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The Dynamics of Tobin's q

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Abstract
In this paper I propose a general-equilibrium model with proportional adjustment costs and industry-specific capital to study the firm migration phenomenon across market-to-book ratio. In my model, investors' desire to diversify their portfolios and in- vestment frictions generate a mean-reverting dynamics of Tobin's *q* consistent with the probabilities of migration found in the data, and a nonlinear pattern in the conditional volatility of Tobin's *q*. In addition, since firms' market-to-book ratios are function of the state of the economy and contain information about stock returns, stock prices inherit these properties, yielding asset-pricing implications in line with the empirical evidence, namely the value premium and a non-monotone relationship between the volatility of stock returns and the Tobin's *q*.

Keywords: Tobin's *q*, Investment, General equilibrium, Firm migration, Cross-section of returns

JEL Classification: G12, D92, D51, D21, D24.

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I Introduction

Since the pioneering evidence of Fama and French (1992), various researchers have proposed several investment-based asset pricing models to rationalize the observed value premium, that is firms with low market-to-book ratios earn on average higher returns than firms with high market-to-book ratios, and the size effect, that is small firms earn on average higher returns than large firms.[1] This literature, despite providing different explanations for the cross-section of stock returns based either on security mis-pricing, beta mis-measurement[2] or omitted state variables, mainly concentrates upon the optimal investment policy, its link with firm characteristics, in particular the market-to-book ratio, and the implications for stock returns.

However, not much attention has been devoted to the dynamics of Tobin's q over time. In this regard, Fama and French (2007a) showed that each year some growth stocks cease to be highly growing firms, lose their growth opportunities and become value stocks, whereas, conversely, some value stocks restructure their assets in place, acquire growth opportunities and migrate towards growth stocks. This tendency of book-to-market ratios to become less extreme after firms are allocated to value and growth portfolios is what they call convergence or firm migration. Fama and French (2007b) quantified the speed of convergence computing the empirical-transition probabilities of migration.

In this paper I propose a general-equilibrium model with proportional adjustment costs and industry-specific capital to study the firm migration phenomenon across value. In my model, investors' desire to diversify their portfolios and investment frictions generate a mean-reverting dynamics of Tobin's q consistent with the empirical probability of migration documented by Fama and French (2007b), and a nonlinear pattern in the conditional

[1] See Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Giammarino (2004), Zhang (2005), Cooper (2006), Gala (2010), and Gomes and Schmid (2010) among the others.

[2] Potential sources of mis-measurement could depend on the fact that the econometric methods employed in estimation do not capture the conditional nature of the pricing model (see among the others Ferson, Kandel, and Stambaugh 1987; Ferson and Harvey 1999; Campbell and Cochrane 2000; Lettau and Ludvigson 2001) or that the market proxy used in estimation is not the mean-variance efficient portfolio (Roll 1977).

volatility of Tobin's q. In addition, since firms' market-to-book ratios are function of the state of the economy and contain information about stock returns, stock prices inherit these properties, yielding asset-pricing implications in line with the empirical evidence, namely the value premium and a non-monotone relationship between the volatility of stock returns and the Tobin's q.

The streamlined capital-market general-equilibrium model of my paper contains the following features. The production side of my economy consists of two industries, each grouping a large number of competitive firms using identical production technologies with constant-returns-to-scale but subject to uncorrelated productivity shocks. The consumption side, instead, i.e. the pool sector, features a storable technology which accumulates capital at a fixed rate of return and finances consumption. I assume that capital is 'industry-specific', that is once it is invested in the production technology of a given industry, capital acquires industry-specific peculiarities which do not render it neither immediately investable in the other industry nor immediately consumable by consumers-investors. First it has to lose those specificities (at a cost), and only afterwards it can be stored in the pool (and consumed) or can acquire the other industry's characteristics. I model these investment/disinvestment costs to be proportional to the amount of capital exchanged. By contrast, the capital stored in the pool sector is not marked by any technological peculiarity. As a result, each industry can only acquire or sell capital from the consumption sector but not (directly) from the other industry.

In the absence of investment/disinvestment costs, for reasons of portfolio diversification, consumers-investors would like the two industries' stocks of capital to be equal and in a fixed proportion with the capital accumulated in the pool (Merton, 1971). However, in the presence of investment/disinvestment costs, an imbalance may develop and persist as a result of cumulated productivity shocks. In this case, in fact, it could not be optimal to correct such imbalance immediately by exchanging resources among the sectors. In equilibrium, I show that there exists a region, i.e. the no-investment region, within which the imbalance of capital stocks is still tolerated and firms do not trade capital with the pool since the investment/disinvestment costs are larger than their expected future benefits. On the contrary, at the boundary of such region, the imbalance of capital stock is not tolerated any more and firms find optimal to invest/disinvest.

More importantly, the investment/disinvestment costs break the perfect substitutability for consumption purposes between the capital stored in the pool and the one invested in a given industry. As a consequence, the relative price of productive capital starts fluctuating according to its scarcity or abundance in the economy, making the Tobin's q deviate from one. Specifically, in my model a firm becomes value when its capital is abundant relative to the one stored in the pool sector and growth in the opposite case. Moreover, the instantaneous expected change of the Tobin's q exhibits a mean-reverting dynamics, switching sign from positive (for value firms) to negative (for growth firms) and vice-versa in a way that is consistent with the transition probabilities of migration found in the data. The conditional volatility of the Tobin's q, instead, is larger in the inside of the no-transaction region than in proximity of the boundaries.

From an economic standpoint, these properties stem from the general-equilibrium characteristics of my model. In fact, utility maximization requires that the Tobin's q tapers off and exhibits a curvature as one moves towards the boundaries of the no-investment region, capturing the anticipation that the investment/disinvestment behavior will prevent the market-to-book ratio from escaping its optimal range. This effect drives mean reversion and significantly decreases the conditional volatility at the boundaries. My model suggests that firm migration can be generated as a result of a pure discount effect, i.e. determined by the stochastic discount factor, and not necessarily as a cash-flow effect. In fact, since firm's price (and thus the market-to-book ratio) is the product of cash flows times the discount factor, the convergence of market-to-book ratios must ensue from the convergence of either the cash flows or the stochastic discount factor. However, since my economy assumes that firms' profitability exhibits constant-returns-to-scale, the firm migration property ensues from the discount term.

Finally, I investigate the asset-pricing implications of my economy. My model generates a negative relationship between market-to-book ratios and risk premia consistent with the empirical evidence, that is the expected returns earned by firms when they are value are higher than those earned when they are low. In addition, I find a non-monotone relationship between the conditional volatility of stock returns and Tobin's q consistent with the finding of Kogan (2004). Specifically, value and growth firms exhibit a higher volatility than neutral firms. The economic mechanism behind these results stems directly from the mean-reverting property of the Tobin's q (together with its nonlinear pattern) and depends on the consumption-smoothing objective of consumers-investors. The latter, in fact, price stocks according to the firm's ability to provide consumption insurance. In this regard, value firms are less able to provide consumption smoothing over time since the abundance of their capital cannot be directly used for consumption purposes and it is costly to transfer it. As a consequence, investors require a higher expected return to hold these stocks. On the contrary, growth firms exhibits a lower equity premium since the scarcity of their capital does not represent a hindrance to smooth consumption over time, given the abundance of capital stored in the pool.

My work is part of a growing line of research, pioneered by Berk, Green and Naik (1999), which relates asset prices to firm's investment decisions. The partial equilibrium model of Berk, Green and Naik (1999) features exogenous project-level cash flows and systematic risk. In their model, multiple sources of risk are used to explain the observed cross-sectional variation of returns. On the contrary, Gomes, Kogan and Zhang (2003) establish an explicit economic relation between firm-level characteristics and expected returns in a dynamic general equilibrium model. My work differs from these papers along several dimensions. First, my paper is a multiple-industries economy which features costly reversibility of investment to study the dynamics of Tobin's q. Second, I model firms whereas they model "projects". In their economy, all "projects" have ex-ante identical productivity and, once adopted, variation in the project-specific productivity only affects that project capital. In contrast, in my model, variation in the profitability of the assets in place affects the firm investment decisions and its entire stock of capital, as in the standard Q - theory of investment.

Kogan (2004) develops a two-goods general equilibrium model with investment constraints: real investment is irreversible, as assumed by a strand of the investment literature (e.g. Dixit and Pindyck, 1994), and the investment rate is bounded from above, representing a special case of the standard convex adjustment costs specification. He shows that investment frictions entail time variation in stock returns and generate high nonlinear patterns between the market-to-book ratio and the conditional volatility of stock returns. In contrast to his paper, I focus on the firm migration phenomenon across value and assume that across-industries investment is reversible at a cost.

Zhang (2005) also links expected returns to size and book-to-market in a dynamic-equilibrium model with convex adjustment costs and costly reversibility of capital, using the neoclassical q-theory approach and an exogenous countercyclical market price of risk. He solves the industry equilibrium by applying the "approximate-aggregation" idea of Krusell and Smith (1998). Moreover, Carlson, Fisher and Giammarino (2004) analyze the effect of operating leverage on expected returns, whereas Cooper (2006) studies the asset pricing implications of non-convex adjustment costs. All these models use a partial-equilibrium framework to explain the asset pricing anomalies, while in my work I endogenize the role of consumption, and thus, the pricing kernel.

Gomes and Schmid (2010) investigate the theoretical relationship between financial leverage and stock returns in a dynamic world where both corporate investment and financing decisions are endogenous. They find that, in the presence of market imperfections, leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities.

Papanikolaou (2011) proposes a two-sector equilibrium model with heterogeneity in the type of firm output and provides evidence that investment-specific technological change is a source of systematic risk that is responsible for some of the cross-sectional variation in risk premia between value and growth firms. In contrast to Papanikolaou (2011) who concentrates on firm heterogeneity, that is heterogeneity arising because of differences between capital good and final good producers, my model focuses on differences in productivity or accumulated capital. In addition, in my economy capital is reversible among sectors, whereas he assumes a fixed level of capital in the investment-good sector.

Much of the methodology of the present article is borrowed from the literature dealing with portfolio choice under transaction costs. Grossman and Laroque (1990) consider fixed transaction costs, while Dumas and Luciano (1991) consider proportional costs, but allowing for terminal consumption only. Liu (2004) proposes a model of optimal consumption and investment with transactions costs and multiple risky assets, whereas Delgado, Dumas and Puopolo (2015) solve a portfolio-choice problem with proportional transaction costs and mean reversion in expected returns. Finally, Dumas (1992) constructs a general-equilibrium model with proportional costs in segmented commodity markets.[3]

The rest of the paper is organized as follows. Section II presents the general-equilibrium model whereas the optimal investment policy is described in Section III. Section IV discusses the calibration and the empirical strategy. The dynamics of the Tobin's q is investigated in Section V whereas Section VI studies the relationship between Tobin's q and stock returns. Section VII concludes. Finally, the appendixes provides technical details on the model computation, the risk-free rate and the optimal investment policy.

