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Transparency and Product Differentiation with Competing Vertical Hierarchies

Matteo Bassi[*], Marco Pagnozzi[**] and Salvatore Piccolo[***]

Abstract
We revisit the choice of product differentiation by competing firms in the Hotelling model, under the assumption that firms are vertically separated, and that retailers choose products' characteristics. We show that retailers with private information about their marginal costs choose to produce less differentiated products than retailers with no private information, in order to increase their information rents. Hence, information asymmetry increases social welfare because it induces firms to sell products that appeal to a larger number of consumers. The socially optimal level of transparency between manufacturers and retailers depends on the weight assigned to consumers' surplus and trades of two effects: higher transparency reduces price distortion but induces retailers to produce excessively similar products.

Keywords: product differentiation, vertical relations, transparency, regulation.

JEL Classification: D43, D82, L13, L51.

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1. Introduction

We consider two firms that sell substitute products and choose the degree of "horizontal" differentiation among their products. Following the classic Hotelling (1929) model, we assume that firms choose where to locate on a line; consumers are distributed along the line and pay a transportation cost to reach a firm and purchase its product. Alternatively, each point on the line may be interpreted as a possible *variety* of a product (e.g., a different amount of some product characteristic). The point at which each consumer is located denotes its most preferred variety while a firm's location represents the variety that it chooses to produce. Transportation cost can be seen as the loss of utility of a consumer that purchases a variety that is different from its most preferred one. This model captures the idea that different consumers prefer different varieties of a product, and firms can choose the degree of differentiation among their products by choosing their specific characteristics.

According to the *principle of differentiation*, firms want to differentiate their products in order to soften price competition. In various contexts, firms want to maximize differentiation and locate as far as possible from their rivals (e.g., with quadratic transportation costs — see D'Aspremont *et al.*, 1979). This is consistent with the observation that firms often search for market niches to position their products with respect to competitors' products. However, there are a number of factors that limit firms' incentive to differentiate their products: concentration of demand on a particular variety of the product (that induces firms to locate where the demand is — see Economides, 1986, and De Palma *et al.*, 1985), fixed costs of production (that limit the number of different varieties that firms are willing to produce), positive externalities in production or demand among firms producing similar products, etc.

We introduce a novel reason that may induce vertically separated firms to produce less differentiated products: asymmetric information between manufacturers and retailers.[1] Retailers often undertake non-market activities such as product design, advertising campaigns, and quality enhancing investments, that determine the degree of product differentiation with respect to competitors. But retailers who have private information about their marginal costs of production also have an incentive to choose product characteristics that appeal to a larger number of consumers, in order to increase sales and, hence, their information rent. Therefore, when retailers choose products' characteristics before prices, asymmetric information induces them to choose less differentiated products (than without private information), even though this increases competition. The availability of less differentiated products increases consumers' welfare because it reduces transportation costs (which can be interpreted as more consumers acquiring a product that is relatively similar to their most preferred ones).

Building on this result, we consider a regulator who can control the level of asymmetric information between manufacturers and retailers (i.e., the dispersion of retailers private infor-

[1] There is a large empirical literature showing that asymmetric information affects pricing decision in industries where firms are vertically separated (see, e.g., Lafontaine and Slade, 1997, and Lanfontaine and Shaw, 1999).

mation), which we interpret as transparency. In practice, firms' accounting rules are subject to regulation that, directly or indirectly, determines the quality and the quantity of information that firms need to report to the public. For instance, when they are subject to stricter accounting standards, retailers must provide more detailed accounting reports, which can be observed by their suppliers, whereby reducing retailers' private information.

It is often argued that a regulator who aims to maximize social welfare should always reduce asymmetric information between manufacturers and retailers, because asymmetric information induces firms to choose inefficiently high prices.[2] By contrast, we prove that, when a regulator can control the dispersion of retailers' private information, he never chooses to impose full transparency — i.e., to eliminate private information altogether — although this would eliminate the distortion in retail prices due to private information. The reason is that, without asymmetric information, firms maximize product differentiation, thus reducing welfare, while retailers with private information produce less differentiated products.

The socially optimal level of transparency depends on the weight assigned to consumers' surplus. Reducing transparency has two opposite effects on social welfare. On the one hand, lower transparency tends to reduce welfare because it induces manufacturers to charge inefficiently high prices: *a price distortion effect*. On the other hand, lower transparency induces retailers to produce products that are relatively more differentiated (but not maximally differentiated), thus reducing consumers' transportation costs and increasing welfare: *a product differentiation effect*.[3] When the regulator assigns a low weight to consumers' surplus, he chooses the maximal degree of asymmetric information compatible with the market being fully covered. In this case, the price distortion effect is weaker than the product differentiation effect. By contrast, when the weight assigned to consumers' surplus takes intermediate values, the optimal degree of asymmetric information is positive, but not maximal, so as to balance the price distortion and the product differentiation effects. Finally, when the regulator assigns a high weight to consumers' surplus, he minimizes asymmetric information because the price distortion effect prevails.

On the normative ground, our analysis offers a justification for regulatory policies that allow for lower (or imperfect) standards of transparency.[4] Although we analyze a stylized IO model, our results are more general and may be applied to procurement contracting, executive compensations, patent licensing or credit relationships, when there are competing hierarchies and non-contractable product differentiation activities, such as investments in product design, advertising campaigns, R&D etc.

The rest of the paper is organized as follows. Section 2 presents the model. After discussing

[2] The literature on supply chains, for instance, shows how the presence of privately informed retailers leads to equilibrium prices higher than marginal costs — see Blair and Lewis (1994), Gal-Or (1991, 1999), Khun (1997), Martimort (1996), and Martimort and Piccolo (2007).

[3] When transparency increases, retailers choose products that are too similar from a social welfare's point of view, because their characteristics are most preferred by fewer potential consumers.

[4] Bennardo et al. (2010), Calzolari and Pavan (2006), Taylor (2004), and Maier and Ottaviani (2009) analyze transparency in vertical contracting. In contrast to our paper, this literature does not consider competing vertical hierarchies.

the benchmark case of full transparency in Section 3, in Section 4 we consider the case of asymmetric information. Section 5 analyzes the choice of transparency by a regulator who maximizes social welfare. Finally, Section 6 concludes. All proofs are in the appendix.

2. The Model

Players and environment. We consider the "linear city" model introduced by Hotelling (1929). There is a unit mass of consumers uniformly distributed with density 1 over $[0,1]$. Two vertical structures, each composed by one manufacturer and one retailer, produce a homogeneous good and are located at a_1 and $(1-a_2)$, respectively, where without loss of generality $a_1+a_2 \leq 1$ ($a_1 = a_2 = 0$ implies maximal differentiation). Specifically, manufacturer M_1 and retailer R_1 are located at a_1, while manufacturer M_2 and retailer R_2 are located at $(1-a_2)$. The location of a vertical structure is chosen by the retailer. We assume that each vertical structure can choose only one location — i.e., can produce a single variety of the good — because of fixed costs.[5]

Each consumer has a valuation v for a single unit of the good. For simplicity, we assume that $v \to +\infty$, so that each consumer always buys one unit, regardless of the price. Consumers pay a quadratic transportation cost to reach the vertical structures. Specifically, a consumer located at $x \in [0,1]$ pays $t(x-a_1)^2$ to buy from R_1 and $t(1-a_2-x)^2$ to buy from R_2.

