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Partnership, Reciprocity and Team Design

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Partnership, Reciprocity and Team Design

Giuseppe De Marco[*], **Giovanni Immordino**[**]

Abstract
This paper studies the impact of intention-based reciprocity preferences on the free-riding problem arising in partnerships. Our results suggest a tendency of efficient partnerships to consist of members whose sensitivity to reciprocity is – individually or jointly – sufficiently high. Sufficient conditions for the implementation of the efficient strategy profile require a reciprocity based sharing rule such that each partner gets a fraction of the output that is a percentage of his own reciprocity with respect to the overall reciprocity in the team. Finally, we introduce the concept of psychological strong Nash equilibrium and show that it allows for the unique and collusion-proof implementation of the efficient strategy profile.

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1 Introduction

It is widely believed and suggested by experimental evidence that social preferences exhibit an important intention-based component.[1] In this paper, we examine the impact of intention-based reciprocity preferences on the partnership framework. We study partnerships where partners jointly produce according to a non-stochastic technology, and share the resulting output among themselves. Partnerships appear one of the most natural environment for reciprocity to play a central role. In the partnership game, where the action sets represent comparable choices (efforts), we use the level of efforts as a measure of kindness. In this case, the reciprocity term in the psychological utility function does not depend on the material payoffs, capturing the idea that a partner may react badly to other partners free-riding regardless of everyone's rewards. It is generally accepted that partnerships are inefficient if the partners' actions are not verifiable. The argument is that some partner will shirk because he must share the marginal benefit of his effort, but he alone bears its cost. Holmstrom (1982) formalizes this argument showing that in certain differentiable, monotonic partnerships, no sharing rule can elicit an efficient set of actions.

We present necessary and sufficient conditions to implement the symmetric efficient strategy profile as a psychological Nash equilibrium.[2] Our first result, gives useful insights for the optimal design of the team by describing the psychological attitudes of the team members required to sustain a given strategy profile – for any given sharing rule. The second result, instead, takes a psychological characteristic of the partners (namely the minimal level of individual reciprocity) as given and finds a condition on the overall level of reciprocity and a sharing rule which implement the symmetric strategy profile as a psychological equilibrium. This reciprocity based sharing rule is such that each partner gets a fraction of the output that is a percentage of his own reciprocity with respect to the overall reciprocity in the team. However, even if the efficient strategy profile is sustained, there could be other (inefficient) strategy profiles which are sustained as a Nash equilibrium or as a psychological Nash equilibrium for the same set of sharing rules. Moreover, the conditions used to prove the previous results are not sufficient to sustain efficiency if some partners can collude. We introduce the concept of psychological strong Nash equilibrium and show that it solves both problems allowing for the unique and collusion-proof implementation of the efficient strategy profile.

Our paper builds on and extend three different literatures. First, after the seminal contribution by Holmstrom (1982) successive papers on partnership have shown that efficiency or near efficiency can be obtained in partnerships with a random technology (Matsushima 1989, Legros and Matsushima 1991, and Williams and Radner 1995), with risk-averse partners and random-sharing rules (Rasmusen 1987), with repeated play (Radner 1986), or finally through the use of mixed strategies, provided the partners have unlimited liability (Legros and Matthews 1993).

This is however, the first paper in which equilibrium-implementation results for efficient action profiles are obtained in one-shot, non-stochastic partnerships with symmetric

[1] See for instance Charness and Rabin (2002); Falk, Fehr and Fischbacher (2003a) and (2003b); Falk and Fischbacher (2006).

[2] Symmetric efficient strategy profiles arise naturally in partnerships with symmetric production functions.

production functions[3].

Second, the experimental evidence in Fehr, Gächter, and Kirchsteiger (1997) suggests that reciprocal motives contribute to the enforcement of contracts. In this vein, Dufwenberg and Kirchsteiger (2000), Englmaier and Leider (2008) and Netzer and Schmutzler (2010) all show that efficiency is generally increased when a materialistic principal interacts with a reciprocal agent[4]. We show that the efficiency-enhancing role of reciprocity, extends also to partnerships.

Finally, we contribute to the theoretical literature on psychological games (Geanakoplos, Pearce and Stacchetti 1989, Rabin 1993, Dufwenberg and Kirchsteiger 2004, Falk and Fischbacher 2006, Battigalli and Dufwenberg 2009) by extending the definition of strong Nash equilibrium that has been introduced by Aumann (1959) to environments with reciprocal players.

The paper is organized as follows. Section 2 introduces the general framework. In section 3 we provide sufficient and necessary conditions to implement the efficient strategy profile as a psychological Nash equilibrium. Section 4 deals with multiplicity, collusion and unique implementation of the efficient strategy profile. Section 5 concludes. All proofs are relegated to the Appendix.

2 The model

We first introduce our general set-up.

Material Payoffs and Efficiency. A partnership consists of a set of partners $N = \{1, ..., n\}$ with $n \geq 2$, a set of effort levels A_i for each partner, a disutility function $v_i : A_i \to \mathbb{R}$ for each partner and a symmetric production function $f : A \to \mathbb{R}$, where $A = \times_{i \in N} A_i$. Effort profile a results in output $y = f(a)$. Moreover, we assume that $A_i = \{1, ..., m\}$ for any i[5]. A sharing rule is a map $s : f(A) \to \mathbb{R}^n$ which determines each partner's share of output $s_i(y)$, and satisfies budget balance for all possible outputs,

$$\sum s_i(y) = y \text{ for all } y \in f(A).$$

The material payoff of partner i, is the function $u_i : A \to \mathbb{R}$ defined by $u_i(a) = s_i(f(a)) - v_i(a_i)$ for all $a \in A$.[6] Efficient actions are those which maximize the following welfare criterion,

$$W(a) \equiv f(a) - \sum v_i(a_i).$$

An efficient effort profile always exists in our setting and for the sake of simplicity is assumed to be unique[7]. We denote it as a^*, and the corresponding output as $y^* = f(a^*)$. Finally, a^* is said to be sustainable if there exists a sharing rule s such that a^* is a Nash equilibrium of the

[3] The assumpton of symmetric production functions makes the problem interesting implying that neither the identity of a shirker nor the one of a non-shirker is revealed after a deviation.

[4] Even if, as shown by Netzer and Schmutzler (2010), firms may not want to employ reciprocal workers.

[5] Indeed, with the only exception of Proposition 3, this assumption is not necessary to our analysis; in fact, what we really need is that $\cap_{i \in N} A_i \neq \emptyset$.

[6] To simplify notation we do not include in the utility function the dependence on the sharing rule s.

[7] When the psychological utility components are taken into account, the notion of overall utility efficiency

normal form game $\Gamma = \{N; (A_i)_{i\in N}; (u_i)_{i\in N}\}$. Notice that since the effort is not observable, compensation cannot be a direct function of a_i. The assumption of a symmetric production function implies that neither the identity of a shirker nor the one of a non-shirker is revealed after a deviation. As shown by Legros and Mathews (1993), partnerships with finite action sets and symmetric production functions cannot sustain efficiency in general.[8]

Next, we introduce some psychological features of our model as a first step in the construction of the partners psychological utility functions.

Beliefs and Reciprocity. Following the literature on intention-based reciprocity preferences, we denote by $b_{ij} \in A_j$ partner $i's$ beliefs about partner $j's$ strategy, and by $c_{iji} \in A_i$ partner $i's$ beliefs about partner $j's$ beliefs about partner $i's$ strategy. We denote by $\kappa_{ij}(a_i, b_{ij})$ the kindness of i to j and by $\lambda_{iji}(b_{ij}, c_{iji})$ partner $i's$ belief about how kind j is to him. In the definition of κ_{ij} and λ_{iji} we will deviate from the previous literature that uses the concept of player's equitable payoff (see for instance Rabin (1993) or Dufwenberg and Kirchsteiger (2004)) and we will instead use the partners' level of effort as a more direct measure of each agent contribution to the output of the partnership. Then, the kindness terms are defined to be

$$\kappa_{ij}(a_i, b_{ij}) = a_i - b_{ij}$$

and

$$\lambda_{iji}(b_{ij}, c_{iji}) = b_{ij} - c_{iji},$$

so that positive kindness from i to j arises if partner i contributes to the output with an effort level larger than the one he expects from partner j. Moreover, partner i will believe that j is kind to him if the effort he expects from partner j is larger than the one he believes partner j expects from partner i. In the general model of reciprocity (Rabin 1993, Dufwenberg and Kirchsteiger 2004), the strategy sets may represent choices of different nature so that the natural way to measure kindness is to look at the players' payoffs. Instead, in the partnership game, where the action sets represent comparable choices (efforts), we adopt the simpler approach to use the level of efforts as a more direct – and easier to test – measure of kindness. In this case, the reciprocity term in the psychological utility function does not depend on the material payoffs, capturing the idea that a partner may react badly to other partners free-riding regardless of everyone's rewards.[9]

Psychological Utility. To specify the psychological utility function for each player, we now introduce for every pair of players (i, j) the reciprocity term of i with respect to j. This

within the set of strategy profiles is not unequivocally defined since the psychological utility depends also on beliefs. For this reason the previous literature has focused on material efficiency. While we also follow this approach, Remark 1 below points out that for the functional form of the psychological payoffs used in our model, a symmetric strategy profile is efficient regardless of the psychological terms whenever outcomes are achieved by equilibrium beliefs. The previous argument implies that efficiency of a symmetric strategy profile is an unambiguous concept in our framework.