[3] The literature on investment in general equilibrium using a real-option approach includes Kogan (2001) and Hugonnier, Morellec and Sundaresan (2005). These papers mainly examine the impact of irreversibility on the investment behavior and do not attempt to investigate neither the convergence of price-to-book ratios nor the cross-section of stock returns. Additional contributions include the works of Gomes, Yaron and Zhang (2006) and Gala (2010).

II The Model

I consider an economy populated by identical risk-averse consumers-investors. The production side of the economy consists of two industries, each grouping a large number of competitive, all equity-financed, firms. All firms employ identical constant-returns-to-scale production technologies with expected rate of return μ[4] and standard deviation σ of rate of return, but are subject to industry-specific productivity shocks.[5] Given this assumption, in the rest of the paper I will simply refer to the representative firm of each industry and denote K^i the amount of capital accumulated by such firm in industry i, with $i = 1, 2$. The rest of the economy, that is the consumption side, is characterized by a riskless technology which accumulates capital at the rate r and finances consumption. I call this sector the pool sector.

There exists a single good which can be consumed, accumulated in the consumption sector, or invested in the two representative firms' production processes. In this regard, I assume that capital is 'industry-specific', that is the investment in the production technology of a given industry bestows upon the good some industry-specific peculiarities which do not render it neither immediately investable in the other industry nor immediately consumable by consumers-investors. In order to be stored in the pool and then consumed, or invested in the other firm's production technology, the capital accumulated in a given industry has first to lose its technological specificities (at a cost), and only afterwards it can acquire the other industry's characteristics. On the contrary, the capital stored in the pool sector is not marked by any technological peculiarity. The assumption of industry-specific capital implies that, in my model, each firm can only acquire or sell capital from the consumption sector but not (directly) from the other firm, and that consumers-investors can only consume the

[4] The expected rate of return μ can be interpreted net of all costs, including for example the costs of re-investing in the own production technology and capital depreciation.

[5] The assumption that the two industries are symmetric on the one hand prevents an industry to systematically dominate the other and (which is the same thing) guarantees that no firm can sustain indefinitely a higher productivity, while on the other, it enormously simplifies the computation of the optimal consumption and investment policies, without being crucial in driving the results.

good available in the pool.

I suppose that, when firms purchase or sell resources to the pool sector, they incur investment costs that are proportional to the amount exchanged. I model these costs to be asymmetric also, that is, firms pay $1 - s$ per unit of capital invested and $1 - s_d$ per unit of capital disinvested to the pool, with $0 \leq s_d < s \leq 1$, to capture the intuition of costly reversibility of capital. Thus, firms face higher costs in contracting than in expanding their capacity. Equivalently, $1/s$ can be interpreted as the price (in units of capital) at which firms can purchase one unit of capital (i.e. buying price), and s_d as the price at which they can sell one unit of capital (i.e. selling price).

For reasons of portfolio diversification, given the symmetric characteristics of the production technologies, consumers-investors would like the two industries' stocks of capital to be equal and in a fixed proportion with the capital accumulated in the pool. However, in the presence of investment/disinvestment costs, an imbalance may develop and persist as a result of cumulated productivity shocks. In this case, in fact, it could not be optimal to correct such imbalance immediately by exchanging resources with the pool sector.

I assume that financial markets are complete and that there are no costs or frictions to trade financial securities. This assumption guarantees that consumers-investors can achieve a Pareto-optimal allocation of consumption, or equivalently that the capital-market and the good-market equilibrium can be replicated by an appropriate central-planning problem. Therefore, for tractability reasons, I focus on the central-planner problem rather than solving the decentralized version of the present economy. Implicit prices, which would prevail explicitly in decentralized markets, can be obtained from the derivatives of the appropriate indirect utility function. Finally, in the rest of the paper, I will use the term 'transaction costs' only to denote the costs of exchanging capital between the sectors, i.e. the investment/disinvestment costs, being absent any friction to trade financial securities.

Accordingly, the central-planner optimization problem is

$$V\left(K^0, K^1, K^2\right) = \max_{\{c, I^1, I^{01}, I^2, I^{02}\}} E\left[\int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt\right] \tag{1}$$

s.t.

$$dK_t^0 = rK_t^0 - c_t dt + s_d dI_t^1 + s_d dI_t^2 - dI_t^{01} - dI_t^{02}, \tag{2}$$

$$dK_t^1 = \mu K_t^1 dt + \sigma K_t^1 dB_t^1 + s dI_t^{01} - dI_t^1, \tag{3}$$

$$dK_t^2 = \mu K_t^2 dt + \sigma K_t^2 dB_t^2 + s dI_t^{02} - dI_t^2, \tag{4}$$

$$c_t \geq 0,\ dI_t^{01} \geq 0,\ dI_t^{02} \geq 0,\ dI_t^1 \geq 0,\ dI_t^2 \geq 0,\ K_t^0 \geq 0,\ K_t^1 \geq 0,\ K_t^2 \geq 0, \tag{5}$$

where γ is the degree of risk aversion, ρ the rate of impatience and dB_t^1 and dB_t^2 are two standard independent Brownian motions. K_t^0 denotes the amount of capital accumulated in the pool sector whereas c_t is consumption. The investment and disinvestment decisions of the representative firm i are captured by the terms I_t^{0i} and I_t^i which are non-decreasing processes increasing only when, respectively, firm i purchases or sells capital to the pool sector.[6]

While the optimal consumption policy is continuous over time, the industries' investments decisions are significantly lumpy. In fact, given the nature of the investment costs considered in the model, there will exist a region Ω of the state space within which the firms do not trade capital with the pool. Considering the linear nature of the constraints and the homogeneity property of the utility function, the variables K^1/K^0 and K^2/K^0 are sufficient state variables to fully characterize the no-investment region Ω.[7] The boundary of Ω, instead, should be viewed as a barrier or a trigger point for the industries' investment

[6] My model, and more precisely when $s_d = 0$, also encompasses the possibility of investment irreversibility across industries, as in Kogan (2001, 2004).

[7] See Dumas (1992) for a detailed explanation of the existence and the properties of the no-transaction region.

decisions. In the paper, I look for the optimal positioning of the barrier.

Consistently with the literature on portfolio choice problems with transaction costs, inside the no-transaction region Ω the imbalance of capital accumulated in the different sectors of the economy is still tolerated and no investment/disinvestment takes place. On the contrary, whenever the state variable K^i/K^0 hits the barrier $\underline{\omega}^i$ from the inside of Ω, for $i = 1, 2$, a lower edge of the region is reached. There, the capital employed in i-th production process is too scarce compared to that stored in the consumption sector and the corresponding imbalance of resources is not tolerated any more. Therefore, an investment decision takes place to ensure consumption-smoothing: firm i finds optimal to increase its capital size and purchases new capital from the pool, thus bringing the stocks of capital back instantaneously inside Ω. Similarly, when the variable K^i/K^0 hits the barrier $\overline{\omega}^i$ (again from the inside of Ω), the upper boundary of the region is reached. Therefore, a disinvestment decision takes place: firm i finds optimal to contract its capital capacity and sells resources to the pool.[8] In general, the barriers $\underline{\omega}^i$ and $\overline{\omega}^i$ are not constant but functions of the other state variable of the economy.[9]

When no investment/disinvestment takes place, using the martingale property, I get that

$$-\rho V + \max_c \left\{ \frac{E_t\left[dV_t\right]}{dt} + \frac{c_t^{1-\gamma}}{1-\gamma} \right\} = 0 \tag{6}$$

$$\rho V = \max_c \left\{ \begin{array}{c} V_{K^0}(rK^0 - c) + \mu K^1 V_{K^1} + \mu K^2 V_{K^2} \\ +0.5\left(\sigma K^1\right)^2 V_{K^1K^1} + 0.5\left(\sigma K^2\right)^2 V_{K^2K^2} + \dfrac{c^{1-\gamma}}{1-\gamma} \end{array} \right\}.$$

Substituting the first order condition for consumption, that is,

$$c\left(K^0, K^1, K^2\right) = \left[V_{K^0}\left(K^0, K^1, K^2\right)\right]^{-1/\gamma}, \tag{7}$$

[8] In fact, at the boundary $\overline{\omega}^i$, the capital stored in the consumption sector is too scarce compared to that employed in i-th production process. As a result, a transfer of resources takes place in order to bring the stocks of capital back instantaneously inside Ω.

[9] In his model of optimal consumption and investment with transaction costs, Liu (2004) shows that the no-transaction region is not an ellipse as was suspected before, but rather does have "corners".

the Hamilton-Jacobi-Bellman equation inside the no-investment region Ω can be written as follows:

$$\begin{aligned}\rho V &= \frac{\gamma\left(V_{K^0}\right)^{\frac{\gamma-1}{\gamma}}}{1-\gamma}+rK^0V_{K^0}+\mu K^1V_{K^1}+\mu K^2V_{K^2} \\ &\quad +0.5\left(\sigma K^1\right)^2V_{K^1K^1}+0.5\left(\sigma K^2\right)^2V_{K^2K^2}.\end{aligned} \tag{8}$$

When the firm invests, the movement to the target position is instantaneous. Hence, the values of the discounted utility before and after the purchase of new capital must be the same, that is, when $K^i/K^0=\underline{\omega}^i$, for $i=1,2$,

$$V\left(K^0,K^i,K^j\right)=V\left(K^0-dI^{0i},K^i+sdI^{0i},K^j\right)\quad\Rightarrow V_{K^0}=sV_{K^i}. \tag{9}$$

Value matching must also hold when a disinvestment takes place, that is, when $K^i/K^0=\overline{\omega}^i$,

$$\left(K^0,K^i,K^j\right)=V\left(K^0+s_d dI^i,K^i-dI^i,K^j\right)\quad\Rightarrow s_dV_{K^0}=V_{K^i}, \tag{10}$$

for $i=1,2$.

