{-\frac{1}{2}\left(\beta - \bar{\beta}\right)' \bar{V}_\beta^{-1}\left(\beta - \bar{\beta}\right)\right\} \end{aligned} \tag{16}$$

where

$$\begin{aligned} \bar{V}_\beta &= \left(\sum_t (I_n \otimes W_t)' \Sigma^{-1} (I_n \otimes W_t) + \underline{V_\beta}^{-1}\right)^{-1} \\ \bar{\beta} &= \bar{V}_\beta \left(\sum_t (I_n \otimes W_t)' \Sigma^{-1} y_t + \underline{V_\beta}^{-1} \underline{\beta}\right) \end{aligned}$$

Similarly, the conditional posterior for Σ is derived by multiplying (13) and (15). Ignoring the terms that do not involve Σ, we have

$$\begin{aligned} p\left(\Sigma^{-1} \mid D, \beta\right) &= W\left(S^{*-1}, \nu^*\right) \\ &\propto |\Sigma|^{-(\nu^* - n - 1)/2} \exp\left\{-\frac{1}{2} tr\left(S^* \Sigma^{-1}\right)\right\} \end{aligned} \tag{17}$$

where

$$
\begin{aligned}
S^* &= S + \sum_t \left[y_t - (I_n \otimes W_t)\, \beta \right] \left[y_t - (I_n \otimes W_t)\, \beta \right]' \\
\nu^* &= \nu + T
\end{aligned}
$$

Starting from arbitrary values of Σ, a Gibbs algorithm samples alternately from (16) and (17). For each draw of the posterior the minimization problem is solved and the empirical distributions of the policy rules parameters and the losses are computed.

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Table 1: Properties of Impulse Response Functions. Effects on unemployment gap of a 100 basis-point contractionary monetary policy shock

Optimal Feedback rule

	Euro Area				US			
	10th	median	wgt. ave.	90th	10th	median	wgt. ave.	90th
1 Quarter	-0.035	-0.022	-0.014	0.003	-0.029	-0.013	-0.014	-0.006
2 Quarters	-0.063	-0.044	-0.027	-0.001	-0.047	-0.030	-0.031	-0.018
3 Quarters	-0.075	-0.053	-0.034	-0.004	-0.059	-0.042	-0.043	-0.028
4 Quarters	-0.076	-0.053	-0.035	-0.006	-0.065	-0.048	-0.048	-0.034
5 Quarters	-0.071	-0.045	-0.033	-0.007	-0.066	-0.048	-0.048	-0.035
6-8 Quarters	-0.053	-0.028	-0.024	-0.003	-0.058	-0.038	-0.040	-0.027
9-12 Quarters	-0.021	-0.005	-0.008	0.013	-0.033	-0.016	-0.019	-0.010
cumulative impact after 2 years	-0.479	-0.303	-0.197	-0.026	-0.439	-0.297	-0.270	-0.203
variance decomposition	2.313	4.322	3.885	5.248	3.182	4.949	5.248	7.216

Taylor Rule

	Euro Area				US			
	10th	median	wgt. ave.	90th	10th	median	wgt. ave.	90th
1 Quarter	-0.036	-0.016	-0.011	0.002	-0.032	-0.015	-0.018	-0.001
2 Quarters	-0.056	-0.030	-0.022	-0.005	-0.050	-0.027	-0.031	-0.013
3 Quarters	-0.067	-0.038	-0.029	-0.012	-0.062	-0.034	-0.038	-0.018
4 Quarters	-0.070	-0.044	-0.033	-0.019	-0.066	-0.038	-0.042	-0.021
5 Quarters	-0.070	-0.046	-0.035	-0.023	-0.063	-0.042	-0.043	-0.023
6-8 Quarters	-0.061	-0.042	-0.034	-0.023	-0.056	-0.039	-0.038	-0.021
9-12 Quarters	-0.041	-0.022	-0.024	-0.010	-0.035	-0.023	-0.022	-0.012
cumulative impact after 2 years	-0.482	-0.298	-0.201	-0.127	-0.442	-0.271	-0.251	-0.140
variance decomposition	2.304	3.285	4.605	5.084	3.344	5.141	5.869	8.156

Note: The table reports the posterior impulse responses of unemployment gap to a 100 basis-point contractionary monetary policy shock. The top and the bottom panel refer to the responses under the Optimal Feedback Rule and the Taylor Rule, respectively. Column (2) to (4) refer to the euro area results. Column (6) to (9) refer to the US results. Rows from (1) to (5) report the quantiles of the simple responses. Rows (6) and (7) report a time average of the quantiles over the second half of the second year and over the third year respectively. Row (8) reports the cumulative impact after 8 quarters. Row (9) reports the percentage of the variance of the unemployment gap 24-quarter-ahead forecast errors explained by the monetary policy shock. The reported quantiles (10th, median, average and 90th) are computed over the distribution of the posterior median responses across the 224 models. The column "average" reports a weighted average over all models with weights given by the relative marginal likelihood computed as in Eq. 11 of the paper.

