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Abstract
The article examines the impact of transaction costs on the trading strategy of informed institutional investors in a sequential trading market where traders can choose to transact a large or a small amounts of stock. The analysis shows how the trading strategy of informed investors and the price impact of their trades depends on market conditions. The main prediction of the model is that institutional buyers are, on average, more aggressive than institutional sellers in bearish markets and less aggressive in bullish markets. Hence, the price impact is higher for purchases when market conditions are bearish, while it is higher for sales when market conditions are bullish. However, this asymmetry vanishes during strongly bearish or bullish phases, when information-based orders stop because the informational advantage of institutional investors becomes too small with respect to the transaction costs.

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1 Introduction

Over the past few decades, the importance of block trading in common stocks has increased significantly.[1] Extant empirical literature shows that block trades have a large and persistent impact on stock prices. Theoretical rationale for this behaviour are liquidity[2] and information[3] effects resulting from the execution of large orders. However, an important body of empirical work has documented an interesting "puzzle" which is difficult to explain using extant theoretical models. Buyer and seller initiated block trades seem to influence stock prices differently. Chan and Lakonishok [4] analyze the price behavior around institutional transactions for securities listed on the New York and American Stock Exchanges during the period 1986 to 1988. They find that institutional purchases have a larger permanent price impact than sales. Gemmill [6] documents a similar asymmetric price impact for block trades executed on the London Stock Exchange between 1987 and 1992. Again, Aitken and Frino [1] examine the determinants of transaction costs associated with institutional trades on the Australian Stock Exchange between 1991 and 1993. Their results also confirm that the permanent price effect is larger for buyer than seller initiated block trades. The more widely accepted explanation is based on the common belief that purchases are more informative than sales. A rationale behind this explanation is that institutional traders can choose among many potential assets to buy but usually they sell only those assets that are already in their portfolio because of restrictions on short sales. Therefore, sales are not necessarily driven by negative information.[4]

This paper aims to contribute to this research area by developing a theoretical model which explains the price impact asymmetry of block trades in financial markets with asymmetric information, sequential trading, competitive price mechanisms and transaction costs increasing in trade size.[5]

For this purpose, we develop a sequential trading model where traders are allowed to transact different trade sizes. The ability to transact orders for large or small quantities introduces a strategic element in the trading game. Traders first observe the price schedule and then choose their optimal trading strategy. If risk neutral informed traders wish to trade, they prefer to trade a larger amount at any given price. Consequently, the market maker sets a greater spread for larger trades in every equilibrium with information based trading.

[1] In June 2008, the New York Stock Exchange executed over 500 million daily shares of block volume on average, representing 17 percent of total NYSE volume (*www.nyse.com/pdfs/5672–Block–Trading.pdf*); in 2004, over 50 percent of total FTSE 100 trading volume was executed in parcels of more than 10,000 shares (Gregoriou [8]).

[2] See Ho and Stoll [9], and Stoll [14].

[3] See Easley and O'Hara [5], and Seppi [13].

[4] This explanation appears in Keim and Madhavan [10]. Saar [12] develops a theoretical model to show how the trading strategy of institutional investors produces a difference between the information content of buy and sell orders that generates the asymmetric price impact.

[5] Our model differs from Saar [12] in that we analyze the trading strategy of risk neutral informed traders in a market for only one asset, while in Saar informed traders are assumed to hold diversified portfolios consisting of many assets, are prevented from concentrating their holding in only a few assets and are adverse to short selling.

The crucial assumption is that traders bear an exogenous cost in transacting which is increasing in trade size. This cost gives informed traders an incentive to transact the small, rather than the large quantity, and can lead to a no-trading informational cascade, which is a situation where all informed traders choose to refrain from trading, regardless of their private signal. [6]

We show that, if transaction costs are sufficiently low and the market width (the distance between the large and the small trade size) is sufficiently large, three types of equilibria can arise depending on underlying market conditions. If transaction costs exceed the informational profit of traders observing private signals about the true asset value, all informed traders prefer to refrain from trading and orders do not convey any information about fundamental value. If the informational advantage of traders is large enough with respect to the average transaction cost, either a separating or a pooling equilibrium can arise, depending on the market width. In a separating equilibrium, informed traders place only large orders. Hence, small orders are uninformative and the spread for small quantities only reflects exogenous transaction costs. This outcome occurs if the difference between the size of large and small orders is sufficiently large. Finally, if the market width is not large enough a pooling equilibrium prevails. In this equilibrium, informed traders trade both large and small amounts. The large trade spread reduces with respect to the separating equilibrium, whereas the spread for small quantities increases because of information costs.

The information content of large orders reaches a maximum in the separating equilibrium. It reduces when a pooling equilibrium prevails in the market and tends to zero in a no-trading equilibrium, as an informational cascade develops. Moreover, trading volume is shown to gradually decrease before a cascade occurs, and reaches a minimum as the cascade develops.

Our analysis also shows that the informational advantage of traders is high when there is extreme uncertainty about the asset's fundamental value and decreases as the public belief converges on the low or on the high asset value. Since the asset's price is positively related to public belief, a no trading equilibrium can prevail both in a strongly bearish and in a strongly bullish market, where the informational profit of traders is lower than transaction costs, whilst a separating or a pooling equilibrium can prevail in a flat market or when market conditions are weakly bearish or bullish.

Finally, in our model, contrarian signals are more valuable than confirming signals. Indeed, traders getting an adverse signal on the true asset value have a larger informational advantage than traders getting a favorable signal if the market is in a bullish phase and lower informational advantage if the market is in a bearish phase. We show that this yields several interesting empirical

[6] More generally, an informational cascade occurs when it is optimal for individuals, after observing the actions of previous agents, to ignore their own information and mimic the decisions of their predecessors. Informational cascades first appeared in Banerjee [2] and in Bikhchandani, Hirshleifer and Welch [3]. Banerjee [2] and Bikhchandani, Hirshleifer and Welch [3] independently show in different settings that informational cascades (or "herding", in Banerjee's terminology) eventually occurs with certainty. The result that trading costs lead to informational cascades in financial markets is due to Lee [11].

implications. First, large information based buy orders are more numerous than large information based sell orders in a bearish market and less numerous in a bullish market. Second, buys have a greater price impact than sells during a bearish phase and lower price impact during a bullish phase. Finally, in a neutral market the model predicts a U-shaped relationship between the price impact asymmetry - defined as the the difference between the absolute value of the price impact of a large buy and of a large sell order - and the asset price. In a strongly bearish market the price impact asymmetry is increasing in the asset price. In a strongly bullish market it is decreasing in the asset price.

The remainder of this paper is structured as follows. In section 2 we describe the model. In section 3 we define and derive equilibrium conditions and discuss the impact of transaction costs on the learning process. In Section 4 we examine the asymmetric price impact of block trades in bearish and bullish markets. Section 5 concludes. All proofs of propositions are in the appendix.

2 The Economy

We consider a sequential trading model analogous to Glosten and Milgrom [7], and modified to account for transaction costs and different trade sizes.[7]

The market is for a risky asset which is exchanged among a sequence of risk neutral traders and competitive market makers who are responsible for quoting prices. The value $\widetilde{V}$ of the asset can be low ($\widetilde{V} = \underline{V}$) or high ($\widetilde{V} = \overline{V}$). The ex-ante probability of $\widetilde{V} = \overline{V}$ is $\pi_0 \in (0,\ 1)$. To simplify the analysis we assume, without loss of generality, $\underline{V} = 0$ and $\overline{V} = 1$.