Given the retail prices of the goods produced by the two vertical structures, p_1 and p_2, a consumer located at x buys from R_1 if and only if

$$p_1 + t(x-a_1)^2 < p_2 + t(1-a_2-x)^2 .$$

Therefore, in an interior solution, the demand for the good sold by R_i is

$$D^i(p_i, p_j) = \frac{1+a_i-a_j}{2} + \frac{p_j - p_i}{2t(1-a_i-a_j)}, \quad i,j = 1,2, \quad i \neq j.$$

Before being offered a contract from his manufacturer, R_i privately observes his constant marginal cost of production θ_i, which is distributed uniformly on $\Theta \equiv [\mu-\sigma, \mu+\sigma]$, so that its c.d.f. is $F(\theta_i) = \frac{\theta_i-(\mu-\sigma)}{2\sigma}$ with mean μ and variance $\frac{\sigma^2}{3} > 0$. We assume that: *(i)* $\sigma \in \{0\} \cup [\underline{\sigma}, \overline{\sigma}]$; *(ii)* $\underline{\sigma} > 0$ and $\underline{\sigma} \simeq 0$; *(iii)* $\overline{\sigma} < \frac{t}{4}$ and $\overline{\sigma} \simeq \frac{t}{4}$.[6] When $\sigma = 0$ there is full transparency and retailers' marginal costs are equal to μ. When $\sigma \neq 0$ there is asymmetric information between manufacturers and retailers. We also assume that $\overline{\sigma} < \mu$, so that marginal costs are always positive.

Contracts. Contracts between manufacturers and retailers are private — i.e., they cannot be

[5] Matsushima and Matsumura (2003) analyze an Hotelling model with two vertically integrated firms and cost uncertainty.

[6] The assumption on $\underline{\sigma}$ ensures the existence of a solution to the regulator's problem in Section 5 (see the proof of Proposition 2). The assumption on $\overline{\sigma}$ ensures that retailers' marginal costs cannot differ too much, so that in a symmetric equilibrium each retailer's demand is always strictly positive (see the proof of Proposition 1).

observed by competitors. We consider "resale price maintenance" (RPM) contracts and use the Revelation Principle to characterize the equilibrium of the game. Hence, M_i offers a menu $(p_i(\hat{\theta}_i), T_i(\hat{\theta}_i))_{\hat{\theta}_i \in \Theta}$ to R_i, where given R_i's report $\hat{\theta}_i$ to M_i, $p_i(\hat{\theta}_i)$ represents the retail price at which R_i has to sell to final consumers and $T_i(\hat{\theta}_i)$ is the franchise fee paid by R_i to M_i. In the appendix we prove that, with private contracts, our model with RPM contracts is equivalent to a model in which each manufacturer offers menus of two-part tariffs composed by a wholesale price and a franchise fee, contingent on the retailer's report.

Timing. The timing of the game is as follows:

1. Retailers simultaneously and independently choose their locations: a_1 and a_2. Locations are publicly observable.
2. Retailers privately observe their costs.
3. Manufacturers offer contracts and retailers choose whether to accept them. If R_i accepts M_i's offer, then he reports his type and pays the franchise fee.
4. Retailers announce retail prices and the market clears.

Equilibrium concept. The solution concept is Perfect Bayesian Equilibrium (PBE). Since contracts are private, we have to make an assumption on retailers' beliefs about their competitors' behavior. We assume that, regardless of the contract offered by his own manufacturer, a retailer always believes that the other manufacturer offers the equilibrium contract,[7] and that each retailer expects the rival retailer to truthfully report his type to the manufacturer in a separating equilibrium.

3. Full Transparency Benchmark

Suppose first that there is full transparency ($\sigma = 0$). Hence, information is complete since retailers' costs are deterministic and equal to μ, and M_i offers a contract (p_i, T_i) that extracts the whole surplus from her retailer — i.e., she charges

$$T_i = D^i(p_i, p_j)(p_i - \mu),$$

where p_j is the equilibrium price chosen by M_j. Given retailers' choice of location, M_i chooses the retail price to maximize profit — i.e., to solve

$$\max_{p_i} D^i(p_i, p_j^*)\,(p_i - \mu)\,.$$

[7] See Pagnozzi and Piccolo (2010) for an analysis of the roles of beliefs when contracts between manufacturers and retailers are private.

Since retailers obtain no rent when they have no private information, they are indifferent among any location choice. Following a standard convention, we assume that, when indifferent, retailers' choose the location that maximizes manufacturers' profit.

Lemma 1. *In a symmetric equilibrium firms locate at* 0 *and* 1 *— i.e., they choose maximal differentiation — and manufacturers choose a retail price equal to* $t + \mu$.

The intuition for this result is straightforward. By producing a differentiated product, a manufacturer reduces price competition and enjoys some market power over consumers distributed around her location. This is the principle of differentiation. Of course, firms' prices are increasing in t, because when the transportation cost is higher products are perceived as more differentiated by consumers, and hence manufacturers compete less fiercely to attract consumers by charging lower prices. Prices are also increasing in μ, because when marginal costs are higher, manufacturers choose higher prices in equilibrium.

4. Equilibrium with Asymmetric Information

Suppose that $\sigma \neq 0$. We focus on a symmetric separating equilibrium. Let $p_i^*(\theta_i)$, $i = 1, 2$, be the retail price chosen by M_i when R_i's cost is θ_i, given the locations chosen by retailers. In a separating equilibrium, a manufacturer's contract must be incentive feasible — i.e., it must satisfy the retailer's participation and incentive compatibility constraints.

Given an incentive compatible menu $(p_i(.), T_i(.))$, R_i's information rent is

$$U_i(\theta_i) = (p_i(\theta_i) - \theta_i) \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i(\theta_i), p_j^*(\theta_j)) dF(\theta_j) - T_i(\theta_i), \quad i = 1, 2.$$

Incentive compatibility implies that

$$U_i(\theta_i) = \max_{\hat{\theta}_i \in \Theta} \left\{ (p_i(\hat{\theta}_i) - \theta_i) \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i(\hat{\theta}_i), p_j^*(\theta_j)) dF(\theta_j) - T_i(\hat{\theta}_i) \right\},$$

which yields the first- and second-order local conditions for incentive compatibility

$$\dot{U}_i(\theta_i) = -\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i(\theta_i), p_j^*(\theta_j)) dF(\theta_j), \tag{4.1}$$

and

$$-\frac{\partial \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i(\theta_i), p_j^*(\theta_j)) dF(\theta_j)}{\partial p_i} \dot{p}_i(\theta_i) \geq 0 \quad \Rightarrow \quad \dot{p}_i(\theta_i) \geq 0. \tag{4.2}$$

Conditions (4.1) and (4.2), together with the participation constraint

$$U_i(\theta_i) \geq 0, \quad \forall \theta_i \in \Theta, \tag{4.3}$$

define the set of incentive-feasible allocations for M_i and R_i. Therefore, M_i solves the following optimization program

$$\max_{\{p_i(.),U_i(.)\}} \int_{\mu-\sigma}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} \left\{D^i(p_i(\theta_i), p_j^*(\theta_j))\,(p_i(\theta_i)-\theta_i) - U_i(\theta_i)\right\} dF(\theta_j)\,dF(\theta_i)\,,$$

subject to conditions (4.1), (4.2) and (4.3).