[8] Legros and Mathews (1993) also show that if partners can commit to pay fines, approximate efficiency is sustainable. None of our results relies on the assumption that partners can adopt a sharing rule which imposes a liability.

[9] Our definition of kindness, besides being better suited to our partnership application, has also the advantage to be completely unrelated to Pareto efficiency.

function assigns to each combination of kindness κ_{ij} and belief about reciprocated kindness λ_{iji} the disutility caused to player i by the mismatch between these two intentions. Let

$$h_{ij}(\kappa_{ij}, \lambda_{iji}) = -(\omega_i \kappa_{ij} - \lambda_{iji})^2,$$

be the functional form of such reciprocity terms. Those single-peaked preferences depend only on the distance between the kindness of i to j, and partner $i's$ belief about how kind j is to him[10]. Then, the overall (psychological) utility function of player i is defined by

$$U_i(a_i, a_{-i}, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) = s_i(f(a_i, a_{-i})) - v_i(a_i) + \rho_i \left[\sum_{j\neq i} h_{ij}(\kappa_{ij}(a_i, b_{ij}), \lambda_{iji}(b_{ij}, c_{iji})) \right], \tag{1}$$

and is made up by the sum of the material payoff $s_i(f(a_i, a_{-i})) - v_i(a_i)$ and the reciprocity term $\rho_i \left[\sum_{j\neq i} h_{ij}(\kappa_{ij}(a_i, b_{ij}), \lambda_{iji}(b_{ij}, c_{iji}) \right]$.

Parameters ρ_i and ω_i summarize the psychological characteristics of each player i: $\rho_i > 0$ measures the relative importance (the weight) of the psychological term with respect to the material payoff; instead, the parameter ω_i relates the relative importance of player i's intentions – towards the others – to his beliefs about other players' intentions. Each ω_i is assumed to be positive, meaning that, for a given λ_{iji}, the optimal kindness of player i (taking into account only h_{ij}) is $\kappa_{ij} = \lambda_{iji}/\omega_i$, which reciprocates kind behavior ($\lambda_{iji} > 0$) with kind behavior ($\kappa_{ij} > 0$) and unkind behavior ($\lambda_{iji} < 0$) with unkind behavior ($\kappa_{ij} < 0$). Moreover, the magnitude of ω_i affects the optimal kindness which increases in absolute value as ω_i decreases to zero and, when $\omega_i = 1$, perfectly reciprocates the believed kindness, i.e. $\kappa_{ij} = \lambda_{iji}$.

Finally, key for our results is how those parameters affect the psychological disutilities caused by deviating from the optimal kindness for each pair of players (i, j). These disutilities are related to the products $\rho_i \omega_i^2$ as follows: fix a pair of players (i, j) and the size ε of a deviation from the optimal kindness $\kappa_{ij} = \lambda_{iji}/\omega_i$, then the player i's disutility with respect to j is given by

$$-\rho_i(\omega_i(\lambda_{iji}/\omega_i + \varepsilon) - \lambda_{iji})^2 = -\rho_i \omega_i^2 \varepsilon^2$$

and it decreases to zero as $\rho_i \omega_i^2$ decreases to zero. Therefore, $\rho_i \omega_i^2$ measures the 'sensitivity' of partner i's reciprocity. In other words, the greater is $\rho_i \omega_i^2$ the more sensible to the psychological reciprocity is partner i. To keep the analysis as simple as possible we normalize the parameters ρ_i to one for every player i. Hence, the psychological utility function that will be considered in this paper is

$$U_i(a_i, a_{-i}, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) = s_i(f(a_i, a_{-i})) - v_i(a_i) + \left[\sum_{j\neq i} h_{ij}(\kappa_{ij}(a_i, b_{ij}), \lambda_{iji}(b_{ij}, c_{iji})) \right]. \tag{2}$$

Assuming that $\rho_i = 1$ does not alter qualitatively the results (see the concluding remarks) and allows us to focus on the parameters ω_i which capture the relative sensibility of each

[10] In Dufwenberg and Kirchsteiger (2004), $h_{ij}(\kappa_{ij}, \lambda_{iji}) = \kappa_{ij}\lambda_{iji}$, while Rabin (1993) chooses $h_{ij}(\kappa_{ij}, \lambda_{iji}) = \lambda_{iji}(1 + \kappa_{ij})$. In these functional forms the psychological term would disappear when λ_{iji} is equal to zero – indipendently from κ_{ij}. Since in our model λ_{iji} is equal to zero in the focal case where partners choose to exert the same level of effort, multiplicative (in λ_{iji}) functional forms are not well suited.

player with respect to others' kindness. Therefore, from now on, we will simply refer to ω_i^2 as the measure of partner's i reciprocity.

Psychological Equilibria. We conclude this section by defining our equilibrium concepts. We will mainly use the following definition of psychological Nash equilibrium[11]:

Definition 1 (Geanakoplos, Pearce and Stacchetti 1989). *A strategy profile* $(\overline{a}_1, \ldots, \overline{a}_n) \in A$ *is a psychological Nash equilibrium if for all* $i \in N$
i) $\overline{a}_i \in \arg\max_{a_i \in A_i} U_i(a_i, \overline{a}_{-i}, (\overline{b}_{i,j})_{j \neq i}, (\overline{c}_{i,j,i})_{j \neq i})$,
ii) $\overline{b}_{ij} = \overline{a}_j$ *and* $\overline{c}_{iji} = \overline{a}_i$ *for all* $j \neq i$.

Moreover, to allow for the unique and collusion-proof implementation of the efficient strategy profile, we will also be interested in the concept of strong Nash equilibrium, that has been introduced by Aumann (1959) for environments in which players can agree privately upon a joint deviation. In that case, any meaningful agreement by the whole set of players must be stable against deviations by all possible coalitions of players. Then, an equilibrium is said to be a strong Nash equilibrium if no subset of players, taking the actions of the others as fixed, can jointly deviate in a way that benefits all of them. More precisely:

Definition 2 (Aumann 1959). *A strategy profile* $(\overline{a}_1, \ldots, \overline{a}_n)$ *is a strong Nash equilibrium of the material game* Γ *if for all subset of players* $J \subseteq N$ *and for all* $a_J \in \prod_{j \in J} A_j$ *there exists a player* $i \in J$ *such that* $u_i(\overline{a}_J, \overline{a}_{-J}) \geq u_i(a_J, \overline{a}_{-J})$, *with* $\overline{a}_{-J} = (\overline{a}_j)_{j \notin J}$.

Building upon the work of Aumann we introduce the definition of psychological strong Nash equilibrium. The following definition is based on the idea that – since players commit ex-ante to a deviation – the deviants' beliefs should be consistent with the deviation itself.

Definition 3. *A strategy profile* $(\overline{a}_1, \ldots, \overline{a}_n)$ *is a psychological strong Nash equilibrium if for all subset of players* $J \subseteq N$ *and for all* $a_J \in \prod_{j \in J} A_j$ *there exists a player* $i \in J$ *such that*
i) $U_i(\overline{a}_i, \overline{a}_{-i}, (\overline{b}_{i,j})_{j \neq i}, (\overline{c}_{i,j,i})_{j \neq i}) \geq U_i(a_J, \overline{a}_{-J}, (b_{ij})_{j \neq i}), (c_{iji})_{j \neq i}))$, *with* $\overline{a}_{-J} = (\overline{a}_j)_{j \notin J}$
ii) $\overline{b}_{ij} = \overline{a}_j$ *and* $\overline{c}_{iji} = \overline{a}_i$ *for all* $j \neq i$,
iii) $b_{ij} = \overline{a}_j$ *and* $c_{iji} = \overline{a}_i$ *for all* $j \in N \setminus J$,
iv) $b_{ij} = a_j$ *and* $c_{iji} = a_i$ *for all* $j \in J \setminus \{i\}$.

Note that when we consider only deviations by singletons, this definition boils down to the definition of psychological Nash equilibrium. Moreover, it generalizes the Aumann's definition of strong Nash equilibrium which can be easily obtained removing the psychological term from the payoffs. In the next two sections we will introduce our results on implementation and unique implementation, respectively.

Remark 1: Turning back to the concept of efficiency, we now clarify why in our framework, when outcomes are achieved by equilibrium beliefs, this concept does not depend on psychological terms. The reason being that for the functional form of the psychological payoffs used in our model a symmetric strategy profile a^* remains efficient even when, in the definition of the welfare criterion $W(\cdot)$, material payoffs are replaced by psychological payoffs with correct beliefs. Indeed, when beliefs are correct (that is, $a_i = c_{iji}$ and $a_j = b_{i,j}$ for every

[11] In the rest of the paper $\overline{a}$ will usually denote an equilibrium and a^* a symmetric equilibrium.

pair of players $i, j \in N$), the psychological terms disappear for symmetric strategy profiles while they are negative for all other (asymmetric) strategy profiles. Moreover, the definitions of equilibrium above imply that beliefs are always correct when outcomes are achieved in equilibrium. Hence, material efficiency of symmetric profiles coincides to psychological efficiency (achieved by equilibrium beliefs) and therefore provides an unambiguous efficiency notion in our framework.