Trades occur sequentially, with one trader allowed to transact at any point in time. The trader whose turn it is to transact may either buy a small or a large quantity, or sell a small or a large quantity, or refrain from trading.[8]

We denote order size using Q_S and Q_L for small and large orders, respectively (hence $Q_S < Q_L$). $A \equiv \{SQ_L,\ SQ_S,\ BQ_S,\ BQ_L,\ NT\}$ is the traders' action space, with SQ_i and BQ_i indicating, respectively, a sell and a buy order for quantity Q_i, with $i = S, L$, and NT indicating an order of zero volume.

There are two types of traders: liquidity traders (fraction $1-\mu$) and informed traders (fraction μ). To simplify the analysis, we assume that liquidity traders choose to submit large or small sell and buy orders and to refrain from trading with equal probability $1/5$. We denote the probability that an uninformed trader submits a given order as $\gamma \equiv \frac{1-\mu}{5}$.[9]

Informed traders privately observe a signal θ correlated with the asset value. The set of private signals is $\Theta = \{\underline{\theta}, \overline{\theta}\}$; the signal $\underline{\theta}$ indicates $\underline{V}$ and the signal

[7] The approach taken involves a sequential trade model similar to that of Easley and O'Hara [5].

[8] The two tradable sizes can be interpreted as two markets: one market for small orders and one for large orders.

[9] For simplicity we assume that the behavior of liquidity traders does not depend on transaction costs. However, the more realistic assumption that the probability of no trade is an increasing function of transaction costs does not affect the qualitative results of the paper.

$\overline{\theta}$ indicates $\overline{V}$. We assume that signals are symmetric and let $p = Pr(\underline{\theta}|\underline{V}) = Pr(\overline{\theta}|\overline{V}) > \frac{1}{2}$. We denote the likelihood ratio of signals by λ_θ.

The expected asset value of an informed trader at time t is denoted by $E_t[\widetilde{V} \mid \theta]$ and the market maker's expectation is $E_t[\widetilde{V}]$. Finally, we denote the probability that the market maker attaches to $\overline{V}$ at time t by π_t.[10]

Before a trader arrives, the market maker sets bid and ask prices at which he is willing to trade each asset quantity. Finally, we assume that market making is costly and traders pay a fee $c(Q)$ to transact quantity $Q \in \{Q_S,\, Q_L\}$. We also assume that $c(Q_S) \leq c(Q_L)$ and $AC(Q_S) \geq AC(Q_L)$, where $AC(Q_i) \equiv c(Q_i)/Q_i$ is the average cost to trade quantity Q_i.

3 Equilibrium Strategies and Prices

At the beginning of any trading round t, the market maker sets competitive quote prices. We denote the price schedule at time t by P_t. Clearly:

$$P_t = \{B_{L,\,t},\ B_{S,\,t},\ A_{S,\,t},\ A_{L,\,t}\},$$

where $B_{L,\,t}$ is the bid price for a large quantity, $B_{S,\,t}$ the bid price for a small quantity, $A_{S,\,t}$ the ask price for a small quantity, and $A_{L,\,t}$ the ask price for a large quantity.

After prices are set, a trader is randomly selected to trade, observes the price schedule and executes his strategy. If the selected trader is a liquidity trader, he acts in an ex-ante specified probabilistic way. If the selected trader is informed, he chooses the strategy which maximizes his expected profit given the price schedule and the transaction fee.

The market maker anticipates the strategies of informed traders and announces his price schedule. Bertrand competition restricts the market maker to earn zero expected profit from each trade. Hence, the trader arriving at t faces a price schedule which satisfies:

$$\begin{aligned} B_{i,\,t} &= E_t[\widetilde{V}\,|SQ_i] \quad \forall\, i \in \{S,\ L\} \\ A_{i,\,t} &= E_t[\widetilde{V}\,|BQ_i] \quad \forall\, i \in \{S,\ L\}. \end{aligned} \tag{1}$$

Since $E_t[\widetilde{V}|\underline{\theta}] \leq E_t[\widetilde{V}] \leq E_t[\widetilde{V}|\overline{\theta}]$ for any π_t and bid and ask prices are included between $E_t[\widetilde{V}|\underline{\theta}]$ and $E_t[\widetilde{V}|\overline{\theta}]$, traders observing a good signal never prefer to sell the asset and traders with a bad signal never prefer to buy the asset. Thus, by the assumption of competitive market making, it follows that in equilibrium, bid and ask prices always straddle the unconditional expected asset value $E_t[\widetilde{V}]$ and are included between $E_t[\widetilde{V}|\underline{\theta}]$ and $E_t[\widetilde{V}|\overline{\theta}]$. Depending on the

[10] Since $\underline{V} = 0$ and $\overline{V} = 1$, $E_t[\widetilde{V}] = \pi_t$. Moreover, by the Bayes rule:

$$E_t[\widetilde{V} \mid \theta] = \frac{\pi_t}{\pi_t + (1-\pi_t)\frac{Pr(\theta|\underline{V})}{Pr(\theta|\overline{V})}},$$

which exceeds $E_t[\widetilde{V}]$ if $\theta = \overline{\theta}$, and which is lower than $E_t[\widetilde{V}]$ if $\theta = \underline{\theta}$.

parameters of the model, different outcomes may prevail. If informed traders prefer to trade only a large quantity, they are separated from small liquidity traders. We call this a separating equilibrium. If informed traders submit either small or large orders with positive probability, a pooling equilibrium occurs. If they prefer to refrain from trading, a no trading equilibrium occurs. Finally, if they are indifferent between submitting an order and refraining from trading, a pooling equilibrium with no trading arises.

It is useful to note that the equilibrium on one side of the market may differ from the equilibrium on the other side. For example, traders observing $\underline{\theta}$ may prefer to sell only large quantities and traders observing $\overline{\theta}$ may choose to buy, with positive probability, either small or large quantities. In the following sections we analyze in more detail the different outcomes that can occur in equilibrium.

3.1 The no-trading equilibrium

In the no-trading equilibrium, no sell or buy order arises from informed traders. Since transactions are not information based, they do not affect the market maker's expected asset value. Therefore, the competitive price schedule is $P^{ne} = \{B_L^{ne}, B_S^{ne}, A_S^{ne}, A_L^{ne}\}$ such that:

$$B_L^{ne} = B_S^{ne} = A_S^{ne} = A_L^{ne} = E[\widetilde{V}]$$

Define $\Pi_\theta^{ne}(\pi) \equiv |E[\widetilde{V} \mid \theta] - E[\widetilde{V}]|$ as the informational advantage of a trader observing θ who does not submit any order. P^{ne} is an equilibrium price schedule when the expected profit from informed trading is strictly negative. Since the average cost is decreasing in Q (by assumption), the condition for a no-buying equilibrium to prevail is $\Pi_{\overline{\theta}}^{ne}(\pi) < AC(Q_L)$ and the condition for a no-selling equilibrium is $\Pi_{\underline{\theta}}^{ne}(\pi) < AC(Q_L)$.

The following Proposition summarizes conditions for the occurrence of a no-trading equilibrium on the bid and on the ask side of market.