To solve this program, we first assume that $\dot{p}_i(\theta_i) \geq 0$, and then check that this condition holds ex post. It follows that $U_i(\theta_i)$ is decreasing and the participation constraint is binding when $\theta_i = \mu + \sigma$. Hence, R_i's information rent is

$$U_i(\theta_i) = \int_{\theta_i}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i(x), p_j^*(\theta_j))dF(\theta_j)dx. \tag{4.4}$$

This rent is increasing in consumers' demand for the good sold by R_i. The reason is that a retailer with a low marginal cost obtains a higher utility by mimicking retailers with higher marginal costs when those retailers sell a higher quantity on average — i.e., the information rent of a type is increasing in the quantity sold by less efficient types. Since the demand for the good sold by R_i is increasing in a_i (because locating closer to the center attracts more customers), this provides an incentive for a retailer to produce a product that is more similar to his competitor's product.

Using expression (4.4), integrating by parts, and substituting in M_i's objective function yield the simplified program

$$\max_{p_i(.)} \int_{\mu-\sigma}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} \left\{D^i(p_i(\theta_i), p_j^*(\theta_j)) \left(p_i(\theta_i) - \theta_i - \frac{F(\theta_i)}{f(\theta_i)}\right)\right\} dF(\theta_j)\,dF(\theta_i)\,.$$

The first-order condition for this program is

$$p_i^*(\theta_i) = \theta_i + \frac{F(\theta_i)}{f(\theta_i)} - \frac{\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(\theta_i), p_j^*(\theta_j))dF(\theta_j)}{\int_{\mu-\sigma}^{\mu+\sigma} \frac{\partial}{\partial p_i} D^i(p_i^*(\theta_i), p_j^*(\theta_j))dF(\theta_j)}. \tag{4.5}$$

Lemma 2. *Given retailers' locations a_1 and a_2, M_i chooses the retail price*

$$p_i^*(\theta_i) = \theta_i + \sigma + \tfrac{t}{3}(1-a_i-a_j)(3-a_i+a_j)\,, \quad i = 1,2. \tag{4.6}$$

By equation (4.5), the retail price distortion with respect to marginal cost is increasing in the hazard rate $\frac{F(\theta_i)}{f(\theta_i)}$. The reason is that, at a best-response to the price schedule $p_j^*(\theta_j)$, M_i chooses p_i so as to equalize her virtual marginal revenue to zero. Under asymmetric information, the cost parameter θ_i is replaced by a higher virtual cost parameter $\theta_i + \frac{F(\theta_i)}{f(\theta_i)}$, so that the allocation of a high-cost type becomes less attractive to a low-cost type, and the latter's incentive to misreport his marginal cost is mitigated. In other words, increasing the price of an agent with type θ_j reduces the information rents of agents with types lower than θ_j.

Consider now retailers' choice of locations. Before observing his marginal cost, in order to maximize his expected information rent, R_i solves

$$\max_{a_i} \int_{\mu-\sigma}^{\mu+\sigma} \int_{\theta_i}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(x), p_j^*(\theta_j)) dF(\theta_j) dx dF(\theta_i).$$

Two opposite effects determine retailers' choice. On the one hand, if a retailer reduces the degree of differentiation of his product vis-à-vis his rival, the rival firm reacts by reducing his own price: price competition is more intense with less differentiated products. This (standard) strategic effect tends to reduce demand and, hence, the retailer's information rent. On the other hand, however, choosing a location closer to the center allows a retailer to attract more customers, since they have to pay a lower transportation cost to acquire the retailer's product. This sales effect tends to increase information rent.

Proposition 1. *In a symmetric PBE, both retailers choose*

$$a^*(\sigma) = \tfrac{1}{2} - \tfrac{1}{2}\sqrt{\tfrac{\sigma}{t}},$$

and set a retail price equal to

$$p^*(\theta_i) = \theta_i + \sigma + \sqrt{t\sigma}.$$

Compared to the complete information benchmark, where the principle of differentiation applies, retailers choose a lower degree of differentiation among their products, in order to increase their information rents. Retailers would jointly prefer to choose maximal differentiation — i.e., $a_1 = a_2 = 0$ — since these are the symmetric locations that maximize total retailers' rents. These are the locations chosen by retailers when $\sigma = 0$. However, when $\sigma \neq 0$, if one retailer chooses $a_i \simeq 0$, the other retailer has an incentive to locate closer to the center in order to increase his information rent. In other words, when product are very differentiated, the sales effect dominates. Anticipating this, the first retailer moves closer to the center as well: the choices of locations are strategic complements for retailers. Hence, retailers face a "prisoners' dilemma" when choosing their locations.

Notice, however, that a^* is decreasing in σ, for $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. A higher σ implies a higher retail price, because more private information creates more price distortion, and a lower a^* — i.e., more product differentiation. The reason is that, when σ is high, retailers expect retail price to be higher and, hence, have a lower incentive to produce less differentiated product to increase sales. So retailers can produce more differentiated products, which increase profit for the strategic effect described above. As $\sigma \to 0$, $a^* \to \frac{1}{2}$: when asymmetric information vanishes, retailers tend to eliminate product differentiation altogether, since they have the strongest incentive to increase sales in order to obtain an information rent. Figure 4.1 summarizes firms' choices of product differentiation as a function of σ.

A higher t implies a higher a^* — i.e., less differentiation — and a higher retail price, because

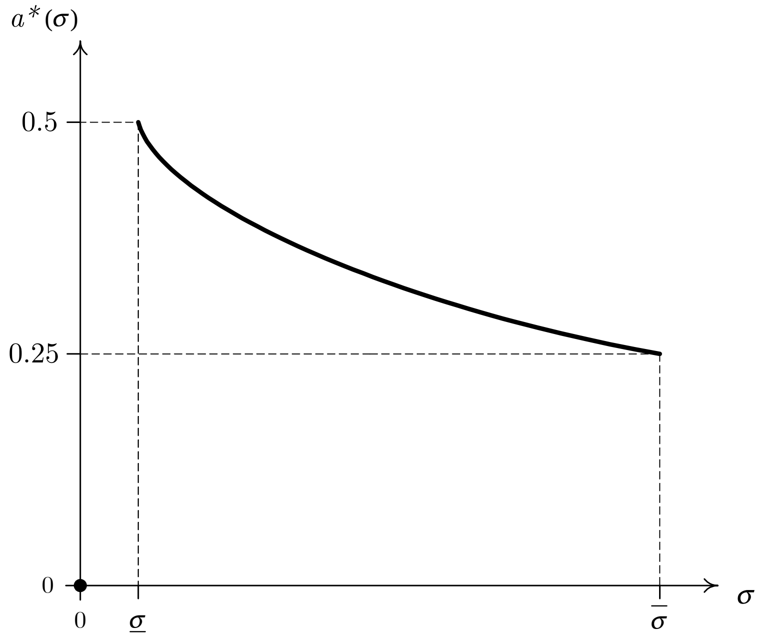

Figure 4.1: Retailers' choice of location

when the transportation cost is high, consumers perceive products as more differentiated, thus reducing price competition among firms.