3 Implementation

In this section we provide necessary and sufficient conditions for a symmetric strategy profile (in particular the efficient symmetric strategy profile) to be implemented as a psychological Nash equilibrium of the partnership game with psychological utility functions defined as in (2).

We begin the section with an example which illustrates how reciprocity sustains efficiency.

Example 1: Consider a 2-player partnership model where each player has only two levels of effort $A_i = \{1, 2\}$ for $i = 1, 2$, the production function is defined by $f(2, 2) = 1/2$, $f(1, 2) = f(2, 1) = 1/3$ and $f(1, 1) = 0$, and the disutility functions are defined by $v_i(2) = 1/10$ and $v_i(1) = 0$ for $i = 1, 2$. Denote with $(s_1(1/2), s_2(1/2)) = (\alpha_1/2, \alpha_2/2)$, $(s_1(1/3), s_2(1/3)) = (\beta_1/3, \beta_2/3)$ with $\alpha_1 + \alpha_2 = 1$ and $\beta_1 + \beta_2 = 1$ and $(s_1(0), s_2(0)) = (\gamma, -\gamma)$. Then the partnership game is

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$\frac{\alpha_1}{2} - \frac{1}{10}, \frac{\alpha_2}{2} - \frac{1}{10}$	$\frac{\beta_1}{3} - \frac{1}{10}, \frac{\beta_2}{3}$
$a_1 = 1$	$\frac{\beta_1}{3}, \frac{\beta_2}{3} - \frac{1}{10}$	$\gamma, -\gamma.$

The welfare function is given by $W(2, 2) = 9/30$, $W(1, 2) = W(2, 1) = 7/30$ and $W(0, 0) = 0$. Now we show that the efficient strategy profile $(a_1^*, a_2^*) = (2, 2)$ cannot be sustained by any sharing rule, i.e., there are no $\alpha_1, \alpha_2, \beta_1, \beta_2$ satisfying $\alpha_1 + \alpha_2 = 1$ and $\beta_1 + \beta_2 = 1$ such that (a_1^*, a_2^*) is a Nash equilibrium of the game above. In fact suppose (a_1^*, a_2^*) is a Nash equilibrium then

$$\frac{\alpha_1}{2} - \frac{1}{10} \geq \frac{\beta_1}{3} \quad \text{and} \quad \frac{\alpha_2}{2} - \frac{1}{10} \geq \frac{\beta_2}{3}. \tag{3}$$

Taking the sum of the two inequalities we get $1/2 - 1/5 \geq 1/3$ which is impossible.
However, the strategy profile (a_1^*, a_2^*) is a psychological Nash equilibrium for suitable parameters ω_1 and ω_2. For the sake of simplicity assume that the two players have identical psychological payoffs, i.e. $\omega_1 = \omega_2 = \omega$, then, given the beliefs $b_{ij}^* = a_j^*$ and $c_{iji}^* = a_i^*$, the payoffs in the psychological game are

$$U_i(a_i, a_{-i}^*, b_{ij}^*, c_{iji}^*) = s_i(f(a_i, a_{-i}^*)) - v_i(a_i) - (\omega(a_i - 2) - (2 - 2))^2 \quad \forall a_i \in A_i.$$

Then, being

$$U_i(2, a_{-i}^*, b_{ij}^*, c_{iji}^*) = \frac{\alpha_i}{2} - \frac{1}{10} \quad \text{and} \quad U_i(1, a_{-i}^*, b_{ij}^*, c_{iji}^*) = \frac{\beta_i}{3} - \omega^2,$$

the efficient effort profile $(a_1^*, a_2^*) = (2, 2)$ is sustained if and only if $\omega^2 \geq \frac{1}{60}$.

Now we present some sufficient conditions to implement any symmetric strategy profile (in particular the symmetric efficient strategy profile) as a psychological Nash equilibrium. The first result concern the case of a given sharing rule while in the second result the sharing rule is endogenously given by the psychological characteristics of the team members. These results give useful insights for the design of the team and for the design of the sharing rule, respectively. Specifically, the first proposition describes the psychological attitudes of the team members required to sustain a given strategy profile for any given sharing rule. The second result, on the other hand, takes the minimal level of individual reciprocity as given and finds a condition on the overall level of reciprocity and a sharing rule which implement the symmetric strategy profile as a psychological equilibrium.

For every sharing rule s we define the maximum utility gain from partner's i unilateral deviation by

$$M_i^s(a^*) = \max_{a_i \in A_i} \left[u_i(a_i, a_{-i}^*) - u_i(a_i^*, a_{-i}^*) \right].$$

Then, the following result follows.

Proposition 1. *Let a^* be a symmetric strategy profile. If $\omega_i^2 \geq M_i^s(a^*)/(n-1)$ for every player i then a^* is a psychological Nash equilibrium of the game Γ with reciprocity parameters $(\omega_i)_{i \in N}$.*

Intuitively, if the parameter measuring how reciprocator is each partner is sufficiently high, then shirking – providing an effort level different from the one of the other team members – will cause a negative psychological effect for each of the team members cheated, so that the aggregated effect will more than compensate the benefit from deviation measured by $M_i^s(a^*)$.

To introduce the next sufficient condition for the sustainability of the symmetric strategy profile we introduce the following notation. First, the maximum production gain from unilateral deviation which – for a symmetric production function – is the same for all players and is equal to

$$F(a^*) = \max_{a_i \in A_i} \left[f(a_i, a_{-i}^*) - f(a_i^*, a_{-i}^*) \right] \quad \forall i \in N. \tag{4}$$

Second, let

$$\nu(a^*) = \max_{i \in N,\ a_i \in A_i} \left[v_i(a_i^*) - v_i(a_i) \right],$$

provides an upper bound to the incremental cost from unilateral deviation for the team's partners. Finally, we call *reciprocity based sharing rule* the one defined by

$$s_j(y) = \frac{\omega_j^2}{\sum_{i \in N} \omega_i^2} y \quad \forall j \in N,\ \forall y \in f(A), \tag{5}$$

where each partner j gets a fraction of the output y that is a percentage of his own reciprocity with respect to the overall reciprocity in the team.

Proposition 2. *Let a^* be a symmetric strategy profile and $\overline{\omega}^2 = \min\{\omega_1^2, \ldots, \omega_n^2\}$. If*

$$\sum_{i \in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}^2 (n-1)} \right) \geq \frac{F(a^*)}{n-1}, \tag{6}$$

then a^* is a psychological equilibrium for the reciprocity based sharing rule.

The previous proposition gives a sufficient condition on the overall reciprocity of the team (6) for the sustainability of a symmetric strategy profile with the reciprocity based sharing rule. Note that the left-hand side of (6) is always decreasing in $\nu(a^*)$, while it is increasing in the overall reciprocity iff the ratio $\nu(a^*)/\overline{\omega}^2$ is sufficiently small. This means that more incremental costs $\nu(a^*)$ are needed to compensate lower maximum production gain from unilateral deviation, fixed the overall reciprocity. Moreover, if the ratio $\nu(a^*)/\overline{\omega}^2$ is sufficiently small (free riding does not reduce costs too much or the minimal level of reciprocity is high enough), then more overall reciprocity is required to compensate a larger maximum production gain from unilateral deviation. While, if the ratio $\nu(a^*)/\overline{\omega}^2$ is large, the percentage of the output to the player with the minimal level of reciprocity must increase to compensate higher maximum production gain from unilateral deviation, implying that the overall reciprocity has to decrease (remaining above $n\overline{\omega}^2$). In the next example we show how those sharing rules can be constructed in a simple game.

Example 2: Consider the game in Example 1 and the efficient strategy profile $(a_1^*, a_2^*) = (2,2)$. One can check that $F(a^*) = -1/6$ and $\nu(a^*) = 1/10$. Fix for example $\overline{\omega}^2 = 1/20$, then condition (6) becomes

$$\sum_{i\in N} \omega_i^2 \left(1 - \frac{1/10}{1/20}\right) \geq -1/6 \implies \sum_{i\in N} \omega_i^2 \leq 1/6.$$

Therefore, if $\omega_j^2 = \overline{\omega}^2 = 1/20$ then

$$\omega_i^2 + \frac{1}{20} \leq \frac{1}{6} \implies \omega_i^2 \leq \frac{7}{60}.$$

Fix $\omega_i^2 = 7/60$, then the following sharing rule satisfies (5) and therefore sustains the efficient strategy profile a^*:[12]

$$\frac{\alpha_i}{2} = \frac{\omega_i^2}{\omega_i^2 + \omega_j^2} f(2,2) = \frac{7}{20}, \quad \frac{\alpha_j}{2} = \frac{\omega_j^2}{\omega_i^2 + \omega_j^2} f(2,2) = \frac{3}{20}$$

and

$$\frac{\beta_i}{3} = \frac{\omega_i^2}{\omega_i^2 + \omega_j^2} f(2,1) = \frac{7}{30}, \quad \frac{\beta_j}{2} = \frac{\omega_j^2}{\omega_i^2 + \omega_j^2} f(2,1) = \frac{3}{30}.$$

Finally, if we consider $\omega_1 = \omega_2 = \omega$ condition (6) becomes

$$2\omega^2 \left(1 - \frac{1/10}{\omega^2}\right) \geq -1/6,$$

which gives back the sufficient condition $\omega^2 \geq 1/60$ for the efficient strategy profile to be implemented by the equal sharing rule.