Proposition 1 *For the bid side of the market a no-trading equilibrium prevails if* $\pi \in [0; \underline{\pi}_{\underline{\theta},S}] \cup [\overline{\pi}_{\underline{\theta},S}; 1]$, *with* $\Pi_{\underline{\theta}}^{ne}(\underline{\pi}_{\underline{\theta},S}) = \Pi_{\underline{\theta}}^{ne}(\overline{\pi}_{\underline{\theta},S}) = AC(Q_L)$ *and* $\underline{\pi}_{\underline{\theta},S} \leq \overline{\pi}_{\underline{\theta},S}$. *Similarly, for the ask side of the market a no-trading equilibrium prevails if:* $\pi \in [0; \underline{\pi}_{\overline{\theta},S}] \cup [\overline{\pi}_{\overline{\theta},S}; 1]$, *with* $\Pi_{\overline{\theta}}^{ne}(\underline{\pi}_{\overline{\theta},S}) = \Pi_{\overline{\theta}}^{ne}(\overline{\pi}_{\overline{\theta},S}) = AC(Q_L)$ *and* $\underline{\pi}_{\overline{\theta},S} \leq \overline{\pi}_{\overline{\theta},S}$.

Proof See Appendix.

Prices are positively related to the market maker's assessment of the fundamental asset value, π. A high π leads to high prices and low π implies low prices. Moreover, if a trade occurs at time $t-1$, the public belief about the asset value at time t, π_t, is equal to the transaction price at $t-1$. Thus, Proposition 1 states that informed traders choose to refrain from trading when the asset price is particularly low or high because, in those cases, their informational advantage is too small with respect to transaction costs (see Figure 1).

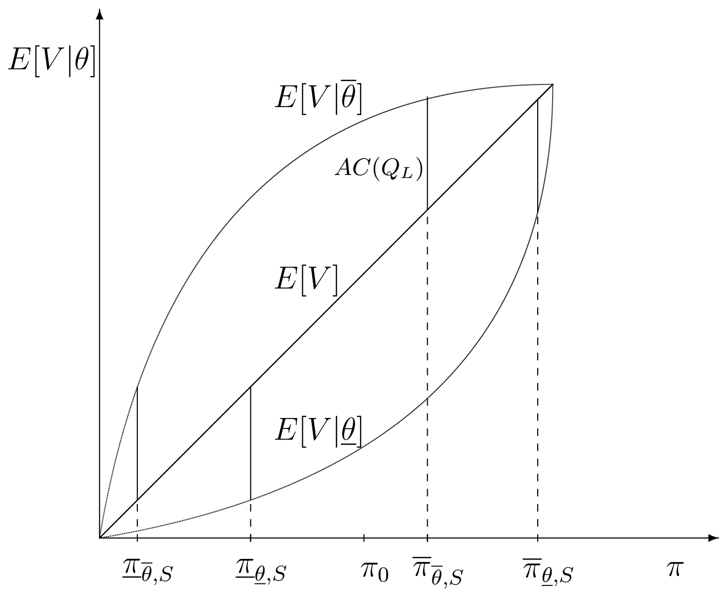

Figure 1: ***The no-trading equilibrium.***
$E[V|\overline{\theta}] - E[V]$ represents the informational advantage of a trader who receives a good signal, as a function of the probability of $\overline{V}$, π; while $E[V|\underline{\theta}] - E[V]$ represents the informational advantage of a trader who receives a bad signal. As π becomes larger than $\overline{\pi}_{\underline{\theta},S}$ or smaller than $\underline{\pi}_{\overline{\theta},S}$ the average transaction cost $AC(Q_L)$ overcomes the informational profit of all traders, resulting in a no trading equilibrium.

Proposition 2 *If $\pi > 1/2$, then a no trading equilibrium for the bid side of the market implies a no trading equilibrium for the ask side of the market. If $\pi < 1/2$, the reverse is true.*

Proof See Appendix.

Proposition 2 implies that in a bull market traders with good news refrain from trading more often than traders with bad news and the reverse arises in a bear market.

To gain some intuition into these results, consider the limiting case of perfect signals ($p = 1$), and suppose that $\widetilde{V} = \overline{V} = 1$. Since signals are perfect, the expected asset value of informed traders is always 1, whatever the public belief. The traders' informational advantage is equal to $(1 - \pi)$. It is low if the probability the market maker attaches to $\overline{V}$, and thus also the asset price, is high, and it grows as this probability approaches 0. The more the valuation of the market differs from 1, the higher the transaction cost that informed traders are willing to pay in order to transact.

In a similar way, when signals are not perfect ($p < 1$), the informational advantage of a trader observing θ is low if the public belief is consistent with θ, because the asset price approaches the trader's expectation, and it grows as the valuation of the market maker moves in the opposite direction. But, unlike the case for perfect signals, the trader's informational advantage decreases also when the market maker attaches a very low probability to the asset value consistent with θ. This occurs because, when signals are not perfect, the trader's expectation depends not only on his private signal, but also on public beliefs. If the past history of trades strictly implies an asset value inconsistent with θ, a trader observing this signal attaches a low weight to his private information with respect to the public belief. Hence, when the market attaches a greater probability to $\underline{V}$ ($\pi < \frac{1}{2}$), the informational advantage of traders observing $\overline{\theta}$ is greater than that of traders observing $\underline{\theta}$; when the market attaches a greater probability to $\overline{V}$ ($\pi > \frac{1}{2}$), the reverse is true (see Figure 1). As a consequence, when prices are low($\pi < \underline{\pi}_{\overline{\theta},S}$), the no trading equilibrium for the ask side of market implies the no trading equilibrium also in the bid side and the converse is true when prices are high ($\pi > \overline{\pi}_{\underline{\theta},S}$).

To conclude, it is interesting to note that in a no-trading equilibrium no new information reaches the market because all informed traders choose to refrain from trading. Therefore, the economy is in an informational cascade.

3.2 The separating equilibrium

A separating equilibrium prevails in a market when the competitive price schedule, $P^{se} = \{B_L^{se}, B_S^{se}, A_S^{se}, A_L^{se}\}$, is such that informed traders place only large orders. Thus, small trades are not information-based and do not affect the public belief about the true asset value, while the information content of large trades is very strong.

This implies that the equilibrium price of small orders is given by:

$$B_S^{se} = A_S^{se} = \pi$$

and the equilibrium price of large orders is given by:

$$\begin{aligned} B_L^{se} = & \quad \pi/(\pi + (1-\pi)\lambda_{SQ_L}^{se}) \\ A_L^{se} = & \quad \pi/(\pi + (1-\pi)\lambda_{BQ_L}^{se}), \end{aligned}$$

where $\lambda_{SQ_L}^{se} \equiv (\gamma + \mu p)/(\gamma + \mu(1-p))$ and $\lambda_{BQ_L}^{se} \equiv (\gamma + \mu(1-p))/(\gamma + \mu p)$ are, respectively, the likelihood ratio of a large sell and of a large buy order. [11].

A separating equilibrium prevails only if, given the price schedule P^{se}, informed traders prefer to trade large quantities. This occurs when the expected profit from trading in size is strictly positive, and the profit due to the larger quantity exceeds the better price available for small trades.

