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Vertical Separation with Private Contracts

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Marco Pagnozzi* **and Salvatore Piccolo***

Abstract
We consider a manufacturer's incentive to sell through an independent retailer, rather than directly to final consumers, when contracts with retailers cannot be observed by competitors. If retailers conjecture that identical competing manufacturers always offer identical contracts (symmetry beliefs), vertical separation by all manufacturers is an equilibrium, and it results in higher consumers' prices and manufacturers' profits. Even with private contracts, vertically separated manufacturers reduce competition by inducing less aggressive behaviour by retailers in the final market. We characterize a condition for manufacturers' profits to be higher with private than with public contracts. Our results hold both with price and with quantity competition, and do not hinge on retailers' beliefs being perfectly symmetric.

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1 Introduction

Can competing manufacturers obtain higher profits by delegating retail decisions to independent agents, rather than selling directly to final consumers? Manufacturers jointly benefit from high retail prices but, when they sell directly to final consumers, competition among them results in low prices and profits. However, a manufacturer can induce an independent retailer to sell at higher prices, by charging a wholesale price higher than marginal cost. And credibly committing to doing so has a 'strategic effect' on rival retailers, who react by selling at higher prices themselves, thus reducing downstream competition. (See, e.g., Bonanno and Vickers, 1988, Vickers, 1995, and Rey and Stiglitz, 1987.)

This insight hinges on the assumption that contracts between manufacturers and retailers are observed by competitors (i.e., public): when contracts are private (or, alternatively, when publicly announced contracts can be secretly renegotiated), a manufacturer's wholesale price cannot affect the strategy of a rival retailer. Therefore, it is often argued that delegation has no strategic effect because manufacturers always charge a wholesale price equal to marginal cost — a *neutrality result* (see, e.g., Coughlan and Wernerfelt, 1989, and Katz, 1991).

We show that the neutrality result rests on a specific assumption about retailers' conjectures on their competitors' contracts — i.e., *passive beliefs* — and that the equilibrium changes when alternative, but equally reasonable, assumptions are considered. The point is that, with private contracts, a retailer's strategy depends on his conjecture about the wholesale price paid by rival retailers, and this conjecture may depend on the contract offered to the retailer. Hence, even if vertical separation cannot directly affect the strategies of rival retailers, it can still affect a retailer's conjecture about his rivals' input cost (as well as the retailer's own input cost).

If retailers conjecture that identical manufacturers always choose the same wholesale price (*symmetry beliefs*), vertical separation by all manufacturers arises in equilibrium, and yields higher manufacturers' profits. Hence, even when contracts with retailers cannot be observed by outsiders, vertical separation can reduce competition by inducing less aggressive behavior by retailers in the final market.

In models with a single principal and multiple agents, the typical assumption is that agents have passive beliefs (e.g., Cremer and Riordan, 1987, Horn and Wolinsky, 1988, Hart and Tirole, 1990, Laffont and Martimort, 2000, Martimort, 1996, and O'Brien and Shaffer, 1992). In contrast to a situation with symmetry beliefs, a retailer who has passive beliefs and receives an offer different from the one he expects in equilibrium does not revise his beliefs about the offers made to rival retailers. In this case, vertical separation affects neither the strategies of rival retailers, nor a retailer's conjectures about these strategies. Hence, vertically separated manufacturers act as if they were integrated with their retailers and always charge a wholesale price equal to marginal cost.

When there are competing manufacturers, however, the assumption of passive beliefs is not necessarily the most natural one. If a manufacturer has an incentive to offer a contract different from the one that the retailer expects, then why should another identical manufacturer not have an incentive to do the same? Arguably, it is reasonable to assume that retailers perceive deviations as symmetric, and conjecture that identical manufacturers always offer the same contract. An alternative interpretation of symmetry beliefs is that retailers are naive, or have 'bounded rationality,' and simply believe that the strategy adopted by a rival manufacturer is always identical to the strategy adopted by the manufacturer with whom they are contracting.[1] Or retailers may be completely uninformed about some private, and common, characteristic of manufacturers — e.g., the manufacturers' production cost — and so be unable to determine the manufacturers' equilibrium contract.[2]

Hart and Tirole (1990) and McAfee and Schwartz (1994) consider symmetry beliefs in a model with a single (monopolistic) manufacturer and two independent and competing retailers. They show that, with private contracts, the manufacturer's profit depends on retailers' beliefs and is higher with symmetry than with passive beliefs.[3] However, they also argue that the assumption of passive beliefs is the most natural one in their model, because if the manufacturer offers a contract different from the equilibrium one to a retailer, she has no incentive to offer the same contract to the other retailer. By contrast, we believe that symmetry beliefs are especially appealing with identical competing vertical chains, because if one of the manufacturers has an incentive to offer a contract different from the equilibrium one, then a rival manufacturer should have an incentive to do the same. So it is natural for a retailer who receives an unexpected offers to conjecture that the same reason that induced his manufacturer to deviate also induced rival manufacturers to make an identical deviation.

To explore the effects of beliefs, we analyze a delegation game with unobservable contracts. First manufacturers publicly choose whether to sell through independent retailers or not. Second, vertically separated manufacturers offer two-part tariffs to retailers. Finally, price competition takes place in the retail market. We compare equilibria with passive and symmetry beliefs. In contrast to the neutrality result with passive beliefs, with symmetry beliefs delegation is a weakly dominant strategy. Specifically, there are two equilibria with symmetry beliefs: one where all manufacturers delegate, and the other where all manufacturers integrate. But the equilibrium where manufacturers sell through independent retailers both Pareto dominates (from the manufacturers' point of view) and risk dominates the one where manufacturers integrate.

The reason for our result is that, if retailers conjecture that other retailers are offered their same contract, vertical separation generates a 'belief effect': the wholesale price charged by

[1] Symmetry beliefs are much simpler than passive ones for retailers, in the following sense. With passive beliefs, a retailer must be capable of computing manufacturers' equilibrium contracts, given retailers' optimal strategies, in order to make a conjecture about his opponent's input cost. By contrast, with symmetry beliefs a retailer simply bases this conjecture on the manufacturer's offer, thus trusting her ability to choose the best contract. So a retailer only needs to compute his own best strategy, given his input cost. Therefore, the assumption of symmetry beliefs appears more natural when retailers face computational or cognitive constraints.

[2] In Appendix B we show that symmetry beliefs arise in a Hotelling model in which manufacturers are privately informed about their common cost of production, and this cost has full support. Symmetry beliefs also arise when retailers have diffuse prior about manufacturers' cost, or about a shock affecting this cost.

[3] See also Rey and Tirole (2007).

a manufacturer affects the retailer's beliefs about the contract offered to competing retailers and, hence, about the retail price charged by the latter. Therefore, by increasing wholesale prices, manufacturers manage to soften downstream competition, because retailers who pay high wholesale prices expect competitors to pay high wholesale prices as well, and respond by charging higher retail prices in equilibrium.[4] Manufacturers can then charge a higher franchise fee and obtain higher profits.

Hence, even with private contracts, manufacturers have an incentive to sell through independent retailers, when retailers have symmetry beliefs.[5] By doing so, manufacturers manage to implicitly coordinate on high wholesale prices, since a manufacturer who charges a lower wholesale price reduces the franchise fee that the retailer is willing to pay.

Our result that manufacturers choose vertical separation even with private contracts does not hinge on retailers having *exactly* symmetry beliefs. Indeed, the belief effect that we have described arises as long as a retailer who is offered a contract different form the equilibrium one assigns a positive probability, which can be arbitrarily small, to a rival retailer being offered the same contract. As with symmetry beliefs, manufacturers can then obtain a strictly higher profit by selling through independent retailers, because they can induce them to sell at high prices.

We also compare manufacturers' profit with private and public contracts. Since each retailer can observe other retailers' contracts when those are public, and choose the preferred retail price based on them, it may be expected that manufacturers always manage to achieve higher profits with public contracts. However, this is not necessarily the case. Although with private contracts a manufacturer can only affect the strategy of her own retailer, she can still charge a higher franchise fee by choosing a high wholesale price. But since rival retailers do not respond by increasing their prices, a high wholesale price also reduces the quantity sold by the retailer, thus lowering the manufacturer's wholesale revenue. On balance, a manufacturer obtains lower profit with private contracts when the strategic effect is not too strong — i.e., when a retailer does not increase his price too much in response to an increase of a rival's price.[6]

Information sharing among firms is usually considered anticompetitive (e.g., Briley, 1994). Our results, however, suggest that, if retailers have symmetry beliefs, manufacturers may prefer to keep information about wholesale prices private, precisely when public contracts would enhance consumer welfare by reducing retail prices. Hence, allowing retailers to obtain information

[4] With public contracts, the strategic effect of a high wholesale price is to induce competitors to charge high prices. By contrast, with private contracts and symmetry beliefs, the effect of offering a high wholesale price is to induce a retailer to believe that his competitors pay high wholesale prices, so that the retailer charges a high retail price and expects high profits.

[5] Koçkesen (2007) analyzes an extensive form game in which principals can sign private contracts with "passive" agents (who only receive lump sum transfers), and shows that principals obtain higher profit with delegation. Delegation has a commitment value because principals can induce agents to play a "minmax strategy" if rival principals do not delegate, regardless of the other agents' action. By contrast, we require agents' choices in the downstream game to be part of a Nash equilibrium.

[6] By contrast, when competing retailers contract with a single monopolistic manufacturer, the manufacturer's profits with public contracts are always higher than those with private contracts (both with passive and with symmetry beliefs). In fact, the commitment value of public contracts allow the manufacturer to obtain the monopoly profit (see, e.g., Rey and Tirole, 2007).

about their rivals' wholesale prices may actually increase competition.

Although we consider price competition in our main model, we obtain similar results with quantity competition: with symmetry beliefs, manufacturers selling through independent retailers obtain higher profits because of the belief effect of high wholesale prices. Moreover, with quantity competition, since a retailer buys the manufacturer's good before observing the realized market price (and hence before observing the quantity sold by competing retailers), manufacturers manage to jointly obtain the monopoly profit. By contrast, the strategic effect of public contracts harms manufacturers with quantity competition, because it induces them to charge lower wholesale prices (e.g., Fershtman and Judd, 1987). So manufacturers always prefer private contracts, rather than public ones, when retailers' choice variables are strategic substitutes.

Our results depend on manufacturers' ability to charge franchise fees before retailers observe the realized demand, when manufacturers can affect the retailers' beliefs about the competitors' choices. A manufacturer can then charge a high franchise fee by choosing a high wholesale price, even if other manufacturers do not choose high wholesale prices. This is consistent with the observation that, in real-world contractual relations, franchise and royalty fees are usually paid ex-ante and do not depend on the quantity sold by retailers.

Besides providing a new rationale for delegation in manufacturer-retailer relationships, our results have implications for a wider range of economic situations involving competing vertical chains. First, they suggest that various types of vertical restraints may soften downstream competition with private contracts and symmetry beliefs. For instance, even with unobservable contracts, exclusive territories may be used to reduce interbrand competition and raise manufacturers' profits. Second, in relation to the literature on the strategic design of managerial incentives (e.g., Fershtman and Judd, 1987, and Sklivas, 1987), our model suggests that incentive schemes different from profit maximization may have a strategic role even when these schemes are private.

