Centre for Studies in Economics and Finance

WORKING PAPER NO. 291

A Note on the Value of Residual Claimancy with Competing Vertical Hierarchies

Riccardo Martina and Salvatore Piccolo

September 2011

University of Naples Federico II | University of Salerno | Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unisa.it

WORKING PAPER NO. 291

A Note on the Value of Residual Claimancy with Competing Vertical Hierarchies

Riccardo Martina and Salvatore Piccolo*

Abstract
In this short paper we study a competing vertical hierarchies model where the allocation of residual claimancy is endogenous and is determined jointly with production and contractual decisions. We .nd a set of circumstances in which the (equilibrium) allocation of residual claimancy is affected by competition in a non trivial manner. More precisely, although revenue-sharing contracts foster agents. (non-contractible) surplus enhancing effort, we show that competing principals dealing with exclusive and privately informed agents might still prefer to retain a share of the surplus from production when dealing with inefficient types. This is because reducing the surplus share of inefficient types reduces the information rent given up to efficient types. Hence, the equilibrium allocation of residual claimancy follows a pro-cyclical rule.

Keywords: Adverse selection, residual claimancy, vertical hierarchies.

Acknowledgements: We thank Carlo Cambini, Elisabetta Iossa, Emanuele Tarantino as well as the audience of the Second IO Workshop at the University of Salento (2011) for useful comments. Usual disclaimers apply.

* Università di Napoli Federico II and CSEF. Corresponding address: Department of Economics, Via Cintia Monte S. Angelo, Università di Napoli Federico II, I-80126 Napoli, Italy. E-mail: salvapiccolo@gmail.com

Table of contents

1. Introduction

2. Set-up

3. The complete information benchmark

4. Asymmetric information

5. Conclusion

6. Appendix

References

1 Introduction

Residual claimancy plays an important role in models with decentralized decision making. The agency literature has explored various aspects related to this issue. For instance, by studying the link between input versus output monitoring and the choice of residual claimancy — see, e.g., Khalil and Lawaree (1995) and Maskin and Riley (1985) — or by emphasizing the beneficial effect of revenue-sharing contracts on managers' incentives to perform non-contractible tasks — see, e.g., Hart (1995).

Existing models, however, mainly consider a single principal-agent set-up, and are silent on the link between competitive forces (e.g., product market competition) and the allocation of residual claimancy within firms. Does competition affect the way organizations distribute the surplus from production across their members? We address this issue within a competing vertical hierarchies model where the equilibrium allocation of residual claimancy is endogenous and is jointly determined with production and contractual decisions. Our objective is to derive basic insights on the interaction between market forces and organization design under asymmetric information.

We consider a model à la Martimort (1996) where two uninformed principals deal with a pair of exclusive agents. Each agent is privately informed about his production costs (type) and produces a verifiable (contractible) output in his principal's behalf. In addition, each agent exerts a surplus-enhancing effort which is non-verifiable in court — i.e., contracts are incomplete. Production generates externalities across the two principal-agent pairs (hierarchies), which can be either positive — i.e., outputs are (strategic) complements — or negative — i.e., outputs are (strategic) substitutes. Agents' types are allowed to be correlated. Principals offer direct revelation mechanisms that specify type-dependent surplus-sharing rules in addition to output decisions and (direct) monetary transfers. Contracts are secret and hence have no strategic value whatsoever.[1]

Within this context, we identify two effects that shape the equilibrium allocation of residual claimancy. On the one hand, by sharing the surplus from production with her agent, a principal is able to increase the agent's non-contractible effort, which makes (ceteris paribus) production more appealing: a *surplus-enhancing* effect. This effect is standard in models with incomplete contracts: it echoes the insights of the property rights approach — see, e.g., Hart (1995, Ch. 3) and Bolton and Dewatripont (2005, Ch. 11) — showing that revenue-sharing contracts are desirable insofar as they provide the right incentives to exert (non-contractible) effort into a project. On the other hand, when agents' costs are correlated, rewarding an agent with a share of the firm's surplus (revenues) generates an informational externality that affects the rent grabbed by efficient types. This effect emerges only if there are production externalities across the hierarchies. In particular, when residual claimancy is endogenous — i.e., it is part of the deal offered by a principal to her agent — the incentive of efficient types to manipulate their costs depends not only on the cost-saving rent that this strategy secures — i.e., to receive a higher transfer — but also on its effect on the firm's expected surplus through the competitive channel. Essentially, when a share of the firm's surplus is

[1] Fersthman and Judd (1987), Bonanno and Vickers (1988), Sklivas (1987) and Vickers (1995) among others, studied the conditions under which revenue-sharing contracts are offered at the equilibrium of games where contractual offers are observable, and thus have a commitment value. This standard strategic effect is not present in our framework.

allocated to the agent directly through the contract offer, the agent's incentive to overstate his cost must weight the effect that this lie produces on the principal's beliefs about the rival agent's type, which in turn affects the surplus that the principal expects to share with the agent, and hence the monetary incentives she is willing to offer: a so called *competing-contracts* effect (Martimort, 1996).

We show that, although efficient types are always made full residual claimants of the firms' surplus, if costs are positively (resp. negatively) correlated and outputs are strategic complements (resp. substitutes) principals may wish to retain a share of the firm's surplus when dealing with inefficient types for rent-extraction reasons. In these cases sharing revenues with inefficient agents increases the mimicking incentives of efficient types. By contrast, if costs are positively (resp. negatively) correlated and outputs are strategic substitutes (resp. complements) full residual claimancy is granted to the agents regardless of their types. In these cases allocating a share of the firm's surplus to the agent invariably reduces his rents.