The partial differential equation (8) and the value-matching conditions (9)-(10) hold for any arbitrary choice of the investment/disinvestment trigger boundaries $(\underline{\omega}^i,\overline{\omega}^i)$. Smooth-pasting conditions have to be satisfied in order for the barriers to be optimal.[10]

This requires that, in the case of investment,

$$\left\{\begin{array}{ll} V_{K^0}\left(K^0,K^i,K^j\right)=V_{K^0}\left(K^0-dI^{0i},K^i+sdI^{0i},K^j\right) & \Rightarrow V_{K^0K^0}=sV_{K^0K^i} \\ V_{K^i}\left(K^0,K^i,K^j\right)=V_{K^i}\left(K^0-dI^{0i},K^i+sdI^{0i},K^j\right) & \Rightarrow V_{K^iK^0}=sV_{K^iK^i}\end{array}\right. , \tag{11}$$

[10] See Dumas (1991) and Dixit (1991) for a discussion of the value-matching and the smooth-pasting conditions.

whereas, in the case of disinvestment,

$$\begin{cases} V_{K^0}\left(K^0, K^i, K^j\right) = V_{K^0}\left(K^0 + s_d dI^i, K^i - dI^i, K^j\right) & \Rightarrow s_d V_{K^0 K^0} = V_{K^0 K^i} \\ V_{K^i}\left(K^0, K^i, K^j\right) = V_{K^i}\left(K^0 + s_d dI^i, K^i - dI^i, K^j\right) & \Rightarrow s_d V_{K^i K^0} = V_{K^i K^i} \end{cases}, \quad (12)$$

for $i = 1, 2$.

The optimal solution to the central planner problem is obtained by solving the differential equation (8) subject to the boundaries conditions (9-12). Since (as far as I know) there exists no closed-form solution to the value function V, I first exploit the homogeneity property of the value function to reduce the dimensionality of the problem, and then apply a numerical technique based on a finite-difference method. In the Appendix I detail the methodology employed.

Specifically, considering the linear nature of the constraints and the isoelastic property of the utility function, the value function $V\left(K^0, K^1, K^2\right)$ is homogeneous of degree $1 - \gamma$. Therefore, the two variables

$$\omega^1 \equiv \frac{K^1}{K^0} \text{ and } \omega^2 \equiv \frac{K^2}{K^0},$$

suffice to fully characterize the state of the economy.

Exploiting this homogeneity property and using the new state variables, I introduce the following (transformed) value function I,

$$(1-\gamma)\log(K^0) + I(\omega^1, \omega^2) \equiv \log V(K^0, K^1, K^2),$$

which will be useful to compute the price-to-book ratios and stock returns.

III Optimal Investment Policy

In this section I compute the optimal investment policy.[11] However, before showing the optimal position of the no-investment region Ω, I investigate the investment behavior of an

[11] The choice of parameter values will be discussed in the next section.

economy similar to the one described above but featuring only one productive industry, to better highlight the economic relevance of all aspects of my model.

Example: A simplified economy with one industry

Clearly, when only one productive industry is considered, the Pareto-planner problem remains equivalent to equations (1-5) except that one industry is removed from the analysis. Without loss of generality, I assume that there exists solely industry 1. In this economy, in the absence of transaction costs (i.e. in a frictionless world)[12], the optimal investment policy would consist in investing/disinvesting infinitesimal amounts of resources to keep the ratio $\frac{K^1}{K^0}$ constantly equal to $\frac{(\mu-r)/\gamma\sigma^2}{1-(\mu-r)/\gamma\sigma^2}$ (Merton, 1971), thus guaranteeing that capital is always perfectly balanced between the pool sector and the risky industry.

However, in the presence of transaction costs, deviations from the above policy may develop and persist as a result of cumulated productivity shocks, since it is costly to correct the imbalance of capital immediately by exchanging resources between the sectors. In this case, the optimal investment/disinvestment behavior would consist in refraining from transacting as long as the ratio $\frac{K^1}{K^0}$ lies within a given interval, i.e. the tolerated imbalance, and investing/disinvesting only at the boundaries of such no-transaction interval (Constantinides, 1986).

FIGURE 1 GOES HERE

As shown in Figure 1 above, in the no-transaction region, that is within the investment and disinvestment lines, the fluctuations of the capital ratio $\frac{K^1}{K^0}$ are still tolerated. For consumption-smoothing purposes, only when $\frac{K^1}{K^0}$ hits the upper line from the inside, the imbalance of resources allocated in the economy is not tolerated any more and the representative firm finds optimal to sell some of its capital to the pool sector, thus disinvesting.

[12] As explained in the previous section, recall that the term 'transaction costs' denotes exclusively the costs of exchanging capital between the sectors, i.e. the investment/disinvestment costs, being absent any friction to trade financial securities.

On the contrary, when $\frac{K^1}{K^0}$ hits the lower boundary from the inside, the firm finds optimal to purchase new resources from the consumption sector, thus investing. As a result, the investment/disinvestment policy adjusts the ratio $\frac{K^1}{K^0}$ and brings it back instantaneously inside the no-transaction region.

Finally, in order to investigate the economic relevance of the investment/disinvestment costs in my economy, I briefly discuss the implications of a very tractable case of convex adjustment costs. In line with Kogan (2004), I introduce convex adjustment costs in my framework in the form of an upper bound on the investment/disinvestment rate, i.e. there exists a maximum rate at which the firm can invest/disinvest.[13] In this case, since the amount of capital that can be exchanged in the economy is capped from above, the fluctuations of the ratio $\frac{K^1}{K^0}$ can escape from the investment and disinvestment lines shown in Figure 1. Therefore, the resulting no-transaction region becomes wider. In other words, compared to the case of proportional investment costs, convex adjustment costs generate a larger tolerated-imbalance of capital.

The general economy with two industries

I now examine the general case with two industries. In the absence of investment/disinvestment costs, for reasons of portfolio diversification, consumers-investors would like the two industries' stocks of capital to be equal and in a fixed proportion with the capital accumulated in the pool. This implies that the optimal investment policy would consist in investing/disinvesting infinitesimal amounts of resources to keep a perfect balance of capital stocks in the economy, that is

$$\frac{K^1}{K^0} = \frac{K^2}{K^0} = \frac{(\mu - r)/\gamma\sigma^2}{1 - 2(\mu - r)/\gamma\sigma^2}, \tag{13}$$

in line with the one-industry case illustrated above.

On the contrary, when the transfer of resources among sectors is costly, consumers-

[13] This specification still allows to exploit the homogeneity property of the value function, which enormously simplifies the numerical solution. See Kogan (2004) for further details.

investors cannot achieve a perfectly-diversified "portfolio", i.e. the frictionless allocation of capital stocks shown in (13), and must allow some imbalance of capital in the economy. Specifically, utility maximization ensures that consumers-investors find optimal to (respectively) buy and sell capital to the pool sector only at the boundaries $\underline{\omega}^i$ and $\overline{\omega}^i$ of the no-transaction region Ω, whereas, inside Ω, no investment activity takes place since the investment/disinvestment costs are larger than their expected future benefits.[14]

FIGURE 2 GOES HERE

Figure 2 above shows the optimal position of the no-transaction region Ω, represented by the interior of "$ABCD$" and bounded by the segments AB, AC, BD and CD. The linear shape of the functions $\underline{\omega}^i$ and $\overline{\omega}^i$ confirms the results obtained by Liu (2004) in a portfolio-choice problem with transaction costs: it is not an ellipse as it was suspected by the previous literature[15], rather it does have "corners". In the graph, firms in industry 1 (respectively 2) find optimal to invest in correspondence of the line $\underline{\omega}^1 = AC$ ($\underline{\omega}^2 = AB$) and disinvest on the segment $\overline{\omega}^1 = BD$ ($\overline{\omega}^2 = CD$). Finally, the assumption of identical production technologies for the two industries implies the symmetry around the 45° degree line. Point F in Figure 2 lies on the diagonal AD and corresponds to the perfectly-diversified "portfolio" shown in Equation (13). Any deviations from this point inside Ω are determined by cumulated productivity shocks – taking also into account the optimal consumption policy – and are due to the existence of proportional investment/disinvestment costs.

Finally, imposing in this framework an upper bound on the rate at which firms can invest/disinvest, that is introducing convex adjustment costs in the spirit of Kogan (2004), would generate a greater region of tolerated-imbalance of capital stocks, in line with the one-industry case. This is because investors would be constrained in the amount of capital that can exchange among the sectors and as a consequence, compared to the case of pro-

[14] I address the issue of the optimality of the investment/disinvestment policy in the Appendix.

[15] See Morton and Pliska (1995).

portional transaction costs, the ratios $\frac{K^1}{K^0}$ and $\frac{K^2}{K^0}$ will have more freedom to deviate from the frictionless policy. As a result, the region "$ABCD$" would broaden.

Before investigating the asset-pricing implications of my model, it is worthwhile to underline that not all literature focusing on the cross-sectional variation of expected stock returns has examined the interaction between the optimal consumption policy and investment behavior.[16] By contrast, in my general-equilibrium economy agents are risk averse and choose their consumption sequence. In turn, this affects the price of financial securities through the endogenous-stochastic discount factor.[17]

IV Calibration and Empirical Strategy

In this section, I first calibrate my model and then briefly discuss the empirical strategy used to generate the asset-pricing implications of my economy. By contrast, the model's ability to replicate important features of the dynamics of Tobin's q and stock returns will be discussed in the next sections.

The calibration of my model is carried out by setting some parameter values to approximately match key unconditional moments of both macroeconomic quantities and asset returns and borrowing others from prior empirical or quantitative studies. Specifically, my economy requires seven parameters to be specified: two for preferences, three for the technological processes and two for the investment/disinvestment decisions. The rate of impatience ρ is set to 0.01 consistently with the related macro-finance literature[18], whereas r is set to 0.018 to capture the average risk-free rate. Regarding the investment/disinvestment

[16] Partial equilibrium models include, among the others, Berk, Green and Naik (1999), Carlson, Fisher and Giammarino (2004), Cooper (2006), Zhang (2005), and Gomes and Schmid (2010).

[17] Consistently with Kogan (2001), the stochastic discount factor is given by

$$\Lambda_{t,s} = e^{-\rho(s-t)} \frac{U'(c_s^*)}{U'(c_t^*)}.$$

[18] In contrast to other models of the investment-based asset pricing literature, which calibrate the rate of impatience to approximately match the average risk-free rate, in my model ρ has no influence on the riskless return since the pool sector shares the characteristics of a riskless technology. Therefore, it is not a surprise that the instantaneous riskfree rate is constant and equal to r. See the Appendix for further details.

costs, s and s_d, I select s equal to 0.95 and s_d equal to 0.8 in line with the 25% gap between the buying and selling price of capital documented by Cooper and Haltiwanger (2006).[19] Finally, I set the degree of relative risk aversion γ equal to 13, the expected rate of return on the production process μ equal to 0.09 and the standard deviation of the production process σ equal to 0.2 to approximately match the following unconditional moments: the aggregate volatility of consumption growth, the average equity premium and the unconditional volatility of the equity premium.