To gain some intuition, define $\Pi_{\overline{\theta}}^{se}(\pi) \equiv E[\widetilde{V} \,|\, \overline{\theta}] - A_L^{se}$ as the informational advantage of a trader observing $\overline{\theta}$ in a separating equilibrium and $\Pi_{\underline{\theta}}^{se}(\pi) \equiv$

[11] Clearly, $\lambda_{SQ_L}^{se} > 1$ because a large sell order can be submitted either by a liquidity trader or by an informed trader observing $\underline{\theta}$, and $\lambda_{\underline{\theta}} > 1$, while $\lambda_{BQ_L}^{se} < 1$ because a large buy order can be submitted either by a liquidity trader or by an informed trader observing $\overline{\theta}$, and $\lambda_{\overline{\theta}} < 1$

$B_L^{se} - E[\widetilde{V} \mid \underline{\theta}]$ the informational advantage of a trader observing $\underline{\theta}$. Suppose that the expected profit from trading large is strictly positive, that is: $\Pi_\theta^{se}(\pi) \geq AC(Q_L)$ for any $\theta \in [\underline{\theta}; \overline{\theta}]$, and consider the ask side of the market. The difference between the expected profit from buying the large and the small quantity can be written as follows:

$$\begin{aligned} \Delta\Pi_{\overline{\theta}}(\pi) = \quad & [\Pi_{\overline{\theta}}^{se}(\pi) - AC(Q_L))](Q_L - Q_S) + \\ & -[(A_L^{se} - A_S^{se}) - (AC(Q_S) - AC(Q_L))]Q_S. \end{aligned} \tag{2}$$

The first term represents the expected gain due to the greater quantity of asset bought and the second term is the loss due to the higher price paid to purchase the first Q_S units of the asset. An informed trader endowed with $\overline{\theta}$ chooses to place a large order if this difference is positive. It is interesting to note that transaction costs have two effects on condition (2). On the one hand, they reduce the benefit of buying a large quantity because they make transactions more expensive.[12] On the other hand, they reduce incentives to buy a small quantity because of the gain due to the lower transaction cost borne from purchasing the first Q_S units of asset.[13] It is easy to verify that, if transaction costs are increasing in the trade size, then the first effect prevails over the second one. Hence, transaction costs reduce the likelihood of a separating equilibrium, despite the assumption of decreasing average costs and even in the limit where they are constant.

The separating equilibrium is more likely to prevail when the distance between the large and the small quantity is greater. A larger Q_L and a lower Q_S have both a direct positive effect on the aptitude to trade large and an indirect positive effect through average costs. Indeed, an increase in the large quantity yields a reduction in $AC(Q_L)$ and an increase in the distance between Q_L and Q_S produces a larger difference in average costs. Finally, if the fraction of liquidity traders is small, the information that the market maker can infer from a large order is very accurate ($\lambda_{SQ_L}^{se}$ is close to $\lambda_{\underline{\theta}}$). This, in turn, implies that the difference between B_S^{se} and B_L^{se} and the difference between A_L^{se} and A_S^{se} are significant and hence the loss due to the worse prices in trading Q_L rather than Q_S are high. Hence, Q_L has to be very large with respect to Q_S in order to encourage informed traders to separate from small liquidity traders.

Since we are interested in studying block trades, in the following we will assume $Q_L > Q_S \left(1 - \lambda_{\overline{\theta}}\right) / \left(\lambda_{BQ_L}^{se} - \lambda_{\overline{\theta}}\right)$. This condition guarantees that a separating equilibrium would always prevail in the absence of transaction costs.[14] It simplifies the analysis but it is not needed to obtain the results of the propositions that follows.

The next Proposition summarizes conditions for the occurrence of a separating equilibrium for the bid and for the ask side of the market.

[12] Transaction costs are assumed to be increasing in trade size.
[13] This effect is due to the assumption of decreasing average costs.
[14] This statement is proved in the Appendix.

Lemma 1 *For any* $\theta \in [\underline{\theta};\overline{\theta}]$*,* $\Pi^{se}_{\theta}(\pi) \geq AC(Q_L)$ *if and only if* $\pi \in [\underline{\pi}_{\theta,L};\overline{\pi}_{\theta,L}]$*, with* $\underline{\pi}_{\theta,L}$ *and* $\overline{\pi}_{\theta,L}$ *such that: i)* $\Pi^{se}_{\theta}(\underline{\pi}_{\theta,L}) = \Pi^{se}_{\theta}(\overline{\pi}_{\theta,L}) = AC(Q_L)$ *and ii)* $\underline{\pi}_{\theta,L} \leq \overline{\pi}_{\theta,L}$*.*

Proposition 3 *A separating equilibrium prevails for the bid side of the market if: i)* $\pi \in [\underline{\pi}_{\underline{\theta},L};\overline{\pi}_{\underline{\theta},L}]$ *and ii)* $\Delta\Pi_{\underline{\theta}}(\pi) \geq 0$*. Similarly, a separating equilibrium prevails for the ask side of the market if: i)* $\pi \in [\underline{\pi}_{\overline{\theta},L};\overline{\pi}_{\overline{\theta},L}]$ *and ii)* $\Delta\Pi_{\overline{\theta}}(\pi) \geq 0$*.*

Proof See Appendix.

Proposition 3 points out that a separating equilibrium never occurs if the asset price is particularly low or high because, in those cases, the transaction costs are higher than the informational advantage of traders. Moreover, in bull markets traders with a bad signal have a larger informational advantage, whilst the informational advantage of traders with a good signal is larger in bear markets. This implies that when prices are high, a separating equilibrium is more likely to prevail for the bid side of the market, while the reverse occurs when prices are low. This result follows from the next proposition.

Proposition 4 *If* $\pi > 1/2$*, then the separating equilibrium for the ask side of the market implies the separating equilibrium for the bid side of the market. If* $\pi < 1/2$*, the reverse is true.*

Proof See Appendix.

3.3 The pooling equilibrium

In a pooling equilibrium traders observing $\underline{\theta}$ choose to play a mixed strategy $\sigma_{\underline{\theta}} \equiv \{\sigma_{\underline{\theta},S};\sigma_{\underline{\theta},L};\sigma_{\underline{\theta},NT}\}$ defined on the simplex $\Delta(SQ_S, SQ_L, NT)$ and traders observing $\overline{\theta}$ choose to play a mixed strategy $\sigma_{\overline{\theta}} \equiv \{\sigma_{\overline{\theta},S};\sigma_{\overline{\theta},L};\sigma_{\overline{\theta},NT}\}$ defined on the simplex $\Delta(BQ_S, BQ_L, NT)$.

A pooling equilibrium where informed traders submit both small and large orders ($\sigma_{\theta,S} > 0$ and $\sigma_{\theta,L} > 0$) prevails when the informational advantage of traders observing θ exceeds the highest average transaction cost, that is: $\Pi^{ne}_{\theta}(\pi) > AC(Q_S)$, but it is not large enough to induce informed traders to separate from small liquidity traders, that is: $\Delta\Pi_{\theta}(\pi) < 0$.

For the competitive price schedule $P^{pe} = \{B^{pe}_L, B^{pe}_S, A^{pe}_S, A^{pe}_L\}$ to describe a pooling equilibrium with both small and large information based orders, informed traders must be indifferent between trading a large or small quantity. This condition requires:

$$(B^{pe}_L - E[\widetilde{V}\,|\,\underline{\theta}])Q_L - c(Q_L) = (B^{pe}_S - E[\widetilde{V}\,|\,\underline{\theta}])Q_S - c(Q_S) \geq 0 \qquad (3)$$

$$(E[\widetilde{V}\,|\,\overline{\theta}] - A^{pc}_L)Q_L - c(Q_L) = (E[\widetilde{V}\,|\,\overline{\theta}] - A^{pc}_S)Q_S - c(Q_S) \geq 0. \qquad (4)$$

It is easy to see that if $B_L > B_S$, traders endowed with $\underline{\theta}$ never choose to sell a small quantity and if $A_L < A_S$, traders observing $\overline{\theta}$ never buy a small quantity. Conditions (3) and (4) can be satisfied only if the price schedule is such that $B_L^{pe} \leq B_S^{pe}$ and $A_L^{pe} \geq A_S^{pe}$. This, in turn, implies that in a pooling equilibrium informed traders are more likely to place a large order.