The rest of the paper is organized as follows. Section 2 presents the model. After discussing the case of passive beliefs in Section 3, in Section 4 we consider symmetry beliefs. Specifically, we first analyze prices and profits when: (*i*) all manufacturers are vertically integrated; (*ii*) all manufacturers are vertically separated; and (*iii*) a vertically integrated manufacturer competes against a vertically separated one. Then, in Section 4.3, we characterize the equilibrium choice of organizational structure by manufacturers. Section 5 describes an example with linear demand function, and Section 6 compares private with public contracts. Quantity competition is discussed in Section 7. In Section 8 we show that our results hold with a more general class of retailers' beliefs. Finally, Section 9 concludes. All proofs are in Appendix A. Appendix B shows how symmetry beliefs can arise with incomplete information.

2 The Model

Players and environment. The game involves two competing vertical structures. There are two (female) manufacturers, M_1 and M_2, that produce substitute goods, and two (male) exclusive retailers, R_1 and R_2.[7] In the downstream market, firms compete by choosing retail prices. (We consider the case of quantity competition in Section 7.) Manufacturers publicly choose their organizational structure: *vertical integration* or *vertical separation.* If M_i is vertically integrated, she chooses the retail price and sells directly to final consumers; if M_i is vertical separated, she sells through retailer R_i, who independently chooses the retail price.

The retail price of the good produced by M_i is p_i, and the (twice continuously differentiable) demand function for this good in the downstream market is $D^i(p_i, p_j)$, with $i, j = 1, 2$ and $i \neq j$. We assume that $D^i(p, q) = D^j(p, q)$ for all prices p and q — i.e., demand functions are symmetric. All firms have constant returns to scale, and manufacturers' marginal cost of production is normalized to zero.

Contracts. With vertical separation, M_i offers a two-part tariff contract $C_i = (w_i, T_i)$ to R_i, specifying a wholesale price w_i and a franchise fee T_i. If R_i accepts the contract, he pays T_i, chooses the retail price, and then pays w_i for each unit sold in the downstream market. R_i's outside option is normalized to zero. We assume that contracts are private, so that a retailer cannot observe the contract offered to his competitor. This assumption captures the idea that manufacturers lack commitment power, because they can recontract and/or offer secret discounts.

Timing. The timing of the game is as follows:

- *Period 1.* Manufacturers simultaneously and publicly choose their organizational structure.
- *Period 2.* A vertically separated manufacturer secretly offers a contract to her exclusive retailer. If the retailer accepts it, he pays the franchise fee and sells the manufacturer's good in period 3.
- *Period 3.* Firms — i.e., an integrated manufacturer, or the retailers of a vertically separated manufacturer — simultaneously choose retail prices in the downstream market and, after observing the realized demand, retailers pay the wholesale price for the quantity they acquire from manufacturers.

Equilibrium concept. Our solution concept is Perfect Bayesian Equilibrium (PBE) (see, e.g., Mas-Colell *et al.*, 1995). A manufacturer's strategy specifies the choice of organizational structure and, depending on this choice, either the contract offered in period 2 or the retail price

[7] R_1 and R_2 can alternatively be interpreted as buyers of an intermediate good, that they transform into a final good through a fixed-coefficient technology.

charged in period 3. A retailer's strategy specifies an acceptance decision in period 2 and the retail price chosen in period 3, contingent on the contract offered by the manufacturer.

In order to describe retailers' 'off-equilibrium' beliefs, define by $\widetilde{w}_j(w_i)$ the belief of R_i regarding the wholesale price offered to R_j, as a function of w_i. We consider three types of beliefs:

- *Passive beliefs:* When a retailer is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the rival retailer. Formally, given an equilibrium with wholesale prices w_1^* and w_2^*, if R_i receives an offer $w_i \neq w_i^*$, then $\widetilde{w}_j(w_i) = w_j^*$.
- *Symmetry beliefs:* Each retailer believes that his competitor is always offered a contract equal to the contract offered by his own manufacturer. Formally, if R_i is offered a wholesale price w_i, then $\widetilde{w}_j(w_i) = w_i$.[8]
- *Mixed beliefs:* Given an equilibrium with wholesale prices w_1^* and w_2^*, if R_i is offered a wholesale price $w_i \neq w_i^*$, he believes that, with probability α, R_j is offered the same wholesale price w_i and, with probability $(1-\alpha)$, R_j is offered the equilibrium wholesale price w_j^*.

In our main analysis we focus on passive and symmetry beliefs. In Section 8, we consider mixed beliefs and show that our qualitative results hold as long as retailers' beliefs are not *exactly* passive — i.e., as long as $\alpha \neq 0$.

Technical assumptions. Let $\Pi_i(p_i, p_j) = D^i(p_i, p_j)(p_i - w_i)$ and $\pi_i(p_i, p_j) = D^i(p_i, p_j)\, p_i$. We make the following assumptions.

A1. $\dfrac{\partial D^i(p_i, p_j)}{\partial p_i} < 0$ and $\dfrac{\partial^2 D^i(p_i, p_j)}{\partial p_i^2} \leq 0$, $\forall p_i, p_j$ — i.e., demand for the good produced by M_i is decreasing and concave in p_i — and $\lim_{p_i \to \infty} D^i(p_i, p_j) = 0$, $\forall p_j$.

A2. $\dfrac{\partial D^i(p_i, p_j)}{\partial p_j} \geq 0$, $\forall p_i, p_j$ — i.e., goods are substitutes. Moreover, $\left| \dfrac{\partial D^i(p_i, p_j)}{\partial p_i} \right| > \dfrac{\partial D^i(p_i, p_j)}{\partial p_j}$.

A3. Stability: $\dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i^2} + \dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} < 0$, for every p_j, w_i and $p_i \geq w_i$.

A4. $\dfrac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} = \dfrac{\partial D^i(p_i, p_j)}{\partial p_j} + (p_i - w_i) \dfrac{\partial^2 D^i(p_i, p_j)}{\partial p_i \partial p_j} > 0$, for every p_j, w_i and $p_i \geq w_i$ — i.e., retailers' prices are strategic complements.

A5. $\dfrac{\partial^2 \pi_i(p_i, p_j)}{\partial p_i^2} + \dfrac{\partial^2 \pi_i(p_i, p_j)}{\partial p_i \partial p_j} < 0$, $\forall p_i, p_j$.

[8] With symmetry beliefs, it is only possible to have symmetric equilibria, in which both manufacturers offer the same wholesale price, if they are vertically separated.

Assumptions A1-A4 are standard in the vertical contracting literature (see for instance Rey and Stiglitz, 1995, and Bonanno and Vickers, 1988). Following Vives (2000, p. 157), Assumption A5 implies that the function $\partial \pi_i(p,p)/\partial p_i$ is downward sloping, and that $\partial \pi_i(p,p)/\partial p_i = 0$ defines the unique (interior) equilibrium, in a game with two symmetric integrated manufacturers.

3 Passive Beliefs

With passive beliefs, when a retailer receives an offer different from the one he expects in equilibrium, he does not revise his beliefs about the offer made to the rival retailer. In this case, each manufacturer chooses a wholesale price equal to zero, regardless of the contract and the organizational structure chosen by the competitor.

To see this, suppose both manufacturers are vertically separated, and denote by p_j the price chosen by R_j in equilibrium in period 3. Because of passive beliefs, R_i's beliefs about R_j's price do not depend on w_i. Hence, R_i's reaction function is

$$p_i(p_j, w_i) \in \arg\max_{p_i} \left\{ D^i(p_i, p_j)(p_i - w_i) - T_i \right\}.$$

Since the franchise fee T_i is a fixed cost, this program yields the standard first-order condition equalizing R_i's marginal revenue to marginal cost

$$\frac{\partial D^i(p_i(p_j,w_i),p_j)}{\partial p_i}(p_i(p_j,w_i) - w_i) + D^i(p_i(p_j,w_i),p_j) \equiv 0. \tag{1}$$

In period 2, M_i offers the contract that maximizes her profit, subject to R_i's participation constraint and given R_j's price — i.e.,

$$C_i \in \arg\max_{(w_i,T_i)\in\mathbb{R}^2} \left\{ D^i(p_i(p_j,w_i),p_j)\, w_i + T_i : \quad D^i(p_i(p_j,w_i),p_j)(p_i(p_j,w_i) - w_i) - T_i \geq 0 \right\}.$$

Since T_i is chosen to satisfy R_i's participation constraint as an equality, this simplifies to

$$w_i \in \arg\max_{w_i} D^i(p_i(p_j,w_i),p_j)\, p_i(p_j,w_i). \tag{2}$$

Differentiating the objective function in (2), and using equation (1),

$$\begin{aligned} & \frac{\partial p_i(p_j,w_i)}{\partial w_i}\left(\frac{\partial D^i(p_i(p_j,w_i),p_j)}{\partial p_i} p_i(p_j,w_i) + D^i(p_i(p_j,w_i),p_j)\right) \\ = & \frac{\partial p_i(p_j,w_i)}{\partial w_i}\frac{\partial D^i(p_i(p_j,w_i),p_j)}{\partial p_i} w_i \leq 0. \end{aligned} \tag{3}$$

Lemma 1 *With passive beliefs, if manufacturers choose vertical separation in period 1, in the unique equilibrium wholesale prices are equal to zero.*

Since a retailer's choice is unaffected by unobserved changes in the rival's wholesale price, each manufacturer acts as if integrated with the retailer and charges a wholesale price equal to marginal cost.[9] The next proposition states the well known *neutrality* that, with private contracts and passive beliefs, vertical separation has no strategic effect (Katz, 1991).

Proposition 2 *With passive beliefs, in any PBE the retail price* p^e *solves*

$$\frac{\partial D^i\left(p^e, p^e\right)}{\partial p_i} p^e + D^i\left(p^e, p^e\right) = 0. \tag{4}$$

Any combination of organizational structures is part of a PBE and yields the same manufacturers' profit.

Hence, with passive beliefs, manufacturers have no incentive to sell through retailers.[10] The neutrality result, however, does not hold when agents have symmetry beliefs.

4 Symmetry Beliefs

Assume now that retailers have symmetry beliefs — i.e., a retailer always believes that his competitor receives the same offer as he does (e.g., Hart and Tirole, 1990, and McAfee and Schwartz, 1994). Hence, when a retailer receives from a manufacturer an offer different from what he expects in equilibrium, he believes that the competing manufacturer has also deviated from equilibrium by making the same offer. Of course, in equilibrium retailers' beliefs must be consistent with manufacturers' strategies.

In games of competing hierarchies, it is usually assumed that beliefs are passive. There seem to be no compelling reason, however, to rule out symmetry beliefs *a priori*, especially when upstream manufacturers are symmetric.[11] Why should a retailer who receives an unexpected, off-equilibrium, offer believe that a rival manufacturer is still offering the equilibrium contract? If one manufacturer has an incentive to offer a different contract, another identical manufacturer should have an incentive to do the same. Arguably, it is reasonable to assume that retailers

[9] As observed by McAfee and Schwartz (1994), this result does not hinge on the nature of downstream production (fixed versus variable proportions) or of downstream competition (strategic substitutes or strategic complements).