Hence, the model shows that the allocation of residual claimancy is affected by competition in a non-trivial manner and that principals are more inclined to share revenues with efficient types (i.e., in good times) rather than with inefficient ones (i.e., in bad times): a pro-cyclical pattern. This result adds to the existing literature in three main respects. First, it extends the competing-contracts effect introduced in Martimort (1996) who considered the case of perfectly correlated types to a more general context in which correlation is not perfect and can be negative. Second, one additional insights of this paper is that once residual claimancy is endogenous, it can potentially play an important role in the welfare comparison between different organizational modes — e.g., common agency versus exclusive deals. Third, it shows that, in addition to considerations regarding the type of screening instruments controlled by principals, competitive forces may contribute to determine the way contracting counterparts share the surplus generated by their relationship via explicit contingent claims on firms' revenues.

Finally, our results also offer simple ready to use guidelines for interpreting and designing future empirical investigations on the link between organization design and competition. Although we will develop the arguments in an abstract principal-agent setting, our analysis can be used to study many standard applications. For instance, it can be used to explain why managerial compensations involve bonuses that vary along the cycle; it may help explaining the rationale of revenue-sharing contracts in models of competing supply chains; and it also provides a few insights on the trade-off between equity and debt contracts in lending relationships where (exclusive) borrowers compete on the product market.

The rest of the paper is organized as follows: Section 2 describes the setting. Section 3 analyzes the complete information benchmark. Section 4 derives the equilibrium characterization under asymmetric information. Section 5 concludes. Proofs are in the Appendix.

2 Set-up

Players. There are two (female) principals, P_1 and P_2, and two (male) exclusive agents, A_1 and A_2. Agent A_i ($i = 1, 2$) produces output q_i in P_i's behalf. Firm i's surplus from production is $S^i(e_i, q_i, q_j)$. Players are risk neutral. Principal P_i's utility is:

$$V^i(.) = (1-\alpha_i)\, S^i(e_i, q_i, q_j) - t_i, \quad i,j = 1,2 \quad i \neq j,$$

where q_i is the output produced by A_i, t_i is the monetary transfer flowing from P_i to A_i, e_i is a non-contractible surplus-enhancing effort exerted by A_i and $\alpha_i \in [0,1]$ denotes the share of the surplus $S^i(.)$ that P_i (eventually) allocates to A_i — i.e., α_i measures the extent to which A_i is made residual claimant of firm-i's surplus. Hence, A_i's utility is:

$$U^i(.) = t_i - \theta_i q_i - \psi(e_i) + \alpha_i S^i(e_i, q_i, q_j), \quad i,j = 1,2 \quad i \neq j,$$

where the parameter $\theta_i \in \Theta_i \equiv \Theta$ $(i = 1,2)$ denotes A_i's marginal cost of production and is private information. The type-space Θ is discrete with $\Theta \equiv \left\{\underline{\theta}, \overline{\theta}\right\}$. The function $\psi(e_i)$ measures the effort cost.

Contracts. Contracts are incomplete — i.e., P_i cannot condition the contract offered to A_i on his effort e_i. We use the version of the Revelation Principle for competing hierarchies to characterize the equilibrium of the game — see, e.g., Martimort (1996). Hence, P_i offers to A_i a direct revelation mechanism:

$$C_i \equiv \{t_i(m_i), q_i(m_i), \alpha_i(m_i)\}_{m_i \in \Theta}$$

that maps A_i's report m_i about θ_i into a monetary transfer $t_i(m_i)$, an output $q_i(m_i)$ and a share of surplus $\alpha_i(m_i)$.[2] Contracts are secret: neither P_j nor A_j can observe C_i. Notice that, the mechanism C_i looks like a revenue-sharing contract as long as $\alpha_i(\theta_i) \neq 0$ for some θ_i.[3]

Timing. The timing of the game is as follows:

- (**T=1**) Agents privately observe their costs.
- (**T=2**) Principals (simultaneously) offer contracts.
- (**T=3**) Agents choose efforts, produce and payments are made.

Each player's outside option is normalized to zero with no loss of insights.

Equilibrium concept. The equilibrium concept is Perfect Bayesian equilibrium (PBE), with the added *passive beliefs* refinement — i.e., when an agent is offered a contract different from the one he expects in

[2] We ruled out the case of relative performance evaluations which would require: (*i*) contract C_i to be contingent on A_j's cost parameter θ_j; (*ii*) lottery contracts that are difficult to enforce in practice — see, e.g., Bertoletti and Poletti (1996) and (1997) on the issue of first-best implementation with correlated types and competing hierarchies.

[3] The commitment and collusive values of revenue-sharing contracts have been analyzed in the traditional IO literature — see, e.g., Fershtman and Judd (1987), Dana and Spier (2001) and Spagnolo (2000). These models assume complete information and do not provide predictions on the evolution of residual claimancy over the cycle neither on the effect of cost correlation and competition on organization design.

equilibrium, he does not revise his beliefs about the contract offered to the other agent — see, e.g., Caillaud et al. (1995), Martimort (1996) and Martin (1993).

Assumptions. The analysis is developed under the following assumptions.