[12] This is easily checked since

$$\frac{7}{20} - \frac{1}{10} = \frac{\alpha_i}{2} - v_i(a_i^*) \geq \frac{\beta_i}{3} - \omega_i^2 = \frac{7}{30} - \frac{7}{60}$$
$$\frac{3}{20} - \frac{1}{10} = \frac{\alpha_i}{2} - v_j(a_j^*) \geq \frac{\beta_j}{3} - \omega_j^2 = \frac{3}{30} - \frac{1}{20}.$$

We now introduce some more notation. First, recall that $|A_i| = m$ for all $i \in N$, then if a^* is a symmetric strategy profile

$$\sum_{a_i \in A_i} \sum_{j \neq i} (a_i - a_j^*)^2 = \sum_{a_i \in A_i} (n-1)(a_i - a_i^*)^2 = m(n-1) \sum_{a_i \in A_i} \frac{(a_i - a_i^*)^2}{m} = m(n-1)V(a^*) \quad \forall i \in N \tag{7}$$

where $V(a^*)$ is a second-order moment with respect to a^*. We also denote with

$$\sum_{a_i \in A_i} [v_i(a_i^*) - v_i(a_i)] = m \sum_{a_i \in A_i} \frac{[v_i(a_i^*) - v_i(a_i)]}{m} = m\Delta v_i(a^*),$$

the mean incremental cost from unilateral deviation for partner i and with

$$\sum_{a_i \in A_i} \left[f(a_i, a_{-i}^*) - f(a_i^*, a_{-i}^*)\right] = mF_M(a^*), \tag{8}$$

the mean production gain from unilateral deviation. Then, the next proposition gives a necessary condition for a symmetric strategy profile to be implemented by a psychological equilibrium.

Proposition 3. *If the symmetric strategy profile a^* is a psychological Nash equilibrium for some sharing rule then*

$$\sum_{i \in N} \omega_i^2 \geq \frac{F_M(a^*) + \sum_{i \in N} \Delta v_i(a^*)}{(n-1)V(a^*)}. \tag{9}$$

Note that the condition in the previous proposition does not depend on a specific sharing rule. The next example shows that the necessary condition (9) is satisfied in the simple game of Example 1.

Example 3: Consider the game in Example 1 and the efficient strategy profile $(a_1^*, a_2^*) = (2, 2)$. One can check that $F_M(a^*) = \frac{1/3 - 1/2}{2} = -1/12$, $\Delta v_i(a^*) = \frac{1/10}{2}$ and $V(a^*) = 1/2$. Condition (9) then becomes

$$\sum_{i \in N} \omega_i^2 \geq \frac{-1/12 + 1/10}{1/2} = \frac{1}{30}.$$

Therefore, if $\omega_1 = \omega_2 = \omega$, the previous necessary condition gives again $\omega^2 \geq 1/60$.

4 Collusion and Unique Implementation

In the previous section, sufficient and necessary conditions were given in order to implement the efficient symmetric strategy profile as a psychological Nash equilibrium. However, even if the efficient strategy profile is sustained, there could be other (inefficient) strategy profiles which are sustained as Nash equilibrium or as a psychological Nash equilibrium for the same set of sharing rules.[13] Moreover, as shown in Example 6, the conditions used to prove

[13] Multiplicity of equilibria obviously implies players' coordination problems on the efficient strategy profile.

Propositions 1 and 2 are not sufficient to sustain efficiency if some partners can collude. In this section, we show that the concept of psychological strong Nash equilibrium solves both problems allowing for the unique and collusion-proof implementation of the efficient strategy profile. More precisely, we show that there exists a set of psychological parameters ω_i, for $i = 1, \ldots, n$ and corresponding sharing rules, such that the efficient strategy profile is the unique psychological strong Nash equilibrium of the game.

We begin the section showing, by way of an example, that reciprocity may destroy asymmetric equilibria.

Example 4: To show that reciprocity destroys asymmetric equilibria, consider again the game in Example 1 and fix, for instance, $\omega^2 = \frac{1}{30}$. It can be checked that the sharing rules $\alpha_1, \alpha_2, \beta_1, \beta_2$ sustain the efficient outcome $(a_1^*, a_2^*) = (2, 2)$ if and only if they satisfy the following conditions

$$\frac{2}{30} \leq \frac{\alpha_1}{2} - \frac{\beta_1}{3} \leq \frac{3}{30} \quad \text{and} \quad \frac{\alpha_2}{2} - \frac{\beta_2}{3} = \frac{5}{30} - \left[\frac{\alpha_1}{2} - \frac{\beta_1}{3}\right]. \tag{10}$$

Moreover, (10) implies that

$$\frac{\alpha_1}{2} - \frac{1}{10} \leq \frac{\beta_1}{3} \quad \text{and} \quad \frac{\alpha_2}{2} - \frac{1}{10} \leq \frac{\beta_2}{3},$$

with at least one strict inequality. Therefore, for every $\alpha_1, \alpha_2, \beta_1, \beta_2$ satisfying the sharing rules implied by (10), it results that – without reciprocity – at least one of the two strategy profiles $(1, 2)$ and $(2, 1)$ is a Nash equilibrium.[14]

Now we show that the psychological term destroys those asymmetric equilibria. Consider for example the asymmetric strategy profile $(1, 2)$, (the same arguments apply to the asymmetric strategy profile $(2, 1)$), and let $\bar{b}_{ij} = \bar{a}_j$ and $\bar{c}_{iji} = \bar{a}_i$ be the beliefs consistent with $(\bar{a}_1, \bar{a}_2)$. Then,

$$U_1(a_1, \bar{a}_2, \bar{b}_{12}, \bar{c}_{121}) = s_1(f(a_1, 2)) - v_1(a_1) - (\omega(a_1 - 2) - (2 - 1))^2 \quad \forall a_1 \in A_1$$

and

$$U_2(a_2, \bar{a}_1, \bar{b}_{21}, \bar{c}_{212}) = s_2(f(a_2, 1)) - v_2(a_2) - (\omega(a_2 - 1) - (1 - 2))^2 \quad \forall a_2 \in A_2.$$

Consider for instance player 1, the payoffs for $a_1 = 1$ and $a_1 = 2$ are, respectively

$$U_1(1, \bar{a}_2, \bar{b}_{12}, \bar{c}_{121}) = \frac{\beta_1}{3} - (\omega + 1)^2 \quad \text{and} \quad U_1(2, \bar{a}_2, \bar{b}_{12}, \bar{c}_{121}) = \frac{\alpha_1}{2} - \frac{1}{10} - 1..$$

Then,

$$U_1(2, \bar{a}_2, \bar{b}_{12}, \bar{c}_{121}) \geq U_1(1, \bar{a}_2, \bar{b}_{12}, \bar{c}_{121}) \iff \frac{\alpha_1}{2} - \frac{\beta_1}{3} \geq \frac{11}{10} - (\omega + 1)^2.$$

For $\omega^2 = \frac{1}{30}$ and for every sharing rule $\alpha_1, \alpha_2, \beta_1, \beta_2$ satisfying (10), it results that

$$\frac{\alpha_1}{2} - \frac{\beta_1}{3} \geq \frac{2}{30} > \frac{11}{10} - (\omega + 1)^2.$$

Hence the strategy profile $(1, 2)$ is not a psychological equilibrium.

[14] Suppose, for instance, that $(1, 2)$ is not a Nash equilibrium. Then, it follows that $\frac{\beta_2}{3} - \frac{1}{10} < \gamma$ and that $\frac{\beta_1}{3} - \frac{1}{10} \geq -\gamma$ (otherwise we would get $1/3 - 1/5 < 0$ which is impossible) so that $(2, 1)$ is a Nash equilibrium. Analogously, it can be shown that if $(2, 1)$ is not a Nash equilibrium then $(1, 2)$ has to be a Nash equilibrium.

The previous example seems to show that the psychological Nash equilibrium concept is sufficient to destroy asymmetric equilibria. However, this is true if the game has only two players. When the game has three or more players, unilateral deviations are not always enough to destroy asymmetric equilibria while, joint deviation of $n-1$ players based on psychological preferences can do the job. Below, we formalize this idea by providing sufficient conditions for the non-existence of asymmetric psychological strong Nash equilibria. In Proposition 4, the sharing rule is fixed and conditions are imposed on the players' reciprocity parameters. In Proposition 5, a condition is imposed on the overall level of reciprocity, given a minimal level of individual reciprocity (which can be considered as an exogenous parameter) and the sharing rule is given endogenously.

Denote with A^a the subset of all asymmetric strategy profiles of A and A^s the subset of all symmetric strategy profiles of A. Given a sharing rule s, denote with

$$\overline{\Phi}_i(s) = \max_{\overline{a}\in A^a} s_i(f(\overline{a})), \quad \underline{\Phi}_i(s) = \min_{a^*\in A^s} s_i(f(a^*)) \quad \text{and } \mu = \max_{a^*\in A} \nu(a^*).$$

Then, the first sufficient condition is the following:

Proposition 4. *Given a sharing rule s, if*

$$(\omega_i+1)^2 > \overline{\Phi}_i(s) - \underline{\Phi}_i(s) + \mu + (n-1)(m-1)^2 \quad \forall i \in N \tag{11}$$

then every asymmetric strategy profile is not a strong psychological equilibrium for the sharing rule s.