[10] Katz (1991) shows that this neutrality result does not hold with agency constraints, and that vertical separation may have a commitment effect when principals and agents have conflicting preferences.

[11] When one (monopolistic) manufacturer contracts with two independent and competing retailers, it is usually argued that symmetry beliefs are unappealing, since the manufacturer's preferred contract with one retailer generally differs from the contract accepted by the other retailer (the 'opportunism problem' in vertical contracting). Moreover, it is argued, since the two retailers represent two separate markets, when the manufacturer changes the offer to one retailer, she has no incentive to also change the offer to the other retailer (e.g., Rey and Tirole, 2007). This criticism is much less compelling in games of competing hierarchies, where a manufacturer may have an incentive to deviate from an equilibrium candidate in order to increase her profit at the expense of the competing manufacturer, but not to harm her own retailer. So if one manufacturer wants to offer a different contract, the other manufacturer should want to do the same.

expect deviations to be symmetric, and conjecture that identical manufacturers always offer identical contracts.

Alternatively, symmetry beliefs capture the idea that retailers are naive or have bounded rationality, and so use the simplest conjecture that the strategy adopted by a rival manufacturer is always identical to the strategy adopted by the manufacturer with whom they are contracting. Retailers may find it too costly, or too difficult, to compute the manufacturers' equilibrium contracts, based on the retailers' optimal strategies (which is required with passive beliefs), and simply prefer to infer the equilibrium contract from the manufacturer's actual offer.

Or retailers may be completely uninformed about some common characteristic of manufacturers that affect their choice of contract — e.g., their production cost — and so be unable to determine the equilibrium contract.[12] We develop this interpretation in Appendix B, where we show that symmetry beliefs arise in the separating equilibrium of a Hotelling model in which manufacturers are privately informed about their common cost of production.[13]

When both manufacturers are vertically integrated, beliefs are irrelevant because no contract is offered. Hence, manufacturers choose the retail price that solves condition (4). In the next two sections, we first analyze the case in which both manufacturers are vertically separated, and then the asymmetric case in which one manufacturer is vertically separated, while the other is not.

4.1 Vertical Separation

Suppose that both manufacturers choose vertical separation in period 1. First notice that the equilibrium with passive beliefs characterized in Lemma 1 is not an equilibrium with symmetry beliefs.

Lemma 3 *If both manufacturers choose vertical separation, with symmetry beliefs there is no PBE in which wholesale prices are equal to zero.*

With passive beliefs, manufacturers cannot coordinate on a positive wholesale price because each manufacturer has an incentive to secretly undercut it, in order to induce her retailer to charge a lower retail price and obtain higher profit. With symmetry beliefs, however, this incentive is weakened because, if a manufacturer reduces the wholesale price, her retailer conjectures that the other manufacturer is doing the same. Hence, the retailer expects to obtain lower profit and is willing to pay a lower franchise fee.

Given a contract $C_i = (w_i, T_i)$, R_i's expected profit, net of the franchise fee, is

$$D^i(p_i, \widetilde{p}_j(w_i))(p_i - w_i) - T_i,$$

[12] White (2007) analyzes the effect of private information in a model with a single monopolistic manufacturer and two retailers.

[13] Another possible rationale for symmetry beliefs is that retailers interpret unexpected offers as trembles, or mistakes, made by manufacturers, and that these trembles are perfectly correlated across identical players.

where $\widetilde{p}_j(w_i) \equiv \widetilde{p}_j(\widetilde{w}_j(w_i))$ is the price that R_i expects R_j to charge, when R_i conjectures that R_j pays the wholesale price w_i.

Let

$$\hat{p}(w_i) \in \arg\max_{p_j} D^j(p_j, \hat{p}(w_i))(p_j - w_i)$$

define the price chosen by R_j, when he is offered the wholesale price w_i and believes that R_i, having received the same offer, also chooses $\hat{p}(w_i)$. By symmetry of the demand functions, $\hat{p}(w_i)$ solves the following first-order condition, which is necessary and sufficient for an optimum under assumptions A1-A4,

$$\frac{\partial D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_i}(\hat{p}(w_i) - w_i) + D^i(\hat{p}(w_i), \hat{p}(w_i)) \equiv 0. \tag{5}$$

Therefore, when a retailer is offered the contract $C_i = (w_i, T_i)$, he expects his rival to choose a retail price equal to $\hat{p}(w_i)$.

In period 2, a manufacturer offers the contract that maximizes her profit subject to the retailer's participation constraint, given the retailer's beliefs and the price charged by the competitor.

Lemma 4 *With symmetry beliefs, if both manufacturers choose vertical separation, in period 2 they offer the contract $C^* = (w^*, T^*)$ such that*

$$C^* \in \arg\max_{(w_i,T_i)\in\mathbb{R}^2} \left\{D^i(\hat{p}(w_i), \hat{p}(w^*))w_i + T_i : \quad D^i(\hat{p}(w_i), \hat{p}(w_i))(\hat{p}(w_i) - w_i) - T_i \geq 0\right\}. \tag{6}$$

Notice that, while M_i takes the competitor's retail price as given (since she expects M_j to offer w^* and R_j to choose $\hat{p}(w^*)$ in equilibrium), R_i's beliefs about the competitor's retail price depend on w_i. Since R_i believes that R_j chooses $\hat{p}(w_i)$, he is willing to pay a franchise fee at most equal to $D^i(\hat{p}(w_i), \hat{p}(w_i))(\hat{p}(w_i) - w_i)$. Therefore, the wholesale price chosen by a manufacturer affects the franchise fee also through its effect on the retailer's conjecture about the competitor's retail price.

Since the retailer's participation constraint is binding, program (6) can be rewritten as

$$w^* \in \arg\max_{w_i} \left\{D^i(\hat{p}(w_i), \hat{p}(w^*))w_i + D^i(\hat{p}(w_i), \hat{p}(w_i))(\hat{p}(w_i) - w_i)\right\}. \tag{7}$$

By the 'envelope theorem' — i.e., using condition (5) — the first-order condition is

$$\frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i}\frac{\partial \hat{p}(w^*)}{\partial w_i}w^* + D^i(\hat{p}(w^*), \hat{p}(w^*)) +$$

$$+\frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_j}\frac{\partial \hat{p}(w^*)}{\partial w_i}(\hat{p}(w^*) - w^*) - D^i(\hat{p}(w^*), \hat{p}(w^*)) \equiv 0. \tag{8}$$

A change in the wholesale price has two effects. First, w_i affects the wholesale revenue — $D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(w^*\right)\right) w_i$ — as reflected by the first two terms in condition (8): a higher w_i increases the wholesale revenue for a given demand, but it also reduces demand because it increases the retail price $\hat{p}\left(w_i\right)$. Second, w_i has a 'belief effect' because it affects R_i's expected profit — $D^i\left(\hat{p}(w_i), \hat{p}(w_i)\right)\left(\hat{p}(w_i) - w_i\right)$ — and, hence, the franchise fee that he is willing to pay, as reflected by the last two terms in condition (8): a higher w_i increases R_i's input cost, which reduces R_i's expected profit, but it also induces R_i to believe that R_j charges a higher retail price, which increases R_i's expected profit.[14]

Simplifying equation (8), we have

$$\underbrace{\frac{\partial D^i\left(\hat{p}\left(w^*\right), \hat{p}\left(w^*\right)\right)}{\partial p_i} w^*}_{<0} + \underbrace{\frac{\partial D^i\left(\hat{p}(w^*), \hat{p}(w^*)\right)}{\partial p_j}\left(\hat{p}(w^*) - w^*\right)}_{>0} = 0, \tag{9}$$

where the second term captures the 'belief effect'.

Denote the (equilibrium) price elasticity of demand by

$$\varepsilon_i^i\left(\hat{p}\left(w^*\right)\right) = -\frac{\partial \log D^i\left(\hat{p}\left(w^*\right), \hat{p}\left(w^*\right)\right)}{\partial \log p_i},$$

and, similarly, the (equilibrium) cross price elasticity of demand by

$$\varepsilon_j^i\left(\hat{p}\left(w^*\right)\right) = \frac{\partial \log D^i\left(\hat{p}\left(w^*\right), \hat{p}\left(w^*\right)\right)}{\partial \log p_j}.$$

Proposition 5 *When retailers have symmetry beliefs and both manufacturers choose vertical separation in period 1:*

- *Given a wholesale price w_i, in period 3 R_i chooses the retail price $\hat{p}\left(w_i\right)$ defined by the first-order condition (5).*
- *In period 2, there is a symmetric PBE where both manufacturers offer the contract $C^* = (w^*, T^*)$ such that*
$$\frac{\hat{p}(w^*) - w^*}{\hat{p}(w^*)} \equiv \frac{\varepsilon_i^i\left(\hat{p}\left(w^*\right)\right) - 1}{\varepsilon_j^i\left(\hat{p}\left(w^*\right)\right)}, \tag{10}$$
and
$$T^* = D^i\left(\hat{p}\left(w^*\right), \hat{p}\left(w^*\right)\right)\left(\hat{p}\left(w^*\right) - w^*\right).$$
- *M_i's profit is $D^i\left(\hat{p}(w^*), \hat{p}(w^*)\right)\hat{p}(w^*)$.*

[14] Out of equilibrium, by choosing an appropriately high wholesale price, a manufacturer can 'fool' the retailer into believing that the other retailer is choosing any high retail price. Of course, the benefit of this must be weighted against the reduction in demand caused by a high wholesale price.

With symmetry beliefs, separated manufacturers charge higher wholesale prices (than integrated manufacturers, or separated ones with passive beliefs) to reduce competition among retailers. Indeed, when a retailer is offered a high wholesale price, he believes that the competing retailer receives the same offer and chooses a high retail price. Hence, he expects high profit and is willing to pay a high franchise fee.

Equation (10) implies that w^* is low when $\varepsilon_i^i(.)$ is large because, if $\varepsilon_i^i(.)$ is large, M_i does not want R_i to charge a very high retail price (since this would cause a large reduction in demand).[15] Moreover, w^* is high when $\varepsilon_j^i(.)$ is large. The reason is that, if $\varepsilon_j^i(.)$ is large, R_i expects a relatively large increase in demand when he is offered a high wholesale price (since he expects his competitor to choose a high retail price), and pays a high franchise fee.[16]

4.2 Asymmetric Vertical Structures

Suppose now that, in period 1, M_i chooses to sell her product through a retailer while M_j does not. In this case, M_i has no incentive to increase her wholesale price, because R_i knows that his competitor's input cost is zero (since M_j is integrated), regardless of the wholesale price offered by M_i. In other words, M_i cannot affect R_i's beliefs in order to obtain a higher franchise fee.