5. Optimal Transparency

In this section, we analyze the choice of the level of transparency between manufacturers and retailers, by a regulator who is interested in maximizing expected welfare. Specifically, we assume that the regulator maximizes a weighted sum of consumer surplus and firms' profits (i.e., the sum of manufacturers' profits and retailers' information rents). Given marginal costs θ_1 and θ_2, retail prices p_1 and p_2, and locations a_1 and a_2, the welfare function is

$$
\begin{aligned}
W(.) \equiv\ & \lambda \left[v - \int_0^{D^1(p_1,p_2)} \left(p_1 + t\,(x-a_1)^2\right) dx - \int_{D^1(p_1,p_2)}^1 \left(p_2 + t\,(1-a_2-x)^2\right) dx \right] + \\
& (1-\lambda)\left[D^1(p_1,p_2)(p_1-\theta_1) + \left(1-D^1(p_1,p_2)\right)(p_2-\theta_2)\right] \\
=\ & \lambda v - D^1(p_1,p_2)\left[(2\lambda-1)p_1 + (1-\lambda)\theta_1\right] - \lambda \int_0^{D^1(p_1,p_2)} t\,(x-a_1)^2\, dx + \\
& -\left(1-D^1(p_1,p_2)\right)\left[(2\lambda-1)\,p_2 + (1-\lambda)\theta_2\right] - \lambda \int_{D^1(p_1,p_2)}^1 t\,(1-a_2-x)^2\, dx,
\end{aligned}
$$

where $\lambda \in [\frac{1}{2}, 1]$ is the weight assigned to consumers' surplus. When $\lambda = \frac{1}{2}$, the regulator treats consumers and firms symmetrically and simply minimizes transportation and production costs (since in our model the total demand is fixed). When $\lambda = 1$, the regulator maximizes consumers' surplus and, hence, minimizes retail prices and transportation costs. Ceteris paribus, a higher λ implies that the regulator is relatively more concerned about reducing retail prices.

We assume that the regulator can choose σ to determine the level of transparency — i.e., the amount of retailers' private information with respect to manufacturers. By choosing $\sigma = 0$, the regulator imposes full transparency; by choosing $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, the regulator controls the level of transparency when retailers have private information: increasing σ reduces transparency since it results in a higher amount of private information. For example, the regulator can increase transparency by imposing more restricting accounting standards to retailers, that allow manufacturers to infer more information about their retailers' marginal costs.

Hence, given the equilibrium choices of locations and retail prices, the regulator solves

$$\max_{\sigma \in \{0\} \cup [\underline{\sigma}, \overline{\sigma}]} \int_{\mu-\sigma}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} W(.)\, dF(\theta_1)\, dF(\theta_2).$$

The analysis of Section 3 suggests that, with asymmetric information, the regulator can prevent firms from choosing maximally differentiated products.

Lemma 3. *The regulator never chooses full transparency — i.e., he always chooses* $\sigma \neq 0$.

Even if the regulator can completely eliminate asymmetric information between manufacturers and retailers by imposing full transparency, he never chooses to do so. The reason is that, with full transparency, retailers choose maximal product differentiation while, with a positive level of asymmetric information, they locate away from the extremes of the interval — i.e., they produce less differentiated products — thus intensifying price competition and increasing welfare.

In order to introduce the effects on expected welfare of asymmetry between manufacturers and private information by retailers, we first consider the choice of a regulator who only minimizes transportation costs or production costs.

Lemma 4. *Assume that the regulator is only interested in minimizing transportation costs. Then the regulator chooses* $\sigma = t - \frac{t\sqrt{3}}{2}$ *and retailers locate (approximately) at* 0.317 *and* 0.683.

Assume that the regulator is only interested in minimizing total production costs. Then the regulator chooses $\sigma = \overline{\sigma}$ *and retailers locate (approximately) at* $\frac{1}{4}$ *and* $\frac{3}{4}$.

With symmetric firms and no uncertainty about marginal costs, the locations that minimize transportation costs are $\frac{1}{4}$ and $\frac{3}{4}$. With asymmetric firms, however, the regulator induces retailers to locate closer to the center to minimize transportation costs, since otherwise contested consumers (located between the two firms) may be forced to pay very high transportation costs in order to purchase from the most efficient firm. By contrast, if the regulator wants to reduce production costs, he induces firms to locate further away from each other in order to increase the number of contested consumers who purchase from the most efficient firm.

We now consider the optimal choice of transparency by the regulator.

Proposition 2. *There exist* $\underline{\lambda}$ *and* $\overline{\lambda}$*, with* $\overline{\lambda} > \underline{\lambda} > \frac{1}{2}$*, such that:*

- *For* $\lambda \in [\frac{1}{2}, \underline{\lambda})$*, the regulator chooses* $\overline{\sigma}$*. Retailers locate approximately at* $\frac{1}{4}$ *and* $\frac{3}{4}$*.*
- *For* $\lambda \in [\underline{\lambda}, \overline{\lambda}]$*, the regulator chooses* $\sigma(\lambda, t)$*, where* $\underline{\sigma} < \sigma(\lambda, t) < \overline{\sigma}$*,* $\frac{\partial \sigma(\lambda,t)}{\partial \lambda} < 0$ *and* $\frac{\partial \sigma(\lambda,t)}{\partial t} > 0$*. Retailers choose* $a^*(\sigma(\lambda, t))$*, where* $\frac{1}{4} < a^*(\sigma(\lambda, t)) < \frac{1}{2}$ *and* $\frac{\partial a^*(\sigma(\lambda,t))}{\partial \lambda} > 0$*.*
- *For* $\lambda \in (\overline{\lambda}, 1]$*, the regulator chooses* $\underline{\sigma}$*. Retailers locate approximately at* $\frac{1}{2}$*.*

Reducing the dispersion of retailers' private information has two opposite effects on social welfare. On the one hand, asymmetric information reduces welfare because it induces manufacturers to distort prices upward in order to minimize retailers' information rents — *a price distortion effect*. On the other hand, asymmetric information affects retailers' choice of product differentiation: minimizing asymmetric information induces retailer to locate too close to the center (thus producing products that appeal to fewer consumers) — *a product differentiation effect*. The relative strength of these effects depends on the weight assigned to consumers' surplus.

As λ increases, the regulator chooses a higher level of transparency and induces retailers to locate closer to the center — i.e., to produce less differentiated products. Hence, the optimal level of transparency is decreasing in λ. Notice, however, that the optimal level of transparency is discontinuous since $\lim_{\lambda \to \overline{\lambda}} \sigma(\lambda, t) > \underline{\sigma} \simeq 0$. Figure 5.1 summarizes the regulator's choice of σ as a function of λ.