Table 2: Change in inference due to labor force participation. Effects on the responses of unemployment gap to a 100 basis-point contractionary monetary policy shock

Optimal Feedback rule

	Euro Area						US					
steps	10th		weighted average		90th		10th		weighted average		90th	
	with	without	with	without	with	without	with	without	with	without	with	without
1 Quarter	-0.035	-0.035	-0.020	-0.013	0.006	0.001	-0.028	-0.029	-0.015	-0.013	-0.007	-0.005
2 Quarters	-0.063	-0.064	-0.036	-0.025	0.004	-0.005	-0.047	-0.048	-0.033	-0.030	-0.020	-0.018
3 Quarters	-0.074	-0.080	-0.043	-0.032	0.000	-0.009	-0.058	-0.059	-0.044	-0.042	-0.029	-0.028
4 Quarters	-0.073	-0.082	-0.042	-0.034	-0.001	-0.012	-0.065	-0.064	-0.049	-0.047	-0.033	-0.034
5 Quarters	-0.066	-0.078	-0.037	-0.033	0.000	-0.013	-0.066	-0.066	-0.048	-0.048	-0.035	-0.035
6-8 Quarters	-0.043	-0.058	-0.022	-0.025	0.002	-0.007	-0.058	-0.057	-0.039	-0.041	-0.027	-0.028
9-12 Quarters	-0.014	-0.025	-0.001	-0.011	0.015	0.007	-0.031	-0.034	-0.017	-0.021	-0.010	-0.010
cumulative impact after 2 years	-0.440	-0.513	-0.244	-0.213	0.014	-0.060	-0.438	-0.437	-0.305	-0.303	-0.205	-0.204
cumulative impact after 6 years	-0.547	-0.730	-0.150	-0.226	0.338	0.278	-0.546	-0.602	-0.297	-0.339	-0.104	-0.108
variance decomposition	2.176	2.689	3.275	4.097	4.504	6.004	2.996	4.202	4.475	5.800	6.040	7.567

Taylor Rule

	Euro Area						US					
steps	10th		weighted average		90th		10th		weighted average		90th	
	with	without	with	without	with	without	with	without	with	without	with	without
1 Quarter	-0.039	-0.033	-0.010	-0.010	0.003	0.000	-0.030	-0.033	-0.018	-0.018	0.000	-0.003
2 Quarters	-0.059	-0.054	-0.021	-0.021	-0.004	-0.007	-0.050	-0.049	-0.030	-0.031	-0.010	-0.014
3 Quarters	-0.068	-0.065	-0.029	-0.028	-0.011	-0.015	-0.062	-0.061	-0.037	-0.039	-0.015	-0.021
4 Quarters	-0.069	-0.071	-0.034	-0.032	-0.018	-0.019	-0.066	-0.067	-0.041	-0.042	-0.018	-0.026
5 Quarters	-0.070	-0.071	-0.037	-0.034	-0.023	-0.023	-0.062	-0.063	-0.042	-0.044	-0.021	-0.027
6-8 Quarters	-0.062	-0.060	-0.036	-0.033	-0.024	-0.023	-0.055	-0.058	-0.037	-0.039	-0.020	-0.024
9-12 Quarters	-0.044	-0.039	-0.025	-0.023	-0.011	-0.010	-0.033	-0.036	-0.021	-0.023	-0.011	-0.013
cumulative impact after 2 years	-0.490	-0.476	-0.239	-0.225	-0.125	-0.133	-0.434	-0.447	-0.278	-0.292	-0.123	-0.161
cumulative impact after 6 years	-0.880	-0.790	-0.358	-0.309	0.078	0.135	-0.615	-0.649	-0.306	-0.332	0.000	-0.042
variance decomposition	2.266	2.323	4.080	4.176	5.091	4.990	3.061	3.950	6.145	7.335	7.108	8.361

Note: The table reports the impulse responses of unemployment gap to a 100 basis-point contractionary monetary policy shock. The top and the bottom panel refer to the responses under the Optimal Feedback Rule and the Taylor Rule, respectively. Column (2) to (7) refer to the euro area results. Column (8) to (13) refer to the US results. Rows from (1) to (5) report the quantiles of the simple responses. Rows (6) and (7) report a time average of the quantiles over the second half of the second year and over the third year respectively. Row (8) and (9) report the cumulative impact after 8 and 24 quarters respectively. Row (10) reports the percentage of the variance of the unemployment gap 24-quarter-ahead forecast errors explained by the monetary policy shock. The reported quantiles (10th, weighted average and 90th) are computed over the distribution of the posterior median responses across the 224 models. The weighted average is taken over all models with weights given by the relative marginal likelihood computed as in Eq. 11 of the paper. Results for each quantile are reported for two classes of models, according to whether the model includes (column "with") or does not include (column "without") the labor force participation rate in the specification. Note that, given the model space described in table A.1, there are 112 models with participation rate and 112 models without.