TABLE I GOES HERE

Table I compares the implied moments of the key aggregate variables in the model with corresponding empirical estimates. My calibration ensures that the simulated data match these key statistics quite well. Obviously, since these parameters are pinned down to tightly match unconditional moments, they provide no degrees of freedom in matching neither the conditional behavior of Tobin's q nor stock returns.

All asset-pricing implications generated by my model, i.e. both the unconditional moments reported above and the conditional results shown in the next sections, are obtained by simulating 100,000 paths of the pair of state-variables $\left(\omega^1, \omega^2\right)$ and then computing the corresponding paths of the relevant asset-prices values.[20] Each path has a length of 1000 months, and the first half observations are dropped to remove the dependence on initial values. To better understand the sense of the empirical strategy, recall that the supply side of my economy consists of two industries, each grouping a large number of identical firms. Therefore, at each point in time there exist only two representative firms in the economy, each featuring a given market-to-book ratio and stock return according to the position of

[19] Recall that, in my model, $1/s$ can be interpreted as the price in units of capital at which firms can purchase one unit of capital (i.e. buying price), and s_d as the price at which they can sell one unit of capital (i.e. selling price).

[20] Recall that the state variables $\left(\omega^1, \omega^2\right)$ suffice to fully characterize the state of the economy. Therefore, each value of the pair $\left(\omega^1, \omega^2\right)$ within the region Ω automatically determines the corresponding values of Tobin's q and stock returns. See also next sections for further details.

the state variables $\left(\omega^1, \omega^2\right)$ within the no-investment region Ω. In this sense, one path represents one possible dynamics of these two firms over time whereas a large number of paths constitutes a representative sample of the firms' behavior (and hence 'values', such as Tobin's q and stock returns) in the different states of nature. As a result, since the distribution of firms in the real world is stationary, the steady-state (asset-pricing) implications of my model are comparable with those observed in the data.

Finally, the empirical-transition probabilities of migration are computed according to Fama and French (2007b). Specifically, I form three value weight portfolios, G, N, V, at the end of each June from 1963 to 2007 based growth (G, firms in the top 30% of NYSE market-to-book ratio), neutral (N, middle 40% of NYSE market-to-book ratio), and value (V, bottom 30% of NYSE market-to-book ratio). In the market-to-book ratio sorts for portfolios formed in June of year t, book equity is for the fiscal year ending in calendar year $t-1$ and market equity is for the end of December of $t-1$. The portfolios for year t include NYSE, Amex, and Nasdaq (after 1972) stocks with positive book equity in $t-1$. The transition vectors are for the firms assigned to a portfolio in June of year t that are also in one of the three portfolios in $t+1$. The year t transition vector for a portfolio is the fraction of firms in the portfolio when formed at the end of June of year t that falls into each of the groups at the end of June of $t+1$.

V Tobin's q and Firm Migration

In this section I investigate the dynamics of Tobin's q over time, focusing in particular on the firm migration phenomenon.

So far the optimization problem has been formulated as a centralized one. Nevertheless, one can infer the prices that would prevail in a decentralized market economy by knowing (the first derivatives of) the value function $V\left(K^0, K^1, K^2\right)$. To start with, I specify the numeraire I use to price all financial assets. Since consumers-investors are constrained to consume only the good physically available in their sector, I choose the pool capital K^0 as the numeraire.

Let q^i denote the price of capital accumulated by firm i relative to capital stored in the pool, i.e. the price of a unit of K^i in units of K^0. As in Kogan (2001),

$$q^i\left(K^0, K^1, K^2\right) = \frac{V_{K^i}\left(K^0, K^1, K^2\right)}{V_{K^0}\left(K^0, K^1, K^2\right)}. \tag{14}$$

Then, the market value of firm i is given by the product of the relative price q^i and its stock of capital, that is $S^i = q^i K^i$. In other words, the shadow price q^i coincides with the Tobin's average q^i of the firm, being the ratio of its market value to the replacement cost of its capital.

In the absence of transaction costs, the Tobin's q^i would be constantly equal to 1. This is because, in a frictionless world, the stocks of capital would always be perfectly balanced in the economy, and thus freely exchangeable between the industries and the pool sector to finance consumption. As a result, the price of a unit of K^i would be identical to the price of a unit of K^0.[21]

On the contrary, as shown in Figure 2, the presence of transaction costs generates imbalances of capital stocks between the sectors, breaking at the same time the perfect substitutability between K^0 and K^i for consumption purposes. As a consequence, the relative price of K^i (in units of K^0) starts fluctuating according to the scarcity or the abundance of the risky capital K^i in the economy (with respect to K^0), making the Tobin's q^i deviate from 1. More precisely, depending on the size of the imbalances, the firm's market-to-book ratio assumes values within the range $[s_d, 1/s]$ reaching exactly $1/s$ and s_d at the boundaries $\underline{\omega}^i$ and $\overline{\omega}^i$ by virtue of the value-matching conditions (9-10). In other words, when an investment (respectively disinvestment) takes place, the value of the

[21] More precisely, in the equivalent frictionless economy, total capital, i.e. $W = K^0 + K^1 + K^2$, would be a sufficient state variable to fully characterize the optimal consumption/investment policy and the indirect value function V. In other words, in such an economy, only total capital would matter, independently on which sector/industry it is accumulated. As a result, $V_{K^i}(W) = V_{K^0}(W)$.

risky capital K^i is $1/s$ (respectively s_d) times the value of the pool capital K^0. Moreover, since the smooth-pasting conditions (11-12) guarantee the optimality of the boundaries, the slopes $\frac{dq^i}{dK^i}$ and $\frac{dq^i}{dK^0}$ are zero in correspondence of the barriers. As a result, moving from the inside of the no-transaction region Ω and approaching the boundaries, the Tobin's q^i tapers off.

To better illustrate these concepts, Figure 3 below displays the typical behavior of the market-to-book ratio within the no-transaction region.[22]

FIGURE 3 GOES HERE

I propose a simple mechanism to explain this behavior. Assume the economy is in a perfectly-balanced allocation of capital stocks (point F in Figure 2), that is consumers-investors are fully able to diversify their capital "portfolios". Then, after a sequence of positive shocks to firm's i output, an imbalance of capital develops as a result of transaction costs. Specifically, there will be an abundance of K^i in terms of K^0, i.e. a high "positive" imbalance, which in turn determines a drop in its relative price. As a result, the Tobin's q^i will get closer to the value s_d and the firm will become a value firm. At the same time, in proximity of the boundary, the probability that the firm sells some of its capital to the consumption sector will increase. On the contrary, when K^i becomes scarce relative to K^0, for example after a sequence of negative output shocks, i.e. when a high "negative" imbalance occurs, the risky capital becomes more valuable (in units of the pool capital) and the corresponding market-to-book ratio increases approaching $1/s$. As a result, the firm will become a growth firm with a higher probability to acquire new capital from the pool sector. It is very easy to locate value and growth firms in the no-transaction region Ω. In fact, from Figure 2, when the state variables are close to the line AB (respectively AC), firms in industry two (one) are growth, whereas in correspondence of BD (CD), firms in industry one (two) are value. This means that the accumulation of capital stocks in the

[22] More precisely, along the main diagonal AD (see Figure 2).

economy identifies automatically the position of the firms within the distribution of the market-to-book ratio.

In Figure 4 I show a randomly-drawn sample path of the Tobin's q^i, together with the corresponding paths of its conditional expected change and its conditional standard deviation, over a period of 1000 months.

FIGURE 4 GOES HERE

Interestingly, as displayed in Figure 4, the instantaneous expected change of the Tobin's q^i exhibits a mean-reverting dynamics, switching sign from positive to negative (and vice-versa) according to the size of the capital stocks' imbalances. More precisely, the drift is positive when the firm's capital K^i is abundant relative to K^0, i.e. when the firm is a value firm, and negative when the capital K^i is instead scarce, i.e. when the firm is a growth firm. In addition, the standard deviation is larger in the inside of the no-transaction region than in proximity of the boundaries. From an economic standpoint, these properties stem from the general-equilibrium characteristics of my model, where the optimal investment/disinvestment policies are aimed at assuring consumption smoothing. In fact, utility maximization requires that the Tobin's q^i tapers off and exhibits a curvature as one moves towards the boundaries (see Figure 3), capturing the anticipation that the investment/disinvestment behavior will prevent the market-to-book ratio from escaping from the range $[s_d, 1/s]$.[23] This effect drives mean reversion and significantly decreases the conditional volatility at the boundaries.

In my paper, consumption plays a crucial role: not only it alters the imbalances of capital

[23] The equivalent explanation for this behavior based in terms of demand and supply is the following. Recall that in my production economy the "tolerated" imbalances of capital are endogenously determined. To ensure market clearing, in response to shocks in the economy, either capital has to be transferred among sectors or prices must change to absorb the shocks. Now, when capital cannot be transferred as a result of transaction costs (i.e. inside the no-transaction region Ω), the Tobin's q must change to absorb the shocks, thus exhibiting a higher volatility. On the contrary, when investment/disinvestment is about to take place (i.e. at the boundaries), the supply of capital is relatively elastic and partially absorbs the shocks, thus reducing the volatility of the Tobin's q.

stocks in the economy and affects their relative price, but also generates an endogenous mean-reverting dynamics for the Tobin's q through the interaction with the investment policy, consistent with the evidence that firms with low market-to-book ratios tend naturally to migrate from value to growth and vice-versa. In other words, consumption acts as a natural regulator of the Tobin's q, pushing it away from its extreme values.