Finally, a pooling equilibrium without small information based orders ($\sigma_{\theta,S} = 0$) prevails when *i)* the average transaction cost for the small quantity is too large with respect to the traders' informational advantage, that is: $\Pi_\theta^{ne}(\pi) \leq AC(Q_S)$, and *ii)* the average transaction cost for the large quantity is below the traders' informational advantage, but is too large to induce traders to submit large orders with probability 1, that is: $\Pi_\theta^{ne}(\pi) > AC(Q_L) \geq \Pi_\theta^{se}(\pi)$.

The next proposition states that in a bull (bear) market where a pooling equilibrium prevails both for the ask and bid side, informed sellers are more (less) aggressive than informed buyers.

Proposition 5 *If $\pi > 1/2$ and a pooling equilibrium prevails in both sides of the market, then the probability of observing a large information based sell order is greater than the probability of observing a large information based buy order. If $\pi < 1/2$, the reverse is true.*

Proof See Appendix.

4 The asymmetric price impact of large trades

The price impact of a large trade in our model is the change in the public belief about the value of the asset due to the large order. It depends both on the orders information content and on the weight the market maker attaches to this information. Since traders observing a good signal never sell and those observing a bad signal never buy, the price impact of a large sell is always negative and the price impact of a large buy is always positive. In the following we will focus on the magnitude of the price impact.

The information content of an order is related to its likelihood ratio. If an order is totally uninformative about the true value of the asset then its likelihood ratio is equal to 1; the more informative the order is, the more its likelihood ratio differs from 1. More specifically, the more informative is a sell order, the more its likelihood ratio is higher than 1 and the more informative is a buy order, the more its likelihood ratio is lower than 1. So we can define the information content of a large sell order as its likelihood ratio and the information content of a large buy order as the reciprocal of its likelihood ratio.

Given the unconditional public belief π, the price impact of a large sell order is:

$$\Delta S(\pi) \equiv |B_L - \pi| = \frac{\pi\,(1-\pi)\,(\lambda_{SQ_L}(\pi) - 1)}{\pi + (1-\pi)\lambda_{SQ_L}(\pi)},$$

where $\lambda_{SQ_L}(\pi) \equiv Pr(SQ_L|\pi, \underline{V}) \backslash Pr(SQ_L|\pi, \overline{V})$ is the equilibrium information content of a large sell order, conditional on the public belief π, and the price impact of a large buy order is:

$$\Delta B(\pi) \equiv |A_L - \pi| = \frac{\pi\,(1-\pi)\,(1-\lambda_{BQ_L}(\pi))}{\pi + (1-\pi)\lambda_{BQ_L}(\pi)},$$

where $1/\lambda_{BQ_L}(\pi) \equiv Pr(BQ_L|\pi, \overline{V}) \backslash Pr(BQ_L|\pi, \underline{V})$ is the equilibrium information content of a large buy order, conditional on π.

Let the price impact asymmetry expression of the asset be defined as:

$$J(\pi) \equiv \Delta B(\pi) - \Delta S(\pi).$$

$J(\pi)$ is larger than, equal to, or lower than 0 if and only if the price impact of a large buy order is, respectively, larger than, equal to, or lower than the impact of a large sell order.

The public belief about an asset value captures the equilibrium price prior to the large trade. The minimum and the maximum value the asset price can achieve in equilibrium are respectively 0 and 1. Hence, a flat market is characterized by $\pi = 1\backslash 2 \equiv \pi_0$, a bullish market is characterized by $\pi > 1\backslash 2$ and a bearish market by $\pi < 1\backslash 2$. [15] This leads to the following result.

Proposition 6 *In a flat market the price impact of a large sell order is equal to the price impact of a large buy order. In a bullish market the price impact of a large sell order is larger than the price impact of a large buy order. In a bearish market the price impact of a large buy order is larger than the price impact of a large sell order.*

Proof See Appendix.

The rationale for the results in Proposition 6 is that the informational advantage of traders observing a signal contrary to the price path, that is, a good signal in a bear market or a bad signal in a bullish market, is larger and induces them to be more aggressive.

Because of exogenous transaction costs, when prices are very low or very high, an informational cascade develops and orders do not have any impact on prices.

In order to study the correlation between the price impact asymmetry and market conditions we will analyze the price impact asymmetry under 4 different market conditions. Depending on the public belief about the true value of the asset, π, we distinguish between strong and weak bearish and bullish markets. A strong bearish market (SBear) is characterized by $\pi \to 0$; a weak bearish market (WBear) is characterized by $\pi \to 1\backslash 2$ and $\pi < 1\backslash 2$; a strong bullish market (SBull) is characterized by $\pi \to 1$; a weak bullish market (WBull) is characterized by $\pi \to 1\backslash 2$ and $\pi > 1\backslash 2$; a neutral market is characterized by $\pi \in (1\backslash 2 - \delta; 1\backslash 2 + \delta)$, with $\delta > 0$ and small enough. Finally, we define the asymmetry in the price impact by $|J(\pi)|$.

[15] Observe that the price in the flat market is the average between 0 and 1.

Proposition 7 *In a weak bear market the asymmetry in the price impact increases as the asset price decreases. In a weak bull market the asymmetry in the price impact increases as the asset price increases.*

Proof See Appendix.

Proposition 8 *In a strong bear market the price impact asymmetry decreases as the asset price decreases. In a strong bull market the price impact asymmetry decreases as the asset price increases.*

Proof See Appendix.

Propositions 7 and 8 state a non-monotone relationship between the price impact asymmetry and the asset price. Indeed, in a strong bearish market the price impact asymmetry is increasing in the asset price; in a neutral market the relationship between the price impact asymmetry and the asset price is U-shaped; in a strong bullish market the relationship becomes decreasing. The intuition for this result is the following. In a flat market ($\pi = 1/2$) the price impact of a large sell order is equal to the price impact of a large buy order. If the asset price increases (reduces), the informational advantage of traders observing the bad news increases (decreases), while that of traders observing the good news decreases (increases). As a consequence, traders with a bad signal become more (less) aggressive and traders with the good signal less (more). This implies that the price impact asymmetry goes up as the asset price moves away from 1/2. However, since signals are imperfect, if the asset price increases (decreases) significantly, the history of trades strictly indicates the high (low) asset value and traders observing the bad (good) news attach to their signal a lower weight than to the public belief. As a consequence, also their informational advantage reduces and they become less aggressive. Thus, the price impact asymmetry decreases when prices become too high or too low and vanishes as an informational cascade develops.

5 Comments and concluding remarks

In this paper we examine the impact of trading costs increasing in order size on price discovery. We show that a no-trading informational cascade develops when market conditions are strongly bearish or bullish (that is, when the asset price is extremely high or low) and, then, the transaction costs exceed the informational profit of traders endowed with private information. Otherwise, in a weakly bearish or bullish market, the informational profit is sufficiently large with respect to transaction costs and two equilibria can arise: a separating equilibrium, where informed traders choose to trade only the large quantity, and a pooling equilibrium, where informed traders choose to submit both large and small orders with positive probability. In the separating equilibrium the spread at the large quantity exceeds the spread at the small quantity. Hence, this outcome prevails if the large quantity is big enough to offset the better price for small orders.