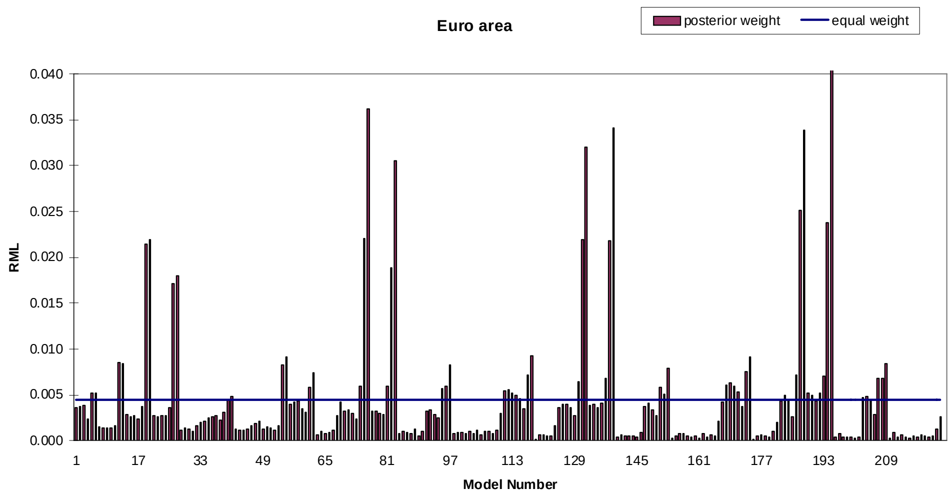
Table A.1: Mapping of the model numbers

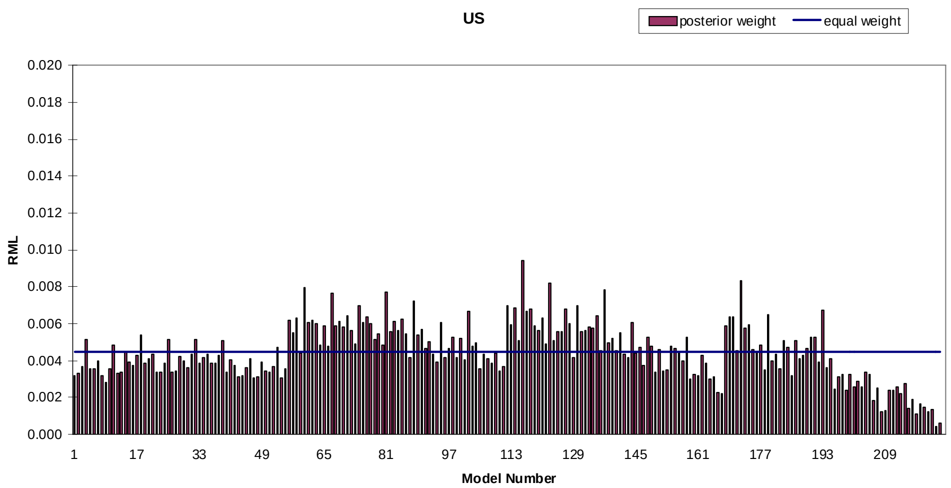
Model number	Variables	Code	Model number	Variables	Code
Model 1	***3 Variables***	3V_1L_1P_1U	Model 57	***3 Variables***	3V_2L_1P_1U
Model 2	$[u, \pi, i]$	3V_1L_2P_1U	Model 58	$[u, \pi, i]$	3V_2L_2P_1U
Model 3		3V_1L_3P_1U	Model 59		3V_2L_3P_1U
Model 4		3V_1L_4P_1U	Model 60		3V_2L_4P_1U
Model 5		3V_1L_5P_1U	Model 61		3V_2L_5P_1U
Model 6		3V_1L_1P_2U	Model 62		3V_2L_1P_2U
Model 7		3V_1L_5P_2U	Model 63		3V_2L_5P_2U
Model 8	***4 Variables***	4V_1L_1P_1U	Model 64	***4 Variables***	4V_2L_1P_1U
Model 9	$[u, pr, \pi, i]$	4V_1L_2P_1U	Model 65	$[u, pr, \pi, i]$	4V_2L_2P_1U
Model 10		4V_1L_3P_1U	Model 66		4V_2L_3P_1U
Model 11		4V_1L_4P_1U	Model 67		4V_2L_4P_1U
Model 12		4V_1L_5P_1U	Model 68		4V_2L_5P_1U
Model 13		4V_1L_1P_2U	Model 69		4V_2L_1P_2U
Model 14		4V_1L_5P_2U	Model 70		4V_2L_5P_2U
Model 15	***4 Variables***	4V_1L_1P_1U	Model 71	***4 Variables***	4V_2L_1P_1U
Model 16	$[u, cp, \pi, i]$	4V_1L_2P_1U	Model 72	$[u, cp, \pi, i]$	4V_2L_2P_1U
Model 17		4V_1L_3P_1U	Model 73		4V_2L_3P_1U
Model 18		4V_1L_4P_1U	Model 74		4V_2L_4P_1U
Model 19		4V_1L_5P_1U	Model 75		4V_2L_5P_1U
Model 20		4V_1L_1P_2U	Model 76		4V_2L_1P_2U
Model 21		4V_1L_5P_2U	Model 77		4V_2L_5P_2U
Model 22	***4 Variables***	4V_1L_1P_1U	Model 78	***4 Variables***	4V_2L_1P_1U
Model 23	$[u, e, \pi, i]$	4V_1L_2P_1U	Model 79	$[u, e, \pi, i]$	4V_2L_2P_1U
Model 24		4V_1L_3P_1U	Model 80		4V_2L_3P_1U
Model 25		4V_1L_4P_1U	Model 81		4V_2L_4P_1U
Model 26		4V_1L_5P_1U	Model 82		4V_2L_5P_1U
Model 27		4V_1L_1P_2U	Model 83		4V_2L_1P_2U
Model 28		4V_1L_5P_2U	Model 84		4V_2L_5P_2U
Model 29	***5 Variables***	5V_1L_1P_1U	Model 85	***5 Variables***	5V_2L_1P_1U
Model 30	$[u, pr, \pi, cp, i]$	5V_1L_2P_1U	Model 86	$[u, pr, \pi, cp, i]$	5V_2L_2P_1U
Model 31		5V_1L_3P_1U	Model 87		5V_2L_3P_1U
Model 32		5V_1L_4P_1U	Model 88		5V_2L_4P_1U
Model 33		5V_1L_5P_1U	Model 89		5V_2L_5P_1U
Model 34		5V_1L_1P_2U	Model 90		5V_2L_1P_2U
Model 35		5V_1L_5P_2U	Model 91		5V_2L_5P_2U
Model 36	***5 Variables***	5V_1L_1P_1U	Model 92	***5 Variables***	5V_2L_1P_1U
Model 37	$[u, pr, \pi, e, i]$	5V_1L_2P_1U	Model 93	$[u, pr, \pi, e, i]$	5V_2L_2P_1U
Model 38		5V_1L_3P_1U	Model 94		5V_2L_3P_1U
Model 39		5V_1L_4P_1U	Model 95		5V_2L_4P_1U
Model 40		5V_1L_5P_1U	Model 96		5V_2L_5P_1U
Model 41		5V_1L_1P_2U	Model 97		5V_2L_1P_2U
Model 42		5V_1L_5P_2U	Model 98		5V_2L_5P_2U
Model 43	***5 Variables***	5V_1L_1P_1U	Model 99	***5 Variables***	5V_2L_1P_1U
Model 44	$[u, cp, \pi, e, i]$	5V_1L_2P_1U	Model 100	$[u, cp, \pi, e, i]$	5V_2L_2P_1U
Model 45		5V_1L_3P_1U	Model 101		5V_2L_3P_1U
Model 46		5V_1L_4P_1U	Model 102		5V_2L_4P_1U
Model 47		5V_1L_5P_1U	Model 103		5V_2L_5P_1U
Model 48		5V_1L_1P_2U	Model 104		5V_2L_1P_2U
Model 49		5V_1L_5P_2U	Model 105		5V_2L_5P_2U
Model 50	***6 Variables***	6V_1L_1P_1U	Model 106	***6 Variables***	6V_2L_1P_1U
Model 51	$[u, pr, \pi, cp, e, i]$	6V_1L_2P_1U	Model 107	$[u, pr, \pi, cp, e, i]$	6V_2L_2P_1U
Model 52		6V_1L_3P_1U	Model 108		6V_2L_3P_1U
Model 53		6V_1L_4P_1U	Model 109		6V_2L_4P_1U
Model 54		6V_1L_5P_1U	Model 110		6V_2L_5P_1U
Model 55		6V_1L_1P_2U	Model 111		6V_2L_1P_2U
Model 56		6V_1L_5P_2U	Model 112		6V_2L_5P_2U