Denote with

$$\Phi = \max_{\overline{a}\in A^a} f(\overline{a}) - \min_{a^*\in A^s} f(a^*).$$

Then, the second sufficient condition is

Proposition 5. *Let $\overline{\omega}^2 = \min\{\omega_1^2,\ldots,\omega_n^2\}$. If*

$$\sum_{i\in N} \omega_i^2 \left(1 - \frac{\mu + (n-1)(m-1)^2}{\overline{\omega}^2}\right) > \Phi \tag{12}$$

then every asymmetric strategy profile is not a strong psychological equilibrium for the reciprocity based sharing rule.

The next example puts Propositions 4 and 5 to work.

Example 5: Consider again the game in example 1. Using the equal sharing rule, then $\overline{\Phi}_i(s) = 1/6$, $\underline{\Phi}_i(s) = 0$, $\mu = 1/10$. Condition (11) becomes

$$(\omega_i+1)^2 > 1/6 + 1/10 + 1 = 76/60 \implies \omega_i^2 > (\sqrt{76/60} - 1)^2 \simeq 16/1000.$$

Note that if $\omega_1^2 = \omega_2^2 = \omega^2 = 1/60$, not only the efficient strategy profile is sustained but there are no asymmetric equilibria since $1/60 > 16/1000$.

Being $\Phi = 1/3$, condition (12) becomes

$$\sum_{i\in N} \omega_i^2 \left(1 - \frac{11/10}{\overline{\omega}^2}\right) > 1/3$$

which, for $\omega_1^2 = \omega_2^2 = \omega^2$, can be rewritten as

$$2\omega^2\left(1 - \frac{11/10}{\omega^2}\right) > 1/3 \implies \omega^2 > 76/60.$$

A condition much stronger than the one arising from (11).

Below we present examples and sufficient conditions showing that the symmetric efficient strategy profile is the unique symmetric psychological strong Nash equilibrium. These results together with the previous results on the non-existence of asymmetric equilibria allow for the unique implementation of the efficient profile. More specifically, the first result shows that – with no additional assumptions – if the overall reciprocity is strong enough then the symmetric efficient profile is a psychological strong Nash equilibrium for the reciprocity based sharing rule. The second result instead, shows that, imposing symmetry of the partnership, the efficient strategy profile is the unique psychological strong Nash equilibrium for the reciprocity based sharing rule. The next example gives the intuition for these results.

Example 6: This example illustrates that if reciprocity is strong enough then the efficient allocation is the unique strong Nash psychological equilibrium of the game. Consider a 3-player partnership model where each player has only two levels of effort $A_i = \{1, 2\}$ for $i = 1, 2, 3$. The production function is defined by $f(2, 2, 2) = 1$, $f(1, 1, 1) = 1/10$ and $f(a) = 1/3$ for every $a \in A \setminus \{(2, 2, 2), (1, 1, 1)\}$, and the disutility functions are defined by $v_i(2) = 1/4$ and $v_i(1) = 0$ for $i = 1, 2, 3$. Denote with $(s_1(1), s_2(1), s_3(1)) = (\alpha_1, \alpha_2, \alpha_3)$, $(s_1(1/3), s_2(1/3), s_3(1/3)) = (\beta_1/3, \beta_2/3, \beta_3/3)$, $(s_1(1/10), s_2(1/10), s_3(1/10)) = (\delta_1/10, \delta_2/10, \delta_3/10)$ with $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\beta_1 + \beta_2 + \beta_3 = 1$ and $\delta_1 + \delta_2 + \delta_3 = 1$. Then the partnership game is

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$\alpha_1 - \frac{1}{4}, \alpha_2 - \frac{1}{4}, \alpha_3 - \frac{1}{4}$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3}, \frac{\beta_3}{3} - \frac{1}{4}$
$a_1 = 1$	$\frac{\beta_1}{3}, \frac{\beta_2}{3} - \frac{1}{4}, \frac{\beta_3}{3} - \frac{1}{4}$	$\frac{\beta_1}{3}, \frac{\beta_2}{3}, \frac{\beta_3}{3} - \frac{1}{4}$

$a_3 = 2$

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3} - \frac{1}{4}, \frac{\beta_3}{3} - \frac{1}{4}$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3}, \frac{\beta_3}{3}$
$a_1 = 1$	$\frac{\beta_1}{3}, \frac{\beta_2}{3} - \frac{1}{4}, \frac{\beta_3}{3}$	$\frac{\delta_1}{10}, \frac{\delta_1}{10}, \frac{\delta_3}{10}$

$a_3 = 1$

The welfare function is given by $W(2, 2, 2) = 1/4$, $W(2, 1, 2) = W(1, 2, 2) = W(2, 2, 1) = -1/12$, $W(1, 1, 2) = W(1, 2, 1) = W(2, 1, 1) = 1/12$ and $W(1, 1, 1) = 1/10$. Now we show that the efficient strategy profile $(a_1^*, a_2^*, a_3^*) = (2, 2, 2)$ cannot be sustained by any sharing rule, that is there are no $(\alpha_1, \alpha_2, \alpha_3)$, $(\beta_1, \beta_2, \beta_3)$ satisfying $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\beta_1 + \beta_2 + \beta_3 = 1$ such that (a_1^*, a_2^*, a_3^*) is a Nash equilibrium of the game above. Suppose that (a_1^*, a_2^*, a_3^*) is a Nash equilibrium then

$$\alpha_i - \frac{1}{4} \geq \frac{\beta_i}{3} \quad \text{for all } i = 1, 2, 3.$$

Taking the sum of the three inequalities we get $1 - 3/4 \geq 1/3$ which is impossible.
However, the strategy profile (a_1^*, a_2^*, a_3^*) is a psychological Nash equilibrium for suitable parameters ω_1, ω_2 and ω_3. Assume for simplicity that the three players have identical psychological payoffs, i.e. $\omega_1 = \omega_2 = \omega_3 = \omega$, then, given the beliefs $b_{ij}^* = a_j^*$ and $c_{iji}^* = a_i^*$, the partners' payoff in the psychological game is

$$U_i(a_i, a_{-i}^*, (b_{ij}^*)_{j \neq i}, (c_{iji}^*)_{j \neq i})$$

$$= s_i(f(a_i, a_{-i}^*)) - v_i(a_i) - (\omega(a_i - 2) - (2 - 2))^2 - (\omega(a_i - 2) - (2 - 2))^2 \quad \forall a_i \in A_i.$$

Then, being

$$U_i(2, a_{-i}^*, (b_{ij}^*)_{j\neq i}, (c_{iji}^*)_{j\neq i}) = \alpha_i - \frac{1}{4} \quad \text{and} \quad U_i(1, a_{-i}^*, (b_{ij}^*)_{j\neq i}, (c_{iji}^*)_{j\neq i}) = \frac{\beta_i}{3} - 2\omega^2,$$

the efficient effort profile $(a_1^*, a_2^*, a_3^*) = (2, 2, 2)$ is sustained if and only if $\omega^2 \geq \frac{1}{72}$. Fix for instance $\omega^2 = \frac{1}{72}$. It can be checked that the sharing rules $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ sustain $(a_1^*, a_2^*, a_3^*) = (2, 2, 2)$ if and only if $U_i(2, a_{-i}^*, (b_{ij}^*)_{j\neq i}, (c_{iji}^*)_{j\neq i})) = U_i(1, a_{-i}^*, (b_{ij}^*)_{j\neq i}, (c_{iji}^*)_{j\neq i}))$ for every player i. Therefore, they sustain efficiency iff

$$\alpha_i = \frac{\beta_i}{3} + \frac{2}{9} \quad \text{for all } i = 1, 2, 3. \tag{13}$$

Replacing the previous sharing rules (13) in the partnership game we get

$a_3 = 2$	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$\frac{\beta_1}{3} - \frac{1}{36}, \frac{\beta_2}{3} - \frac{1}{36}, \frac{\beta_3}{3} - \frac{1}{36},$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3}, \frac{\beta_3}{3} - \frac{1}{4}$
$a_1 = 1$	$\frac{\beta_1}{3}, \frac{\beta_2}{3} - \frac{1}{4}, \frac{\beta_3}{3} - \frac{1}{4}$	$\frac{\beta_1}{3}, \frac{\beta_2}{3}, \frac{\beta_3}{3} - \frac{1}{4}$

$a_3 = 1$	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3} - \frac{1}{4}, \frac{\beta_3}{3}$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3}, \frac{\beta_3}{3}$
$a_1 = 1$	$\frac{\beta_1}{3} - \frac{1}{4}, \frac{\beta_2}{3}, \frac{\beta_3}{3}$	$\frac{\delta_1}{10}, \frac{\delta_1}{10}, \frac{\delta_3}{10}$