A1 The vector of random variables $\boldsymbol{\theta} = (\theta_1, \theta_2)$ is drawn from a joint cumulative distribution function with: $\Pr(\underline{\theta}, \underline{\theta}) = \nu^2 + \rho$, $\Pr(\underline{\theta}, \overline{\theta}) = \Pr(\overline{\theta}, \underline{\theta}) = \nu(1-\nu) - \rho$ and $\Pr(\overline{\theta}, \overline{\theta}) = (1-\nu)^2 + \rho$. The marginal distribution entails: $\Pr(\underline{\theta}) = \nu$ and $\Pr(\overline{\theta}) = 1 - \nu$. Posteriors are computed through the Bayes rule: $\Pr(\underline{\theta}|\underline{\theta}) = \nu + \frac{\rho}{\nu}$, $\Pr(\underline{\theta}|\overline{\theta}) = \nu - \frac{\rho}{1-\nu}$, $\Pr(\overline{\theta}|\underline{\theta}) = 1 - \nu - \frac{\rho}{\nu}$ and $\Pr(\overline{\theta}|\overline{\theta}) = 1 - \nu + \frac{\rho}{1-\nu}$.[4]

The parameter ρ is the correlation index between θ_1 and θ_2 — i.e., $\Pr(\underline{\theta}, \underline{\theta}) \Pr(\overline{\theta}, \overline{\theta}) - \Pr(\underline{\theta}, \overline{\theta})^2 = \rho$. Hence, $\rho > 0$ (resp. $<$) means positive (resp. negative) correlation between types (when $\rho = 0$ types are uncorrelated, see, e.g., Martin, 1993).[5]

A2 The surplus function $S^i(.)$ is symmetric and quadratic:

$$S(e_i, q_i, q_j) = \kappa + e_i q_i + \beta q_i - q_i^2 + \delta q_i q_j \quad i, j = 1, 2 \quad i \neq j. \tag{1}$$

The effort cost is quadratic: $\psi(e_i) = \frac{e_i^2}{2\phi}$, with $0 < \phi < 2$.[6]

Assumption **A2** yields single peaked profit functions. The parameter δ measures the extent of (strategic) complementarity ($\delta > 0$) or substitutability ($\delta < 0$) between outputs.[7]

A3 Non-negative probabilities:

$$\Pr(\underline{\theta}, \overline{\theta}) = \Pr(\overline{\theta}, \underline{\theta}) \geq 0 \quad \Leftrightarrow \quad \nu(1-\nu) \geq \rho \quad if \quad \rho \geq 0,$$

$$\min\left\{\Pr(\overline{\theta}, \overline{\theta}), \Pr(\underline{\theta}, \underline{\theta})\right\} \geq 0 \quad \Leftrightarrow \quad \min\{(1-\nu), \nu\} \geq \sqrt{|\rho|} \quad if \quad \rho < 0.$$

Finally, the next hypothesis rules out countervailing incentives and ensures that outputs are always positive when uncertainty vanishes — i.e., when $\Delta\theta = \overline{\theta} - \underline{\theta} \to 0$.

[4] See also Dequiedt and Martimort (2010) for a similar type space and probabilistic structure.

[5] The case of positive correlation captures those instances where firms' production technologies are affected by aggregate factors such as public expenditures in R&D, changes in the fiscal pressure etc. By contrast, the case of negative correlation can be useful to capture those situations, such as R&D races, where the technological success of a firm — e.g., a patent which allows to reduce marginal costs — excludes that of its rivals.

[6] Quadratic specifications are standard in IO models — see, e.g., Martin (1993), Vives (2000) and Raith (1996) among many others. However, it can be shown that our predictions remain qualitatively the same with nonlinear surplus functions and small uncertainty — i.e., when $\Delta\theta = \overline{\theta} - \underline{\theta}$ is small. Indeed, our results would be obtained in the limiting case where Taylor expansions are valid.

[7] When δ is negative one can think of this model as a Cournot competition set-up. When δ is positive, instead, it can be interpreted as a model of price competition where the strategic variables are complements.

A4 $\beta > \overline{\theta} > \underline{\theta} \geq 0$. Moreover, the effort cost is convex enough — i.e.,

$$\phi < \min\left\{2-\tfrac{\delta\rho}{\nu(1-\nu)^2}, 2-\delta\right\},$$

with $\min\left\{2-\tfrac{\delta\rho}{\nu(1-\nu)^2}, 2-\delta\right\} > 0$.

3 The complete information benchmark

Before studying the case of asymmetric information, in this section we describe the equilibrium of the game when costs are common knowledge within each hierarchy — i.e., when each principal observes her own agent's type but not that of the rival agent.

Lemma 1 *If costs are common knowledge within each hierarchy, there exists a unique symmetric PBE such that agents are full residual claimants of the firms' surplus and are left with no rents — i.e.,* $\alpha^*(\theta_i) = 1$ $\forall \theta_i \in \Theta,\ i, j = 1, 2,$ *and:*

$$t^*(\theta_i) = \theta_i q^*(\theta_i) - \max_{e_i \geq 0}\left\{\sum_{\theta_j \in \Theta} \Pr(\theta_j|\theta_i)\, S(e_i, q^*(\theta_i), q^*(\theta_j)) - \psi(e_i)\right\} \quad \forall \theta_i \in \Theta, \quad i, j = 1, 2 \quad i \neq j,$$

where $q^*(.) : \Theta \to \Re_{++}$ *solves:*

$$\beta - (2-\phi)\, q^*(\theta_i) + \delta \mathbb{E}_{\theta_j}[q^*(\theta_j)\,|\theta_i] = \theta_i \quad \forall \theta_i \in \Theta, \quad i, j = 1, 2 \quad i \neq j.$$

When principals can fully extract rents, letting the agents be full residual claimants of the firms' surplus magnifies their incentives to exert the non-contractible effort. Hence, it maximizes equilibrium aggregate profits, which will be extracted by the principals via the transfer. This effect echoes the insights of the property rights literature — see, e.g., Hart (1995, Ch. 3) and Bolton and Dewatripont (2005, Ch. 11) — which shows that revenue-sharing contracts (or, alternatively, sell-out contracts) are desirable insofar as they provide the right incentives to invest effort into a project.