When λ is sufficiently high, the regulator assigns a high weight to consumers' surplus and, hence, he minimizes σ in order to reduce prices, even though this induces retailers to locate too close to the center, compared to the location that minimizes transportation costs. In this case, the price distortion effect is stronger than the product differentiation effect. By contrast, when λ is sufficiently low, the regulator assigns a lower weight to consumers' surplus and, hence, he maximizes σ and reduces the level of transparency. In this case, the price distortion effect is weaker than the product differentiation effect. When λ takes intermediate values, the regulator chooses a strictly positive, but not maximal, degree of asymmetric information in order to balance the price distortion and the product differentiation effects.

Finally, notice that Lemma 3 proves that the regulator never chooses full transparency, because there is always a non-zero level of transparency that yields higher social welfare. However, the regulator may not be able to fine tune σ and achieve the optimal level of transparency described in Proposition 2. Nonetheless, the next lemma shows that any level of asymmetric information (i.e., any level on transparency different form 0) generates higher social welfare than full transparency.

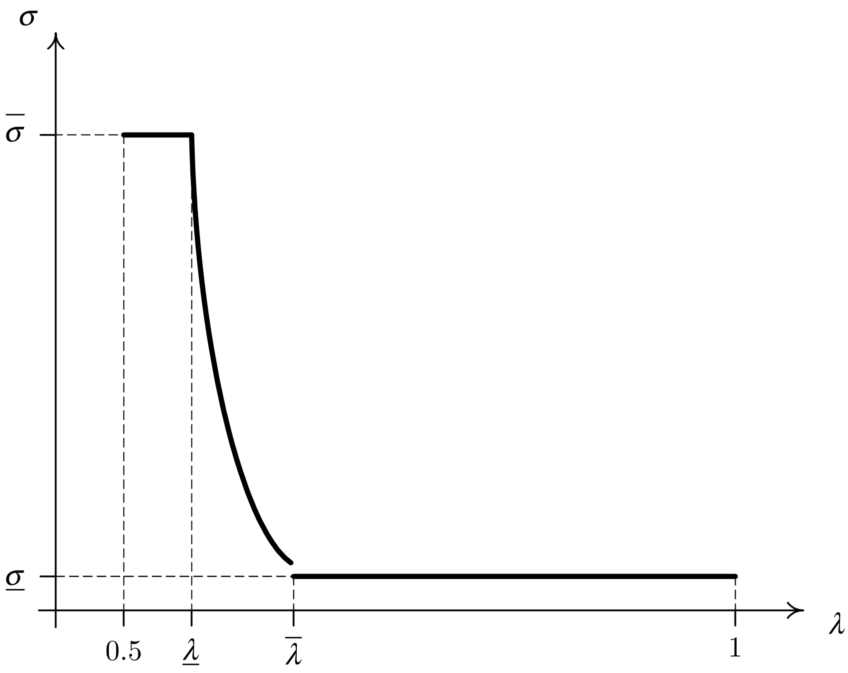

Figure 5.1: Socially optimal transparency

Lemma 5. *For any $\lambda \in [\frac{1}{2}, 1]$, social welfare is always higher with asymmetric information (for any $\sigma \in [\underline{\sigma}, \overline{\sigma}]$) than with full transparency.*

Therefore, even if the regulator is not fully able to select the desired level of transparency, he always prefers that retailers have some private information. This offers a justification for regulatory policies that allow firms to have relatively low standards of transparency in their accounting reports.

6. Conclusions

We have introduced a novel reason that may induce vertically separated firms to produce less differentiated products. When privately informed retailers undertake non-market activities that determine the degree of product differentiation with respect to competitors — such as product design, advertising campaigns, and quality enhancing investments — they have an incentive to choose product characteristics that appeal to a larger number of consumers, in order to increase their information rent. This also increases consumers' surplus.

Market transparency affects social welfare. When a regulator can control the degree of asymmetric information between manufacturers and retailers, he never imposes full transparency, because less strict transparency standards always produce higher welfare than full transparency. Lower transparency may also be interpreted as retailers adopting riskier technologies, that entail higher uncertainty about marginal costs. Therefore, in contrast to common wisdom, asymmetric information may be socially beneficial in our model.

A. Appendix

Proof of Lemma 1. The proof of this result is standard — see, e.g., Tirole (1988), Ch. 7, p. 281. ■

Proof of Lemma 2. The first-order necessary and sufficient condition for M_i's program is

$$\frac{1+a_i-a_j}{2}+\frac{\int_{\mu-\sigma}^{\mu+\sigma} p_j^*\left(\theta_j\right) dF(\theta_j)-2p_i^*(\theta_i)+\theta_i+\frac{F(\theta_i)}{f(\theta_i)}}{2t\left(1-a_i-a_j\right)}=0$$

$$\Leftrightarrow \quad p_i^*(\theta_i)=\tfrac{1}{2}t\left(1-a_i-a_j\right)\left(1+a_i-a_j\right)+\tfrac{1}{2}\int_{\mu-\sigma}^{\mu+\sigma} p_j^*\left(\theta_j\right) dF(\theta_j)+\theta_i-\tfrac{1}{2}\left(\mu-\sigma\right). \qquad \text{(A.1)}$$

Taking expectations with respect to θ_i,

$$\int_{\mu-\sigma}^{\mu+\sigma} p_i^*(\theta_i)dF(\theta_i)=\tfrac{1}{2}t\left(1-a_i-a_j\right)\left(1+a_i-a_j\right)+\tfrac{1}{2}\int_{\mu-\sigma}^{\mu+\sigma} p_j^*\left(\theta_j\right) dF(\theta_j)+\tfrac{1}{2}\left(\mu+\sigma\right).$$

Hence,

$$\int_{\mu-\sigma}^{\mu+\sigma} p_i^*(\theta_i)dF(\theta_i)=\mu+\sigma+\tfrac{2}{3}t\left(1-a_i-a_j\right)\left(2a_i+a_j\right)+t\left(1-a_i-a_j\right)^2, \quad i,j=1,2.$$

Substituting this equation in (A.1) yields equation (4.6). ■

Proof of Proposition 1. Using the equilibrium prices in Lemma 2, R_i's expected rent is

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\theta_i}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(x),p_j^*\left(\theta_j\right))dF(\theta_j)dxdF\left(\theta_i\right)=\frac{\sigma(3-\frac{\sigma}{t}-2a_i-4a_j-a_i^2+a_j^2)}{6\left(1-a_i-a_j\right)}.$$

Hence, R_i's optimization program is

$$\max_{a_i}\frac{3-\frac{\sigma}{t}-2a_i-4a_j-a_i^2+a_j^2}{1-a_i-a_j},$$

yielding the first-order necessary and sufficient condition

$$t(1+a_i^2+a_j^2)-2t(a_i-a_j+a_ia_j)-\sigma=0.$$

The relevant solution for a_i is

$$a_i(a_j)=1-\sqrt{\tfrac{\sigma}{t}}+a_j,$$

which implies that $a_i(a_j)$ is increasing in a_j — i.e., location choices are strategic complements. In a symmetric equilibrium, each retailer chooses $a_i=a_j=a^*=\frac{1}{2}-\frac{1}{2}\sqrt{\frac{\sigma}{t}}$. Using equation

(4.6), the equilibrium retail price is

$$
\begin{aligned}
p^{*}(\theta_i) &= \theta_i + \sigma + t\,(1 - 2a^{*}) \\
&= \theta_i + \sigma + \sqrt{t\sigma}.
\end{aligned} \tag{A.2}
$$