We now show that this material game is not collusion-proof and for every pair of players – say i and j – it is always profitable a joint deviation from the efficient strategy profile $(a_1^*, a_2^*, a_3^*) = (2, 2, 2)$ to the strategy profile in which both i and j exert effort equal to 1 given that their opponent exerts effort equal to 2.[15] In other words, the conditions used to prove Propositions 1 and 2 are not sufficient to sustain efficiency if some partners can collude. Indeed, we show that for $\omega^2 = \frac{1}{72}$ this joint deviation is profitable also in the psychological game. However, we also show that if ω^2 is high enough then (a_1^*, a_2^*, a_3^*) is the unique strong Nash psychological equilibrium of the game. For every joint deviation of pairs of players, the general payoff of the deviants (say i and j) is

$$U_i(a_i = 1, a_j = 1, a_k = 2; b_{i,j} = 1, b_{i,k} = 2; c_{iji} = 1, c_{iki} = 2) =$$

$$s_i(f(1, 1, 2)) - v_i(1) - (\omega(1-1) - (1-1))^2 - (\omega(1-2) - (2-2))^2 = \frac{\beta_i}{3} - \omega^2$$

and

$$U_j(a_j = 1, a_i = 1, a_k = 2; b_{j,i} = 1, b_{j,k} = 2; c_{jij} = 1, c_{jkj} = 2) =$$

$$s_j(f(1, 1, 2)) - v_j(1) - (\omega(1-1) - (1-1))^2 - (\omega(1-2) - (2-2))^2 = \frac{\beta_j}{3} - \omega^2,$$

while the payoff from playing the equilibrium candidate is

$$U_i(a_i^*, a_{-i}^*, (b_{ij}^*)_{j\neq 1}, (c_{iji}^*)_{j\neq 1}) = \alpha_i - \frac{1}{4}.$$

Consider a joint deviation of players 1 and 2 from (a_1^*, a_2^*, a_3^*) and fix $\omega^2 = 1/72$, we have that

$$\frac{\beta_i}{3} - \omega^2 > \frac{\beta_i}{3} - \frac{1}{36}, \qquad for\ i = 1, 2$$

[15] Notice that in this case their payoff would be $\frac{\beta_i}{3} = \frac{\beta_j}{3} > \frac{\beta_i}{3} - \frac{1}{36} = \frac{\beta_j}{3} - \frac{1}{36}$.

then we have that for sharing rules satisfying (13)

$$U_1(1,1,2;b_{1,2}=1,b_{1,3}=2;c_{121}=1,c_{131}=2) > U_1(a_1^*,a_{-1}^*,(b_{1j}^*)_{j\neq 1},(c_{1j1}^*)_{j\neq 1})$$

and

$$U_2(1,1,2;b_{2,1}=1,b_{2,3}=2;c_{212}=1,c_{232}=2) > U_2(a_2^*,a_{-2}^*,(b_{2j}^*)_{j\neq 2},(c_{2j2}^*)_{j\neq 2}).$$

In words, for $\omega^2 = 1/72$, (a_1^*, a_2^*, a_3^*) cannot be a psychological strong Nash equilibrium. More generally, a joint deviation of players i and j is not profitable if

$$\frac{\beta_i}{3} - \omega^2 \leq \alpha_i - \frac{1}{4} \quad \text{or} \quad \frac{\beta_j}{3} - \omega^2 \leq \alpha_j - \frac{1}{4}.$$

This latter condition is achieved, for instance, if $\alpha_i = \beta_i = 1/3$ and $\omega^2 \geq 1/36$. Therefore, for every $\omega^2 \geq 1/36$, the equal sharing rule prevents the joint deviation of any pair of players from the efficient strategy profile. Moreover, in the joint deviation of the grand coalition (the three players) the psychological payoffs consistent with the deviation are equal to the material payoffs in the deviation. Hence, it can be checked that the equal sharing rule prevents also from the deviation of the grand coalition. Finally, since the psychological payoffs tend to destroy asymmetric equilibria, it is possible to find $\omega^2 \geq 1/36$ such that there are no asymmetric psychological equilibria. Thus, for such ω^2 the efficient strategy profile is the unique strong Nash psychological equilibrium for the equal sharing rule.

We now introduce the maximum production gain from deviating for coalitions of size k which – for a symmetric production function – is equal across players, that is

$$F_k(a^*) = \max_{a_J \in A_J} \left[f(a_J, a_{-J}^*) - f(a_J^*, a_{-J}^*) \right] \quad \forall J \subset N \text{ with } |J| = k. \tag{14}$$

Then, the next result follows.

Proposition 6. *Let a^* be a symmetric efficient strategy profile. If*

$$\sum_{i \in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}^2} \right) \geq \max_{k \in \{1,\ldots,n-1\}} F_k(a^*) \tag{15}$$

then a^* is a psychological strong Nash equilibrium for the reciprocity based sharing rule.

Notice that condition (15) is *stronger* than condition (6) since

$$\sum_{i \in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}^2 (n-1)} \right) \geq \sum_{i \in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}^2} \right) \geq \max_{k \in \{1,\ldots,n-1\}} F_k(a^*) \geq \frac{F(a^*)}{n-1},$$

so that for a^* to be a psychological strong Nash equilibrium the condition that must be imposed on the team's psychological traits are more demanding than the ones needed to implement a^* as a simple psychological Nash equilibrium.

The following proposition shows that once we impose symmetry the equilibrium will be unique.

Proposition 7. *Let a^* be a symmetric efficient strategy profile and assume that condition (15) holds. If the following conditions hold:*

i) Costs are symmetric, i.e., $v_i(y) = v(y)$ for every $i \in N$ and for every $y \in f(A)$;

ii) Reciprocity is equal across players, i.e., $\omega_i = \omega$ for every $i \in N$.

then a^* is the unique symmetric psychological strong Nash equilibrium for the reciprocity based sharing rule.

Example 7: Here we apply the conditions of Propositions 6 and 7 to the game in Example (6). Being $F_1(a^*) = F_2(a^*) = -2/3$, $\nu(a^*) = 1/4$, the sufficient condition (6) for symmetric psychological Nash equilibria gives

$$\sum_{i \in N} \omega_i^2 \left(1 - \frac{1/4}{2\overline{\omega}^2}\right) \geq \frac{(-2/3)}{2},$$

which becomes $\omega^2 \geq 1/72$ when $\omega^2 = \omega_i^2$ for all $i \in N$. This previous condition is also necessary since, being $F_M(a^*) = -1/3$, $\Delta v_i(a^*) = 1/8$ and $V(a^*) = 1/2$, the necessary condition (9) gives

$$\sum_{i \in N} \omega_i^2 \geq \frac{-1/3 + 3/8}{2(1/2)}$$

that becomes $\omega^2 \geq 1/72$ when $\omega^2 = \omega_i^2$ for all $i \in N$. The sufficient condition (15) for symmetric psychological strong Nash equilibria gives

$$\sum_{i \in N} \omega_i^2 \left(1 - \frac{1/4}{\overline{\omega}^2}\right) \geq -2/3,$$

which simplifies to $\omega^2 \geq 1/36$ when $\omega^2 = \omega_i^2$ for all $i \in N$. Therefore, in light of the previous Propositions, if $\omega^2 \geq 1/36$ the efficient strategy profile a^* is the unique symmetric strong Nash equilibrium.

5 Concluding remarks

Even the best people do not always work together as well as they could. Simply assigning a group of people to a task, does not magically turn them into a well-functioning team. Basic skills for managing a team may include: how to set agendas, leading priority setting, leading without dominating, project management skills, encouraging and facilitating a discussion, monitoring team progress, and so on. However, our results indicate that, how well a partnership can overcome free riding depends between other things on the set of partners. In a world in which this choice is endogenous, there should be a tendency for the most efficient partnerships to form. Our efficiency results suggest a tendency of partnerships to consist of members whose sensitivity to reciprocity is – individually or jointly – sufficiently high.

Our theory is a first step in understanding the impact of intention-based reciprocity preferences on the design of a team and on the design of sharing rules that lead to efficiency in the simple and relevant partnership framework where the efficient strategy profile is symmetric. The peculiarity of the partnership model is that it allows to consider measures of kindness which depend only on effort and not on the particular material preferences representation. This captures the idea that a partner may react badly to other partners free-riding regardless of rewards. To conclude we discuss some possible extensions.

Contingent reciprocity

To focus on the fact that kindness depends only on effort and not on the particular material preferences representation, in the previous analysis, we considered constant reciprocity parameters and find conditions on such parameters which allow for the (unique) implementation of the efficient strategy profile. A more general analysis could take into account reciprocity parameters which depend on the sharing rule. However, this approach would require to use the concept of dynamic psychological equilibrium (Battigalli and Dufwenberg 2009) and its refinements.

Weighted psychological utility

When, in the psychological utility functions, ρ_i is not normalized to 1 (as in (1)), similar results for the implementation and the unique implementation of the symmetric efficient strategy profile can also be obtained by mimicking the arguments contained in the proofs and imposing the same conditions on the terms $\rho_i \omega_i^2$ (instead of ω_i^2). For instance, in this case, the reciprocity based sharing rule takes the following form

$$s_j(y) = \frac{\rho_j \omega_j^2}{\sum_{i \in N} \rho_i \omega_i^2} y \quad \forall j \in N, \ \forall y \in f(A). \tag{16}$$

Non-symmetric or stochastic production

The assumption of symmetric production functions makes the problem interesting implying that neither the identity of a shirker nor the one of a non-shirker is revealed after a deviation. However, the shirker's identity might remain unrevealed even for some non-symmetric or stochastic production functions. We highlight that the unique, collusion-proof implementation of the symmetric efficient strategy profile does not depend on the symmetry of the production function. Indeed, – mimicking the arguments contained in the proofs – Proposition 1, 2 and 6 can be generalized substituting the new (non-symmetric or stochastic) production function in $M_i^s(a^*)$ and $F_k(a^*)$.