4 Asymmetric information

In this section we consider asymmetric information. Principals must learn their agents' costs through costly contracting — i.e., they must give up an information rent in order to screen types. The minimization of those rents leads to equilibrium outputs and sharing rules that are distorted away from efficiency.

For any cost realization $\theta_i \in \Theta$, let A_i's expected utility (in a truthful equilibrium) be:

$$U_i(\theta_i) \equiv t_i(\theta_i) - \theta_i q_i(\theta_i) + \max_{e_i \geq 0}\left\{\alpha_i(\theta_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j|\theta_i)\, S(e_i, q_i(\theta_i), q_j(\theta_j)) - \psi(e_i)\right\}.$$

Contract C_i is acceptable by agent A_i if and only if it satisfies the ex-ante participation constraints:

$$U_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta.$$

Moreover, agent A_i truthfully reports his type if and only if the following Bayesian incentive compatibility constraints hold:

$$U_i(\theta_i) \geq t_i(m_i) - \theta_i q_i(m_i) + \max_{e_i \geq 0} \left\{ \alpha_i(m_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j|\theta_i) S(e_i, q_i(m_i), q_j(\theta_j)) - \psi(e_i) \right\}, \quad \forall m_i \neq \theta_i.$$

Denote by $q^e(.) : \Theta \to \Re_{++}$ the symmetric output function emerging in a separating PBE. As standard in the screening literature, we assume that efficient types mimic inefficient ones (this conjecture will be verified ex post). Hence, only the participation constraint of the high-cost types and the incentive constraint of the low-cost types matter — see, e.g., Laffont and Martimort (2002, Ch. 2). Formally, the set of incentive feasible allocation is defined by:

$$U_i(\overline{\theta}) \geq 0, \tag{2}$$

and, under **A2**:

$$U_i(\underline{\theta}) \geq U_i(\overline{\theta}) + \Delta\theta q_i(\overline{\theta}) + \delta\alpha_i(\overline{\theta})q_i(\overline{\theta}) \left[\sum_{\theta_j \in \Theta} \Pr(\theta_j|\underline{\theta}) q^e(\theta_j) - \sum_{\theta_j \in \Theta} \Pr(\theta_j|\overline{\theta}) q^e(\theta_j) \right]. \tag{3}$$

Principal P_i's relaxed optimization program is:

$$\mathcal{P}: \quad \max \sum_{\theta_i \in \Theta} \Pr(\theta_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j|\theta_i) \left[S(e_i(\theta_i), q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i) - \psi(e_i(\theta_i)) - U_i(\theta_i) \right],$$

subject to

(2) − (3),

$\alpha_i(\theta_i) \in [0,1] \quad \forall \theta_i \in \Theta,$

$e_i(\theta_i) = \psi'^{-1}(\alpha_i(\theta_i) q_i(\theta_i)) \quad \forall \theta_i \in \Theta.$

As standard, both (2) and (3) bind at the optimum. Moreover, note that under **A1:**

$$\sum_{\theta_j \in \Theta} \Pr(\theta_j|\underline{\theta}) q^e(\theta_j) - \sum_{\theta_j \in \Theta} \Pr\left(\theta_j|\overline{\theta}\right) q^e(\theta_j) = \tfrac{\rho(q^e(\underline{\theta}) - q^e(\overline{\theta}))}{\nu(1-\nu)}.$$

Hence, (3) can be then rewritten as:

$$U_i(\underline{\theta}) = \underbrace{\Delta\theta q_i(\overline{\theta})}_{\textit{Standard rent}} + \underbrace{\tfrac{\alpha_i(\overline{\theta}) q_i(\overline{\theta}) \delta\rho(q^e(\underline{\theta}) - q^e(\overline{\theta}))}{\nu(1-\nu)}}_{\textit{Generalized competing-contracts effect}}. \tag{4}$$

There are few interesting insights to be learned from the incentive compatibility constraint (4). The first term on its right-hand side captures the standard information rent that an efficient type enjoys in a single principal-agent relationship ($\delta = 0$): a low-cost type is tempted to overstate his cost to negotiate a higher transfer with his principal. However, the second term captures an informational externality across the hierarchies that depends not only on the nature of upstream externalities, as reflected by the sign and magnitude of the parameter δ, but also on the degree of correlation between types, as reflected by the sign and magnitude of the parameter ρ: it is a generalized version of the competing-contracts effect emphasized in Martimort (1996) who considers the case of perfectly correlated types in a framework where agents are (by assumption) full residual claimants of the surplus that they share with the principals. In the standard case where (in equilibrium) efficient types produce more than inefficient ones — i.e., $q^e(\underline{\theta}) > q^e(\overline{\theta})$ — this effect mitigates (resp. strengthens) A_i's incentive to overstate his type if $\delta\rho < 0$ (resp. > 0).