To see this, suppose that M_j chooses the retail price p_j. Given a contract $C_i = (w_i, T_i)$, R_i chooses the retail price that solves

$$\max_{p_i} \left\{ D^i(p_i, p_j)\,(p_i - w_i) - T_i \right\}.$$

Therefore, R_i's best response function $p_i^e(w_i)$ is defined by the first-order condition

$$\frac{\partial D^i(p_i^e(w_i), p_j)}{\partial p_i}\,(p_i^e(w_i) - w_i) + D^i(p_i^e(w_i), p_j) \equiv 0.$$

The contract offered by manufacturer M_i is

$$C_i^e \in \underset{(w_i,T_i)\in\mathbb{R}^2}{\arg\max} \left\{ D^i(p_i^e(w_i), p_j) w_i + T_i : \quad D^i(p_i^e(w_i), p_j)\,(p_i^e(w_i) - w_i) - T_i \geq 0 \right\}.$$

Since the retailer's participation constraint is binding, this is equivalent to

$$w_i^e \in \underset{w_i}{\arg\max} D^i(p_i^e(w_i), p_j) p_i^e(w_i).$$

[15] Rearranging equation (10), $\frac{w^*}{\hat{p}(w^*)} \equiv 1 - \frac{\varepsilon_i^i(\hat{p}(w^*)) - 1}{\varepsilon_j^i(\hat{p}(w^*))}$.

[16] This is consistent with the evidence discussed in Lafontaine and Slade (1997), who show that retail prices of delegated outlets are higher when the cross-price elasticity of demand is large relative to the own-price elasticity, and when reaction functions are steep. They also show that delegation is more likely in these cases.

By the 'envelope theorem,' the derivative of M_i's objective function is

$$\frac{\partial p_i^e(w_i)}{\partial w_i}\frac{\partial D^i(p_i^e(w_i),p_j)}{\partial p_i}w_i. \tag{11}$$

Since $\frac{\partial p_i^e(.)}{\partial w_i} > 0$ and $\frac{\partial D^i(.)}{\partial p_i} < 0$, this derivative is strictly negative for every $w_i > 0$. Hence, we have the following result.

Lemma 6 *When one manufacturer is vertically integrated while the other is vertically separated, the separated manufacturer charges a wholesale price equal to zero. In period 3, there is a unique equilibrium in which both goods are sold at the retail price p^e such that*

$$\frac{\partial D^i(p^e,p^e)}{\partial p_i}p^e + D^i(p^e,p^e) = 0.$$

Notice that the equilibrium retail price is equal to the one with two integrated manufacturers, or with passive beliefs (see Proposition 2). Hence, the profit obtained by a vertically separated manufacturer competing against an integrated manufacturer is equal to the profit of an integrated manufacturer.

4.3 Equilibrium

In this section we characterize the equilibrium choice of organizational structure by manufacturers. In order to do so, we start by comparing the retail price when both manufacturers choose separation with the retail price when at least one manufacturer chooses integration.

Lemma 7 *The equilibrium retail price with two vertically separated manufacturers is higher than the equilibrium retail price with at least one vertically integrated manufacturer — i.e., $p^* \equiv \hat{p}(w^*) > p^e$.*

In contrast to an integrated manufacturer, a manufacturer selling through a retailer has an incentive to offer a strictly positive wholesale price, in order to induce the retailer to believe that his competitor also pays a positive wholesale price and, hence, chooses a high retail price. The retailer is then willing to sell at a high retail price and pay a high franchise fee. Therefore, both wholesale and retail prices are higher when manufacturers sell through retailers.

Since manufacturers extract the whole surplus from retailers, manufactures' profits when they choose integration (I) or separation (S) are given by

		M_2: I	M_2: S
M_1	I	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$
	S	$\pi_1(p^e,p^e)$, $\pi_2(p^e,p^e)$	$\pi_1(p^*,p^*)$, $\pi_2(p^*,p^*)$

where $\pi_i(p,p) = D^i(p,p)\,p$.

Proposition 8 *With symmetry beliefs, there are two equilibria: one where both manufacturers choose vertical integration and one where both manufacturers choose vertical separation in period 1. The equilibrium where both manufacturers choose separation Pareto dominates, and risk dominates, the one where they both choose integration.*

In the proof of Proposition 8, we show that $\pi_i(p^*,p^*) > \pi_i(p^e,p^e)$, since $p^* > p^e$ by Lemma 7 and both prices are lower than the price that maximizes the function $\pi_i(p,p)$ (which is strictly concave by Assumption A5). Therefore, vertical separation is also a weakly dominant strategy for manufacturers. The intuition is that, when one manufacturer is vertically separated, the other manufacturer prefers to choose vertical separation too, in order to commit not to undercut the competitor, when the latter charges a high wholesale price.

Hence, we expect both manufacturers to sell through independent retailers, when those retailers have symmetry beliefs. By choosing vertical separation and charging high wholesale prices, manufacturers induce retailers to sell at high retail prices, thus reducing competition and increasing profit.[17]

5 The Linear Example

We analyze a simple example with linear inverse demand function $P_i(q_i,q_j) = \alpha - \beta q_i - \gamma q_j$, where q_i is the quantity of the good produced by M_i that is sold in the retail market. This is a natural and often analyzed demand function (see, e.g., Vives, 2000). We assume that $\alpha > 0$ and $\beta > \gamma \geq 0$, so that inverting the system of inverse demand functions yields direct demand functions

$$D^i(p_i,p_j) = \frac{\alpha(\beta-\gamma) - \beta p_i + \gamma p_j}{\beta^2 - \gamma^2}, \quad i = 1,2.$$

The parameter γ reflects the degree of substitutability among products.

First consider passive beliefs. Vertically separated manufacturers charge a wholesale price equal to zero and, by condition (4), the unique equilibrium retail price is

$$p^e = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma}.$$

Manufacturers' profit is

$$\pi^e = \frac{\alpha^2\beta(\beta-\gamma)}{(2\beta-\gamma)^2(\beta+\gamma)}.$$

Now consider symmetry beliefs. Using equations (5) and (9), when both manufacturers

[17] Clearly, manufacturers' profit when retailers have symmetry beliefs are higher than their profit when retailers have passive beliefs (since in this last case the retail price is p^e).

choose vertical separation the unique equilibrium wholesale price is

$$w^* = \frac{\alpha\gamma\left(\beta-\gamma\right)}{2\beta^2-\gamma^2} > w^e = 0,$$

and the unique equilibrium retail price is

$$p^* = \frac{\alpha\left(\beta^2-\gamma^2\right)}{2\beta^2-\gamma^2}.$$

Therefore, in the Pareto dominant equilibrium, a manufacturer's profit is

$$\pi^* = \frac{\alpha^2\beta^2\left(\beta-\gamma\right)}{\left(2\beta^2-\gamma^2\right)^2}.$$

Finally,

$$p^e - p^* = -\frac{\alpha\beta\gamma\left(\beta-\gamma\right)}{\left(2\beta^2-\gamma^2\right)\left(2\beta-\gamma\right)} < 0,$$

and

$$\pi^e - \pi^* = -\frac{\alpha^2\gamma^2\beta\left(\beta-\gamma\right)\left(\beta^2+\gamma\left(\beta-\gamma\right)\right)}{\left(2\beta^2-\gamma^2\right)^2\left(2\beta-\gamma\right)^2\left(\beta+\gamma\right)} < 0.$$

As expected by Proposition 8, retail prices and manufacturers' profits are higher when they are vertically separated than when they are integrated. Clearly, prices and profits with separation and integration are equal when $\gamma = 0$, since products are independent. Moreover, the difference between prices and profits with separation and integration tends to zero as $\beta \to \gamma$, because products become closer substitutes and manufacturers competing à la Bertrand make zero profit.

6 Private vs. Public Contracts

In this section, we compare retail prices of vertically separated manufacturers with private and public contracts.

With public contracts, a retailer observes both manufacturers' wholesale prices and chooses the retail price that solves

$$\max_{p_i} D^i\left(p_i, p_j\right)\left(p_i - w_i\right),$$

yielding the first-order conditions

$$\frac{\partial D^i\left(p_i, p_j\right)}{\partial p_i}\left(p_i - w_i\right) + D^i\left(p_i, p_j\right) = 0, \qquad i = 1, 2. \tag{12}$$

These conditions define the function $p_i\left(w_i, w_j\right)$.[18]

Since a manufacturer chooses the franchise fee so that the retailer's participation constraint

[18] Of course, under our assumptions, $p_i\left(w_i, w_j\right)$ is increasing in both w_i and w_j.

is binding, M_i solves

$$\max_{w_i} D^i \left(p_i \left(w_i, w_j\right), p_j \left(w_j, w_i\right)\right) p_i \left(w_i, w_j\right).$$

Hence, the (symmetric) equilibrium wholesale price w^{**} is defined by the first-order conditions

$$\left(\frac{\partial D^i (.)}{\partial p_i} p_i (.) + D^i (.)\right) \frac{\partial p_i (.)}{\partial w_i} + \frac{\partial D^i (.)}{\partial p_j} \frac{\partial p_j (.)}{\partial w_i} p_i (.) = 0, \qquad i = 1, 2, \tag{13}$$

and the equilibrium retail price is $p^{**} = p\left(w^{**}, w^{**}\right)$. The second term in equation (13) represents the strategic effect: when choosing the wholesale price, M_i anticipates R_j's reaction, and the resulting effect on his own product's demand (see Bonanno and Vickers, 1988, and Rey and Stiglitz, 1995). Since prices are strategic complements, the strategic effect of an increase in w_i on M_i's profit is positive.

Let $\varphi\left(p_j | w_i\right)$ be R_i's reaction function in period 3, given p_j and w_i, defined by condition (12). Then $\beta\left(w^{**}\right) \equiv \partial \varphi\left(p^{**} | w^{**}\right) / \partial p_j$ is the slope of a retailer's reaction function, in the symmetric equilibrium with wholesale price w^{**}. To simplify the analysis, we assume that there is a unique equilibrium both with public and with private contracts.

Proposition 9 *Assume that manufacturers' profits are single-peaked both with private and with public contracts. With symmetry beliefs, wholesale prices, retail prices and manufacturers' profits are higher (resp. lower) with private contracts than with public contracts if and only if*

$$\frac{p^{**} - w^{**}}{p^{**}} > \beta\left(p^{**}\right) \qquad \textit{(resp. } < \textit{).} \tag{14}$$

Public contracts allow retailers to observe the wholesale price paid by competitors and respond to it, thus creating a strategic effect that facilitates coordination among players. The strategic effect is captured by the right-hand-side of condition (14), and its strength depends on retailers' reaction function. On the other hand, with private contracts, manufactures can induce retailers to expect a high price from their competitors and, hence, to pay a high franchise fee, regardless of the wholesale price that competitors actually pay. The belief effect is captured by the left-hand-side of condition (14), and its strength depends on retailers' price-cost markup. By condition (14), the strategic effect dominates the belief effect when the retailer's reaction function with public contracts is relatively steep — i.e., when an increase in a retailer's price induces a large increase in the competitor's price with public contracts, which in turn increases the manufacturer's wholesale revenue.

To see the intuition for this result, consider the equilibrium wholesale price with public contracts w^{**}. Does a manufacturer have an incentive to charge a price higher than w^{**} when contracts are private? There are two effects. First, a higher wholesale price induces the retailer to expect higher profit (since he expects the competitor's retail price to be higher). Hence, the retailer is willing to pay a higher franchise fee. Second, however, a higher wholesale price also induces the retailer to choose a higher retail price, while the other retailer still chooses p^{**}.