Model 113	***3 Variables***	3V_3L_1P_1U	Model 169	***3 Variables***	3V_4L_1P_1U
Model 114	$[u,\pi,i]$	3V_3L_2P_1U	Model 170	$[u,\pi,i]$	3V_4L_2P_1U
Model 115		3V_3L_3P_1U	Model 171		3V_4L_3P_1U
Model 116		3V_3L_4P_1U	Model 172		3V_4L_4P_1U
Model 117		3V_3L_5P_1U	Model 173		3V_4L_5P_1U
Model 118		3V_3L_1P_2U	Model 174		3V_4L_1P_2U
Model 119		3V_3L_5P_2U	Model 175		3V_4L_5P_2U
Model 120	***4 Variables***	4V_3L_1P_1U	Model 176	***4 Variables***	4V_4L_1P_1U
Model 121	$[u,pr,\pi,i]$	4V_3L_2P_1U	Model 177	$[u,pr,\pi,i]$	4V_4L_2P_1U
Model 122		4V_3L_3P_1U	Model 178		4V_4L_3P_1U
Model 123		4V_3L_4P_1U	Model 179		4V_4L_4P_1U
Model 124		4V_3L_5P_1U	Model 180		4V_4L_5P_1U
Model 125		4V_3L_1P_2U	Model 181		4V_4L_1P_2U
Model 126		4V_3L_5P_2U	Model 182		4V_4L_5P_2U
Model 127	***4 Variables***	4V_3L_1P_1U	Model 183	***4 Variables***	4V_4L_1P_1U
Model 128	$[u,cp,\pi,i]$	4V_3L_2P_1U	Model 184	$[u,cp,\pi,i]$	4V_4L_2P_1U
Model 129		4V_3L_3P_1U	Model 185		4V_4L_3P_1U
Model 130		4V_3L_4P_1U	Model 186		4V_4L_4P_1U
Model 131		4V_3L_5P_1U	Model 187		4V_4L_5P_1U
Model 132		4V_3L_1P_2U	Model 188		4V_4L_1P_2U
Model 133		4V_3L_5P_2U	Model 189		4V_4L_5P_2U
Model 134	***4 Variables***	4V_3L_1P_1U	Model 190	***4 Variables***	4V_4L_1P_1U
Model 135	$[u,e,\pi,i]$	4V_3L_2P_1U	Model 191	$[u,e,\pi,i]$	4V_4L_2P_1U
Model 136		4V_3L_3P_1U	Model 192		4V_4L_3P_1U
Model 137		4V_3L_4P_1U	Model 193		4V_4L_4P_1U
Model 138		4V_3L_5P_1U	Model 194		4V_4L_5P_1U
Model 139		4V_3L_1P_2U	Model 195		4V_4L_1P_2U
Model 140		4V_3L_5P_2U	Model 196		4V_4L_5P_2U
Model 141	***5 Variables***	5V_3L_1P_1U	Model 197	***5 Variables***	5V_4L_1P_1U
Model 142	$[u,pr,\pi,cp,i]$	5V_3L_2P_1U	Model 198	$[u,pr,\pi,cp,i]$	5V_4L_2P_1U
Model 143		5V_3L_3P_1U	Model 199		5V_4L_3P_1U
Model 144		5V_3L_4P_1U	Model 200		5V_4L_4P_1U
Model 145		5V_3L_5P_1U	Model 201		5V_4L_5P_1U
Model 146		5V_3L_1P_2U	Model 202		5V_4L_1P_2U
Model 147		5V_3L_5P_2U	Model 203		5V_4L_5P_2U
Model 148	***5 Variables***	5V_3L_1P_1U	Model 204	***5 Variables***	5V_4L_1P_1U
Model 149	$[u,pr,\pi,e,i]$	5V_3L_2P_1U	Model 205	$[u,pr,\pi,e,i]$	5V_4L_2P_1U
Model 150		5V_3L_3P_1U	Model 206		5V_4L_3P_1U
Model 151		5V_3L_4P_1U	Model 207		5V_4L_4P_1U
Model 152		5V_3L_5P_1U	Model 208		5V_4L_5P_1U
Model 153		5V_3L_1P_2U	Model 209		5V_4L_1P_2U
Model 154		5V_3L_5P_2U	Model 210		5V_4L_5P_2U
Model 155	***5 Variables***	5V_3L_1P_1U	Model 211	***5 Variables***	5V_4L_1P_1U
Model 156	$[u,cp,\pi,e,i]$	5V_3L_2P_1U	Model 212	$[u,cp,\pi,e,i]$	5V_4L_2P_1U
Model 157		5V_3L_3P_1U	Model 213		5V_4L_3P_1U
Model 158		5V_3L_4P_1U	Model 214		5V_4L_4P_1U
Model 159		5V_3L_5P_1U	Model 215		5V_4L_5P_1U
Model 160		5V_3L_1P_2U	Model 216		5V_4L_1P_2U
Model 161		5V_3L_5P_2U	Model 217		5V_4L_5P_2U
Model 162	***6 Variables***	6V_3L_1P_1U	Model 218	***6 Variables***	6V_4L_1P_1U
Model 163	$[u,pr,\pi,cp,e,i]$	6V_3L_2P_1U	Model 219	$[u,pr,\pi,cp,e,i]$	6V_4L_2P_1U
Model 164		6V_3L_3P_1U	Model 220		6V_4L_3P_1U
Model 165		6V_3L_4P_1U	Model 221		6V_4L_4P_1U
Model 166		6V_3L_5P_1U	Model 222		6V_4L_5P_1U
Model 167		6V_3L_1P_2U	Model 223		6V_4L_1P_2U
Model 168		6V_3L_5P_2U	Model 224		6V_4L_5P_2U