The main insight that comes out of the analysis is the relation between market conditions and the empirically well documented price impact asymmetry. In the model, the price impact of a large trade corresponds to the change in the market's expectations about the true asset value due to the arrival of a large order. In bearish markets large buys have, on average, greater price impact than large sells, while in bullish markets the exact opposite occurs. Moreover, the asymmetry increases in absolute value during a weakly bearish or bullish phase and tends to vanish as the asset price becomes extremely high or low.

Our analysis suggests that information effects drive the asymmetry between block purchases and sales in neutral markets, whilst it does not play any role in strongly bearish or bullish markets. Thus our model provides a framework for further empirical analysis of the asymmetry phenomenon.

Appendix

Proof of Proposition 1

We prove the proposition for the ask side of market. The proof for the bid side is similar. The function $\Pi^{ne}_{\overline{\theta}}(\pi) \equiv \frac{\pi(1-\pi)(1-\lambda_{\overline{\theta}})}{(\pi+(1-\pi)\lambda_{\overline{\theta}})}$, defined for all $\pi \in [0;1]$, is a positive concave function equal to zero for π equal to 0 and 1 and with maximum in the point $\pi = \sqrt{\lambda_{\overline{\theta}}}/(1+\sqrt{\lambda_{\overline{\theta}}})$. Thus, if $AC(Q_L)$ is low enough, there are only two points into $[0;1]$, $\underline{\pi}_{\overline{\theta},S}$ and $\overline{\pi}_{\overline{\theta},S}$, such that $\Pi^{ne}_{\overline{\theta}}(\underline{\pi}_{\overline{\theta},S}) = \Pi^{ne}_{\overline{\theta}}(\overline{\pi}_{\overline{\theta},S}) = AC(Q_L)$ and $\Pi^{ne}_{\overline{\theta}}(\pi) \geq AC(Q_L)$ iff $\pi \in [0;\underline{\pi}_{\overline{\theta},S}] \cup [\overline{\pi}_{\overline{\theta},S};1]$. This concludes the proof. □

Proof of Proposition 2

Since $\lambda_{\overline{\theta}} < 1 < \lambda_{\underline{\theta}}$, $\Pi^{ne}_{\overline{\theta}}(\pi) < \Pi^{ne}_{\underline{\theta}}(\pi)$ if and only if $\pi > 1/2$ and $\arg\max \Pi^{ne}_{\overline{\theta}}(\pi) < 1/2 < \arg\max \Pi^{ne}_{\underline{\theta}}(\pi)$. Thus, from the concavity of $\Pi_{\theta,S}(\pi)$ it follows that either

$$\underline{\pi}_{\overline{\theta},S} < \overline{\pi}_{\overline{\theta},S} < 1/2 < \underline{\pi}_{\underline{\theta},S} < \overline{\pi}_{\underline{\theta},S}$$

or

$$\underline{\pi}_{\overline{\theta},S} < \underline{\pi}_{\underline{\theta},S} < 1/2 < \overline{\pi}_{\overline{\theta},S} < \overline{\pi}_{\underline{\theta},S},$$

depending on the magnitude of $AC(Q_L)$. □

Proof of Lemma 1

We prove the lemma for signal $\overline{\theta}$.

The function $\Pi^{se}_{\overline{\theta}}(\pi) \equiv \frac{\pi(1-\pi)(\lambda^{se}_{BQ_L}-\lambda_{\overline{\theta}})}{(\pi+(1-\pi)\lambda^{se}_{BQ_L})(\pi+(1-\pi)\lambda_{\overline{\theta}})}$, defined for all $\pi \in [0;1]$, is a positive concave function equal to zero for π equal to 0 and 1 and with maximum in the point $\pi = \sqrt{\lambda_{\overline{\theta}} \cdot \lambda^{se}_{BQ_L}}/(1+\sqrt{\lambda_{\overline{\theta}} \cdot \lambda^{se}_{BQ_L}})$. Thus, if $c(Q_L)/Q_L$ is low enough, there are only two points into $[0;1]$, $\underline{\pi}_{\overline{\theta},L}$ and $\overline{\pi}_{\overline{\theta},L}$, such that $\Pi^{se}_{\overline{\theta}}(\underline{\pi}_{\overline{\theta},L}) = \Pi^{se}_{\overline{\theta}}(\overline{\pi}_{\overline{\theta},L}) = c(Q_L)/Q_L$ and $\Pi^{se}_{\overline{\theta}}(\pi) \geq c(Q_L)/Q_L$ iff $\pi \in [\underline{\pi}_{\overline{\theta},L};\overline{\pi}_{\overline{\theta},L}]$. □

Proof of Proposition 3

Since the expected profit from trading the large quantity of a trader observing θ is strictly negative, a separating equilibrium never prevails when $\pi \in [0;\underline{\pi}_{\theta,L}] \cup [\overline{\pi}_{\theta,L};1]$. On the other hand, when $\pi \in [\underline{\pi}_{\theta,L};\overline{\pi}_{\theta,L}]$ a separating equilibrium occurs if and only if $\Delta\Pi_\theta(\pi) \geq 0$. □

Proof of Proposition 4

In order to prove the proposition, we first show that $\Pi^{se}_{\underline{\theta}}(\pi) \geq \Pi^{se}_{\overline{\theta}}(\pi)$ iff $\pi \in [1/2;1]$ (*Step 1*) and then, we demonstrate that $\Pi^{se}_{\underline{\theta}}(\pi)Q_L - \Pi^{ne}_{\underline{\theta}}(\pi)Q_S \geq \Pi^{se}_{\overline{\theta}}(\pi)Q_L - \Pi^{ne}_{\overline{\theta}}(\pi)Q_S$ iff $\pi \in [1/2;1]$ (*Step 2*).

Step 1: $\Pi^{se}_{\underline{\theta}}(\pi) \geq \Pi^{se}_{\overline{\theta}}(\pi)$ *iff* $\pi \in [1/2; 1]$. Since signals are symmetric:

$$\begin{aligned}
\Pi^{se}_{\overline{\theta}}(\pi) &= \frac{\pi\,(1-\pi)\left(\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}\right)}{\left(\pi + (1-\pi)\,\lambda^{se}_{BQ_L}\right)\left(\pi + (1-\pi)\,\lambda_{\overline{\theta}}\right)} = \\
&= \frac{\pi\,(1-\pi)\left(\lambda_{\underline{\theta}} - \lambda^{se}_{SQ_L}\right)}{\left(\lambda^{se}_{SQ_L}\pi + (1-\pi)\right)\left(\lambda_{\underline{\theta}}\pi + (1-\pi)\right)} = \Pi^{se}_{\underline{\theta}}(1-\pi)
\end{aligned}$$

This implies that: $\Pi^{se}_{\underline{\theta}}(\pi) - \Pi^{se}_{\overline{\theta}}(\pi) = \Pi^{se}_{\underline{\theta}}(\pi) - \Pi^{se}_{\underline{\theta}}(1-\pi)$, which is positive only if:

$$\begin{aligned}
\left(\pi + (1-\pi)\,\lambda^{se}_{SQ_L}\right)\left(\pi + (1-\pi)\,\lambda_{\underline{\theta}}\right) &\leq \left(\lambda^{se}_{SQ_L}\pi + (1-\pi)\right)\left(\lambda_{\underline{\theta}}\pi + (1-\pi)\right) \\
&\Updownarrow \\
(2\pi - 1)\left(\lambda^{se}_{SQ_L}\lambda_{\underline{\theta}} - 1\right) &\leq 0,
\end{aligned}$$

that is true only if $\pi \geq 1/2$ since $\lambda_{\underline{\theta}} > \lambda^{se}_{SQ_L} > 1$.