Hence, the first retailer sells a lower quantity, and the manufacturer obtains a lower wholesale revenue. This second, negative, effect is stronger when the slope of the reaction functions at p^{**} is larger, because in this case the increase in the retailer's price is larger, resulting in a larger reduction in demand.

Therefore, although public contracts have a commitment values for competing manufacturers, with symmetry beliefs manufacturers' profit may be higher with private contracts than with public ones, and manufacturers may prefer not to share information about their retail contracts with competitors. By contrast, when a single monopolistic manufacturer sells to competing retailers, her profit is maximized by public contracts (e.g., Hart and Tirole, 1990).

In the linear example of Section 5, the unique equilibrium wholesale price with public contracts is

$$w^{**} = \frac{\alpha\gamma^2(\beta-\gamma)}{\beta(4\beta^2-\gamma^2-2\beta\gamma)},$$

and

$$w^{**} - w^{*} = \frac{(\gamma-\beta)^2\left(4\beta^2-\gamma^2\right)\alpha\gamma}{\left(2\beta\gamma-4\beta^2+\gamma^2\right)\left(2\beta^2-\gamma^2\right)\beta} < 0.$$

Hence, wholesale prices are higher with private contracts.[19] Similarly, also retail prices and manufacturers' profits are higher with private contracts.

7 Quantity Competition

Suppose that firms compete by choosing the quantity produced, rather than the retail price. In this case, if a manufacturer chooses vertical separation, in period 3 the retailer first acquires the quantity he chooses to produce, paying the wholesale price, and then sells it to final consumers at the market clearing price.

Let $P(Q)$ be the demand function, where $Q = q_i + q_j$ is the total quantity produced. We assume that $P'(.) < 0$ and $P''(.) \leq 0$.

Private contracts. As in the case of price competition, with private contracts and passive beliefs, vertical separation has no strategic effect (see Proposition 2). In the unique equilibrium with vertically separated manufacturers, the wholesale price is equal to zero and each retailer sells the quantity q^e such that

$$P'(2q^e)\,q^e + P(2q^e) = 0. \tag{15}$$

This is the same quantity produced by each of two integrated manufacturers. Therefore, any combination of organizational structures is a PBE and yields manufacturers' profit equal to $P(2q^e)\,q^e$.

By contrast, with symmetry beliefs, each manufacturer has an incentive to charge a wholesale price greater than zero when she is vertically separated, in order to induce the retailer to produce

[19] With a linear demand, the condition of Proposition 9 is $\frac{p^{**}-w^{**}}{p^{**}} - \beta(p^{**}) = \frac{(2\beta+\gamma)(\beta-\gamma)}{2\beta^2} > 0$.

a lower quantity and to expect his competitor to do the same. The retailer is then willing to pay a higher franchise fee, because he anticipates higher profits. This confirms the insight of our analysis with price competition: a positive wholesale price reduces competition among retailers when they have symmetry beliefs.

Proposition 10 *With symmetry beliefs and quantity competition, if both manufacturers choose vertical separation in period 1:*

- *Given a wholesale price w_i, in period 3 R_i produces the quantity $\hat{q}(w_i)$ such that*

$$P'(2\hat{q}(w_i))\,\hat{q}(w_i) + P(2\hat{q}(w_i)) - w_i \equiv 0. \tag{16}$$

- *In period 2, there is a symmetric PBE where both manufacturers offer the contract $C^* = (w^*, T^*)$ such that*

$$w^* \equiv -P'(2\hat{q}(w^*))\,\hat{q}(w^*) > 0, \tag{17}$$

  and

$$T^* = (P(2\hat{q}(w^*)) - w^*)\hat{q}(w^*). \tag{18}$$

- *M_i's profit is $P(2\hat{q}(w^*))\,\hat{q}(w^*)$, and each retailer produces the quantity $\hat{q}(w^*) < q^e$.*

The quantity produced by a retailer when the manufacturer is vertically separated, $\hat{q}(w^*)$, is equal to half the quantity produced by a monopolist. Hence, vertically separated manufacturers manage to maximize joint profit. Each manufacturer obtains higher profit than with price competition because, with quantity competition, a retailer buys from the manufacturer *before* observing the market price and learning the quantity chosen by his competitor (while with price competition, he only buys from the manufacturer after observing the price chosen by his competitor). Hence, a manufacturer can extract the whole total expected surplus from the retailer ex-ante, via the franchise fee and the wholesale payment.[20]

A vertically separated manufacturer induces the retailer to produce a lower quantity than a vertically integrated manufacturer because of the 'belief effect.' Hence, consider manufacturers' choice between vertical separation and integration.

Proposition 11 *With symmetry beliefs and quantity competition, there are two equilibria: one where both manufacturers choose vertical integration and one where both manufacturers choose vertical separation in period 1. The equilibrium where both manufacturers choose separation Pareto dominates, and risk dominates, the one where they both choose integration.*

[20] In contrast to price competition, a manufacturer has no incentive to reduce the wholesale price in order to increase the wholesale revenue, when she expects the rival manufacturer to charge a high wholesale price: if M_i reduces the wholesale price, R_i conjectures that M_j also reduced the wholesale price and, hence, he does not produce a much larger quantity.

As in the case of price competition, manufacturers' profits with vertical separation exceed those with vertical integration. Therefore, even with quantity competition, it is a weakly dominant strategy for manufacturers to choose vertical separation in order to reduce competition among retailers, when retailers have symmetry beliefs.

Public contracts. In contrast to price competition, with quantity competition and public contracts manufacturers charge *lower* wholesale prices if they are vertically separated (than if they are integrated). The reason is that a lower wholesale price tends to increase the quantity produced by the retailer and, since quantities are strategic substitutes, this induces the competing retailer to respond by reducing his own quantity (Vickers, 1983). *Ceteris paribus*, this strategic effect increases manufacturer's profit. Therefore, both manufacturers have an incentive to choose vertical separation, but they obtain lower profits by doing so, since the total quantity produced is higher (Fershtman and Judd, 1987, and Vickers, 1983).

Proposition 12 *With symmetry beliefs and quantity competition, wholesale prices and manufacturers' profits are higher with private than with public contracts.*

In the proof of Proposition 12, we show that the equilibrium wholesale price with public contracts is lower than manufacturers' marginal cost (hence, lower than zero in our model). Therefore, retailers produce larger quantities and charge lower prices with public contracts. This reduces manufacturers' profits compared to private contracts.

Our analysis suggests that, when retailers compete by choosing the quantity produced and have symmetry beliefs, manufacturers *always* prefer to maintain contracts private, rather than disclose them to competitors. Indeed, with quantity competition, the strategic effect of public contracts harms manufacturers, while private contracts have a positive belief effect.

8 Mixed Beliefs

In Section 4 we showed that, if retailers have symmetry rather than passive beliefs, manufacturers are not indifferent between vertical separation and vertical integration. Passive and symmetry beliefs, however, may be considered two opposite and extreme assumptions. It is worth asking how robust is the neutrality result of passive beliefs to a small change in retailers' beliefs. In order to answer this question we consider mixed beliefs, a more general class of beliefs that includes passive and symmetry beliefs as special cases (when $\alpha = 0$ and $\alpha = 1$, respectively). For $\alpha \in (0, 1)$, mixed beliefs capture the idea that, after being offered a contract different from the equilibrium one, a retailer is uncertain about the contract offered to the rival retailer and assigns a positive probability α, which can be arbitrarily small, to the other manufacturer offering the same contract, rather than the equilibrium one.

Consider a symmetric equilibrium with wholesale price w^*_α and retail price p^*_α. With mixed beliefs, if R_i is offered a wholesale price $w_i \neq w^*_\alpha$, he believes that, with probability α, R_j is offered the same wholesale price w_i while, with probability $(1-\alpha)$, R_j is offered the equilibrium wholesale price w^*_α and therefore chooses the equilibrium retail price p^*_α. Hence, R_i's objective function is

$$(p_i - w_i)\left[(1-\alpha) D^i\left(p_i, p^*_\alpha\right) + \alpha D^i\left(p_i, \hat{p}_j\right)\right], \tag{19}$$

where $\hat{p}_j$ is the retail price that R_i expects R_j to choose when R_j is offered w_i. In this case, R_j has exactly the same beliefs as R_i when he is offered w_i, and therefore has the same objective function (19).

Let

$$\hat{p}_\alpha\left(w_i\right) \in \arg\max_{p_i} \left(p_i - w_i\right)\left[(1-\alpha) D^i\left(p_i, p^*_\alpha\right) + \alpha D^i\left(p_i, \hat{p}_j\right)\right], \tag{20}$$

define the retail price chosen by R_i if he is offered the wholesale price w_i. By symmetry of the demand functions, $\hat{p}_\alpha\left(w_i\right)$ is also the price chosen by R_j when he is offered w_i and, by definition, $p^*_\alpha = \hat{p}_\alpha\left(w^*_\alpha\right)$. Therefore, the first-order condition for (20) is

$$\begin{gathered}(1-\alpha) D^i\left(\hat{p}_\alpha\left(w_i\right), p^*_\alpha\right) + \alpha D^i\left(\hat{p}_\alpha\left(w_i\right), \hat{p}_\alpha\left(w_i\right)\right) + \\ \left(\hat{p}_\alpha\left(w_i\right) - w_i\right)\left[(1-\alpha)\frac{\partial D^i\left(\hat{p}_\alpha\left(w_i\right), p^*_\alpha\right)}{\partial p_i} + \alpha\frac{\partial D^i\left(\hat{p}_\alpha\left(w_i\right), \hat{p}_\alpha\left(w_i\right)\right)}{\partial p_i}\right] \equiv 0.\end{gathered} \tag{21}$$

In a symmetric equilibrium in period 2, M_i offers the contract $C^*_\alpha = \left(w^*_\alpha, T^*_\alpha\right)$ such that

$$\begin{gathered}w^*_\alpha \in \arg\max_{w_i}\left\{D^i\left(\hat{p}_\alpha\left(w_i\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right) w_i + \right. \\ \left.\left[\alpha D^i\left(\hat{p}_\alpha(w_i), \hat{p}_\alpha(w_i)\right) + (1-\alpha) D^i\left(\hat{p}_\alpha\left(w_i\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right)\right]\left(\hat{p}_\alpha(w_i) - w_i\right)\right\},\end{gathered}$$

and T^*_α satisfies R_i's participation constraint as an equality. Therefore, by the Envelope Theorem — i.e., using condition (21) — the equilibrium wholesale price w^*_α solves

$$\begin{gathered}\frac{\partial D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right)}{\partial p_i}\frac{\partial \hat{p}_\alpha\left(w^*_\alpha\right)}{\partial w_i} w^*_\alpha + D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right) + \\ +\alpha\frac{\partial D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right)}{\partial p_j}\frac{\partial \hat{p}_\alpha\left(w^*_\alpha\right)}{\partial w_i}\left(\hat{p}_\alpha(w^*_\alpha) - w^*_\alpha\right) - D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right) \equiv 0,\end{gathered}$$

$$\Leftrightarrow \quad \frac{\partial D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right)}{\partial p_i} w^*_\alpha + \alpha\frac{\partial D^i\left(\hat{p}_\alpha\left(w^*_\alpha\right), \hat{p}_\alpha\left(w^*_\alpha\right)\right)}{\partial p_j}\left(\hat{p}_\alpha(w^*_\alpha) - w^*_\alpha\right) \equiv 0. \tag{22}$$

The second term of condition (22) represents the belief effect. Comparing this with condition (9), the belief effect is weaker with mixed than with symmetry beliefs. Moreover, by inspection: *(i)* $w^*_\alpha = 0$ for $\alpha = 0$, as with passive beliefs; *(ii)* $w^*_\alpha = w^*$ for $\alpha = 1$, as with symmetry beliefs; and *(iii)* $w^*_\alpha > 0$ for every $\alpha \neq 0$.