Coming back to the optimization program, P_i maximizes:

$$\sum_{\theta_i \in \Theta} \Pr(\theta_i) \sum_{\theta_j \in \Theta} \Pr(\theta_j|\theta_i) \left[S(e_i(\theta_i), q_i(\theta_i), q^e(\theta_j)) - \theta_i q_i(\theta_i) - \psi(e_i(\theta_i))\right] - \nu q_i(\overline{\theta}) \left[\Delta\theta + \tfrac{\alpha_i(\overline{\theta})\delta\rho(q^e(\underline{\theta}) - q^e(\overline{\theta}))}{\nu(1-\nu)}\right],$$

$$\text{subject to } \alpha_i(\theta_i) \in [0,1], \quad e_i(\theta_i) = \psi'^{-1}(\alpha_i(\theta_i) q_i(\theta_i)) \quad \forall \theta_i \in \Theta.$$

Using **A2**, the necessary and sufficient first-order conditions with respect to outputs are:

$$\beta + 2\alpha_i(\underline{\theta}) q_i(\underline{\theta}) \phi - 2q_i(\underline{\theta}) + \delta \mathbb{E}_{\theta_j}[q^e(\theta_j)|\underline{\theta}] - \alpha_i(\underline{\theta})^2 q_i(\underline{\theta})\phi = \underline{\theta}, \tag{5}$$

$$\beta + 2\alpha_i(\overline{\theta}) q_i(\overline{\theta}) \phi - 2q_i(\overline{\theta}) + \delta \mathbb{E}_{\theta_j}[q^e(\theta_j)|\overline{\theta}] - \alpha_i(\overline{\theta})^2 q_i(\overline{\theta})\phi = \overline{\theta} + \tfrac{\nu}{1-\nu}\left[\Delta\theta + \tfrac{\alpha^e(\overline{\theta})\delta\rho(q^e(\underline{\theta}) - q^e(\overline{\theta}))}{\nu(1-\nu)}\right]. \tag{6}$$

Low-cost types' output is chosen so as to equalize (expected) marginal revenues to marginal costs: the efficient rule in the standard Bayesian sense. By contrast, high-cost types are forced to produce a downward distorted output for rent extraction reasons. Note that this distortion increases (resp. decreases) in $\alpha_i(\overline{\theta})$ when $\delta\rho > 0$ (resp. < 0). Using **A2** again, the first-order necessary and sufficient conditions with respect to the shares $\alpha_i(\underline{\theta})$ and $\alpha_i(\overline{\theta})$ are:

$$\nu(1 - \alpha_i(\underline{\theta})) q_i(\underline{\theta})\phi - \lambda_i(\underline{\theta}) + \mu_i(\underline{\theta}) = 0, \tag{7}$$

$$(1-\nu)(1 - \alpha_i(\overline{\theta})) q_i(\overline{\theta})\phi - \tfrac{\delta\rho(q^e(\underline{\theta}) - q^e(\overline{\theta}))}{1-\nu} + \lambda_i(\overline{\theta}) - \mu_i(\overline{\theta}) = 0, \tag{8}$$

with complementary slackness conditions:

$$\lambda_i(\theta_i)\alpha_i(\theta_i) = 0, \quad \lambda_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta, \tag{9}$$

$$\mu_i(\theta_i)(1 - \alpha_i(\theta_i)) = 0 \quad \mu_i(\theta_i) \geq 0 \quad \forall \theta_i \in \Theta, \tag{10}$$

where $\lambda(\theta_i)$ and $\mu(\theta_i)$ are the multipliers associated with the inequality constraints $\alpha_i(\theta_i) \geq 0$ and

$\alpha_i(\theta_i) \leq 1$, respectively.

Proposition 1 *Assume **(A1)-(A4)** and that $\Delta\theta$ is small enough. There exists a unique symmetric PBE such that $\alpha^e(\underline{\theta}) = 1$ and*

- $\alpha^e(\overline{\theta}) = 1$ *if and only if* $\delta\rho \leq 0$;
- $\alpha^e(\overline{\theta}) \in (0,1)$ *if and only if* $\delta\rho > 0$, *with*

$$\alpha^*(\overline{\theta}) \approx 1 - \frac{\nu\delta\rho(2-\phi-\delta)\Delta\theta}{\phi(\beta-\underline{\theta})(1-\nu)(\nu(\nu-1)^2(2-\phi)-\delta\rho)}.$$

Moreover, the solution of the relaxed program $\mathcal{P}$ also solves the general program.

There are two forces that shape the equilibrium allocation of residual claimancy. First, since agents choose their effort to equalize the marginal benefit $\alpha_i q_i$ to the marginal cost of effort $\psi'(e_i)$, a higher α_i promotes more effort and thus increases the surplus that P_i and A_i share in equilibrium. Second, as shown by the incentive constraint (4), the allocation of residual claimancy also changes the agents' mimicking incentives via the generalized competing-contracts effect. To understand the logic of this effect two different cases must be considered.

Case 1: $\delta\rho < 0$. Suppose first that $\delta < 0$ and $\rho > 0$. Because types are positively correlated, A_i anticipates that if he overstates his cost, P_i will believe that A_j is more likely to be inefficient too and that, ceteris paribus, hierarchy-i's expected surplus is high because outputs are strategic substitutes. But this belief will induce P_i to reduce the monetary transfer flowing to A_i.

Next, assume that $\delta > 0$ and $\rho < 0$. Because types are negatively correlated, A_i anticipates that if he overstates his cost, P_i will believe that A_j is less likely to be inefficient too and that, ceteris paribus, hierarchy-i's expected surplus is high because outputs are strategic complements. Again, this belief induces P_i to reduce the monetary transfer flowing to A_i.