Finally, I examine the implications of my model on the speed of convergence of the market-to-book ratio. In this regard, Fama and French (2007b) provided a better understanding of the firm migration phenomenon by quantifying the empirical-transition probabilities of migration. Using their methodology (see Section IV), I construct the average transition frequencies given by the data, Table II, and generated by my model, Table III, of three portfolios formed on market-to-book ratios.[24]

TABLE II GOES HERE

TABLE III GOES HERE

The migration probabilities shown in Table III capture quite well the average transition densities found in the data. In this regard, it is important to underline that this result stems from a general-equilibrium framework in which production processes are characterized by constant-returns-to-scale technologies. More precisely, my model suggests that mean reversion can be generated as a result of a pure discount effect, i.e. determined by the stochastic discount factor, and not necessarily as a cash-flow effect. In fact, since firm's price (and thus the market-to-book ratio) is the product of cash flows times the discount factor, the convergence of market-to-book ratios must ensue from the convergence of either

[24] As highlighted in the discussion of the empirical strategy, the transition densities shown in Table III are generated by the steady-state migration of two representative firms. Therefore, they represent the steady-state probability that a portfolio, conditional on any initial state of nature, will be in a given state of nature after one year. In fact, since the distribution of firms in the real world is stationary, it is possible to assimilate the probability of migration of these two firms with the transition densities that have been observed in the data (Table II).

the cash flows or the stochastic discount factor. However, since my economy assumes that firms' profitability exhibits constant-returns-to-scale, the firm migration property ensues from the discount term. On the contrary, several papers of the related literature are set in partial equilibrium (exogenous consumption) and impose some mean-reverting properties for aggregate and idiosyncratic state variables, thus forcing the cross-sectional distribution of firms.[25]

VI Tobin's q and Stock Returns

In my model, firms' market-to-book ratio is function of the state of the economy and therefore contains information about the behavior of stock returns. In this section, I precisely investigate this relationship, concentrating on the cross section of stock returns and on the properties of the conditional volatility of returns.

Recall that, in my economy, the market value of firm i, S_t^i, is equal to the product of the Tobin's q^i and its stock of capital, that is $S_t^i = q_t^i K_t^i$. In the absence of transaction costs, as underlined in the previous sections, the Tobin's q^i would be constantly equal to 1, implying that the firm price S_t^i would be equal to K_t^i. In other words, in a frictionless world, q^i would be totally unrelated to the firm value and stock returns would exhibit a constant expected rate of return equal to μ and a constant volatility equal to σ, exactly as in the dynamics of the capital stock K_t^i. On the contrary, the presence of transaction costs, by generating a region of tolerated-imbalances of capital stocks among the sectors, makes the relative price of K^i (in units of K^0) changing over time. As a result, the Tobin's q^i becomes function of the state of the economy and strongly affects stock returns. In turn, the latter exhibit interesting time-varying dynamics, in contrast to the frictionless world.[26]

To start with, I investigate the implications of my model on expected stock returns.

[25] In fact, Gomes, Kogan and Zhang (2003) report that, at the aggregate level, mean reversion is necessary to ensure that the growth rate of output does not explode, whereas, at the firm level, mean reversion is required to obtain a stationary distribution of firms in equilibrium.

[26] Note that in my model stock returns depend on both state variables ω^1 and ω^2, which implies that both shocks dB_t^1 and dB_t^2 are systematic.

In Table IV below I show the cross-section of expected returns, whereas Figure 5 shows a randomly-drawn sample path of firm'i expected return over a period of 1000 months (together with the corresponding path of the Tobin's q^i, for comparison).[27]

TABLE IV GOES HERE

FIGURE 5 GOES HERE

My model generates a negative relationship between market-to-book ratios and risk premia in line with the empirical evidence, i.e. the expected returns earned by firms when they are value are higher than those earned when they are growth. The economic mechanism behind this result stems directly from the mean-reverting property of the Tobin's q and is based on a "consumption insurance" explanation.[28] More precisely, recall that the objective of consumers-investors is to smooth consumption over time (and states of nature). Hence, they price stocks according to the firm's ability to provide consumption insurance: the more able a firm is, the higher its market value and therefore the lower its expected return.[29] In my model, a sequence of positive shocks to firm's i output increases its capital K^i in terms of K^0, thus decreasing its Tobin's q^i. As a result, the firm migrates towards value and is less able to provide consumption smoothing over time since the abundance of capital K^i (with respect to K^0) cannot be directly used for consumption purposes and it is costly to transfer it. Therefore, as shown in Table IV, investors require a higher expected return to hold its stocks. On the contrary, negative shocks to the firm's output decrease its capital

[27] As explained in Section IV, it is important to underline that the relationship between market-to-book ratios and risk premia shown in Table IV is not a real cross-section of stock returns, rather it is the evidence of the steady-state expected returns earned by portfolios formed with the two industries at ten different states of nature defined by the market-to-book ratio.

[28] Recall, in fact, that risk premia mainly depend on the expected return on the production technology and the expected change in the Tobin's q (plus their quadratic covariation). Given the assumption of constant-return-to-scale production functions, the cross-section of stock returns ensues from the (expected) variation of Tobin's q which, as underlined in the previous section, is positive for value firms and negative for growth firms.

[29] Similarly, the more the firm's hindrance to smooth consumption, the lower its stock price and the higher its equity premium.

K^i (in terms of K^0) and increase its Tobin's q^i. As a result, the firm becomes a growth firm and exhibits a lower equity premium since the relative scarcity of capital K^i does not represent a hindrance to smooth consumption over time, given the abundance of K^0.

I now concentrate on the second moments of stock returns. In Figure 6 I show a randomly-drawn sample path of firm'i conditional volatility over a period of 1000 months (together with the corresponding path of Tobin's q^i, for comparison).

FIGURE 6 GOES HERE

My model generates a non-monotone relationship between Tobin's q and conditional volatility consistent with the finding of Kogan (2004). Specifically, value and growth firms exhibit a higher volatility than neutral firms. The explanation behind this result is pretty intuitive. Recall, in fact, that in my production economy the "tolerated" imbalances of capital are endogenously determined. To ensure market clearing, in response to shocks in the economy, either capital has to be transferred among sectors or prices must change to absorb the shocks. Now, when capital cannot be transferred as a result of transaction costs (i.e. inside the no-transaction region Ω), the Tobin's q must change to absorb the shocks, thus exhibiting a higher volatility. On the contrary, when investment/disinvestment is about to take place (i.e. at the boundaries), the supply of capital is relatively elastic and partially absorbs the shocks, thus reducing the volatility of Tobin's q.

Stock price S^i inherits this behavior since it is the product of Tobin's q^i times capital K^i, but exhibits it in a different manner because of the different reactions of q^i and K^i to shocks realizations, which go in opposite directions and generate two contrasting forces simultaneously at work. In fact, recall that positive shocks to the firm's output on the one hand increase its capital K^i but, on the other hand, decrease its relative valuation (in units of K^0), i.e. the Tobin's q^i. The resulting effect is that, close to the boundaries, i.e. in the case of value and growth firms, stock returns are relatively volatile because they mainly depend on the volatility σ of the technology process whereas the volatility of Tobin's q is nearly zero. On the contrary, far from the boundaries, i.e. in the case of neutral firms, stock returns are less volatile because the response of Tobin's q to economic shocks partially offsets the reaction of K^i.

FIGURE 7 GOES HERE

TABLE V GOES HERE

More importantly, the non-monotone relationship highlighted in Figure 6 is confirmed by the evidence found in the data. In fact, Figure 7 shows that the pattern of the volatility of ten portfolios sorted on book-to-market ratio, obtained using the CRSP database for the years 1963 through 2007, is pretty in line with the prediction of my model. In addition, the differences in total risk (i.e. in volatility) between value and growth firms on the one hand and neutral firms on the other are statistically significant, as shown in Table V above.

VII Conclusion

I propose a general-equilibrium model with proportional adjustment costs and industry-specific capital to study the migration of firms across their market-to-book ratio. The production side of my economy consists of two industries, each grouping a large number of competitive firms using identical production technologies with constant-returns-to-scale and facing higher costs in selling rather than acquiring capital from the consumption sector. The latter features a riskless technology which stores capital and finances consumption. I assume that capital is 'industry-specific', i.e. once invested in the production technology of a given industry, it acquires industry-specific peculiarities which do not render it neither immediately investable in the other industry nor (immediately) consumable by agents.

In equilibrium, I find that there exists a region, namely the no-investment region, within which firms do not trade capital with the pool sector since the investment/disinvestment costs are larger than their expected future benefits. On the contrary, they find optimal to invest/disinvest only at the boundary of such region. Furthermore, I find that the instantaneous expected change of the Tobin's q exhibits a mean-reverting dynamics, switching sign from positive (for value firms) to negative (for growth firms) and vice-versa according to the size of the capital stocks' imbalances, in a way that is consistent with the transition probabilities of migration found in the data. The conditional volatility of the Tobin's q, instead, is larger in the inside of the no-transaction region than in proximity of the boundaries.

Finally, the asset-pricing implications of my economy are in line with the empirical evidence. The first moment of stock returns is negatively correlated with the market-to-book ratio, that is the expected returns earned by firms when they are value are higher than those earned when they are growth. Moreover, my model suggests a non-monotone relationship between Tobin's q and conditional volatility of stock returns consistent with the findings of Kogan (2004). Specifically, value and growth firms exhibit a higher volatility than neutral firms.

Appendixes

Appendix A provides technical details on the solution of the model, focusing first on the reduction of the dimensionality of the problem from three to two state variables, and then on the numerical approach used to determine the no-transaction region. In Appendix B, instead, I show that the instantaneous risk-free rate is constant and equal to r. Finally, Appendix C discusses the optimality of the investment policy.

A The Homogeneity Property and Computational Details

As shown in Section II, inside the no-transaction region Ω, the HJB-equation (i.e. equation 8) is

$$\begin{aligned}\rho V &= \frac{\gamma}{1-\gamma}\left(V_{K^0}\right)^{\frac{\gamma-1}{\gamma}} + rK^0 V_{K^0} + \mu K^1 V_{K^1} + \mu K^2 V_{K^2} \\ &\quad + 0.5\left(\sigma K^1\right)^2 V_{K^1K^1} + 0.5\left(\sigma K^2\right)^2 V_{K^2K^2}.\end{aligned}$$

Considering the linear nature of the constraints and the isoelasticity of the period utility function, the value function $V\left(K^0, K^1, K^2\right)$ is homogeneous of degree $1-\gamma$. Exploiting this homogeneity and defining $\omega^i \equiv \frac{K^i}{K^0}$, I can rewrite the value function V as

$$V(K^0, K^1, K^2) \equiv (K^0)^{1-\gamma} G(\omega^1, \omega^2). \tag{A.1}$$

Taking the logarithm, I get

$$(1-\gamma)\log(K^0) + I(\omega^1, \omega^2) \equiv \log V(K^0, K^1, K^2). \tag{A.2}$$

Computing the appropriate derivatives of Equation (A.2), it is possible to rewrite the P.D.E. (8) as

$$\begin{aligned}\rho &= \frac{\gamma}{1-\gamma}\left(1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2}\right)^{\frac{\gamma-1}{\gamma}}\left(e^I\right)^{\frac{-1}{\gamma}} + r\left(1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2}\right) \\ &\quad + \mu\left[\omega^1 I_{\omega^1}+\omega^2 I_{\omega^2}\right] + 0.5\sigma^2\left[\omega^1\omega^1 I_{\omega^1\omega^1} + \left(\omega^1 I_{\omega^1}\right)^2 + \omega^2\omega^2 I_{\omega^2\omega^2} + \left(\omega^2 I_{\omega^2}\right)^2\right]\end{aligned} \tag{A.3}$$