Proposition 13 *Assume that retailers have mixed beliefs and $\alpha \in (0,1)$. When both manufacturers are vertically separated, each manufacturer offers the wholesale price w^*_α defined by equation (22), where $0 < w^*_\alpha < w^*$, and each retailer chooses the retail price $\hat{p}_\alpha\left(w^*_\alpha\right)$, where*

$\hat{p}_\alpha(.)$ is defined by equation (21) and $p^e < \hat{p}_\alpha(w^*_\alpha) < \hat{p}(w^*)$. In period 1, vertical separation is a weakly dominant strategy for manufacturers, for every $\alpha \neq 0$.

With mixed beliefs, vertically separated manufacturers charge strictly positive wholesale prices and obtain higher profit than integrated ones, although their profit is not as high as with symmetry beliefs. Therefore, our qualitative results hold as long as, when a manufacturer offers a contract different from the equilibrium one, the retailer is not certain that the other manufacturer is still offering the equilibrium contract and assigns some positive probability to the other manufacturer offering the same contract. An arbitrarily small uncertainty is sufficient to generate the belief effect and allows manufacturers to obtain higher profit by selling through retailers. The neutrality result hinges on retailers' beliefs being *exactly* passive.

9 Conclusions

Manufacturers strictly prefer to sell through independent retailers who have symmetry (or at least not completely passive) beliefs, even if contracts are private and regardless of the nature of competition in the retail market. The reason is that, by charging high wholesale prices, manufacturers manipulate retailers' beliefs about competitors' strategies, thus reducing competition among retailers and increasing profit. Manufacturers may even prefer to keep contracts private, rather than disclose them to competitors, precisely because private contracts allow manufacturers to affect retailers' beliefs about the contracts offered to competitors.

With private contracts, vertical separation can also arise because of asymmetric information between manufacturers and retailers — see Caillaud, Jullien and Picard (1995) for the case of adverse selection, and Katz (1991) for the case of moral hazard. Our analysis, however, shows that vertical separation does not require asymmetric information. In future work we plan to analyze vertical separation with both asymmetric information and symmetry beliefs.

A Appendix. Proofs

Proof of Lemma 1. The proof follows from (3) and the fact that $\frac{\partial D^i(.)}{\partial p_i} < 0$ and $\frac{\partial p_i(.)}{\partial w_i} > 0$. ■

Proof of Proposition 2. When manufacturers are vertically separated, condition (4) follows from Lemma 1 and equation (1). Under assumptions A1-A4, this condition is necessary and sufficient for an optimum. Clearly, condition (4) also defines the retail price chosen by an integrated manufacturer. Hence, the equilibrium retail price and the manufacturers' profit do not depend on the organizational structure chosen by manufacturers in period 1. ■

Proof of Lemma 3. We show that $w_i = w_j = 0$ is not an equilibrium with symmetry beliefs. To see this, suppose that $w_j = 0$. Then M_i solves

$$\max_{w_i} D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(0\right)\right) w_i + D\left(\hat{p}(w_i), \hat{p}(w_i)\right)\left(\hat{p}(w_i) - w_i\right).$$

The derivative of the objective function evaluated at $w_i = 0$ is

$$\left.\begin{array}{c}\frac{\partial D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(0\right)\right)}{\partial p_i}\frac{\partial \hat{p}\left(w_i\right)}{\partial w_i} w_i + D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(0\right)\right) - D^i\left(\hat{p}(w_i), \hat{p}(w_i)\right) + \\ + \frac{\partial D^i\left(\hat{p}(w_i), \hat{p}(w_i)\right)}{\partial p_j}\frac{\partial \hat{p}\left(w_i\right)}{\partial w_i}\left(\hat{p}(w_i) - w_i\right)\end{array}\right|_{w_i=0} =$$

$$= \frac{\partial D^i\left(\hat{p}(0), \hat{p}(0)\right)}{\partial p_j}\frac{\partial \hat{p}\left(0\right)}{\partial w_i}\hat{p}(0) > 0.$$

Therefore, when M_j charges a wholesale price equal to 0, it is not a best reply for M_i to choose $w_i = 0$. ■

Proof of Lemma 4. If M_i expects her rival to offer the contract $C^* = (w^*, T^*)$, she expects R_j to choose price $\hat{p}\left(w^*\right)$ (since R_j believes that R_i pays his same wholesale price w^*). Hence, M_i's objective function is $D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(w^*\right)\right) w_i + T_i$. Moreover, R_i's participation constraint is

$$D^i\left(\hat{p}\left(w_i\right), \hat{p}\left(w_i\right)\right)\left(\hat{p}\left(w_i\right) - w_i\right) - T_i \geq 0,$$

because he believes that R_j pays the wholesale price w_i and sells at price $\hat{p}\left(w_i\right)$. ■

Proof of Proposition 5. From equation (5), it follows that, given wholesale prices w_1 and w_2, retail prices are $p_1 = \hat{p}\left(w_1\right)$ and $p_2 = \hat{p}\left(w_2\right)$. Using the implicit function theorem,

$$\frac{d\hat{p}\left(w_i\right)}{dw_i} = \frac{\frac{\partial D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_i}}{2\frac{\partial D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_i} + \frac{\partial D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_j} + \left(\hat{p}\left(w_i\right) - w_i\right)\left(\frac{\partial^2 D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_i^2} + \frac{\partial^2 D^i(\hat{p}(w_i), \hat{p}(w_i))}{\partial p_i \partial p_j}\right)}.$$

This is strictly positive by Assumption A5.

Consider a symmetric equilibrium with wholesale contract $C^* = (w^*, T^*)$ and retail price $\hat{p}(w^*)$. By equation (5), which defines the function $\hat{p}(.)$,

$$\frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i} w^* = \frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i} \hat{p}(w^*) + D^i(\hat{p}(w^*), \hat{p}(w^*)).$$

Substituting this in equation (9), that defines w^*, we have

$$\frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i} \hat{p}(w^*) + D^i(\hat{p}(w^*), \hat{p}(w^*)) + \frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_j} (\hat{p}(w^*) - w^*) = 0 \quad (23)$$

$$\Leftrightarrow \quad \frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i} \frac{\hat{p}(w^*)}{D^i(\hat{p}(w^*), \hat{p}(w^*))} + 1 +$$

$$+ \frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_j} \frac{\hat{p}(w^*)}{D^i(\hat{p}(w^*), \hat{p}(w^*))} \frac{\hat{p}(w^*) - w^*}{\hat{p}(w^*)} = 0.$$

$$\Leftrightarrow \quad \frac{\hat{p}(w^*) - w^*}{\hat{p}(w^*)} = -\frac{1 + \frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_i} \frac{\hat{p}(w^*)}{D^i(\hat{p}(w^*), \hat{p}(w^*))}}{\frac{\partial D^i(\hat{p}(w^*), \hat{p}(w^*))}{\partial p_j} \frac{\hat{p}(w^*)}{D^i(\hat{p}(w^*), \hat{p}(w^*))}} \equiv \frac{\varepsilon_i^i(\hat{p}(w^*)) - 1}{\varepsilon_j^i(\hat{p}(w^*))}.$$

To prove the existence of an equilibrium satisfying this condition, notice that: (*i*) the first-order condition (9) is continuous in w since $\hat{p}(.)$ is continuous and $D^i(.)$ is twice continuously differentiable by assumption; (*ii*) the derivative of the manufacturer's profit is strictly positive at $w_i = w_j = 0$ (see equation (9)); (*iii*) by equation (23), the derivative of the manufacturer's profit tends to $-\infty$ as w_i and w_j tend to $+\infty$ because

$$\lim_{w \to \infty} D^i(\hat{p}(w), \hat{p}(w)) = 0$$

by Assumption A1 and since $\hat{p}(w)$ is increasing in w, and

$$\lim_{w \to \infty} \left(\frac{\partial D^i(\hat{p}(w), \hat{p}(w))}{\partial p_j} + \frac{\partial D^i(\hat{p}(w), \hat{p}(w))}{\partial p_i} \right) \hat{p}(w) < 0$$

by Assumption A2.

Finally, M_i extracts the whole retailer's surplus by charging a franchise fee equal to

$$T^* = D^i(\hat{p}(w^*), \hat{p}(w^*)) (\hat{p}(w^*) - w^*),$$

and obtains a profit equal to $D^i(\hat{p}(w^*), \hat{p}(w^*)) \hat{p}(w^*)$. ■

Proof of Lemma 6. Given the equilibrium price p_i^e chosen by R_i, the price chosen by the integrated manufacturer M_j satisfies

$$\frac{\partial D^j(p_j^e, p_i^e)}{\partial p_i} p_j^e + D^j(p_j^e, p_i^e) = 0. \quad (24)$$

Since M_i chooses $w_i = 0$, as implied by equation (11), R_i also chooses a retail price satisfying condition (24). Therefore, under Assumption A5, there is a unique equilibrium in which both the integrated manufacturer and R_i choose the same retail price p^e. (This is the price that would be chosen in the unique equilibrium with two integrated firms.) ■

Proof of Lemma 7. From Lemma 6, recall that p^e is defined by

$$\frac{\partial D^i\left(p^e,p^e\right)}{\partial p_i}p^e + D^i\left(p^e,p^e\right) = 0. \tag{25}$$

From Section 4.2, recall that the first order condition for the choice of $p^* \equiv \hat{p}\left(w^*\right)$ is

$$\frac{\partial D^i\left(p^*,p^*\right)}{\partial p_i}\left(p^* - w^*\right) + D^i\left(p^*,p^*\right) = 0, \tag{26}$$

where w^* is defined by

$$\frac{\partial D^i\left(p^*,p^*\right)}{\partial p_i}w^* + \frac{\partial D^i\left(p^*,p^*\right)}{\partial p_j}\left(p^* - w^*\right) = 0. \tag{27}$$

Hence, substituting condition (26) in (27), p^* must satisfy

$$\frac{\partial D^i\left(p^*,p^*\right)}{\partial p_i}p^* + D^i\left(p^*,p^*\right) = -\frac{\partial D^i\left(p^*,p^*\right)}{\partial p_j}\left(p^* - w^*\right). \tag{28}$$

Consider the function $\phi\left(p\right) \equiv \frac{\partial D^i(p,p)}{\partial p_i}p + D^i\left(p,p\right)$, which is decreasing by assumption A5. By condition (25), p^e is such that $\phi\left(p^e\right) = 0$. By condition (28), and since $\frac{\partial D^i(.)}{\partial p_j} > 0$ and $p^* > w^*$, p^* is such that $\phi\left(p^*\right) < 0$. Therefore, it must be that $p^* > p^e$. ■

Proof of Proposition 8. Consider the function $\Phi\left(p\right) \equiv \pi_i\left(p,p\right) = D^i\left(p,p\right) \cdot p$. The profit obtained by an integrated manufacturer competing against another integrated manufacturer is equal to $\Phi\left(p^e\right)$. When one manufacturer is vertically separated while the other is not, the profit obtained by each manufacturer is also equal to $\Phi\left(p^e\right)$ by Lemma 6. Therefore, there exists an equilibrium in which both manufacturers choose vertical integration (since given that a manufacturer chooses integration, the other is indifferent between integration and separation).