In both cases the informational externality created by the interaction between competition and types' correlation goes in the same direction of the effort-enhancing effect. Hence, it is optimal to award full residual claimancy to the agents regardless of their costs.

Case 2: $\delta\rho > 0$. Suppose first that $\delta < 0$ and $\rho < 0$. Because types are negatively correlated, A_i anticipates that if he overstates his cost, P_i believes that A_j is less likely to be inefficient too and that, ceteris paribus, hierarchy-i's expected surplus is low (because outputs are strategic substitutes). This will, in turn, induce P_i to increase the monetary transfer flowing to A_i in order to compensate him for the reduction of surplus induced by a tougher competitor.

Next, assume that $\delta > 0$ and $\rho > 0$. Because types are positively correlated, A_i anticipates that if he overstates his cost, P_i believes that A_j is more likely to be inefficient too and that, ceteris paribus, hierarchy-i's expected surplus is low because outputs are complements and A_j is inefficient. But this induces P_i to increase the monetary transfer flowing to A_i in order to compensate him for the reduction of surplus due A_j's low (expected) output.

In both cases there is a trade-off between the effort-enhancing effect of revenue-sharing contracts and their perverse effect on the efficient types' information rent. To balance out those two countervailing forces, in equilibrium, principals will retain a fraction of the firms' surplus when dealing with inefficient types.

Of course, if $\delta = 0$ or $\rho = 0$ there is no competing-contracts effect whatsoever. Hence, agents are made full residual claimants of the firms' revenues as in the complete information benchmark.

5 Conclusion

We showed that the division of surplus between two contracting counterparts might be affected by competitive forces in a non trivial manner and that residual claimancy might follow a pro-cyclical pattern — i.e., principals are more inclined to share revenues with good types rather than with bad types. This result adds to the previous theoretical literature and offers simple ready to use guidelines for interpreting and designing future empirical investigations on the link between organization design and competition.

6 Appendix

Proof of Lemma 1. Agent A_i chooses the effort according to the following condition:

$$e(\theta_i) = \alpha(\theta_i)\, q(\theta_i)\, \phi \quad \forall \theta_i \in \Theta, \quad i, j = 1, 2, \quad i \neq j,$$

Hence, for any θ_i, principal P_i maximizes

$$\kappa + \beta q_i(\theta_i) + \alpha_i(\theta_i)\, q_i(\theta)^2 \phi - q_i(\theta_i)^2 + q_i(\theta_i)\, \delta \mathbb{E}_{\theta_j}[q^*(\theta_j)\,|\theta_i] - \theta_i q_i(\theta_i) - \tfrac{\phi \alpha_i(\theta_i)^2 q_i(\theta_i)^2}{2}.$$

The derivative of this function with respect to $\alpha_i(\theta_i)$ is:

$$q_i(\theta_i)\,(1 - \alpha_i(\theta_i))\, \phi \geq 0 \quad \forall \theta_i \in \Theta.$$

Hence, $\alpha^*(\theta_i) = 1\ \forall \theta_i \in \Theta^2$ $(i = 1, 2)$. The participation constraint is clearly binding, hence the first order conditions with respect to outputs are immediately derived from the objective above. ■

Proof of Proposition 1. Using the first-order condition (7) it is straightforward to show that $\alpha^e(\underline{\theta}) = 1$. Hence, under **A2** the system of equations that, in an interior solution, identifies $q^e(\underline{\theta})$, $q^e(\overline{\theta})$ and $\alpha^e(\overline{\theta})$ is

$$\beta - (2 - \phi)\, q^e(\underline{\theta}) + \delta \mathbb{E}_\theta[q^e(\theta)\,|\underline{\theta}] = \underline{\theta}, \tag{A1}$$

$$\beta + 2\alpha^e(\overline{\theta}) q^e(\overline{\theta})\phi - 2q^e(\overline{\theta}) + \delta \mathbb{E}_\theta[q^e(\theta)\,|\overline{\theta}] - \alpha^e(\overline{\theta})^2 q^e(\overline{\theta})\phi = \overline{\theta} + \tfrac{\nu}{1-\nu}\left[\Delta\theta + \tfrac{\alpha^e(\overline{\theta})\delta\rho\left(q^e(\underline{\theta}) - q^e(\overline{\theta})\right)}{\nu(1-\nu)}\right], \tag{A2}$$

$$(1 - \nu)\left(1 - \alpha^e(\overline{\theta})\right) q^e(\overline{\theta})\phi - \tfrac{\rho\delta\left(q^e(\underline{\theta}) - q^e(\overline{\theta})\right)}{1-\nu} = 0. \tag{A3}$$

Note that for $\Delta\theta$ small, the solution of this system is such that:

$$\alpha^e(\overline{\theta})\big|_{\Delta\theta=0} = 1, \qquad q^e(\underline{\theta})\big|_{\Delta\theta=0} = q^e(\overline{\theta})\big|_{\Delta\theta=0} = q^* = \tfrac{\beta-\theta}{2-\delta-\phi} > 0.$$

It then follows that:

$$\alpha^e(\overline{\theta}) \approx 1 + \Delta\theta \lim_{\Delta\theta\to 0} \frac{\partial \alpha^e(\overline{\theta})}{\partial \Delta\theta},$$

$$q^e(\underline{\theta}) \approx q^* + \Delta\theta \lim_{\Delta\theta\to 0} \frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta}, \qquad q^e(\overline{\theta}) \approx q^* + \Delta\theta \lim_{\Delta\theta\to 0} \frac{\partial q^e(\overline{\theta})}{\partial \Delta\theta}.$$