Note: The table reports the composition of the model space, with the total number of models (column 1), the variables included (column 2) and the codification (column 3). Each model is characterized by 4 elements: number of variables (V), number of lags (L), type of prior (P), and type of detrending method used in the calculation of the natural rate of unemployment (U). The VAR can have from three (3V) to six (6V) endogenous variables, and from one (1L) to four (4L) lags. Five types of priors are possible. With the first prior (1P), both the inflation persistence and the persistence of the labor market variables are unrestricted. With the second (2P), third (3P) and fourth (4P) prior, inflation is assumed to be a Random Walk, an Autoregressive process and a White Noise, respectively, while the persistence of the labor market variables is unrestricted. With the fifth prior (5P), there is no restriction on the inflation persistence and the labor market variables are assumed to follow an Autoregressive process. Two types of detrending methods are used to compute the natural rate of unemployment. The first one (1U) uses the Baxter and King band pass filter. The second one (2U) is a Phillips-curve-based method estimated with Kalman Filter techniques. Therefore, in model 196 (coded as 4V_4L_5P_2U) there are four variables (unemployment, inflation rate and interest rate and exchange rate), four lags, the prior on inflation and unemployment is the fifth one, and the natural rate of unemployment has been computed with a Phillips-curve-based method estimated with Kalman Filter techniques.