Since $\dot{p}^{*}(\theta_i) > 0$, the local second-order incentive compatibility constraint (4.2) is satisfied. Moreover, consider the global incentive compatibility constraint. Let the equilibrium franchise fee be

$$T^{*}(\theta_i) = (p^{*}(\theta_i) - \theta_i) \int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(\theta_i), p^{*}(\theta_j)) dF(\theta_j) - \int_{\theta_i}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(x), p^{*}(\theta_j)) dF(\theta_j) dx.$$

The equilibrium contract must satisfy the following inequality

$$U_i(\theta_i) \geq U_i(\theta_i, \theta'), \quad \forall (\theta_i, \theta') \in \Theta^2.$$

$$\Leftrightarrow \quad \int_{\theta_i}^{\theta'} \left\{ \dot{T}^{*}(x) - \frac{\partial (p^{*}(x) - \theta_i) \int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(x), p^{*}(\theta_j)) dF(\theta_j)}{\partial x} \right\} dx \geq 0,$$

$$\Leftrightarrow \quad \int_{\theta_i}^{\theta'} \left\{ \dot{T}^{*}(x) - \dot{p}^{*}(x) \left(\int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(x), p^{*}(\theta_j)) dF(\theta_j) - \frac{p^{*}(x) - \theta_i}{2t\,(1 - 2a^{*})} \right) \right\} dx \geq 0. \tag{A.3}$$

Since $\dot{p}^{*}(\theta_i) = 1$ and the local first-order incentive compatibility implies

$$
\begin{aligned}
\dot{T}^{*}(\theta_i) &= -\frac{\partial (p^{*}(\theta_i) - \theta_i) \int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(\theta_i), p^{*}(\theta_j)) dF(\theta_j)}{\partial \theta_i} \\
&= \dot{p}^{*}(\theta_i) \left(\int_{\mu-\sigma}^{\mu+\sigma} D^{i}(p^{*}(\theta_i), p^{*}(\theta_j)) dF(\theta_j) - \frac{p^{*}(\theta_i) - \theta_i}{2t\,(1 - 2a^{*})} \right), \quad \forall \theta_i \in \Theta,
\end{aligned}
$$

the left-hand side of (A.3) is

$$\int_{\theta_i}^{\theta'} \frac{x - \theta_i}{2t\,(1 - 2a^{*})} dx. \tag{A.4}$$

Suppose, without loss of generality, that $\theta' > \theta_i$. Then $x > \theta_i$ and (A.4) is positive. Hence, the global incentive compatibility constraint holds at equilibrium.

Finally, we need to check that retailers' demand is (strictly) positive regardless of realized costs. Note that, since the equilibrium price is increasing with respect to marginal costs,

$$D^{i}(p^{*}(\mu + \sigma), p^{*}(\mu - \sigma)) > 0 \quad \Rightarrow \quad D^{i}(p^{*}(\theta_i), p^{*}(\theta_j)) > 0, \quad \forall (\theta_i, \theta_j) \in \Theta^2.$$

Using the equilibrium price and the equilibrium localization choice,

$$D^{i}(p^{*}(\mu + \sigma), p^{*}(\mu - \sigma)) = \tfrac{1}{2} - \sqrt{\tfrac{\sigma}{t}}.$$

Hence, our assumption that $\frac{\sigma}{t} < \frac{1}{4}$ ensures that demand is always positive. ■

Proof of Lemma 3. In order to prove that the regulator never chooses full transparency, we show that: (i) if $\lambda \neq \frac{1}{2}$, welfare when $\sigma = \underline{\sigma}$ is strictly higher than when $\sigma = 0$; (ii) if $\lambda = \frac{1}{2}$, welfare when $\sigma = \underline{\sigma}$ is the same as when $\sigma = 0$.

First, when $\sigma = 0$, retailers choose maximal differentiation. Using the retail prices in Lemma 1, the welfare function is

$$\begin{aligned} W(.)|_{\sigma=0} &= \lambda v - (2\lambda - 1)(\mu + t) - (1-\lambda)\mu - \lambda t \int_0^{\frac{1}{2}} x^2 dx - \lambda t \int_{\frac{1}{2}}^1 (1-x)^2 dx \\ &= \lambda v + t - \lambda\left(\mu + \tfrac{25}{12}t\right). \end{aligned}$$

Second, when $\sigma = \underline{\sigma} \approx 0$, retailers locate at $a^e = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\sigma}{t}} \approx \frac{1}{2}$. Using the retail prices in equation (A.2), R_1's demand is

$$\begin{aligned} D^1(p^*(\theta_1), p^*(\theta_2)) &= \frac{1}{2} + \frac{p^*(\theta_2) - p^*(\theta_1)}{2t(1-2a^*)} \\ &= \frac{1}{2} + \frac{\theta_2 - \theta_1}{2t(1-2a^*)}. \end{aligned}$$

Hence, the (expected) welfare function is

$$\left.\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} W(.)\,\tfrac{1}{4\sigma^2} d\theta_1 d\theta_2\right|_{\sigma=\underline{\sigma}} =$$

$$\begin{aligned} = \lambda v - \int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} &\left\{ \int_0^{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}} \left[(2\lambda-1)p^*(\theta_1) + \lambda t(x-a^*)^2 + (1-\lambda)\theta_1\right] dx + \right.\\ &\left. - \int_{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}^1 \left[(2\lambda-1)p^*(\theta_2) + \lambda t(1-a^*-x)^2 + (1-\lambda)\theta_2\right] dx \right\} \tfrac{1}{4\sigma^2} d\theta_1 d\theta_2 \end{aligned}$$

$$\approx \lambda v - \lambda\left(\mu + \tfrac{t}{12}\right) \geq W(.)|_{\sigma=0}.$$

The inequality is strict for $\lambda \neq \frac{1}{2}$. ■

Proof of Lemma 4. In order to minimize transportation costs, the regulator minimizes

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left[\int_0^{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}} t(x-a^*)^2 dx + \int_{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}^1 t(1-a^*-x)^2 dx\right] \tfrac{1}{4\sigma^2} d\theta_1 d\theta_2$$

$$= \tfrac{\sigma}{6}\sqrt{\tfrac{\sigma}{t}} - \tfrac{\sqrt{t\sigma}}{4} + \tfrac{\sigma}{4}.$$

The first-order necessary and sufficient condition is

$$\tfrac{1}{4} + \tfrac{1}{4}\sqrt{\tfrac{\sigma}{t}} - \tfrac{1}{8}\sqrt{\tfrac{t}{\sigma}} = 0 \quad \Leftrightarrow \quad \sigma = t - \tfrac{t\sqrt{3}}{2}.$$

Hence, retailers locate at $\frac{3}{4} - \frac{\sqrt{3}}{4}$.