Step 2: $\Pi^{se}_{\underline{\theta}}(\pi)Q_L - \Pi^{ne}_{\underline{\theta}}(\pi)Q_S \geq \Pi^{se}_{\overline{\theta}}(\pi)Q_L - \Pi^{ne}_{\overline{\theta}}(\pi)Q_S$ *iff* $\pi \in [1/2; 1]$. The claim is equivalent to:

$$\Pi^{se}_{\overline{\theta}}(\pi)\left[\frac{Q_L}{Q_S} - \frac{\Pi^{ne}_{\overline{\theta}}(\pi)}{\Pi^{se}_{\overline{\theta}}(\pi)}\right] \leq \Pi^{se}_{\underline{\theta}}(\pi)\left[\frac{Q_L}{Q_S} - \frac{\Pi^{ne}_{\underline{\theta}}(\pi)}{\Pi^{se}_{\underline{\theta}}(\pi)}\right].$$

Notice that:

$$\begin{aligned}
\frac{\Pi^{ne}_{\overline{\theta}}(\pi)}{\Pi^{se}_{\overline{\theta}}(\pi)} &= \frac{\left(1 - \lambda_{\overline{\theta}}\right)\left(\pi + (1-\pi)\,\lambda^{se}_{BQ_L}\right)}{\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}} = \frac{\left(1 - 1/\lambda_{\underline{\theta}}\right)\left(\pi + (1-\pi)\,1/\lambda^{se}_{SQ_L}\right)}{1/\lambda^{se}_{SQ_L} - 1/\lambda_{\underline{\theta}}} \\
&= \frac{\left(\lambda_{\underline{\theta}} - 1\right)\left(\lambda^{se}_{SQ_L}\pi + (1-\pi)\right)}{\lambda_{\underline{\theta}} - \lambda^{se}_{SQ_L}} = \frac{\Pi_{\underline{\theta},S}(1-\pi)}{\Pi_{\underline{\theta},L}(1-\pi)},
\end{aligned}$$

since signals are symmetric. This implies that $\Pi^{ne}_{\overline{\theta}}(1/2)/\Pi^{se}_{\overline{\theta}}(1/2) = \Pi^{ne}_{\underline{\theta}}(1/2)/\Pi^{se}_{\underline{\theta}}(1/2)$ and $\Pi^{ne}_{\overline{\theta}}(\pi)/\Pi^{se}_{\overline{\theta}}(\pi) > \Pi^{ne}_{\underline{\theta}}(\pi)/\Pi^{se}_{\underline{\theta}}(\pi)$ for all $\pi > 1/2$, since $\Pi^{ne}_{\underline{\theta}}(\pi)/\Pi^{se}_{\underline{\theta}}(\pi)$ is decreasing in π and $\pi > 1 - \pi$ iff $\pi > 1/2$.

The proposition follows immediately by combining claims in Step 1 and 2.

Proof of Proposition 5

For any $\theta \in \{\underline{\theta}; \overline{\theta}\}$, $\sigma \in [0, 1]$ and $\pi \in [0, 1]$, we define the following functions:

- $\lambda_\theta(\sigma) \equiv \frac{\gamma + \mu\,\sigma\ \Pr(\theta|\underline{V})}{\gamma + \mu\,\sigma\ \Pr(\theta|\overline{V})}$,
- $\Pi_\theta(\pi, \sigma) \equiv \left|E[\widetilde{V}\,|\,\theta] - \frac{\pi}{\pi + (1-\pi)\lambda_\theta(\sigma)}\right|$, and

- $G_\theta(\pi,\sigma) \equiv \Pi_\theta(\pi,\sigma)\,Q_L - \Pi_\theta(\pi,1-\sigma)\,Q_S$.

Assume that the market is in a pooling equilibrium and $\sigma_{\underline{\theta},NT} = \sigma_{\overline{\theta},NT} = 0$. The equilibrum strategies of informed traders are $\sigma_{\underline{\theta}} = \{1-\sigma_{\underline{\theta},L};\sigma_{\underline{\theta},L};0\}$ and $\sigma_{\overline{\theta}} = \{1-\sigma_{\overline{\theta},L};\sigma_{\overline{\theta},L};0\}$ such that:

$$G_{\underline{\theta}}(\pi,\sigma_{\underline{\theta},L}) = G_{\overline{\theta}}(\pi,\sigma_{\overline{\theta},L}) = c(Q_L) - c(Q_S)$$

Observe that *i)* $\partial G_\theta(\pi,\sigma)/\partial\sigma < 0$ for all θ and *ii)* $G_{\underline{\theta}}(\pi,\sigma) = G_{\overline{\theta}}(1-\pi,\sigma)$. Hence, $\sigma_{\underline{\theta},L} > \sigma_{\overline{\theta},L}$ if $G_{\overline{\theta}}(1-\pi,\sigma) > G_{\overline{\theta}}(\pi,\sigma)$ for all σ, and $\sigma_{\underline{\theta},L} < \sigma_{\overline{\theta},L}$ if $G_{\overline{\theta}}(1-\pi,\sigma) < G_{\overline{\theta}}(\pi,\sigma)$ for all σ. In the following we will show that $G_{\overline{\theta}}(1-\pi,\sigma) > G_{\overline{\theta}}(\pi,\sigma)$ for all σ iff $\pi > 1/2$ and $G_{\overline{\theta}}(1-\pi,\sigma) < G_{\overline{\theta}}(\pi,\sigma)$ for all σ iff $\pi < 1/2$. Let define $H(\pi,\sigma) \equiv \frac{\Pi_{\overline{\theta}}(\pi,1-\sigma)}{\Pi_{\overline{\theta}}(\pi,\sigma)}$ and notice that:

$$\begin{gathered} G_{\overline{\theta}}(1-\pi,\sigma) > G_{\overline{\theta}}(\pi,\sigma) \Leftrightarrow \\ \Pi_{\overline{\theta}}(1-\pi,\sigma)\left[\tfrac{Q_L}{Q_S} - H(1-\pi,\sigma)\right] > \Pi_{\overline{\theta}}(\pi,\sigma)\left[\tfrac{Q_L}{Q_S} - H(\pi,\sigma)\right]. \end{gathered} \tag{5}$$

First observe that, for any σ, $\Pi_{\overline{\theta},L}(1-\pi,\sigma) > \Pi^{pe}_{\overline{\theta},L}(\pi,\sigma)$ iff $\pi > 1/2$. Indeed, simple algebraic calculus shows that:

$$\Pi_{\overline{\theta}}(1-\pi,\sigma) > \Pi_{\overline{\theta}}(\pi,\sigma) \Leftrightarrow (2\pi-1)\left(1-\lambda_{\overline{\theta}}(\sigma)\lambda_{\overline{\theta}}\right) > 0 \Leftrightarrow \pi > 1/2, \tag{6}$$

since both $\lambda_{\overline{\theta}}(\sigma)$ and $\lambda_{\overline{\theta}}$ are lower that 1. Second, some algebraic manipulation gives:

$$H(\pi,\sigma) = \frac{\left(\lambda_{\overline{\theta}}(1-\sigma) - \lambda_{\overline{\theta}}\right)\left(\pi + (1-\pi)\lambda_{\overline{\theta}}(\sigma)\right)}{\left(\lambda_{\overline{\theta}}(\sigma) - \lambda_{\overline{\theta}}\right)\left(\pi + (1-\pi)\lambda_{\overline{\theta}}(1-\sigma)\right)},$$