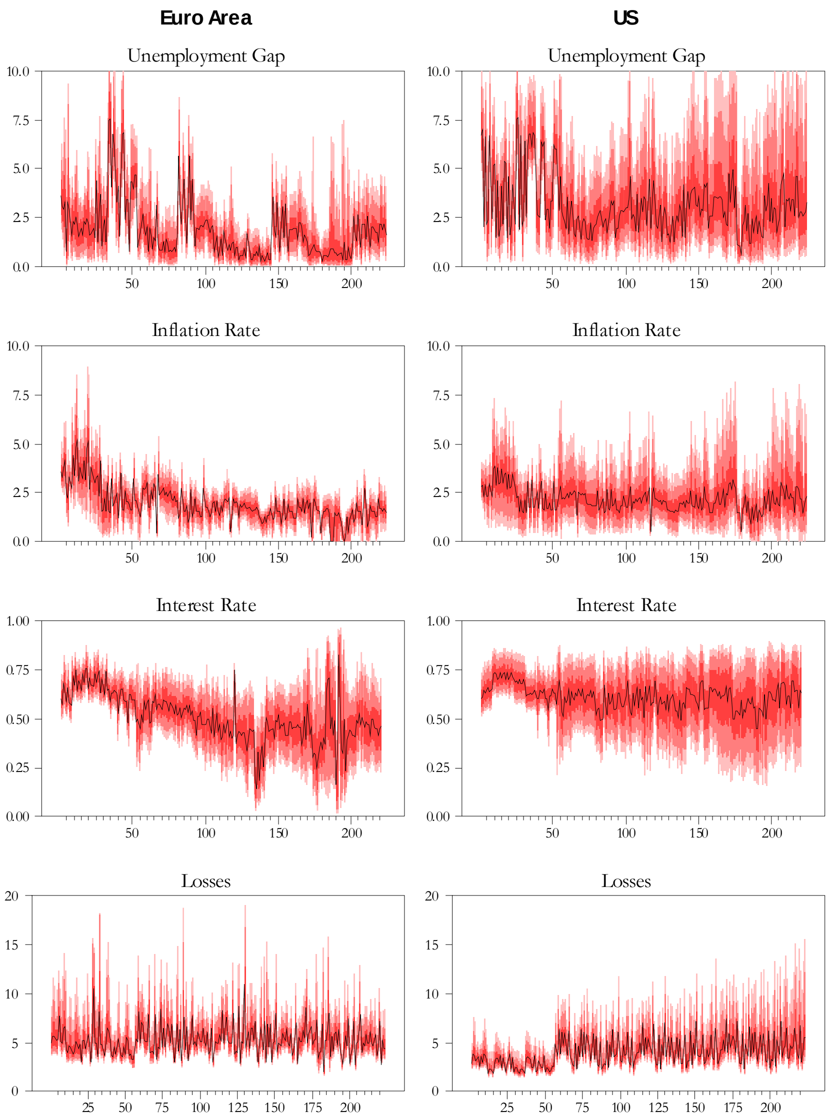
Figure 1: Relative Marginal Likelihoods





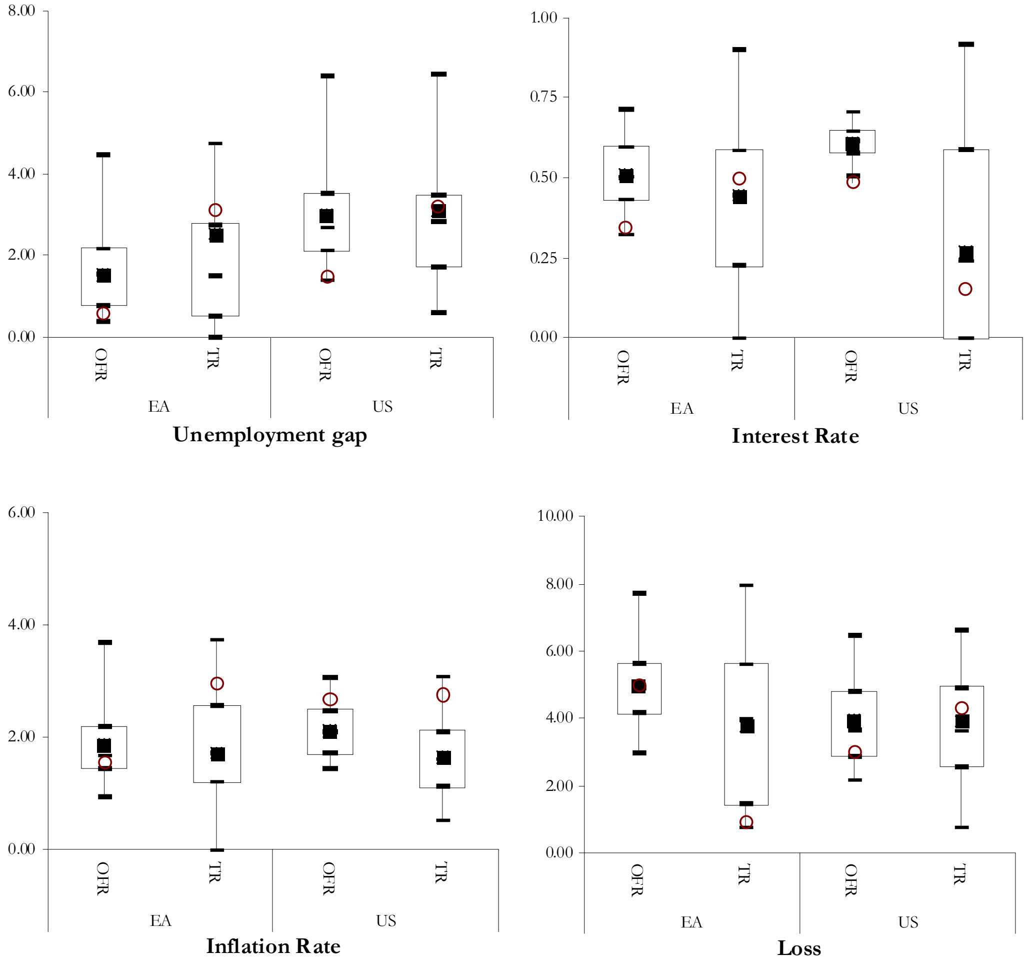
Note: The charts report the Relative Marginal Likelihood (RML) of the 224 models (bars) and the fixed equal weight (horizontal line). The RML is defined as the ratio of the Marginal Likelihood (ML) of a given model over the sum of all MLs (Eq. 11 in the paper). The ML is numerically computed from the Gibbs output using the harmonic mean of the likelihood values at each draw of the posterior distribution of the parameter vector. In the computation of the harmonic mean all marginal likelihoods have been computed on the basis of equations for the same three endogenous variables, namely unemployment, inflation and interest rate. The models on the x-axis are ordered according to the scheme described in Table A1.

Figure 2: Posterior distributions of policy parameters and expected losses



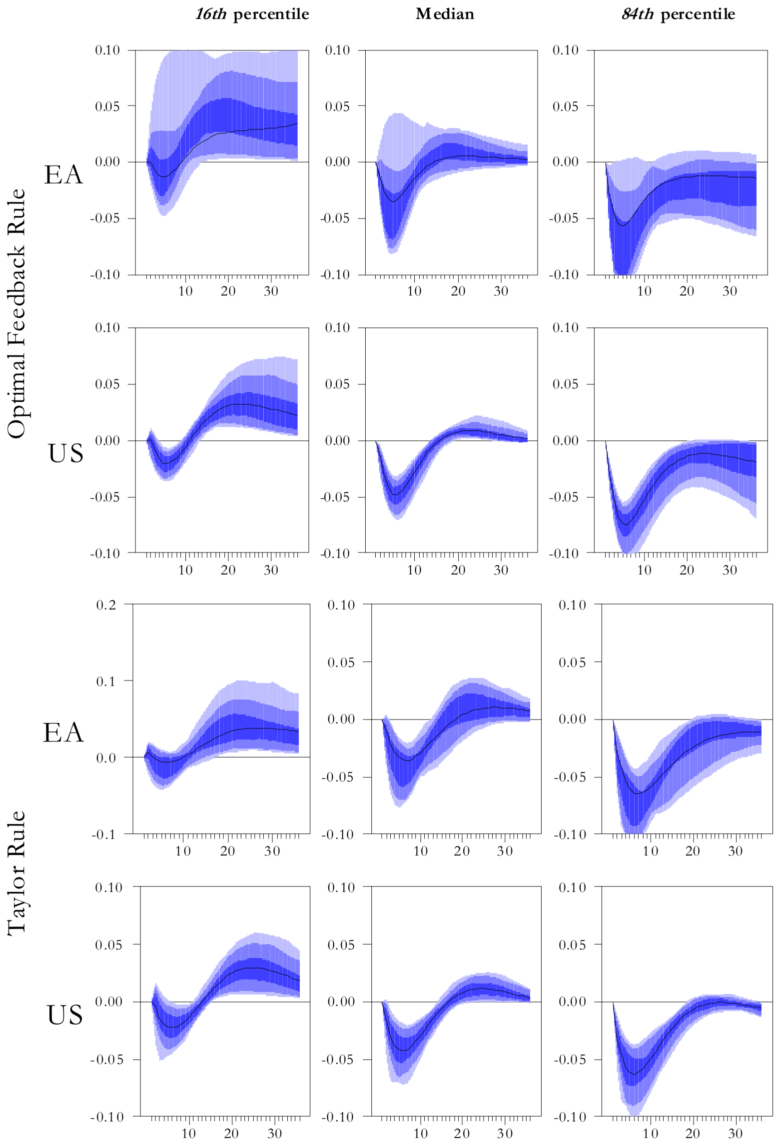
Note: The charts report the posterior distributions of the long run reaction coefficient of unemployment gap and inflation rate, the smoothing parameter of the interest rate and the loss values, for the OFR and all models. Column (1) refers to the euro area results. Column (2) refers to the US results. The solid black line that goes through the areas is the posterior median of each model. The shaded areas comprise the 95 percent of the posterior distribution around it, as in a fan chart representation: there is an equal number of bands on either side of the central band. The latter covers the interquartile range and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on, until the 95% of the distribution is covered. Models on the x-axes are organized according to two layers of complexity: they are first sorted in ascending lag length order and then by the number of variables. Therefore, the models with one lag come first, then the models with two lags, and so on. Among the specifications with the same number of lags, the models with three variables come first, followed by the models with four variables, and so on. Thereafter, the ordering is the same as in table A1, i.e., first we have the specifications with priors from 1 to 5 and the first detrending method, and then the specifications with priors 1 and 5 and the second detrending method.