Recall that an investment takes place only when there is abundance of K^0 with respect to K^i, that is when $\omega^i = \underline{\omega}^i$. On the contrary, when ω^i reaches the upper boundary $\overline{\omega}^i$ from the inside of Ω, a disinvestment takes place. Therefore, exploiting again Equation (A.2), value-matching conditions (9) and (10) can be rewritten as

$$1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2} = sI_{\omega^i}, \qquad \omega^i = \underline{\omega}^i, \tag{A.4}$$

$$s_d \left[1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2}\right]=I_{\omega^i}, \qquad \omega^i{=}\overline{\omega}^i, \tag{A.5}$$

whereas smooth-pasting conditions (11) and (12) become, when $\omega^i = \underline{\omega}^i$,

$$\left\{ \begin{array}{c} s\left[\omega^i\omega^j I_{\omega^i\omega^i}+\left(\omega^i I_{\omega^i}\right)^2\right]= \\ \omega^i\left[\left(1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2}\right)\left(\omega^i I_{\omega^i}\right)-\left(\omega^i I_{\omega^i}\right)-\omega^i\omega^i I_{\omega^i\omega^i}-\omega^i\omega^j I_{\omega^i\omega^j}\right], \end{array} \right. , \tag{A.6}$$

and when $\omega^i = \overline{\omega}^i$,

$$\left\{ \begin{array}{c} \left[\omega^i\omega^j I_{\omega^i\omega^i}+\left(\omega^i I_{\omega^i}\right)^2\right]= \\ s_d\omega^i\left[\left(1-\gamma-\omega^1 I_{\omega^1}-\omega^2 I_{\omega^2}\right)\left(\omega^i I_{\omega^i}\right)-\left(\omega^i I_{\omega^i}\right)-\omega^i\omega^i I_{\omega^i\omega^i}-\omega^i\omega^j I_{\omega^i\omega^j}\right], \end{array} \right. , \tag{A.7}$$

for $i = 1, 2$.

From a mathematical point of view, my model represents a free-boundary-problem in which both the value function $I(\omega^1, \omega^2)$ and the position of the no-trading region Ω are unknown. Thus, in order to solve the system (A.3-A.7) and compute the optimal position of the investment/disinvestment boundaries $\underline{\omega}^i$ and $\overline{\omega}^i$ (together with the function $I(\omega^1, \omega^2)$), I proceed in three steps.

First, I determine the coordinates of points A and D of the no-investment region Ω (see Figure 2) using a 'shooting' numerical technique. More precisely, recalling the symmetry of the model, I pick a trial value for the coordinates of A and apply value matching (A.4) and smooth-pasting (A.6) conditions at this extreme point to get the values of I_{ω^i} and $I_{\omega^i\omega^i}$ there, for $i = 1, 2$. Starting from these initial conditions, and using the Runge-Kutta method of order 4, I iterate the discretized version of the PDE (A.3) all along the diagonal AD until point D, computing at the same time the function $I(\omega^1, \omega^2)$ and its derivatives along the way (AD). In point D, both conditions (A.5) and (A.7) must hold for $i = 1, 2$. Therefore, if this is true (i.e., if there exists a point such that these conditions hold simultaneously), I have found the optimal position of points A and D, otherwise I pick a new trial value for point A and repeat the entire procedure.

In the second step, I determine the coordinates of point B (see again Figure 2) using a similar shooting technique. In contrast to the first step, here I conjecture a trial linear-shape for the boundary AB. Then, given the same initial conditions of the first step, and using the Runge-Kutta method of order 4, I iterate the discretized version of the PDE (A.3) all along the segment AB until point B (determining also the function $I(\omega^1, \omega^2)$ and its derivatives along the way). As before, in point B, conditions (A.5) and (A.7) must hold for $i = 1$ (see Sections II and III for further details). Again, if this is true (i.e. if there exists a point such that these conditions hold simultaneously), I have found the optimal position of the segment AB, otherwise I pick a new trial value for the slope of AB and restart the procedure.

The assumption of identical production technologies implies the symmetry of the no-transaction region around the diagonal line AD. This gives the coordinates of point C.

In the last step, I compute the missing values of the function $I(\omega^1, \omega^2)$ using the information provided by the optimal position of the no-investment region Ω and the knowledge of the value function $I(\omega^1, \omega^2)$ (and its derivatives) along the diagonal AD and the boundaries AB,AC,BD, and CD. Specifically, I first discretize the values of ω^1 and ω^2 within Ω. Then, using a finite difference method, I solve the PDE (A.3) and compute the function $I(\omega^1, \omega^2)$ (and its derivatives) strictly inside Ω.

B The Risk-free rate

In this Appendix I prove that the instantaneous riskfree rate is constant and equal to r. Applying Ito's lemma to $V_{K^0}(K^0, K^1, K^2)$ gives, in the no-investment region Ω,

$$\begin{aligned} dV_{K^0} &= \begin{bmatrix} V_{K^0K^0}(rK^0 - c) + \mu K^1 V_{K^0K^1} + \mu K^2 V_{K^0K^2} \\ +0.5\left(\sigma K^1\right)^2 V_{K^0K^1K^1} + 0.5\left(\sigma K^2\right)^2 V_{K^0K^2K^2} \end{bmatrix} dt + \\ & \quad \sigma K^1 V_{K^0K^1} dB^1 + \sigma K^2 V_{K^0K^2} dB^2. \end{aligned}$$

As shown in Section II, using the martingale property, I have that

$$\rho V = \max_c \left\{ \begin{array}{c} V_{K^0}(rK^0 - c) + \mu K^1 V_{K^1} + \mu K^2 V_{K^2} \\ +0.5\left(\sigma K^1\right)^2 V_{K^1K^1} + 0.5\left(\sigma K^2\right)^2 V_{K^2K^2} + \dfrac{c^{1-\gamma}}{1-\gamma} \end{array} \right\}.$$

Differentiating the previous equation with respect to K^0, and using the envelope theorem, yields:

$$-\rho V_{K^0} + \left\{ \begin{array}{c} V_{K^0K^0}(rK^0 - c) + rV_{K^0} + \mu K^1 V_{K^0K^1} + \mu K^2 V_{K^0K^2} \\ +0.5\left(\sigma K^1\right)^2 V_{K^0K^1K^1} + 0.5\left(\sigma K^2\right)^2 V_{K^0K^2K^2} \end{array} \right\} \equiv 0$$

Therefore,

$$dV_{K^0} = (\rho - r)V_{K^0} dt + \sigma K^1 V_{K^0K^1} dB^1 + \sigma K^2 V_{K^0K^2} dB^2. \tag{B.1}$$

Since K^0 is used as a numeraire, the price $P_\theta(t)$ of an asset with stochastic dividend stream $\theta(u)$ in consumption units is: $P_\theta(t) = E_t\left[\int_t^\infty e^{-\rho(u-t)} \frac{V_{K^0}(u)}{V_{K^0}(t)} \theta(u) du\right]$. Applying this to price an instantaneously riskless bond yields, as in Cox, Ingersoll, and Ross (1985),

$$\frac{E_t\left[dV_{K^0}\right]}{V_{K^0}} = \left[\rho - r(t)\right] dt.$$

Finally, equation (B.1) implies that in my model $r(t) = r$.

C Optimality of the investment policy

Since the optimal control problem studied in Section II involves continuous consumption and discrete investment/disinvestment at stopping times, it belongs to the class of *combined stochastic control* as studied by Brekke and Øksendal (1998). In this appendix I do not provide a formal proof of the existence of the value function $V(K^0, K^1, K^2)$ satisfying the system (A.3-A.7), and of the optimality of the investment/disinvestment policy described in Sections II and III, because the verification theorem provided by Liu (2004) encompasses the model outlined in my paper.[30] In fact, his Lemma 1 applies to any well-behaved utility function $U(c)$, including the power utility function considered in my framework, and to the dynamics of capital shown in Equations 2-4. On the contrary, here I simply verify that the combined stochastic control implied by the optimal consumption/investment policies satisfy the conditions of his verification theorem.

Let τ_j, $j \in \mathbb{N}$ denote the time when firms invest/disinvest according to the policy specified in Sections II and III. Since this strategy consists in buying and selling the minimal amount of capital necessary to maintain ω_t^i between $\underline{\omega}^i(\omega^j)$ and $\overline{\omega}^i(\omega^j)$, where ω_t^i is the value of the state variable ω^i at time t, the investment time is clearly a stopping time, with $0 \leq \tau_j \leq \tau_{j+1}$ a.s., $\forall\ j \in \mathbb{N}$.

For all $j \in \mathbb{N}$, define χ_i^j the amount invested or disinvested at time τ_j by firm i. More precisely,

$$\chi_i^j = \begin{cases} \underline{\omega}^i - \omega_{\tau_j}^i & \text{if } \omega_{\tau_j}^i \leq \underline{\omega}^i(\omega^j) \\ \overline{\omega}^i - \omega_{\tau_j}^i & \text{if } \omega_{\tau_j}^i \geq \overline{\omega}^i(\omega^j) \\ 0 & \text{otherwise.} \end{cases}$$

Obviously, χ_i^j is $\mathcal{F}_{\tau_j}-$measurable. Finally, since $\forall t \in (0, \infty)$, $P\left\{\omega_t^i \in [\underline{\omega}^i(\omega^j), \overline{\omega}^i(\omega^j)]\right\} = 1$, it follows that $P\left(\lim_{m\to\infty} \tau_m \leq L\right) = 0, \forall L \geq 0$, satisfying the conditions stated in Definition 1 of Liu (2004).

[30] Liu (2004) provides a modified version of the verification theorems of Brekke and Øksendal (1998) and Korn (1998).

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Table I: **Aggregate Moments**

	Data		Model	
	Mean	Std.	Mean	Std.
Consumption Growth	1.72	3.28	1.24	3.71
Risk-free Rate	1.80	3.00	1.80	-
Equity Premium	6.00	18.00	6.20	17.94

Table I reports the unconditional means and standard deviations generated from the simulated data of key variables of the model. The numbers reported in columns denoted "Data" are taken from Campbell, Lo, and MacKinlay (1997). In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$. All data are annualized and in percentages.