The profit obtained by a vertically separated manufacturer when competing against another vertically separated manufacturer is equal to $\Phi\left(p^*\right)$. In order to show that there is also an equilibrium in which both manufactures choose vertical separation, we need to show that $\Phi\left(p^*\right) \geq \Phi\left(p^e\right)$.

By Assumption A5, $\Phi\left(p\right)$ is strictly concave and has a unique maximum. Let

$$p^M \equiv \arg\max_p \Phi\left(p\right),$$

so that

$$\Phi'\left(p^M\right) \equiv \frac{\partial D^i\left(p^M,p^M\right)}{\partial p_i}p^M + \frac{\partial D^i\left(p^M,p^M\right)}{\partial p_j}p^M + D^i\left(p^M,p^M\right) = 0.$$

Clearly, $\Phi'\left(p\right) > 0$ if and only if $p < p^M$.

By condition (28), p^* is such that

$$\Phi'(p^*) = \frac{\partial D^i(p^*, p^*)}{\partial p_j} w^*.$$

Since $\frac{\partial D^i(.)}{\partial p_j} > 0$ by assumption, $\Phi'(p^*) > 0$ and, therefore, $p^M > p^*$. Moreover, by Lemma 7, $p^* > p^e$. Summing up, $p^M > p^* > p^e$ and, therefore, $\Phi(p^M) > \Phi(p^*) > \Phi(p^e)$. This also proves that manufacturers obtain higher profits in the equilibrium where they both choose vertical separation than in the equilibrium where they both choose integration. By inspection, the former equilibrium is also risk dominant. ■

Proof of Proposition 9. From equation (12) and the implicit function theorem,

$$\beta(p^{**}) = \frac{\partial \varphi(p^{**}|w^{**})}{\partial p_i} = -\frac{\frac{\partial^2 D^i(p^{**},p^{**})}{\partial p_i \partial p_j}(p^{**} - w^{**}) + \frac{\partial D^i(p^{**},p^{**})}{\partial p_j}}{\frac{\partial^2 D^i(p^{**},p^{**})}{\partial^2 p_i}(p^{**} - w^{**}) + 2\frac{\partial D^i(p^{**},p^{**})}{\partial p_i}}.$$

By Assumption A5, $0 \leq \beta(p^{**}) \leq 1$. Moreover, it is straightforward to show that

$$\frac{\frac{\partial p_j(w^{**},w^{**})}{\partial w_i}}{\frac{\partial p_i(w^{**},w^{**})}{\partial w_i}} = \beta(p^{**}).$$

Therefore, dividing equation (13) by $\frac{\partial p_i(w^{**},w^{**})}{\partial w_i}$ yields

$$\frac{\partial D^i(p^{**}, p^{**})}{\partial p_i} p^{**} + D^i(p^{**}, p^{**}) = -\frac{\partial D^i(p^{**}, p^{**})}{\partial p_j} \beta(p^{**}) p^{**}. \tag{29}$$

Consider now private contracts. From equation (28), the derivative of M_i's objective function with symmetry beliefs, evaluated at $p_i = p_j = p^{**}$ and $w_i = w_j = w^{**}$, is

$$\frac{\partial D^i(p^{**}, p^{**})}{\partial p_i} p^{**} + D^i(p^{**}, p^{**}) + \frac{\partial D^i(p^{**}, p^{**})}{\partial p_j}(p^{**} - w^{**}). \tag{30}$$

Substituting equation (29) in (30), we have

$$-\frac{\partial D^i(p^{**}, p^{**})}{\partial p_j} \beta(p^{**}) p^{**} + \frac{\partial D^i(p^{**}, p^{**})}{\partial p_j}(p^{**} - w^{**}) =$$

$$= \frac{\partial D^i(p^{**}, p^{**})}{\partial p_j}((1 - \beta(p^{**})) p^{**} - w^{**}). \tag{31}$$

Uniqueness of the equilibrium with private contracts implies that $p^* > p$ if and only if the derivative of the manufacturer's objective function evaluated at p is greater than zero. (By the assumptions on $D^i(.)$, this derivative is continuous.) Therefore, since $\frac{\partial D^i(.)}{\partial p_j} > 0$, $p^* > p^{**}$ if and only if equation (31) is positive — i.e.,

$$((1-\beta(p^{**}))\,p^{**}-w^{**})>0 \quad\Leftrightarrow\quad \frac{p^{**}-w^{**}}{p^{**}}>\beta(p^{**}).$$

Clearly, $p^* < p^{**}$ if and only if

$$\frac{p^{**}-w^{**}}{p^{**}}<\beta(p^{**}).$$

Finally, it is immediate to show that the same condition also ranks wholesale prices and manufacturers' profits with private and public contracts.

Notice that manufacturers' profits are single-peaked with private contracts if

$$\frac{d}{dp}\left[\frac{\partial D^i(p,p)}{\partial p_i}p+D^i(p,p)+\frac{\partial D^i(p,p)}{\partial p_j}(p-w)\right]<0, \qquad \forall w\le p.$$

See Rey and Stiglitz (1995) for conditions that guarantee that manufacturers' profits are single-peaked with public contracts. ■

Proof of Proposition 10. With symmetry beliefs, if M_i offers the wholesale price w_i, R_i conjectures that: (i) M_j offered w_i to R_j, and (ii) R_j believes that M_i offered w_i to R_i. Hence, since for every w_i R_i expects R_j to choose his same quantity, R_i chooses $\hat{q}(w_i)$ such that

$$\hat{q}(w_i)\in\arg\max_{q_i}(P(q_i+\hat{q}(w_i))-w_i)\,q_i.$$

This immediately implies condition (16).

Consider a symmetric equilibrium in which both manufacturers charge a franchise fee T^* defined by equation (18). Then each manufacturer chooses the wholesale price to solve

$$\max_{w_i}\{\hat{q}(w_i)\,w_i+(P(2\hat{q}(w_i))-w_i)\hat{q}(w_i)\}.$$

Using the envelope theorem, the first-order condition of this problem is

$$w^*+P'(2\hat{q}(w^*))\hat{q}(w^*)=0.$$

This immediately implies equation (17). Moreover, $w^*>0$. Equation (18) hold because, to extract the whole retailers' surplus, manufacturers charge a franchise fee such that

$$(P(2\hat{q}(w^*))-w^*)\hat{q}(w^*)-T^*=0.$$

Given contract $C^*=(w^*,T^*)$, each retailer produces the quantity $\hat{q}(w^*)$ defined by (16). Comparing this with equation (15) it follows that $\hat{q}(w^*)<q^e$. Finally, a manufacturer's profit is equal to $P(2\hat{q}(w^*))\,\hat{q}(w^*)$. ■

Proof of Proposition 11. The proof follows the same logic of the proof of Proposition 8. Indeed, it is straightforward to show that: (i) when both manufacturers choose integration, their marginal cost is zero by assumption; (ii) when one manufacturer chooses integration while the other chooses separation, since the integrated manufacturer's marginal cost is zero, the separated manufacturer charges a wholesale price equal to zero. In both cases, each retailer produces the quantity q^e defined by condition (15). Hence, there is an equilibrium where both manufacturers choose integration.

To prove that there is also an equilibrium where both manufacturers choose separation, and that this equilibrium Pareto dominates (and also risk dominates) the equilibrium where both manufacturers choose integration, we show that manufacturers' profits with separation — i.e., $P\left(2\hat{q}\left(w^*\right)\right)\hat{q}\left(w^*\right)$ — are larger than manufacturers' profits with integration — i.e., $P\left(2q^e\right)q^e$. Let $\theta\left(q\right)=P\left(2q\right)q$. The function $\theta\left(q\right)$ is strictly concave by the assumption on $P\left(.\right)$, and has a unique maximum at q^* such that

$$2P'\left(2q^*\right)q^*+P\left(2q^*\right)=0.$$

By equation (16) and (17) it follows that

$$2P'\left(2\hat{q}\left(w^*\right)\right)\hat{q}\left(w^*\right)+P\left(2\hat{q}\left(w^*\right)\right)=0.$$

Hence, $\hat{q}\left(w^*\right)$ maximizes $\theta\left(q\right)$, and $P\left(2\hat{q}\left(w^*\right)\right)\hat{q}\left(w^*\right)>P\left(2q\right)q$ for every $q\neq\hat{q}\left(w^*\right)$. Notice that $2\hat{q}\left(w^*\right)$ is the quantity produced by a monopolist. ■

Proof of Proposition 12. First consider public contracts. Given contracts $C_i=\left(w_i,T_i\right)$ and $C_j=\left(w_j,T_i\right)$, equilibrium quantities are determined by the first-order conditions

$$P'\left(q_i+q_j\right)q_i+P\left(q_i+q_j\right)-w_i=0,\qquad i=1,2. \tag{32}$$

These conditions define the quantities $q_1\left(w_1,w_2\right)$ and $q_2\left(w_2,w_1\right)$ produced by the two retailers, as a function of the wholesale prices.

Hence, M_i solves

$$\max_{w_i}\left\{q_i\left(w_i,w_j\right)w_i+\left(P(q_i\left(w_i,w_j\right)+q_j\left(w_j,w_i\right))-w_i\right)q_i\left(w_i,w_j\right)\right\}.$$

Using the envelope theorem, the first-order condition is

$$\frac{\partial q_i\left(w_i,w_j\right)}{\partial w_i}w_i+P'(q_i\left(w_i,w_j\right)+q_j\left(w_j,w_i\right))\frac{\partial q_j\left(w_j,w_i\right)}{\partial w_i}q_i\left(w_i,w_j\right)=0.$$

Therefore, a symmetric equilibrium with wholesale price w^{**} and quantity $q_i(w^{**},w^{**})=q^{**}$ is characterized by

$$w^{**}=-P'(2q^{**})\frac{\frac{\partial q_j(w^{**},w^{**})}{\partial w_i}}{\frac{\partial q_i(w^{**},w^{**})}{\partial w_i}}q^{**}.$$

It is immediate to verify that $\frac{\partial q_i(.)}{\partial w_i}<0$ and that $\frac{\partial q_j(.)}{\partial w_i}>0$, so that $w^{**}<0$.

As shown by Vickers (1985) and Fershtman and Judd (1987), regardless of the organizational structure chosen by the competitor, with public contracts each manufacturer obtains a higher profit with vertical separation than with integration. Hence, manufacturers choose vertical separation with public contracts.