6 Appendix

Proof of Proposition 1. From the assumptions it follows that, for every player i,

$$\sum_{j \neq i} \omega_i^2 (a_i - b_{i,j}^*)^2 \geq (n-1)\omega_i^2 \geq M_i^s(a^*) \geq u_i(a_i, a_{-i}^*) - u_i(a_i^*, a_{-i}^*) \quad \forall a_i \in A_i.$$

Hence

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) - \sum_{j \neq i} \omega_i^2 (a_i - b_{i,j}^*)^2 \quad \forall a_i \in A_i.$$

Being

$$U_i(a_i^*, a_{-i}^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) = u_i(a_i^*, a_{-i}^*)$$

and

$$U_i(a_i, a_{-i}^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) = u_i(a_i, a_{-i}^*) - \sum_{j\neq i} \omega_i^2 (a_i - b_{i,j}^*)^2$$

we get the assertion. ■

The following Lemma is required for the proofs of the main Propositions.

Lemma 1. *Let a^* be a symmetric strategy profile, $1 \leq k \leq n-1$ and $\overline{\omega}_k^2 = \min\{\omega_1^2, \ldots, \omega_{n-k+1}^2\}$. If*

$$\sum_{i\in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}_k^2 (n-k)}\right) \geq \frac{F_k(a^*)}{n-k} \tag{17}$$

then, given the sharing rule defined by

$$s_j(y) = \frac{\omega_j^2}{\sum_{i\in N} \omega_i^2} y \quad \forall j \in N,\ \forall y \in f(A).$$

for all subset of players $J \subseteq N$ with $|J| = k$ and for all $a_J \in \prod_{j\in J} A_j$ there exists a player $i \in J$ such that i)-iv) in Definition 3 are satisfied.

Proof of Lemma 1. Let $J \subseteq N$ with $|J| = k$, then there obviously exists a player $i \in \{1, \ldots, n-k+1\} \cap J$. From (17), it follows that

$$(n-k) \sum_{i\in N} \omega_i^2 \geq F_k(a^*) + \frac{\nu(a^*) \sum_{i\in N} \omega_i^2}{\overline{\omega}_k^2}$$

Then

$$(n-k) \geq \frac{F_k(a^*)}{\sum_{i\in N} \omega_i^2} + \frac{\nu(a^*)}{\overline{\omega}_k^2}.$$

Let $(b_{i,j}^*)_{j\neq i}$ and $(c_{i,j,i}^*)_{j\neq i}$ be the beliefs of player i consistent with the symmetric strategy profile a^*, i.e., $b_{ij}^* = a_j^* = c_{iji}^* = a_i^*$ for all $j \neq i$. Let $a_i \neq a_i^*$, then $(n-k) \leq \sum_{j\in N\setminus J} (a_i - b_{i,j}^*)^2$. It follows that for every player $i \in \{1, \ldots, n-k+1\} \cap J$,

$$\omega_i^2 \sum_{j\in N\setminus J} (a_i - b_{i,j}^*)^2 \geq \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} F_k(a^*) + \frac{\nu(a^*)\omega_i^2}{\overline{\omega}_k^2} \geq \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} F_k(a^*) + \nu(a^*). \tag{18}$$

By definition

$$F_k(a^*) \geq f(a_J, a_{-J}^*) - f(a_J^*, a_{-J}^*) \quad \forall a_J \in A_J \quad \text{and} \quad \nu(a^*) \geq v_i(a_i^*) - v_i(a_i) \quad \forall a_i \in A_i.$$

Hence from (18)

$$\omega_i^2 \sum_{j\in N\setminus J} (a_i - b_{i,j}^*)^2 \geq \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} \left[f(a_J, a_{-J}^*) - f(a_J^*, a_{-J}^*)\right] + v_i(a_i^*) - v_i(a_i) \quad \forall a_J \in A_J.$$

Therefore

$$\frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a_J^*, a_{-J}^*) - v(a_i^*) \geq \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a_J, a_{-J}^*) - v(a_i) - \sum_{j\in N\setminus J} \omega_i^2 (a_i - b_{i,j}^*)^2 \quad \forall a_J \in A_J.$$

Being

$$U_i(a_J^*, a_{-J}^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a_J^*, a_{-J}^*) - v(a_i^*)$$

and being

$$U_i(a_i, a_{-i}^*, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a_i, a_{-i}^*) - v(a_i) - \sum_{j\in N\setminus J} \omega_i^2 (a_i - b_{i,j}^*)^2 \quad \forall a_i \in A_i,$$

where
1)$b_{ij} = a_j^*$ and $c_{iji} = a_i^*$ for all $j \in N \setminus J$
2) $b_{ij} = a_j$ and $c_{iji} = a_i$ for all $j \in J \setminus \{i\}$,
the assertion follows. ■

Proof of Proposition 2. The proof is a direct application of Lemma 1 with $k = 1$, $F_1(a^*) = F(a^*)$ and $\overline{\omega}_1^2 = \overline{\omega}^2$. ■

Proof of Proposition 3. Suppose a^* is a symmetric psychological equilibrium for a sharing rule s then, for player $i \in J$ it results that

$$\omega_i^2 \sum_{j\neq i} (a_i - b_{i,j}^*)^2 \geq s_i(f(a_i, a_{-i}^*)) - s_i(f(a_i^*, a_{-i}^*)) + v_i(a_i^*) - v_i(a_i) \quad \forall a_i \in A_i$$

therefore

$$\sum_{a_i\in A_i} \omega_i^2 \sum_{j\neq i} (a_i - b_{i,j}^*)^2 \geq \sum_{a_i\in A_i} \left[s_i(f(a_i, a_{-i}^*)) - s_i(f(a_i^*, a_{-i}^*))\right] + \sum_{a_i\in A_i} \left[v_i(a_i^*) - v_i(a_i)\right].$$

Hence

$$\sum_{i\in N}\left[\sum_{a_i\in A_i} \omega_i^2 \sum_{j\neq i} (a_i - b_{i,j}^*)^2\right] \geq \sum_{i\in N}\left[\sum_{a_i\in A_i} \left[s_i(f(a_i, a_{-i}^*)) - s_i(f(a_i^*, a_{-i}^*))\right] + \sum_{a_i\in A_i} \left[v_i(a_i^*) - v_i(a_i)\right]\right].. \tag{19}$$

Since

$$\sum_{i\in N}\left[\sum_{a_i\in A_i} \omega_i^2 \sum_{j\neq i} (a_i - b_{i,j}^*)^2\right] = \sum_{i\in N} \omega_i^2 \left[m(n-1)V(a^*)\right],$$

$$\sum_{i\in N}\left[\sum_{a_i\in A_i} \left[s_i(f(a_i, a_{-i}^*)) - s_i(f(a_i^*, a_{-i}^*))\right]\right] = \sum_{a_i\in A_i} \left[f(a_i, a_{-i}^*) - f(a_i^*, a_{-i}^*)\right] = mF_M(a^*)$$

and

$$\sum_{i\in N}\left[\sum_{a_i\in A_i} \left[v_i(a_i^*) - v_i(a_i)\right]\right] = \sum_{i\in N} m\Delta v_i(a^*),$$

from (19) it follows that

$$\sum_{i\in N}\omega_i^2\left[(n-1)V(a^*)\right] \geq F_M(a^*) + \sum_{i\in N}\Delta v_i(a^*).$$

Hence the assertion follows. ■

Proof of Proposition 4. Consider an asymmetric strategy profile $\overline{a}$, let $\overline{a}_q = \max_{i\in N}\overline{a}_i$ and $J = \{i \in N \mid \overline{a}_i \neq \overline{a}_q\}$. Consider the joint deviation a^*_J of coalition J where $a^*_j = \overline{a}_q$ for all $j \in J$ and, with an abuse of notation, denote with a^* the symmetric strategy profile in which $a^*_i = \overline{a}_q$ for every $i \in N$. From (11) it follows that for every player i

$$(\omega_i+1)^2 > \overline{\Phi}_i(s) - \underline{\Phi}_i(s) + \mu + (n-1)(m-1)^2 \geq s_i(f(\overline{a})) - s_i(f(a^*)) + \mu + (n-1)(m-1)^2 \geq$$

$$s_i(f(\overline{a})) - s_i(f(a^*)) + v_i(a^*_i) - v_i(\overline{a}_i) + (n-1)(m-1)^2$$

which finally implies that

$$s_i(f(a^*)) - v_i(a^*_i) - (n-1)(m-1)^2 > s_i(f(\overline{a})) - v_i(\overline{a}_i) - (\omega_i+1)^2 \tag{20}$$