In order to minimize total production costs, the regulator minimizes

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left[\left(\frac{1}{2}+\frac{\theta_2-\theta_1}{2t\left(1-2a^*\right)}\right)\theta_1+\left(\frac{1}{2}-\frac{\theta_2-\theta_1}{2t\left(1-2a^*\right)}\right)\theta_2\right]\tfrac{1}{4\sigma^2}d\theta_1 d\theta_2$$

$$= \mu - \tfrac{\sigma^{\frac{3}{2}}}{3\sqrt{t}}.$$

Since this function is strictly decreasing in σ, the regulator chooses the lowest possible level of transparency. ■

Proof of Proposition 2. The regulator chooses σ to maximize

$$V(\sigma) \equiv \int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} W\left(.\right)\tfrac{1}{4\sigma^2}d\theta_1 d\theta_2$$

$$= \lambda v - \int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left[\int_0^{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}\Big((2\lambda-1)p^*\left(\theta_1\right)+\lambda t\left(x-a^*\right)^2+(1-\lambda)\theta_1\Big)\,dx+\right.$$
$$\left.-\int_{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}^{1}\Big((2\lambda-1)p^*\left(\theta_2\right)+\lambda t\left(1-a^*-x\right)^2+(1-\lambda)\theta_2\Big)\,dx\right]\tfrac{1}{4\sigma^2}d\theta_1 d\theta_2$$

$$= \lambda v - \lambda\mu + \sigma - \tfrac{\lambda t}{12} - \sqrt{\sigma t}\left(\tfrac{21}{12}\lambda - 1\right) + \lambda\sigma\left(\tfrac{1}{6}\sqrt{\tfrac{\sigma}{t}} - \tfrac{9}{4}\right). \qquad \text{(A.5)}$$

The first-order necessary condition for an internal maximum is

$$\frac{\partial V(\sigma)}{\partial\sigma} = 0 \quad\Leftrightarrow\quad \tfrac{2\lambda\sigma}{t} + (8-18\lambda)\sqrt{\tfrac{\sigma}{t}} + 4 - 7\lambda = 0, \qquad \text{(A.6)}$$

which has a unique positive solution

$$\sigma\left(\lambda,t\right) = \tfrac{t}{4} - \tfrac{t}{4\lambda^2}\left(\lambda\left(152-175\lambda\right) - 32 + \left(18\lambda-8\right)\sqrt{95\lambda^2-80\lambda+16}\right).$$

Evaluating the second-order sufficient condition at $\sigma\left(\lambda,t\right)$,

$$\left.\frac{\partial^2 V\left(\sigma\right)}{\partial\sigma^2}\right|_{\sigma=\sigma(\lambda,t)} < 0 \quad\Leftrightarrow\quad 7\lambda - 4 + \tfrac{2\lambda\sigma(\lambda,t)}{t} < 0 \quad\Leftrightarrow\quad \lambda < \bar{\lambda} \approx 0.571.$$

Hence, for $\lambda < \bar{\lambda}$ the function $V\left(\sigma\right)$ has a unique maximum at $\min\left\{\overline{\sigma}, \sigma\left(\lambda,t\right)\right\}$, and $\sigma\left(\lambda,t\right) \geq \overline{\sigma}$ for $\lambda \leq \underline{\lambda} \approx 0.516$. Notice that, since by Lemma 3 social welfare is always higher at $\underline{\sigma}$ than at $\sigma = 0$, social welfare is also higher at $\min\left\{\overline{\sigma}, \sigma\left(\lambda,t\right)\right\}$ than at $\sigma = 0$, when $\lambda \leq \underline{\lambda}$. Moreover, simple computations show that $\frac{\partial\sigma(\lambda,t)}{\partial t} > 0$ and $\frac{\partial\sigma(\lambda,t)}{\partial\lambda} < 0$.

Consider now $\lambda > \bar{\lambda}$. The first order condition (A.6) identifies a unique local minimum of $V\left(\sigma\right)$ since $\left.\frac{\partial^2 V(\sigma)}{\partial\sigma^2}\right|_{\sigma=\sigma(\lambda,t)} > 0$. Hence, the regulator's program has a corner solution — i.e., he chooses either $\underline{\sigma}$ or $\overline{\sigma}$. Using equation (A.5),

$$V(0) - V\left(\tfrac{t}{4}\right) = t\left(\tfrac{17}{12}\lambda - \tfrac{3}{4}\right).$$

This difference is strictly positive for $\lambda > \bar{\lambda}$. Since, by assumption, $\underline{\sigma}$ is arbitrarily close to 0 and $\overline{\sigma}$ is arbitrarily close to $\frac{t}{4}$, continuity of $V(\sigma)$ and Lemma 3 imply that the social welfare is maximized at $\underline{\sigma}$, when $\lambda > \bar{\lambda}$. Retailers' locations are obtained by substituting the optimal σ in $a^*(\sigma)$. ■

Proof of Lemma 5. First suppose that $\lambda \leq \bar{\lambda}$. In the proof of Proposition 2 we showed that $V(\sigma)$ is strictly concave. By Lemma 3, $V(\underline{\sigma}) > W(.)|_{\sigma=0}$. Hence, we only need to prove that $V(\overline{\sigma}) > W(.)|_{\sigma=0}$. This is true since, for $\lambda \geq \frac{1}{2}$,

$$V(\overline{\sigma}) - W(.)|_{\sigma=0} = \tfrac{t}{12}(7\lambda - 3) > 0.$$

Suppose now that $\lambda > \bar{\lambda}$. By Proposition 2, $V(.)$ is minimized at $\sigma(\lambda, t)$. Simple computations show that

$$V(\sigma(\lambda, t)) - W(.)|_{\sigma=0} =$$

$$= \tfrac{257}{2} + \tfrac{8}{\lambda^2} - \tfrac{56}{\lambda} - 97\lambda + \tfrac{(9\lambda-4)^2}{8\lambda^2}\sqrt{95\lambda^2 - 80\lambda + 16}$$
$$- \tfrac{64\lambda + \sqrt{95\lambda^2-80\lambda+16}(9\lambda-4) - 67\lambda^2 - 16}{12\lambda^2}\sqrt{8 + 44\lambda^2 - 38\lambda - \tfrac{(9\lambda-4)\sqrt{95\lambda^2-80\lambda+16}}{2}}.$$

This is positive for $\lambda > \bar{\lambda}$. ■

Two-part tariffs. We show that, with secret contracts, RPM contracts are equivalent to two-part tariff contracts composed by a wholesale price and a franchise fee. In this framework, M_i offers a menu $(w_i(\theta_i), T_i(\theta_i))_{\theta_i \in \Theta}$ to R_i, where $w_i(\theta_i)$ is the wholesale price paid by R_i for each unit of the good produced by M_i and $T_i(\theta_i)$ is the franchise fee, given R_i's report.

The timing of the game is modified as follows:

1. Retailers simultaneously and independently choose their locations: a_1 and a_2.
2. Retailers privately observe their costs.
3. Manufacturers offer contracts and retailers choose whether to accept them. If R_i accepts M_i's offer, then he reports his type and pays the wholesale price and the franchise fee.
4. Retailers choose retail prices and the market clears.