Now consider private contracts. Since $w^* > 0$, comparing equations (15) and (32), it follows that the quantity produced by retailers is lower with private contracts than with public contracts — i.e., $\hat{q}(w^*) < q^{**}$. Finally, manufacturers' profits with private contracts — i.e., $P(2\hat{q}(w^*))\,\hat{q}(w^*)$ — are higher than with private contracts — i.e., $P(2q^{**})\,q^{**}$ — since the function $\theta(q) = P(2q)\,q$ has a unique maximum at $\hat{q}(w^*)$ (see the proof of Proposition 11). ■

Proof of Proposition 13. By inspection of the first order condition (22), $0 < w^*_\alpha < w^*$ for $\alpha \in (0,1)$. To analyze how the equilibrium retail price changes as w_i changes, apply the implicit function theorem to condition (21) to obtain

$$\frac{d\hat{p}_\alpha(w_i)}{dw_i} = \frac{(1-\alpha)\frac{\partial D^i(\hat{p}_\alpha(w_i),p^*_\alpha)}{\partial p_i} + \alpha\frac{\partial D^i(\hat{p}_\alpha(w_i),\hat{p}_\alpha(w_i))}{\partial p_i}}{\Delta(\alpha, w_i, p^*_\alpha)},$$

where

$$\Delta(\alpha, w_i, p^*_\alpha) = 2(1-\alpha)\frac{\partial D^i(\hat{p}_\alpha(w_i),p^*_\alpha)}{\partial p_i} + 2\alpha\frac{\partial D^i(\hat{p}_\alpha(w_i),\hat{p}_\alpha(w_i))}{\partial p_i} + \alpha\frac{\partial D^i(\hat{p}_\alpha(w_i),\hat{p}_\alpha(w_i))}{\partial p_j}$$
$$+(\hat{p}_\alpha(w_i) - w_i)\left((1-\alpha)\frac{\partial D^i(\hat{p}_\alpha(w_i),p^*_\alpha)}{\partial p_i^2} + \alpha\left(\frac{\partial^2 D^i(\hat{p}_\alpha(w_i),\hat{p}_\alpha(w_i))}{\partial p_i^2} + \frac{\partial^2 D^i(\hat{p}_\alpha(w_i),\hat{p}_\alpha(w_i))}{\partial p_i \partial p_j}\right)\right).$$

Hence, under assumptions A1, A2 and A5, $\frac{d\hat{p}_\alpha(w_i)}{dw_i} > 0$.

When $w_i = w^*_\alpha = 0$, condition (21) is identical to condition (4) and, hence, $\hat{p}_\alpha(0) = p^e$. In equilibrium, when $w_i = w^*_\alpha$, condition (21) is also identical to condition (5) and, hence, $\hat{p}_\alpha(w_i) = \hat{p}(w_i)$. Therefore, the retail price with vertical separation and mixed beliefs is higher than the retail price with vertical integration — i.e., $\hat{p}_\alpha(w^*_\alpha) > p^e$ for all $\alpha > 0$ — and lower than the retail price with vertical separation and symmetry beliefs — i.e., $\hat{p}_\alpha(w^*_\alpha) < \hat{p}(w^*)$ for all $\alpha < 1$.

The proof that delegation is a weakly dominant strategy for manufacturers, for every $\alpha > 0$, follows the proof of Proposition 8: since equilibrium retail prices when both manufacturers choose vertical separation are higher than equilibrium retail prices when one or more manufacturers choose vertical integration, and equilibrium retail prices when only one manufacturer chooses vertical separation are equal to equilibrium retail prices when both manufacturers choose vertical integration, a manufacturer obtains a (weakly) higher profit if she chooses vertical separation. As with symmetry beliefs, for every $\alpha > 0$, there are two equilibria: one where both manufacturers choose vertical separation and one where they both choose vertical integration. But the former equilibrium Pareto dominates, and risk dominates, the latter equilibrium. ■

B Hotelling Model with Uncertainty about Manufacturers' Cost

In this appendix, we show that symmetry beliefs naturally arise in the separating equilibrium of a Hotelling model of differentiated products in which manufacturers are privately informed about their common cost of production. Moreover, as in our model with complete information about manufacturers' cost, manufacturers choose vertical separation in equilibrium.

There is a unit mass of consumers uniformly distributed over $[0, 1]$. Two vertical structures produce a homogeneous good and are located at the extremes of the interval; specifically, retailer R_1 is located at 0 and retailer R_2 is located at 1. Each consumer has a valuation v for a single unit of the good. For simplicity, we assume $v \to +\infty$, so that each consumers always buys one unit, regardless of the price. The transportation cost paid by a consumer located at $x \in [0, 1]$ who buys from R_1 (resp. R_2) is tx^2 (resp. $t(1-x)^2$).

Manufacturers have a constant marginal cost of production c distributed on $(-\infty, +\infty)$ — i.e., the cost has "full support."[21] The marginal cost is private information to manufacturers. Manufacturers offer a two-part tariff contract to retailers: M_i charges R_i a wholesale price $w_i \in \mathbb{R}_+$ and a fixed fee $T_i \in \mathbb{R}$. R_i chooses the retail price $p_i \in \mathbb{R}_+$, $i = 1, 2$.

Given prices p_1 and p_2, a consumer located at x buys from R_1 if and only if

$$p_1 + tx^2 < p_2 + t(1-x)^2.$$

Therefore, in an interior solution, the demand for the good sold by R_i is

$$\frac{p_j - p_i + t}{2t}, \quad i, j = 1, 2, \quad i \neq j.$$

Consider a symmetric separating equilibrium in which manufacturers offers a wholesale price defined by the function $w^*(c)$. Define the set of wholesale prices that a manufacturer can offer in equilibrium by

$$\Omega = \{w : \exists c \in (-\infty, +\infty) \text{ such that } w^*(c) = w\}.$$

Because retailers' beliefs must be consistent with manufacturers' strategies in equilibrium, when R_i is offered a wholesale price $w_i \in \Omega$, he expects that R_j is offered the same wholesale price w_i.

Therefore, as in a standard Hotelling model with two firms having marginal cost w_i, R_i chooses the retail price $\widehat{p}_i = w_i + t$ in equilibrium (and retailers' markup does not depend on wholesale prices). Hence, retailers' expected profit is $\frac{t}{2}$ and M_i charges a franchisee fee $T^* = \frac{t}{2}$.

Moreover, M_i chooses the wholesale price w_i to solve

$$\max_{w_i} \left\{ T^* + (w_i - c)\left[\frac{\widehat{p}_j - \widehat{p}_i + t}{2t}\right] \right\} \quad \Leftrightarrow \quad \max_{w_i} (w_i - c)\left[\frac{w_j - w_i + t}{2t}\right].$$

This is the standard problem of two Hotelling firms with marginal cost c that choose prices w_i and w_j. The solution to this problem is $w^*(c) = c + t$: the equilibrium wholesale price is higher than marginal cost. It follows that $\Omega = (-\infty, +\infty)$ — i.e., every wholesale price can be offered in equilibrium by manufacturers. Therefore, retailers have symmetry beliefs in the symmetric separating equilibrium of the model. Finally, the equilibrium retail price is $p^*(c) = c + 2t$ and manufacturers' profit is t.

[21] A negative marginal cost can be interpreted as a subsidy to the manufacturer by the government.

We now consider the choice of organizational structure by manufacturers. First, when the two vertical structures are integrated, it is straightforward to show that the equilibrium retail price is $c+t$ and manufacturers' profit is $\frac{t}{2}$. Hence, both the retail price and manufacturers' profit are lower than with two separated vertical structures.

Second, suppose there are two asymmetric vertical structures: M_i sells directly to final consumers, while M_j sells through R_j. Consider a separating equilibrium in which M_j offers a wholesale price $w(c)$. Define the set of wholesale prices that M_j can offer in equilibrium by

$$\Omega^a = \{w : \exists c \in (-\infty, +\infty) \text{ such that } w(c) = w\}.$$

Because R_j's beliefs must be consistent with M_j's strategy in equilibrium, when R_j is offered a wholesale price $w_j \in \Omega$, he believes M_i's marginal cost to be $w^{-1}(w_j)$.

Let $p_i(c)$ be the retail prices charged by M_i in equilibrium, and $p_j(w_j)$ be the retail prices charged by R_j in equilibrium, given the wholesale price w_j. When he is offered the wholesale price $w_j \in \Omega^a$, R_j chooses the retail price to solve

$$\max_{p_j}(p_j - w_j)\frac{p_i\left(w^{-1}(w_j)\right) - p_j + t}{2t},$$

and expects M_i to choose the retail price to solve

$$\max_{p_i}(p_i - w^{-1}(w_j))\frac{p_j(w_j) - p_i + t}{2t}.$$

The solution of these problems yields

$$p_j(w_j) = t + \frac{2}{3}w_j + \frac{1}{3}w^{-1}(w_i), \tag{33}$$

and

$$p_i\left(w^{-1}(w_j)\right) = t + \frac{2}{3}w^{-1}(w_j) + \frac{1}{3}w_j. \tag{34}$$

Notice that $p_i\left(w^{-1}(w_j)\right)$ is the retail price that R_j expects M_i to charge, when R_j is offered the wholesale price w_j.

Therefore, M_j solves

$$\max_{w_j}\left\{T_j(w_j) + (w_j - c)\,\frac{p_i(c) - p_j(w_j) + t}{2t}\right\},$$

where, in order to satisfy R_j's participation constraint,

$$T_j(w_j) = (p_j(w_j) - w_j)\,\frac{p_i\left(w^{-1}(w_j)\right) - p_j(w_j) + t}{2t}.$$

The (necessary and sufficient) first-order condition of this problem is

$$\frac{p_i(c) - p_j(w_j) + t}{2t} - \frac{\partial p_j(w_j)}{\partial w_j}\frac{(w_j - c)}{2t} - \frac{p_i(c) - p_j(w_j) + t}{2t} + \frac{(p_j(w_j) - w_j)}{2t}\frac{\partial p_i\left(w^{-1}(w_j)\right)}{\partial w_j} = 0.$$

Evaluating this condition at the equilibrium wholesale price $w_j = w(c)$ — i.e., where $p_i\left(w^{-1}(w(c))\right) = p_i(c)$ and $p_j(w_j) = p_j(w(c))$ — and using equations (33) and (34), we have

$$-\frac{2}{3}(w(c) - c) + \frac{1}{3}\left(t - \frac{1}{3}w(c) + \frac{1}{3}c\right) \equiv 0 \quad \Leftrightarrow \quad w(c) = c + \frac{3}{7}t.$$

Therefore, equilibrium retail prices are $p_i(c) = c + \frac{8}{7}t$ and $p_j(w(c)) = c + \frac{9}{7}t$, and manufacturers' profits are $\pi_i = \frac{32}{49}t$ and $\pi_j = \frac{27}{49}t$. Hence, with asymmetric vertical structures, retail prices and manufacturers' profits are lower than with separated vertical structures.

Summing up, manufacturers' profit when they choose integration (I) or separation (S) are

		M_2: I	M_2: S
M_1	I	$\frac{1}{2}t$, $\frac{1}{2}t$	$\frac{32}{49}t$, $\frac{27}{49}t$
	S	$\frac{27}{49}t$, $\frac{32}{49}t$	t, t

By inspection, separation is a strictly dominant strategy for each manufacturer. As in our main model, vertical separation allow manufacturers to sell at higher retail price and obtain higher profit.

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