Now, let $i \in J$ and consider the psychological payoff of player i consistent with the strategy profile $\overline{a}$:

$$U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) = s_i(f(\overline{a})) - v(\overline{a}_i) - \sum_{j\neq i}(\omega_i(\overline{a}_i - \overline{b}_{ij}) - (\overline{b}_{ij} - \overline{c}_{iji}))^2$$

with $\overline{b}_{ij} = \overline{a}_j$ and $\overline{c}_{iji} = \overline{a}_i$ for all $j \neq i$, therefore

$$U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) = s_i(f(\overline{a})) - v(\overline{a}_i) - \sum_{j\neq i}(\omega_i(\overline{a}_i - \overline{a}_j) - (\overline{a}_j - \overline{a}_i))^2 =$$

$$= s_i(f(\overline{a})) - v(\overline{a}_i) - \sum_{j\neq i}((\omega_i+1)(\overline{a}_i - \overline{a}_j))^2 \leq s_i(f(\overline{a})) - v(\overline{a}_i) - (\omega_i+1)^2. \tag{21}$$

Consider the psychological payoff of player i consistent with the deviation a^*:

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) = s_i(f(a^*)) - v(a^*_i) - \sum_{j\neq i}(\omega_i(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2$$

where $a^*_i = \overline{a}_q$, $b^*_{ij} = \overline{a}_q$ and $c^*_{iji} = \overline{a}_q$ if $j \in J$ and $b^*_{ij} = \overline{a}_j$ and $c^*_{iji} = \overline{a}_i$ if $j \notin J$. Therefore

$$\sum_{j\neq i}(\omega_i(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2 = \sum_{j\in J}(\omega_i(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2 + \sum_{j\notin J}(\omega(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2 =$$

$$\sum_{j\notin J}(\omega_i(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2 = \sum_{j\notin J}(-(\overline{a}_q - \overline{a}_j))^2 \leq \sum_{j\notin J}(m-1)^2 \leq (n-1)(m-1)^2$$

Hence

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) \geq s_i(f(a^*)) - v(a^*_i) - (n-1)(m-1)^2 \tag{22}$$

Hence from (20,21,22) it follows that

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) > U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) \quad \forall i \in J$$

and therefore $\overline{a}$ is not a strong Nash psychological equilibrium. ∎

Proof of Proposition 5. Consider an asymmetric strategy profile $\overline{a}$, let $\overline{a}_q = \max_{i\in N} \overline{a}_i$ and $J = \{i \in N \mid \overline{a}_i \neq \overline{a}_q\}$. Consider the joint deviation a^*_J of coalition J where $a^*_j = \overline{a}_q$ for all $j \in J$ and, with an abuse of notation, denote with a^* the symmetric strategy profile in which $a^*_i = \overline{a}_q$ for every $i \in N$. From (12) it follows that

$$1 > \frac{\Phi}{\sum_{i\in N}\omega_i^2} + \frac{\mu + (n-1)(m-1)^2}{\overline{\omega}^2} \implies \omega_i^2 > \frac{\omega_i^2\Phi}{\sum_{i\in N}\omega_i^2} + \frac{\omega_i^2(\mu + (n-1)(m-1)^2)}{\overline{\omega}^2}$$

Since $(\omega_i + 1)^2 > \omega_i^2$ then

$$(\omega_i + 1)^2 > \frac{\omega_i^2\Phi}{\sum_{i\in N}\omega_i^2} + \frac{\omega_i^2(\mu + (n-1)(m-1)^2)}{\overline{\omega}^2} \geq$$

$$\frac{\omega_i^2}{\sum_{i\in N}\omega_i^2}\left[f(\overline{a}) - f(a^*)\right] + v_i(a^*_i) - v_i(\overline{a}_i) + (n-1)(m-1)^2$$

which finally implies that

$$\frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a^*) - v_i(a^*_i) - (n-1)(m-1)^2 > \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(\overline{a}) - v_i(\overline{a}_i) - (\omega_i+1)^2 \tag{23}$$

Following the same steps in the proof of Proposition (4), let $i \in J$ and consider the psychological payoff of player i consistent with the strategy profile $\overline{a}$:

$$U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(\overline{a}) - v_i(\overline{a}_i) - \sum_{j\neq i}(\omega_i(\overline{a}_i - \overline{b}_{ij}) - (\overline{b}_{ij} - \overline{c}_{iji}))^2$$

with $\overline{b}_{ij} = \overline{a}_j$ and $\overline{c}_{iji} = \overline{a}_i$ for all $j \neq i$; then

$$U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) \leq \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(\overline{a}) - v_i(\overline{a}_i) - (\omega_i+1)^2. \tag{24}$$

The psychological payoff of player i consistent with the deviation a^* is

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a^*) - v_i(a^*_i) - \sum_{j\neq i}(\omega_i(a^*_i - b^*_{ij}) - (b^*_{ij} - c^*_{iji}))^2$$

where $a^*_i = \overline{a}_q$, $b^*_{ij} = \overline{a}_q$ and $c^*_{iji} = \overline{a}_q$ if $j \in J$ and $b^*_{ij} = \overline{a}_j$ and $c^*_{iji} = \overline{a}_i$ if $j \notin J$. Hence

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) \geq \frac{\omega_i^2}{\sum_{i\in N}\omega_i^2} f(a^*) - v_i(a^*_i) - (n-1)(m-1)^2 \tag{25}$$

Hence from (23,24,25) it follows that

$$U_i(a^*, (b^*_{i,j})_{j\neq i}, (c^*_{i,j,i})_{j\neq i}) > U_i(\overline{a}, (\overline{b}_{i,j})_{j\neq i}, (\overline{c}_{i,j,i})_{j\neq i}) \quad \forall i \in J$$

and therefore $\overline{a}$ is not a strong Nash psychological equilibrium. ■

Proof of Proposition 6. Since $\overline{\omega}^2 = \overline{\omega}_1^2 \leq \overline{\omega}_2^2 \leq \cdots \leq \overline{\omega}_{n-1}^2$ and $\nu(a^*) \geq 0$ then for every $1 \leq k \leq n-1$ it follows that

$$1 - \frac{\nu(a^*)}{\overline{\omega}^2} \leq 1 - \frac{\nu(a^*)}{\overline{\omega}_k^2(n-k)}.$$

Hence condition (15) implies that for every $k \in \{1, \ldots, n-1\}$

$$\sum_{i\in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}_k^2(n-k)}\right) \geq \sum_{i\in N} \omega_i^2 \left(1 - \frac{\nu(a^*)}{\overline{\omega}^2}\right) \geq \max_{k\in\{1,\ldots,n-1\}} F_k(a^*) \geq F_k(a^*).$$

Therefore condition (17) in Lemma holds and therefore for all subset of players $J \subseteq N$ with $|J| \leq n-1$ and for all $a_J \in \prod_{j\in J} A_j$ there exists a player $i \in J$ such that *i)-iv)* in Definition 3 are satisfied for the sharing rule defined by

$$s_j(y) = \frac{\omega_j^2}{\sum_{i\in N} \omega_i^2} y \quad \forall j \in N,\ \forall y \in f(A).$$

Now we prove that for this sharing rule any deviation of the grand coalition (coalition N)is not profitable, in fact suppose that there exist a strategy profile a with $a_i \neq a_i^*$ for every player i such that for every player $i \in N$

$$U_i(a_J^*, a_{-J}^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) < U_i(a_i, a_{-i}, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) \quad \forall i \in N$$

where

$$U_i(a^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} f(a^*) - v_i(a_i^*)$$

with $b_{ij}^* = a_j^*$ and $c_{iji}^* = a_i^*$ for all $j \neq i$ and

$$U_i(a, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} f(a) - v_i(a_i) \quad \forall a_i \in A_i,$$

where $b_{ij} = a_j$ and $c_{iji} = a_i$ for all $j \neq i$; therefore

$$\sum_{i\in N} U_i(a^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) < \sum_{i\in N} U_i(a, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) \implies$$

$$f(a^*) - \sum_{i\in N} v_i(a_i^*) < f(a) - \sum_{i\in N} v_i(a_i)$$

which is a contradiction since a^* is an efficient strategy profile. ■

Proof of Proposition 7. In light of Proposition 6, a^* is a psychological strong Nash equilibrium. Let a be any other symmetric strategy profile, then for every player i, denote with

$$U_i(a^*, (b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} f(a^*) - v_i(a_i^*) = \frac{f(a^*)}{n} - v(a_i^*)$$

where $b_{ij}^* = a_j^*$ and $c_{iji}^* = a_i^*$ for all $j \neq i$, the psychological utility of player i consistent with a^* and with

$$U_i(a, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) = \frac{\omega_i^2}{\sum_{i\in N} \omega_i^2} f(a) - v_i(a_i) = \frac{f(a)}{n} - v(a_i)$$

where $b_{ij} = a_j$ and $c_{iji} = a_i$ for all $j \neq i$, the psychological utility of player i consistent with a. Since a^* is the efficient strategy profile, it follows that

$$f(a^*) - nv(a_i^*) > f(a) - nv(a_i) \implies \frac{f(a^*)}{n} - v(a_i^*) > \frac{f(a)}{n} - v(a_i)$$

which obviously implies that

$$U_i(a^*(b_{i,j}^*)_{j\neq i}, (c_{i,j,i}^*)_{j\neq i}) > U_i(a, (b_{i,j})_{j\neq i}, (c_{i,j,i})_{j\neq i}) \quad \forall i \in N$$

and hence a is not a psychological strong Nash equilibrium. ■

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