Table II: Empirical Transition Probabilities of Migration

Durables				**Healthcare**			
	G(t)	N(t)	V(t)		G(t)	N(t)	V(t)
G(t-1)	63.5	33.1	3.4	G(t-1)	64.8	28.0	7.2
N(t-1)	7.3	68.9	23.8	N(t-1)	8.1	66.7	25.2
V(t-1)	0.2	7.3	92.5	V(t-1)	0.6	6.9	92.5
Manufacturing				**High Technology**			
	G(t)	N(t)	V(t)		G(t)	N(t)	V(t)
G(t-1)	79.6	18.9	1.5	G(t-1)	76.6	21.3	2.1
N(t-1)	5.9	72.5	21.6	N(t-1)	5.7	71.1	23.2
V(t-1)	0.2	9.2	90.6	V(t-1)	0.2	8.8	91.0

Table II shows the average transition vectors for stocks that migrate within the group of three market-to-book ratio portfolios, for portfolio formation years 1963-2007. Specifically, I form three value weight portfolios, G, N, V, at the end of each June from 1963 to 2007 based growth (G, firms in the top 30% of NYSE market-to-book ratio), neutral (N, middle 40% of NYSE market-to-book ratio), and value (V, bottom 30% of NYSE market-to-book ratio). Industry sort based on Compustat/CRSP SIC codes. In the market-to-book ratio sorts for portfolios formed in June of year t, book equity is for the fiscal year ending in calendar year $t-1$ and market equity is for the end of December of $t-1$. The portfolios for year t include NYSE, Amex, and Nasdaq (after 1972) stocks with positive book equity in $t-1$. The transition vectors are for the firms assigned to a portfolio in June of year t that are also in one of the three portfolios in $t+1$. Compared to Fama and French (2007b) I decided to exclude four categories of firms because my model does not generate them: (i) Good Delists, which stop trading between June of t and June of $t+1$ because they are acquired by another firm (CRSP delist codes 200 to 399); (ii) Bad Delists, which stop trading because they no longer meet listing requirements (CRSP delist codes below 200 and above 399), (iii) firms with negative book equity for the fiscal year ending in calendar year t (Neg); and (iv) firms missing book equity for year t or market equity for December of t or June of $t+1$ (NA). The year t transition vector for a portfolio is the fraction of firms in the portfolio when formed at the end of June of year t that falls into each of the groups at the end of June of $t+1$. The table reports averages of the annual transition vectors. Each row shows the average transition vector for a particular portfolio. Up to rounding error, the overall sum of the transition percents for a portfolio is 100.

Table III: Theoretical Transition Probabilities of Migration

Transition Probabilities

	Growth	Neutral	Value
Growth	68.2	21.7	10.1
Neutral	5.9	74.8	19.3
Value	0.3	4.5	95.2

Table III shows the average annual transition probabilities generated from the simulated data of my model for stocks that migrate within the group of three market-to-book ratio portfolios, as a percent of firms in a portfolio. Specifically, Growth portfolio is composed by firms in the top 30% of the steady-state distribution of Tobin's q, Neutral is composed by firms in the middle 40% of the steady-state distribution of Tobin's q and, finally, Value in the bottom 30%. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$. Up to rounding error, the overall sum of the transition percents for a portfolio is 100.

Table IV: The Cross-section of Expected Stock Returns

Growth to Value											
Growth											Value
1A	1B	2	3	4	5	6	7	8	9	10A	10B
Mean Excess Return (% per year): Empirical Data											
2.99	3.64	4.53	5.45	5.25	6.02	6.76	8.16	7.25	10.74	12.99	13.66
Mean Excess Return (% per year): Implied Returns											
3.36	3.56	4.00	4.61	5.53	5.91	6.57	7.21	7.87	8.51	8.64	8.70

Table IV shows the cross-section of expected excess returns for 10 market-to-book portfolios, from Growth (decile 1) to Value (decile 10). The bottom and top two portfolios (1A, 1B, 10A and 10B) split the bottom and top deciles in half. The numbers reported in the row denoted "Empirical Data" are taken from Gala (2010). In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$. All data are annualized and in percentages.

Table V: Volatility-Equality test among portfolio deciles sorted on market-to-book ratio

market-to-book deciles	F-statistics	p-value
Growth - decile 1 vs decile 5	1.398	0.000
decile 2 vs decile 5	1.170	0.07
decile 9 vs decile 5	1.1105	0.226
Value - decile 10 vs decile 5	1.491	0.000

Table V tests the null hypothesis that the volatility of growth and value portfolios (respectively deciles 1-2 and 9-10) is the same as the volatility of growth firms (decile 5). Portfolios are obtained using the CRSP database for the years 1963 through 2007. The statistics used is the ratio of the sample-returns variances of the portfolio deciles considered. Such statistics has an F-distribution and is equal to 1 under the null hypothesis of equality of variances. The null hypothesis is rejected if the F-statistics is either too large or too small.

Figure 1: The no-investment region: a simplified economy with one industry

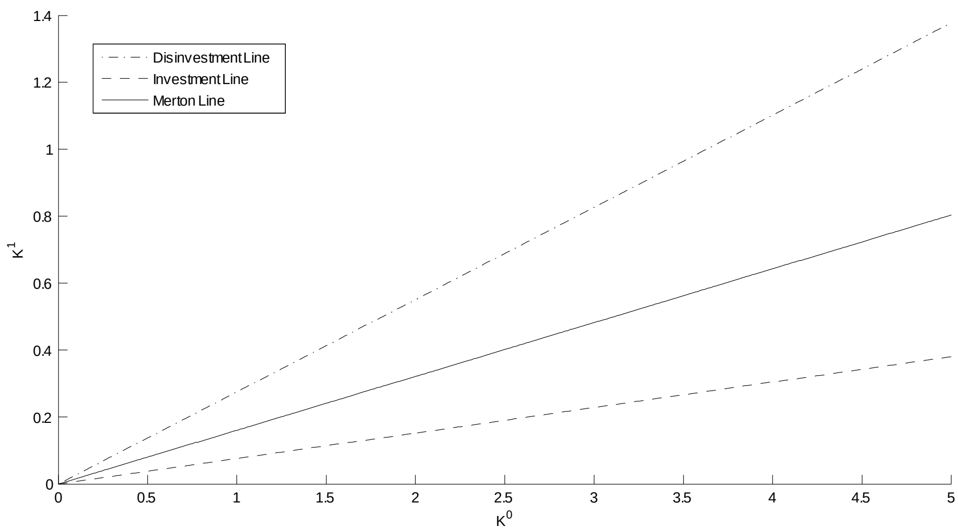

Figure 1 shows the optimal position of the no-investment region in a simplified economy with only one productive industry. The slopes of the three lines are: 0.275 (disinvestment line), 0.076 (investment line) and 0.160 (Merton line). In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$.

Figure 2: The no-investment region Ω: the general economy with two industries

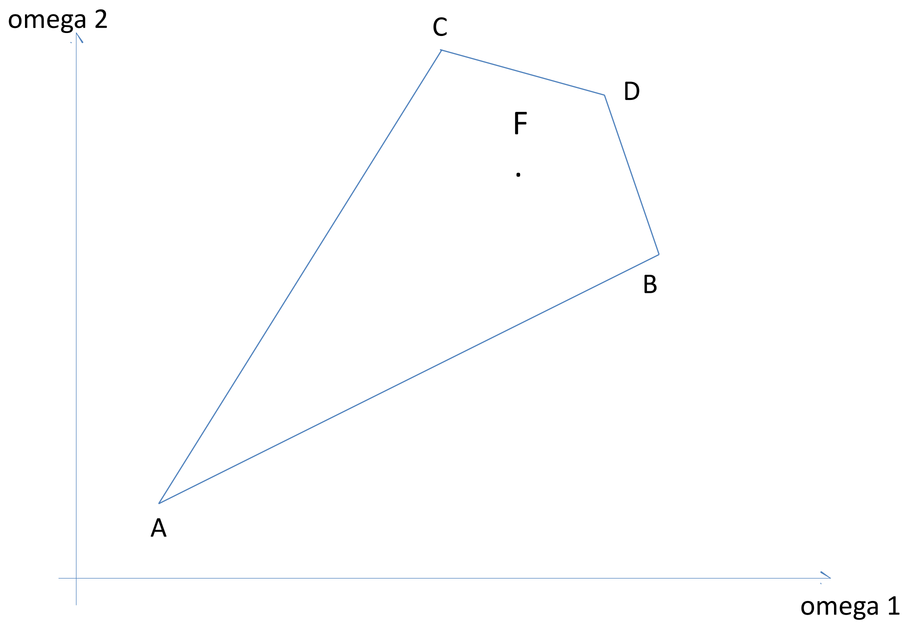

Figure 2 shows the optimal position of the no-investment region Ω in the space (ω^1, ω^2). The coordinates of the corners are: $A = (0.091, 0.091)$, $B = (0.333, 0.146)$, $D = (0.289, 0.289)$ and $C = (0.146, 0.333)$. Point F corresponds to the frictionless allocation of capital stocks shown in Equation (13). Its coordinates are $(0.16, 0.16)$. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$. Growth firms in industry two (respectively one) are located close the segment AB (AC). Value firms in industry one (respectively two) are located close to segment BD (CD).

Figure 3: Tobin's q

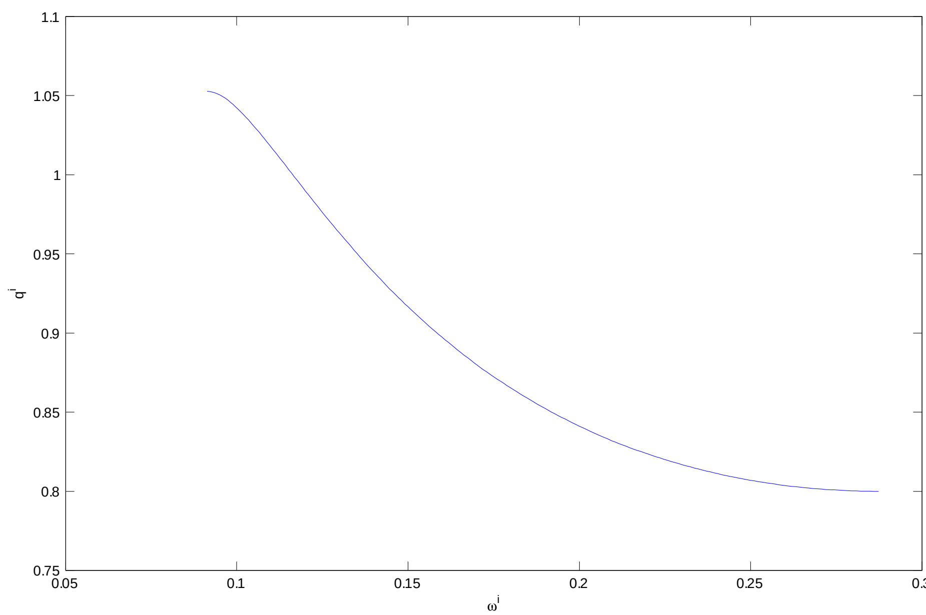

Figure 3 displays the behavior of firm's i market-to-book ratio along the diagonal line AD of the no-investment region Ω as a function of the state variable ω^i. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$.