and since:

$$\frac{\partial H(\pi,\sigma)}{\partial\pi} = \frac{\sigma\lambda_{\overline{\theta}}\left(\lambda_{\overline{\theta}}(\sigma) - \lambda_{\overline{\theta}}(1-\sigma)\right)}{\left(\lambda_{\overline{\theta}}(\sigma) - \lambda_{\overline{\theta}}\right)\left(\pi + (1-\pi)\lambda_{\overline{\theta}}(1-\sigma)\right)^2} > 0,$$

because $\lambda_{\overline{\theta}}(\sigma) > \lambda_{\overline{\theta}}$ for any σ, we can conclude that $H(\pi,\sigma) > H(1-\pi,\sigma)$ iff $\pi > 1/2$ and, then:

$$\begin{aligned} \tfrac{Q_L}{Q_S} - H(1-\pi,\sigma) &< \tfrac{Q_L}{Q_S} - H(\pi,\sigma) \quad \forall\,\pi < 1/2 \\ \tfrac{Q_L}{Q_S} - H(1-\pi,\sigma) &> \tfrac{Q_L}{Q_S} - H(\pi,\sigma) \quad \forall\,\pi > 1/2. \end{aligned}$$

By combining this result with (6) and (5), we obtain that $G_{\underline{\theta}}(\pi,\sigma) = G_{\overline{\theta}}(1-\pi,\sigma)$ is larger than $G_{\overline{\theta}}(\pi,\sigma)$ iff $\pi > 1/2$.

This proves the claim for the case $\sigma_{\underline{\theta},NT} = \sigma_{\overline{\theta},NT} = 0$. The proof of the claim for other cases is similar and will be omitted. □

Proof of Proposition 6

In a flat market, both $\Pi^{se}_{\overline{\theta}}(1/2) = \Pi^{se}_{\underline{\theta}}(1/2)$ and $\Pi^{ne}_{\overline{\theta}}(1/2) = \Pi^{ne}_{\underline{\theta}}(1/2)$. As a consequence, $\lambda_{SQ_L}(1/2) = 1/\lambda_{BQ_L}(1/2)$ and $J(1/2) = 0$.

Consider now a bullish market. The price impact asymmetry expression can be written as:

$$J(\pi) = \frac{\pi(1-\pi)\phi}{(\pi + (1-\pi)\lambda_{SQ_L}(\pi))(\pi + (1-\pi)\lambda_{BQ_L}(\pi))},$$

where:

$$\phi \equiv \pi(2-\lambda_{SQ_L}(\pi)-\lambda_{BQ_L}(\pi))-(1-\pi)(2\lambda_{SQ_L}(\pi)\cdot\lambda_{BQ_L}(\pi)-\lambda_{SQ_L}(\pi)-\lambda_{BQ_L}(\pi)).$$

Notice that $J(\pi) \geq 0$ iff $\phi \geq 0$.

If the separating equilibrium prevails on both sides, the information content of a large sell is $\lambda_{SQ_L}(\pi) = \lambda^{se}_{SQ_L}$; the information content of a large purchase is $1\backslash\lambda_{BQ_L}(\pi) = 1/\lambda^{se}_{BQ_L} = \lambda^{se}_{SQ_L}$. The price impact asymmetry expression is negative since:

$$\phi^{se} = \frac{(1-2\pi)\left(\lambda^{se}_{SQ_L}-1\right)^2}{\lambda^{se}_{SQ_L}},$$

which is negative for all $\pi > 1/2$.

If the separating equilibrium prevails on the bid side and the pooling equilibrium prevails on the ask side of the market (from Proposition 4 we know that the reverse is not possible when $\pi > 1/2$), then $\lambda^{se}_{BQ_L} < \lambda^{pe}_{BQ_L}(\pi)$. Since:

$$\partial\phi/\partial\lambda_{BQ_L} = -\pi - (1-\pi)(2\lambda^{se}_{SQ_L}-1) < 0 \;\; \forall\pi,$$

and $\phi^{se} < 0$, we have also in this case $\phi^{pse} < \phi^{se} < 0$.

If the pooling equilibrium prevails on both sides of the market, from Proposition 5 we know that $\lambda^{pe}_{SQ_L}(\pi) > 1/\lambda^{pe}_{BQ_L}(\pi)$ for all $\pi > 1/2$. Moreover, when $\lambda_{SQ_L} = 1/\lambda_{BQ_L}$ then $\phi < 0$ for all $\pi > 1/2$, and $\partial\phi/\partial\lambda_{BQ_L} < 0$. As a consequence, $\phi^{pe} < 0$. □

Proof of Proposition 7

If the separating equilibrium prevails on both sides, the derivative of the price impact asymmetry expression is:

$$\frac{\partial J^{se}(\pi)}{\partial\pi} = \frac{\left(\lambda^{se}_{SQ_L}-1\right)^2}{\left(\pi\lambda^{se}_{SQ_L}+(1-\pi)\right)^2\left(\pi+(1-\pi)\lambda^{se}_{SQ_L}\right)^2}\cdot$$

$$\left[\lambda^{se}_{SQ_L}(1-6\pi+6\pi^2) - 2(\lambda^{se}_{SQ_L}-1)^2\pi^2(1-\pi)^2\right]$$

which is negative into a neighborhood of 1/2.

To conclude the proof, observe that in a weak bear market, the probability to observe a large buy order increases as π decreases, while the probability to observe a large sell order decreases as π decreases. Since the asset price is an increasing function of π, the difference in the information content of large buy and sell orders increases as the asset price goes down. □

Proof of Proposition 8

Since $\Pi^{ne}_{\overline{\theta}}(\pi) \leq \Pi^{ne}_{\underline{\theta}}(\pi)$ for all $\pi > 1/2$, we have that $\underline{\pi}_{\overline{\theta},S} \leq \underline{\pi}_{\underline{\theta},S}$. In a strong bear market, when $\pi \in \left[\underline{\pi}_{\overline{\theta},S}, \underline{\pi}_{\underline{\theta},S}\right]$, the probability to observe an information based large sell order is zero and the probability to observe an information based large buy order decreases as π decreases. □

Proposition 9 *If $Q_L/Q_S > \left(1 - \lambda_{\overline{\theta}}\right) / \left(\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}\right)$, then a separating equilibrium always prevails in a market with zero transaction costs.*

Proof

By condition 2 it follows that if a separating equilibrium prevails in the ask side of market then:

$$\frac{Q_L}{Q_S} \leq \frac{1 - \lambda_{\overline{\theta}}}{\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}}(\pi + (1-\pi)\lambda^{se}_{BQ_L}) \equiv f_{\overline{\theta}}(\pi). \tag{7}$$

If it prevails in the bid side of market then:

$$\frac{Q_L}{Q_S} \leq \frac{\lambda_{\underline{\theta}} - 1}{\lambda_{\underline{\theta}} - \lambda^{se}_{SQ_L}}(\pi + (1-\pi)\lambda^{se}_{SQ_L}) \equiv f_{\underline{\theta}}(\pi). \tag{8}$$

The proposition is proved immediately by noting that

$$\max_{\pi \in [0,\,1]} f_{\overline{\theta}}(\pi) = \max_{\pi \in [0,\,1]} f_{\underline{\theta}}(\pi) = \frac{1 - \lambda_{\overline{\theta}}}{\lambda^{se}_{BQ_L} - \lambda_{\overline{\theta}}}.$$

□

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