Hence, linearizing the system (A1)-(A3) around $\Delta\theta$ close to 0, one gets:

$$-(2-\phi) \lim_{\Delta\theta\to 0} \frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} + \delta \mathbb{E}_\theta \left[\lim_{\Delta\theta\to 0} \frac{\partial q^e(\theta)}{\partial \Delta\theta} \middle| \underline{\theta} \right] = 0, \tag{A4}$$

$$-2(1-\phi) \lim_{\Delta\theta\to 0} \frac{\partial q^e(\overline{\theta})}{\partial \Delta\theta} + \delta \mathbb{E}_\theta \left[\lim_{\Delta\theta\to 0} \frac{\partial q^e(\theta)}{\partial \Delta\theta} \middle| \overline{\theta} \right] = 1 + \frac{\nu}{1-\nu} + \frac{\delta\rho \lim_{\Delta\theta\to 0}\left[\frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} - \frac{\partial q^e(\overline{\theta})}{\partial \Delta\theta}\right]}{(1-\nu)^2}, \tag{A5}$$

$$-(1-\nu)\,\phi q^* \lim_{\Delta\theta\to 0} \frac{\partial \alpha^e(\overline{\theta})}{\partial \Delta\theta} - \frac{\rho\delta \lim_{\Delta\theta\to 0}\left(\frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} - \frac{\partial q^e(\overline{\theta})}{\partial \Delta\theta}\right)}{1-\nu} = 0. \tag{A6}$$

The solution for $\lim_{\Delta\theta\to 0} \frac{\partial \alpha^e(\overline{\theta})}{\partial \Delta\theta}$ entails:

$$\lim_{\Delta\theta\to 0} \frac{\partial \alpha^e(\overline{\theta})}{\partial \Delta\theta} = -\frac{\nu\delta\rho(2-\phi-\delta)}{\phi(\beta-\underline{\theta})(1-\nu)(\nu(\nu-1)^2(2-\phi)-\delta\rho)},$$

which directly implies:

$$\alpha^*(\overline{\theta}) \approx \min\left\{1, 1 - \frac{\nu\delta\rho(2-\phi-\delta)\Delta\theta}{\phi(\beta-\underline{\theta})(1-\nu)(\nu(1-\nu)^2(2-\phi)-\delta\rho)}\right\}.$$

Hence, for $\Delta\theta$ small and $\delta\rho > 0$, there exists an interior solution $\alpha^*\left(\overline{\theta}\right) < 1$, which is also greater than zero for $\Delta\theta$ small. For $\delta\rho \leq 0$, instead, the solution involves $\alpha^*(\overline{\theta}) = 1$ — i.e., $\mu^e(\overline{\theta}) \geq 0$.

Finally, we show that under **A4** and for $\Delta\theta$ small enough, high-cost types do not find it convenient to under-report their type. Hence, the solution of $\mathcal{P}$ also solves P_i's general optimization problem.

Using standard techniques, the incentive constrain of a high-cost type can be rewritten as:

$$U^e\left(\overline{\theta}\right) \geq U^e(\underline{\theta}) - \Delta\theta q^e(\underline{\theta}) - \frac{q^e(\underline{\theta})\alpha^e(\underline{\theta})\delta\rho(q^e(\underline{\theta})-q^e(\overline{\theta}))}{\nu(1-\nu)}. \tag{A7}$$

Since the solution of the relaxed program $\mathcal{P}$ entails $\alpha^e(\underline{\theta}) = 1$, $U^e(\overline{\theta}) = 0$ and

$$U^e(\underline{\theta}) = q^e(\overline{\theta})\left[\Delta\theta + \frac{\alpha^e(\overline{\theta})\delta\rho(q^e(\underline{\theta})-q^e(\overline{\theta}))}{\nu(1-\nu)}\right],$$

the incentive constraint (A7) rewrites as

$$0 \geq -\left(q^e(\underline{\theta}) - q^e(\overline{\theta})\right)\left[\Delta\theta + \frac{\delta\rho\left(q^e(\underline{\theta}) - q^e(\overline{\theta})\alpha^e(\overline{\theta})\right)}{\nu(1-\nu)}\right], \tag{A8}$$

where, by Proposition 1, $\alpha^e(\overline{\theta}) = 1$ for $\delta\rho \leq 0$ and $\alpha^e(\overline{\theta}) < 1$ for $\delta\rho > 0$.

Consider first the case where $\delta\rho > 0$. Solving the linearized system (A4)-(A6) for $\lim_{\Delta\theta\to 0} \frac{\partial q^e(\overline{\theta})}{\partial \Delta\theta}$ and

$\lim_{\Delta\theta\to 0} \frac{\partial q^e(\underline{\theta})}{\partial \Delta\theta}$ we have:

$$\lim_{\Delta\theta\to 0} \tfrac{\partial q^e(\overline{\theta})}{\partial \Delta\theta} = -\frac{(1-\nu)\left(\nu\left(2-\phi-\delta\nu\right)-\rho\delta\right)}{\nu\left(1-\nu\right)^2\left(2-\phi\right)-\delta\rho}, \qquad \lim_{\Delta\theta\to 0} \tfrac{\partial q^e(\underline{\theta})}{\partial \Delta\theta} = -\frac{\delta\left(1-\nu\right)\left(\nu\left(1-\nu\right)-\rho\right)}{\nu\left(1-\nu\right)^2\left(2-\phi\right)-\delta\rho}.$$

Hence, under **A4**,

$$q^e(\underline{\theta}) - q^e(\overline{\theta}) = \tfrac{\nu(1-\nu)(2-\phi-\delta)\Delta\theta}{\nu(1-\nu)^2(2-\phi)-\rho\delta} > 0. \tag{A9}$$

This directly implies that (A7) holds for $\delta\rho > 0$ and $\Delta\theta$ small enough.