Suppose that manufacturers and retailers are vertically separated. We focus on a symmetric separating equilibrium. Given a report $\hat{\theta}_i$ and the locations chosen by retailers, R_i chooses p_i to solve

$$\max_{p_i}(p_i - w_i(\hat{\theta}_i) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i, \tilde{p}_j(\theta_j))dF(\theta_j),$$

where $\tilde{p}_j(\theta_j)$ is the price that R_i expects R_j to choose in equilibrium as a function of his type. This yields the first-order conditions

$$\int_{\mu-\sigma}^{\mu+\sigma} D^i(\tilde{p}_i(w_i(\hat{\theta}_i),\theta_i),\tilde{p}_j(\theta_j))dF(\theta_j) - \frac{\tilde{p}_i(w_i(\hat{\theta}_i),\theta_i) - w_i(\hat{\theta}_i) - \theta_i}{2t(1-a_i-a_j)} = 0$$

$$\Rightarrow \quad \tilde{p}_i(\theta_i, w_i(\hat{\theta}_i)) = \frac{\theta_i + \int_{\mu-\sigma}^{\mu+\sigma} \tilde{p}_j(\theta_j)\, dF(\theta_i) + w_i(\hat{\theta}_i)}{2} +$$
$$+\tfrac{t}{2}(1-a_i-a_j)(1+a_i-a_j), \quad i,j=1,2. \tag{A.7}$$

Taking expectations for $\hat{\theta}_i = \theta_i$,

$$\int_{\mu-\sigma}^{\mu+\sigma} \tilde{p}_i(\theta_i)dF(\theta_i) = \frac{\mu + \int_{\mu-\sigma}^{\mu+\sigma} \tilde{p}_j(\theta_j)\, dF(\theta_j) + \int_{\mu-\sigma}^{\mu+\sigma} w_i(\theta_i)\, dF(\theta_i)}{2} +$$
$$+\tfrac{t}{2}(1-a_i-a_j)(1+a_i-a_j), \quad i,j=1,2.$$

Solving the system

$$\int_{\mu-\sigma}^{\mu+\sigma} \tilde{p}_i(\theta_i)dF(\theta_i) = \mu + \tfrac{2}{3}\int_{\mu-\sigma}^{\mu+\sigma} w_i(\theta_i)\, dF(\theta_i) + \tfrac{1}{3}\int_{\mu-\sigma}^{\mu+\sigma} w_j(\theta_j)\, dF(\theta_j) +$$
$$+\tfrac{t}{3}(3+a_i-a_j)(1-a_i-a_j), \quad i,j=1,2.$$

Hence, substituting in (A.7),

$$\tilde{p}_i(w_i(\hat{\theta}_i),\theta_i) = \tfrac{1}{2}\left(\mu + \theta_i + w_i(\hat{\theta}_i)\right) + \tfrac{1}{6}\int_{\mu-\sigma}^{\mu+\sigma} w_i(\theta_i)\, dF(\theta_i) +$$
$$+\tfrac{1}{3}\int_{\mu-\sigma}^{\mu+\sigma} w_j(\theta_j)\, dF(\theta_j) + \tfrac{t}{3}(3+a_i-a_j)(1-a_i-a_j).$$

In a separating equilibrium, the contract offered by M_i must be incentive feasible — i.e., it must satisfy R_i's participation and incentive compatibility constraints. R_i's information rent is

$$U_i(\theta_i) = (\tilde{p}_i(w_i(\theta_i),\theta_i) - w_i(\theta_i) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j(\theta_j))dF(\theta_j) - T_i(\theta_i).$$

Incentive compatibility implies

$$U_i(\theta_i) = \max_{\hat{\theta}_i\in\Theta}\left\{(\tilde{p}_i(w_i(\hat{\theta}_i),\theta_i) - w_i(\hat{\theta}_i) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(\tilde{p}_i(w_i(\hat{\theta}_i),\theta_i),\tilde{p}_j(\theta_j))dF(\theta_j) - T_i(\hat{\theta}_i)\right\},$$

which yields the first- and second-order local conditions for incentive compatibility

$$\dot{U}(\theta_i) = -\int_{\mu-\sigma}^{\mu+\sigma} D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j(\theta_j))dF(\theta_j). \tag{A.8}$$

and

$$-\left[\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial\theta_i}+\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial w_i}\right]\int_{\mu-\sigma}^{\mu+\sigma}\tfrac{\partial}{\partial\tilde{p}_i}D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j\left(\theta_j\right))dF\left(\theta_j\right)\geq 0$$

$$\Rightarrow\quad\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial\theta_i}+\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial w_i}\geq 0. \tag{A.9}$$

Conditions (A.8) and (A.9), together with the participation constraint

$$U_i\left(\theta_i\right)\geq 0,\quad\forall\theta_i\in\Theta, \tag{A.10}$$

define the set of incentive feasible allocations for M_i and R_i.

M_i maximizes

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left\{D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j\left(\theta_j\right))\left(\tilde{p}_i(w_i(\theta_i),\theta_i)-w_i(\theta_i)-\theta_i\right)-U_i\left(\theta_i\right)\right\}dF(\theta_j)dF\left(\theta_i\right),$$

subject to conditions (A.8), (A.9) and (A.10). We first assume, and then check ex post, that (A.10) is satisfied. Then $U_i\left(\theta_i\right)$ is decreasing and the participation constraint binds when $\theta_i=\mu+\sigma$. Hence,

$$U_i\left(\theta_i\right)=\int_{\theta_i}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}D^i(\tilde{p}_i(w_i(x),x),\tilde{p}_j\left(\theta_j\right))dxdF(\theta_j). \tag{A.11}$$

Using expression (A.11) integrating by parts and substituting into M_i's objective function, yield the relaxed program

$$\max_{w_i(.)}\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left\{D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j\left(\theta_j\right))\left(p_i(\theta_i)-w_i(\theta_i)-\theta_i-\frac{F\left(\theta_i\right)}{f\left(\theta_i\right)}\right)\right\}dF(\theta_j)dF\left(\theta_i\right).$$

The first-order necessary and sufficient condition for M_i's program is

$$\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial w_i(\theta_i)}\int_{\mu-\sigma}^{\mu+\sigma}D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j\left(\theta_j\right))dF(\theta_j)+$$

$$-\frac{1}{2t\left(1-a_i-a_j\right)}\left(\tilde{p}_i(w_i(\theta_i),\theta_i)-\theta_i-\frac{F\left(\theta_i\right)}{f\left(\theta_i\right)}\right)\frac{\partial\tilde{p}_i(w_i(\theta_i),\theta_i)}{\partial w_i(\theta_i)}=0.$$

Simplifying,

$$\int_{\mu-\sigma}^{\mu+\sigma}D^i(\tilde{p}_i(w_i(\theta_i),\theta_i),\tilde{p}_j\left(\theta_j\right))dF(\theta_j)-\frac{1}{2t\left(1-a_i-a_j\right)}\left(\tilde{p}_i(w_i(\theta_i),\theta_i)-\theta_i-\frac{F\left(\theta_i\right)}{f\left(\theta_i\right)}\right)=0.$$

This is exactly the same first-order condition of the maximization problem with RPM. By setting $w_i(\theta_i)=\frac{F(\theta_i)}{f(\theta_i)}$, R_i chooses a retail price $\tilde{p}_i(w_i(\theta_i),\theta_i)=p_i^*(\theta_i)$, as defined in Lemma 2, and the locations with a two-part tariff contract are the same as the ones with RPM.

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