Figure 4: Sample paths of Tobin's q^i, its conditional expected change and its conditional standard deviation

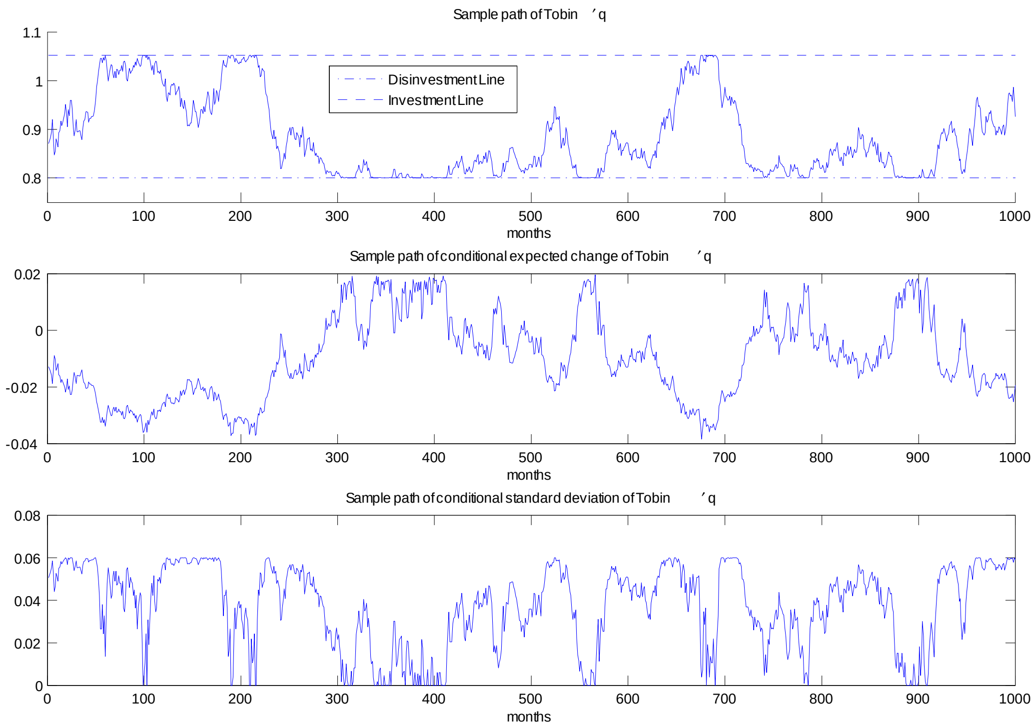

Figure 4 shows a randomly-drawn sample path of the Tobin's q^i, together with the corresponding paths of its conditional expected change and its conditional standard deviation, over a period of 1000 months. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$.

Figure 5: Sample path of expected stock returns

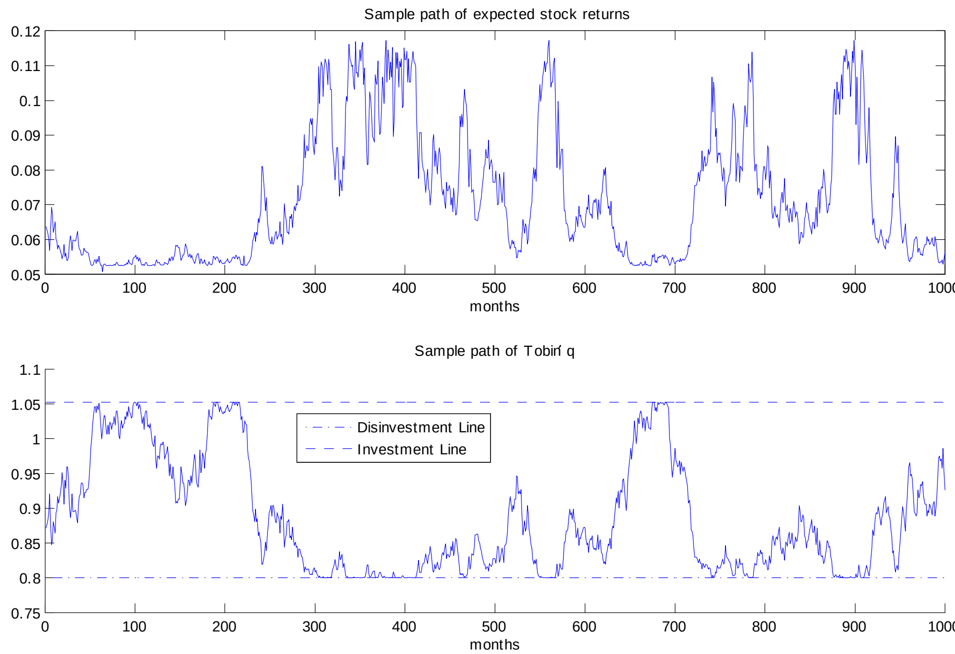

Figure 5 shows a randomly-drawn sample path of expected stock returns, together with the corresponding path of the Tobin's q, over a period of 1000 months. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$.

Figure 6: Sample path of conditional volatility of stock returns

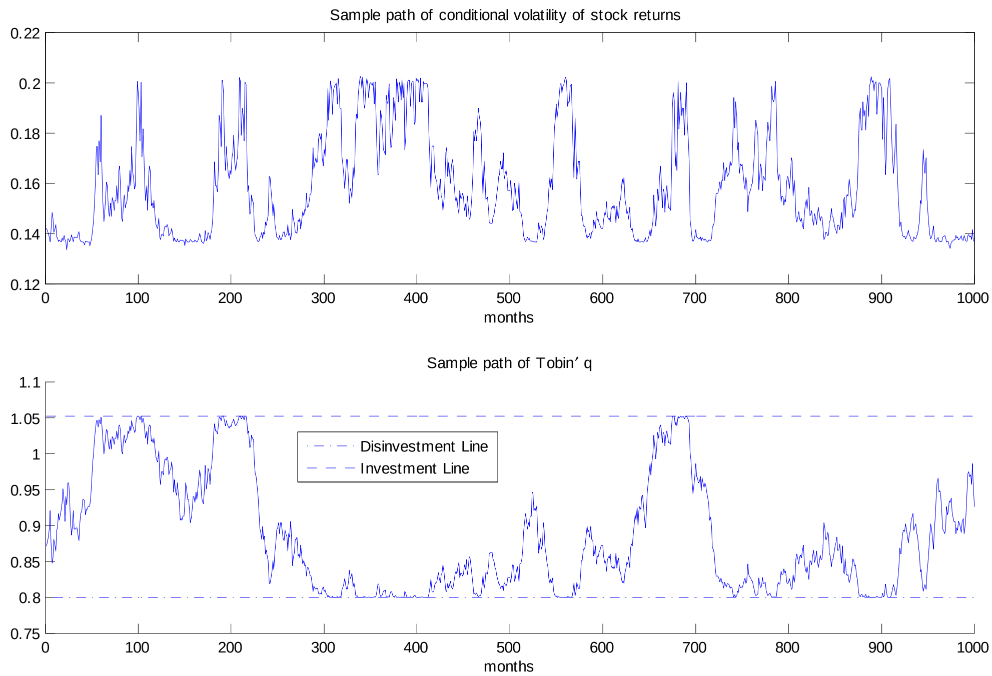

Figure 6 shows a randomly-drawn sample path of the conditional volatility of stock returns, together with the corresponding path of the Tobin's q, over a period of 1000 months. In the model, the riskfree rate is constant and equal to $r = 0.018$. The discount rate ρ is set to 0.01, while the risk aversion γ is 13. The investment cost parameter s is 0.95, whereas the disinvestment cost s_d is 0.8. Finally, the expected rate of return μ and the standard deviation σ of the productivity process are given by $\mu = 0.09$ and $\sigma = 0.2$.

Figure 7: **Conditional volatility of equity returns and Tobin's q - empirical evidence**

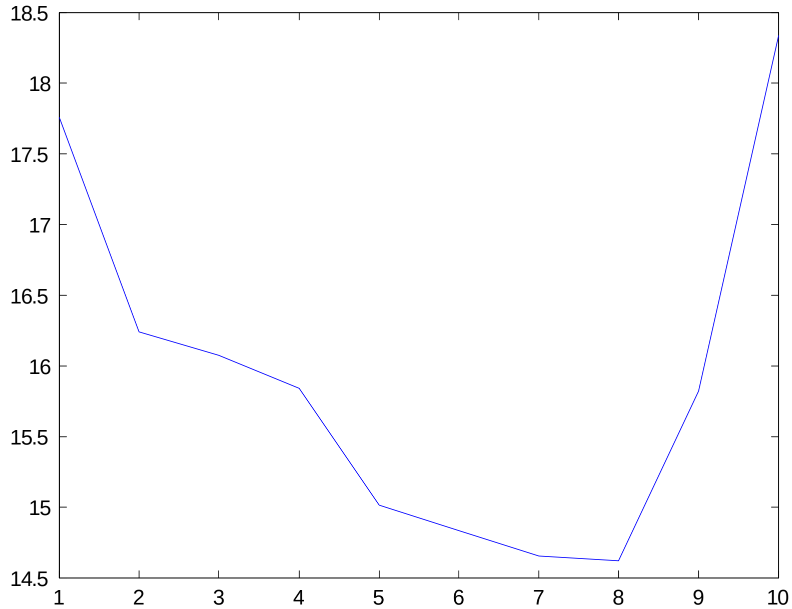

Figure 7 shows the conditional volatility (vertical axis) of ten portfolios sorted on market-to-book ratio, from Growth (decile 1) to Value (decile 10). Portfolios are obtained using the CRSP database for the years 1963 through 2007. In the book-to-market sorts for portfolios formed in June of year t, book equity is for the fiscal year ending in calendar year $t-1$ and market equity is for the end of December of $t-1$. The portfolios for year t include NYSE, Amex (after 1963), and Nasdaq (after 1972) stocks with positive book equity in $t-1$. All data are annualized and in percentages.