Figure 3: Distributions across models of the median policy parameters and expected losses



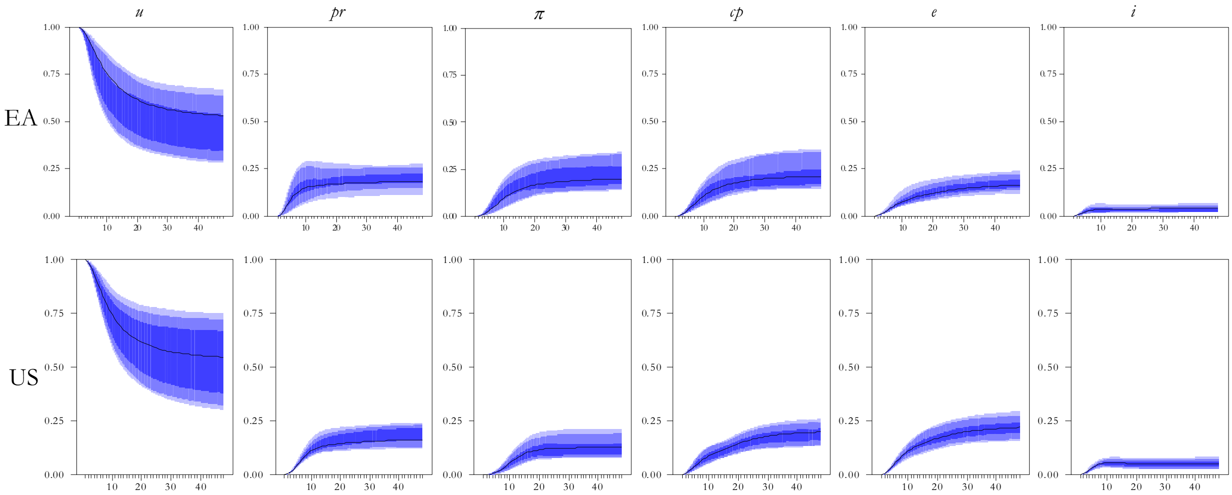
Note: The box plots report the extreme values, the median and the interquartile ranges of the relevant (long-run) policy parameters and the expected losses computed over the posterior medians of the 224 models for the Optimal Feedback Rule (OFR) and the Taylor Rule (TR). Each chart is divided in two parts: on the left hand side the euro area box plots are reported, and on the right hand side the US box plots are reported. The interest rate coefficient is simply the smoothing parameter in the TR, and the sum over $p-1$ lags of the autoregressive coefficients in the OFR. The long-run response coefficients for unemployment gap and inflation rate are computed as $f_u/(1-f_i)$, $f_\pi/(1-f_i)$, for the TR and as $\sum_{j=0}^{p-1} f_u/(1-\sum_{j=1}^{p-1} f_i)$ and $\sum_{j=0}^{p-1} f_\pi/(1-\sum_{j=1}^{p-1} f_i)$ for the OFR, respectively, where p represents the order of autoregression of the estimated model. The dark squares in the box plot are the weighted averages of the results, where the weights are given by the RML. The empty circles represent the results associated with the best models (i.e. Model 196 and Model 117 of Table A.1, for the Euro Area and for the US, respectively).