Consider now $\delta\rho \leq 0$. The system of equations that defines the equilibrium outputs $q^e(\overline{\theta})$ and $q^e(\underline{\theta})$ is:

$$\beta - (2-\phi)\, q^e\left(\underline{\theta}\right) + \delta \mathbb{E}_\theta[q^e\left(\theta\right)|\underline{\theta}] = \underline{\theta},$$

$$\beta - (2-\phi)\, q^e(\overline{\theta}) + \delta \mathbb{E}_\theta[q^e\left(\theta\right)|\overline{\theta}] = \overline{\theta} + \tfrac{\nu}{1-\nu}\left[\Delta\theta + \tfrac{\delta\rho\left(q^e(\underline{\theta})-q^e(\overline{\theta})\right)}{\nu(1-\nu)}\right].$$

Taking the difference between the first and second equation, using the fact that $\overline{\theta} = \underline{\theta} + \Delta\theta$, it follows that:

$$-\left(2-\phi\right)\left(q^e\left(\underline{\theta}\right) - q^e(\overline{\theta})\right) + \tfrac{\delta\rho\left(q^e(\underline{\theta})-q^e(\overline{\theta})\right)}{\nu(1-\nu)} = -\frac{\Delta\theta}{1-\nu} - \tfrac{\delta\rho\left(q^e(\underline{\theta})-q^e(\overline{\theta})\right)}{(1-\nu)^2}$$

from which we immediately have:

$$q^e\left(\underline{\theta}\right) - q^e(\overline{\theta}) = \tfrac{\nu(1-\nu)\Delta\theta}{\nu(\nu-1)^2(2-\phi)-\delta\rho} > 0. \tag{A10}$$

Under **A4** it then follows that:

$$\Delta\theta + \tfrac{\delta\rho\left(q^e(\underline{\theta})-q^e(\overline{\theta})\alpha^e(\overline{\theta})\right)}{\nu(1-\nu)} = \tfrac{\nu(1-\nu)^2(2-\phi)\Delta\theta}{\nu(\nu-1)^2(2-\phi)-\delta\rho} > 0.$$

Hence, the incentive constraint (A7) holds also for $\delta\rho \leq 0$.

Hence, the solution of $\mathcal{P}$ also solves P_i's general optimization problem. ■

References

[1] Bolton, P., and M., Dewatripont, (2005), *Contract Theory.* The MIT Press.

[2] Bertoletti, P., and Poletti, C., (1996), "A Note on Endogenous Firm Efficiency in Cournot Models of Incomplete Information", *Journal of Economic Theory,* 71: 303-310.

[3] Bertoletti, P., and Poletti, C., (1997), (1997), "X-Inefficiency, Competition and Market Information", *Journal of Industrial Economics*, 45: 359-375.

[4] Bonanno, G., and J. Vickers, (1988), "Vertical Separation." *Journal of Industrial Economics*, 36: 257-65.

[5] Caillaud, B., B. Jullien and P. Picard, (1995), "Competing Vertical Structures: Precommitment and Renegotiation", *Econometrica*, 63: 621-646.

[6] Dequiedt, V. and Martimort, D., (2010), “Mechanism Design with Bilateral Contracts.” mimeo IDEI.

[7] Dana, J. and Spier, K., (2001), “Revenue Sharing and Vertical Control in the Video Rental Industry”, *Journal of Industrial Economics*, 49: 223-45.

[8] Fershtman, C., and K. Judd, (1987), “Equilibrium Incentives in Oligopoly.”, *American Economic Review*, 77: 927-940.

[9] Hart, O., (1995), *Firms Contracts and Financial Structure.* Clarendon Lectures in Economics, Clarendon Press. Oxford.

[10] Khalil, F. and Lawaree, J., (1995), “Input versus output monitoring: who is the residual claimant?”, *Journal of Economic Theory*, 66: 139-157.

[11] Laffont, J.J., Martimort, D., (2002). *The Theory of Incentives, The Principal–Agent Model.* Princeton University Press.

[12] Maskin, E. and Riley, J., (1985), “Input versus output incentive schemes”, *Journal of Public Economics,* 28: 1-23.

[13] Martimort, D., (1996), “Exclusive Dealing, Common Agency and Multiprincipals Incentive Theory”, *RAND Journal of Economics*, 27: 1-31.

[14] Martin, S., (1993), “Endogenous Firm Efficiency in a Cournot Principal-Agent Model”, *Journal of Economic Theory*, 59: 445-450.

[15] Sklivas, S. D., (1987), “The Strategic Choice of Managerial Incentives”, *RAND Journal of Economics*, 18: 452-458.

[16] Spagnolo G., (2000), “Stock-related compensation and product-market competition”, *RAND Journal of Economics*, 31: 22-42.

[17] Vickers, J., (1985), “Delegation and the Theory of the Firm”, *Economic Journal*, 95: 138-47.