Figure 4: Posterior distributions of impulse response functions - Responses of unemployment gap to a 100 basis-point contractionary monetary policy shock



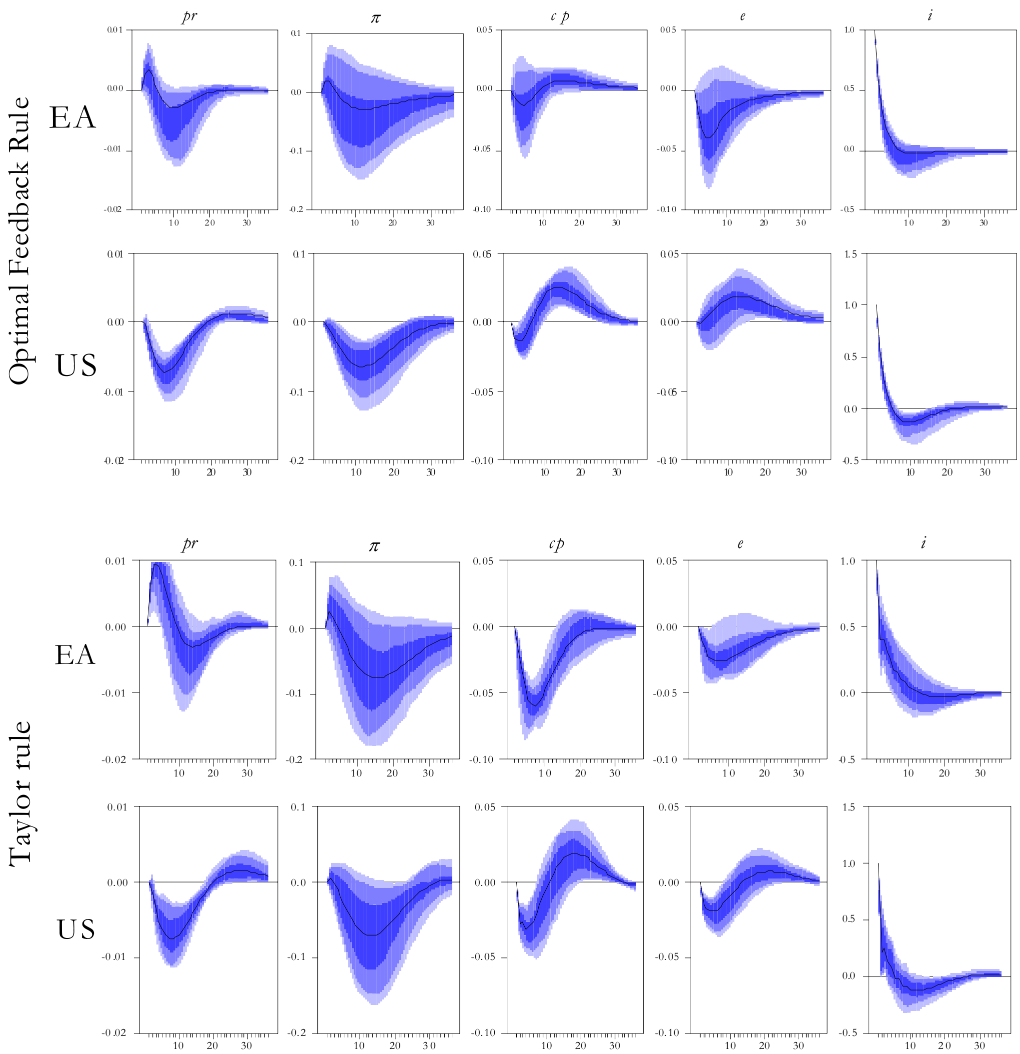
Note: The charts report three quantiles – the median, the 16th percentile and the 84th percentile – of the posterior Impulse Response Functions of unemployment gap to a 100 basis-point contractionary monetary policy obtained from the Gibbs sampler. For each quantile the distribution across the models has been 'fan-charted'. Results are reported for the Optimal Feedback Rule and the Taylor rule, and for the euro area and the US. Hence, in the charts with the title 'median' we plot the distribution across models of the median responses. In each chart, the shaded areas represent the dispersion across models. There is an equal number of bands on either side of the central band. The latter covers the interquartile range across models and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on up to the 95%. The solid black line that goes through the areas is the weighted average across models, where the weights are given by the relative marginal likelihoods of each model computed as in Eq. 11 of the paper.

Figure 5: Forecast errorr variance decomposition. Percentage of the variance of unemployment gap explained by all variables



Note: The charts report the posterior medians of the percentage of the unemployment gap forecast errors variance explained by the monetary policy shock (column "i") and by all other endogenous variables of the VAR. More precisely, u is the unemployment gap; pr is the participation rate; π is the inflation rate; cp is the commodity price inflation; e is the exchange rate; and, i stands for the nominal interest rate. The distributions – which are obtained under the Optimal Feedback Rule for both economies – are reported with the same "fan-chart" principle as in Figure 4. Hence, in each chart, the shaded areas represent the dispersion across models of the portion of variance explained by each variable. There is an equal number of bands on either side of the central band. The latter covers the interquartile range across models and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on, until the 95% is covered. The solid black line that goes through the areas is the weighted average across models, where the weights are given by the relative marginal likelihoods of each model computed as in Eq. 11 of the paper. The average variances explained by each variable cannot sum up to one as not all variables appear always in the same models. Therefore, the variance attributed to the single variables refers to the fraction of the variance explained by these variables only in models whose specification contains them.

Figure 6: The transmission mechanism. Distribution across models of the posterior median impulse responses of all variables to a 100 basis-point contractionary monetary policy shock



Note: The charts report the posterior medians of the Impulse Response Functions of all variables to a 100 basis-point contractionary monetary policy. The acronyms of the variables are the same as in Figure 5, that is: pr is the participation rate; π is the inflation rate; cp is the commodity price inflation; e is the exchange rate; i stands for the nominal interest rate. The distributions across models are reported for the Optimal Feedback Rule and the Taylor rule, and for the euro area and the US. The 'fan-chart' principle is the same as in Figures 4 and 5. Therefore, in each chart, the shaded areas represent the dispersion across models of the median responses. There is an equal number of bands on either side of the central band. The latter covers the interquartile range across models and is shaded with the deepest intensity. The next deepest shade, on both sides of the central band, takes the distribution out to 80%; and so on until the 95% is covered. The solid black line that goes through the areas is the weighted average of each quantile (median, 16th and 84th percentile) across models, where the weights are given by the relative marginal likelihoods of each model computed as in Eq. 11 of the paper.